Fiscal Policy and the Term Structure of Interest Rates

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Abstract

Macroeconomists want to understand the effects of fiscal policy on interest rates, while financial economists look for the factors that drive the dynamics of the yield curve. To shed light on both issues, we present an empirical macro-finance model that combines a no-arbitrage affine term structure model with a set of structural restrictions that allow us to identify fiscal policy shocks, and trace the effects of these shocks on the prices of bonds of different maturities. Compared to a standard VAR, this approach has the advantage of incorporating the information embedded in a large cross-section of bond prices. Moreover, the pricing equations provide new ways to assess the model's ability to capture risk preferences and expectations. Our results suggest that (i) government deficits affect long term interest rates: a one percentage point increase in the deficit to GDP ratio, lasting for 3 years, will eventually increase the 10-year rate by 40–50 basis points; (ii) this increase is partly due to higher expected spot rates, and partly due to higher risk premia on long term bonds; and (iii) the fiscal policy shocks account for up to 12% of the variance of forecast errors in bond yields.

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Introduction

Empirical macroeconomic research has not been able to establish if and how government deficits affect interest rates (Elmendorf and Mankiw (1999)). Yet, the issue is of crucial importance for policy making and for academic research. One reason for this lack of success is that macroeconomists have not fully incorporated long term interest rates into their empirical models. Instead, the literature has mainly relied upon simple least-squares estimates (see Gale and Orszag (2003) and Engen and Hubbard (2004) for surveys of the existing literature). Recently, Canzoneri, Cumby, and Diba (2002) and Laubach (2003) have presented clever regressions that suggest that deficits matter, and Evans and Marshall (2002) have studied the response of the yield curve to a range of macroeconomic shocks, identified separately. The common feature of these papers is that they do not model the kernel that prices long term bonds, and, therefore, do not provide an explicit decomposition of long rate changes into expected short rates and risk premia.

On the other hand, recent theoretical and empirical research in finance has led to a better understanding of the dynamic properties of the term structure of interest rates: The models are parsimonious, financially coherent, and are able to capture some important stylized facts. (see Dai and Singleton (2003) for a recent survey of this literature). Most existing models, however, are based on unobserved or latent risk factors, which are not easy to interpret. The next step, currently under way, (see, e.g., Piazzesi (2003), Ang and Piazzesi (2003), Rudebusch and Wu (2003), and Hördahl, Tristani, and Vestin (2003)) is to draw explicit connections between latent risk factors that drive the term structure dynamics and observed macro-economic variables characterizing the state of the economy.

In this paper, we develop a dynamic term structure model that emphasizes the role of fiscal policy. We start by estimating an affine model that combines observable macroeconomic variables with one latent factor. We then identify fiscal policy shocks using the restrictions proposed by Blanchard and Perotti (2002). Finally, we examine the impact of policy shocks on the economic system and the yield curve.

Our work contributes to both the macroeconomic and empirical finance literature in three ways. First, we introduce a fiscal policy variable into a no-arbitrage dynamic term structure model. It has been known at least since Taylor (1993) that there is enough information in inflation and the output gap to account for changes in the short term interest rate. Ang and Piazzesi (2003) confirm this finding, and also show that these same macroeconomic factors do not capture the dynamics of long term rates very well. We go some way toward addressing this issue by showing that fiscal policy can account for some (but not all) of the unexplained long rate dynamics.

Second, we argue that bond pricing equations provide useful over-identifying restrictions to empirical macroeconomic models. The number of variables one can include in a VAR is limited, but how can we be sure that a small state space is actually able to capture technology, preferences and the relevant information sets of economic agents? We show how one can use bond prices to address this key issue. Bond prices are observable and bond returns are predictable (see Cochrane and Piazzesi (2004) for some recent results). Empirical models should be able to price bonds and predict returns. A failure to do so means that the model does not capture risk aversion, or expectations, or both. These ideas guide our preliminary analysis, and in particular our choice of the variables to be included in the state space. We then conduct a maximum likelihood estimation of the model that incorporate all of the over-identifying restrictions offered by bond prices and returns. We find that a model with four observable macroeconomic variables (federal funds rate, inflation, deficit, real activity) and one latent factor can price bonds, capture return predictability and explain the deviations from the expectations hypothesis.

Third, we show that fiscal *policy* matters for interest rates. Research in finance has focused on finding a kernel that can price various bonds, but it has not tried to *identify* the economic shocks driving the kernel. To do so, one must impose theoretically motivated restrictions on the covariance matrix of reduced form shocks. We use an identification strategy similar to the one in Blanchard and Perotti (2002) to compute fiscal policy shocks. We find that a fiscal shock that increases the deficit to GDP ratio by 1% leads to 40-50 basis points increase in the 10 year interest rate.

When we decompose this increase into risk premia and expected future short rates, we find that the risk premia explain one third to one half of the increase in long term interest rates. This finding sheds new light on the conflicting results reported in Gale and Orszag (2003) and Engen and Hubbard (2004), because previous macroeconomic research has systematically neglected risk premia. Finally, we decompose deficit changes into public spending and taxes, and we find that taxes matter independently from spending.

The rest of the paper is organized as follows. Section 1 introduces the bond pricing model. In, Section 2, we conduct a preliminary analysis of the data using a set of excess returns and yield regressions, and we argue that these regressions can help us choose the state space of the model. Section 3 presents the estimation of the macro-finance model by maximum likelihood. Section 4 discusses identification and presents the impulse responses to fiscal policy shocks.

1 The Affine Pricing Model

We begin with a description of the main features of the discrete-time dynamic term structure model. Technical details can be found in the appendices. We assume that the state vector y_t follows a Vector Autoregressive process[1] of finite order $L+1$, $y_t = \phi_0 + \sum_{l=1}^{L+1} \phi_l y_{t-l} + u_t$. We defer the discussion of which variables should be included in the state space to section 2. By expanding the state space to the companion form $Y_t = [y_t \; .. \; y_{t-L}]$, we can rewrite the state dynamics in the more convenient VAR(1) form (after normalizing the unconditional mean to 0):

$$Y_t = \Phi Y_{t-1} + U_t, \tag{1}$$

where the shocks $U_t = [u_t; 0]$ are jointly normally distributed with constant covariance matrix $\Omega = E\left[U_t U_t'\right]$.

To price the government bonds, we assume that the pricing kernel takes the form:

$$\frac{M_{t+1}}{M_t} = \exp\left(-r_t - \frac{\Lambda_t' \Omega \Lambda_t}{2} - \Lambda_t' U_{t+1}\right) \; , \tag{2}$$

where the vector of market prices of risk is given by

$$\Lambda_t = \Omega^{-1}\left(\Lambda_0 + \Lambda Y_t\right), \tag{3}$$

[1]The observation interval is arbitrary at this stage, and is quarterly in the empirical implementation.

and the short rate (1-quarter) is given by

$$r_t = \delta_0 + \delta' Y_t. \tag{4}$$

We assume that the government will never default on its nominal obligations. Real defaults are possible through high inflation, however. We believe that these are sensible assumptions for the US in the post-war period. By definition of the pricing kernel, the price of a n-period zero-coupon default-free bond at time t must satisfy:

$$P_t^n = E_t \left[\frac{M_{t+1}}{M_t} P_{t+1}^{n-1} \right].$$

In an affine setup, one can easily show that bond prices are given by

$$P_t^n = \exp\left(-A_n - B_n' Y_t\right), \tag{5}$$

where A_n and B_n solve recursive equations

$$\begin{aligned} A_n &= \delta_0 + A_{n-1} - B_{n-1}'\Lambda_0 - \frac{B_{n-1}'\Omega B_{n-1}}{2}, \\ B_n &= \delta + (\Phi - \Lambda)' B_{n-1}. \end{aligned} \tag{6}$$

with initial conditions $A_0 = B_0 = 0$. Clearly, $A_1 = \delta_0$, and $B_1 = \delta$.

1.1 Relation to Existing Work on the Term Structure

Equations (1), (2), (3), and (4) constitute a full-fledged term structure model, which belongs to the class of affine term structure models (see, e.g., Duffie and Kan (1996), Dai and Singleton (2000), Dai and Singleton (2002), and Duffee (2002)). In section 2, we will argue that in order to give a reasonable description of both the economic environment and the term structure dynamics, the state vector y_t should include the federal fund rate (f_t), the logarithm of spending over taxes (d_t), the log growth rate of the GDP deflator (π_t), the help wanted index (h_t) [2], and a latent variable.

Existing works that are most closely related to our model are Ang and Piazzesi (2003), Rudebusch and Wu (2003), and Hördahl, Tristani, and Vestin (2003). Ang and Piazzesi (2003) use a no-arbitrage VAR where the maintained assumption is

[2] This is an index of help wanted advertising in newspapers, available on FRED® II (Federal Reserve Economic Data).

that latent factors (which presumably include monetary and fiscal policies) do not affect output or inflation. Their model is most useful to understand how much of the dynamics of the yield curve can be accounted for by inflation and real activity, but it is not suitable for identifying the effects of monetary and fiscal policies.

In contrast, Rudebusch and Wu (2003) and Hördahl, Tristani, and Vestin (2003) start from a simple textbook model of the macro-economy, with a price setting equation for firms, and a linearized Euler equation for consumption and output. We do not follow this strategy, for two reasons. First, while the price setting equation that governs the inflation process in the textbook model appears to be quite reasonable (see, e.g., Gali and Gertler (1999)), the Euler equation, that supposedly links aggregate dynamics to asset prices suffers from known failures (the most well-known being the equity premium puzzle or the risk-free rate puzzle). Indeed, for the purpose of pricing bonds, Rudebusch and Wu (2003) and Hördahl, Tristani, and Vestin (2003) posit a reduced-form pricing kernel on top of the marginal rate of substitution underlying the Euler equation. Second, to examine the effects of fiscal policy shocks on the term structure of interest rates, it would be necessary to introduce the fiscal variables into either the pricing equation, or the aggregate demand equation, or both. There is hardly any consensus in the macro literature on how this should be achieved.

2 Choosing the State Space

We now turn to the choice of the variables to be included in the state space. As one can see from equation (5), the affine model predicts linear relations between the components of the state space and bond yields. As it turns out, the same is true for bond returns. One can therefore use simple OLS regressions to assess the performance of different candidate variables for the state space.

We focus on two sets of linear regressions: yield regressions that relate changes in the yield levels to (contemporaneous) changes in the state vector; and excess return regressions that relate the predictable component of bond returns to the current state of the economy. The idea here is simply that, if the state space is correctly specified, it should be able to explain bond prices as well as bond returns through the reduced-form bond pricing equations implied by the model. If the proposed state space fails this test, there is no point going further. If the proposed state space passes the test, then it makes sense to estimate a term structure model that imposes cross-sectional restrictions (which arise from the no-arbitrage assumption) on the regression coefficients. The implication of such restrictions will be discussed following the results from the unrestricted OLS regressions.

