Time-Series Estimation of Aggregate Corporate Bond Credit Spreads*

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Abstract

This paper examines the daily time-series properties of aggregate corporate bond credit spreads, using nine Merrill Lynch option-adjusted spread indices with ratings from AAA to CCC for the period of January 1997 through August 2002. The paper introduces an econometric model of credit spreads that incorporates autocorrelation, conditional heteroscedasticity, time-varying jumps, Treasury bond and/or equity market factors, and index rebalancing effects. The time-series of credit spread indices are found to be mean-reverting in the long-run through the index rebalancing effect. We also find that the lagged Russell 2000 index return and the lagged changes in the slope of the Treasury yield curve are predictive in forecasting the conditional distribution of credit spreads. Meanwhile, the lagged level of the CBOE VIX index is found to be a good indicator of the probability of jumps in the logarithm of credit spreads. The model diagnostic test shows that the jump specification is crucial in capturing the leptokurtic behavior in the daily time-series of log-credit spreads. Finally, the paper finds that the ARCH-jump specification outperforms the specification without jumps in the out-of-sample, one-step-ahead forecast of credit spreads.

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1 Introduction

Assessing and managing the credit risk of risky corporate debt instruments has become a major area of interest and concern to academics, practitioners and regulators in the past decade, especially in the aftermath of a series of credit crises such as the Russian default and the Enron and the WorldCom collapses. In the academic field, there has been a fast growing literature on models of credit risk measurement and management.[1] In the industry, the market in credit risk transfer, especially the market of corporate credit derivatives, has been expanding rapidly, and is becoming increasingly relevant in the business of commercial banks, insurance companies, pension/investment funds and hedge funds. The market in credit risk transfer is evolving into an important fragment of the global financial markets.

To help investors keep up with the latest information in the different segments of the credit markets and assist client in trading and managing portfolios, major investment banks and rating agencies have constructed various credit indices to serve as market indicators and as references for structured credit products. These credit indices are either based on the corporate-Treasury credit spread (such as the credit spread indices constructed by Lehman Brothers, Salomon Smith Barney, Merrill Lynch) or the newly available credit default swap spread data. For risk management purposes, investors holding a large corporate bond portfolio may find it more convenient to hedge themselves against changes in credit risk at the aggregate level instead of at individual issue level. For example, the group of banks launching the iBoxx company in 2001 have integrated a variety of iBoxx indices, including the credit default swap (CDS) indices, into their research products and used those indices as a basis for research recommendation. Given the surging demand for portfolio credit risk management, it would not surprising to see these credit index based products gaining more popularity in the near future.

Given the rapid development of the credit risk product market, a thoughtful understanding of the time-series behavior of credit indices, especially under market turbulence, would serve the interest of a variety of investors. The paper of Prigent, Renault and Scaillet (2001) in *Risk* magazine demonstrates the increasing interest from practitioners on this issue. Capturing the distributional characteristics of credit spreads and its interaction with equity markets and Treasury markets over various market conditions would provide financial decision makers with the needed insights about the nature of the risk in this market and the risk in the various newly developed credit risk products. In particular, as illustrated by the daily credit spread indices from Merrill Lynch in Figure 1, similar to the dynamics

[1] See, for example, Caouette, Altman, and Narayanan (1998), Saunders and Allen (2002), Duffie and Singleton (2003) and references therein.

of exchange rates and interest rates, the systematic variation in U.S. corporate bond credit spreads is very sensitive to financial turmoils around the world, and experiences fairly large movements surrounding these events such as the outbreak of the Eastern Asian financial crisis, the Russian bond default and the collapse of LTCM, and the Eron and WorldCom defaults. The leptokurtic behavior in financial time-series surmounts paramount importance in the pricing and hedging of related derivatives products and managing the associated risk. In the past decades, there has been substantial efforts by researchers to understand and model the leptokurtic properties in the time-series of interest rates and exchange rates through time varying volatility or/and jump models.[2] By contrast, despite the increasing interest in credit risk from practitioners, regulators and academics, little has been known about the nature of the leptokurtosis in the credit index time-series and the role of jumps and time varying volatility in characterizing the systematic variation in credit risk.

Existing studies on the time-series of credit indices are limited. Using daily investment-grade option-adjusted spread indices from Bloomberg between October, 1995 and March, 1997, Pedrosa and Roll (1999) model the leptokurtic behavior in the unconditional distribution of daily log-spread changes as a mixture of Gaussian distributions with constant probabilities. They also find evidence of GARCH-type autocorrelation in the volatility of log-spread changes. The Gaussian mixture and the GARCH model are estimated separately as two alternative specifications. Prigent, Renault and Scaillet (2001) measure the level of aggregate credit spreads in the market as the difference between yields on Moody's corporate indices (Aaa and Baa) and the 10 year (constant maturity) Treasury yield. This data is used to estimate parameters in a CKLS type (Chan et al., 1992) CEV model and a model of an Ornstein-Uhlenbeck process with jumps. Both studies have not investigated whether GARCH model or jumps alone can fully describe the leptokurtic behavior in credit spread indices, and have not considered the information contained in equity market and Treasury markets in forecasting the conditional distribution of credit spread indices. Given the intuition implied by the Merton (1974) type structural corporate bond pricing models, it is implausible to study the time-series behavior of aggregate credit spreads without considering its relationship with the changing environment in equity markets and Treasury markets. In summary, the limited existing studies on credit spreads have suffered from serious data problems, and the role of time varying volatility, jumps, and the information content of equity market and Treasury market variables have not been investigated yet.

In this paper, we investigate the daily time-series behavior of the option-adjusted cor-

[2] Examples of this type of empirical studies include Das (2002) on short-term interes rates, and Vlaar and Palm (1993) and Bekaert and Gray (1996) on exchange rates.

porate bond credit spread indices from Merrill Lynch[3] for nine rating/maturity categories over January 1997 through August 2002. This includes the AA-AAA rated corporate bond indices with maturity of 1-10 years, 10-15 years, and 15+ years, the BBB-A rated corporate bond indices with maturity of 1-10 years, 10-15 years, and 15 years, and the BB rated, B rated and CCC-Lower rated high-yield bond indices. We propose an econometric model for the time-series of aggregate credit spreads that incorporates autocorrelation, conditional heteroscedasticity, time varying jumps, Treasury bond and/or equity market factors, and index rebalancing effects. Our specification offers several advantages. First, the use of a model with both time varying volatility and jumps allows a comparison of these two specifications in capturing the leptokurtic behavior in credit spread indices. For example, studies on the time-series of equity index, interest rates and exchange rates all have shown that both components are needed in describing the dynamics of those financial time series.

Second, the information contained in equity market variables and Treasury market variables is used to forecast the movement in credit spread indices, including the probability of jumps. The literature has documented close comovements between aggregate credit spreads and changes in shape of the yield curve (e.g., Duffee, 1998), equity index return and equity index return volatility (e.g., Huang and Kong, 2003) at monthly frequency. Given this evidence, an interesting and important question to ask is whether the shape of the Treasury yield curve and the equity index movement are predictive in forecasting the distribution of aggregate credit spreads.

Finally, like equity indices, a corporate bond index is often rebalanced, usually at a monthly frequency, to maintain the qualifying criteria for the index. Credit spreads on index rebalancing days are implicitly bounded by some absorbing boundaries for rating based corporate bond indices. We propose a bounded transformation of the log-normal distribution for credit spreads on rebalancing days. The rebalancing day credit spreads are dependent on the credit spread level right before the rebalancing and also on equity and Treasury market variables. Under certain regularity conditions, this specification allows for credit spreads to behavior like a random walk in between index rebalancing days while maintain mean-reverting in the long-run through the rebalancing effect. This reconciles the near unit-root property of log-spreads observed in the Merrill Lynch credit spread data (also reported by Pedrosa and Roll, 1999), and the strong economic argument for credit spreads to be mean-reverting. Beyond these improvements in model specification

[3]For a corporate bond with embedded options such as call provisions, the option-adjusted spread calculation begins by using statistical methods to generate a large number of possible interest rate paths that can occur over the term of the bond and measures the resulting impact of the scenarios on the bond's value. By averaging the results of all the scenarios, the implied spread over the Treasury yield curve is determined.

over previous studies, we also use a variety of statistical measures to assess the in-sample and out-of-sample performance of the model.

We use the Merrill Lynch credit spread indices for a few reasons. First, Merrill Lynch is a major market maker in the corporate bond market and a reliable source of high quality bond prices. Daily prices of corporate bonds used to calculate various Merrill Lynch corporate bond indices and the associated option-adjusted credit spread indices are based on the bid side of the market at 3:00 PM New York time, and obtained from the Merrill Lynch trading desk. Yields on several Merrill Lynch corporate bond indices are in fact quoted in the *Wall Street Journal* every day as market indicators. Accordingly we are confident about the quality of the data that form all the Merrill Lynch credit spread indices.

Second, our objective is to examine the time-series properties of aggregate credit spreads under different market conditions. Hence, we seek credit spread indices that are at high-frequency and span both tranquil and volatile market conditions. The Merrill Lynch credit spread indices are incepted on December 31 of 1996, and calculated daily since then. The indices are classified based on ratings and maturity. By contrast, the option-adjusted spread series in the Salomon Yield Book are only available at monthly frequency, and there is no easily accessible option-adjusted spread data on the various Lehman Brothers corporate bond indices. The Merrill Lynch data cover a horizon that is featured with several major financial and credit crises such as the Eastern Asian financial crisis, Russian bond default and the collapse of LTCM, and the Eron and WorldCom defaults. The behavior of aggregate credit spreads under these extremely severe downside markets would provide valuable insights on the risk embedded in those newly developed portfolio credit products, such as those based on the iBoxx credit default swap indices. Moreover, as pointed out by Black (1976), if we want to study changes in volatility, data at daily frequency are required to provide accurate estimation of volatility within a short period of time so that significant changes in volatility could be detected. The Merrill Lynch data stand out as the best available data for studying the time-series properties of aggregate corporate bond credit spreads.

Third, the Merrill Lynch index is a market-value weighted average of the option-adjusted credit spread on each individual component bond. The option-adjusted spread for each individual bond is calculated based on the bid price from the Merrill Lynch trading desk. Accordingly, the Merrill Lynch credit spread indices offers a relatively clean measure of the level of credit spreads within a given maturity, industry, and credit rating category. Moreover, the ML high-yield indices each cover a substantial amount of high-yield issues (see Appendix C). Existing studies on the time-series of aggregate credit spreads have only focused on investment grade bonds because of data constraints. A study of high-yield indices may help understand the general dynamics of credit spreads in the high-yield bond market.

The empirical findings reported in this paper can be summarized as follows. First, we find ample support for time varying volatility and jumps in the time series of credit spread indices. In particular, incorporating jumps greatly improves the model's in sample fit. When the model only incorporates the ARCH type time varying volatility, the model produces standardized residuals with large sample leptokurtosis in the range of 7 to 24. When jumps are included in the specification, the sample leptokurtosis in the standardize residuals is less than one. Results from a one-step ahead point forecast of credit spreads using a sub-sample of the data show evidence that on average incorporating jumps yields smaller forecasting errors as measured by the root mean squared error and the mean absolute error.

Second, we find that the one-day lag of the level of equity index option-implied volatility, as measured by the Chicago Board of Exchange (CBOE) VIX index, is a reliable indicator of the time varying probability of jumps in log-spreads for both investment-grade and high-yield corporate bonds. There are two reasons why the VIX index helps capture the variation in the jump probability in credit spreads. First, there might be information spill-over effect from the equity index option market to the credit market, as the equity index option market is more liquid and the new information regarding the state of the economy might be first revealed there. Meanwhile, large movements in log-spreads have the tendency to cluster over time, especially after major financial crises. Although the GARCH type volatility persistence model can partly capture this high volatility cluster surrounding market turmoils, the estimated GARCH parameters reflect the degree of volatility persistence over the whole sample period, and could not recognize the extra persistence in volatility after the crisis. However, the equity index option-implied volatility typically increases enormously following these crises. It improves the model's fit by allowing the jump probability in aggregate credit spreads depend on the one-day lagged level of the VIX index so that some of the subsequent large movements are recognized as jumps. Evidence reported here suggest a close relationship between equity index option-implied volatility and the volatility of aggregate credit spreads. This is consistent with the implications from structural corporate bond pricing models.

Third, the one-day lags of the Russell 2000 index return and the change in the slope of the Treasury yield curve are found to be predicative in forecasting the movements of aggregate credit spreads. There is a negative relationship between returns on the Russell 2000 index and the subsequent changes in log-spreads. This relationship is statistically significant at the 5% level for the AA-AAA rated 15+ Years index, the BBB-A rated indices, and all high-yield indices, and is both statistically and economically stronger for lower rated bonds. If, say, the Russell 2000 index dips by 1% on day t, credit spreads on BB rated corporate bonds are expected to climb up by about 0.11% (both are using continuous compound) on day t+1. If the credit spread on BB rated corporate bond is at 300 basis points on day t, the expected increase on day t+1 is around 0.3 basis point. Again, this negative relationship is consistent with the intuition from the structural corporate bond pricing models. A steepening Treasury yield curve typically indicates an economic recovery and is accompanied by lower level of credit spreads. Using the one-day lagged value of changes in the Treasury yield curve as a predictor, we find some evidence that increases in the slope of the Treasury yield curve on day t are followed by lower credit spreads on day t+1. This negative relationship is significant at the 5% level for the AA-AAA rated 1-10 Years index and 10-15 Years index, and the BBB-A rated 1-10 years index and 15+ Years index. If the slope of the Treasury yield curve increases 10 basis points on day t, the expected drop in credit spreads is about 0.3% for the AA-AAA rated indices and 0.15% for the BBB-A rated indices. For high-yield bonds, however, the parameter estimate is all positive and insignificant. This intriguing difference between investment-grade bonds and high-yield bonds calls for more investigations. Changes in the level of interest rates are found not predictive for log-spread changes.

