The size of venture capitalists' portfolios*

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Abstract

This paper contributes to the emerging literature on the optimal size of venture capitalists' (VCs') portfolios of entrepreneurial firms. We develop a model in which a VC maximizes his expected portfolio value, net of effort costs, with respect to the number of projects he invests in and to the share of each project's profits that he leaves to entrepreneurs. The relationship between the VC and entreprneurs is characterized by double-sided moral hazard, which causes the VC to trade-off larger portfolios against lower profit shares. We analyze the relation between the VC's optimal portfolio structure and exogenous factors, such as entrepreneurs' and VC's productivities, their disutilities of effort, the value of a successful project, and the required initial investment in each venture. The analysis results in ambiguous predictions for the reduced-form effects of the exogenous factors on the optimal VC's portfolio size. We demonstrate that the testing of the theory should focus on the unambiguous structural relations, while accounting for the inherent endogeneity of the profit sharing rule. We test the predictions of the model empirically using a proprietary dataset collected through a survey targeted to VC funds worldwide, and find some support for the theory.

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1 Introduction

Venture capital provides an opportunity for young, innovative firms to develop and grow. Unlike regular investors, venture capitalists (VCs) are actively involved in the management of their portfolio companies. VCs provide assistance with strategic and operational planning, management recruitment, marketing, and obtaining additional capital.[1] However, the relationship between VCs and entrepreneurs is characterized by double-sided moral hazard.[2,3] In most cases, the parties' effort levels are unobservable/nonverifiable and, thus, noncontractible. There is a large literature concerned with the design of financial contracts that mitigate the agency problems between VCs and entrepreneurs[4] that are caused by the information asymmetry and by the

[1] See Sahlman (1990) for a discussion of the nature of the relationship between VCs and entrepreneurs, and Gorman and Sahlman (1989) for some survey evidence of VCs' activities. In addition to VCs, there are various types of investors in the venture capital markets, such as banks and corporations (see Gompers and Lerner (1999b)). The objectives of the latter are not limited to maximizing the values of entrepreneurial ventures. In this paper we concentrate on pure VCs.

[2] Double-sided moral hazard has been analyzed in various other contexts. See, for example, Bhattacharyya and Lafontaine (1995) for an analysis of double-sided moral hazard in franchising, and Cooper and Ross (1985), and Dybvig and Lutz (1993) for models of double-sided moral hazard in the context of product warranties. Double-sided moral hazard in the venture capital setting has been analyzed, most recently, by Casamatta (2003) and Schmidt (2003).

[3] In addition to moral hazard, the relationship is characterized by asymmetric information. A VC's ability to reduce the information asymmetry by screening projects is an important determinant of his success. See Houben (2002) for a simultaneous analysis of double-sided asymmetric information and double-sided moral hazard in the context of optimal structure of financial contracts between VCs and entrepreneurs.

[4] In addition, there are potential agency problems between VCs (general partners) and investors in the fund (limited partners). Limited partnership contracts usually include provisions that reduce the possibility that VCs' decisions would harm limited partners' interests. See Gompers and Lerner (1996) for an empirical analysis of contractual restrictions imposed on general partners (fund managers) by limited partners, and Gompers and Lerner (1999a) for a theoretical and an empirical analyses of VCs' compensation structures. There is empirical evidence that sources of VC funds af-

unobservability/nonverifiability of VCs' and entrepreneurs' actions.

Unlike contracting issues between VCs and entrepreneurs,[5] the optimal size of VCs' portfolios has received little attention both in the theoretical and the empirical venture capital literatures. Investing in multiple ventures, as opposed to a single project, allows VCs to make the best use of their funds, while reducing the risk of their investment. However, because VCs invest time and effort in advising their portfolio firms, as opposed to just providing funds, increasing the number of firms in the portfolio dilutes the quantity and quality of the managerial advice that VCs can provide to each entrepreneurial venture. Because the number of experts is limited, venture capital is not easily scalable. In addition, the increasing cost of a VC's advice or, analogously, the "decreasing return to scale" of the advice may stem from their specialization in certain industries. The higher the number of ventures financed and advised by the VC, the larger the potential competition between the portfolio firms, and the lower the expected return on each venture and the value of VC's advice.[6] If VC's and entrepreneurs' efforts are complementary to ventures' success, reduced VC's advice may also lead to a lower commitment of entrepreneurs to their ventures.[7] On

fect the ways the funds are invested (see Mayer, Schoors and Yafeh (2004), VCs' profits (see Lerner, Schoar and Wong (2004)), and the performance of VC funds (see Jones and Rhodes-Kropf (2004)).

[5] See Bascha and Walz (2001), Bergemann and Hege (1998), Cornelli and Yosha (2003), Garmaise (2001), Houben (2002), Kaplan and Strömberg (2004), Marx (1998), Repullo and Suarez (2004), and Schmidt (2003), among others, for models of optimal structure of contracts between VCs and enterpreneurs. See Gompers (1995) and Gompers and Lerner (1999b) for empirical investigations of venture capital contracts.

[6] Cestone and White (2003) show that it may be beneficial for a VC to commit to not fund too many ventures, precisely because of the potential competition that may diminish expected returns.

[7] Anecdotal evidence suggests that, in addition to complementarities between VCs' and entrepreneurs' efforts, there are network externalities between efforts of different entrepreneurs. These externalities may mitigate the effect of reduced VC's effort devoted to each venture. It is not clear, however, how the number of entrepreneurial ventures in the VC's portfolio affects the complementarities between the efforts of different entrepreneurs.

the other hand, if VC's wealth is unconstrained, investing in more ventures increases the expected total dollar return of VC's portfolio. Thus, the choice of the optimal portfolio size involves a trade-off between the number of projects and each project's expected value net of the cost of VC's effort devoted to it.[8]

To the best of our knowledge, Kanniainen and Keuschnigg (2003, 2004), KK henceforth, provide the only theoretical analysis of the optimal VC's portfolio size. Fulghieri and Sevilir (2004) examine the effects of the relatedness between entrepreneurial ventures (portfolio focus) on VCs' incentives to concentrate on a single venture or to invest in two ventures. Cumming (2004) provides empirical evidence on the factors affecting VCs' portfolio size. Kaplan and Strömberg (2003) and Hege, Palomino and Schwienbacher (2003) provide some evidence on the allocation of cash flows between VCs and entrepreneurs. In what follows, we use the terms "portfolio size" and "number of portfolio firms" interchangeably.

In this paper, we develop a model of the optimal size of VC's portfolio. The VC chooses not only the number of firms in his portfolio, but also the shares of the ventures' profits that the entrepreneurs retain. Profit sharing affects the optimal effort levels exerted by the VC and the entrepreneurs, which, in turn, determine the expected values of entrepreneurial ventures. In addition, we analyze a setting in which the portfolio size and the profit sharing rule are chosen so as to maximize social welfare, and compare this welfare-maximizing equilibrium with an equilibrium in which the portfolio structure is chosen by the VC.

The complexity of the model allows us to derive only the equilibrium effort levels of the VC and the entrepreneurs in a general form. We make two compromises when we solve for the optimal portfolio structure. First, we assume specific functional forms for the probability of a venture's success, as well as for the costs of VC's and

[8] In our model, the effect of the portfolio structure on the parties' optimal effort levels is through the allocation of cash flow rights (profits). Kaplan and Strömberg (2003) find that VCs often separately allocate cash flow rights, board rights, liquidation rights, and other control rights.

entrepreneurs' efforts. Second, we assume that VC's and entrepreneurs' efforts are additive and, thus, not complementary to the probability of projects' success.[9] The model results in unambiguous predictions for the effect of the exogenous variables on the optimal portfolio size only when the sharing rule is held constant.[10] However, when we endogenize the sharing rule, the signs of the reduced-form relations between most of the factors and the optimal portfolio size become ambiguous.

Our theoretical analysis shows that the reduced-form theoretical relations between VC's portfolio size and exogenous factors, such as VC's and entrepreneurs' qualities and their effort costs, derived in KK, are an artifact of the assumption that entrepreneurs' effort is a binary choice variable. To derive testable predictions, we focus on partial effects of the exogenous factors on VC's portfolio size and the profit sharing rule. We demonstrate that one should account for the endogeneity of both the profit sharing rule and the portfolio size while examining these relations empirically.

The framework of our model of effort choices is most similar to Bhattacharyya and Lafontaine (1995), who analyze franchising agreements, which also pose a double-sided moral hazard problem for a principal facing multiple agents.[11] The framework of our model in the portfolio structure choice stage is more similar to that in Casamatta

[9]This is done for analytical tractability only. We show in a general setting that the complementarity of efforts leads to VC's and entrepreneurs' optimal effort levels being affected by their counterparties' quality and disutility of effort. Existence of complementarity, however, does not change the signs of the relations between other exogenous factors and the parties' effort levels.

[10]When the profit sharing rule is exogenous, our results for the relations between the optimal portfolio size and various factors are quite similar to those in KK, despite the differences in the models' setup.

[11]The difference between our model and that of Bhattacharyya and Lafontaine (1995) is that in their setting, the assumption of a constant marginal cost of effort of a franchizor (the party choosing the number of "companies" in his portfolio of franchisees) makes the optimal choice of the number of portfolio firms irrelevant. While it seems reasonable that a franchisor faces constant marginal cost of effort, this assumption seems far-fetched in the context of venture capital, given the different nature of services provided by franchisors and VCs.

(2003), who investigates the rationale for the joint provision of funds and advice by VCs to entrepreneurs.

To test the predictions of the model we obtained comprehensive data for 42 VC funds through a survey and follow-up phone interviews in 2004 with venture capital and private equity funds in Europe and North America. A multivariate analysis of the determinants of VCs' portfolio sizes and entrepreneurs' profit shares accounts for the inherent endogeneity of the profit sharing rule by using instrumental variables. The results of the analysis provide some support for the predictions of our model. In particular, we find strong support for the prediction that VC's portfolio size should vary non-monotonically with the profit shares held by entrepreneurs. This finding is robust to including various control variables, as well as to different specifications of the non-linear relation between the portfolio size and the profit sharing rule. In addition, consistent with the model, entrepreneurs' profit shares are found to be positively, albeit insignificantly, related to the number of firms in VC's portfolio. Some of the exogenous factors in the model, however, turn out not to have a statistically significant impact on the portfolio size and the profit sharing rule, or have an impact opposite to that predicted by the model. This is possibly due to the small size of our sample, which is a fairly common characteristic across most empirical studies of the venture capital industry.

The remainder of the paper is organized as follows. The next section lays out the framework of the model. We derive the relations between the optimal VC's and entrepreneurs' effort levels, and the structure of VC's portfolio in a general setting in Section 3. Section 4 provides the solution of the model under certain assumptions on functional forms. In Section 5, we establish empirical implications for the system of relations between the portfolio size, the profit shares given to entrepreneurs, and the exogenous factors mentioned above, while accounting for the endogeneity of entrepreneurs' profit shares, and present the data and the empirical tests of the model's predictions. We conclude and discuss the limitations of this study and possible directions for future research in Section 6. All proofs are found in the Appendix.

2 The setup of the model

Consider a single venture capitalist, who finances multiple risky entrepreneurial ventures.[12] There is an infinitely elastic supply of identical projects, each requiring an initial investment of I and providing a (discounted) cash flow of R if successful, and zero if unsuccessful. Assuming that the value of a project in an unfavorable state is zero simplifies the analysis, but prevents us from analyzing security design issues - for example, the expected payoff from holding a fraction x of a firm's equity is identical to a debt position with a promised repayment of xR. However, optimal security design is not the focus of this paper.[13] This setting assumes one financing round.[14]

Entrepreneurs are assumed to be identical. This assumption may seem unrealistic, since VCs' screening ability is limited, and they typically have both "good" and "bad" projects in their portfolios. However, this distinction between "good" and "bad" projects is made ex-post, while the optimal choices of portfolio structure are based on entrepreneurial ventures' ex-ante characteristics. In addition, this assumption

[12]Syndication is an important aspect of the venture capital industry. Incorporating into the model two or more VC investors having different characteristics and simultaneously investing in the same projects is not a prohibitive task. However, considering the goal of our analysis, the costs in terms of analytical tractability seem to outweigh the benefits of additional empirical predictions.

[13]Security design is the focus of recent papers by Casamatta (2003) and Cestone and White (2003) among others. They assume that the project value can take two realizations: R_{high} and R_{low}, and are able to examine the effects of the form of financing on entrepreneurs' and VCs' actions.

[14]The no-staging assumption precludes dynamic considerations. With multiple rounds of financing, the optimal structure of VC's portfolio would be affected by his past decisions, as well as by past shocks to projects' values. While staging is an important phenomenon, it is beyond the objectives of our analysis. See Fulghieri and Sevilir (2004) for a model in which a VC can re-allocate resources from a failing venture to a successfull venture in the second financing stage. We control for the effects of staging in our empirical tests.

reflects the largely documented specialization of the majority of VCs, who tend to invest in similar projects.[15]

The probability of venture i's success, p_i, depends on the effort exerted by entrepreneur i, e_i, the effort (the amount of advice to venture i) by the VC, E_i, the productivity (quality) of the entrepreneurs, α, and that of the VC, β.[16] Empirically, the variation in VCs' qualities may be attributed to both the quality of general partners, and to the characteristics of a venture capital fund, such as limited partnership covenants and the affiliation of a fund (government versus corporate). Because VC's and entrepreneurs' effort levels only affect the probabilities of a projects' success, and are not verifiable, they are not contractible.

We assume that all agents in the model are risk neutral. This assumption has the following implications. First, it prevents the portfolio structure from being affected by risk sharing considerations, which are interesting, but are beyond the scope of this paper. Second, the expected utility of the VC depends exclusively on the expected value of his portfolio of entrepreneurial ventures, net of his effort costs, and no assumption on the joint probability of the ventures' success is necessary.

We assume that all the functions in the model are twice continuously differentiable. The probability of venture i's success is assumed to be strictly increasing and weakly concave in both entrepreneur i's and VCs' effort levels: $p_{i_{e_i}} > 0$, $p_{i_{E_i}} > 0$, $p_{i_{e_i e_i}} \leq 0$, $p_{i_{E_i E_i}} \leq 0$, where $p_{i_\cdot}$ denotes the partial derivative of p_i with respect to $\cdot$, $\frac{\partial p_i}{\partial \cdot}$, and similarly for $p_{i_{\cdot\cdot}}$. The effort levels of entrepreneur i and the effort of the VC devoted to project i are assumed to be weakly complementary: $p_{i_{e_i E_i}} \geq 0$. The probability of

[15]Moreover, the results of our model for the case of identical entrepreneurs can be easily extended to a case where a VC faces entrepreneurs with different characteristics, as long as there is no information asymmetry.

[16]Effort choices are continuous. The modelling of an entrepreneur's effort level as a continuous variable is one of the differences between our approach and that of Kanniainen and Keuschnigg (2003, 2004). In KK, entrepreneurs' effort choices are binary, and the equilibrium profit shares given to the entrepreneurs are the lowest ones that ensure effort exertion on their part.

project i's success is positively related to the qualities of both the entrepreneur and the VC: $p_{i_\alpha} > 0$ and $p_{i_\beta} > 0$. The marginal contributions of entrepreneur's and VC's efforts to the project's success are also positively related to the respective agents' qualities, i.e. $p_{i_{e_i\alpha}} > 0$ and $p_{i_{E_i\beta}} > 0$. To ensure the existence of an equilibrium in effort levels, we assume that $p_{i_{e_ie_i}} + p_{i_{e_iE_i}} < 0$ and $p_{i_{E_iE_i}} + p_{i_{e_iE_i}} < 0$.[17] In addition, in order to interpret $p_i(e_i, E_i, \alpha, \beta)$ as a probability function, we assume that $0 \leq p \leq 1$ for all e_i, E_i, α and β.

