Stock Returns and Volatility: Pricing the Long-Run and Short-Run Components of Market Risk

Tobias Adrian and Joshua Rosenberg*

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*Both authors are with Capital Markets Research, Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045.

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Corresponding Author: Tobias Adrian, email tobias.adrian@ny.frb.org, web http://nyfedeconomists.org/adrian/.

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Abstract

In this paper, we examine the importance of market volatility dynamics for asset pricing, focusing on a decomposition of volatility into a short-run, quickly mean reverting component and a long-run, slowly evolving component. Within an ICAPM model, we show that the stochastic discount factor is a function of both the long- and short-run volatility components as well as the market return.

The major results of the paper concern the pricing of short- and long-run volatility components in the 25 size and book-to-market sorted portfolios. In monthly cross-sectional regressions for 1963-2003, we find that both the long-run and short-run components of market volatility are highly significant asset pricing factors with a negative price of risk. As the long- and short-run volatility components are negatively correlated with the market return, this finding is consistent with the prediction of the ICAPM that investors hedge innovations in volatility. The price of risk of the long-run volatility component is six times higher than the price of risk of the short-run component. When we include the Hml and Smb factors of Fama and French (1993) in the cross-sectional regression, the volatility components stay highly significant, whereas Hml and Smb are insignificant.

We also split our sample to study the period 1986-2003, which allows us to compare our three-factor model to the one based on market implied volatility (VIX) as proposed by Ang, Hodrick, Xing, and Zhang (2004). We find that our volatility estimates are highly correlated with implied volatility. However, in cross-sectional pricing, our three-factor model generates a 22% lower J-statistic than the model with the VIX and the market return as pricing factors. In addition, over the shorter sample period, our model significantly outperforms the Fama-French three-factor model, with a J-statistic that is 15% smaller.

The long-run volatility factor is highly correlated with macroeconomic measures such as the growth rate of industrial production (-29%), changes in the unemployment rate (23%), and measures of macroeconomic uncertainty, showing that the long-run component is counter-cyclical. The short-run volatility component is highly correlated with measures of market liquidity and interest rates.

Key words: asset pricing, stochastic volatility, ICAPM
JEL classification: G10, G12

1 Introduction

It is well documented that the volatility of the stock market is stochastic (see Bollerslev, Engle, Nelson (1994) and Ghysels, Harvey, Renault (1996)). In equilibrium settings such as Merton's (1973) ICAPM or the CIR model by Cox, Ingersoll, Ross (1985), shocks to the volatility process become pricing kernel state variables. The relationship between expected market returns and market volatility is then determined by two forces. From a static point of view, there is the risk-return trade-off: risk-averse investors demand a higher risk premium if volatility is higher. However, from a dynamic point of view, investors price shocks to volatility that are correlated with shocks to the market return.

Only few papers have closely examined volatility as a pricing factor in a cross-sectional pricing context (see, in particular, Ang, Hodrick, Xing, and Zhang (2004)). We extend this analysis by modeling log-volatility as the sum of a short-run and a long-run component, each of which may have its own risk premium. Our equilibrium ICAPM setting predicts that investors hedge volatility risk, and that asset expected returns depend on their covariance with innovations to the short-run and the long-run volatility components as well as the market return. Intuitively, investors might react differently to volatility shocks that are expected to be short-lived (e.g. news announcements, transitory liquidity events) compared to long-lived shocks (e.g. changes in the economic outlook, structural changes). Our approach makes it possible to identify and analyze long-run volatility shocks that are likely to be most relevant for expected returns.

Using a variety of estimation methods, Engle and Lee (1999), Alizadeh, Brandt and Diebold (2002), Bollerslev and Zhou (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), and Chacko and Viceira (2003) find that two-factor volatility specifications significantly outperform one-factor models. Two-factor models are designed to allow for shocks with different levels of persistence to drive the volatility process. Hence, this feature is potentially important if underlying economic forces that determine volatility operate at different frequencies. In a two-factor model, volatility can exhibit persistent deviations from the unconditional average, while also allowing for fast mean reversion to recent volatility levels.