Finally, the distribution of credit spreads on index rebalancing days is special and should be modeled carefully. Treating the distribution of credit spreads on rebalancing days just like any other non-rebalancing days is likely to significantly bias the parameter estimate of some credit spread series, since part of the change in spreads is from the changes in the index component that is not exactly homogenous.[4] The results from our bounded distribution model for rebalancing day spreads show that the distribution of credit spreads within the bounds depends on the level of spread before rebalancing, but not on Russell 2000 index returns or changes in the slope of the Treasury yield curve. Meanwhile, the estimates of the autocorrelation coefficient all fall into the range that makes the credit spread indices mean-reverting in the long-run. We also find that the estimation results for non-rebalancing days are robust with respect to different assumptions about the degree of memory refreshing in the autocorrelations after rebalancing days.

The remainder of the paper is organized as follows. Section 2 describes the Merrill Lynch credit spread data set used in our empirical analysis. Section 3 outlines the econometric

[4] In an exercise not reported here, we estimate the econometric model using all observations, assuming rebalancing day spreads are no difference from normal days, and compare the results with those from an estimation using only non-rebalancing days. The estimates of jump volatility of the BBB-A 10-15 Years index, the BB rated index and the C rated index drop from 5.45%, 6.83%, and 6.41% to 4.16%, 5.01% and 2.19% respectively. The estimates of the coefficient on the VIX index are also significantly different for most indices.

framework we propose for modeling the time-series of aggregate credit spreads. Section 4 contains the estimation results and diagnostic tests. Section 5 discusses the out-of-sample forecast issue and the implications for the credit spread risk measurement and management. Section 6 concludes.

2 The Credit Spread Data

The credit spread data used in this study are the daily Merrill Lynch option-adjusted spreads of corporate bond indices. Each index is a market value weighted average of individual credit spreads on component bonds within a given maturity, industry, and credit rating category. Daily prices of corporate bonds used to calculate the credit spread are based on the bid side of the market at 3:00 P.M. New York time, and obtained from the Merrill Lynch trading desk. Each index is rebalanced on the last calendar day of each month to maintain its qualifying criteria; see Appendix B for a detailed description of the rebalancing procedure and for the number of issues included in each index on rebalancing days. Except for the AA-AAA rated 10-15 Years index, all the other indices typically contain a large number of bond issues. Issues that no longer meet qualifying criteria for a given index are dropped from the index and new issues that meet the qualifying criteria are included for the following month. The dynamics of credit spreads of a given index thus reflects the spread risk of a corporate bond portfolio that is regularly rebalanced to maintain its characteristics on rating, maturity, and amount outstanding. We believe that these ML data are of high quality since as mentioned earlier, yields on several Merrill Lynch corporate bond indices are quoted in the *Wall Street Journal*.

For investment-grade corporate bonds, we obtain industrial corporate credit spread indices for three maturities, 1-10 years, 10-15 years, and 15+ years and two rating groups, AA-AAA and BBB-A, and as a result, have six indices in total. These indices track the performance of US dollar-denominated investment-grade public debts of industrial sector corporate issuers, issued in the US domestic bond market. For high-yield corporate bonds, Merrill Lynch has credit spread indices for three credit ratings: BB, B and CCC-Lower. The industry composition of these high-yield indices is not available. We have in total 9 series of credit spreads with different rating and maturity categories. The sample period is from December 31, 1996 (the inception date of the Merrill Lynch option-adjusted credit spread indices) through August 30, 2002.

The original credit spread series we obtained contain data on weekends or holidays. To ensure that the credit spread data we use reflect market information, we restrict our analysis to NYSE trading days. This is done by matching the credit spread data with the $S\&P$ 500 index data over the sample period. When no match is found, we drop the corresponding observation(s) from the credit spread series. This results in 1,425 daily observations for each ML credit spread index. Figures 1-1, 1-2 and 1-3 plot credit spreads of the AA-AAA rated indices, the BBB-B rated indices and the high-yield indices respectively. Figure 2 plots the changes in log-credit spreads on non-rebalancing days (multiplied by 100). One can see from the figure that spread changes exhibit volatility cluster and large spikes, especially during the Asian financial crisis, the Russian and LTCM defaults and the September 11 event. The changes in investment-grade credit spreads are clearly more volatile during the first two years of the sample period.

2.1 Summary Statistics

Before estimating the model, we look at the statistical properties of the credit spread series first. Table 1 shows the basic statistical properties of the 9 credit spread series on non-rebalancing days. Panels A through C contain respectively the summary statistics on the credit spread level, the basis point changes and the changes in log-credit spreads. Credit spreads are given in basis points.

The mean and standard deviation of each credit spread series reported here are comparable to those reported in other studies using monthly option-adjusted spreads (e.g., Caouette, Altman, and Narayanan, 1998; Kao, 2000). The mean and volatility of credit spreads are generally higher for indices of lower quality and longer maturity. The sample mean of credit spread changes is all insignificantly different from zero. Credit spread changes of all rating/maturity categories show large excess kurtosis.

Because of the index rebalancing, we calculate the first-order serial correlation coefficient $\rho(1)$ of credit spread changes as defined by

$$\rho(1) = \sum_{t=2}^{T_{nr}} \left(\Delta s_t - \overline{\Delta s}\right) \left(\Delta s_{t-1} - \overline{\Delta s}\right) (1 - d_t) / \sum_{t=1}^{T} \left(\Delta s_t - \overline{\Delta s}\right)^2, \tag{1}$$

where $T_n r$ is the total number of non-rebalancing days, $\overline{\Delta s}$ is the sample mean of credit spread changes, and d_t is a dummy variable that takes on the value one if t is the day right after a rebalancing day, and zeros otherwise. The first order autocorrelation in credit spread changes (both in basis point changes and in log spread changes) is significantly negative for investment-grade credit spreads and significantly positive for high-yield credit spreads at 95% significant level. The first-order serial correlation coefficient of squared credit spread changes, which is defined in the same way as Eq. (1), is significantly positive at 95% significant level for all investment-grade credit spread series and the CCC-Lower rated spread series.

2.2 On Mean-Reversion of Credit Spreads

There is strong economic argument for the aggregate credit spread time-series to be mean-reverting in the long-run, as they fluctuate with the state of the economy. For credit spread indices based on certain rating categories and rebalanced regularly, even though in principle the logarithm of credit spreads could fluctuate randomly between positive and negative infinity, they are repositioned to certain implicit bounds upon rebalancing. This implies that the credit spread index time-series is intrinsically mean-reverting in the long-run. However, the aggregate credit spread time-series might be very persistent and takes many years to fully revert to its long-run mean, especially since rebalancing only happens once each month. Using the Augmented Dickey-Fuller (ADF) test, Pedrosa and Roll (1998) could not reject the unit root hypothesis for daily investment-grade option-adjusted credit spread indices of Bloomberg over the period October 5, 1995 to March 26, 1997. Their failure to reject could be due to the relatively short sample period in their study or the fact that the ADF test is known to have a very low power against near unit root alternatives.

To detect the degree of persistence in the aggregate credit spreads, below, we perform a comprehensive unit root analysis, using either unit root or stationarity as the null hypothesis, and allowing for structural breaks in the time series. When doing so, we use the whole sample period, including rebalancing day observations. If there is obvious mean-reverting in credit spreads in between rebalancing days, together with the bounded credit spreads distribution on rebalancing days, we would expect more evidence against unit root through these tests.

2.2.1 Standard Unit Root Tests

We begin with two widely used unit root tests in the literature. Let s_t denote the logarithm of the credit spread on day t. We use the log-credit spreads in all the tests so that the time series under study is not bounded from below by zero.

The augmented Dickey-Fuller (ADF) test of unit root hypothesis against the stationarity hypothesis is based on the following regression:

$$s_t - s_{t-1} = \alpha + \beta s_{t-1} + \sum_{j=1}^{p} \nu_j \left(s_{t-j} - s_{t-j-1}\right) + \epsilon_t. \tag{2}$$

where ϵ_t is white noise. The null hypothesis of unit root is that $\beta = 0$, while the alternative hypothesis of mean-reverting is $\beta < 0$. Following Said and Dickey (1984) the initial autocorrelation lag p is selected as a function of the sample size: $p = 5N^{1/4}$ where N is the number of observation in the regression. Based on the regression with this p, the optimal p is then selected under the null hypothesis using the Schwartz information criterion (SIC).

Since the assumption made in the ADF test that ϵ_t is white noise may be violated in the credit spread data, we consider another widely used unit root test, the Phillips-Perron (1988) test. Consider

$$s_t = \alpha + \beta s_{t-1} + \epsilon_t, \tag{3}$$

where ϵ_t is a zero-mean stationary process. The null hypothesis of unit root corresponds to $\beta = 1$ and the alternative hypothesis is $\beta < 1$. This test employs a Newey-West type variance estimator of the long-run variance of ϵ_t and is robust to a wide variety of serial correlation and heteroscedasticity.

The estimate of the β coefficient in the ADF test and the Phillips-Perron test are reported respectively in Panels A and B of Table 2. One can see from the table that the unit root hypothesis could not be rejected in any of the 9 credit spread series. The mean-reversion coefficients β in the ADF test are all negative, but insignificantly different from zero. The estimates of β in the Phillips-Perron test are all above 0.99 and the unit root hypothesis is not rejected at the 95% significance level.

2.2.2 Stationarity Tests

It is a well-known empirical fact that the standard unit root tests such as the ADF and Phillips-Perron tests fail to reject the null hypothesis of a unit root in a near unit root economic time series. The null hypothesis is always accepted unless there is strong evidence against it. To avoid this problem, tests have been designed under the null hypothesis that the time series under test is stationary around a long-term mean, against the alternative that the time series has a unit root. We employ two such stationarity tests as a robustness check of the conclusion reached from the ADF and the Phillips-Perron tests.

The first stationarity test we use is developed by Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS hereafter). The KPSS test assumes that the time series under test can be decomposed into a random walk and a stationary error term as follows:

$$s_t = r_t + \epsilon_t, \tag{4}$$

$$r_t = r_{t-1} + u_t, \tag{5}$$

where the u_t's are $i.i.d\,(0, \sigma_u^2)$. Under the null hypothesis that $\sigma_u^2 = 0$, the process under test is stationary around a long-term mean. A Lagrange multiplier test statistic is designed under the null hypothesis of stationarity and a large value of this statistic leads to the rejection of stationarity hypothesis.

Another stationarity test we use here is proposed by Bierens and Guo (1993). The test is designed with the null hypothesis

$$s_t = \mu + \epsilon_t, \tag{6}$$

against the alternative

$$\Delta s_t = s_t - s_{t-1} = \epsilon_t \tag{7}$$

where ϵ_t is a zero-mean stationary process and μ is the long-term mean. Bierens and Guo (1993) design four types of Cauchy tests against unit root hypothesis, based on an auxiliary linear time trend regression. Large values of these tests would lead to the rejection of the stationarity null hypothesis.

The results of the two stationarity tests are contained in Panel C and Panel D of Table 2. In the KPSS test, the null hypothesis of stationarity is rejected at the 95% significance level for all credit spread series. The Bierens and Guo Cauchy tests exhibit similar pictures. The only evidence of stationarity is from the type 3 and type 4 Cauchy tests on the credit spread of the AA-AAA 10-15-year index.

2.2.3 Nonlinear Augmented Dickey-Fuller Test

One possible reason for the non-stationarity shown above could be the presence of structural breaks in the credit spread time series. Perron (1989, 1990) and Perron and Vogelsang (1992) have shown that when a time series has structural breaks in the mean, the unit root hypothesis is often accepted before structure breaks are taken into account, while it is rejected after structural breaks are considered. The fact that our sample includes extraordinary financial and credit events as mentioned earlier makes it very likely to have some structural breaks.

A few unit root tests have been developed for time series with structural breaks. We use the Bierens (1997) nonlinear augmented Dickey-Fuller (NLADF) test here since it allows the trend to be an almost arbitrary deterministic function of time. The test is based on an ADF type auxiliary regression model where the deterministic trend is approximated by a linear function of Chebishev polynomials:

$$\Delta s_t = \beta s_{t-1} + \sum_{j=1}^{p} \nu_j \Delta s_{t-j} + \theta^T P_{t,n}^{(m)} + \epsilon_t, \tag{8}$$

where $P_{t,n}^{(m)} = \left(P_{0,n}^{*}(t), P_{1,n}^{*}(t), ..., P_{m,n}^{*}(t)\right)^T$ is a vector of orthogonal Chebishev polynomials. Under the null hypothesis of unit root, $\beta = 0$ and $\theta^T = 0$. The unit root hypothesis is tested based on the t-statistic of β, the test statistic $Am = ((n-p-1)\beta) / \left|1 - \sum_{j=1}^{p} \nu_j\right|$, and the $F-$test of the joint hypothesis that β and the last m components of θ^T are zero.