We use quarterly time-series observations of the federal funds rate, the log spending to taxes ratio[3], the quarterly growth rate of the GDP deflator, the change in the help wanted index, and zero-coupon bond yields with maturities ranging from 1 to 40 quarters. Yield data are constructed by extending the Fama-Bliss smoothed data set to the recent quarters. The macro-variables are obtained from the National Income and Product Accounts. The sample period is from the first quarter of 1970 to the third quarter of 2003 and the summary statistics are reported in **Table 1.**

2.1 Yields regressions

Consider first the dynamics of bond prices of different maturities. We can rewrite the pricing equation as

$$r_t^n \equiv -\frac{\log(P_t^n)}{n} = a_n + b_n' Y_t, \tag{7}$$

where r_t^n is the yield at time t on a zero-coupon Treasury with remaining maturity n, and $a_n \equiv A_n/n$ and $b_n \equiv B_n/n$.

Table 2 presents the yields regressions (7) estimated by OLS independently for each maturity n. The first four regressions do not include the 2-year rate, while the last one does. There are three main findings. First, the R^2 are high. Second, the fiscal variables become more relevant as maturity increases, while the federal fund rate, help and inflation become relatively less important. Not including the deficit reduces the R^2 by more than 10 percentage points for the 10-year rate. Looking at the fourth column, one cannot reject the hypothesis that taxes and spending enter with opposite coefficients of similar magnitude. The last regression includes also the

[3]The definition for spending and taxes follows Blanchard and Perotti (2002) (see their appendix for details). Spending is the purchase of goods and services by federal, state and local governments. Taxes are taxes minus transfers.

two year rate. Obviously, the 2-year rate is correlated with the 10-year rate, but note that the deficit remains very significant. Finally, we have introduced the debt to GDP ratio in our regressions, and we have found it to be systematically insignificant[4]. This may or may not be surprising, depending on which model one has in mind (see Mankiw (2000) for a discussion).

2.2 Excess return regressions

Alternatively, we can examine holding-period returns on bonds of various maturities. By definition, the holding-period return on an n-period zero-coupon bond for τ period, in excess of the return or yield on a τ-period zero-coupon bond, is given by

$$xr^n_{t\to t+\tau} \equiv \log\left(P^{n-\tau}_{t+\tau}\right) - \log\left(P^n_t\right) - r^\tau_t = A_n + B'_n Y_t - A_{n-\tau} - B'_{n-\tau} Y_{t+\tau} - \tau r^\tau_t, \quad (8)$$

so that the expected excess return is given by

$$E_t\left[xr^n_{t\to t+\tau}\right] = \alpha_n + \beta'_n Y_t,$$

where $\alpha_n = A_n - A_{n-\tau} - A_\tau$, and $\beta_n = B'_n - B'_\tau - B'_{n-\tau}\Phi^\tau$. Using the recursion for B_n, the slope coefficients can be computed explicitly and are given by

$$\beta'_n = B_{n-\tau}\left[(\Phi - \Lambda)^\tau - \Phi^\tau\right].$$

Clearly, the risk premium is constant for all n and τ if and only if Λ vanishes. **Table 3** presents the excess return regressions.[5] The observable state space can predict 20% of the excess returns on 5-year bonds, slightly more on 2-year bonds, and slightly less on 10-year bonds. When we add the 2-year rate to the state space, we can account for more than 36% of excess returns. As a benchmark, Cochrane and Piazzesi (2004) report predictability of around 40% using all the forward rates (although their study is restricted to maturities of five years or less).

The help wanted index is clearly the preferred choice for pricing bonds. Columns 4 and 5 show that neither the growth rate of GDP, nor the growth rate of non-durable

[4]If one introduces additional lags, the data chooses to recreate the deficit by putting coefficients of similar magnitudes and opposite signs on the log of debt and its lag.

[5]Again, these are "unrestricted" regressions in the sense that cross-sectional restrictions on α_n and β_n for different n are ignored.

consumption, are statistically significant. Note however that one qualitative feature appears very robust: excess returns on long term bonds go down when the state of the economy improves. This is consistent with the view that risk aversion varies over time in a counter-cyclical fashion. The regressions simply indicate that the help wanted index is a better proxy for this time varying risk aversion. The regressions also suggest that excess returns are high when real interest rates are high. Finally, and as expected, the last column shows that the slope between the two-year rate and the federal funds rate predicts excess returns.

2.3 Taking stock

We draw two main conclusion from this exercise. First, the observable variables that we propose can potentially account for a large fraction of the dynamics of interest rates. Second, we need a latent factor to capture some movements in the levels and excess returns of long term bonds. The fact that the two-year rate is very useful in predicting excess returns means that there is extra information in the current yield curve about agents' expectations and/or risk aversion. A formal economic interpretation of the latent variable is beyond the scope of this paper. For our purpose, it suffices to say that economic agents presumably form their expectations using a larger information set than by simply looking at past values of inflation, output and the federal funds rate. These expectations are then embedded in the term structure, and, therefore, in our latent variable. Other factors that can affect the supply and demand of long term bonds, but are not adequately captured by the macro-economic variables included here, range from "liquidity preference" to central bank intervention in the currency market.[6]

The yields and returns regressions that we have presented do not enforce the restrictions (on the time-series behavior of yields or returns of individual bonds) implied by our pricing model. The unrestricted coefficients, however, can be used to construct restricted estimates of the parameters of interest. Specifically, since the

[6]For an example, the rally of the 10-year market in the first half of 2004 was strongly influenced by the dollar purchase by Asian central banks, which increased the demand for long-term U.S. Treasuries as the banks' dollar reserve was cycled into the Treasury markets.

slope coefficients from the yield regressions must satisfy the recursions (6), and since the mean reversion matrix Φ can be estimated directly by OLS, the state-dependent portion of the market price of risk can be estimated by choosing $\Lambda = \Lambda^{yield}$ that gives the best cross-sectional fit of the unrestricted yields coefficients. Alternatively, since the excess returns are proportional to the market prices of risk, we can obtain another estimate, Λ^{return}, by regressing the coefficients from the excess return regressions on the coefficients from the yield regressions. Λ^{yield} and Λ^{return} emphasize different aspects of the data. If the model is correctly specified, the two estimates should be similar. Indeed, an important reason why we find it necessary to include a latent variable in the state space is that Λ^{yield} and Λ^{return} are much closer when the latent variable (proxied by the two-year rate in our OLS analysis) is present than when it is absent. As a practical matter, when Λ^{yield} and Λ^{return} are sufficiently close, we can use either one of the estimates as starting value for the MLE, and the model has a good chance of explaining the violation of the expectations puzzle. Otherwise, the model tends to be bimodal in the sense that there are (at least) two local optima that emphasize different aspects of the model fit (yield levels versus returns).

3 Maximum Likelihood Estimation

Having settled on a state space, we will now estimate the model using maximum likelihood based on the Kalman Filter. Details on the construction of the Kalman Filter and the likelihood are presented in the appendices.

At a technical level, the maximum likelihood approach allows us to replace the bond yield in the OLS analysis by a latent variable in order to impose no-arbitrage restrictions in a proper and natural manner. It also allows us to compute asymptotic standard errors for the parameter estimates based on standard inference procedures. At a substantive level, the maximum likelihood approach allows the model to achieve the best trade-off between the time-series properties of the state variables and the cross-sectional behavior of bond yields and returns. This is critically important for our purpose because, as we will elaborate in the next section, fiscal policies affect the term structure through both expectations and risk premia. In order to identify the two effects separately, we do not wish to skew the model toward one channel at the expense of another through arbitrary choices of moment conditions and weighting schemes. It is worth pointing out that, by design, the MLE estimation is independent of the structural restrictions needed to identify the effects of fiscal shocks.

Based on the analysis presented in section 3, we choose the state space $y_t = (f_t, d_t, \pi_t, h_t, q_t)$, where f_t is the federal funds rate, d_t is (one tenth of) log of spending over taxes, π_t is realized inflation, h_t is help wanted index, and q_t is latent. We specify the dynamics of y_t as a $VAR(2)$ using an information criterion to select the number of lags. In addition to the macro variables, we assume that eight bonds (with 1, 2, 4, 8, 16, 20, 30, 40 quarters of maturity) are observed and used in the estimation.[7] The measurement errors on these bonds have a multi-variate normal distribution with zero mean and arbitrary correlation.

The sample period, from 1970:1Q to 2003:3Q, consists of 135 quarters. In each quarter, we observe 12 variables (8 bonds and 4 observable macro variables), for a total of 1620 observations. We restrict the short-rate equation and the market price of risk in such a way that current bond prices depend only upon current values of the state space, i.e., such that A_n and B_n load only on y_t, not y_{t-1}.[8] The parameters of interest characterize the dynamics of the system (Φ, Ω), the short rate (δ_0, δ) and the market price of risk (Λ_0, Λ).[9] Given the model parameters and other necessary normalizations[10], an unbiased estimate of the latent variable q_t can be obtained through

[7]We allow f_t to depend on all the macroeconomic variables as well as the latent factor. This specification accommodates backward and forward looking monetary policy rules, and allows the monetary authority to react to the information contained in long term bonds.

[8]This restriction is again motivated by OLS regressions where we have found that past values of macroeconomic variables did not contain a significant amount of information for current yields. Technically, this is achieved by imposing the restriction that (i) the short rate does not load on the lagged state variables, and (ii) the dynamics of y_t is VAR(1) under the risk-neutral measure. This reduces the number of free parameters by 30. As noted earlier, the forecast model for y_t under the physical measure is VAR(2).

[9]The MLE also estimates the covariance matrix of the measurement errors. Intuitively, one of the observed bond yields identifies the latent factor. The resulting state space identifies the VAR parameters. The remaining bond yields identify the pricing kernel and the covariance matrix of the measurement errors through the pricing restrictions and zero-mean restrictions on the measurement errors.

[10]The presence of a latent factor means that the model is invariant to certain affine transformations. We normalize the model by imposing the following restrictions: (i) the loading of the short rate on the latent factor is 1; and (ii) the latent factor is conditionally uncorrelated with the observed

the Kalman filter.[11]

Table 4 presents the estimated coefficients and **Table 5** presents the t-statistics.[12] We find that many parameters are sharply identified. For instance, look at the fourth column of the loadings of the market prices of risk on the current state space. The point estimate for the (4,4) element 0.705 means that a positive shock to h_t increases Λ_t and therefore decreases the expected excess returns on long term bonds. This captures the time varying, counter-cyclical risk aversion of the economy. The t-statistic for this coefficient is 2.233.