In the first stage, the VC chooses the number of projects to finance, n, the profit share in each venture offered to entrepreneurs, x_i for project i, and makes an irreversible investment of I in each project. We assume that VC's wealth is unbounded (the supply of financing by limited partners is perfectly elastic). This assumption is supported by the empirical evidence that wealth constraints are not binding for most VCs. For example, Cumming (2004) reports that, on average, venture capital funds have about half of their resources invested in entrepreneurial ventures at a given point in time. The VC maximizes the expected value of his portfolio of entrepreneurial ventures (fund) net of his effort cost.[18]

Because a project generates zero cash flows when it fails, a linear profit sharing rule is as good as any other sharing rule.[19] The VC makes a take-it-or-leave-it (TIOLI)

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[17]This condition is similar to the one ensuring uniqueness and global stability of the solution to a Cournot duopoly game: $\pi_{i,i} + \pi_{i,-i} < 0$, where $\pi_{i,i}$ is the second partial derivative of firm i's profit with respect to its own quantity, and $\pi_{i,-i}$ is the cross-partial derivative of firm i's profit with respect to its own and its rival's quantities. (See Chapter 4 in Vives (2000)).

[18]In reality, it is not uncommon for VCs to operate more than one fund. See Cumming (2004) for a discussion of possible effects of operating more than one fund on the optimal structure of the funds.

[19]A large body of literature argues that linear profit sharing contracts are not optimal in the venture capital setting (see, for example, Cornelli and Yosha (2003), and Schmidt (2003)), and that convertible securities should be used instead. Kaplan and Strömberg (2003) find that in their sample of 213 financing rounds, convertible securities are used in 206 rounds, and 170 rounds use solely convertible preferred stock. They find, however, that the state contingencies of cash flow rights

offer to each entrepreneur, specifying the share of the profits he retains in exchange for his investment and future advice ($1 - x_i$ for venture i). Each entrepreneur accepts the TIOLI offer if his participation constraint is satisfied. We assume, for simplicity, that if an entrepreneur rejects the offer, his reservation utility is zero. Hence, an entrepreneur will accept any offer that leaves him with a positive profit share.[20] The assumption that entrepreneurs have no alternatives to forming a partnership with a specific VC implies, in particular, that each entrepreneur is approached by at most one VC. This is consistent with the assumption of a perfectly elastic supply of entrepreneurs.[21]

In the second stage, the entrepreneurs and the VC choose their nonverifiable effort levels. Effort is costly. Its cost to entrepreneur i, K_i, is increasing and convex in the effort level: $K_{i_{e_i}} > 0$ and $K_{i_{e_i e_i}} > 0$. The cost of effort and the contribution of a marginal unit of effort to the cost of effort are assumed to be increasing in an entrepreneur's "disutility of effort" parameter, γ: $K_{i_\gamma} > 0$ and $K_{i_{e_i \gamma}} > 0$. Similarly, the cost of effort of the VC, L, is increasing and convex in the total amount of advice provided to the projects: $L_{\sum_{i=1}^{n} E_i} > 0$ and $L_{\sum_{i=1}^{n} E_i \sum_{i=1}^{n} E_i} > 0$, for $i = 1, ..., n$. Moreover, we assume that because of "coordination costs", the disutility and the marginal disutility of VC's effort are increasing and weakly convex in the number of firms he invests in, holding the total effort level constant: $L_n > 0$, and $L_{nn} \geq 0$.

are not very large. Bhattacharyya and Lafontaine (1995) show that the optimal sharing rule can be represented by a linear contract in a setting with double-sided moral hazard, but no asymmetric information and risk neutral agents.

[20] An entrepreneur does not commit to exert any effort by accepting the offer. Therefore, the worst he can do is receive zero expected utility, since the expected value of a project is non-negative, and the entrepreneur does not invest any funds in the project.

[21] This seems consistent with the empirical evidence. For example, Sahlman (1990) finds that venture capital funds evaluate more than 1,000 potential projects each year, investing in a tiny fraction of them. Thus, VCs' funding and expertise, and not entrepreneurs' knowledge and ideas, are a scarce resourse.

The cost of VC's effort, the contribution of the marginal unit of effort to the total cost of effort, and the coordination costs are increasing in the VC's disutility of effort parameter, δ: $L_\delta > 0$, $L_{\sum_{i=1}^{n} E_i \delta} > 0$ and $L_{n\delta} > 0$. The VC and the entrepreneurs maximize the expected values of their respective shares of the projects net of effort costs, U_{vc} and U_{en_i} respectively, given the sharing rules, x_i for project i, and the size of the VC's portfolio, n:

$$U_{en_i} = x_i p_i(e_i, E_i, \alpha, \beta)R - K_i(e_i, \gamma) \text{ for } i = 1, ..., n, \tag{1}$$

$$U_{VC} = \sum_{i=1}^{n} [[1 - x_i]p_i(e_i, E_i, \alpha, \beta)R - I] - L(\sum_{i=1}^{n} E_i, n, \delta). \tag{2}$$

An implicit assumption in (2) is that each venture's success is equally valuable to the VC.[22] The game is solved using the concept of sub-game perfect equilibrium. In the second stage, n entrepreneurs and the VC maximize their expected values, net of effort costs, with respect to their own effort levels, given the size of the VC's portfolio and the (linear) profit sharing rules. In the first stage, the VC chooses the number of portfolio companies and the shares given to the entrepreneurs, while accounting for his own and entrepreneurs' second-stage optimal effort levels choices.

In order to ensure interior solutions for the parties' optimal effort levels, we assume that the marginal benefits of VC's and entrepreneurs' efforts are higher than their respective marginal effort costs, as the effort levels approach zero: $x_i p_{i_{e_i}} R > K_{i_{e_i}}$ as $e_i \to 0$ for any $x_i > 0$, $E_i > 0$, α, β, and γ, and $[1 - x_i]p_{i_{E_i}} R > L_{E_i}$ as $E_i \to 0$ for any $x_i > 0$, $n > 0$, $e_i > 0$, α, β, and δ.

[22] This means that general partners invest identical amounts of the fund's wealth as well as their personal wealth in each project. Gompers and Lerner (1996) find that the contracts between general and limited partners often include absolute restrictions on the ability of general partners to invest private funds in entrepreneurial firms. In this case, all the projects are equally valuable for a VC.

3 Optimal effort level choices

In this section we derive the second-stage optimal effort levels of the VC and the entrepreneurs, taking the portfolio structure as given. Maximizing each entrepreneur's expected value, U_{en_i} in (1), with respect to his effort level, e_i, and VC's expected value, U_{vc} in (2), with respect to the level of advice given to each entrepreneur, E_i, results in a system of $2n$ first-order conditions that have to hold simultaneously in equilibrium. The n FOC's of entrepreneurs are

$$x_i p_{i_{e_i}} R - K_{i_{e_i}} = 0 \text{ for } i = 1, ..., n, \tag{3}$$

indicating that each entrepreneur chooses his effort so that its marginal benefit equals its marginal cost. The n FOC's of the VC are

$$[1 - x_i] p_{i_{E_i}} R - L_{E_i} = 0 \text{ for } i = 1, ..., n, \tag{4}$$

assuring that in equilibrium the marginal cost of VC's effort devoted to venture i equals the marginal increase in the expected value of venture i.

While multiple asymmetric equilibria may exist, in order to obtain tractable results, we restrict our attention to symmetric equilibria. The following lemma establishes the existence of a symmetric equilibrium in the second stage, in which all entrepreneurs choose identical effort levels, and the VC devotes the same effort to each project. The symmetric equilibrium in effort levels is conditional on the VC offering identical profit shares to all entrepreneurs in the first stage. We prove, in the next section, that there exists an equilibrium in the first stage of the game, in which the VC gives equal profit shares to all entrepreneurs.

Lemma 1 *If the profit shares given to all entrepreneurs are equal, $x_i = x$ for all $i = 1, ..., n$, then there exists a symmetric equilibrium in entrepreneurs' and VC's effort levels, where $e_i^*(x, n) = e^*(x, n)$ and $E_i^*(x, n) = E^*(x, n)$ for all $i = 1, ..., n$.*

In this symmetric equilibrium, the VC's effort is evenly distributed across n firms in his portfolio, and all entrepreneurs exert identical effort levels. Therefore, for the symmetric case, the system of FOC's in (3) and (4) may be rewritten as the following system of 2 equations:

$$xp_e R - K_e = 0, \tag{5}$$

$$[1-x]p_E R - L_E = 0, \tag{6}$$

where p denotes the probability of success of a typical entrepreneurial project, e is the effort level of a typical entrepreneur, and E is the effort level exerted by the VC on a typical project.

Differentiating this system of equations with respect to the number of portfolio firms, n, while assuming fixed profit share, x, equating the resulting expressions to zero, and solving the system of two equations leads to the following proposition:

Proposition 1 *For any given $x = \overline{x}$, VC's equilibrium effort devoted to each project, $E^*(n, \overline{x})$, is strictly decreasing in the number of firms in his portfolio, n. If there are no complementarities between entrepreneurs' and VC's efforts, $p_{eE} = 0$, then the equilibrium effort level of a typical entrepreneur, $e^*(n, \overline{x})$, is independent of n. If the efforts are complementary, $p_{eE} > 0$, then $e^*(n, \overline{x})$ is strictly decreasing in n.*

We illustrate the relations between E^*, e^*, and n, described in Proposition 1 in Figures 1 and 2. Figure 1 describes the case of no complementarities between entrepreneurs' and VC's efforts, while Figure 2 presents the case of complementary efforts.

Insert Figures 1 and 2 here

Increasing the number of portfolio firms stretches VC's effort over more projects. If, as we assumed, the total cost of effort is convex in n, then the optimal level of VC's advice to each firm decreases as n increases. In addition, if entrepreneurs' and VC's efforts are complementary, reduced advice by the VC induces the entrepreneurs to optimally exert less effort. This latter effect, in turn, reinforces VC's incentive to exert less effort. Therefore, the direct and the indirect effects of a change in n on the effort levels of the entrepreneurs and the VC are reinforcing. The indirect effect is zero in the absence of complementarities, in which case the optimal effort of a typical entrepreneur, e^*, is not affected by n.

We now turn to the relation between the equilibrium effort levels and the sharing rule offered to the entrepreneurs, x. Differentiating the system in (5) and (6) with respect to x, while assuming that the number of firms in the portfolio is fixed, equating the resulting expressions to zero, and solving the system of two equations, leads to the following result:

Proposition 2 *1) If VC's and entrepreneurs' efforts are not complementary,* $p_{eE} = 0$*, then for any given* $n = \overline{n}$*, the equilibrium effort of a typical entrepreneur,* $e^*(\overline{n}, x)$*, is strictly increasing in the share given to entrepreneurs,* x*, and the equilibrium effort of the VC devoted to a typical project,* $E^*(\overline{n}, x)$*, is strictly decreasing in* x*.*
2) If VC's and entrepreneurs' efforts are complementary, $p_{eE} > 0$*, then for any given* $n = \overline{n}$*,* $e^*(\overline{n}, x)$*, and* $E^*(\overline{n}, x)$ *do not vary monotonically with the share given to entrepreneurs,* x*. For* $x \to 1$*, both* $e^*(\overline{n}, x)$ *and* $E^*(\overline{n}, x)$ *are decreasing in* x*. For* $x \to 0$*, both* $e^*(\overline{n}, x)$ *and* $E^*(\overline{n}, x)$ *are increasing in* x*.*

Figure 3 illustrates the relations between x and the equilibrium effort levels of the VC and a typical entrepreneur for the case of complementary efforts.

Insert Figure 3 here

Changes in the profit share given to the entrepreneurs, x, have a direct and, possibly, an indirect effect on the optimal levels of effort chosen by the VC and the entrepreneurs. The direct effect of an increase in x is the reduced (increased) incentive of the VC (the entrepreneur) to exert effort for any given effort level of the entrepreneur (the VC). This direct effect on VC's effort level is reflected by a shift to the left of VC's best response function, $E^*(e)$. If entrepreneurs' and VC's efforts are complementary, then there is also an indirect effect, caused by a shift up of a typical entrepreneur's best response function, $e^*(E)$. Because the two effects have opposite implications for the equilibrium VC's effort level, the sign of the relation between E^* and x depends on their relative magnitudes. The same discussion applies, of course, to entrepreneurs' effort levels, which are affected by x directly and indirectly (through the change in the optimal VC's effort).

We illustrate the two parts of Proposition 2 in Figures 4 and 5. Figure 4 presents the case of no complementarities, while Figures 5A and 5B describe the case where efforts are complementary for $x \to 1$ and $x \to 0$ respectively.

Insert Figures 4, 5A and 5B here

If efforts are complementary, then when the share given to a typical entrepreneur is close to one, a further increase in his share does not (directly) increase his optimal effort level enough to offset the indirect negative effect of the reduced VC's effort. Thus, for x close to one, the indirect and the direct effects on the entrepreneurs' efforts have opposite signs, with the former more than offsetting the latter. This results in a negative effect of increasing x on both VC's and entrepreneurs' effort levels when x is close to one. On the other hand, when x is close to zero, the direct negative effect of increasing x on the optimal VC's effort level is more than offset by the indirect positive effect of increased entrepreneur's effort level. Thus, for x close to zero, both entrepreneurs' and VC's effort levels are increasing in x.

In addition to studying the effects of changes in x and n on the optimal effort choices of the entrepreneurs and the VC, our general framework allows us to analyze the effects of changes in the exogenous parameters of the model on the equilibrium effort level choices. The set of exogenous parameters, Φ, includes the quality parameters of the VC and a typical entrepreneur, their disutility of effort parameters, the value of a successful project, and the required initial investment: $\Phi = \{\alpha, \beta, \gamma, \delta, R, I\}$. The next proposition establishes these relations.

Proposition 3 *For any given $n = \overline{n}$ and $x = \overline{x}$, the equilibrium effort level of a typical entrepreneur, $e^*(\overline{n}, \overline{x}, \Phi)$, and the equilibrium effort devoted to a typical project by* *the VC , $E^*(\overline{n}, \overline{x}, \Phi)$, are monotonically increasing in their own quality; are monotonically decreasing in their own disutility of effort; are monotonically increasing in the value of a successful project; and are independent of the required investment. In addition, if efforts are complementary, $p_{eE} > 0$, $e^*(\overline{n}, \overline{x}, \Phi)$ and $E^*(\overline{n}, \overline{x}, \Phi)$ are monotonically increasing in entrepreneurs'/VC's counterparty's quality and are monotonically decreasing in their counterparty's disutility of effort.*

This result is intuitive. VC's and entrepreneurs' efforts are increasing (decreasing) in their own quality (disutility of effort) because of the trade-off between the marginal costs and the marginal benefits of exerting effort. If efforts are complementary, the same logic holds for VC's and entrepreneurs' counterparty's quality and disutility of effort. In the absence of complementarities, each party's effort level is independent of the counterparty's characteristics. The equilibrium effort levels do not depend on the initial investment because in the effort choice stage this investment represents a sunk cost.

4 Optimal portfolio size and profit sharing rule

In this section, we solve the first stage of the game, in which the size of VC's portfolio and the profit sharing rule are chosen. First, we solve a partial equilibrium model, in which the profit sharing rule is pre-determined (and identical across projects), and the VC maximizes his net expected value with respect to the number of projects he invests in. This allows us to derive the partial effects of the exogenous factors on the optimal portfolio size. We perform a similar analysis for the profit sharing rule. That is, we assume a fixed portfolio size, and study the partial effect of the exogenous factors on the optimal sharing rule. Then, we integrate the two partial equilibrium models to derive the relations between the optimal portfolio size and the exogenous factors, when both the portfolio size and the profit sharing rule are endogenous. Finally, we analyze the welfare maximizing portfolio structure and compare it to that chosen by the VC.