Panel E of Table 2 presents the results of NLADF tests and the associated critical values. The results show that even after any nonlinear trend breaks are taken into consideration, the unit root hypothesis still could not be rejected.

In summary, we can conclude that there is no empirical evidence of clear mean-reversion in the 9 credit spread series over our sample period. The behavior of aggregate credit spreads in between index rebalancing days is close to a random walk. And even though some type of mean-reverting behavior happens on the index rebalancing days, it could not be picked up by any of the unit root/stationarity test. Accordingly, we need a model specification that recognizes both the empirically observed random walk property of credit spreads in between rebalancing days and the intrinsically mean-reverting property of credit spreads on rebalancing days.

3 An Econometric Model of Corporate Bond Credit Spreads

In this section we describe our model of the time-series of credit spreads on corporate bond indices. We will discuss the models for spreads on rebalancing days and on non-rebalancing days separately. The rebalancing day model recognizes the absorbing boundaries that exist on the distribution of rebalancing day credit spreads and the mean-reverting properties of rebalancing day credit spreads. While for credit spreads on non-rebalancing days, we focus on the changes in the logarithm of credit spreads. (Non-negativity of credit spreads makes it natural to focus the time series study of credit spreads on the logarithm.)

3.1 Model Specifications

The unconditional distribution of aggregate credit spread changes exhibits large leptokurtosis, as reported in Table 1 and in Pedrosa and Roll (1998). Empirical studies have provided ample evidence that the GARCH type specification is generally insufficient to describe the dynamics of financial time series that are featured by occasional large discontinuous movements. Models that incorporate both GARCH feature and jumps have been shown to result in significant model improvements in the studies of exchange rates (e.g., Vlaar and Palm, 1993; Bekaert and Gray, 1996) and interest rates (e.g., Das ,2002). The credit market is subject to substantial surprises that would induce significant jumps in the credit spread movement. The default of the Russian government, Enron and WorldCom are just typical examples of information surprises that would induce jumps in the systematic risk of corporate bond credit spreads. This makes out a case for the time varying volatility and jumps model. However, the monthly rebalancing leads to changes in the index components, and potentially in the memory of the volatility process as well. It is not clear whether the

GARCH model that assumes long memory dependence is the appropriate default specification. For these reasons, we use a specification with jumps and ARCH type time varying volatility for log-spread changes on non-rebalancing days.

Another aspect of our model specification is related to the information content of general market conditions on the systematic variation of corporate bond credit spreads. The BIS (1998) requirements for controlling "spread risk," "downgrade risk" and "default risk" call for credit risk models that fully integrate market risk and credit risk. Jarrow and Turnbull (2000) are among the first to incorporate general market conditions, as reflected in changes in the spot interest rate and equity market indices, into the reduced-form models of corporate bond pricing (e.g., Duffie and Singleton, 1999; Jarrow and Turnbull, 1995). Huang and Kong (2003) document that changes in the ML index credit spreads are closely correlated with the concurrent changes in the interest rates and the equity market index. A more interesting issue for credit risk measurement and management purposes is to look at the information content of the lagged general market information on the movement of credit spreads. Understanding the predictable component in the daily movement of corporate bond credit spread would help credit risk measurement and management. In our specification, we allow for the movement of credit spreads to depend on the lagged market information.

As mentioned in the introduction, since large movements in credit spreads of an issuer are often accompanied by changes in credit ratings, credit spreads on index rebalancing days are implicitly bounded by some absorbing boundaries for rating based corporate bond indices. Below we propose a specification of rebalancing day credit spreads that recognizes this feature.

Let S_t be the credit spread of a given credit index/portfolio on day t, and Ω_t denote the information set available at t. Consider first the model specification of credit spreads on rebalancing days. The credit spread S_t when t is a rebalancing day is assumed to be given by:

$$\begin{aligned} S_t|_{\Omega_{t-1}} &= \frac{\beta + \alpha\,(\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1}) - u_{r,t} - \epsilon_{r,t}\right)}{1 + (\beta-\alpha)\exp\left(-\gamma\ln\left(S_{t-1}\right) - u_{r,t} - \epsilon_{r,t}\right)}, \qquad (9) \\ u_{r,t} &= \mu_r + \sum_{k=1}^{K} \kappa_{r,k} x_{k,t-1}, \quad \epsilon_{r,t} \sim N\left(0, \sigma_r^2\right) \end{aligned}$$

where α and β are the lower and upper bounds ($0 \leq \alpha \leq \beta$), x_k ($k = 1, \ldots, K$) are exogenous variables representing market factors such as interest rates, and μ_r is a constant. (the subscription r refers to rebalancing days). It is easy to see that $\alpha < S_t < \beta$ and

$$\lim_{u_{r,t}\uparrow -\infty} S_t = \alpha \qquad (10)$$

$$\lim_{u_{r,t}\uparrow\infty} S_t = \beta \tag{11}$$

$$\lim_{\alpha\downarrow 0,\beta\uparrow\infty} S_t = S_{t-1}\exp\left(\gamma + u_{r,t} + \epsilon_{r,t}\right). \tag{12}$$

It can be proved that (see Appendix A), under the specification given in Eq. (9), the credit spread index time-series is mean-reverting in the long-run if $\mid \gamma \mid < \frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1}$. This specification recognizes the connection of the rebalancing day spread to the history of spreads prior to the rebalancing date and to other state variables such as interest rates as well.

Consider next the model specification of credit spreads on non-rebalancing days. Suppose day t is the Jth ($J \geq 1$) day after the last rebalancing day. We will often work with the logarithm of credit spreads on non-rebalancing days because it guarantees the non-negativity of predicted credit spreads from the model. We assume that the log-spread, $\ln(S_t)$, conditional on the information set at $t-1$, takes the following ARX-ARCH-Jump specification:

$$\begin{aligned}\ln(S_t)|_{\Omega_{t-1}} &= \mu_0 + \gamma \ln\left(S_{t-1}\right) + \sum_{j=1}^{J} \phi_j \left(1 - D_{1,t-j}\right) \ln\left(S_{t-j}/S_{t-j-1}\right) \\ &\quad + \sum_{k=1}^{K} \beta_k x_{k,t-1} + \lambda_t \mu_J + \epsilon_t,\end{aligned} \tag{13}$$

where

$$\epsilon_t \sim \begin{cases} N\left(\left(1-\lambda_t\right)\mu_J, h_t^2 + \sigma_J^2\right) & w.p.\ \lambda_t \\ N\left(-\lambda_t \mu_J, h_t^2\right) & w.p.\ \left(1-\lambda_t\right) \end{cases} \tag{14}$$

$$h_t^2 = \varpi_0 + \sum_{p=1}^{P} b_p \left(1 - D_{1,t-p}\right) \epsilon_{t-p}^2. \tag{15}$$

In the above specification, $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise. This essentially assumes that index rebalancing completely wipes out all memory in the volatility process. Admittedly, this assumption is rather strong, as a considerable number of bonds will likely remain in the index upon rebalancing. However, given the difficulty in quantifying the degree of memory refreshing, this assumption can be regarded a good benchmark for practical purposes. We will check the robustness of our major empirical finds with respect to this assumption later. The exogenous variables $x_k, k = 1, \ldots, K$, represent market factors such as interest rates. h_t^2 is the conditional variance of ϵ_t in the no-jump state and follows an ARCH(P) process where $P \leq J$. For the jump intensity in log-credit spreads, we concentrate on the Bernoulli distribution, first introduced in Ball and Torous (1983) and also used in Vlaar and Palm (1993), Bekaert and

Gray (1996), and Das (2002). In this structure, the probability that a jump occurs on day t is λ_t and the probability of no jumps on day t is $1-\lambda_t$. Various studies have shown that this structure is tenable for daily frequency data. Conditional on a jump occurrence, we assume the jump size J to be $i.i.d$ and normally distributed with $J \sim N\left(\mu_J, \sigma_J^2\right)$.[5] We also allow the jump probability to depend on lagged exogenous variables z_1,...,z_L such as the volatility of interest rates and the volatility of equity market indices. Specifically, the jump probability is assumed to be a logistic function augmented by lagged exogenous variables z_1,...,z_L as follows

$$\lambda_t = \frac{\exp\left(p_0 + p_1 z_{1,t-1} + p_2 z_{2,t-1} + ... + p_L z_{L,t-1}\right)}{1+\exp\left(p_0 + p_1 z_{1,t-1} + p_2 z_{2,t-1} + ... + p_L z_{L,t-1}\right)} \tag{16}$$

where $p_\ell, \ell = 0, \ldots, L$, are parameters to be estimated.

3.2 Estimation Method

Consider first the model of spreads on rebalancing days. We will show that the lower and upper bounds are identified and can be estimated by maximum likelihood. It follows from Eq. (9) that

$$\epsilon_t = \ln\left(S_t - \alpha\right) + \ln\left(\beta - \alpha\right) - \ln\left(\beta - S_t\right) - \gamma \ln\left(S_{t-1}\right) - u_r, \tag{17}$$

and thus

$$\frac{d\epsilon_t}{dS_t} = \frac{(\beta - \alpha)}{\left(\beta - S_t\right)\left(S_t - \alpha\right)}. \tag{18}$$

Under the normality assumption of ϵ_t, the probability density function of credit spread on rebalancing day t is given by

$$f\left(S_t\right)_r = \frac{(\beta-\alpha)}{\left(\beta - S_t\right)\left(S_t - \alpha\right)} \times \frac{\exp\left[-\frac{1}{2\sigma_r^2} \times \left(\ln\left(\frac{(S_t-\alpha)(\beta-\alpha)}{(\beta-S_t)}\right) - \gamma\ln\left(S_{t-1}\right) - u_r\right)^2\right]}{\sqrt{2\pi}\sigma_r}. \tag{19}$$

The estimation of the parameter set $\theta_r = [\alpha, \beta, \gamma, (\kappa_{r,k}), \mu_r, \sigma_r]$ involves maximizing the log-likelihood function as follows

$$\max_{\theta_r = \left[\alpha,\beta,\gamma,(\kappa_{r,k}),\mu_r,\sigma_r\right]} \quad \mathcal{L} = \sum_{t=1}^{T_r} \ln\left(f\left(S_t|\theta_{\mathbf{r}}\right)_r\right). \tag{20}$$

where T_r is the number of rebalancing day observations in the sample. Of course, the ML estimators of the lower (upper) bound α (β) should be strictly less (greater) than the sample minimum (maximum) of spreads on rebalancing days for a given credit index. In

[5] In principle, the mean and variance of the jump size could be allowed to depend on lagged exogenous variables. Our empirical analysis finds that this does not provide any model improvement.

Appendix A we show that the parameters can be identified by the first-order condition $E\left(\partial \mathcal{L}/\partial \theta_{\mathbf{r}}\right) = 0$.

Consider next the model of spreads on non-rebalancing days. As can be seen from Eq. (13), our model of credit spreads on non-rebalancing days is specified in terms of the conditional distribution of the log-credit spreads. Estimation will be done by maximizing the conditional likelihood function.

Let $\theta_{nr} = [\mu_0, \gamma, (\phi_j), (\beta_k), \varpi_0, (b_p), (p_\ell), \mu_J, \sigma_J]$ (the subscription nr refers to non-rebalancing days). Let also $I_{1,t}$ be an indicator function that equals one in the event of jump on day t and zero otherwise. It follows from Eq. (13) that the conditional density of the credit spread S_t on non-rebalancing days can be written as the following:

$$\begin{aligned} f\left(S_t|\Omega_{t-1}, \theta_{\mathbf{nr}}\right)_{nr} &= (1-\lambda_t) f\left(S_t|\Omega_{t-1}, I_{1,t}=0\right) + \lambda_t f\left(S_t|\Omega_{t-1}, I_{1,t}=1\right) \\ &= (1-\lambda_t)\exp\left(\frac{-\left(\ln\left(S_t\right) - \Psi_{t-1} - \mu_0\right)^2}{2h_t^2}\right)\frac{1}{\sqrt{2\pi h_t^2}S_t} \\ &\quad +\lambda_t \exp\left(\frac{-\left(\ln\left(S_t\right) - \Psi_{t-1} - \mu_0 - \mu_J\right)^2}{2\left(h_t^2+\sigma_J^2\right)}\right)\frac{1}{\sqrt{2\pi\left(h_t^2+\sigma_J^2\right)}S_t} \end{aligned} \tag{21}$$

where

$$\Psi_{t-1} \equiv \gamma \ln\left(S_{t-1}\right) + \sum_{j=1}^{J} \phi_j \left(1 - D_{1,t-j}\right) \ln\left(S_{t-j}/S_{t-j-1}\right) + \sum_{k=1}^{K} \beta_k x_{k,t-1}$$

As a result, under our model specification the density function of credit spreads on day t can be written as follows:

$$f\left(S_t|\Omega_{t-1}\right) = \begin{cases} f\left(S_t|\Omega_{t-1}, \theta_{\mathbf{r}}\right)_r, & if\ D_{1,t} = 1 \\ f\left(S_t|\Omega_{t-1}, \theta_{\mathbf{nr}}\right)_{nr}, & if\ D_{1,t} = 0 \end{cases} \tag{22}$$

Since the parameter sets θ_r and θ_{nr} do not intersect, the estimation of the model can be done separately. Namely, the model in (9) can be estimated using only the rebalancing day sub-sample and the model in (13) estimated using only the non-rebalancing day sub-sample.