Figure 1 presents the yield loadings b_n implied by the MLE estimates of Φ and Λ. The loadings do not have a structural interpretation because the variables in the system are jointly endogenous, and also because we can rotate the model by adding any linear combination of the observed variables to the latent variable. The shapes of the loading curves, however, are still informative. As expected from the OLS regressions, we find that the loadings on the deficit increase with maturity. This is an interesting property since all the observed factors we are aware of tend to display the opposite pattern when they are embedded in an affine model. Panels (a) and (b) of **Figure 2** shows that the model does a good job at matching the mean and volatility of the yield curve. In each subplot, we include the sample moment (circles), moment computed from the model-implied yields (crosses), and the population moment evaluated at the MLE estimates (solid line) together with one standard-error bands (dashed lines). For the most part, the sample moments for both observed and model-implied yields are within one standard error of the population moments. **Figure 3** shows that the observed factors account for 80% to 95% of the variance of interest rates, that the latent factor explains most of the remaining variance, and that the pricing errors are small.[13]

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factors. See Dai and Singleton (2000) for a more general discussion of these issues.

[11]We impose that the eigenvalues of $I - \sum_{j=1}^{L+1} \phi_j$ lie within the unit circle, so that the state process is stationary under the physical measure. We also rule out complex eigen-values for the mean reversion matrix under the risk-neutral measure to avoid oscillating behavior in the yield loadings.

[12]In computing the t-statistics, we fixed some of the parameters to their point estimates if their t-ratios are less than 1. Essentially, these parameters are numerically under-identified even though they are identified in theory.

[13]By definition, our model implies that $r_t^n = a_n + b_n' y_t + \text{error}_t^n$. The orthogonal part of the latent

An important test on whether a model can explain the conditional distribution of the yield curve is to check whether it can explain the violation of the expectations hypothesis. Under the expectations hypothesis, the slope coefficients in the following regressions,

$$r_{t+1}^{n-1} - r_t^n = \text{constant} + c^n \times \frac{r_t^n - r_t^1}{n-1} + \text{errors}, \tag{9}$$

should be equal to 1. Campbell and Shiller (1991) show, however, that the c^n coefficients are negative for all maturities, suggesting that the expectations hypothesis is violated. For our sample, the slope coefficients c^n range from -0.5 to -3, as indicated by circles in **Figure 4**. The solid line in the same graphs represents the population values of the slope coefficients from our model (solid line), which is computed as 1 minus the linear projection coefficient from the expected excess return on the slope of the yield curve.[14] The fact that the Campbell-Shiller coefficients lie within the predicted standard-error bands (dotted lines) of the population coefficients means that our model explains the expectations puzzle. For comparison, we also plot the coefficients (stars) implied by the filtered yields from MLE. The latter are even closer to their sample counterparts, indicating that the remaining difference between the sample coefficients and the population coefficients may be explained by small sample biases.

We conclude that we have a decent bond pricing model to work with. In particular, the fact that our model explains the expectations puzzle should allow us to separate the effects of fiscal policy shocks on risk premia from their effects on expectations of future short rates. This does not mean that the structural restrictions that we are going to impose in the next section are warranted. We are simply going to use an off-the-shelf procedure. One can agree or disagree with the identifying restrictions

factor is the residual from projecting the latent factor onto the observed variables. This ordering gives as much explanatory power as possible to the observed factors.

[14]See Dai and Singleton (2002), who show that, by definition, the following equation must hold:

$$r_{t+\tau}^{n-\tau} - r_t^n + \frac{E_t[xr_{t\to t+\tau}^n]}{n-\tau} = \frac{r_t^n - r_t^\tau}{n-\tau}.$$

It follows that the downward bias from 1 (representing the expectations hypothesis) is equal to the linear projection coefficient from the expected excess return, $E_t[xr_{t\to t+\tau}^n]$, on the slope of the yield curve, $r_t^n - r_t^\tau$.

independently from the bond pricing model. However, if one accepts the identifying restrictions, then, given that our bond pricing model is reasonably successful, one should take seriously the impulse responses and variance decomposition derived from the model.

4 Fiscal Policy and Interest Rates

Our goal in this paper is to study the effects of fiscal policy on the term structure of interest rates. In the previous sections, we have presented an affine model that seems to capture expectations and risk premia reasonably well. We now turn to the issue of identifying the policy shocks. Identification is the central issue in empirical macroeconomics, but it has not received the same attention in empirical finance. Following Blanchard and Perotti (2002), and without loss of generality, we write the reduced form shocks to spending (u^g) as the sum of the reduced form shocks to real activity $\left(u^h\right)$ and inflation (u^π), as well as the fiscal policy shocks (ε^g)

$$u_t^g = \varepsilon_t^g + \alpha^g u_t^h + \gamma^g u_t^\pi \ , \tag{10}$$

and similarly for taxes

$$u_t^\tau = \varepsilon_t^\tau + \alpha^\tau u_t^h + \gamma^\tau u_t^\pi \ . \tag{11}$$

Equation (11) emphasizes the fact that not all changes in tax revenues reflect genuine policy shocks. When the economy expands, tax revenues increase mechanically. The problem is that one cannot use macroeconomic data to identify the elasticities ($\alpha^g, \alpha^\tau, \gamma^g, \gamma^\tau$). Fortunately, using detailed knowledge of the US tax system, Blanchard and Perotti (2002) and Perotti (2004) have already *calibrated* the automatic responses of spending and taxes to shocks to inflation and output. They find that:

	Blanchard and Perotti (2002)		Perotti (2004)	
Elasticity	α^g	α^τ	γ^g	γ^τ
Calibrated Value	0	2 in 1970, 3 in 2000	-0.5	1.2

Intuitively, spending does not react to news about real activity within the quarter ($\alpha^g = 0$), and half of it is not directly indexed on inflation ($\gamma^g = -0.5$). On the other hand, tax revenues increase with real activity ($\alpha^\tau = 2$), and also with inflation since the tax system is not inflation neutral ($\gamma^\tau = 1.2$). Since the deficit is simply $d = \log(G) - \log(T)$, we have $u^d = u^g - u^\tau$ and therefore[15]

$$\alpha^d = \alpha^g - \alpha^\tau \ ; \ \gamma^d = \gamma^g - \gamma^\tau$$

Finally, note that the value of α^τ increases steadily over our sample period, as explained in Blanchard and Perotti (2002).

There is an issue in using the Blanchard and Perotti (2002) approach in our setup because we do not include GDP in our state space. The tension is the following. On the one hand, as explained earlier, the help wanted index does a better job at pricing bonds than GDP growth would. On the other hand, the elasticities (α^g, α^τ) apply to GDP, not h. The problem is that innovation to h and to GDP are not exactly the same. In practice, fortunately, the innovations $\left(u_t^h\right)$ explain 70% of the innovations to GDP, and more than 90% if we also include lagged innovations $\left(u_{t-1}^h\right)$. We have conducted extensive checks, and found that all of our results are robust to using one specification or the other. We have also estimated a model with GDP growth. In this model, the performance of the bond pricing model deteriorates, but the main qualitative and quantitative features of the impulse responses to identified fiscal shocks remain the same.

Given the values of these elasticities, we can identify the structural shocks ε^g and ε^τ and, therefore, the impulse responses to the fiscal shocks. We present the impulse responses of the state space, and then the implied responses of the short rate and the 10-year rate. We present first our baseline specification using only the deficit (log of spending over taxes). We discuss later the extension to a state space with 6 variables, where we consider spending and taxes separately.

4.1 Responses to Deficit Shocks

Figure 5 presents the responses to the deficit shock. The initial shock is $\varepsilon^d = 1\%$. Because of the automatic stabilizers, the initial increase in deficit is only 0.8%.

[15] For purely aesthetic reasons, we scale the deficit variable by a factor of 10, so that $d = (\log(G) - \log(T))/10$. Correspondingly, the elasticities to deficit are also scaled by the same factor.

Inflation and real activity increase, while the federal fund rate does not react initially. Eventually, the federal fund rate increases by substantially more than inflation. To get a sense of the magnitudes involved, remember that spending is roughly 20% of GDP, and that we have normalized $d = \frac{1}{10}\log(G/T)$. We are therefore looking at a shock that would increase spending by 10%, or the deficit to GDP ratio by 2 percentage points before macroeconomic feedbacks.

Figure 6 presents the response of different yields. The response of the 10-year rate ranges from 5 to 90 basis points. This suggests an maximum elasticity of long rates to deficits of around 40-50 basis points, which is consistent with the numbers reported in Gale and Orszag (2003). **Figure 6** also shows the response of the 10-year rate under the expectations hypothesis, i.e. under the assumption that the 10-year at any point is the average of future short rates over the following 10 years. The difference between the actual 10-year rate and the 10-year rate under EH reflects the risk premia on long-term bonds. The risk premium is initially negative before becoming positive. After 5 years, the risk premium explains between a third and a half of the increase in long term rates. The initial drop and subsequent increase in the risk premium come from the dynamics of the state space and the market price of risk. For instance, deficit spending is expansionary, so h_t goes up. As discussed above, this reduces risk aversion and lowers the premium on long term bonds. Similarly, we see that the real rate is below its long run mean for at least one year, which contributes to the low risk premia. **Figure 7** shows the impulse response of the 10-year rate together with its asymptotic standard error bands, while **Figure 8** does the same for the 10-year term premium.

This has several new and important implications. First, it justifies our use of an explicit term structure model for studying the fiscal policy[16]. Second, it explains why previous research has reached inconsistent results. More specifically, researchers have estimated large coefficients when regressing long rates on current deficits, but small (and sometimes insignificant) coefficients when regressing current short rates

[16] There are no mechanical reasons for the model to imply a time varying risk premia in response to a fiscal shock. Our specification of the market price of risk is flexible enough that, in case fiscal shocks did not move risk premia, we would not have found it.

on current deficits or debt to GDP ratios. Our results suggest that part of the reason is that high long rates do not necessarily turn into high future short rates.

Figures 9 and **10** show the forecast error variance decomposition of the factors and selected yields that one can attribute to fiscal policy shocks. Fiscal shocks matter more at longer horizons, and they explain roughly 12% of the variance of interest rates beyond 5 years.

A potentially important issue that we have not yet discussed is the stability of our estimates across different subsamples. Of particular concern is the change in monetary policy after the 1981 recession. We have therefore estimated our model on the post 1981 sample. While some of the estimates change, the main features of the responses to fiscal policy shocks are unaffected.