For reasons of analytical tractability, in this section we make specific assumptions about the functional forms of the probability of a project's success and entrepreneurs' and VC's costs of effort. In particular, we assume that the probability of project i's success equals

$$p_i = \frac{\alpha e_i + \beta E_i}{\overline{p}_i}. \tag{7}$$

The functional form of p_i in (7) weakly satisfies the assumptions of the general model. The probability of the project's success is linearly increasing in VC's and entrepreneur's effort levels. Their efforts are not complementary, in the sense that the cross-partial of p_i with respect to efforts is zero. The value of $\overline{p}_i$ in (7) is chosen so that the equilibrium probability of a project's success is lower than one.[23]

The effort cost function of entrepreneur i, K_i, takes the following form:

$$K_i = \gamma e_i^2, \tag{8}$$

and the total cost of effort of a VC, L, is given by:

$$L = \delta \left[\left[\sum_{i=1}^{n} E_i \right]^2 + n^2 \right]. \tag{9}$$

The last element in (9) reflects the assumption that coordination costs are increasing and convex in the number of portfolio firms. This functional form is parallel to that in Bhattacharyya and Lafontaine (1995), where the effort is separated into "private" and "public". In our framework, $\delta \left[\sum_{i=1}^{n} E_i \right]^2$ is the total cost of private effort, while δn^2 is the cost of public effort. Before solving the model, we establish that there exists an equilibrium in which the profit shares that the VC offers to the n entrepreneurs are identical. We focus on a symmetric equilibrium in profit shares thereafter.

[23]One possible value of $\overline{p}_i$ is $\alpha \widehat{e}_i + \beta \widehat{E}_i$, where $\widehat{e}_i$ and $\widehat{E}_i$ are social welfare-maximizing effort level choices (which maximize the expected value of venture i net of the costs of parties' efforts devoted to the venture). The proof of this statement is available upon request.

Lemma 2 *There exists a symmetric equilibrium in the profit shares given to the entrepreneurs, where* $x_i^*(n, \Phi) = x^*(n, \Phi)$ *for all* $i = 1, ..., n$.

The expected values of a typical entrepreneur and the VC, net of effort costs, in (1) and (2) may be rewritten as

$$U_{en_i} = xR\left[\frac{\alpha e_i + \beta E_i}{\overline{p}}\right] - \gamma e_i^2, \tag{10}$$

$$U_{VC} = \sum_{i=1}^{n}\left[[1-x]R\left[\frac{\alpha e_i + \beta E_i}{\overline{p}}\right]\right] - nI - \delta\left[\left[\sum_{i=1}^{n} E_i\right]^2 + n^2\right]. \tag{11}$$

Applying the FOC's in (5) and (6) results in the following equilibrium effort levels devoted to a typical venture by the VC and the entrepreneur:

$$e^* = \frac{x\alpha R}{2\gamma\overline{p}}, \tag{12}$$

$$E^* = \frac{[1-x]\beta R}{2\delta n\overline{p}}. \tag{13}$$

The effect of the exogenous variables on the equilibrium effort levels of a typical entrepreneur and of the VC are consistent with those derived in the general framework discussed in the previous section for the case of no complementarities. The optimal effort levels of the VC and the entrepreneurs are independent of the characteristics of their counterparty. Moreover, the equilibrium effort level of the entrepreneurs is unrelated to the number of firms in VC's portfolio, while VC's optimal per-project effort is decreasing in n. Finally, as argued in Section 3, VC's effort level devoted to each venture is decreasing in x, while entrepreneurs' optimal efforts are increasing in x.

4.1 Optimal choice of the portfolio size when the profit sharing rule is fixed

In this section, we assume that the profit sharing rule, x, is fixed, and examine the effects of the exogenous factors and x on the optimal number of firms in VC's portfolio,

$n^*(x, \Phi)$. Focusing on a symmetric equilibrium, we can rewrite the expected value of VC's portfolio in (11) as:

$$U_{VC} = n \left[[1-x] \frac{R}{\overline{p}^2} \left[\frac{x\alpha^2}{2\gamma} + \frac{[1-x]\beta^2}{2\delta n} \right] - I - \delta \left[\frac{[1-x]^2 \beta^2 R^2}{4\delta^2 n \overline{p}^2} + n \right] \right]. \tag{14}$$

Differentiating VC's expected value in (14) with respect to n, and equating the resulting expression to zero, gives the optimal size of VC's portfolio:

$$n^*(x, \Phi) = \frac{1}{4} \frac{[1-x]\, x R^2 \alpha^2}{\gamma \delta \overline{p}^2} - \frac{1}{2} \frac{I}{\delta}. \tag{15}$$

Analyzing the expression in (15) leads to the following proposition:

Proposition 4 *The optimal number of firms in VC's portfolio, $n^*(x, \Phi)$, does not vary monotonically with the profit share given to a typical entrepreneur, x. For $x < \frac{1}{2}$, $n^*(x, \Phi)$ is increasing in x. For $x > \frac{1}{2}$, $n^*(x, \Phi)$ is decreasing in x. Moreover, holding the profit share given to entrepreneurs, x, constant, the optimal number of firms in VC's portfolio, $n^*(x, \Phi)$, is increasing in the quality of entrepreneurs; is independent of the quality of the VC; is decreasing in the disutilities of effort of the entrepreneurs and the VC; is increasing in the value of a successful project; and is decreasing in the required investment in each project.*

The direct effect of increasing entrepreneurs' profit share on the value of VC's portfolio is negative. In addition, increasing x reduces the optimal VC's per-project effort level, which further reduces the expected value of each project. However, it also increases the optimal entrepreneurs' efforts, increasing the value of VC's portfolio. VC's optimal reaction to the change in x in terms of portfolio size may take two forms. The VC can invest in fewer projects and increase his level of advice to each venture, thus increasing each project's value. Alternatively, the VC can decide to invest in a larger number of ventures, possibly increasing the total value of his portfolio. Which of these two alternatives is optimal depends on the profit share retained by the entrepreneurs. For large x, the positive (indirect) effect of decreasing n on VC's expected profits through his increased effort level is larger in absolute magnitude than the negative (direct) effect of the reduced number of projects (i.e. $\frac{dn^*}{dx} < 0$). However, for small enough x, the reverse is true: the positive direct effect of increasing n dominates the negative indirect effect (i.e. $\frac{dn^*}{dx} > 0$). This non-monotonicity plays an important role in the next subsection, where we investigate the total effects of changes in the exogenous factors on the optimal VC's portfolio size.[24]

Intuitively, the portfolio size is positively related to the profitability of each venture. Thus, higher quality of entrepreneurs, lower disutility of the parties' efforts, higher value of a successful project, and lower required initial investment result in a higher value of each venture and in a larger optimal portfolio size.

VC's quality does not affect the optimal portfolio size in (15). Higher VC's quality increases the value of each venture, given VC's per-project effort level. This, in turn, leads the VC to fund a larger number of projects. However, the optimal VC's per-project effort level decreases as the number of firms in his portfolio increases, which reduces the value of each venture. Holding everything else constant, this would reduce the optimal number of projects to be funded. In our model, the two effects exactly offset each other. Thus, the optimal number of firms chosen by the VC is independent of his quality parameter.[25]

This result is different from the conclusion in KK, who show that the equilibrium portfolio size is increasing in the VC's quality. Except for this discrepancy, Proposition 4 is consistent with the results in KK. The crucial assumption underlying Proposition 4, however, is that the profit sharing rule is determined exogenously. In Subsection 4.3 we derive the relations between the optimal portfolio size and the

[24]It follows from (16) below that the equilibrium profit share given to entrepreneurs, x^*, can not exceed $\frac{1}{2}$. However, Proposition 4 discusses the effect of x on the optimal choice of the number of portfolio firms, n^*, which does not require x to be in equilibrium.

[25]This result seems to be influenced by the specific setting of the model. In a more general setting, the effect of the VC's quality on the optimal number of portfolio firms may be positive or negative, depending on the relative strengths of the two effects.

exogenous factors when the optimal sharing rule is also endogenous. We show that, in such setting, one cannot make unambiguous predictions regarding the relations between the optimal portfolio size and the exogenous factors.

4.2 Optimal choice of the profit sharing rule when the portfolio size is fixed

In this subsection we assume that the number of firms in VC's portfolio is fixed, and investigate the effects of the exogenous factors on the optimal profit share given by the VC to entrepreneurs, $x^*(n, \Phi)$. Differentiating VC's expected value in (14) with respect to x, and equalizing the resulting expression to zero, provides the following expression for the optimal profit sharing rule, given the portfolio size:

$$x^*(n, \Phi) = \frac{\frac{\alpha^2}{\gamma}n - \frac{\beta^2}{\delta}}{2\frac{\alpha^2}{\gamma}n - \frac{\beta^2}{\delta}}. \tag{16}$$

Of course, the fact that $x^*(n, \Phi)$ has to be between 0 and 1, requires $\frac{\alpha^2}{\gamma}n - \frac{\beta^2}{\delta} \geq 0$ to hold. The next proposition establishes how $x^*(n, \Phi)$ varies with the number of portfolio firms, n, and other exogenous factors, Φ.

Proposition 5 *The optimal profit share given to the entrepreneurs, $x^*(n, \Phi)$, is increasing in the number of portfolio firms, n. Moreover, holding the portfolio size, n, constant, the optimal share given to the entrepreneurs, $x^*(n, \Phi)$, is increasing in entrepreneurs' quality parameter and in VC's disutility of effort parameter; is decreasing in VC's quality parameter and in entrepreneurs' disutility of effort parameter; and is independent of the value of a successful project and of the required investment.*

Increasing the number of portfolio firms reduces the equilibrium per-firm effort level of the VC, thus reducing the expected value of each project. In order to mitigate the negative direct effect of the reduced VC's per-project effort, the VC optimally increases the share offered to the entrepreneurs. This increases entrepreneurs' optimal efforts, which, in turn, positively affects the projects' expected values.[26]

The relations between the exogenous factors and $x^*(n, \Phi)$ follow from their direct effects on the expected value of the ventures, and from the indirect effects through the optimal efforts of the VC and the entrepreneurs. The relations in Proposition 5 reflect the relative magnitudes of the direct and the indirect effects. Because both VC's and entrepreneurs' effort levels, and the expected value of the ventures are linear in the value of a successful project, R, the latter does not affect the optimal sharing rule. Because the parties' equilibrium effort levels, e^* and E^*, are independent of the required initial investment, I, the latter does not interact with x and, thus, does not affect the equilibrium sharing rule. The relations between the equilibrium profit share and the qualities and the disutilities of effort of the entrepreneurs and the VC follow from the trade-off between the marginal costs and benefits of exerting effort.[27]

4.3 The total effects of exogenous factors on the optimal portfolio size and the profit sharing rule

The exogenous factors affect the choice variables of the model in two ways. First, there is a direct effect, outlined in Propositions 4 and 5. In addition, the exogenous factors influence each endogenous variable indirectly, through changes induced into the other choice variable. Using the envelope theorem to examine the total effect of each of the exogenous factors on the optimal profit sharing rule leads to the following result:

Proposition 6 *When both the profit sharing rule and the portfolio size are endoge-*

[26] This result differs from the one in Bhattacharyya and Lafontaine (1995), where the profit sharing rule is independent of the number of firms because the marginal cost of effort is assumed constant. The positive relation between $x^*(n, \Phi)$ and n is consistent with that in KK.

[27] It is worth pointing out how the prediction that $x^*(n, \Phi)$ decreases with VC's quality is consistent with the empirical regularity of more reputable VCs retaining larger shares of a venture's profits.

nous, the optimal profit share given to the entrepreneurs is increasing in entrepreneurs' quality and in the value of a successful project; is decreasing in the quality of the VC, in entrepreneurs' disutility of effort, and in the required initial investment; and is independent of VC's disutility of effort.

The difference between the relations in Proposition 6 and those in Proposition 5 are due, of course, to the indirect effects of the exogenous factors on the optimal entrepreneurs' profit share through changes in the optimal portfolio size. In the case of the quality of entrepreneurs and their disutility of effort, the direct and the indirect effects are reinforcing. In the case of VC's quality, there is no indirect effect, since $n^*(x, \Phi)$ was shown in Proposition 4 to be independent of β. The optimal sharing rule is not affected by the quality of the VC because the direct and the indirect effect of δ exactly offset each other. Finally, the relations between $x^*(n, \Phi)$, the value of a successful project, and the required initial investment are due solely to the indirect effects of these parameters on the optimal portfolio size.

Proposition 6 results in testable empirical predictions regarding the reduced-form relations between the exogenous factors and the profit shares of entrepreneurs. Some of the predictions are consistent with the conclusions in Casamatta (2003), who analyzes effort exertion by the VC and entrepreneurs in a setting where outside financing arises endogenously. For example, Casamatta shows that VC's involvement and profit share in a venture is higher, the less competent is the entrepreneur. This is consistent with the positive relation between entrepreneurs' quality and their shares in the ventures, demonstrated in Proposition 6.

The analysis of the optimal profit sharing rule is not the main focus of this paper, however. As its title suggests, the main goal of our model is to investigate the determinants of the size of a VC's portfolio. Proposition 7 provides the relations between the optimal number of firms in VC's portfolio and the exogenous factors, when their indirect effects through changes in the optimal profit sharing rule are taken into account.

Proposition 7 *When both the portfolio size and the profit sharing rule are endogenous, the relations between the portfolio size and the qualities and the disutilities of effort of the VC and the entrepreneurs are ambiguous. The optimal portfolio size is increasing in the value of a successful project, and is decreasing in the required initial investment.*

This may be seen as one of the main results of the paper. It is impossible to make empirical predictions regarding the effects of the majority of the exogenous variables on the optimal VC's portfolio size. The reason is that the relation between the optimal portfolio size and the profit share retained by the entrepreneurs can not be signed unambiguously. Thus, it is unclear how various exogenous factors affect the optimal portfolio size. Conversely, the indirect effect on $n^*(\Phi)$ is absent in the case of changes in R and I. Thus, it is possible to make unambiguous predictions regarding these relations.

The main conclusion from this subsection is that the testing of the determinants of the optimal VC's portfolio size can not be performed in a reduced form setting. Any test of a model of the optimal size of VC's portfolio has to be done in a framework that accounts for the simultaneous effects of various factors on the optimal profit sharing rule and on the interrelation between the optimal portfolio size and the optimal profit sharing rule. We discuss such a framework in Section 5.

4.4 Socially efficient VC's portfolio

The previous analysis assumes that the equilibrium portfolio size and profit sharing rule are chosen by the VC to maximize his own expected profit net of effort costs. The assumption that VCs' expertise is the scarce resource seems reasonable. This, in turn, gives the VC all the bargaining power and, thus, the ability to choose the portfolio structure that maximizes his utility. Nevertheless, it is instructive to compare VC's portfolio structure choices with those that maximize social welfare. This analysis, for instance, may be relevant for the portfolio structure of government-sponsored venture capital funds (see Cumming (2004)).

We first compare the optimal choices of the portfolio size and the profit sharing rule that maximize the social welfare to those that maximize VC's utility in a reduced-form setting, in which one of the choice variables is held constant. We then make the same comparison for the case in which the number of portfolio firms and the sharing rule are jointly determined in equilibrium.