4 Empirical Results

The model given in Eq. (13) is rather general. The summary statistics reported in Table 1 suggest that a particular specification of the general model may be adequate for our sample of data. In particular, results of $\rho(1)_{\Delta s}$ and $\rho(1)_{\Delta s^2}$ shown in Table 1 indicate that a specification with AR(1) and ARCH(1) in Eq. (13) may be a good first attempt to capture the autocorrelation in spread changes and conditional variance. As a result, we will estimate an ARX(1)-ARCH(1)-Jump model of log-credit spread changes in this section, and also perform a number of diagnostic and robustness tests.

4.1 Estimated Model

The econometric model introduced in section 3 allows the dynamics of credit spreads to depend on lagged exogenous variables. It has been well recognized that changes in corporate bond credit spreads are closely correlated with the contemporaneous changes in the general market and economic conditions, as reflected by changes in the interest rate and stock market indices. The exogenous variables we consider include lagged interest rate changes, changes in the slope of the yield curve, the Russell 2000 index returns and the CBOE VIX implied volatility index. Specifically, we allow the conditional mean of log -credit spread changes to depend on lagged interest rate changes Δr, the yield curve slope changes $\Delta slope$ and the Russell 2000 index return ret_{rus}. We also allow for the conditional jump probability to depend on the lagged level of the CBOE VIX index since we expect to observe more extreme movements in credit spreads in a more volatile equity market.

Changes in credit spreads are generally considered to be negatively correlated with the contemporaneous changes in interest rates and changes in the slope of the Treasury yield curve, as has been shown in Duffee (1998). We use the change in the Merrill Lynch Treasury Master Index yield (%) as a proxy for the change in the interest rate. The slope of the Treasury yield curve is approximated by the difference between the ML 15+ year Treasury Index yield (%) and the ML 1-3-year Treasury Index yield (%). Credit spreads also tend to rise when returns on the stock market index are low. We choose the Russell 2000 index return ($ret_{rus,t} = \ln(P_{rus,t}/P_{rus,t-1})$) here because it has been shown to be more closely related to credit spread changes than a large-cap index such as the S&P 500 index (e.g., Kao, 2000; Huang and Kong, 2003).

Based on the above discussion, we estimate the following model with the sub-sample of credit spreads on rebalancing days:

$$
\begin{aligned}
S_t &= \frac{\beta + \alpha(\beta - \alpha)\exp(-\gamma \ln(S_{t-1}) - u_{r,t} - \epsilon_{r,t})}{1 + (\beta - \alpha)\exp(-\gamma \ln(S_{t-1}) - u_{r,t} - \epsilon_{r,t})}, \qquad (23)\\
u_{r,t} &= \mu_r + \kappa_1\, ret_{rus,t-1} + \kappa_2\, slope_{t-1} + \kappa_3 \Delta r_{t-1}, \quad \epsilon_{r,t} \sim N\left(0, \sigma_r^2\right)
\end{aligned}
$$

and the following ARX(1)-ARCH(1)-Jump model with the sub-sample of credit spreads on non-rebalancing days:

$$
\begin{aligned}
\ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1(1 - D_{1,t-1})\ln(S_{t-1}/S_{t-2}) \qquad (24)\\
&\quad + \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t,\\
h_t^2 &= \varpi_0 + b_1(1 - D_{1,t-1})\epsilon_{t-1}^2, \qquad (25)\\
\lambda_t &= \exp(p_0 + p_1 * VIX_{t-1}) / (1 + \exp(p_0 + p_1 * VIX_{t-1})). \qquad (26)
\end{aligned}
$$

To compare the relative importance of the ARCH specification and jumps in explaining the leptokurtic behavior of spread changes, we estimate both the ARX-ARCH-Jump model and the nested ARX-ARCH model and report results separately. The estimation is done via the (quasi) maximum likelihood method using the GAUSS MAXLIK and CML modules. Both the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm, and the Berndt, Hall, Hall, and Hausman (BHHH) algorithm are used in the estimation and give the same results.

4.2 Results from the Model for Rebalancing Days

Table 3 contains the estimation results of the credit spread distribution on rebalancing days for the nine Merrill Lynch credit spread series. The parameter estimates of the lower and upper bounds, α and β, and the autoregression coefficient γ, show that credit spreads on rebalancing days exhibit mean-reverting behavior for all the nine indices, as the estimated γ are all positive but less than $\frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1}$. The level of credit spreads on rebalancing days depends on the level of credit spreads before the rebalancing action, but not on the lagged value of other state variables such as Russell 2000 index returns, and changes in the term structure of the yield curve. This evidence suggests that the aggregate credit spread time-series can be described as a process that follows a random walk on regular days and is reverted to a long-term mean slowly only through the rebalancing process.

4.3 Results from the Model in between Rebalancing Days

Estimation results from the nested ARX(1)-ARCH(1) model for log-credit spreads in between rebalancing days are reported in Table 4. Results from the complete ARX(1)-ARCH(1)-Jump model are presented in Table 5.

As shown in the tables, the drift term μ_0 and the mean of the jump size μ_J are largely insignificant. Even though in the model with jumps, the estimate of μ_J is positive for 8 indices, it is only significant for the BBB-A 15+ year index. This implies that jumps affect credit spreads mainly through the conditional volatility of log-credit spreads. There is autocorrelation in the changes of log-credit spreads, as suggested by the estimate of the AR(1) coefficient ϕ_1. The positive autocorrelation in log-credit spread changes of high-yield indices is clearly a result of the slightly upward trend exhibited by high-yield credit spread indices over the sample period. The negative autocorrelation in log-credit spread changes of investment-grade indices suggests the existence of short-run mean-reversion. The estimates of the autocorrelation term are more significant when jumps are considered in the model.

The return on the equity market index provides useful information in forecasting the changes of credit spread on the next trading day. The estimated coefficients of the lagged

Russell 2000 index returns are significantly negative for all indices in the ARX-ARCH model. In the ARX-ARCH-Jump model, the lagged Russell 2000 index returns lose significance for the AA-AAA 1-10-year and 10-15-year indices, but are still significant at the 5% level for all the other seven indices. This negative relationship is both statistically and economically stronger for lower rated bonds. Based on the estimated parameter, if the Russell 2000 index dips by 1% on day t, credit spreads on BB rated corporate bonds are expected to climb up by about 0.11% (both are using continuous compound). If the credit spread on BB rated corporate bond is at 300 basis points on day t, the expected increase on day t+1 is around 0.3 basis point. This negative relationship is consistent with the intuition from the structural corporate bond pricing models, which implies a negative relationship between the return on a firm's equity and the changes in the bond credit spread of the firm.

A steepening Treasury yield curve typically indicates an economic recovery and is accompanied by lower level of credit spreads. Using the one-day lagged value of changes in the Treasury yield curve as a predictor, we find some evidence that increases in the slope of the Treasury yield curve on day t are followed by lower credit spreads on day t+1. This negative relationship is significant at the 5% level for the AA-AAA rated 1-10 Years index and 10-15 Years index, and the BBB-A rated 1-10 years index and 15+ Years index. If the slope of the Treasury yield curve increases 10 basis points on day t, the expected drop in credit spreads for these four indices ranges is about 0.3% for the AA-AAA rated indices and 0.15% for the BBB-A rated indices. For high-yield bonds, however, the parameter estimate is all positive and insignificant. This intriguing difference between investment-grade bonds and high-yield bonds calls for more investigations.

We do not find any convincing evidence that changes in the interest rate provide any useful information on the next day movement of credit spreads. The estimated coefficients on lagged interest rate changes are all insignificant at the 5% level.

In summary, the conditional mean of log-credit spread changes depends on the lagged log-credit spread changes, the lagged Russell 2000 index returns and the lagged changes in the slope of the Treasury yield curve.

We now discuss results on jumps. The estimated mean of the jump size is not significantly different from zero for most indices. Jumps affect credit spreads mainly through the conditional volatility. Consequently, including jumps in the movement of credit spreads results in a sharp decrease in the constant term ϖ_0 and the persistent coefficient b_1 in the ARCH(1) specification. Clearly, jump models help explain the large spread movement surrounding financial and economic crises. Otherwise, the model would result in ARCH parameter estimates that imply implausibly high volatility persistence.

The conditional jump probability is clearly time-varying and can be measured through the lagged volatility in the equity market. The coefficient on the lagged CBOE VIX index in the time-varying jump probability specification is significantly positive for all indices. The sensitivity of conditional jump probability to the lagged equity market volatility tends to increase as the credit quality gets lower. This is consistent with the implication of structural models of corporate bond pricing that high-yield bonds behave more like equity. The sample mean of the CBOE VIX index over the sample period is about 26%. When evaluated at the sample mean of the VIX index, the daily jump probability in log-credit spreads ranges from 11.2% for the AA-AAA 10-15-year index to 3.1% for the BB index.

Shown at the bottom of Table 5, results of the Schwartz and Akaike information criteria indicate that the ARX-ARCH-Jump model outperforms the ARX-ARCH model in terms of the overall goodness-of-fit. A potential problem might arise when using the likelihood ratio test for the statistical significance of the jump behavior in log-credit spreads. This is because the parameters associated with jumps cannot be identified under the null hypothesis of no jumps. Hansen (1992) states that unless the likelihood surface is locally quadratic with respect to the nuisance parameters, the LRT statistic is no longer distributed χ^2 under the null hypothesis.[6] A formal test on the null hypothesis would require a series of optimizations over a grid of the nuisance parameters and the computation would be extremely burdensome. In our case, the fact that the coefficient on the lagged equity market volatility is highly significant does provide a certain amount of confidence in the existence of jump behavior in credit spreads. In the next subsection, we present more model diagnostic tests on the ARX-ARCH-Jump model and the nested ARX-ARCH model based on in-sample residuals.

4.4 Model Diagnostic Tests

Several specification tests based on in-sample residuals are performed to test the conditional normality of the innovation. We summarize the results in Table 6. Under the ARX-ARCH specification, the standardized residual of the model is standard normally distributed. In the specification involving jumps, the residual of the estimated model is actually a mixture of two normal distributions. To compare the residual distribution from the estimation of the ARX-ARCH-Jump model and the nested ARX-ARCH model, we use the method of Vlaar and Palm (1993). We first calculate the probability of observing a value smaller than the standardized residual. In the jump specifications, this would be a weighted average of the normal cumulative distribution function under the jump state and the no-jump state. Under the null hypothesis of normal mixtures, these probabilities should be identically uniformly

[6] The LRT statistic ranges from 256 for the B rated index to 1011 for the BBB-A 1-10-year index.

distributed between 0 and 1. A Pearson chi-squared goodness-of-fit test is then performed on these transformed residual series of each model by classifying the series into g groups based on their magnitudes. Under the null hypothesis, this test statistic is chi-squared distributed with $g-1$ degree of freedom.

Column 4 of Table 6 presents the associated Pearson chi-squared goodness-of-fit test statistic of each model when g equals 20. It is quite clear from the large $\chi^2(19)$ value that the ARX-ARCH normal model is inappropriate for index credit spreads. The smallest value of the $\chi^2(19)$ statistic in the ARX-ARCH model, that of the B credit spread index, is as high as 74.88. The results improve significantly when the specification includes jumps. In the ARX-ARCH-Jump model, the null hypothesis is not rejected at the 5% level for 4 out of the 9 indices, and is not rejected at the 1% level for 7 out of the 9 indices.

We now conduct two diagnostic tests based on the autocorrelation in the standardized residuals of the estimated models. In the specification with jumps, we again follow Vlaar and Palm (1993). Specifically, the standardized residuals are obtained by inverting the standard normal cumulative distribution function based on the probability series in calculating the Pearson chi-squared test. We compute the first-order sample autocorrelation coefficient of the standardized residuals $\rho_\epsilon(1)$, and of the squared standardized residuals $\rho_{\epsilon^2}(1)$ based Eq. (1). The results are presented in columns 5 and 6 of Table 6. It turns out that our specification has removed most of the first-order autocorrelation in spread innovations as reported in Table 1. Although the first-order sample autocorrelation coefficient of the standardized residuals is significant for the AA-AAA rated indices, they are typically between 0 and -0.1. The ARCH(1) specification also seems successful in capturing the time dependence of volatility. Except for the AA-AAA 1-10-year index and the BBB-A 10-15-year index, the first-order sample autocorrelation coefficient of the squared standardized residuals is insignificant. In the two cases where they are significant, the estimates are both less than 0.06. Overall, correlation in the residuals and the squared residuals does not pose any challenge to our model specification.

For five indices, the ARX-ARCH-Jump model passes the Pearson $\chi^2(19)$ goodness-of-fit test marginally. We explore the possible misspecification by looking at the empirical skewness and kurtosis of the standardized residuals from the model estimation. The first four central moments of the standardized residuals are computed in a joint GMM-system. The normality of the standardized residuals is then tested based on a Wald-test that both the skewness and kurtosis coefficients are jointly equal to zero. The estimated sample skewness, kurtosis and the GMM normality test statistic are reported in the last three columns of Table 6. The results in Table 6 tell us that there is no significant skewness in the standardized residuals from both specifications, and the two specifications differ primarily in modelling the fatness in the tails of the distribution. There is still substantial leptokurtosis in the standardized residuals from the ARX-ARCH specification. The minimum is 7.2 for the C rated index and the maximum is 24.8 for the BBB-A 1-10-year index. The leptokurtosis in the standardized residuals from the ARX-ARCH-Jump specification is also significant for all indices, but the maximum is only 0.84. Again, the transformed residuals from the ARX-ARCH-Jump specification pass the normality test marginally. Since the existing leptokurtosis in the transformed residuals is at such a small magnitude, we are comfortable to say that the ARCH-Jump specification well captures the fat tails in the original distribution of credit spread changes.