4.2 Responses to Spending and Tax Shocks

While separating government purchases of goods and services from tax revenues and transfers is important for macroeconomics, it is not the main focus of our paper. Moreover, existing papers have already investigated the issue (see Blanchard and Perotti (2002)). Here, we simply wish to present some evidence that we hope will be informative for future research. Introducing taxes and spending separately is not straightforward because the two series are non-stationary and need to be either detrended, or introduced as growth rates. Growth rates do not have much explanatory power for yields: the data want the deficit (the difference between log spending and taxes), not the change in the deficit, and using additional lags does not solve the problem. In other words, the model in growth rates is misspecified. Detrending is also problematic, however, since it assumes that economic agents know the actual trends. This does not seem like an ideal assumption[17], especially for the purpose of pricing bonds. For lack of a better alternative, we will nonetheless proceed with linearly detrended series. Another issue is that the number of free parameters increases substantially when we move to a six variable VAR, and the MLE becomes hard to

[17] The deficit is stationary, has been and is expected to be. But whether the government will choose to satisfy his budget constraint by adjusting taxes or spending is far from obvious, and certainly hard to forecast.

implement.

For all these reasons, the results in this section are not based on MLE. Rather, we use the simpler, two-steps, "matching moments" approach described at the end of section 2. We use a state space with 6 variables: (f_t, π_t, h_t) plus detrended spending (g_t) and taxes (τ_t), and the 2-year interest rate (r_t^8). The state space is fully observable since there is no latent factor, and we can estimate $\hat{\Phi}$ and $\hat{\Omega}$ as in a standard VAR. In the first step, we also run 40 yields regressions (7), and 36 one-year excess returns regressions (8). In the second step, we choose the parameters $(\delta_0, \delta, \lambda_0, \lambda)$ to minimize the distance between the coefficients predicted by the recursive equations (6), using $\hat{\Phi}$ and $\hat{\Omega}$ from the VAR, and the estimates from the the 76 yields and returns regressions. This method is clearly less efficient than MLE, since $\left(\hat{\Phi}, \hat{\Omega}\right)$ and $(\delta_0, \delta, \lambda_0, \lambda)$ are not jointly estimated, and since the choice of the 2-year interest rate is arbitrary. To build some confidence, we have checked that it delivers broadly similar results for the five-variables model of the previous section. Finally, the policy shocks are constructed using the elasticicities $(\alpha^\tau, \gamma^\tau, \alpha^g, \gamma^g)$ as described above.

Figures 11 and **12** present the impulse responses to spending and taxes separately. The initial shocks are always 1% and the initial response of the other fiscal variable (of spending to taxes or taxes to spending) is set to 0[18]. The results look very similar to the ones presented in Figure 5, but the responses are somewhat larger for spending shocks. Spending does not seem react to the tax shock. It turns out that changes in tax policies are not systematically followed by changes in government spending. We did not impose or expect this result, but we note that it allows us to talk about the reactions of the economy to changes in the timing of tax revenues, while keeping the path of spending roughly constant.

Figures 13 and **14** present the impulse responses of different nominal yields to spending and tax shocks. Again, we find the initial drop and subsequent increase in risk premia. The drop is stronger for spending shocks, and the eventual increase is smaller. The response of the economy to tax shocks, in terms of both prices and

[18]Changing the ordering changes nothing to our results since the two reduced form shocks are almost uncorrelated. See Blanchard and Perotti (2002) for similar results and a more detailled discussion.

quantities, does not appear consistent with Ricardian equivalence in the short and medium run. In the long run, everything is neutral by assumption, since our VAR is stationary in the level of interest rates and detrended taxes and spending.

5 Conclusion

We have presented and estimated an empirical macro-finance model of the term structure. Based on bond pricing equations, we have chosen a state space that includes the federal funds rate, the government deficit, inflation, real activity and one latent factor. The model successfully explains the dynamics of the term structure of interest rates, and deviations from the expectation hypothesis. The model shows that risk-premia are counter-cyclical and increasing with the level of real rates.

We have found that government deficits increase interest rates, especially long ones, and that the fiscal shocks affect long rates through expectations of future spot rates as well as risk premia. Following an expansionary fiscal shock, the response of the risk premium is initially small or negative before turning positive after 5 years, where it accounts for one third to one half of the increase in the 10-year rate. Thus the initial response of interest rates to fiscal shocks is muted, while the long-run response is amplified. Our results emphasize that the usual macroeconomic approach of equating long rates with average future short rates is rejected by the data, and that not recognizing this fact can lead to inconsistent estimates of the effects of fiscal policy. Finally, we have provided some evidence that taxes affect interest rates for a given path of government spending, which suggests that the Ricardian equivalence may not hold in the medium run.

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A Model Specification and Parameterization

In this appendix, we collect all of the assumptions and analytical results needed for a complete description of the dynamic term structure model.

The model is based on $L+1$ lags of a $N \times 1$ state vector. Let y_t be the state vector. We assume that

$$y_t = \phi_0 + \sum_{l=1}^{L+1} \phi_l y_{t-l} + u_t,$$

where u_t is multi-variate normal with zero mean and covariance matrix ω. It is convenient to re-write the dynamics in terms of the expanded vector: $Y_t = (y_t, y_{t-1}, \ldots, y_{t-L})'$, which is VAR(1):

$$Y_t = \Phi_0 + \Phi Y_{t-1} + U_t,$$

where

$$\begin{aligned} \Phi_0 &= \begin{pmatrix} \phi_0 \\ 0_{NL\times 1} \end{pmatrix}, \; \Phi = \begin{bmatrix} \phi_1 \ldots \phi_{L+1} \\ I_{NL\times NL} \quad 0_{NL\times N} \end{bmatrix}, \\ U_t &= \begin{pmatrix} u_t \\ 0_{NL\times 1} \end{pmatrix}, \; \Omega = \begin{bmatrix} \omega & 0_{N\times NL} \\ 0_{NL\times N} & 0_{NL\times NL} \end{bmatrix} \equiv cov(U_t). \end{aligned}$$

A.1 Pricing Kernel and Risk-Neutral Dynamics

Let δ_1 be a $N(L+1)\times 1$ vector, λ_0 be a $N \times 1$ vector, and λ_1 be a $N \times N(L+1)$ matrix. We assume that the pricing kernel takes the following form:[19]

$$\begin{aligned} \frac{M_{t+1}}{M_t} &= e^{-r_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t' u_{t+1}}, \\ r_t &= \delta_0 + \delta' Y_t, \quad \Lambda_t = \Lambda_0 + \Lambda Y_t, \end{aligned}$$

where

$$\Lambda_0 = \begin{pmatrix} \omega^{-1}\lambda_0 \\ 0_{NL\times 1} \end{pmatrix}, \; \Lambda = \begin{bmatrix} \omega^{-1}\lambda \\ 0_{NL\times N(L+1)} \end{bmatrix}.$$

[19] The zero-restrictions in Λ_t implies that the pricing kernel can be alternatively written as

$$\frac{M_{t+1}}{M_t} = e^{-r_t - \frac{1}{2}\lambda_t'\omega\lambda_t - \lambda_t'\sigma\epsilon_{t+1}},$$

where $\lambda_t = \omega^{-1}(\lambda_0 + \lambda Y_t)$. This captures the idea that only the shocks at $t+1$ is priced. Dependence of lagged shocks can be normalized away even if allowed.

It follows that under the risk-neutral measure Q, the state-dynamics follows:

$$
\begin{aligned}
Y_t &= \Phi_0^Q + \Phi^Q Y_{t-1} + U_t^Q, \\
\Phi_0^Q &= \Phi_0 - \Omega\Lambda_0 = \begin{pmatrix} \phi_0 - \lambda_0 \\ 0_{NL\times 1} \end{pmatrix}, \\
\Phi^Q &= \Phi - \Omega\Lambda = \begin{bmatrix} (\phi_1 ... \phi_{L+1}) - \lambda \\ I_{NL\times NL} \quad 0_{NL\times N} \end{bmatrix}.
\end{aligned}
$$

and U_t^Q is multi-variate normal with zero mean and covariance matrix Ω under Q.

A.2 Bond Pricing

Under the above assumptions, the price of a zero-coupon bond with maturity n periods is given by $P_t^n = e^{-A_n - B_n' Y_t}$, where $A_0 = 0$, $B_0 = 0_{N(L+1)\times 1}$, and for $n \geq 0$,

$$
A_{n+1} = \delta_0 + A_n + (\Phi_0^Q)' B_n, \;\; B_{n+1} = \delta_1 + (\Phi^Q)' B_n.
$$

It follows that the zero-coupon bond yields are given by

$$
r_t^n = a_n + b_n' Y_t,
$$

where $a_n \equiv A_n/n$ and $b_n \equiv B_n/n$.

B Kalman Filter and Likelihood Function

In this section, we collect all of the assumptions and analytical results for constructing the Kalman Filter and the likelihood function.

Suppose that we include K bonds in the estimation, with maturities n_k, $k = 1, 2, \ldots, K$, then the observed time-series variables can be collected in the vector:

$$
X_t \equiv \begin{pmatrix} r_t^{n_1} & r_t^{n_2} & \ldots & r_t^{n_K} & z_t' \end{pmatrix}',
$$

where z_t is equal to y_t excluding any latent variables. Let's assume that, out of N state variables, M are observed. Without loss of generality, we assume that y_t is ordered in such a way that all latent variables follow the observed variables. Then the observation equation can be written as[20]

$$X_t = G' + H'Y_t + v_t,$$

where,

$$\begin{aligned} G &= \begin{pmatrix} a_{n_1} & a_{n_2} & \ldots & a_{n_K} & 0_{1\times M} \end{pmatrix}, \\ H &= \begin{bmatrix} b'_{n_1} & b'_{n_2} & \ldots & b'_{n_K} & \begin{bmatrix} I_{M\times M} & 0_{M\times N(L+1)-M} \end{bmatrix}' \end{bmatrix}. \end{aligned}$$

As part of the econometric specification, we assume that the "measurement errors" v_t are *i.i.d.*, multi-variate normal, with zero mean and covariance matrix R. In addition, we assume that the observed state variables do not contain measurement errors, so that the last M elements of the $K + M$ vector v_t are identically zero, and R is identically zero except the upper-left $K \times K$ sub-matrix, which represents the covariance matrix of the measurement errors in the observed yields.[21]

Let $\mathcal{I}_t = (X_s : s \leq t)$ be the current information set, and let

$$\hat{Y}_{t+1|t} \equiv E\left(Y_{t+1}|\mathcal{I}_t\right), \; P_{t+1|t} \equiv E\left[(Y_{t+1} - \hat{Y}_{t+1|t})^2|\mathcal{I}_t\right],$$

be the optimal forecast of the state vector and the associated mean square forecast errors (MSE). The Kalman-Filter algorithm allows us to compute the forecasts and the associated MSE recursively as follows:

$$\begin{aligned} \hat{Y}_{t+1|t} &= \Phi\hat{Y}_{t|t-1} + \Phi P_{t|t-1}H(H'P_{t|t-1}H+R)^{-1}\left(X_t - G' - H'\hat{Y}_{t|t-1}\right), \\ P_{t+1|t} &= \Phi\left[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H+R)^{-1}H'P_{t|t-1}\right]\Phi' + \Omega, \end{aligned}$$

starting with the unconditional mean and covariance matrix $\hat{Y}_{1|0} = E\left(Y_t\right)$ and $P_{1|0} = cov\left(Y_t\right)$. Under our VAR specification, the unconditional covariance matrix is given by $\text{vec}(P_{1|0}) = [I - \Phi \otimes \Phi]^{-1} \times \text{vec}(\Omega)$.