Our focus on low-frequency movements is related to the recent literature examining the impact of permanent components of consumption growth and dividend growth on asset pricing (see, in particular, Barsky and DeLong (1993), Bansal and Lundblad (2002), Bansal and Yaron (2004), Bansal, Dittmar, Lundblad (2004), Bansal, Khatchatrian and Yaron (2004)). In comparison, an advantage of our work is that we only use financial market data as asset pricing factors, which is available at a high frequency for long time periods.

Previous papers have had difficulty empirically identifying the risk-return trade-off in the time-series. This does not come as a surprise from a theoretical point of view: Abel (1988) and Gennotte and Marsh (1993) show in equilibrium settings with one-factor stochastic volatility processes that the market return is not necessarily positively related to the market variance in the time-series. In their theory, this is due to the dynamic optimization of rational investors who hedge changes in the investment opportunity set. In particular, volatility is predictable and, in equilibrium, provides information about expected returns. Therefore, changes in volatility change the investment opportunity set and should be priced. We extend this intuition to a two-factor ICAPM that we discuss in section 2 and analyze in detail in the appendix.

When we estimate a two-factor volatility model over the period 1963-2003 in section 3, we do find that the market return is positively related to its variance in time-series estimation, which is the standard risk-return trade-off. We also find it is negatively related to the short- and long-run volatility components, consistent with the prediction of the ICAPM that investors hedge changes in volatility and price volatility risk. The half-life of a shock to the long-run component is 8.5 months, whereas the half-life of a shock to the short-run component is only 5.2 days.

Return innovations are found to be negatively correlated with both the short- and long-run volatility components. This asymmetry (sometimes called the leverage effect) has been documented in one-factor contexts by French, Schwert and Stambaugh (1987), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Zakoian (1994), Andersen, Benzoni, and Lund (2002), Eraker, Johannes, and Polson (2003), and Brandt and Kang (2004) among others.

We also decompose daily squared returns into short- and long-run component using the Hodrick-Prescott (1997) filter. This provides a non-parametric measure of the volatility components that we use for robustness checks of our results. A key insight from this exercise is that the non-parametric long-run component is very highly correlated with our model-based estimate. Thus, our empirical results do not appear to be particularly sensitive to the volatility model specification used for the decomposition.

Our main empirical results concern the pricing of short and long-run volatility in the 25 size and book-to-market sorted portfolios. The empirical model that emerges from our theory is a three-factor model with the market excess return and the short- and long-run volatility components as pricing factors.

In section 4, we discuss the findings from monthly cross-sectional regressions for 1963-2003. We find that both the long- and short-run components of market volatility are highly significant asset pricing factors consistent with the prediction of the ICAPM that investors hedge innovations in volatility. The ICAPM also predicts that factors that are negatively correlated with the market (such as short- and long-run volatility) have a negative risk premium, which is what we find in our estimation results.

The price of risk of the long-run volatility factor is orders of magnitudes higher than the price of risk of the short-run component (as it is expected to last longer). The large price of risk of the long-run component implies a large Sharpe ratio for investment strategies taking advantage of this component's low-frequency movements.

We also split our sample to study the period 1986-2003, which allows us to compare our three-factor model to the one based on market implied volatility (VIX) as proposed by Ang, Hodrick, Xing, and Zhang (2004). We find that our volatility estimates are highly correlated with implied volatility. However, in cross-sectional pricing, our three-factor model generates 22% lower J-statistic than the model with the VIX and the market return as pricing factors. We report the findings of the split sample in section 5.

Our cross-sectional asset pricing results are closest to those reported by Ang, Hodrick, Xing, and Zhang (2004). They use the VIX as a pricing factor in cross-sectional regressions (see our section 5 for a comparison). They also decompose volatility into systematic and idiosyncratic components, while we decompose volatility into short- and long-run components. Other papers that focus on the cross-sectional pricing implications of stochastic volatility include Engle, Bollerslev and Wooldridge

(1988), Harvey (1989) and Schwert and Seguin (1990). These authors estimate static CAPMs with stochastic market return volatility. They specify time-variation in CAPM betas as a function of aggregate volatility, but do not examine the pricing implications of the hedging demands that result from stochastic volatility environments, which is the focus of our paper.