Overall, there is clear evidence that both jumps and time-varying volatility exist in the daily movement of credit spreads of different credit quality and maturity corporate bond indices. Model diagnostic tests show that the ARX-ARCH-Jump model is strongly preferred over the nested ARX-ARCH and performs rather well for the dynamics of both investment-grade and high-yield indices.

4.5 Robustness Tests

In this section, we check the robustness of our major empirical results. First, as mentioned earlier, for practical reasons, we have started with a specification assuming that index rebalancing completely wipes out all memory in the volatility process. This may not always be realistic, as a considerable number of bonds will likely remain in the index upon rebalancing. We now check the robustness of our estimation results with respect to this assumption. Assuming that rebalancing will not affect the memory of the credit spread time-series on non-rebalancing days at all, we estimate an AR(1)X-ARCH(1)-Jump model without the rebalancing day dummy D_1 and a AR(1)X-GARCH(1,1)-Jump model, and compare with the previous estimated AR(1)X-ARCH(1)-Jump model with the rebalancing dummy.

Dropping the rebalancing day dummy D_1 in the AR(1)X-ARCH(1)-Jump model has little impact on the parameter estimates and results in small drop (less than 8) in the log-likelihood function. To save space, the estimation results are not reported here. The estimation results of the AR(1)X-GARCH(1,1)-Jump model are reported in Table 7. The GARCH coefficient is significant at the 5% level for the AA-AAA rated 1-10 Years index, the three BBB-A rated indices and all three high-yield indices. The VIX index is still a significant indicator of the variation in jump probabilities for most indices, although it is not significant any more for the AA-AAA rated 15+ Years index, the BBB-A rated 10-15 Years index, and the BB rated index. However, using the GARCH(1,1) specification has little impact on the estimated jump volatility, and the jump frequency. Meanwhile, we find that the GARCH(1,1) type conditional volatility alone is not able to capture the leptokurtosis in the changes of log-spread changes. When the jump component is dropped from the specification, the standardized residuals still exhibit large leptokurtosis. In summary, the different assumptions of the memory refreshing impact of rebalancing has little impact on the overall in-sample performance of the model. This is not particularly surprising given the fact that rebalancing only happens once each month. However, the different assumptions about the volatility persistence through rebalancing might have an impact on the forecast of the credit spread right after the rebalancing. This aspect of the rebalancing impact is checked when we conduct the out-of-sample forecast in the next section.

The aggregate corporate bond credit spreads experienced dramatic increase in the aftermath of the September 11 terrorist attack. To test the impact of this extreme event on the robustness of the parameter estimates, especially the jump parameters, we re-estimate the AR(1)X-ARCH(1)-Jump model in Eq. (26) without the observations on September 17 and September 18 of 2001. Dropping these two observations has little impact on the parameter estimates of the six investment-grade bond indices, although the exclusion of these two observations has a significant impact on the estimated jump volatility of high-yield bond indices. The estimated jump volatility for the three high-yield bond indices decreases from about 5%, 4.16%, and 2.19% to 2.81%, 2.59% and 1.9% respectively.

On balance, our major empirical results are robust with respect to the different assumptions about the memory refreshing impact of rebalancing, and the exclusion of the extreme event of the September 11 terrorist attack.

5 Out-of-Sample Forecast and Implications

In this section we seek to further explore the economic implication of allowing for lagged exogenous variables, conditional heteroscedasticity, and jumps in the modeling of credit spreads. We first derive the one-step-ahead prediction formula for the ARX-ARCH-Jump model. We then demonstrate that the model with jumps performs well in forecasting out-of-sample credit spreads. We also discuss the implication of our findings for the measuring and pricing of credit risk.

5.1 Out-of-Sample Specification Tests

To avoid over-parameterization of the ARX-ARCH-Jump model, and to establish the economic significance of allowing for jumps in the dynamics of credit spread, we compare the out-of-sample forecast ability of the model with jumps and that of the model without jumps.

Given the model specification in Eqs. (24) and (26), we can form the following one-step-ahead forecast conditional on the information set Ω_{t-1} at time $t-1$:

$$\begin{aligned} E_{t-1}(S_t) &= E_{t-1}\Big[\exp(\ln(S_{t-1}) + \mu_0 + \phi_1(1-D_{1,t-1})\ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t)\Big] \\ &= S_{t-1}\exp(\phi_1(1-D_{1,t-1})\ln(S_{t-1}/S_{t-2}))\exp(\mu_0 + \lambda_t\mu_J) \\ &\quad \times \exp(\beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3\Delta r_{t-1})\ E_{t-1}(\exp(\epsilon_t)). \end{aligned} \tag{27}$$

It follows from (14) that

$$\begin{aligned} E_{t-1}(\exp(\epsilon_t)) &= (1-\lambda_t)\exp\left(-\lambda_t\mu_J + 0.5h_t^2\right) \\ &\quad + \lambda_t \exp\left((1-\lambda_t)\mu_J + 0.5\left(h_t^2 + \sigma_J^2\right)\right). \end{aligned} \tag{28}$$

The forecasting procedure for the model without jumps can be obtained from the above equation by setting λ_t to zero, and re-estimating the model.

In performing the out-of-sample test, we first estimate the ARX-ARCH-Jump model and the nested ARX-ARCH model using the data from January 1997 through December 1999. The estimated parameters are used in the one-step ahead out-of-sample prediction for the non-rebalancing day credit spread in January 2000. The same procedure is repeated each month over the subsequent period. That is, starting with January 2000, on the first non-rebalancing day of each month, the parameters of the model are estimated using all past observations, and the parameters are then used for the credit spread forecast within this month without being updated. In principle, the parameter could be updated each day using past observations. However, this practice will be computationally burdensome. The approach we have adopted is a trade-off between computational convenience and timely updating of new information. Because the estimates of the drift term μ_0, the mean of the jump size μ_J, and the coefficient on the lagged interest rate changes are mostly insignificant, we have dropped the lagged changes in interest rates, and allowed for both μ_0 and μ_J to be zero in the model we used for forecast. For comparison, we also include the prediction performance of a simple random walk model of credit spreads just using credit spreads of previous day. The difference between the forecast and the actual credit spreads over the out-of-sample period is summarized in the form of root mean squared error (RMSE) and mean absolute errors (MAE).

The changes in log-credit spreads are much more volatile over the first three years than the most recent three years of the sample. This makes it important to allow for a time-varying jump probability. Results in Table 8 show that in the out-of-sample forecast race, the model with time-varying jumps outperform the model without jumps in eight out of the nine different indices. The complicated models outperform the simple random walk model in six (seven) indices in terms of MAE (RMSE).[7] Given the fact that the parameter estimates are only updated monthly for the complicated models, these results are quite encouraging. However, the absolute difference in the prediction error is relatively small. For investment-grade credit spreads, the difference of the three model in RMSE is less than 0.1 basis point. To summarize, using out-of-sample prediction as a model evaluation criterion, the complicated model with jumps also outperform the model without jumps. The out-of-sample forecast results alleviate the fear of over-parameterization and demonstrate the economic significance of allowing for jumps in the dynamics of credit spreads.

5.2 Practical Implications

Our empirical findings from the proposed ARX-ARCH-Jump model of credit spreads have a number of practical implications. First, the econometric model we have proposed for the systematic credit spread risk in corporate bond portfolios directly incorporates the information on the general market condition into the forecasts of the conditional mean and variance of the credit spread. Furthermore, rare extreme movements in credit spreads have been captured by jumps with a time-varying jump probability that depends on equity market volatility. These information could be incorporated into the calculation of the 'value at risk' measure for the credit spread risk.

Second, our results shed some light on the effort of reaching superior investment performance through exploring the information content of the equity market index returns and volatility on general credit spread movements. Asset price predictability could arise as a result of time-varying risk premium, and not necessarily from information inefficiency. Whatever the reason is, our findings call for further studies on the interaction of the equity markets and the corporate bond credit spread risk, and the corresponding strategies that could make use of this information content.

Third, our model could be used for the valuation of credit derivatives written on credit indices. For instance, the model could be potentially used in the valuation of corporate bond credit spread options.

6 Conclusions

We propose an econometric model to describe the dynamic behavior of credit spreads of corporate bond portfolios. In particular, we develop a method to capture the fact that such

[7] We also compared the forecast errors excluding the month of September 2001 in the sample. The relative performance of the models under consideration is not affected by this although it reduces the RMSE and MAE by 0.1-2 basis points.

portfolios are subjected to rebalancing on a regular basis – an issue that has been ignored in the literature. The proposed model integrates together portfolio rebalancing, changes in general market conditions, conditional heteroscedasticity and jumps. We test the model using daily option-adjusted credit spreads of the Merrill Lynch credit spread indices from December 31, 1996 through August 30, 2002. Empirical results indicate that changes in credit spreads of both investment-grade and high yield bond portfolios exhibit autoregression, conditional heteroscedasticity and jumps. Lagged equity market index returns and changes in the slope of the Treasury yield curve are shown to help predict credit spread changes. The time-varying jump probability is found to be related to the lagged option-implied volatility in the equity market. The statistical and economic significance of jumps and the information content of general market conditions are supported both by in-sample and out-of-sample data.

Given the importance of credit risk management in practice, this study may serve the needs of both investors in corporate bond markets and related regulatory agencies. The estimation method developed here that takes into account the rebalancing of a corporate bond portfolio may be extended to deal with similar issues in equity portfolios.

A Mean-reverting of Credit Spreads

Let β and α denote the implicit upper and lower bound, respectively, of credit spreads of a given index on re-balancing days. In our model, the distribution of credit spreads on rebalancing days is

$$\begin{aligned} S_t &= \frac{\beta + \alpha\,(\beta-\alpha)\exp\left(-\omega_{r,t}\right)}{1+(\beta-\alpha)\exp\left(-\omega_{r,t}\right)}, \qquad (29) \\ \omega_{r,t} &= \gamma\ln(S_{t-1}) + \mu_r + \sum_{k=1}^{K}\kappa_{r,k}x_{k,t-1} + \epsilon_{r,t}, \quad \epsilon_{r,t}\sim N\left(0,\sigma_r^2\right) \end{aligned}$$

where α and β are the lower and upper bounds ($0 \le \alpha \le \beta$), $x_k, k = 1,\ldots,K$ are exogenous variables representing market factors such as interest rates, and μ_r is a constant. (the subscription r refers to rebalancing days).

To simply the notations, let's denote $\mu_r + \sum_{k=1}^{K}\kappa_{r,k}x_{k,t-1}$ as $u_{r,t}$ and $\gamma\ln(S_{t-1}) + \mu_r + \sum_{k=1}^{K}\kappa_{r,k}x_{k,t-1}$ as $\nu_{r,t}$. In the following, we will derive the regularity condition under which the rebalancing day credit spreads are stationary. To check this condition, we start with the first-order derivative of $\ln(S_t)$ with respect to $\ln(S_{t-1})$. Given the specification in Eq. (29), we have

$$\begin{aligned} \ln(S_t) &= \ln(\alpha) + \ln\left(\beta/\alpha + (\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1}) - u_{r,t} - \epsilon_t\right)\right) \\ &\quad - \ln\left(1 + (\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1}) - u_{r,t} - \epsilon_t\right)\right). \qquad (30) \end{aligned}$$

Without loss of generality, let's ignore the $u_{r,t}$ and the ϵ_t terms for the time being. Accordingly, the first-order derivative goes as

$$\begin{aligned} \frac{\partial\ln(S_t)}{\partial\ln(S_{t-1})} &= \frac{(-\gamma)(\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)}{\beta/\alpha + (\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)} \\ &\quad + \frac{\gamma(\beta+\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)}{1+(\beta+\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)} \\ &= \gamma(\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)\left\{\frac{1}{1+(\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)}\right. \\ &\quad \left. - \frac{1}{\beta/\alpha + (\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)}\right\} \qquad (31) \end{aligned}$$

Denote the expression $(\beta-\alpha)\exp\left(-\gamma\ln(S_{t-1})\right)$ as ξ. The right-hand side of the above equation can be written as $\gamma\left(\frac{\xi}{1+\xi} - \frac{\xi}{\beta/\alpha+\xi}\right)$. Because $\beta > \alpha$, $\left(\frac{\xi}{1+\xi} - \frac{\xi}{\beta/\alpha+\xi}\right)$ is strictly positive, but less than 1. Denoting $\left(\frac{\xi}{1+\xi} - \frac{\xi}{\beta/\alpha+\xi}\right)$ as Y and using the FOC of Y with respect to ξ yields the solution for the maximum of Y at $\xi = \sqrt{(}\beta/\alpha)$. Hence, we have

$$\begin{aligned} \left|\frac{\partial\ln(S_t)}{\partial\ln(S_{t-1})}\right| &\le |\gamma|\left(\frac{\sqrt{\beta/\alpha}}{1+\sqrt{\beta/\alpha}} - \frac{1}{1+\sqrt{\beta/\alpha}}\right) \\ &= |\gamma|\frac{\sqrt{\beta/\alpha}-1}{\sqrt{\beta/\alpha}+1} < 1 \quad if \quad |\gamma| < \frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1}. \qquad (32) \end{aligned}$$