[20]For the Kalman Filter, we will follow closely the notation and algorithms developed in *Time Series Analysis* by James D. Hamilton. Accordingly, we set, without loss of generality, $\phi_0 = 0$ and therefore $\Phi_0 = 0$ by taking out the unconditional (or sample) means of the state variables througout the paper.

[21]In principle, we can allow the observed state variables z_t to contain measurement errors, in which case the matrix R has full rank.

The likelihood function can be constructed by noting that, given the information set $\mathcal{I}_t$, the conditional distribution of the observed vector X_{t+1} is multi-variate normal.[22] That is,

$$X_{t+1}|\mathcal{I}_t \sim N\left(G' + H'\hat{Y}_{t+1|t}, H'P_{t+1|t}H + R\right), \ t \geq 0.$$

All of the parameters (Φ, Ω, G, H, R) that determine the behavior of the Kalman Filter are completely determined by the primitive parameters $\beta \equiv (\phi_j, j = 0, 1, \ldots, L+1, \omega, \delta_0, \delta, \lambda_0, \lambda)$ through deterministic transformations and the no-arbitrage pricing restrictions. In particular, the no-arbitrage pricing restrictions are encapsulated in the vector G and matrix H, which are completely determined by the yield loadings.

[22] By convention, $\mathcal{I}_0$ means no information and therefore X_1 is drawn from the unconditional distribution.

Table 1: Summary Statistics. Sample Period is 1970:1 to 2003:3

	Obs	Mean	Std. Dev.	Min	Max
1-quarter nominal rate	135	0.0632	0.0291	0.00678	0.154
2-year nominal rate	135	0.0709	0.0273	0.0132	0.158
5-year nominal rate	135	0.0753	0.0246	0.0255	0.152
10-year nominal rate	135	0.0784	0.0226	0.0381	0.150
Federal Fund Rate	135	0.0667	0.0313	0.0100	0.175
Log(Spending/Taxes)	135	0.281	0.121	0.0141	0.551
Detrended Real Taxes	135	0	0.0339	-0.0583	0.0724
Detrended Real Spending	135	0	0.114	-0.275	0.264
Inflation	135	0.0409	0.0259	0.00782	0.122
Help Wanted Index	135	0.0763	0.0172	0.0370	0.105

Table 2: Yield Regressions

Maturity	2-year	5-year	10-year	10-year	10-year	10-year
Federal Fund Rate	0.861	0.752	0.669	0.665	0.661	-0.144
	0.032	0.034	0.034	0.034	0.044	0.039
Log(Spending/Taxes)/10	0.319	0.56	0.681			0.38
	0.067	0.071	0.07			0.034
Inflation	-0.044	-0.048	-0.056	-0.088	-0.104	-0.015
	0.039	0.041	0.041	0.049	0.053	0.018
Δ Help Wanted Index	0.602	0.56	0.444	0.342	0.666	-0.124
	0.175	0.187	0.185	0.187	0.24	0.087
Detrended Real Taxes				-0.706		
				0.074		
Detrended Real Spending				0.412		
				0.13		
2-year Rate						0.944
						0.041
# of Observations	135	135	135	135	135	135
R^2	0.893	0.85	0.826	0.829	0.7	0.965

Notes: Standard errors are under regression coefficients. Sample period: 1970:1 to 2003:3

Table 3: One Year Excess Return Regressions

Maturity	2-year	5-year	10-year	10-year	10-year	10-year
Federal Fund Rate	0.146	0.27	0.458	0.559	0.583	-4.479
	0.067	0.216	0.417	0.421	0.419	0.925
Log(Spending/Taxes)/10	0.421	1.152	1.989	1.787	1.285	-0.101
	0.137	0.44	0.848	0.856	0.897	0.836
Inflation	-0.328	-1.022	-1.979	-1.808	-1.558	-1.759
	0.079	0.253	0.489	0.545	0.551	0.437
Change in Help Wanted Index	-1.29	-3.142	-5.008			-8.22
	0.357	1.15	2.218			2.05
Growth Rate of GDP				-0.747		
				1.208		
Growth Rate of Non Durable Consumption					-2.089	
					1.391	
2-year Rate						5.921
						1.015
# of Observations	131	131	131	130	131	131
R^2	0.236	0.204	0.191	0.168	0.173	0.364

Notes: Standard errors are under regression coefficients. Sample period: 1970:1 to 2003:3

Table 4: MLE Estimates

$$
\begin{aligned}
r_t &= 1.5812\% + \begin{pmatrix} 0.153 \\ -0.069 \\ 0.008 \\ 0.113 \\ 0.250 \end{pmatrix}' y_t + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}' y_{t-1}, \text{ (per quarter)} \\
\Lambda_t &= \begin{pmatrix} -0.014 & -0.006 & 0.038 & -0.017 & 0.011 \end{pmatrix}' \\
&+ \begin{pmatrix} -0.449 & 0.048 & 0.200 & 1.441 & 0.092 \\ -0.077 & -0.202 & 0.039 & -0.392 & 0.169 \\ 0.495 & -0.841 & -0.869 & -0.396 & 0.184 \\ -0.222 & 0.141 & 0.270 & 0.705 & -0.414 \\ 0.118 & -0.146 & -0.200 & -0.339 & -0.351 \end{pmatrix} y_t + \begin{pmatrix} 0.163 & -0.147 & 0.148 & -0.874 & -0.027 \\ 0.032 & 0.281 & 0.009 & 0.433 & -0.154 \\ -0.030 & 0.017 & 0.234 & -0.488 & 0.001 \\ 0.034 & 0.167 & 0.028 & -0.288 & 0.228 \\ 0.060 & 0.035 & 0.086 & 0.015 & 0.179 \end{pmatrix} y_{t-1} \\
y_{t+1} &= \begin{pmatrix} 0.530 & 0.005 & 0.058 & 1.296 & 0.352 \\ -0.022 & 0.686 & -0.003 & -0.477 & 0.180 \\ 0.170 & -0.071 & 0.631 & 0.384 & -0.303 \\ -0.126 & -0.141 & 0.015 & 1.355 & -0.090 \\ 0.055 & 0.028 & -0.051 & -0.131 & 0.463 \end{pmatrix} y_t + \begin{pmatrix} 0.163 & -0.147 & 0.148 & -0.874 & -0.027 \\ 0.032 & 0.281 & 0.009 & 0.433 & -0.154 \\ -0.030 & 0.017 & 0.234 & -0.488 & 0.001 \\ 0.034 & 0.167 & 0.028 & -0.288 & 0.228 \\ 0.060 & 0.035 & 0.086 & 0.015 & 0.179 \end{pmatrix} y_{t-1} \\
&+ \begin{pmatrix} 2.230 & \mathbf{-0.195} & \mathbf{0.139} & \mathbf{0.175} & \cdot \\ -0.131 & 0.657 & \mathbf{-0.184} & \mathbf{-0.209} & \cdot \\ 0.306 & -0.353 & 2.155 & \mathbf{-0.031} & \cdot \\ 0.121 & -0.124 & -0.059 & 0.668 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0.842 \end{pmatrix} \epsilon_{t+1} \times 10^{-2} \\
R &= \left(\begin{array}{c|cccccccc} \mathbf{Q} & \mathbf{1} & \mathbf{2} & \mathbf{4} & \mathbf{8} & \mathbf{16} & \mathbf{20} & \mathbf{30} & \mathbf{40} \\ \hline \mathbf{1} & 47.165 & \mathbf{0.849} & \mathbf{0.304} & \mathbf{-0.411} & \mathbf{-0.838} & \mathbf{-0.766} & \mathbf{-0.550} & \mathbf{-0.406} \\ \mathbf{2} & 27.933 & 17.394 & \mathbf{0.701} & \mathbf{-0.121} & \mathbf{-0.933} & \mathbf{-0.944} & \mathbf{-0.771} & \mathbf{-0.626} \\ \mathbf{4} & 7.263 & 19.998 & 10.854 & \mathbf{0.567} & \mathbf{-0.582} & \mathbf{-0.806} & \mathbf{-0.880} & \mathbf{-0.825} \\ \mathbf{8} & -7.317 & 7.687 & 12.962 & 6.021 & \mathbf{0.298} & \mathbf{-0.080} & \mathbf{-0.481} & \mathbf{-0.600} \\ \mathbf{16} & -13.421 & -6.733 & 0.867 & 4.554 & 3.122 & \mathbf{0.917} & \mathbf{0.622} & \mathbf{0.443} \\ \mathbf{20} & -13.823 & -10.028 & -4.300 & 1.009 & 3.597 & 1.282 & \mathbf{0.878} & \mathbf{0.753} \\ \mathbf{30} & -13.640 & -14.261 & -12.588 & -6.497 & 2.679 & 3.304 & 2.581 & \mathbf{0.971} \\ \mathbf{40} & -12.902 & -16.932 & -17.893 & -11.975 & 2.695 & 7.245 & 5.379 & 2.248 \end{array} \right) (bp)
\end{aligned}
$$

Parameters that are fixed to 0 are represented by a "·". The lower triangle of the volatility matrix for y_t contains the Cholesky decomposition of its conditional covariance matrix and the upper triangle contains the correlation matrix (which is not scaled by 10−2). Similarly, the lower triangle of R represents the Cholesky decomposition of the covariance matrix of the measurement errors, and the upper triangle represents the correlation matrix.