The joint utility of the VC and the entrepreneurs, U_W, is obtained by combining (10) and (11):

$$U_W = n\left[[1-x]\frac{R}{\overline{p}^2}\left[\frac{x\alpha^2}{2\gamma}+\frac{[1-x]\beta^2}{2\delta n}\right]-I-\delta\left[\frac{[1-x]^2\beta^2R^2}{4\delta^2 n\overline{p}^2}+n\right]\right]+ \\ +n\left[x\frac{R}{\overline{p}^2}\left[\frac{x\alpha^2}{2\gamma}+\frac{[1-x]\beta^2}{2\delta n}\right]-\frac{x^2\alpha^2R^2}{4\gamma}\right]. \tag{17}$$

Maximizing the social welfare in (17) first with respect to x while holding n constant, and then, in turn, with respect to n while holding x constant produces the following values of $x^*(n)$ and $n^*(x)$:

$$x_W^*(n,\Phi) = \frac{\frac{\alpha^2}{\gamma}n}{\frac{\alpha^2}{\gamma}n+\frac{\beta^2}{\delta}}, \tag{18}$$

$$n_W^*(x,\Phi) = \frac{1}{8}\frac{[2-x]\,xR^2\alpha^2}{\gamma\delta\overline{p}^2}-\frac{1}{2}\frac{I}{\delta}. \tag{19}$$

Comparing the values of $x^*(n)$ and $n^*(x)$ in (18) and (19) with the respective values in (16) and (15) that were obtained as a result of VC's utility maximization, results in the following proposition:

Proposition 8 *The number of firms in VC's portfolio that maximizes social welfare, $n_W^*(x,\Phi)$, is higher than the number of firms that maximize VC's value, $n^*(x,\Phi)$, for any given profit sharing rule. Entrepreneurs' profit share that maximizes social welfare, $x_W^*(n,\Phi)$, is higher than the profit share that maximizes VC's value, $x^*(n,\Phi)$, for any given number of portfolio firms.*

The intuition for this result is straightforward. Incorporating entrepreneurs' utilities into the maximization problem increases the marginal benefit of adding a firm to the portfolio (by the net value of entrepreneur's share of the marginal firm net of his effort cost). Similarly, holding n constant, taking entrepreneurs' utilities into account increases the net benefit of a marginal increase in x by entrepreneurs' share of the projects, compared with the case of VC's utility maximization, while not changing the marginal cost of a marginal increase in x.

Proposition 8 established reduced form comparative statics for the cases of VC's utility maximization and social welfare maximization. The next proposition combines the two partial equilibrium results above, and establishes the differences between $x^*_W(\Phi)$ and $x^*(\Phi)$, and between $n^*_W(\Phi)$ and $n^*(\Phi)$, when they are jointly determined in equilibrium.

Proposition 9 *When both the profit sharing rule and the portfolio size are endogenous, social welfare maximization results in a higher number of firms and larger entrepreneurs' profit shares than those that maximize VC's value.*

This result is illustrated in Figure 6.

Insert Figure 6 here

The dotted lines represent the optimal profit share given the portfolio size, and the optimal portfolio size given the profit sharing rule for the case of VC's maximization problem. The equilibrium is obtained at the intersection of the two dotted lines, denoted Eq_{VC}. (There are two points at which the solid lines intersect, but only the upper intersection, denoted Eq_{VC}, constitutes a stable equilibrium, as shown in the proof to Proposition 9.) The solid lines represent the optimal profit share given the portfolio size, and the optimal number of portfolio firms given the sharing rule when social welfare is maximized. The stable equilibrium for this case is denoted Eq_W. Moving from the equilibrium that maximizes VC's utility to the social welfare equilibrium results in both a higher number of portfolio firms and a larger share of ventures' values given to entrepreneurs.

5 Empirical tests

Propositions 4 and 5, discussing the reduced-form relations between the optimal number of portfolio firms, and entrepreneurs' optimal profit shares respectively, and exogenous variables, assume that one of the two choice variables in the first stage of the model is held constant. Barclay, Marx and Smith (2003) show that a theory implies monotonic comparative statics, providing unambiguously signed predictions for the reduced-form regression coefficients, if and only if the theory implies that the direct effect of a change in an exogenous variable on an endogenous variable and the indirect effect (through another endogenous variable) are reinforcing.

As demonstrated in Propositions 6 and 7, this is not the case in our model. Therefore, the relation between the portfolio size, the profit sharing rule, and the exogenous factors can not be tested in a reduced form. A proper testing of our model's predictions would involve an estimation of the following system of two equations using an instrumental variables approach:

$$x^* = \zeta_0 + \zeta_1\alpha + \zeta_2\beta + \zeta_3\gamma + \zeta_4\delta + \zeta_5 n + \overline{\zeta_6\Psi} + \epsilon_x, \tag{20}$$

$$n^* = \eta_0 + \eta_1\alpha + \eta_2\gamma + \eta_3\delta + \eta_4 R + \eta_5 I + \eta_6 f(x) + \overline{\eta_7}\overline{\Psi} + \epsilon_n, \tag{21}$$

where $\overline{\Psi}$ is a vector of factors that may affect the VC's optimal portfolio structure, but were omitted from the model, such as staging, syndication, voting rights, and control rights, among others,[28] and $f(x)$ is a non-linear function of entrepreneurs' profit share, that is expected to capture the non-monotonicity of the relation between n^* and x.[29]

[28] While these factors may be determined endogenously, the theory is not rich enough to specify a structural model, where all VC's choice variables are endogenous. Thus, $\overline{\Psi}$ is simply included in the right-hand side of (20) and (21).

[29] Since I and R do not appear in equation (20), while β does not appear in equation (21), the

The predictions of our model regarding the signs of coefficients $\widehat{\zeta}_i$ and $\widehat{\eta}_i$ are summarized in Table 1. Column 1 presents the signs of the predicted relations between n^*, x, and exogenous factors. Column 2 shows how x^* is expected to depend on n and exogenous factors. For comparative purposes, in parentheses we provide the reduced-form relations obtained in a setup in which x is omitted from the n^* equation, and n is omitted from the x^* equation, as discussed in the previous section.

Insert Table 1 here

This section provides some tests of the predictions of the model developed in the previous section. We make use of a newly constructed dataset, which we describe in Subsection 5.1. Subsection 5.2 provides summary statistics for the variables of interest. Multivariate empirical tests controlling for the endogeneity of the number of portfolio firms and entrepreneurs' profit shares, and their mapping to the theory are presented in Subsection 5.3.

5.1 Data source

The data were obtained from a survey of and follow-up phone interviews with venture capital and private equity funds, hereafter referred to as VC funds, in Europe and North America. The survey and the interviews were conducted in Spring and Summer of 2004. We were able to obtain a complete set of reliable, private, and confidential data from 42 limited partnership VC funds from Canada (2 funds), Czech Republic (1 fund), Denmark (1 fund), France (3 funds), Germany (5 funds), Israel (2 funds), Italy (1 fund), the Netherlands (1 fund), Switzerland (1 fund), the U.K. (3 funds), and the U.S. (22 funds).

These 42 funds have financed a total of 668 entrepreneurial firms by June 2004. Some of the funds made investments in later stage and buyout investments, as dis-

system is identified. Another possible way to achieve identification, while including I, R, and β in both equations (20) and (21), is to omit some of the elements of $\overline{\Psi}$ from (20) and (21).

cussed in Subsection 5.2 below. We control for stages of investment in our analysis. We do not exclude later stage focused funds from the dataset, since all funds indicated that they would consider investments at all investment stages (subject to the quality of the investment, regardless of the stated focus of the fund).

Two reasons motivated us to obtain data from a diverse set of countries in Europe and North America. First, there does not exist any international evidence on VC portfolio sizes. Second, by considering a diverse set of countries, we avoid any bias that might be attributable to any single country's legal or institutional structures that could affect the VC industry in that country.[30] Our data comprise very specific details on each investment from each VC fund that allow us to test the relations, predicted by the model. The rich details we have gathered from the funds go far beyond those available from VC associations.

The funds in our data almost invariably have the typical finite horizon of 10 years, with the option to continue for 2-3 years. One of the funds, however, was an exception in that it was structured as an open-ended fund without a finite horizon. Our results are robust the inclusion/exclusion of this fund in the data. The scope of the data is broadly similar to other venture capital datasets. For example, Lerner and Schoar (2004) make use of data from 208 transactions by 23 private equity funds. Kaplan and Strömberg (2003) use a similar scope of data.

As discussed in Lerner and Schoar (2004), it is difficult to ascertain the representativeness of the data in view of the lack of completeness of any international private equity or venture capital dataset. However, it is important to note that the response rate to our survey was low. Specifically, three rounds of surveys were e-mailed to over 8,000 funds, indicating a response rate of about 0.5%. Because of the diverse char-

[30] At the time this paper was prepared, there exists only one empirical study of VC portfolio sizes (Cumming, 2004), and that study is based on a Canadian-only sample. The Canadian VC market is dominated by government sponsored funds, which could distort the results of examining VCs' portfolio structures.

acteristics of respondent funds, and because the summary statistics discussed below are generally in line with other empirical VC studies, we do not think that our results are influenced by a self-selection bias. A low number of observations precludes us, however, from obtaining results that are highly statistically significant.

5.2 Summary statistics

We group the variables in the analysis into three broad categories: (1) entrepreneurs' ownership percentage and VCs' portfolio sizes,[31] (2) proxies for factors that are predicted to affect portfolio size and profit sharing, and (3) control variables. Group (1) encompasses the endogenous variables in the model, and groups (2) - (3) contain the explanatory variables. Table 2 provides definitions of the variables and basic summary statistics.[32]

Insert Table 2 here

The median number of entrepreneurial firms per VC fund (NUM_FIRMS) is 9.5, and the mean is 15.9. VC funds employed a median of 5 fund managers (FND_MGRS). The (unreported) median number of entrepreneurial firms per VC fund manager was 2.0, and the mean was 3.1. Our summary statistics on VCs' portfolio sizes are similar to those reported in Cumming (2004), once the number of fund managers is accounted for.

[31] Our data encompass many other "choice" variables, including, but not limited to, different control rights, voting rights, and security design choices. In the empirical analysis, we do consider accounting for the endogeneity of other factors, but this more elaborate approach does not materially affect the results of interest. Additional statistics on these other variables, and other regression specifications, are available upon request.

[32] To conserve space, we do not report the correlation matrix between the variables used in the analysis. In general, the correlations among most explanatory variables are relatively low in absolute value, which reduces the likelihood of the results being influenced by multicollinearity.

The profit share of entrepreneurs ($OWNER\_ENT$) typically varies over the life of the investment, subject to entrepreneurs' performance (see, for example Kaplan and Strömberg (2003)). In gathering the data, we therefore asked VC fund managers to indicate the typical ownership percentage held in their investee companies (given a median performance outcome of the entrepreneurial firm). Median entrepreneurs' ownership percentage is 80% in our sample, and the mean is 70.26%. These numbers are somewhat higher than the typical ownership percentage reported by Kaplan and Strömberg (2003), whose study indicates that U.S. VCs typically hold about 50% equity in the investee companies. For U.S. VCs in our sample, the average entrepreneurs' ownership is 67.64%, and the median is 80%.[33]

We use two proxies for entrepreneurs' quality, α. The first one is a subjective assessment of VCs of entrepreneurs' commitment to their ventures (ENT_COMMIT). On the scale of 1 to 10, median commitment is 9, and the mean is 8.86. The second proxy for α is the number of years of post-high school education of entrepreneurs (ENT_EDU). Entrepreneurs' education can also serve as an inverse proxy for their disutility of effort. We expect both these variables to be positively associated with both the number of portfolio firms and entrepreneurs' profit shares. Another proxy for entrepreneurs' cost of effort, γ, is the average age of entrepreneurs (ENT_AGE). It is expected to be negatively related to both choice variables in the model. A typical entrepreneur in our sample is 44 years old.

We proxy for VC fund managers' quality, β, by the number of year of their post-high school education (MGR_EDU), which can also be an inverse proxy for managers' effort costs. An alternative proxy for managers' quality is the number of years of their experience in managing VC funds (MGR_EXP). These proxies are predicted

[33] Our sample does not include a direct control for the typical ownership percentage of syndicated investors. For firms with reliable data on syndicated investors' ownership, it is positively correlated with, and typically about one half of the typical ownership percentage of the VCs. This limitation for some of the funds in our sample is attributable to the inaccessibility of confidential information.

not to affect the portfolio size, and to be negatively related to entrepreneurs' profit shares. A typical manager has 7 years of post- high school education and 13 years of experience. In addition to MGR_EDU, managerial disutility (cost) of effort, δ, can be (inversely) proxied by managers' involvement in the ventures (MGR_INVO), measured by weekly hours spent assisting a typical entrepreneurial firm. This variable is expected to be positively related to portfolio size and to be negatively related to entrepreneurs' profit shares. The average required investment in a project, I, is proxied by the average capital invested in a venture (CAP_INV) as of June 2004. We expect CAP_INV to be negatively associated with the number of portfolio firms. The expected return in case the project is successful, R, is proxied by the VCs' perception of the proportion of projects with expected IRR above 100% (IRR_100). IRR_100 is expected to be positively related to portfolio size.

Our control variables are as follows. First, we expect the number of firms to be positively related to the total capital raised by a fund (CAP_RAISED). Second, the number of firms is expected to be positively related to the total investment duration from the date of first investment to June 2004 (DUR). Third, we expect it to be affected by the amount of government guarantees for failed investments ($CGOVT$), both because of a potential negative effect of government guarantees on VCs' effort level choices, and because of the potentially different maximization function of a fund (see, for example, Keuschnigg (2004), Keuschnigg and Nielsen (2003), and Lerner (1999)). As our analysis of welfare maximization demonstrates, maximizing social welfare as opposed to VC's utility results in a larger equilibrium portfolio and larger entrepreneurs' profit shares. Thus, government sponsorship has two contrasting effects on the portfolio structure.[34] In addition, we expect the profit sharing rule to

[34] None of the funds in our sample were "pure" government funds with 100% government support; however, 11 funds did receive some capital from government bodies, and 2 of the funds received more than 50% of their capital (one received 60% and the other 70%) from government sources in our sample.

be affected by the stage of the fund ($SEED$, $EARLY$, AVG_STG). About 7% of firms in our sample are in the seed stage, while 22% of the firms are in the early stage. The average number of stages of a firm in a fund is 2.3. We also expect entrepreneurs' profit shares to be negatively related to the number of financing rounds ($FINANCE$). The average VC fund managers' assessment of risk of their entrepreneurial ventures (AVG_RISK), measured on a scale of 1 to 10, is also expected to affect profit sharing. In addition, we expect the percent of firms in which a VC is the lead investor ($PERCENT\_LEAD$) to be positively (negatively) associated with VC's (entrepreneurs') profit shares. Finally, since our data comes from different countries, we control for the institutional environment in their country by the "legality index" ($LEGALITY$). The legality index, based on Berkowitz, Pistor and Richard (2003), is affected by the following factors: civil versus common law systems, efficiency of judicial system, rule of law, corruption, risk of expropriation, risk of contract repudiation, and shareholder rights.[35,36]

5.3 Multivariate empirical analysis

The model developed in the previous section demonstrated that it is impossible to test the relation between portfolio size and various exogenous factors without accounting for the endogeneity of the profit sharing rule. Thus, we estimate regressions of VCs' portfolio sizes using 2SLS, instrumenting for entrepreneurs' profit shares by their pre-

[35] We also considered GNP per capita and country dummy variables as controls in our multivariate tests. However, GNP per capita is highly correlated with $LEGALITY$ (the correlation coefficient is 0.85), and country dummy variables are generally insignificant and immaterial to the results of interest.

[36] Our data encompass a plethora of other fund characteristics, such as fund covenants, fixed management fees (as in Gompers and Lerner (1996, 1999a)), the type of advice provided to entrepreneurs (marketing, financial, strategic, administrative), and controls for different industries, among other things. However, we do not find these variables to be material to our empirical analysis. Additional details are available upon request.

dicted values from first-stage regressions, and estimate regressions of entrepreneurs' profit shares, instrumenting for VCs' portfolio sizes by their predicted values from first-stage regressions. (The first-stage regressions include most of the variables that are expected to affect portfolio size and profit sharing. The results of these regressions are not reported, but are available upon request.) Table 3 presents the results of the second-stage regressions of the number of portfolio firms, according to (21).

Insert Table 3 here

The model predicts a non-monotonic relation between entrepreneurs' profit shares, x, and the number of portfolio firms, n^*. n^* is expected to be increasing in x when x is low, while it is expected to be decreasing in x when it is high. We use two ways of modelling this non-monotonicity, in order to demonstrate the robustness of the results.

In Panel 1 of Table 3, we model the non-monotonicity of the relation between n^* and x by regressing the number of firms in a VC fund (NUM_FIRMS) on the predicted entrepreneurs' ownership percentage ($INSTR\_OWNER\_ENT$) and on the squared predicted entrepreneurs' ownership percentage ($INSTR\_OWNER\_ENT^2$). The coefficient on the squared ownership percentage is negative and significant at a 10% level in all five specifications in which it is included, which, coupled with the positive coefficient estimate on the predicted ownership percentage, indicates that n is increasing in x when it is low, while it is decreasing in x when x becomes high. Back-of-the-envelope calculations show that the change in the slope from positive to negative occurs when entrepreneurs' ownership percentage is between 60 and 70 percent, depending on regression specification.