Now let's assume log-spreads in-between rebalancing days follow a random walk, and $\mid \gamma \mid < \frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1}$. Then because $\mid \frac{\partial \ln(S_t)}{\partial \ln(S_{t-1})} \mid < 1$, $\ln(S_t)$ as a function of $\ln(S_{t-1})$ is a contraction mapping, which is a crucial conditional for mean-reverting of credit spreads, as will be shown as follow. Assume S_{t-1} is the level of credit spreads right before the next rebalancing day t, $t-J-1$ is the previous rebalancing day and ψ_{t-j} is the log-spread change on day $t-j$. Then we have

$$\ln(S_{t-1}) = \ln(S_{t-J-1}) + \sum_{j=2}^{J} \psi_{t-j}. \tag{33}$$

If $0 < \gamma < \frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1}$, we have $0 < \frac{\partial \ln(S_t)}{\partial \ln(S_{t-1})} \leq \rho < 1$. This implies that $\ln(S_t) \leq \rho \ln(S_{t-1}) + C$ where C is a constant. Substituting the expression of $\ln(S_{t-1})$ as in Eq. (33) into this inequality, we get $\ln(S_t) \leq \rho \ln(S_{t-J-1}) + \rho \sum_{j=2}^{J} \psi_{t-j} + C$. The same inequality holds for $\ln(S_{t-J-1})$. In general, assuming that t is the Kth $(K \geq 1)$ index rebalancing day and the initial credit spread is S_0, we have the following general expression for $\ln(S_t)$:

$$E \mid \ln(S_t) \mid < \rho^K E \mid \ln(S_0) \mid + \sum_{\ell=1}^{K} \rho^{K-\ell+1} \sum_{j=1}^{J_{\ell-1,\ell}} E \mid \psi_{\ell-1,\ell,j} \mid + C^*, \tag{34}$$

where $J_{\ell-1,\ell}$ is the number of days in between the $(\ell-1)th$ and the ℓth rebalancing days and $\psi_{\ell-1,\ell,j}$ is the jth log-spread changes for the $\ell-1$ to ℓ rebalancing period, and C^* is a constant. Because $\sum_{j=1}^{J_{\ell-1,\ell}} E \mid \psi_{\ell-1,\ell,j} \mid + C^* < M$ where M is a constant, and $0 < \rho < 1$, we have

$$\lim_{t\to\infty} \sup E \mid \ln(S_t) \mid \leq M/(1-\rho). \tag{35}$$

The same argument applies if $-\frac{\sqrt{\beta/\alpha}+1}{\sqrt{\beta/\alpha}-1} < \gamma < 0$, so that $-1 < -\rho \leq \frac{\partial \ln(S_t)}{\partial \ln(S_{t-1})} \leq < 0$.

B Identification of the Upper and Lower Bounds

Given the specification of rebalancing day credit spreads as in Eq. (29), it follows that

$$\frac{d\epsilon_{r,t}}{dS_t} = \frac{(\beta-\alpha)}{(\beta-S_t)(S_t-\alpha)}. \tag{36}$$

Under the assumption that $\epsilon_{r,t}$ is normally distributed with $N(0, \sigma_r^2)$, the density function of spreads on rebalancing days is,

$$h(S_t) = \frac{(\beta-\alpha)}{(\beta-S_t)(S_t-\alpha)} \times \frac{\exp\left[-\frac{1}{2\sigma_r^2} \times \left(\ln\left(\frac{(S_t-\alpha)(\beta-\alpha)}{(\beta-S_t)}\right) - \nu_{r,t}\right)^2\right]}{\sqrt{2\pi}\sigma_r}, \tag{37}$$

as follows from the well-known transformation formula for densities. The first-order condition for a maximum of the likelihood function of spread distribution is satisfied, as will be shown as follows.

Taking the first order derivative of the log-density function with respect to β given the true parameters α and β, we have:

$$\frac{\partial \ln\left(h\left(S_t\right)\right)}{\partial \beta} = -\frac{\left(\ln\left(\frac{(S_t-\alpha)(\beta-\alpha)}{(\beta-S_t)}\right) - \nu_{r,t}\right)}{\sigma_r^2} \times \left(\frac{1}{\beta-\alpha} - \frac{1}{\beta-S_t}\right) + \frac{1}{\beta-\alpha} - \frac{1}{\beta-S_t}. \tag{38}$$

Because $\ln\left(\frac{(S_t-\alpha)(\beta-\alpha)}{(\beta-S_t)}\right) = \omega_{r,t}$, the above equation could also be written as

$$\frac{\partial \ln\left(h\left(S_t\right)\right)}{\partial \beta} = -\frac{\left(\omega_{r,t} - \nu_{r,t}\right)}{\sigma_r^2} \times \left(\frac{1}{\beta-\alpha} - \frac{1}{\beta-S_t}\right) + \frac{1}{\beta-\alpha} - \frac{1}{\beta-S_t}. \tag{39}$$

From Eq. (29), we have

$$\frac{1}{\beta-S_t} = \frac{\exp\left(\omega_{r,t}\right) + (\beta-\alpha)}{(\beta-\alpha)^2}.$$

It then follows that

$$\frac{\partial \ln\left(h\left(S_t\right)\right)}{\partial \beta} = \frac{1}{(\beta-\alpha)^2} \times \frac{\left(\omega_{r,t} - \nu_{r,t} - \sigma_r^2\right)}{\sigma_r^2} \exp\left(\omega_{r,t}\right). \tag{40}$$

Integrating $\omega_{r,t}$ out then yields

$$\begin{aligned} (\beta-\alpha)^2 E\left[\frac{\partial \ln\left(h\left(S_t\right)\right)}{\partial \beta}\right] &= \int_{-\infty}^{\infty} \frac{\left(\omega_{r,t} - \nu_{r,t} - \sigma_r^2\right)}{\sigma_r^2} \times \exp\left(\omega_{r,t}\right) \frac{\exp\left(-\frac{\left(\omega_{r,t}-\nu_{r,t}\right)^2}{2\sigma_r^2}\right)}{\sqrt{2\pi}\sigma_r} d\omega_{r,t} \\ &= \frac{\exp\left(\frac{2\nu_{r,t}\sigma_r^2+\sigma_r^4}{2\sigma_r^2}\right)}{\sigma_r} \int_{-\infty}^{\infty} y \frac{\exp\left(-\frac{y^2}{2}\right)}{\sqrt{2\pi}} dy = 0. \end{aligned} \tag{41}$$

We can show in a similar fashion that

$$E\left[\frac{\partial \ln\left(h\left(S_t\right)\right)}{\partial \alpha}\right] = 0. \tag{42}$$

The first-order conditions also have a unique solution, as will be shown as follows. Assume there is another set of parameters where $\alpha^* < \alpha$, $\beta^* > \beta$ and $\sigma_r^* \neq \sigma_r$ and the corresponding log-likelihood function is $\ln h^*\left(S_t\right)$. For α^*, β^* and σ_r^* to be a solution of the first order condition, there should be $\ln h\left(S_t\right) = \ln h^*\left(S_t\right)$ for all S_t. However, as $S_t \uparrow \beta$, $\ln h\left(S_t\right)$ approaches infinity while $\ln h^*\left(S_t\right)$ will be finite. This is also true as $S_t \downarrow \alpha$. Other cases can be proved in a similar manner.

C Rebalance of the Merrill Lynch Corporate Bond Credit Spread Indices

In this appendix, we describe certain rules used in rebalance of the Merrill Lynch Corporate Bond Credit Spread Indices. The information is based on a publication from Merrill Lynch (2000). The publication contains information on the Merrill Lynch High Grade U.S. Industrial Corporate Index, the Merrill Lynch U.S. High Yield Master II Index, and a detailed description about the general rebalancing rules used by Merrill Lynch to maintain the qualifying criteria of each index. We believe that the same criteria should hold for the sub-indices we use in this study.

To be included in an index, qualifying bonds must have a fixed coupon schedule and at least one year to maturity. The amount of outstanding required for being on a high-grade index is a minimum of \$150 million, while that for being on a high-yield index is a minimum of \$100 million.

Rebalancing takes place on the last calendar day of each month. The adding or dropping decision of any issue will be based on information that is available in the marketplace "up to and including the third business day prior to the last business day of the month." There are 62 rebalancing days in total including the inception date of the indices, December 31, 1996, in our sample.

The table below contains the number of issues that were included in each index on rebalancing days, which are supposed to be the last calendar day of each month since December 31, 1996. If the last calendar day is not a business day of New York Stock Exchange, we use the next available observation.

Number of Issues Included in Each Merrill Lynch Credit Spread Index on Rebalancing Days

This table contains the number of issues that were included in each Merrill Lynch index on rebalancing day. Rebalancing is done on the last calendar day of each month since December 31, 1996 (the inception day of the indexes). When the last calendar day is not a business day of New York Stock Exchange, the first trading day of the next month is used.

Rebalance days	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
19961231	74	7	46	556	83	363	376	467	65
19970131	76	6	46	558	83	364	356	461	66
19970228	75	6	46	570	82	367	358	478	74
19970331	74	6	47	573	81	376	361	482	80
19970430	75	6	47	586	84	381	383	465	79
19970602	75	6	47	598	83	384	378	458	79
19970630	76	6	46	604	84	389	367	454	81
19970731	76	6	45	612	85	397	367	456	76
19970902	80	7	46	621	84	411	367	446	73
19970930	90	7	49	669	91	437	361	451	69
19971031	91	7	51	683	92	447	370	468	69
19971201	89	7	51	691	92	454	372	474	74
19971231	89	7	53	695	92	466	380	476	78
19980202	91	8	55	705	93	488	376	493	84
19980302	91	8	57	707	86	478	408	503	86
19980331	90	8	57	723	88	484	419	515	90
19980430	91	8	59	738	89	488	402	528	90
19980601	88	8	59	755	92	502	384	531	93
19980630	86	7	58	758	91	514	384	523	103
19980731	87	7	58	750	95	517	390	530	107
19980831	86	8	59	766	100	520	392	530	110
19980930	86	8	59	765	98	521	398	541	124
19981102	92	8	60	777	95	526	388	546	137
19981130	89	7	58	786	93	528	390	554	134
19981231	84	8	50	813	94	542	377	537	141
19990201	83	8	51	821	92	542	370	538	141
19990301	81	10	45	831	95	552	380	534	144
19990331	82	9	45	834	93	556	382	532	149
19990430	83	9	47	829	87	552	394	536	155

Rebalance days	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
19990601	82	9	47	841	87	557	386	529	158
19990630	81	9	47	849	87	556	379	529	150
19990802	66	7	44	590	56	423	359	537	139
19990831	70	6	45	611	58	427	361	531	142
19990930	72	6	45	615	57	430	370	552	145
19991101	75	6	45	613	52	426	372	564	147
19991130	73	6	45	630	52	433	373	568	140
19991231	71	6	41	764	61	493	365	563	144
20000131	76	1	36	817	66	530	365	566	136
20000229	78	1	38	858	66	543	382	569	138
20000331	78	1	38	867	66	545	397	600	147
20000501	79	3	43	869	64	539	407	591	148
20000531	77	3	40	870	64	545	402	588	161
20000630	78	3	39	881	64	547	390	600	159
20000731	79	3	39	877	67	546	391	609	165
20000831	78	3	39	890	65	551	396	624	165
20001002	79	3	39	895	67	554	406	619	174
20001031	78	3	39	898	69	552	409	692	207
20001130	74	3	38	896	67	548	396	706	214
20010102	74	4	41	889	65	537	384	703	224
20010131	74	5	35	896	71	534	386	684	253
20010228	77	5	37	923	66	535	398	659	264
20010402	78	4	37	928	67	524	425	651	255
20010430	80	4	37	932	63	521	456	612	276
20010531	81	4	37	942	68	524	453	605	282
20010702	83	4	37	943	65	519	458	578	301
20010731	91	4	37	949	67	522	472	568	298
20010831	89	4	37	963	67	525	495	519	318
20011001	87	4	37	970	64	523	508	525	318
20011031	95	4	44	979	67	511	490	537	313
20011130	89	4	41	1005	68	513	489	523	331
20011231	89	5	44	1005	67	506	495	529	336
20020131	88	5	44	1004	64	508	509	529	328
20020228	82	4	44	1017	62	509	517	564	305
20020331	84	3	42	1028	63	512	523	546	309
20020430	85	2	42	1037	64	509	538	523	315
20020531	86	2	43	1037	62	494	603	520	297
20020630	84	2	44	1041	64	493	663	662	308
20020731	85	2	43	1026	61	480	656	698	303
20020831	85	2	41	1042	63	481	656	702	315
Average	81	6	46	791	77	494	401	547	162

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Table 1
Summary Statistics on Merrill Lynch Option-adjusted Credit Spreads

This table reports the summary statistics on the daily Merrill Lynch option-adjusted credit spread (OAS) indices on non-rebalancing days. Panels A through C present respectively the summary statistics on credit spread level, changes in credit spreads and changes in log-credit spreads. $\rho(1)$ is the first order autocorrelation coefficient. The sample period of daily credit spreads is from 1/02/1997 through 8/30/2002. There are 1358 observations on non-rebalancing days.