In section 6, we relate the volatility factors to macroeconomic and financial variables. It is well known that the Fama-French Hml and Smb factors are not strongly related to macroeconomic variables such as production growth, unemployment, inflation, and measures of credit risk. Furthermore, macroeconomic risk factors do not generally yield satisfactory cross-sectional asset pricing results. We find that the long-run component of market volatility is highly negatively correlated with the growth rate of industrial production (29%), highly negatively correlated with changes in the unemployment rate (23%), and positively correlated with the credit spread (14%) All of these correlations are significant at the 1% level. The long-run volatility factor thus appears to be countercyclical. The short-run volatility factor is more highly correlated with market liquidity measures and the Hml and Smb factor. Section 7 concludes with a review of the main results and suggestions for future research.

2 An ICAPM with two-factor volatility

In this section, we present an ICAPM with a two-factor stochastic volatility process that is developed in more detail in the appendix. We assume that the instantaneous market excess return dp_t^M evolves according to the following diffusion:

$$dp_t^M = \mu_t^M dt + \sqrt{v_t} dZ_t^M \tag{1}$$

where Z_t^M is a standard Brownian motion, v_t is the instantaneous, stochastic variance of the market return, and μ_t^M is the drift. Log-volatility is the sum of two components s and q:

$$\log \sqrt{v_t} = s_t + q_t \tag{2}$$

$$ds_t = -\kappa^s s_t dt + \chi^s dZ_t^s \tag{3}$$

$$dq_t = \kappa^q \left(\bar{q} - q_t\right) dt + \chi^q dZ_t^q \tag{4}$$

where Z^q and Z^s are Brownian motions that are correlated with each other and the market innovation Z^M. The two components of log-volatility have potentially different rates of mean reversion. Without loss of generality, let q be the slowly mean-reverting component and s be the quickly mean reverting component. Both s and q are Ornstein-Uhlenbeck processes and are therefore conditionally normal. As a consequence, log-volatility is conditionally normal. The persistence of the short-run component s is given by the parameter of mean-reversion, $\kappa^s \geq 0$. Higher parameter values correspond to faster mean-reversion back to zero. The long-run component q mean-reverts to a constant $\bar{q}$ at rate κ^q. When $\kappa^q = 0$, q_t is non-stationary.

By summing equations (3) and (4), we obtain an expression for the evolution of the log-volatility of the market excess return:

$$d \ln \sqrt{v_t} = \kappa^s \left(q_t^* - \ln \sqrt{v_t}\right) dt + \chi^s dZ_t^s + \chi^q dZ_t^q \tag{5}$$

where $q_t^* = \frac{\kappa^q}{\kappa^s}\bar{q} + \left(1 - \frac{\kappa^q}{\kappa^s}\right) q_t$. The logarithm of the standard deviation of the market is a process that is reverting around the stochastic mean q_t^* at rate κ^s. Our model can be thought of as a generalized one-factor volatility model with a (slowly evolving) stochastic mean.

The drift of the market return process μ^M is an endogenous variable. In order to show how it depends on the state variables of the economy, additional assumptions have to be made.

Investors are assumed to have HARA preferences, so that the equilibrium arguments of Merton (1973) can be directly applied. Adrian and Rosenberg (2004) show that the asset pricing implications that we derive apply up to a first-order approximation if investors have Epstein-Zin-Weil preferences.

For simplicity, it is assumed that the goods market clears, i.e. investor consumption equals dividends at any point in time, and that the risk-free asset is in zero net supply. This assumption simplifies the asset pricing implications somewhat relative to a model with positive supply of the risk-free rate and no goods-market clearing, as Adrian and Rosenberg (2004) show.