The other predictions of the model are partially supported by the data. Managers' quality, as proxied by the combined number of years of education (TOT_MGR_EDU) is significantly positively related to portfolio size. (The model predicts no relation between these two variables, but this prediction is likely to be an artifact of the specific functional forms in the model). In addition, TOT_MGR_EDU can be interpreted as an inverse proxy for the costs of managerial effort. This interpretation is consistent with the model. An estimate of entrepreneurs' commitment, serving as a proxy for α, is significantly positively related to the number of firms, as predicted by the model. Also, consistent with the model, entrepreneurs' average age, which proxies for their effort costs, is negatively, albeit insignificantly related to n^*. The average capital investment in a venture is negatively and significantly related to n^* in all specifications. An estimated proportion of high IRR ventures, proxying for R, is positively, but insignificantly, related to n^*. Consistent with managers being pressured to invest funds that have been raised, n^* is positively associated with the amount of capital raised. The coefficient on government involvement is insignificant, perhaps following from its two opposite effects on portfolio size, discussed above. As expected, fund duration is positively related to the number of firms a fund invests in.

In Panels 2-4 of Table 3, we model the non-monotonicity of the relation between n^* and x by constructing a variable equalling the absolute value of the deviation of the predicted first-stage value of x from a threshold, below which the effect of increasing x on n^* is expected to be positive, and above which this effect is expected to be negative. We use 70% as the threshold value in Panel 2, 60% in panel 2 and 50% in panel 4.[37] Consistent with the model's predictions, the coefficient on the absolute deviations from thresholds are negative and significant in the vast majority of specifications in Panels 2-4, indicating that when entrepreneurs' ownership percentage is low, increasing it (reducing the absolute deviation from a threshold) would increase the number of portfolio firms, while when x is large, increasing it (increasing the deviation from a threshold) would reduce n^*. Increasing the deviation of entrepreneurs' ownership percentage from a threshold by one percentage point reduces the number of portfolio firms by a range of 0.5-0.6 firms in Panel 2 to 0.2-0.3 firms in Panel 4.

[37] These values are chosen as a compromise between the predicted threshold in the model (50%) and a typical profit share of entrepreneurs (between 70% and 80%). Other threshold values in this range provide similar results.

The associations between n^* and other variables are broadly consistent with those documented in Panel 1.

Table 4 presents the results of the second-stage regressions of entrepreneurs' profit shares on predicted first-stage portfolio sizes and control variables.

Insert Table 4 here

First, and most importantly, the percentage of entrepreneurs' ownership is positively (although in most cases insignificantly) associated with the instrument for n ($INSTR\_NUM\_FIRMS$). The relation is marginally significant in the sixth specification, where all the control variables are included. Managers' experience is negatively related to x^*, consistent with the model. The results for managers' disutility of effort, as inversely proxied by MGR_INVO, are inconsistent with the model. Managers' involvement in ventures is negatively and significantly related to x^*. Consistent with the model, the quality of entrepreneurs is positively and significantly related to x, while their effort cost, as proxied by their average age, is negatively related to their profit shares. Most control variables, such as those describing the stage of ventures and their risk are insignificant. The exception is the percentage of government subsidies. There is little evidence that legal conditions affect profit sharing.

Overall, the data provide some support for the model developed in Section 4. The strongest support is provided for the non-monotonic relation between the portfolio size and entrepreneurs' ownership percentage. The weakness of some of the other results is possibly attributable to the scope of the available data and our ability to simultaneously control for numerous factors with a limited number of observations. The main results reported above are quite robust to regression specifications and to an inclusion of other control variables.[38]

[38] Some specifications, which are not reported for reasons of brevity but are available upon request, include a simultaneous estimation of other explanatory variables (such as control rights), but these extra equations do not materially affect the main results.

6 Conclusions, limitations, and future research

6.1 Summary and conclusions

This paper contributes to the emerging literature on the optimal size of venture capitalists' (VCs') portfolios of entrepreneurial firms. We develop a model of the optimal structure of VC's portfolio in the presence of double-sided moral hazard. The VC maximizes his expected portfolio value, net of his effort costs, with respect to the number of projects he invests in, and to the share of the projects' expected values that he gives to entrepreneurs. The portfolio structure affects the unobservable effort levels of the entrepreneurs and the VC, thus influencing the value of the VC's portfolio.

We examine the effects of exogenous factors, such as VC's and entrepreneurs' qualities, their disutilities of effort, the profitability of the projects, and the required initial investment in the projects, on the optimal portfolio structure. One contribution of the paper is showing that when both the optimal VC's portfolio size and the profit share given to entrepreneurs are determined endogenously, one can not make unambiguous predictions regarding the reduced-form relations between the optimal portfolio size and most of the exogenous factors. Thus, the predictions of the model of optimal VC's portfolio size by Kanniainen and Keuschnigg (2003, 2004) are not empirically testable in a reduced-form setting. The empirical analysis of VCs' portfolio sizes has to be performed while adjusting for the endogeneity of the profit sharing rule. Our analysis results in predictions for the partial effects of the exogenous factors on VC's portfolio size, provided that one controls for the endogenous profit sharing rule. In this setting, the optimal VC's portfolio size is predicted to be positively related to the quality of the entrepreneurs and to the value of a successful project, and to be negatively related to the disutilities that the VC and the entrepreneurs have from exerting effort, and to the required initial investment in the projects. Finally, the relation between the optimal portfolio size and the profit sharing rule is predicted to be non-monotonic: the optimal number of firms first increases and then decreases with the share of the profits retained by a typical entrepreneur.

We test the predictions of the model using data collected through a survey of venture capital and private equity funds in Europe and North America. Our sample includes 42 VC funds. Consistent with the predictions of the model, we perform a multivariate analysis of VCs' portfolio structures using instrumental variables, in order to control for the endogeneity of the profit sharing rule. Consistent with the model, we find that VC's portfolio size varies non-monotonically with the profit share retained by entrepreneurs. This finding is quite robust to various specifications of tests. In addition, consistent with the model's prediction, there is a positive, although statistically insignificant, relation between entrepreneurs' profit shares and the number of firms in VCs' portfolios. However, the effects of some of the exogenous factors in the model on VCs' portfolio structures do not conform to the predictions of the model.

Given the small size of our sample of VC funds, we view the empirical results reported here as preliminary. We believe that further research, both theoretical and empirical, is warranted in order to determine whether the few inconsistencies between the model and the empirical results are an artifact of the small size of our sample, or the fact that our theory is not rich enough. With regard to this last possibility, in the closing section of this paper we suggest some avenues for future research.

6.2 Limitations and future research

This study focuses exclusively on the optimal structure of VCs' portfolios, and ignores various important aspects of VCs' activities. We focus on double-sided moral hazard, while abstracting from the information asymmetry between entrepreneurs and VCs. The extent of information asymmetry is one of the most important characteristics of the venture capital industry.

Since the focus of this paper is the optimal VCs' portfolio sizes and not the structure of the contracts between VCs and entrepreneurs, we assumed that the return on a project in a failure state is zero. This precluded us from analyzing the structure of an optimal contract between the parties. It would be interesting to incorporate contracting issues into the model, for example by allowing financing that takes a form of convertible preferred equity. Such a contract could be incorporated in our model by using the framework in Casamatta (2003), where the value of an unsuccessful project is positive, and the share of the project given to an entrepreneur depends on whether the project is successful or not.

In addition, the model ignored the empirical regularity that VCs often invest in ventures in different stages of development. Ventures in more advanced stages of development have higher success probabilities, but offer smaller returns conditional on success. This is just one example of the limitations following from assuming complete symmetry in entrepreneurs' and projects' characteristics, and focusing exclusively on symmetric equilibria.

Risk-neutrality of the entrepreneurs and the VC improved the tractability of our analysis, but is unrealistic. It is likely that one of the reasons why VCs invest in multiple entrepreneurial ventures is risk reduction through diversification. We also ignored syndication and the possibility that venture capital firms operate more than one fund. VCs frequently operate multiple funds and/or join forces and co-invest in entrepreneurial ventures. There are other simplifying assumptions. Obvious examples are zero participation constraint of the entrepreneurs, zero VC's wealth constraint, and perfectly elastic supply of projects.

Introducing information asymmetry between the VC and the entrepreneurs, and allowing for more realistic contracts between the parties seem as the two most interesting extensions of the model. It should also be possible to relax some of the model's other assumptions (such as no-syndication, one firm equals one fund restriction, zero entrepreneurs' participation constraint and VC's wealth constraint), at a cost of reducing analytical tractability.

While the model clearly has its limitations, we think that the most interesting and potentially rewarding area is a further empirical examination of the determinants of venture capitalists' portfolio structures using larger and more representative datasets of VC funds. We believe that such tests would greatly improve our understanding of the choices venture capitalists make.

A Appendix

Proof of Lemma 1

Without loss of generality, we restrict our attention to two entrepreneurs, i and j. $x_i = x_j = x$ by assumption. In addition, assume that VC chooses to devote identical efforts to both ventures: $E_i = E_j = E$. Then, the FOC's for entrepreneurs i's and j's effort level choices in (3) may be rewritten as

$$xp_{i_{e_i}}(e_i, E, \alpha, \beta)R - K_{i_{e_i}}(e_i, \delta) = 0, \tag{A.1}$$

$$xp_{j_{e_j}}(e_j, E, \alpha, \beta)R - K_{j_{e_j}}(e_j, \delta) = 0. \tag{A.2}$$

Assume $e_i = e^*$ solves (A.1). Then, since $p_{i_{e_i}}(e^*, E, \alpha, \beta) = p_{j_{e_j}}(e^*, E, \alpha, \beta)$ and $K_{i_{e_i}}(e^*, \delta) = K_{j_{e_j}}(e^*, \delta)$, $e_j = e^*$ also solves (A.2). Therefore, $e_i^*(E) = e_j^*(E)$, implying that if the VC devotes the same effort level to each project, then the optimal effort levels of entrepreneurs are identical.

Now, we relax the assumption of VC devoting the same effort to each venture, and assume instead that the two entrepreneurs choose identical effort levels: $e_i = e_j = e$. Then, the two FOC's of VC in (4) may be rewritten as

$$[1 - x]\, p_{i_{E_i}}(e, E_i, \alpha, \beta)R - L_{i_{E_i}}(E_i, E_j, n, \delta) = 0, \tag{A.3}$$

$$[1 - x]\, p_{i_{E_j}}(e, E_j, \alpha, \beta)R - L_{i_{E_j}}(E_i, E_j, n, \delta) = 0. \tag{A.4}$$

Assume that $E_i = E^*$ solves (A.3). Then, similar to the arguments above, $E_j = E^*$ also solves (A.4). Thus, $E_i^*(e) = E_j^*(e)$, meaning that if the two entrepreneurs choose identical effort levels, then the VC chooses to devote identical efforts to the two ventures.

Given the symmetric equilibrium above, the FOC's of entrepreneurs and VC in (3) and (4) respectively may be rewritten as

$$xp_e(e, E, \alpha, \beta)R - K_e(e, \delta) = 0, \tag{A.5}$$

and

$$[1 - x]\, p_E(e, E, \alpha, \beta)R - L_E(E, n, \delta) = 0. \tag{A.6}$$

Differentiating the FOC in (A.5) with respect to E results in

$$\frac{de^*(E)}{dE} = -\frac{xp_{i_{eE}}R}{xp_{i_{ee}}R - K_{i_{ee}}}.$$

Given the assumptions on the derivatives, this implies that $\frac{de^*(E)}{dE} \geq 0$. Similarly, differentiating the FOC in (A.6) with respect to e results in $\frac{dE^*(e)}{de} \geq 0$. In addition, it follows from the assumptions that $e^*(E) > 0$ when $E \to 0$. Also, $p_{i_{ee}} + p_{i_{eE}} < 0$ implies $\frac{de^*(E)}{dE} < 1$. Similarly, $E^*(e) > 0$ when $e \to 0$, and $\frac{dE^*(e)}{de} < 1$. Therefore, the reaction functions $e^*(E)$ and $E^*(e)$ intersect at least once, and there is a symmetric equilibrium in effort levels where the two equations

$$e^* = \arg\max_e U_{en}(e, E^*, \alpha, \beta, \delta),$$

$$E^* = \arg\max_E U_{vc}(e^*, E, \alpha, \beta, \gamma)$$

hold simultaneously. ■

Proof of Proposition 1

Totally differentiating the FOC's in (5) and (6) with respect to n, while holding x constant, gives the following system of two equations:

$$p_{ee}xR\frac{de^*(E)}{dn} + p_{eE}xR\frac{dE^*(e)}{dn} - K_{ee}\frac{de^*(E)}{dn} = 0, \quad \text{(A.7)}$$

$$p_{EE}[1-x]R\frac{dE^*(e)}{dn} + p_{eE}[1-x]R\frac{de^*(E)}{dn} - L_{En} - L_{EE}\frac{dE^*(e)}{dn} = 0. \quad \text{(A.8)}$$

Solving the system of (A.7) and (A.8), while substituting in the FOC's in (5) and (6) results in

$$\frac{dE^*}{dn} = -\frac{[K_{ee} - p_{ee}xR](L_{En})}{\Gamma},$$

$$\frac{de^*}{dn} = -\frac{p_{eE}xR(L_{En})}{\Gamma},$$

where

$$\Gamma = [L_{EE} - p_{EE}(1-x)R][K_{ee} - p_{ee}xR] - [1-x]xR^2p_{eE}^2. \quad \text{(A.9)}$$

Given the assumptions about the derivatives, it is straightforward to show that $\Gamma > 0$ and that $\frac{dE^*}{dn} < 0$. Moreover, if there are no complementarities ($p_{eE} = 0$), then it is clear that $\frac{de^*}{dn} = 0$, while, if $p_{eE} > 0$, then, given the assumptions about the derivatives, it follows that $\frac{de^*}{dn} < 0$. ■

Proof of Proposition 2

Totally differentiating the FOC's in (5) and (6) with respect to x, while holding n constant, provides the following system of equations:

$$p_e R + p_{ee} x R \frac{de^*(E)}{dx} + p_{eE} x R \frac{dE^*(e)}{dx} - K_{ee} \frac{de^*(E)}{dx} = 0, \tag{A.10}$$

$$-p_E R + p_{EE}[1-x] R \frac{dE^*(e)}{dx} + p_{eE}[1-x] R \frac{de^*(E)}{dx} - L_{EE} \frac{dE^*(e)}{dx} = 0. \tag{A.11}$$

Solving the system in (A.10) and (A.11), while substituting in the FOC's in (5) and (6) gives

$$\frac{dE^*}{dx} = \frac{\Delta}{\Gamma},$$
$$\frac{de^*}{dx} = \frac{\Theta}{\Gamma},$$

where Γ is defined in (A.9) and was shown to be positive, and

$$\Delta = R[K_e p_{eE} \frac{[1-x]}{x} - p_E[K_{ee} - p_{ee} x R]],$$
$$\Theta = -R[L_E p_{eE} \frac{x}{[1-x]} - p_e[L_{EE} - p_{EE}[1-x]R]].$$

If there is no complementarity between the effort of the entrepreneur and that of the VC ($p_{eE} = 0$), then, given the assumptions about the derivatives, it is easy to show that Θ is strictly positive, while Δ is strictly negative, which proves the first part of this proposition.