Statistics	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Panel A: Option-adjusted Spreads (Basis Points)									
Mean	62.51	72.03	87.38	136.32	137.45	157.25	319.5	552.33	1317.7
Median	63.668	69.383	90.502	138.756	135.666	153.442	308.45	531.358	1262.645
Std Dev.	20.39	24.49	27.08	54.21	49.96	52.25	125.6	205.77	533.44
Max	103.248	133.32	138.088	239.189	279.223	258.472	725.166	1082.642	2358.557
Min	27.138	16.64	36.17	50.384	52.76	73.994	136.141	280.837	504.05
Panel B: Changes in Credit Spreads ($\Delta S_t = S_t - S_{t-1}$)									
Mean	-0.004	-0.056	0.001	0.127	0.038	0.037	0.34	0.549	1.06
Std Dev.	1.76	3.03	1.83	2.01	2.47	2.03	6.08	8.96	13.79
Skewness	-0.62	-1.79	0.38	1.35	0.68	1.23	7.02	3.36	1.65
kurtosis	20.63	34.27	12.83	15.76	7.85	16.89	116.16	53.7	25.53
Max	10.859	20.23	14.267	18.81	14.77	20.93	106.67	145.81	182.47
Min	-18.7	-41.37	-12.589	-12.43	-12.61	-12.86	-24.8	-48.84	-91.17
$\rho(1)_{\Delta S}$	-0.28	-0.17	-0.17	-0.02	-0.04	-0.01	0.15	0.17	0.17
$\rho(1)_{\Delta S^2}$	0.36	0.05	0.3	0.23	0.31	0.15	0.02	0.01	0.04
Panel C: Changes in Log-Credit Spreads ($\Delta s_t = \ln(S_t/S_{t-1}) * 100)$)									
Mean	-0.014	-0.063	0.007	0.077	0.024	0.027	0.065	0.082	0.074
Std Dev.	3.74	5.19	2.51	2.03	2.22	1.52	1.54	1.47	1.01
Skewness	-1.78	0.37	-0.31	0.001	0.49	-0.25	3.13	1.81	1.35
kurtosis	47.88	55.6	20.91	27.94	21.31	22.8	41.63	18.91	14.13
Max	29.83	74.91	18.99	15.41	22.88	11.79	22.75	17.47	10.24
Min	-52.43	-60.75	-21.1	-18.36	-16.25	-14.59	-6.24	-5.89	-6.18
$\rho(1)_{\Delta s}$	-0.29	-0.29	-0.22	-0.16	-0.2	-0.14	0.1	0.14	0.2
$\rho(1)_{\Delta s^2}$	0.37	0.4	0.37	0.41	0.42	0.27	0.01	0.01	0.04

Table 2
Unit Root/Stationarity Tests on the Merrill Lynch Option-adjusted Credit Spread Series

This table presents the results of various unit root/stationarity tests on Merrill Lynch option-adjusted credit spreads over the period of 12/31/1996 - 8/30/2002. The estimates and associated t-statistics from the Augmented Dickey-Fuller test and the Phillips-Perron test are reported in Panels A and B. Panel C contains the Lagrangian Multiplier (LM) test statistic from the stationarity test of Kwiatkowski, Phillips, Schmidt and Shin (1992). Panel D reports the four types of Cauchy tests of Bierens and Guo (1993) stationarity test. Panel E presents the results of Bierens (1997) non-linear Augmented Dickey-Fuller tests for unit root.

Test Statistics	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Panel A: Augmented Dickey-Fuller Test (Critical Value: (5%)=-2.89; (10%)=-2.58)									
β	-0.0042	-0.009	-0.003	-0.001	-0.001	-0.002	-0.005	-0.001	-0.0004
$t-stat$	-1.51	-2.2	-1.53	-0.97	-0.63	-1.18	-0.39	-0.66	-0.33
Panel B: Phillips-Perron Test (Critical Value: (5%)=-14.51; (10%)=-11.65)									
β	0.9925	0.9842	0.996	0.9986	0.9985	0.9985	0.9996	0.9995	0.992
$t-stat$	-5.41	-13.68	-3.6	-1.46	-1.37	-2.33	-1.17	-0.92	-12.04
Panel C: KPSS (1992) Stationarity Test (Critical Value: (5%)=0.463; (10%)=0.347)									
$LM-Stat$	3.23	1.9	2.87	4.12	3.99	3.87	4.14	3.79	4.18
Panel D: Bierens-Guo (1993) Stationarity Tests (Critical Value: (5%)=12.706; (10%)=6.314)									
Type 1	46.33	2.48	374.54	748.52	367.7	607.54	805.6	887	718.59
Type 2	49.61	2.5	541.15	1391.8	1386	1376	1424.7	1424	1425
Type 3	13.62	1.63	29.84	149.36	90.75	122.1	157.6	175.89	205.95
Type 4	30.01	2.82	44.14	189.17	128.28	111.26	118.9	73.88	121.6
Panel E: Bierens (1997) Nonlinear ADF test (Critical Value of t-stat (5%)=-3.97; (10%)=-3.46) (Critical Value of Am (5%)=-27.2; (10%)=-23) (Critical Value of F-test (5%)=4.88; (10%)=5.68)									
β	-0.006	-0.009	-0.003	-0.004	-0.008	-0.003	-0.008	-0.005	-0.007
$t-stat$	-1.32	-1.93	-1.03	-1.35	-2.1	-1.26	-2.36	-2.13	-2.37
Am	-5.1	-9.04	-3.1	-4.84	-9.16	-4.74	-11.34	-9.64	-11.28
F-test	1.25	1.77	1.91	1.69	2.58	1.35	3.06	1.89	3.43

Table 3
Maximum Likelihood Estimates of Credit Spread Distribution on Rebalancing Days

This table presents results of the estimation of Merrill Lynch option-adjusted credit spread indices on index rebalancing days. The distribution of credit spreads S_t for a given index on rebalancing day t takes the following form:

$$S_t = \frac{\beta + \alpha\,(\beta - \alpha)\exp\left(-\mu_r - \gamma\ln(S_{t-1}) - \kappa_1\, ret_{rus,t-1} - \kappa_2\, slope_{t-1} - \kappa_3\Delta r_{t-1} - \epsilon_{r,t}\right)}{1 + (\beta - \alpha)\exp\left(-\mu_r - \gamma\ln\left(S_{t-1}\right) - \kappa_1\, ret_{rus,t-1} - \kappa_2\, slope_{t-1} - \kappa_3\Delta r_{t-1} - \epsilon_{r,t}\right)} \tag{43}$$

where $\alpha < S_t < \beta$, μ_r is a constant, and ϵ_t is normally distributed with $N\left(0, \sigma_r^2\right)$. Maximum likelihood estimates of the parameters for each index and the heterodscadesticity-consistent standard errors are reported below. There are 66 observations on rebalancing days.

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
α	25.98	21.08	26.30	41.09	36.31	55.15	92.88	223.99	486.88
	(3.85)	(3.44)	(6.18)	(2.41)	(5.74)	(5.69)	(15.70)	(19.47)	(17.49)
β	137.71	249.10	210.80	282.07	398.81	331.98	844.85	1369.07	2624.45
	(29.58)	(67.85)	(46.51)	(18.21)	(62.75)	(34.44)	(103.96)	(97.04)	(79.23)
σ_r	0.22	0.20	0.13	0.13	0.10	0.11	0.14	2.61	3.04
	(0.09)	(0.05)	(0.05)	(0.04)	(0.03)	(0.04)	(0.05)	(0.30)	(0.17)
μ_r	-7.07	-3.74	-5.22	-7.26	-4.80	-8.08	-6.29	-10.41	-14.57
	(2.81)	(1.14)	(2.24)	(0.90)	(1.13)	(1.69)	(1.60)	(1.96)	(1.22)
γ	2.68	1.86	2.19	2.52	1.99	2.61	2.09	2.62	3.04
	(0.68)	(0.27)	(0.50)	(0.19)	(0.23)	(0.34)	(0.27)	(0.30)	(0.17)
ret_{rus}	-0.06	-0.04	-0.04	-0.03	-0.02	-0.03	0.00	-0.01	0.00
	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)
$slope$	-0.15	0.32	-0.38	-0.36	-0.09	-0.25	-0.34	0.25	-0.39
	(0.28)	(0.28)	(0.22)	(0.23)	(0.15)	(0.17)	(0.22)	(0.29)	(0.47)
Δr	0.09	0.23	-0.25	0.26	-0.09	-0.07	-0.04	-0.51	0.65
	(0.64)	(0.64)	(0.38)	(0.44)	(0.30)	(0.29)	(0.34)	(0.41)	(0.80)
$\ln(L)$	-187.76	-225.52	-198.52	-209.99	-219.38	-214.08	-285.77	-297.99	-389.60

Table 4
Maximum Likelihood Estimates of the ARX(1)-ARCH(1) Model of Credit Spreads on Non-rebalancing Days

This table reports the maximum likelihood estimates of the ARX(1)-ARCH(1) model of log-credit spreads for the period 01/02/1997 through 08/30/2002. The estimated model is

$$\begin{aligned} \ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1(1 - D_{1,t-1})\ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3 \Delta r_{t-1} + \epsilon_t, \end{aligned} \tag{44}$$

where $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise, $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is the lagged changes in the slope of the yield curve, and Δr_{t-1} is the lagged changes in the interest rates. The disturbance ϵ_t has mean zero and conditional variance h_t^2, where h_t^2 is specified as an ARCH(1) process:

$$h_t^2 = \varpi_0 + b_1(1 - D_{1,t-1})\epsilon_{t-1}^2. \tag{45}$$

The asymptotic heteroscedasticity-consistent standard errors are reported in parentheses. Bold numbers indicate significance at the 10% level

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
μ_0	0.121	-0.074	0.014	0.082	0.031	0.002	0.066	**0.075**	**0.074**
	(0.133)	(0.143)	(0.061)	(0.049)	(0.057)	(0.034)	(0.083)	(0.039)	(0.037)
β_1	**-0.226**	**-0.287**	**-0.127**	**-0.175**	**-0.137**	**-0.106**	**-0.169**	**-0.165**	**-0.100**
	(0.075)	(0.094)	(0.040)	(0.047)	(0.040)	(0.030)	(0.053)	(0.031)	(0.037)
$\beta_2(*10^2)$	**-4.742**	-4.113	**-3.043**	-2.150	**-4.372**	**-2.035**	2.707	0.413	1.396
	(2.293)	(3.012)	(1.821)	(1.371)	(1.477)	(1.051)	(5.619)	(1.200)	(0.937)
$\beta_3(*10^2)$	-0.805	5.267	-0.228	-0.395	0.964	-0.183	0.095	2.003	0.785
	(3.513)	(3.445)	(1.460)	(1.266)	(1.288)	(1.031)	(1.129)	(1.319)	(0.768)
ϕ_1	-0.261	-0.094	**-0.126**	**-0.151**	-0.096	0.047	0.137	**0.196**	**0.265**
	(0.195)	(0.067)	(0.059)	(0.068)	(0.081)	(0.044)	(0.180)	(0.055)	(0.075)
ϖ_0	**4.226**	**11.104**	**3.351**	**1.660**	**2.290**	**1.227**	**1.936**	**1.980**	**0.786**
	(0.892)	(2.349)	(0.684)	(0.392)	(0.368)	(0.286)	(0.370)	(0.291)	(0.093)
b_1	**0.961**	**0.790**	**0.602**	**0.743**	**0.571**	**0.579**	0.233	0.041	0.236
	(0.288)	(0.355)	(0.195)	(0.246)	(0.175)	(0.194)	(0.450)	(0.038)	(0.223)
$\ln(L)$	-3211.12	-3799.04	-2951.22	-2508.74	-2717.15	-2262.90	-2479.04	-2412.45	-1878.81
BIC	6472.750	7648.577	5952.934	5067.986	5484.810	4576.294	5008.570	4875.404	3808.112
AIC	6436.259	7612.086	5916.443	5031.495	5448.319	4539.803	4972.079	4838.913	3771.621

Table 5
Maximum Likelihood Estimates of the ARX(1)-ARCH(1)-Jump Model of Credit Spreads on Non-rebalancing Days

This table presents the maximum likelihood estimates of the ARX(1)-ARCH(1)-Jump model of log-credit spreads for the period 01/02/1997 through 08/30/2002. The estimated model is

$$\begin{aligned}\ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 (1 - D_{1,t-1}) \ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t, \end{aligned} \tag{46}$$

where $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise, $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is the lagged changes in the slope of the yield curve, and Δr_{t-1} is the lagged changes in the interest rates. The disturbance ϵ_t has mean zero and is a mixture of two normal distributions: one is $N\left(-\lambda_t \mu_J, h_t^2\right)$ with probability $(1-\lambda_t)$ in the event of no jumps and the other is $N\left((1-\lambda_t)\mu_J, h_t^2 + \sigma_J^2\right)$ with probability λ_t. h_t^2, the conditional variance of ϵ_t in the no-jump state, is assumed to follow an ARCH(1) process:

$$h_t^2 = \varpi_0 + b_1 (1 - D_{1,t-1})\, \epsilon_{t-1}^2. \tag{47}$$

The jump probability $\lambda_t = \exp(p_0 + p_1 * VIX_{t-1}) / (1 + \exp(p_0 + p_1 * VIX_{t-1}))$. The asymptotic heteroscedasticity-consistent standard errors are in parentheses. Bold numbers indicate significance at the 10% level.