The key insight of Merton (1973) and Cox, Ingersoll, Ross (1984) is that state variables of the return generating process become state variables of the pricing kernel. In the appendix, we demonstrate that the equilibrium pricing kernel m is:

$$\frac{dm_t}{m_t} = -\left(\gamma_t dp_t^M + F_s ds_t + F_q dq_t\right) \tag{6}$$

where $F(s,q)$ is a function that depends on preferences and γ is the coefficient of relative risk aversion. The pricing kernel of our ICAPM economy consists therefore of three factors: the market return as well as the long- and short-run components of market volatility. As we assumed that the risk-free asset is in zero net supply, it follows that the risk-free rate is a function of the volatility factors, justifying our implicit assumption that the risk-free rate is not a state variable.

In a static setting that involves only the absence of arbitrage, Ross (1976a) shows that state variables of the payoff distribution become state variables of the return process. We emphasize the equilibrium models proposed by Merton (1973) and Cox, Ingersoll, Ross (1985) as the APT is inherently static, whereas our focus is the dynamic evolution of volatility risk premia.

For any asset i, expectation of the equilibrium excess return dp_t^i is then:

$$E_t\left[dp_t^i\right] = \gamma_t E\left[dp_t^i dp_t^M\right] + F_s E_t\left[dp_t^i ds_t\right] + F_q E_t\left[dp_t^i dq_t\right] \tag{7}$$

The expected excess return of the market portfolio thus depends on the variance of equilibrium excess returns (v) as well as the covariance of the market portfolio with the two volatility factors s and q. In general, the dependence of $E_t\left[dp_t^M\right]$ on s and q can be non-linear. In one-factor stochastic volatility setups, Abel (1988) and March and Genotte (1993) derive closed form solutions to the equilibrium market return with stochastic volatility. In our two-component setup, we can solve the model in closed form if we make the assumption that the two volatility components follow Ornstein-Uhlenbeck processes. In that case, both F_s and F_q are constants. For more general processes such as the ones specified in equations (3) and (4), F_s and F_q are both functions of s and q.

In work in progress, Tauchen (2004) derives the general equilibrium of an economy with a two-factor volatility process. The main difference to our model is the specification of the volatility process. Furthermore, Tauchen (2004) relates the stochastic volatility process to the volatility of the dividend process, whereas we take the evolution of the variance-covariance matrix of asset excess returns as given.

For the market return, the pricing relationship (7) reduces to:

$$E_t\left[dp_t^M\right] = \gamma_t v_t + F_s E_t\left[dp_t^M ds_t\right] + F_q E_t\left[dp_t^M dq_t\right] \tag{8}$$

The covariance of the market return with the first pricing kernel factor is the variance of the market return. Expected returns of the market thus depend of three elements: the static risk-return trade-off, and the risk premium due to the pricing of the hedging components of the short-run and long-run volatility factors. It will turn out that the market return is positively correlated with innovations in both s and q, reflecting the leverage effect. The relationship between expected returns and volatility is therefore ambiguous. From a static point of view, there is the risk-return trade-off: risk-averse investors demand a higher risk premium if volatility is higher. However, from a dynamic point of view, investors price shocks to volatility that are correlated with shocks to the market return.

3 The time-series of market risk and its components

In order to estimate the stochastic volatility process and its short-run and long-run components, we specify the following Egarch-components-in-mean model:

$$\begin{aligned} R_{t+1}^M &= \alpha_0 + \alpha_1 v_t + \alpha_2 q_t + \alpha_3 s_t + \sqrt{v_t}\varepsilon_{t+1} \qquad \text{(Egarch)} \\ &\varepsilon_t \sim N(0,1) \qquad \sqrt{v_t} = \exp(s_t + q_t) \\ s_{t+1} &= \alpha_4 s_t + \alpha_5 \varepsilon_{t+1} + \alpha_6\left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right) \\ q_{t+1} &= \alpha_7 + \alpha_8 q_t + \alpha_9 \varepsilon_{t+1} + \alpha_{10}\left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right) \end{aligned}$$