If, instead, there are complementarities ($p_{eE} > 0$), using the assumptions about the derivatives, it is straightforward to show that

$$\begin{aligned}
\lim_{x\to 0}\Delta &= \infty, \\
\lim_{x\to 0}\Theta &= Rp_e[L_{EE} - p_{EE}R] > 0, \\
\lim_{x\to 1}\Delta &= -Rp_E[K_{ee} - p_{ee}R] < 0, \\
\lim_{x\to 1}\Theta &= -\infty.
\end{aligned}$$

It follows that

$$\begin{aligned}
\lim_{x\to 0}\frac{dE^*}{dx} &> 0, \\
\lim_{x\to 1}\frac{dE^*}{dx} &< 0, \\
\lim_{x\to 0}\frac{de^*}{dx} &> 0, \\
\lim_{x\to 1}\frac{de^*}{dx} &< 0.
\end{aligned}$$

which proves the second part of the proposition. ■

Proof of Proposition 3

Totally differentiating the FOC's in (5) and (6) with respect to α, while holding n and x constant, yields the following system of equations:

$$p_{ee}xR\frac{de^*(E)}{d\alpha} + p_{eE}xR\frac{dE^*(e)}{d\alpha} + p_{e\alpha}xR - K_{ee}\frac{de^*(E)}{d\alpha} = 0, \quad \text{(A.12)}$$

$$p_{EE}[1-x]R\frac{dE^*(e)}{d\alpha} + p_{eE}[1-x]R\frac{de^*(E)}{d\alpha} + p_{E\alpha}[1-x]R - L_{EE}\frac{dE^*(e)}{d\alpha} = 0. \quad \text{(A.13)}$$

Solving the system in (A.12) and (A.13), while substituting in the FOC's in (5) and (6) gives

$$\begin{aligned}
\frac{dE^*}{d\alpha} &= \frac{(1-x)R}{\Gamma}[p_{E\alpha}[K_{ee} - p_{ee}xR] + p_{e\alpha}p_{eE}xR], \\
\frac{de^*}{d\alpha} &= \frac{xR}{\Gamma}[p_{e\alpha}[L_{EE} - p_{EE}[1-x]R] + p_{E\alpha}p_{eE}[1-x]R],
\end{aligned}$$

where Γ is defined in (A.9) and was shown to be positive. Using the assumptions about the derivatives, it is straightforward to show that, for any α

$$\frac{de^*}{d\alpha} > 0.$$

In addition, if the efforts are complementary, $p_{eE} > 0$, then

$$\frac{dE^*}{d\alpha} > 0,$$

otherwise

$$\frac{dE^*}{d\alpha} = 0.$$

By following the same approach, it can be easily shown that the derivatives of E^* and e^* with respect to β, γ, δ, and R are equal to the following expressions:

$$\frac{dE^*}{d\beta} = \frac{(1-x)R}{\Gamma}[p_{E\beta}[K_{ee} - p_{ee}xR] + p_{e\beta}p_{eE}xR],$$
$$\frac{de^*}{d\beta} = \frac{xR}{\Gamma}[p_{e\beta}[L_{EE} - p_{EE}[1-x]R] + p_{E\beta}p_{eE}[1-x]R],$$

$$\frac{dE^*}{d\gamma} = -\frac{p_{eE}[1-x]R}{\Gamma}K_{e\gamma},$$
$$\frac{de^*}{d\gamma} = -\frac{[L_{EE} - p_{EE}[1-x]R]}{\Gamma}K_{e\gamma},$$

$$\frac{dE^*}{d\delta} = -\frac{[K_{ee} - p_{ee}xR]}{\Gamma}L_{E\delta},$$
$$\frac{de^*}{d\delta} = -\frac{p_{eE}xR}{\Gamma}L_{E\delta},$$

$$\frac{dE^*}{dR} = \frac{[1-x]}{\Gamma}[p_E[K_{ee} - p_{ee}xR] + p_e p_{eE}xR],$$
$$\frac{de^*}{dR} = \frac{x}{\Gamma}[p_e[L_{EE} - p_{EE}[1-x]R] + p_E p_{eE}[1-x]R].$$

Using the assumptions about the derivatives, it is easily shown that

$$\frac{dE^*}{d\beta} > 0,$$
$$\frac{de^*}{d\beta} > 0 \text{ if } p_{eE} > 0 \text{ and } \frac{de^*}{d\beta} = 0 \text{ if } p_{eE} = 0,$$
$$\frac{dE^*}{d\gamma} < 0 \text{ if } p_{eE} > 0 \text{ and } \frac{dE^*}{d\gamma} = 0 \text{ if } p_{eE} = 0,$$
$$\frac{de^*}{d\gamma} < 0,$$
$$\frac{dE^*}{d\delta} < 0,$$
$$\frac{de^*}{d\delta} < 0 \text{ if } p_{eE} > 0 \text{ and } \frac{de^*}{d\delta} = 0 \text{ if } p_{eE} = 0,$$
$$\frac{dE^*}{dR} > 0,$$
$$\frac{de^*}{dR} > 0.$$

In addition,

$$\frac{dE^*}{dI} = 0,$$
$$\frac{de^*}{dI} = 0.$$

This concludes the proof. ■

Proof of Lemma 2

Without loss of generality, we focus on two entrepreneurs, i and j. It was shown in Lemma 1 that $x_i = x_j = x$ is consistent with $e_i^*(x) = e_j^*(x)$ and $E_i^*(x) = E_j^*(x)$. We now show that, given the best responses $e^*(E^*, x)$ and $E^*(e^*, x)$, obtained in the last stage of the game, $x_i = x_j = x^*$ is indeed a symmetric equilibrium. Given the best responses e_i^* and e_j^*, maximizing U_{VC} with respect to x_i and x_j, yields the following 2 FOC's:

$$p_{i_E}[1 - x_i]R\frac{dE_i^*(e_i)}{dx_i} + p_{i_{e_i}}[1 - x_i]R\frac{de_i^*(E_i)}{dx_i} - L_{E_i} - p_i R = 0, \tag{A.14}$$

$$p_{j_E}[1 - x_j]R\frac{dE_j^*(e_j)}{dx_j} + p_{j_{e_j}}[1 - x_j]R\frac{de_j^*(E_j)}{dx_j} - L_{E_j} - p_j R = 0. \tag{A.15}$$

Due to the symmetry in effort levels, if x^* solves (A.14), then it must solve (A.15) as well, which proves the claim. ■

Proof of Proposition 4

Differentiating $n^*(x, \Phi)$ in (15) with respect to x results in

$$\frac{\partial n^*(x,\Phi)}{\partial x} = \frac{\alpha^2 R^2 [1-2x]}{4\gamma\delta\overline{p}^2}.$$

For $x < \frac{1}{2}$

$$\frac{\partial n^*(x,\Phi)}{\partial x} > 0,$$

and for $x > \frac{1}{2}$,

$$\frac{\partial n^*(x,\Phi)}{\partial x} < 0.$$

Moreover, partially differentiating $n^*(x, \Phi)$ in (15) with respect to α, β, γ, δ, R, and I respectively gives

$$\frac{\partial n^*(x,\Phi)}{\partial \alpha} = \frac{\alpha R^2 x [1-x]}{2\gamma\delta\overline{p}^2} > 0,$$
$$\frac{\partial n^*(x,\Phi)}{\partial \beta} = 0,$$
$$\frac{\partial n^*(x,\Phi)}{\partial \gamma} = -\frac{\alpha^2 R^2 x [1-x]}{4\gamma^2\delta\overline{p}^2} < 0,$$
$$\frac{\partial n^*(x,\Phi)}{\partial \delta} = -\frac{1}{\delta}\left[\frac{1}{4}\frac{[1-x]\, xR^2\alpha^2}{\gamma\delta\overline{p}^2} - \frac{1}{2}\frac{I}{\delta}\right] = -\frac{1}{\delta}n^*(x,\Phi) < 0,$$
$$\frac{\partial n^*(x,\Phi)}{\partial R} = \frac{\alpha^2 R x [1-x]}{2\gamma\delta\overline{p}^2} > 0,$$
$$\frac{\partial n^*(x,\Phi)}{\partial I} = -\frac{1}{2\delta} < 0.$$

This concludes the proof. ■

Proof of Proposition 5

Partially differentiating the expression for $x^*(n, \Phi)$ in (16) with respect to n results in

$$\frac{\partial x^*(n,\Phi)}{\partial n} = \frac{\alpha^2\beta^2\gamma\delta}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} > 0.$$

Furthermore, partially differentiating $x^*(n,\Phi)$ in (16) with respect to α, β, γ, and δ gives

$$\begin{aligned}
\frac{\partial x^*(n,\Phi)}{\partial \alpha} &= \frac{2\alpha\beta^2\gamma\delta n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} > 0,\\
\frac{\partial x^*(n,\Phi)}{\partial \beta} &= -\frac{2\alpha^2\beta\gamma\delta n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} < 0,\\
\frac{\partial x^*(n,\Phi)}{\partial \gamma} &= -\frac{\alpha^2\beta^2\delta n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} < 0,\\
\frac{\partial x^*(n,\Phi)}{\partial \delta} &= \frac{\alpha^2\beta^2\gamma n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} > 0.
\end{aligned}$$

Because $x^*(n,\Phi)$ does not depend on R and I, its derivative with respect to these variables is equal to zero. ■

Proof of Proposition 6

Using the envelope theorem, we can write

$$\frac{dx^*(\Phi)}{d\alpha} = \frac{\partial x^*(n,\Phi)}{\partial \alpha} + \frac{\partial x^*(n,\Phi)}{\partial n}\frac{\partial n^*(x,\Phi)}{\partial \alpha}, \tag{A.16}$$

and similarly for the case of other parameters. Using the relations derived in Propositions 4 and 5, it is straightforward to show that

$$\begin{aligned}
\frac{dx^*(\Phi)}{d\alpha} &> 0,\\
\frac{dx^*(\Phi)}{d\beta} &< 0,\\
\frac{dx^*(\Phi)}{d\gamma} &< 0,\\
\frac{dx^*(\Phi)}{dR} &> 0,\\
\frac{dx^*(\Phi)}{dI} &< 0.
\end{aligned}$$

In addition, using an expression analogous to (A.16), we can write

$$\frac{dx^*(\Phi)}{d\delta} = \frac{\alpha^2\beta^2\gamma n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} + \frac{\alpha^2\beta^2\gamma\delta}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2}\left[-\frac{1}{\delta}n\right] = 0.$$

This concludes the proof. ■

Proof of Proposition 7

Using the envelope theorem and the results in Propositions 4 and 5, we can write

$$\frac{dn^*(\Phi)}{d\alpha} = \frac{\alpha R^2 x\,[1-x]}{2\gamma\delta\overline{p}^2} + \frac{\alpha^2 R^2\,[1-2x]}{4\gamma\delta\overline{p}^2}\frac{2\alpha\beta^2\gamma\delta n}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2}. \tag{A.17}$$

For $x \to 0$, (A.17) is positive. For $x \to 1$, (A.17) is negative. Similarly, for $x \to 0$,

$$\begin{aligned} \frac{dn^*(\Phi)}{d\beta} &< 0, \\ \frac{dn^*(\Phi)}{d\gamma} &< 0, \\ \frac{dn^*(\Phi)}{d\delta} &> 0, \end{aligned}$$

while for $x \to 1$,

$$\begin{aligned} \frac{dn^*(\Phi)}{d\beta} &> 0, \\ \frac{dn^*(\Phi)}{d\gamma} &> 0, \\ \frac{dn^*(\Phi)}{d\delta} &< 0. \end{aligned}$$

Thus the relations between $n^*(\Phi)$ and α, β, γ, and δ can not be signed unambiguously. For the cases of R and I, we use Proposition 4 to write

$$\begin{aligned} \frac{dn^*(\Phi)}{dR} &= \frac{\partial n^*(x,\Phi)}{\partial R} > 0, \\ \frac{dn^*(\Phi)}{dI} &= \frac{\partial n^*(x,\Phi)}{\partial I} < 0. \end{aligned}$$

This concludes the proof. ■

Proof of Proposition 8

Subtracting the optimal number of portfolio firms for the case of VC's utility maximization in (15) from that for the case of social welfare maximization in (19) results in

$$n_W^*(x,\Phi) - n^*(x,\Phi) = \frac{\alpha^2 R^2 x^2}{8\gamma\delta\overline{p}^2} > 0.$$

Subtracting the optimal entrepreneurs' profit share for the case of VC's utility maximization in (16) from that for the case of social welfare maximization in (18) gives

$$x_W^*(n,\Phi) - x^*(n,\Phi) = \frac{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2 + \alpha^2\delta n\beta^2\gamma}{\left[2\alpha^2\delta n - \beta^2\gamma\right]\left[\alpha^2\delta n + \beta^2\gamma\right]} > 0.$$

This concludes the proof. ■

Proof of Proposition 9

First, we will establish that the stable equilibrium portfolio size and entrepreneurs' profit shares is given by the right-most intersection between $n^*(x,\Phi)$ and $x^*(n,\Phi)$ for the case of VC's utility maximization and the right-most intersection between $n_W^*(x,\Phi)$ and $x_W^*(n,\Phi)$ for the case of social welfare maximization. The stable equilibrium in both case is such that the slope of $n^*(x,\Phi)$ ($n_W^*(x,\Phi)$) is higher than the slope of $x^*(n,\Phi)$ ($x_W^*(n,\Phi)$). Twice differentiating $n^*(x,\Phi)$ with respect to x gives

$$\frac{\partial n^*(x,\Phi)}{\partial x} = \frac{[1-2x]\,R^2\alpha^2}{4\gamma\delta\overline{p}^2},$$
$$\frac{\partial^2 n^*(x,\Phi)}{\partial x^2} = -\frac{R^2\alpha^2}{2\gamma\delta\overline{p}^2} < 0.$$

Twice differentiating $x^*(n,\Phi)$ with respect to n results in

$$\frac{\partial x^*(n,\Phi)}{\partial n} = \frac{\alpha^2\beta^2\gamma\delta}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} > 0$$
$$\frac{\partial^2 x^*(n,\Phi)}{\partial n^2} = -\frac{4\alpha^4\beta^2\gamma^2\delta}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^3} < 0.$$

Thus, both $n^*(x,\Phi)$ and $x^*(n,\Phi)$ are concave in their respective arguments. Thus, the intersection between $n^*(x,\Phi)$ and $x^*(n,\Phi)$ for which the slope of $n^*(x,\Phi)$ is higher than the slope of $x^*(n,\Phi)$ is the one the right one in Figure 6 (denoted by Eq_{VC}), and it constitutes the stable equilibrium for the case of VC's utility maximization.