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
μ_0	**-0.070**	-0.001	-0.018	-0.016	-0.029	-0.027	0.006	0.039	0.025
	(0.040)	(0.057)	(0.033)	(0.024)	(0.032)	(0.019)	(0.035)	(0.034)	(0.023)
β_1	-0.049	-0.048	**-0.063**	**-0.051**	**-0.074**	**-0.062**	**-0.108**	**-0.133**	**-0.076**
	(0.034)	(0.042)	(0.025)	(0.020)	(0.025)	(0.016)	(0.031)	(0.030)	(0.018)
$\beta_2(*10^2)$	**-3.054**	**-3.68**	**-1.567**	**-1.683**	-1.524	**-1.282**	0.316	0.030	0.572
	(1.277)	(1.735)	(0.978)	(0.662)	(0.988)	(0.553)	(1.026)	(0.824)	(0.614)
$\beta_3(*10^2)$	-1.148	**2.458**	**-1.268**	-0.714	0.624	-0.529	0.072	1.670	**0.989**
	(0.932)	(1.386)	(0.683)	(0.522)	(0.694)	(0.447)	(1.047)	(1.341)	(0.535)
ϕ_1	**-0.243**	**-0.133**	**-0.114**	**-0.133**	-0.045	-0.015	**0.092**	**0.168**	**0.215**
	(0.033)	(0.04)	(0.04)	(0.047)	(0.034)	(0.068)	(0.047)	(0.052)	(0.035)
ϖ_0	**1.311**	**2.258**	**0.771**	**0.411**	**0.798**	**0.280**	**1.122**	**1.305**	**0.478**
	(0.136)	(0.236)	(0.076)	(0.037)	(0.082)	(0.026)	(0.119)	(0.139)	(0.05)
b_1	**0.380**	**0.324**	**0.369**	**0.365**	**0.373**	**0.383**	**0.140**	**0.047**	**0.073**
	(0.063)	(0.06)	(0.082)	(0.07)	(0.05)	(0.079)	(0.049)	(0.02)	(0.028)
p_0	**-4.634**	**-3.604**	**-3.345**	**-5.032**	**-4.132**	**-4.502**	**-5.679**	**-6.124**	**-5.471**
	(0.649)	(0.588)	(0.554)	(0.658)	(0.685)	(0.565)	(1.195)	(1.383)	(1.000)
p_1	**0.077**	**0.059**	**0.049**	**0.090**	**0.065**	**0.087**	**0.086**	**0.103**	**0.112**
	(0.023)	(0.02)	(0.02)	(0.022)	(0.024)	(0.021)	(0.03)	(0.035)	(0.027)
μ_J	1.140	-0.09	0.216	0.557	0.386	**0.769**	1.207	1.070	0.443
	(0.912)	(0.827)	(0.426)	(0.584)	(0.466)	(0.329)	(1.205)	(1.203)	(0.312)
σ_J	**8.708**	**10.146**	**5.283**	**5.027**	**4.667**	**3.346**	**5.015**	**4.152**	**2.189**
	(1.497)	(1.286)	(0.725)	(0.762)	(0.664)	(0.533)	(2.182)	(1.586)	(0.551)
$\ln(L)$	-2803.19	-3329.02	-2564.66	-2003.22	-2428.47	-1844.43	-2265.01	-2279.74	-1736.64
BIC	5685.738	6737.38	5208.669	4085.796	4936.287	3768.203	4609.365	4638.831	3552.626
AIC	5628.395	6680.03	5151.326	4028.453	4878.944	3710.860	4552.022	4581.487	3495.283

Table 6
Model Diagnostic Tests

Table 6 presents various model diagnostic tests for the estimated ARX(1)-ARCH(1)-Jump model and the nested ARX(1)-ARCH(1) model as reported in Table 5. The marginal significance level of the corresponding test statistics are reported in square brakets.

Spread Index	Model	χ_2 (19)	ρ_ϵ (1)	ρ_{ϵ^2} (1)	Skewness	Kurtosis	Normality Test
	ARCH	403.1	-0.1	0.071	0.28	15.95	25.66
AA-AAA rated		[0.00]	[0.00]	[0.005]	[0.73]	[0.00]	[0.00]
1-10 Yrs	Jump-ARCH	34.41	-0.096	0.059	-0.037	0.779	4.52
		[0.02]	[0.00]	[0.015]	[0.76]	[0.06]	[0.1]
	ARCH	530.1	-0.072	0.035	0.00	11.29	31.18
AA-AAA rated		[0.00]	[0.004]	[0.102]	[0.99]	[0.00]	[0.00]
10-15 Yrs	Jump-ARCH	60.61	-0.07	0.032	0.06	0.424	6.11
		[0.00]	[0.006]	[0.117]	[0.52]	[0.024]	[0.05]
	ARCH	424.3	-0.038	0.004	0.24	13.32	24.36
AA-AAA rated		[0.00]	[0.08]	[0.44]	0.72	[0.00]	[0.00]
15+ Yrs	Jump-ARCH	31.96	-0.048	0.028	0.059	0.484	6.24
		[0.03]	[0.038]	[0.15]	[0.52]	[0.01]	[0.04]
	ARCH	489.6	-0.05	0.034	0.33	24.8	31.3
BBB-A rated		[0.00]	[0.029]	[0.11]	[0.77]	[0.00]	[0.00]
1-10 Yrs	Jump-ARCH	40.24	-0.022	0.058	-0.024	0.618	7.55
		[0.003]	[0.21]	[0.02]	[0.8]	[0.01]	[0.02]
	ARCH	242.1	-0.04	0.058	0.26	11.99	26.8
BBB-A rated		[0.00]	[0.06]	[0.02]	[0.67]	[0.00]	[0.00]
10-15 Yrs	Jump-ARCH	26.48	-0.016	0.046	0.004	0.65	7.2
		[0.12]	[0.28]	[0.04]	[0.97]	[0.014]	[0.027]
	ARCH	407.5	-0.074	0.024	0.17	20.93	19.16
BBB-A rated		[0.00]	[0.003]	[0.188]	[0.87]	[0.00]	[0.00]
15+ Yrs	Jump-ARCH	31.05	-0.008	0.038	0.045	0.842	8.19
		[0.04]	[0.38]	[0.08]	[0.7]	[0.01]	[0.02]
	ARCH	123.5	-0.058	-0.005	-0.035	21.55	7.63
BB rated		[0.00]	[0.02]	[0.43]	[0.98]	[0.006]	[0.02]
	Jump-ARCH	21.56	-0.013	0.006	-0.047	0.417	4.65
		[0.31]	[0.32]	[0.41]	[0.6]	[0.03]	[0.098]
	ARCH	74.88	-0.042	0.005	-0.302	11.72	6.62
B rated		[0.00]	[0.06]	[0.43]	[0.7]	[0.01]	[0.04]
	Jump-ARCH	13.13	-0.009	0.003	-0.086	0.304	4.16
		[0.83]	[0.37]	[0.45]	[0.26]	[0.05]	[0.12]
	ARCH	90.65	-0.079	-0.004	-0.01	7.2	9.57
C rated		[0.00]	[0.002]	[0.43]	[0.98]	[0.002]	[0.008]
	Jump-ARCH	17.46	-0.034	0.024	0.022	0.314	2.2
		[0.56]	[0.1]	[0.19]	[0.8]	[0.14]	[0.33]

Table 7
Maximum Likelihood Estimates of the AR(1)X-GARCH(1,1)-Jump Model of Credit Spreads on Non-rebalancing Days

This table presents the maximum likelihood estimates of the AR(1)X-GARCH(1,1)-Jump model of log-credit spreads for the period 01/02/1997 through 08/30/2002. The estimated model is

$$\begin{aligned}\ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 \ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3\, \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t,\end{aligned}$$

where $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is the lagged changes in the slope of the yield curve, and Δr_{t-1} is the lagged changes in the interest rates. The disturbance ϵ_t has mean zero and is a mixture of two normal distributions: one is $N\left(-\lambda_t\mu_J, h_t^2\right)$ with probability $(1-\lambda_t)$ in the event of no jumps and the other is $N\left((1-\lambda_t)\,\mu_J, h_t^2+\sigma_J^2\right)$ with probability λ_t. h_t^2, the conditional variance of ϵ_t in the no-jump state, is assumed to follow an GARCH(1) process:

$$h_t^2 = \varpi_0 + b_1\,\epsilon_{t-1}^2 + b_2\, h_{t-1}^2. \tag{48}$$

The jump probability $\lambda = \exp\left(p_0 + p_1 * VIX_{t-1}\right) / \left(1 + \exp\left(p_0 + p_1 * VIX_{t-1}\right)\right)$. The asymptotic heteroscedasticity-consistent standard errors are in parentheses. Bold numbers indicate significance at the 5% level.

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
μ_0	-0.096	0.014	-0.013	-0.014	-0.011	-0.005	0.007	0.035	0.027
	(0.040)	(0.057)	(0.034)	(0.023)	(0.032)	(0.020)	(0.035)	(0.034)	(0.023)
Rus	**-0.062**	-0.049	**-0.064**	**-0.051**	**-0.076**	**-0.069**	**-0.117**	**-0.134**	**-0.077**
	(0.030)	(0.043)	(0.025)	(0.019)	(0.025)	(0.017)	(0.030)	(0.029)	(0.018)
$slope$	**-3.354**	**-3.646**	-1.597	**-1.977**	**-2.494**	**-1.369**	0.126	0.036	0.720
	(1.202)	(1.740)	(1.008)	(0.653)	(0.950)	(0.561)	(1.062)	(1.22)	(0.617)
r	-1.451	**3.011**	**-1.527**	-0.730	0.960	-0.701	0.456	1.202	0.677
	(0.935)	(1.343)	(0.654)	(0.504)	(0.670)	(0.443)	(1.059)	(1.332)	(0.545)
$AR(1)$	**-0.269**	**-0.144**	**-0.132**	**-0.113**	-0.058	-0.025	**0.093**	**0.142**	**0.187**
	(0.027)	(0.029)	(0.029)	(0.030)	(0.032)	(0.034)	(0.046)	(0.05)	(0.034)
ω	**0.696**	**2.018**	**0.416**	**0.151**	**0.245**	**0.148**	**0.336**	**0.458**	**0.109**
	(0.226)	(0.263)	(0.196)	(0.037)	(0.094)	(0.036)	(0.144)	(0.142)	(0.039)
$ARCH(1)$	**0.296**	**0.288**	**0.234**	**0.225**	**0.220**	**0.346**	**0.099**	**0.037**	**0.077**
	(0.067)	(0.053)	(0.115)	(0.047)	(0.059)	(0.081)	(0.047)	(0.015)	(0.023)
$GARCH(1)$	**0.331**	0.051	0.344	**0.472**	**0.561**	**0.271**	**0.625**	**0.621**	**0.688**
	(0.150)	(0.031)	(0.243)	(0.082)	(0.112)	(0.118)	(0.129)	(0.108)	(0.074)
P_0	-4.666	-3.570	-3.009	-4.615	-3.745	-4.399	-4.862	-5.916	-4.974
	(0.804)	(0.598)	(0.621)	(0.734)	(0.799)	(0.654)	(1.194)	(1.421)	(1.000)
VIX	**0.069**	**0.058**	0.033	**0.070**	0.031	**0.080**	0.052	**0.094**	**0.092**
	(0.028)	(0.020)	(0.024)	(0.025)	(0.029)	(0.023)	(0.030)	(0.035)	(0.028)
μ_J	1.506	-0.084	-0.044	0.742	0.456	0.721	1.030	1.134	0.436
	(1.151)	(0.767)	(0.445)	(0.587)	(0.687)	(0.399)	(1.282)	(1.329)	(0.334)
σ_J	**9.049**	**10.044**	**5.285**	**4.869**	**5.226**	**3.039**	**5.362**	**4.275**	**2.171**
	(1.956)	(1.319)	(0.789)	(0.879)	(0.878)	(0.502)	(2.374)	(1.739)	(0.600)
$\ln(L)$	-2788.64	-3331.78	-2557.77	-1987.57	-2403.78	-1820.96	-2259.45	-2281.07	-1731.90
BIC	5663.847	6750.139	5202.105	4061.705	4894.138	3728.485	4605.48	4648.71	3550.376
AIC	5601.282	6687.574	5139.540	3999.140	4831.573	3665.920	4542.911	4586.15	3487.811

Table 8
Out-of-Sample Forecast Comparison

This table presents the root mean squared error (RMSE) and the mean absolute error of the actual credit spread and the one-step-ahead predicted credit spread from the ARX-ARCH-Jump model and the nested ARX-ARCH model. Starting from January of 2000, on the first non-rebalancing trading day of each month, the parameters of the model are estimated using all past observations. The parameters are held constant for the one-step-ahead prediction within the month. The initial sample period runs from January, 1997 through December, 1999 and the forecast period is from January, 2000 through August, 2002. The data used are daily observations of Merrill Lynch credit spread indices on non-rebalancing days. RW indicates the simple random walk model of credit spreads. Bold numbers indicate the smallest value.

Panel A: Root Mean Squared Error

Model	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Jump-ARCH	**1.447**	**2.605**	**1.909**	2.181	2.708	**2.233**	8.077	**11.081**	**17.494**
ARCH	1.465	2.634	1.919	2.204	2.769	2.24	**8.021**	11.101	17.815
RM	1.497	2.657	1.929	**2.118**	**2.701**	2.255	8.219	11.318	17.686

Panel B: Mean Absolute Error

Model	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Jump-ARCH	**0.905**	**1.559**	1.17	1.319	1.783	**1.341**	4.543	**7.08**	**11.8**
ARCH	0.949	1.618	1.185	1.389	1.844	1.357	**4.525**	7.092	12.011
RW	0.915	1.561	**1.166**	**1.267**	**1.756**	1.356	4.56	7.268	11.974