where R^M_{t+1} denotes the market excess return. To our knowledge, estimates for this specification have not yet been reported in the literature. Brandt and Jones (2002) propose a range-based two-component Egarch model that has two latent components similar to our s and q, but they do not specify that either of the components appear independently of market variance in the mean equation. The specification LL2V of Chernov, Gallant, Ghysels and Tauchen (2003) is very similar to the one that we are presenting here, but they only allow one of their volatility components to appear in the mean equation. Compared to the Garch-components model proposed by Engle and Lee (1999), our model allows for more skewness in the distribution of volatility as volatility is conditionally log-normal. In addition, our specification guarantees positive volatility estimates without any parameter restrictions, which facilitates estimation.

In section A.4 of the appendix, we use a result of Nelson (1990) to show that the Egarch model converges to the system of diffusions (3) and (4) together with the mean equation

$$dp^M_t = \alpha_0 + \alpha_1 v_t + \alpha_2 q_t + \alpha_3 s_t + \sqrt{v_t} dZ^M_t \tag{9}$$

In the continuous time model, there are three times-series shocks: Z^M, Z^s, Z^q. The beauty of Nelson's (1990) result is that this can be approximated by a discrete time filter using a single innovation. In the continuous time limit, the single shock of the Egarch approximation converges to three imperfectly correlated shocks: one for the mean equation, and one for each of the volatility equations, subject to a covariance restriction of equation (see equation (30) in the appendix).

In general, the hedging component of expected returns to the market depends on the preference specification of the economy (see equation (8)). The return equation of the continuous-time limit (9) can be interpreted as a first-order Taylor approximation to the true relationship, i.e.

$$\gamma_t v_t + F_s \rho^{Ms} + F_q \rho^{Mq} \simeq \alpha_0 + \alpha_1 v_t + \alpha_2 q_t + \alpha_3 s_t$$

Note that this approximation has a number of implications that are important for the interpretation of the results. First, the coefficient α_1 cannot be directly interpreted as the coefficient of absolute risk-aversion. Second, the estimates of α_2 and α_3 are likely to be imprecise, as per definition, v_t is highly correlated with s_t and q_t. Nevertheless, the specification makes clear that in a world with stochastic volatility, the mean-equation of the ICAPM includes terms related to the hedging demand in addition to the static-risk-return trade-off.

We estimate the model using daily market excess return data. We use the value-weighted cum-dividend CRSP portfolio as our measure of the stock market return, and the three month Treasury rate as the proxy for the risk-free rate. We estimate the Egarch-components model using daily data in order to improve the estimation precision.

The results of estimating the stochastic volatility model using the Egarch-components specification are presented in Table 1. Table 2 presents comparisons of the Egarch-components model to other commonly used Garch specifications. Our specification tests indicate that the model adequately captures volatility dynamics with a p-value greater than 10% for the Ljung-Box test on the standardized squared errors.

In the market excess return equation, the intercept α_0 is insignificant. Excess returns are positively related to the market return variance, with a coefficient estimate of 0.049. Both the coefficient on the long-run volatility component α_2 and the short-run volatility component α_3 have negative signs, but only the coefficient of the short-run component is statistically significantly different from zero.

Consistent with previous papers, we find a significant negative correlation between lagged returns and volatility. In our model, we uncover this "leverage effect" for both the short-run and long-run volatility components (the coefficients α_5 and α_9 respectively). It is the significance of the leverage effect that ultimately gives rise to the negative correlation between market returns and volatility innovations.

This asymmetry has been documented in one-factor contexts by French, Schwert and Stambaugh (1987), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Zakoian (1994), Andersen, Benzoni, and Lund (2002), Eraker, Johannes, and Polson (2003), and Brandt and Kang (2004) among others. Engle and Lee (1999) allow for an asymmetric relation between returns and the short-run component of volatility, but not the long-run component. To our knowledge, we are the first to document that both the short-run and long-run components of stock market volatility exhibit a leverage effect.[1]

The short-run volatility component has a coefficient on its own lag of 0.867, while the long-run component has a coefficient on its own lag of 0.996 (both are significant at the 1% level). The long-run component is therefore highly persistent, but not permanent (we reject the hypothesis that $\alpha_8 = 1$ at the 1% level). These estimates imply a half life of the permanent component of 179 trading days (or 8.5 months), whereas the half life of the transitory component is only 5.2 trading days.