Similarly, twice differentiating $n_W^*(x,\Phi)$ with respect to x and $x_W^*(n,\Phi)$ with respect to n gives

$$\frac{\partial n_W^*(x,\Phi)}{\partial x} = \frac{[1-x]\,R^2\alpha^2}{4\gamma\delta\overline{p}^2} > 0,$$
$$\frac{\partial^2 n_W^*(x,\Phi)}{\partial x^2} = -\frac{R^2\alpha^2}{4\gamma\delta\overline{p}^2} < 0,$$
$$\frac{\partial x_W^*(n,\Phi)}{\partial n} = \frac{\alpha^2\beta^2\gamma\delta}{\left[\alpha^2\delta n+\beta^2\gamma\right]^2} > 0,$$
$$\frac{\partial^2 x_W^*(n,\Phi)}{\partial n^2} = -\frac{2\alpha^4\beta^2\gamma^2\delta}{\left[\alpha^2\delta n+\beta^2\gamma\right]^3} < 0.$$

Implying that the stable equilibrium for the case of social welfare maximization is the right intersection between $n^*(x,\Phi)$ and $x^*(n,\Phi)$

The difference between the optimal portfolio size for the case of social welfare maximization problem and that for the case of VC's utility maximization problem, Δx^*, can be expressed as

$$\Delta x^* = x_W^*(\Phi) - x^*(\Phi) = [x_W^*(n^*,\Phi) - x^*(\Phi)] + \int_{n^*(\Phi)}^{n_W^*(\Phi)} \frac{\partial x_W^*(n^*,\Phi)}{\partial n} dn, \qquad \text{(A.18)}$$

and the difference between the optimal entrepreneurs' profit share for the case of social welfare maximization problem and that for the case of VC's utility maximization problem, Δn^*, can be expressed as

$$\Delta n^* = n_W^*(\Phi) - n^*(\Phi) = [n_W^*(x^*,\Phi) - n^*(\Phi)] + \int_{x^*(\Phi)}^{x_W^*(\Phi)} \frac{\partial n_W^*(x^*,\Phi)}{\partial x} dx. \qquad \text{(A.19)}$$

First, note that Δx^* and Δn^* have to be of the same sign. (Assume $\Delta n^* > 0$. Then, it follows from (A.18) that Δx^* can not be negative, since for a given n, $x_W^*(n^*,\Phi) > x^*(n^*,\Phi)$, and $\frac{\partial x_W^*(n,\Phi)}{\partial n} > 0$. Similarly, assume that $\Delta x^* > 0$. Then, it follows from (A.19) that Δn^* can not be negative). Combining this with the fact that the stable equilibria for both the case of social welfare maximization and the case of VC's utility maximization are the right-most intersections in Figure 6, leads to the conclusion that $\Delta x^* > 0$ and $\Delta n^* > 0$. ■

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Table 1 – Empirical predictions for the system of equations

This table summarizes the empirical predictions for the signs for the partial effects of the exogenous variables on the optimal VC's portfolio size (n^*) when one controls for the endogeneity of the optimal profit sharing rule (x), and the partial effects of the exogenous variables on the optimal entrepreneurs' profit shares (x^*) when one controls for the endogeneity of portfolio size, n. α and β are the quality parameters of the entrepreneurs and the VC respectively. γ and δ are the disutility of effort parameters of the entrepreneurs and the VC respectively. R is the value of a successful project, and I is the initial investment required in each project. "+" indicates a positive predicted regression coefficient, "–" indicates a negative coefficient, "0" means that the model predicts no relation between an exogenous and an endogenous variable, and "?" means that the sign of the predicted relation is ambiguous. The signs in parentheses indicate the predictions for the reduced-form relations following from the model in which x^* and n^* are jointly determined but, x is omitted from the n^* regression, and n is omitted from the x^* regression.

Equation/Variable	**n***	**x***
α	+ (?)	+ (+)
β	0 (?)	- (-)
γ	- (?)	- (-)
δ	- (?)	+ (0)
R	+ (+)	0 (+)
I	- (-)	0 (-)
x	+ for small x - for large x	
n		+

Table 2 – Summary statistics

This table presents summary statistics for the variables used in the empirical tests. *NUM_FIRMS* is the number of firms in a VC fund. *OWNER_ENT* is the typical entrepreneurs' ownership percentage, *FND_MGR* is the number of VC fund managers, *ENT_COMMIT* is a subjective measure of entrepreneurs' commitment, *ENT_EDU* is the number of years of a typical entrepreneur's post-high school education, *ENT_AGE* is a typical entrepreneurs' age, *MGR_EDU* is the average number of years of fund managers' education, *MGR_EXP* is the average number of years of managers' experience in managing VC funds, *MGR_INVO* is the average number of hours per week that a fund's managers devote to a typical venture, *CAP_INV* is the average investment in each venture (in $MM), *IRR_100* is the percentage of projects that are expected by a VC to generate IRR above 100%, *CAP_RAISED* is the total funds raised by a fund (in $MM), *DUR* is the duration of a fund, *CGOVT* is the percentage of government guarantees for failed ventures, *SEED* is the percentage of firms in the seed stage, *EARLY* is the percentage of firms in the early stage, *AVG_STG* is the average number of stages, *FINANCE* is the number of financing rounds, *AVG_RISK* is a subjective VC's assessment of the risk of a typical venture in his portfolio, *PERCENT_LEAD* is the percentage of firms in which a VC is a lead investor, and *LEGALITY* is an index of a country's legal conditions, based on Berkowitz, Pistor and Richard (2003).

Variable	Mean	Median	Min	Max	Std Dev
NUM_FIRMS	15.90	9.5	1	85	16.89
OWNER_ENT	70.26	80	0	97.5	24.02
FND_MGR	6.64	5	1	17	4.96
ENT_COMMIT	8.86	9	6	10	0.78
ENT_EDU	9.74	8.75	2.5	23	4.44
ENT_AGE	44.08	45	33	50	3.92
MGR_EDU	7.02	6	4	12	2.04
MGR_EXP	13.53	15	0	30	6.75
MGR_INVO	2.45	1	1	8	2.50
CAP_INV	11.79	4.10	0.22	103.80	20.35
IRR_100	12.08	12	0	38.10	10.26
CAP_RAISED	264.90	101.50	11	3100	496.60
DUR	5.08	3.46	1	33.94	5.93
CGOVT	5.32	0	0	70	14.78
SEED	7.55	0	0	90	17.76
EARLY	22.26	20	0	100	26.28
AVG_STG	2.30	2	0	4	0.91
FINANCE	9.36	12	4	20	3.88
AVG_RISK	5.82	6	2	10	1.76
PERCENT_LEAD	0.5836	0.58	0	1	0.2843
LEGALITY	20.31	20.85	14	21.91	1.50

Table 3 – Portfolio size regressions

This table presents the results of regressions of the number of portfolio firms, *NUM_FIRMS*, on non-linear functions of entrepreneurs' ownership percentage, and on control variables, discussed in Table 2. In Panel 2, the regressions include predicted first-stage values of entrepreneurs' ownership percentages, *INSTR_OWN_ENT*, and squared predicted values of ownership percentages, $INSTR\_OWN\_ENT^2$. Panels 2-4 include absolute deviations of entrepreneurs' ownership percentage from 70% (|*INSTR_OWN_ENT-70%*| in Panel 2), 60% (|*INSTR_OWN_ENT-60%*| in Panel 3), and 50% (|*INSTR_OWN_ENT-50%*| in Panel 4). ***, **, * indicate statistical significance at a 1%, 5%, and 10% levels respectively.

Panel 1 – Regressions with squared entrepreneurs' ownership percentage

Independent Variable	(1) Coeff. (Std.Err)	(2) Coeff. (Std.Err)	(3) Coeff. (Std.Err)	(4) Coeff. (Std.Err)	(5) Coeff. (Std.Err)	(6) Coeff. (Std.Err)
Intercept	22.073*	-83.441	-73.215	-75.771	-73.462	-78.442
	(12.922)	(56.908)	(55.799)	(61.507)	(56.244)	(61.809)
INSTR_OWN_ENT	-0.088	3.072*	2.091*	2.116*	1.944*	1.991*
	(0.180)	(1.672)	(1.203)	(1.243)	(1.113)	(1.122)
$INSTR\_OWN\_ENT^2$		-0.023*	-0.016*	-0.017*	-0.015*	-0.015*
		(0.012)	(0.009)	(0.009)	(0.008)	(0.009)
FND_MGR				-0.102		-0.196
				(0.952)		(0.930)
TOT_MGR_EDU			0.247***	0.258**	0.242***	0.263**
			(0.052)	(0.117)	(0.055)	(0.116)
ENT_COMMIT			4.973*	5.077*	4.254*	4.451*
			(2.584)	(2.798)	(2.563)	(2.666)
ENT_AGE			-0.638	-0.619	-0.502	-0.464
			(0.537)	(0.573)	(0.539)	(0.576)
CAP_INV			-0.593***	-0.584***	-0.543***	-0.525***
			(0.158)	(0.181)	(0.157)	(0.180)
IRR_100			0.121	0.128	0.140	0.153
			(0.194)	(0.207)	(0.195)	(0.208)
CAP_RAISED			0.020***	0.020***	0.018***	0.018***
			(0.007)	(0.007)	(0.006)	(0.007)
DUR					0.593	0.596
					(0.383)	(0.389)
CGOVT					0.011	0.012
					(0.164)	(0.166)
N	42	42	42	42	42	42
$Adj\text{-}R^2$	-0.019	0.044	0.528	0.513	0.550	0.536

Panel 2 – Regressions with absolute deviation of ownership percentage from 70%

Independent Variable	(1) Coeff. (Std.Err)	(2) Coeff. (Std.Err)	(3) Coeff. (Std.Err)	(4) Coeff. (Std.Err)
Intercept	-26.269	-26.802	-25.503	-27.889
	(29.065)	(33.533)	(28.303)	(32.574)
|INST_OWN_ENT-70%|	-0.606**	-0.608**	-0.545**	-0.550**
	(0.260)	(0.267)	(0.253)	(0.259)
FND_MGR		-0.031		-0.141
		(0.941)		(0.906)
TOT_MGR_EDU	0.254	0.258**	0.254	0.269**
	(0.051)	(0.115)	(0.053)	(0.113)
ENT_COMMIT	5.271**	5.304*	4.515*	4.658*
	(2.576)	(2.790)	(2.518)	(2.717)
ENT_AGE	-0.197	-0.191	-0.159	-0.132
	(0.488)	(0.526)	(0.470)	(0.508)
CAP_INV	-0.542***	-0.540***	-0.502***	-0.490***
	(0.155)	(0.177)	(0.150)	(0.172)
IRR_100	0.138	0.140	0.158	0.167
	(0.191)	(0.203)	(0.186)	(0.198)
CAP_RAISED	0.019***	0.019***	0.017***	0.017***
	(0.006)	(0.007)	(0.006)	(0.006)
DUR			0.552	0.554
			(0.379)	(0.385)
CGOVT			0.072	0.072
			(0.154)	(0.157)
N	42	42	42	42
Adj-R^2	0.529	0.515	0.565	0.552

Panel 3 – Regressions with absolute deviation of ownership percentage from 60%

Independent Variable	(1) Coeff. (Std.Err)	(2) Coeff. (Std.Err)	(3) Coeff. (Std.Err)	(4) Coeff. (Std.Err)
Intercept	-4.017	-6.672	-6.329	-9.575
	(30.834)	(34.315)	(30.637)	(33.982)
|INST_OWN_ENT-60%|	-0.409**	-0.416**	-0.326*	-0.334*
	(0.181)	(0.187)	(0.182)	(0.189)
FND_MGR		-0.180		-0.222
		(0.956)		(0.935)
TOT_MGR_EDU	0.240	0.260**	0.236***	0.260**
	(0.052)	(0.116)	(0.055)	(0.115)
ENT_COMMIT	5.127**	5.311*	4.382*	4.604*
	(2.587)	(2.799)	(2.569)	(2.771)
ENT_AGE	-0.696	-0.670	-0.592	-0.559
	(0.500)	(0.526)	(0.493)	(0.519)
CAP_INV	-0.638***	-0.624***	-0.584***	-0.567***
	(0.160)	(0.178)	(0.159)	(0.177)
IRR_100	0.146	0.157	0.154	0.168
	(0.191)	(0.203)	(0.190)	(0.202)
CAP_RAISED	0.021***	0.021***	0.020***	0.019***
	(0.006)	(0.007)	(0.006)	(0.007)
DUR			0.590	0.593
			(0.385)	(0.391)
CGOVT			0.012	0.011
			(0.158)	(0.160)
N	42	42	42	42
Adj-R^2	0.526	0.512	0.548	0.534

Panel 4 – Regressions with absolute deviation of ownership percentage from 50%

Independent Variable	(1) Coeff. (Std.Err)	(2) Coeff. (Std.Err)	(3) Coeff. (Std.Err)	(4) Coeff. (Std.Err)
Intercept	-0.751	1.312	-2.648	-2.303
	(33.481)	(37.238)	(34.005)	(37.561)
|INST_OWN_ENT-50%|	-0.278*	-0.276*	-0.218	-0.218
	(0.160)	(0.162)	(0.168)	(0.171)
FND_MGR		0.131		0.022
		(0.965)		(0.940)
TOT_MGR_EDU	0.246	0.232**	0.235***	0.233**
	(0.053)	(0.118)	(0.057)	(0.117)
ENT_COMMIT	5.133*	4.999*	4.274	4.252
	(2.660)	(2.877)	(2.627)	(2.830)
ENT_AGE	-0.778	-0.799	-0.659	-0.663
	(0.542)	(0.571)	(0.542)	(0.571)
CAP_INV	-0.611***	-0.622***	-0.561***	-0.563***
	(0.163)	(0.184)	(0.161)	(0.182)
IRR_100	0.132	0.124	0.135	0.133
	(0.198)	(0.210)	(0.197)	(0.209)
CAP_RAISED	0.021***	0.022***	0.020***	0.020***
	(0.007)	(0.007)	(0.007)	(0.007)
DUR			0.688*	0.688*
			(0.390)	(0.397)
CGOVT			-0.029	-0.029
			(0.169)	(0.172)
N	42	42	42	42
Adj-R^2	0.499	0.484	0.527	0.512

Table 4 – Entrepreneurs' profit shares regressions

This table presents the results of regressions of entrepreneurs' ownership percentage, $OWNER\_ENT$, on the number of portfolio firms, NUM_FIRMS, and on control variables, discussed in Table 2. ***, **, * indicate statistical significance at a 1%, 5%, and 10% levels respectively.

Independent Variable	(1) Coeff. (Std.Err)	(2) Coeff. (Std.Err)	(3) Coeff. (Std.Err)	(4) Coeff. (Std.Err)	(5) Coeff. (Std.Err)	(6) Coeff. (Std.Err)
Intercept	137.528***	135.481***	146.467***	146.457***	192.348***	189.416***
	(40.200)	(40.872)	(41.670)	(42.316)	(77.472)	(76.565)
INSTR_NUM_FIRMS	0.084	0.122	0.072	0.069	0.391	0.560
	(0.310)	(0.324)	(0.295)	(0.312)	(0.340)	(0.360)
FND_MGR		-0.466		0.038		-1.393
		(0.986)		(1.016)		(1.071)
TOTAL_MGR_EXP	-0.157**	-0.135*	-0.160***	-0.162**	-0.228***	-0.179**
	(0.067)	(0.082)	(0.065)	(0.081)	(0.072)	(0.080)
MGR_INVO	3.191**	3.026**	2.402*	2.412*	2.269*	1.874
	(1.323)	(1.382)	(1.304)	(1.349)	(1.296)	(1.315)
ENT_EDU	1.276*	1.225	1.069	1.073	0.563	0.329
	(0.738)	(0.753)	(0.712)	(0.732)	(0.781)	(0.793)
ENT_AGE	-1.728**	-1.645*	-1.653**	-1.657*	-1.552	-1.279
	(0.855)	(0.882)	(0.839)	(0.859)	(0.941)	(0.953)
SEED			-0.189	-0.188	-0.181	-0.222
			(0.187)	(0.192)	(0.188)	(0.188)
EARLY			0.226*	0.228*	0.142	0.059
			(0.122)	(0.132)	(0.130)	(0.143)
FINANCE			-1.220	-1.218	-0.755	-0.672
			(0.855)	(0.871)	(0.902)	(0.894)
AVG_RISK					2.424	2.943
					(2.552)	(2.553)
AVG_STG					-1.137	-0.424
					(4.679)	(4.655)
PERCENT_LEAD					-17.624	-20.482
					(13.279)	(13.300)
CGOVT					-0.370*	-0.479**
					(0.220)	(0.233)
LEGALITY					-2.291	-2.500
					(2.334)	(2.312)
N	42	42	42	42	42	42
Adj-R^2	0.260	0.244	0.330	0.309	0.386	0.401

Figure 1 – The effects of a change in n on the best response functions and the resulting equilibrium effort levels – the case of no complementarities

This figure presents the best response functions of the VC before an increase in n, $E_0^*(e)$, and after an increase in n, $E_1^*(e)$, and those of a typical entrepreneur before an increase in n, $e_0^*(E)$, and after an increase in n, $e_1^*(E)$. E_0^* and E_1^* are the resulting VC's equilibrium efforts devoted to a typical project before and after a change in n respectively, while e_0^* and e_1^* are a typical entrepreneur's equilibrium efforts before and after the change in n.