Table 5: t-Ratios

$$
r_t = \cdot + \begin{pmatrix} 10.755 \\ -1.848 \\ \cdot \\ 6.039 \\ \cdot \end{pmatrix}' y_t + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}' y_{t-1}
$$

$$
\Lambda_t = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}'
$$

$$
+ \begin{pmatrix} -3.596 & 0.306 & 1.801 & 3.416 & 0.789 \\ -2.233 & -2.141 & 1.037 & -3.356 & 1.544 \\ 1.108 & -1.294 & -2.401 & -0.491 & 0.481 \\ -1.182 & 0.434 & 1.453 & 2.233 & -1.526 \\ 0.942 & -0.760 & -1.652 & -1.817 & -2.455 \end{pmatrix} y_t + \begin{pmatrix} \cdot & \cdot & \cdot & -1.990 & \cdot \\ \cdot & 5.873 & \cdot & 3.903 & -1.434 \\ \cdot & \cdot & 1.732 & -1.226 & \cdot \\ \cdot & \cdot & \cdot & -2.867 & 1.631 \\ \cdot & \cdot & 1.794 & \cdot & \cdot \end{pmatrix} y_{t-1}
$$

$$
y_{t+1} = \begin{pmatrix} 5.445 & \cdot & 0.603 & 2.994 & \cdot \\ \cdot & 14.367 & \cdot & -4.298 & 1.671 \\ 2.138 & \cdot & 5.556 & 0.965 & -1.768 \\ -4.887 & \cdot & 0.691 & 13.505 & -0.673 \\ 1.784 & \cdot & -1.063 & -2.098 & 5.883 \end{pmatrix} y_t + \begin{pmatrix} \cdot & \cdot & \cdot & -1.990 & \cdot \\ \cdot & 5.873 & \cdot & 3.903 & -1.434 \\ \cdot & \cdot & 1.732 & -1.226 & \cdot \\ \cdot & \cdot & \cdot & -2.867 & 1.631 \\ \cdot & \cdot & 1.794 & \cdot & \cdot \end{pmatrix} y_{t-1}
$$

$$
+ \begin{pmatrix} 10.263 & \mathbf{-12.817} & \mathbf{11.920} & \mathbf{13.055} & \cdot \\ \cdot & 12.330 & \mathbf{-1.345} & \mathbf{-1.610} & \cdot \\ \cdot & -1.142 & 11.339 & \mathbf{-0.903} & \cdot \\ \cdot & -1.294 & \cdot & 12.963 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 6.984 \end{pmatrix} \epsilon_{t+1}
$$

$R =$

Q	**1**	**2**	**4**	**8**	**16**	**20**	**30**	**40**
1	9.589	**23.713**	**6.263**	**−7.314**	**−43.835**	**−12.416**	**−8.174**	**−6.640**
2	13.389	7.535	**16.265**	**−0.730**	**−71.397**	**−30.189**	**−6.955**	**−4.255**
4	·	4.512	5.298	**3.707**	**−17.814**	**−20.232**	**−13.409**	**−7.640**
8	·	1.575	4.487	5.227	**2.196**	**−0.477**	**−3.847**	**−5.506**
16	−14.424	·	·	5.852	19.126	**34.377**	**6.480**	**3.543**
20	−13.097	−3.471	−1.914	·	14.654	15.679	**24.335**	**11.257**
30	−14.685	−2.096	−2.125	−3.699	·	·	·	**116.015**
40	·	−1.582	−1.935	−3.619	·	12.465	7.338	·

$$
\log L = -59.28344
$$

Fixed parameters are represetned by a "·". Some of these parameters are fixed as a normalization, and some are fixed to their MLE point estimates because their t-ratios are relatively small even if they are free.