Previous papers have had difficulty in empirically identifying the risk-return trade-off in the time-series of stock index returns. French, Schwert and Stambaugh (1987), Baillie and DeGennaro (1990), Campbell and Hentschel (1992) and Brandt and Kang (2004) find a positive but insignificant relationship between market risk and return; Glosten, Jagannathan and Runkle (1993), Turner, Startz and Nelson (1989) and Harvey (2001) find both a positive and a negative relationship between market risk and return depending on the model specification; whereas Campbell (1987) and Nelson (1991) find a significantly negative relationship. Ghysels, Santa-Clara, and Valkanov (2004) do find a positive risk-return trade-off by specifying the MIDAS estimator of volatility, but they do not take the hedging demand due to the time-variation of volatility into account. Scruggs (1998) finds a positive trade-off by introducing the risk-free rate as additional risk-factor, which is similar to what we are doing as the risk-free rate is a function of the volatility factors. Guo and Whitelaw (2004) find a positive relationship by estimating a structural model, and Lundblad (2004) finds a positive trade-off in the very long-run. Merton (1980) finds a positive trade-off by restricting estimation priors.

Our interpretation of these findings stems from equation (8). The expected market return might depend positively or negatively on the market variance, depending on the relative importance of the risk-return trade-off and the pricing of the hedging demand. All of the specifications find a significant leverage effect, translating into a negative relationship between variance innovations and market return innovations. Investors with a sufficiently large intertemporal elasticity of substitution hedge innovations to volatility risk. Our Egarch-components specification is able

[1] A recent paper focusing on the role of jumps in returns is Pan (2002), a paper studying jumps in both returns and volatility is Eraker, Johannes and Polson (2003).

to (at least partially) separate out these two effects as the volatility components are non-linear transformations of the total variance.

In Table 2, we report our Egarch-components model together with three alternative specifications. For each specification, we report the parameter estimates (together with the standard errors in parenthesis), the log-likelihood function, the Akaike and Schwarz information criteria, and the Ljung-Box Q-statistic of the standardized squared errors. In the Garch-GJR and the Egarch specifications, we include the market variance in the mean equation. In both models, the variance term is negative and insignificant. Nelson (1991), Glosten, Jagannathan and Runkle (1993), Turner, Startz and Nelson (1989), and Harvey (2001) all report a negative (but insignificant) risk-return trade-off in similar specifications. When we add a variance term in the mean equation of the Engle and Lee (1999) model, we find that long-run volatility component estimates are negative for part of the sample period. For this reason, we report the model without a variance term in the mean equation. Our Egarch-components-in-mean specification is the only one that detects a positive and significant risk-return trade-off.

Our Egarch-components specification achieves the highest log-likelihood (-11,664), followed by the Egarch (-11,761), the Garch-GJR (-11,806), and the Garch-components specification (-11,824). It is somewhat surprising that the Garch-components specification achieves a lower log-likelihood than the Garch-GJR model. The reason for that is that the inclusion of the market variance in the mean equation of the Engle-Lee model gives negative estimates of the variance process, leading us to report the specification without the variance in the mean equation.

The four different Garch specifications are then non-nested, so that we cannot report model comparisons based on the likelihood-ratio statistic. In order to compare the models nevertheless, taking the number of parameters into account, we also report the Akaike and Schwarz information criteria. For both criteria, our Egarch-components model achieves the lowest values, indicating that it is preferable to the other three specifications. All four specifications easily pass the Ljung-Box Q-test with p-values above 10% for 10 and 20 lags.

Table 3 reports summary statistics for the market excess return, its estimated variance, and the short-run (s) and long-run volatility components (q). In our sample, the average annual market excess return is 5.29% and average annualized market volatility is 14.02%. Annualized volatility ranges from a minimum of 3.8% to a maximum of 92.6% (the 1987 crash). In Figure 1, it appears that since 1999 volatility has remained above average for a protracted period.