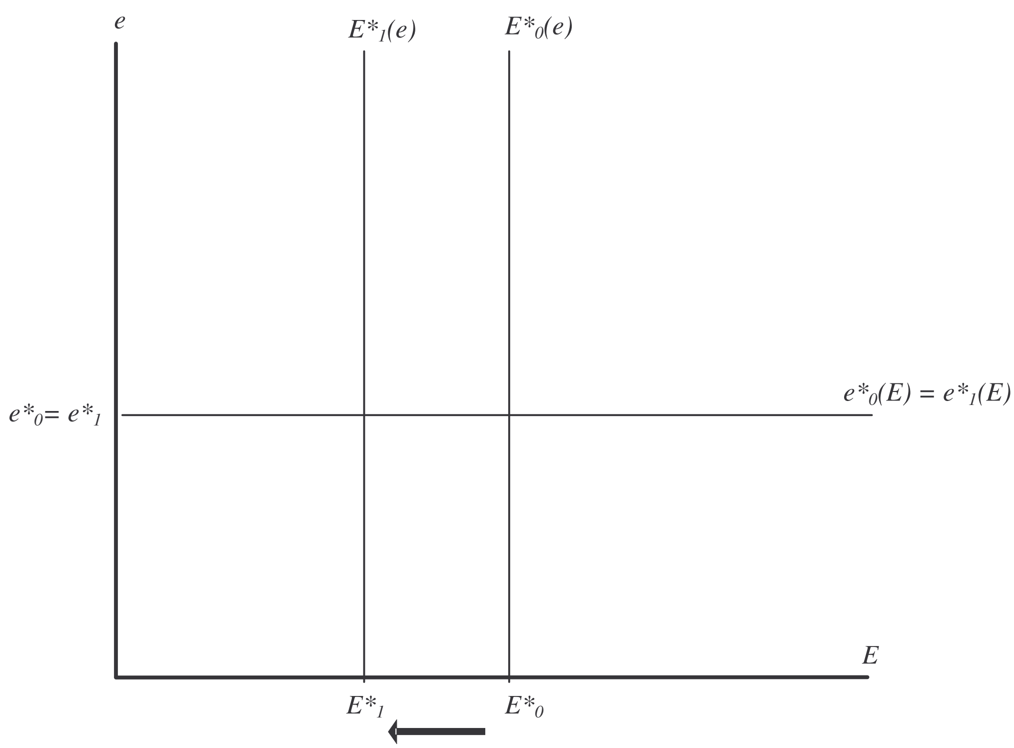

Figure 2 – The effects of a change in n on the best response functions and the resulting equilibrium effort levels – the case of complementary efforts

This figure presents the best response functions of the VC before an increase in n, $E_0^*(e)$, and after an increase in n, $E_1^*(e)$, and those of a typical entrepreneur before an increase in n, $e_0^*(E)$, and after an increase in n, $e_1^*(E)$. E_0^* and E_1^* are the resulting VC's equilibrium efforts devoted to a typical project before and after a change in n respectively, while e_0^* and e_1^* are a typical entrepreneur's equilibrium efforts before and after the change in n. The best responses are depicted as linear functions for illustrative purposes only.

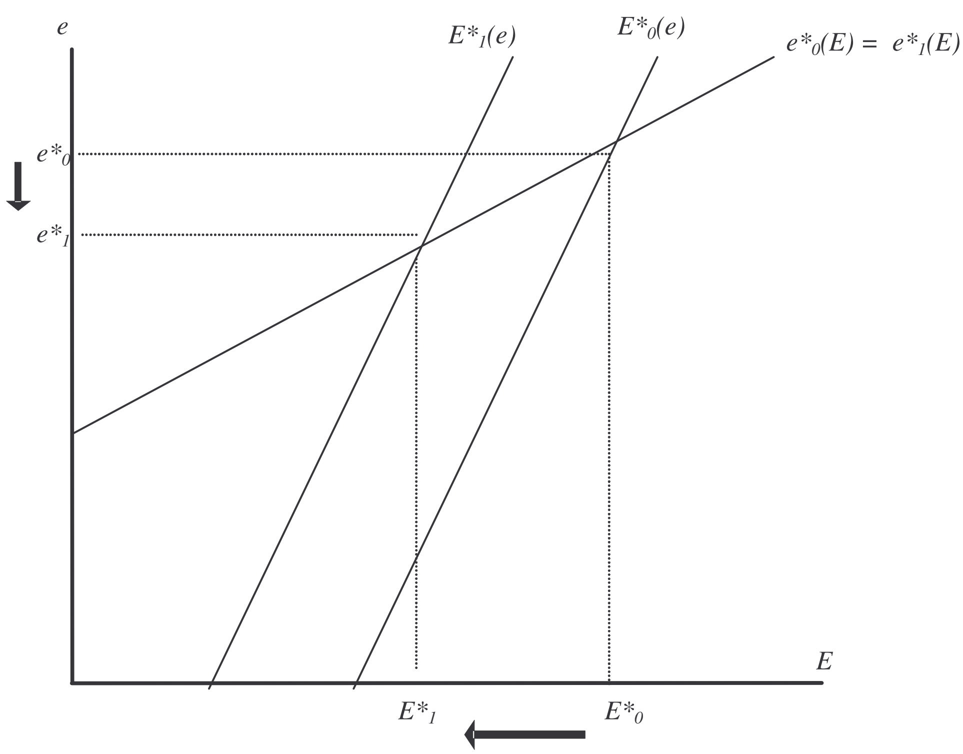

Figure 3 – An illustration of the relation between x and the equilibrium efforts of the VC and a typical entrepreneur

This figure presents an illustration of the relations between the profit share given to a typical entrepreneur, x, and the equilibrium efforts of the VC and a typical entrepreneur, E^* and e^*, respectively.

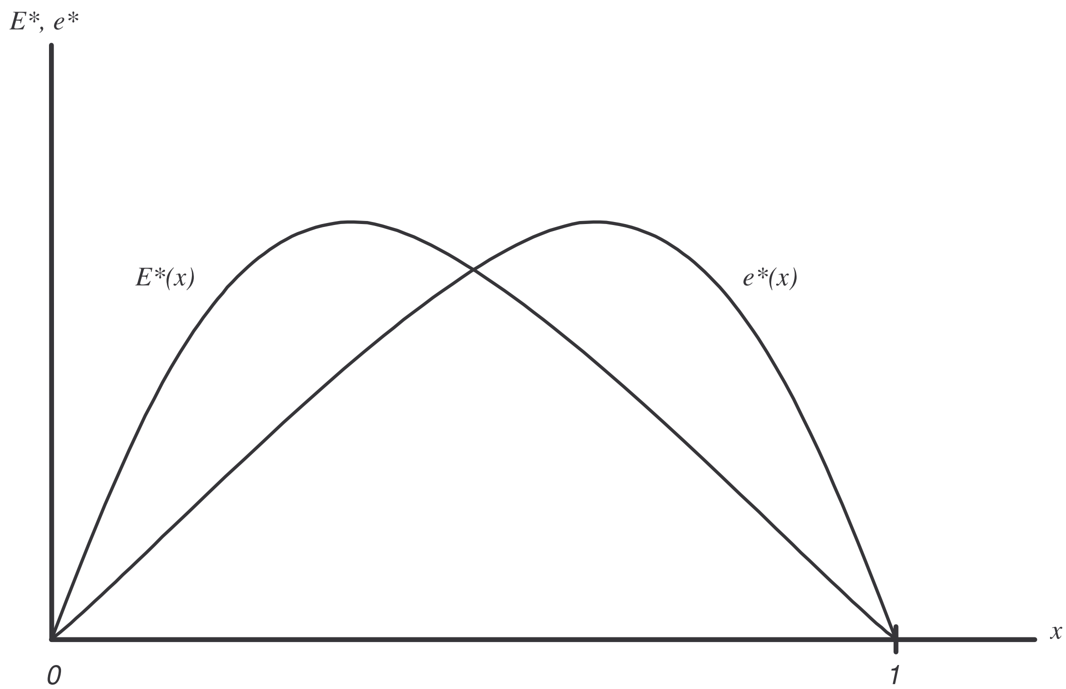

Figure 4 – The effects of a change in x on the best response functions and the resulting equilibrium effort levels – the case of no complementarities

This figure presents the best response functions of the VC before an increase in x, $E_0^*(e)$, and after an increase in x, $E_1^*(e)$, and those of a typical entrepreneur before an increase in x, $e_0^*(E)$, and after an increase in x, $e_1^*(E)$. E_0^* and E_1^* are the resulting VC's equilibrium efforts devoted to a typical project before and after a change in x respectively, while e_0^* and e_1^* are a typical entrepreneur's equilibrium efforts before and after the change in x.

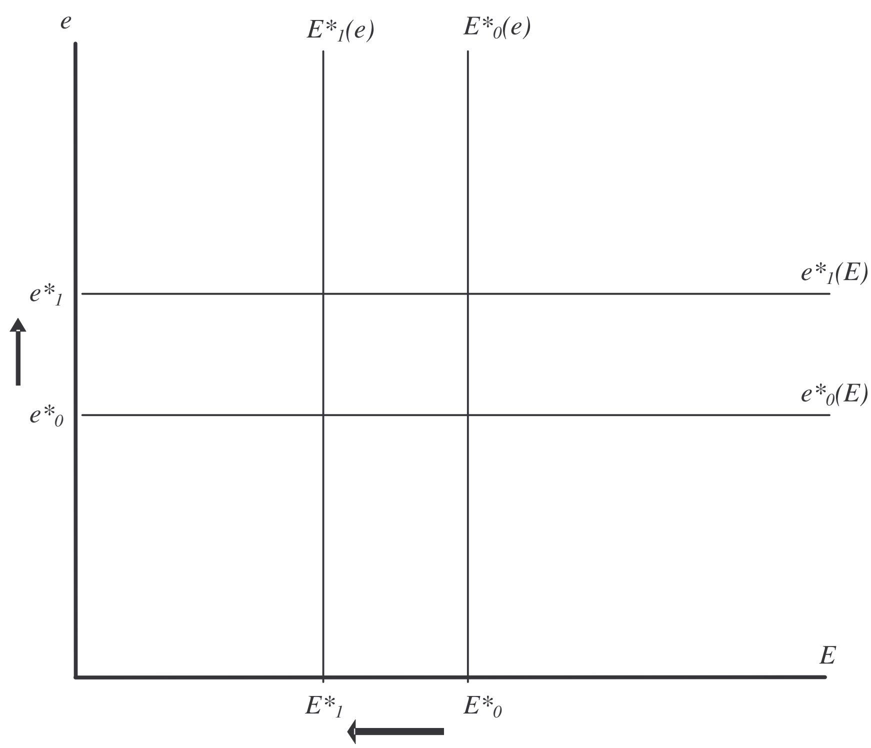

Figure 5 – The effects of a change in x on the best response functions and the resulting equilibrium effort levels – the case of complementary efforts

This figure presents the best response functions of the VC before an increase in x, $E_0^*(e)$, and after an increase in x, $E_1^*(e)$, and those of a typical entrepreneur before an increase in x, $e_0^*(E)$, and after an increase in x, $e_1^*(E)$. E_0^* and E_1^* are the resulting VC's equilibrium efforts devoted to a typical project before and after a change in x respectively, while e_0^* and e_1^* are a typical entrepreneur's equilibrium efforts before and after the change in x. The best responses are depicted as linear functions for illustrative purposes only. Figure 5A presents the case of $x \to 1$, while Figure 5B demonstrates the case of $x \to 0$.

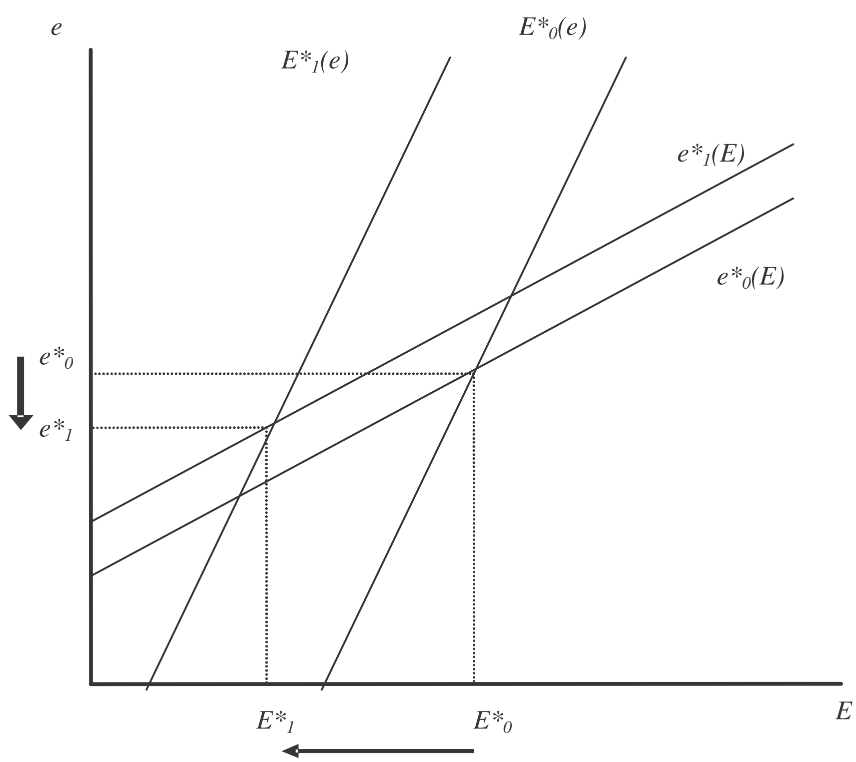

Figure 5A: $x \to 1$

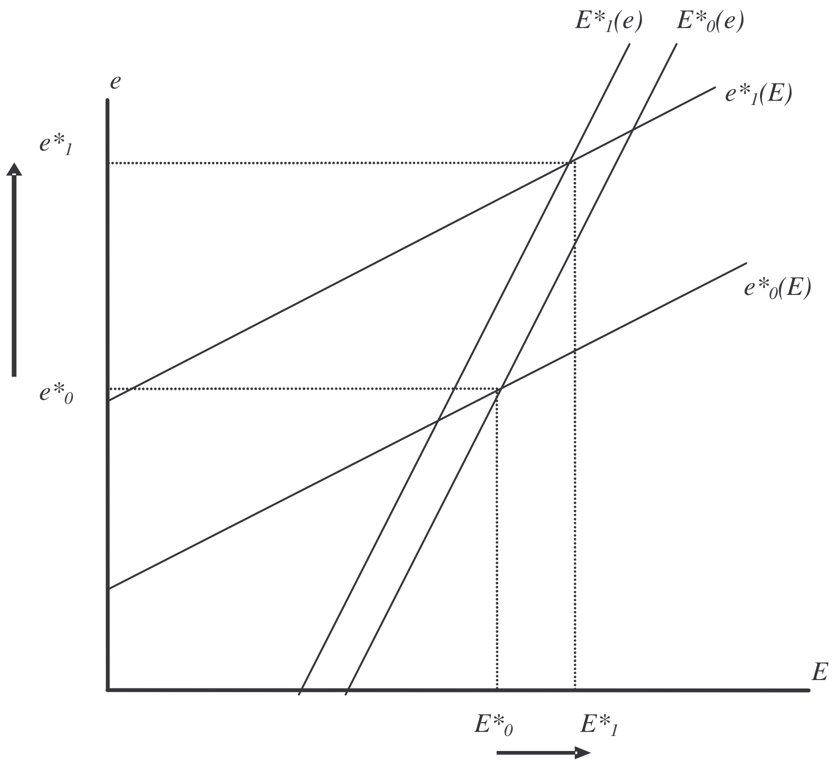

Figure 5B: $x \to 0$

Figure 6 – Comparison of the VC's utility maximization equilibrium with the social welfare maximization equilibrium

This figure presents the optimal number of portfolio firms as a function of entrepreneurs' profit shares and the optimal profit share as a function of portfolio size for the case of VC's utility maximization (dotted curves), $n^*(x)$ and $x^*(n)$ respectively, and social welfare maximization (solid curves), $n^*_W(x)$ and $x^*_W(n)$ respectively. The stable equilibria for the two cases, Eq_{VC} and Eq_W respectively, are depicted by thick dots.