Figure 1: Yields Loadings

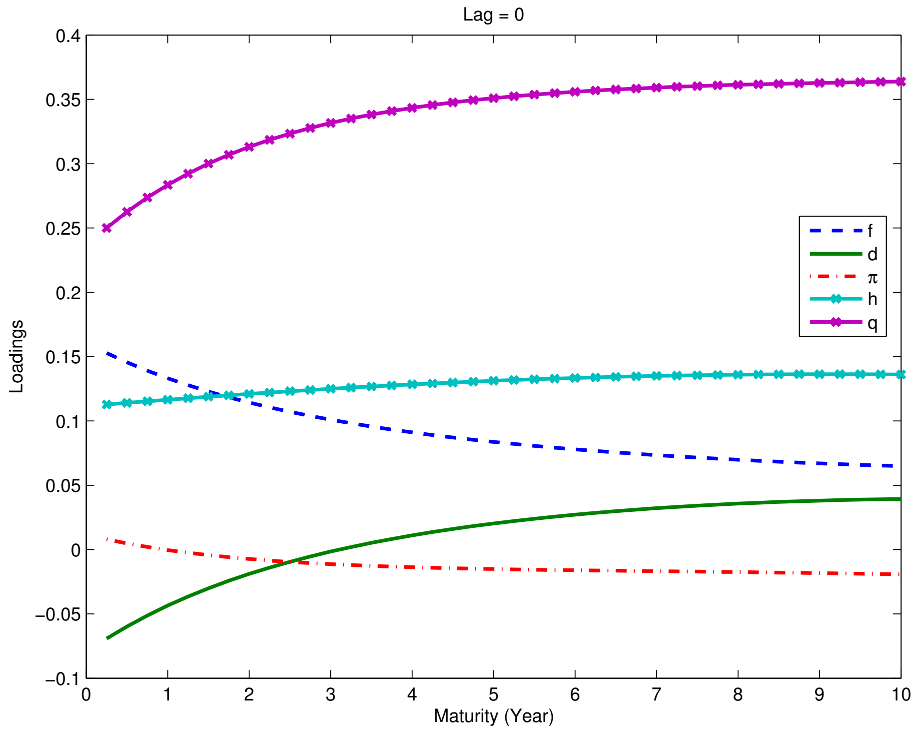

Figure 2: Moments of Yields

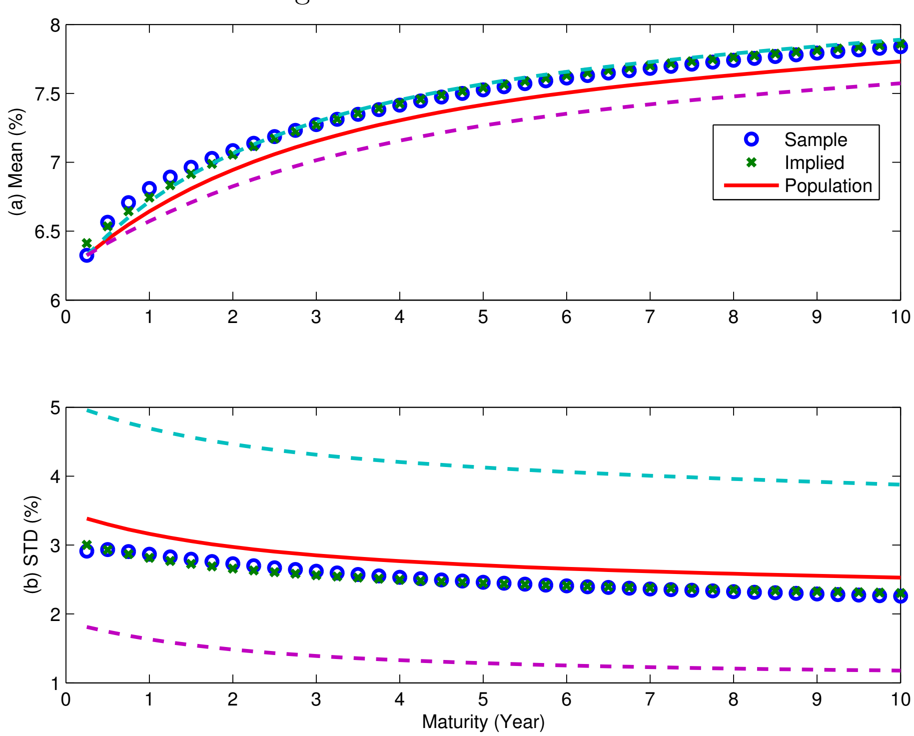

Figure 3: Decomposition of Unconditional Yield Variance

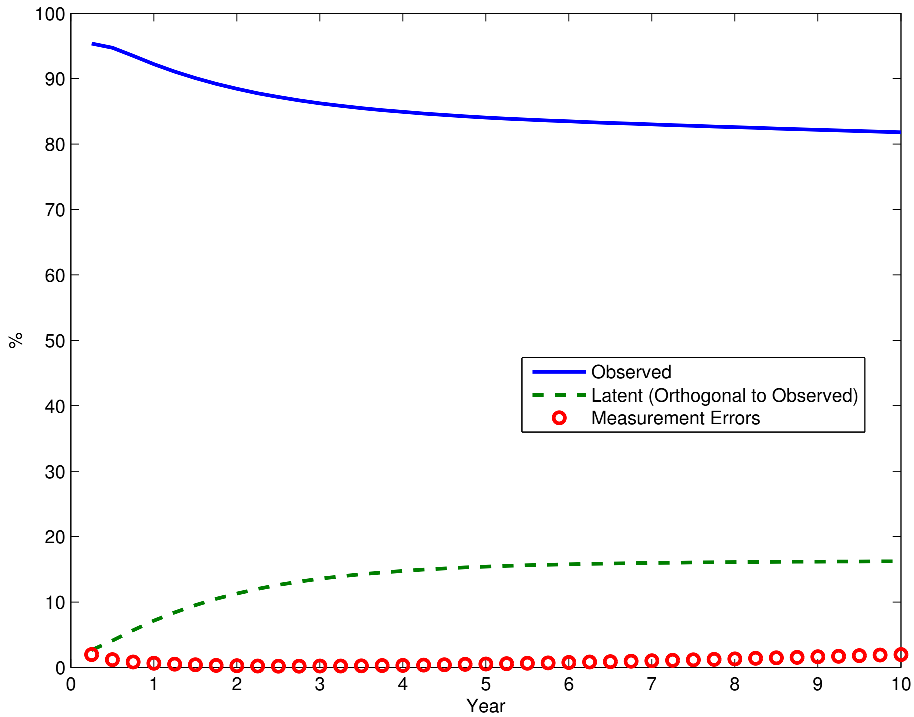

Figure 4: Campbell-Shiller

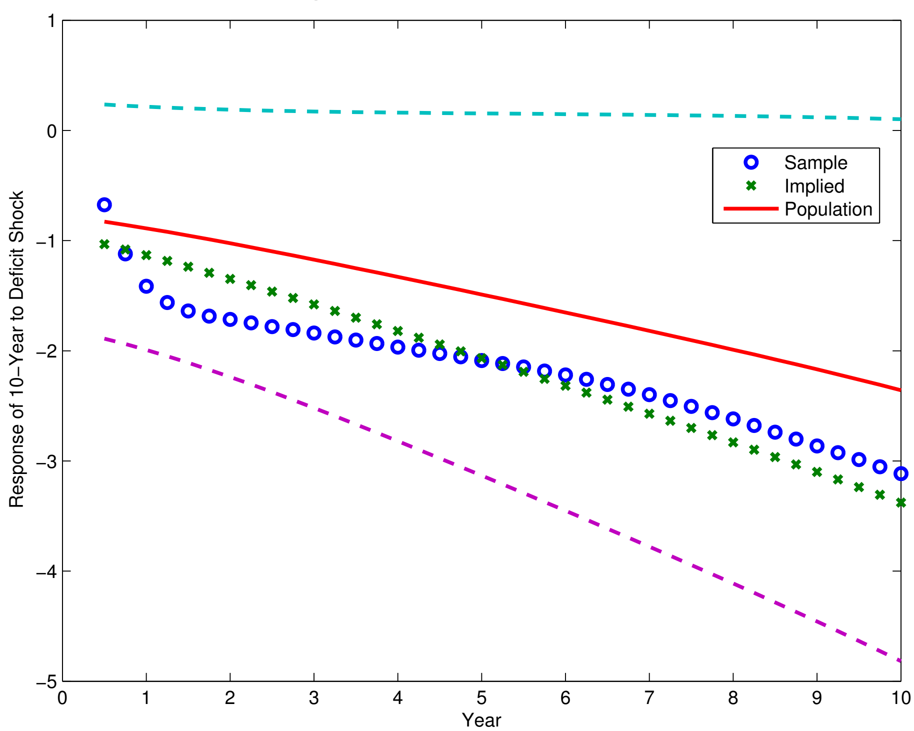

Figure 5: Impulse Response Functions to Deficit Shock

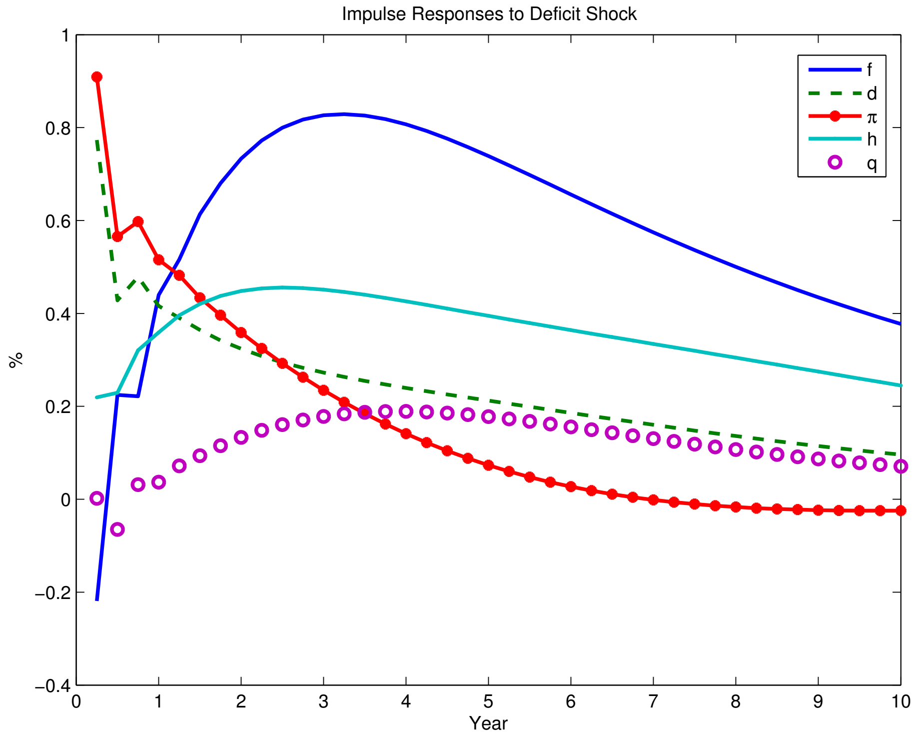

Figure 6: Yield Responses to Deficit Shock

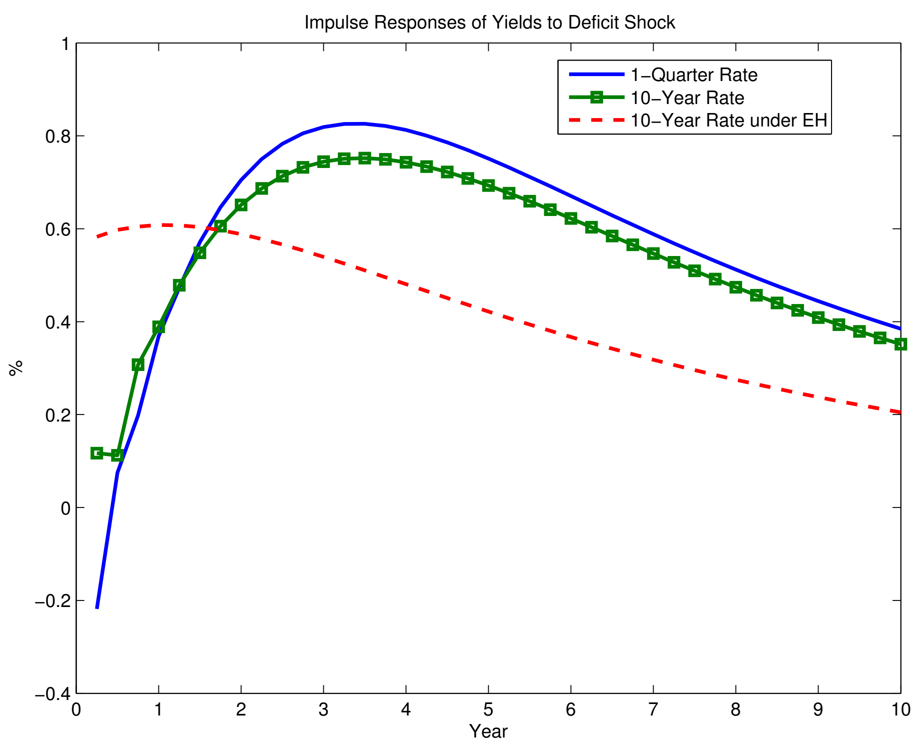

Figure 7: Response of 10-Year

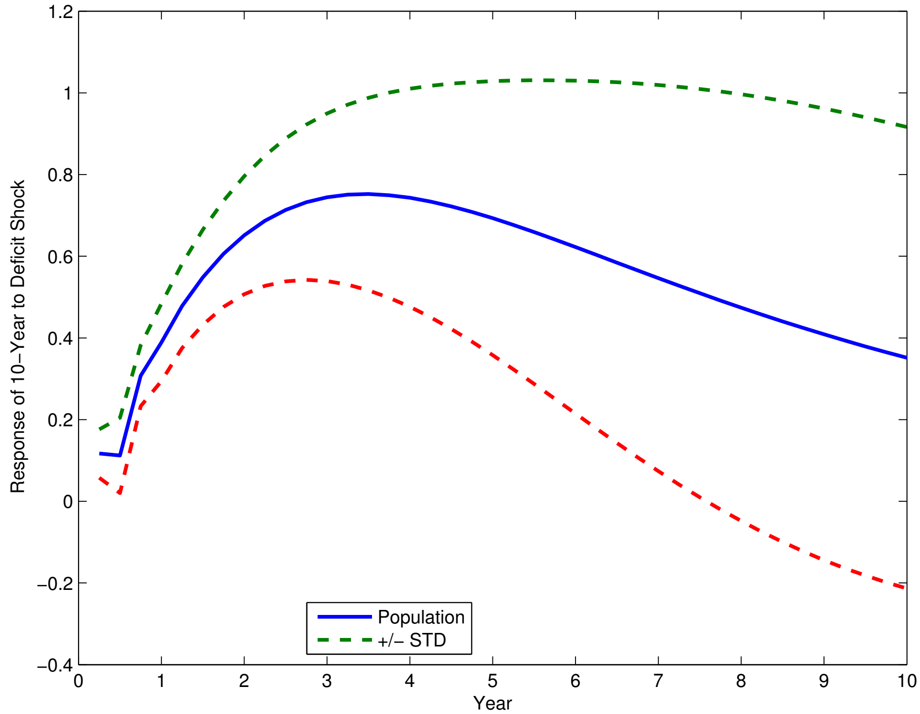

Figure 8: Response of 10-Year Risk Premium

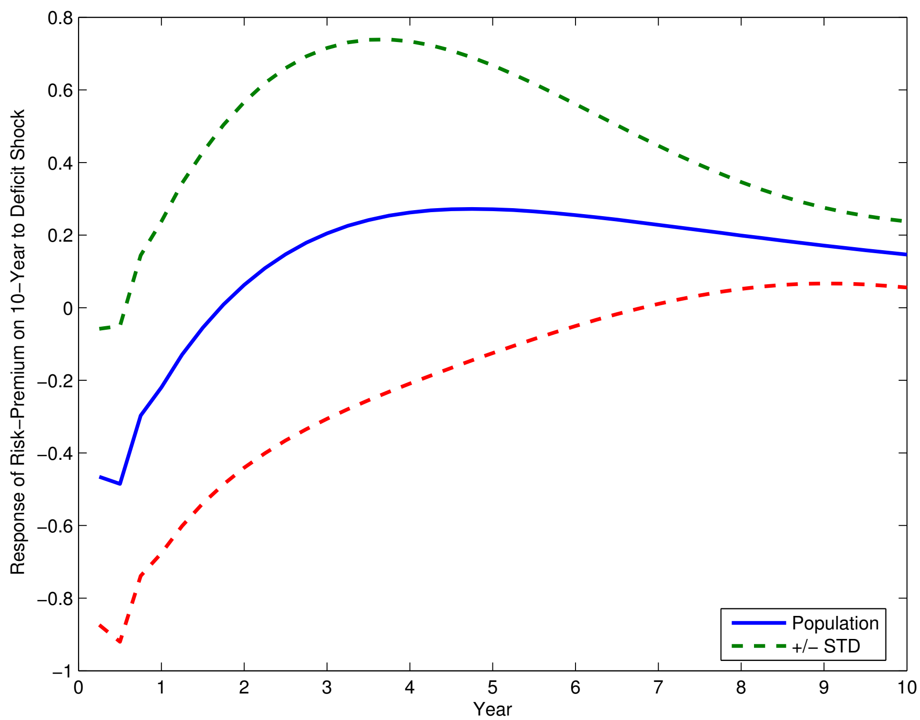

Figure 9: Variance Decomposition for Priced Factors

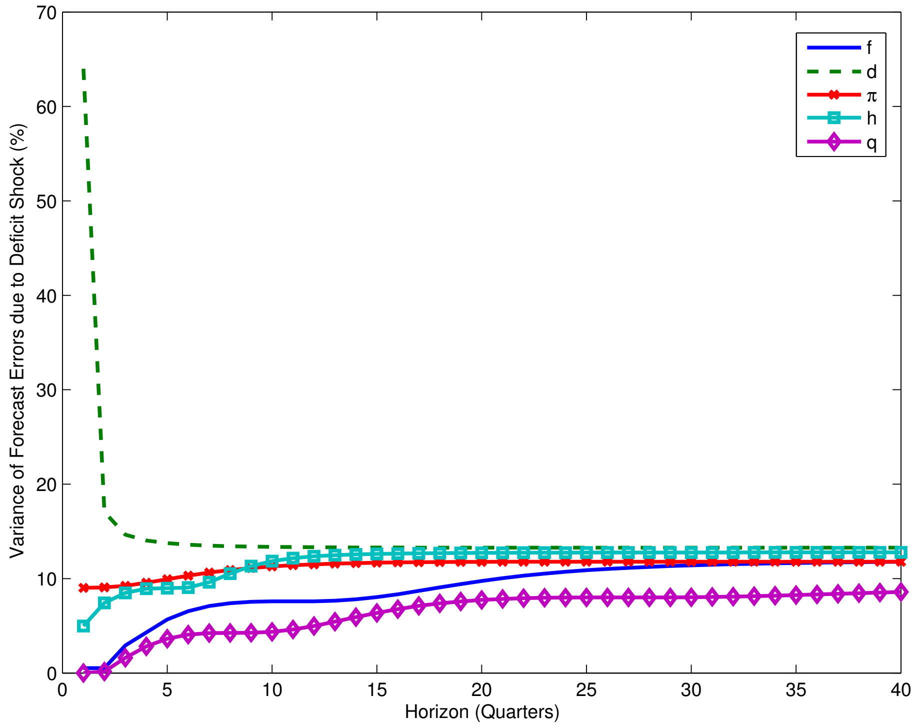

Figure 10: Variance Decomposition for Yields

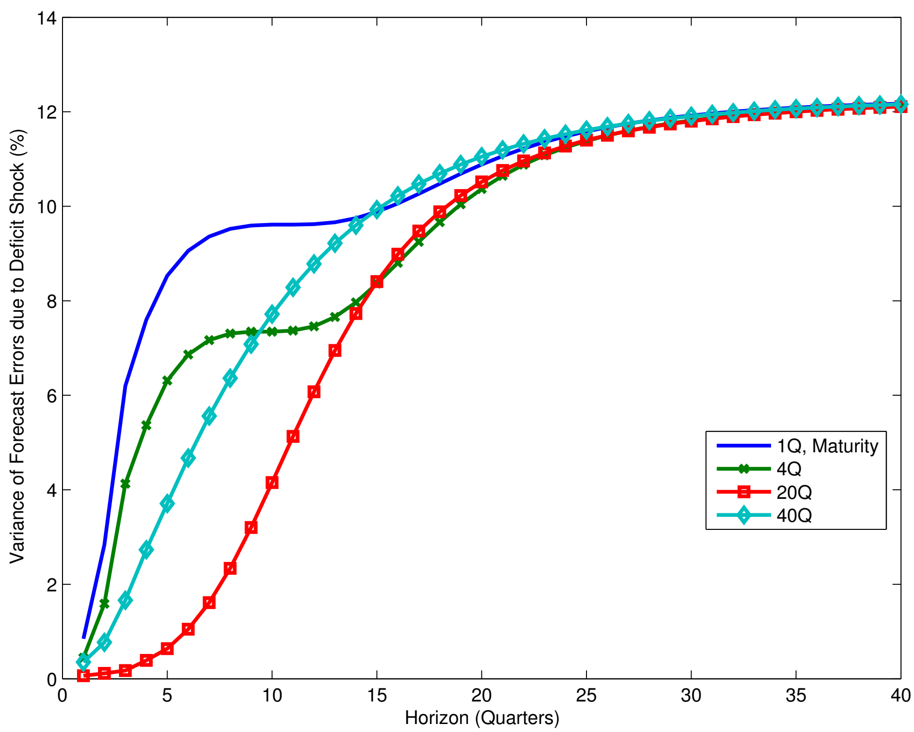

Figure 11: Impulse Responses to Spending Shock

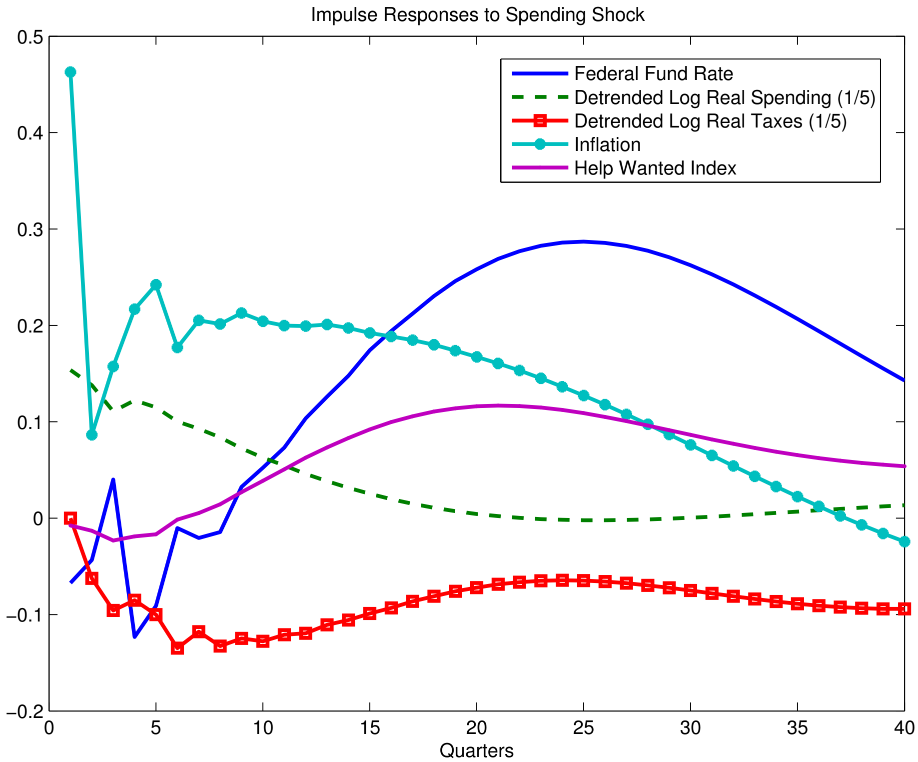

Figure 12: Impulse Responses to Tax Shock

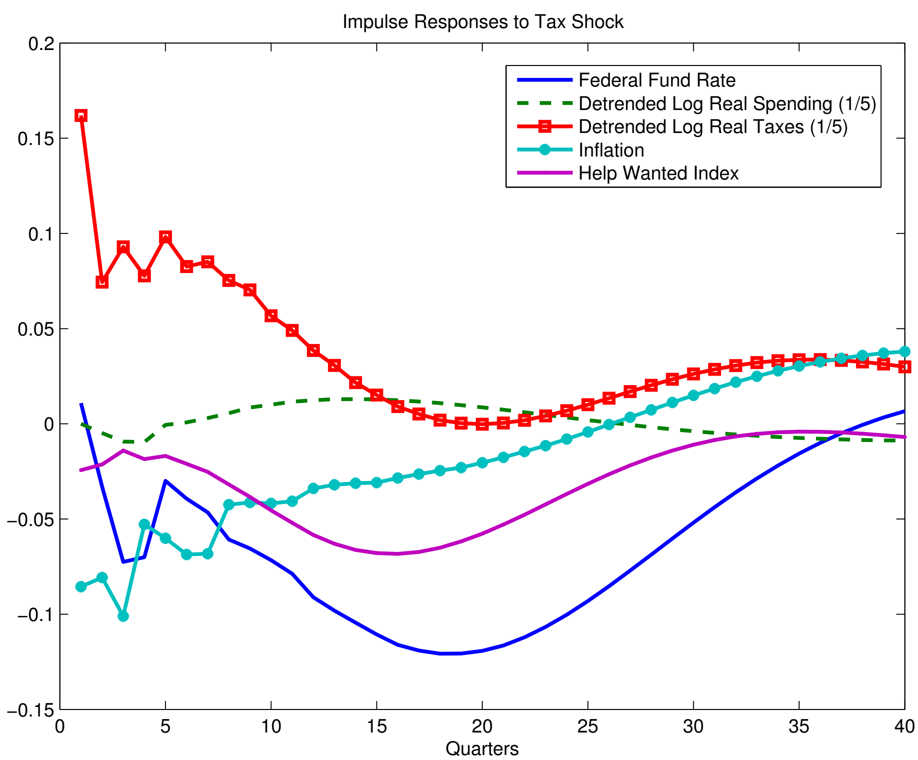

Figure 13: Yield Responses to Spending Shock

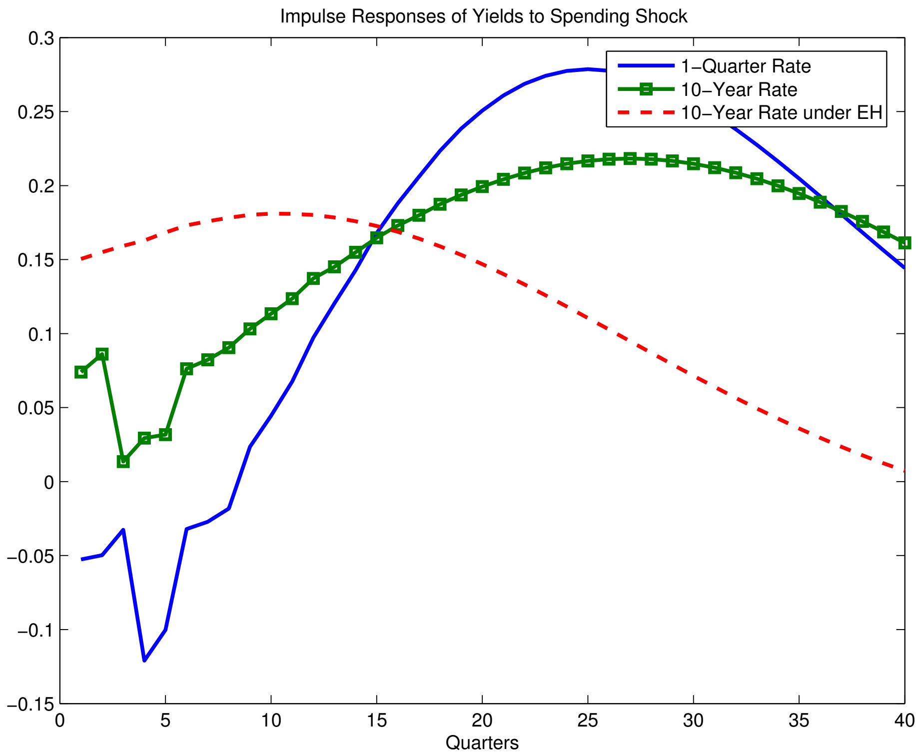

Figure 14: Yield Responses to Tax Shock