While our volatility model is estimated at the daily frequency, our cross-sectional analysis uses monthly data. In order to construct monthly variance and volatility factor estimates, we average v, s, and q each month and multiply the average by 21, which is the average number of trading days in our sample. This time aggregation reduces the skewness and kurtosis of returns and return volatility (e.g. panel A vs. B). In Panel A, we report the correlation matrix of daily market excess returns with daily measures of v, s, and q. The excess market return is strongly negatively correlated with its variance and the short-run factor, but only weakly correlated with the low-frequency volatility factor q. Aggregation to the monthly frequency increases the return correlation with the short-run volatility components, but does not affect the correlation with the long-run component. When we examine volatility innovations, we find that the long-run component is strongly negatively correlated with returns (-.402, not reported). Our model predicts that the innovation correlation is what determines pricing in the cross-section.

The long-run volatility factor q is plotted in Figure 2. Over the sample period 1963-2003, it appears that the low-frequency component of stock market volatility has increased. This finding is in contrast to the fact that the volatility of macroeconomic variables such as GDP and consumption has decreased since the mid-1980's (a fact that is explored in an asset pricing context by Lettau, Ludvigson, and Wachter (2004)). As we are not modelling fundamentals explicitly, we do not address this recent divergence of financial and macroeconomic volatility, but it seems to be an area worth exploring in future research.

The estimates of conditional variance v and the short- and long-run volatility factors s and q are from the Egarch-components specification. In order to ensure that our cross-sectional asset pricing results reported in later sections do not rely on this particular model specification, we construct an alternative measure of variance. Following Andersen, Diebold, and Labys (2003), we compute "realized" variance as daily squared returns. In order to decompose the realized volatility into a short-run and long-run component, we apply the Hodrick-Prescott (1997) filter to the square root of the logarithm of daily squared returns.[2]

Panel A of Table 3 reveals that, on average, daily squared returns are nearly the same as daily conditional variance (0.80 versus 0.78). The annual volatility implied by the daily squared returns is 14%, versus 14.2% for the conditional volatility. The standard deviation, skewness, and kurtosis of daily squared returns are markedly higher than corresponding measures from the conditional model, but these differences in higher order moments become smaller in the monthly sample. The standard deviation of realized variance is roughly twice as high as the standard deviation of conditional variance, the skewness is 3.6 times as high, and the kurtosis is nearly 10 times as high. This is likely due to the fact that daily squared returns are a noisy measure of variance, even when they are aggregated to a monthly variance measure. In contrast, the volatility estimates from the Egarch-components model are conditional expectations.

In Figure 1, the conditional variance is plotted together with the 252-day moving average of realized volatility (to be precise, the square root of the 252-day moving average of daily squared returns). Conditional volatility appears to be unbiased. As shown in Panels A and B of Table 3, the correlation between monthly conditional variance and realized variance is 82.5%, while the correlation is only 28.9% for daily data.

We decompose daily squared returns into a short-run and a long-run volatility component. Figure 2 plots the daily estimates of the long-run component from the Egarch-components model and the HP-filtered daily squared returns. The two series track each other very well, their correlation is 93.3% for the monthly data, and 61.6% for daily data. This finding suggests that the decomposition of log-volatility of market excess returns into a short-run and a long-run component from the Egarch-components model is not specific to our model specification. Decomposing daily squared returns with the HP-filter provides an estimate of the long-run component that is very similar. The HP-filtered long-run component does appear to be slightly more volatile (7.94 versus 5.86 monthly, Table 3 Panel B) and more skewed (0.116 versus 0.113), but has lower kurtosis (2.52 versus 2.86).

[2] For a related application of frequency domain filtering of realized variance to volatility forecasting, see Bollerslev and Wright (2001)

4 The cross section of expected returns 1963-2003

In Table 4, we report summary statistics for 25 value-weighted size and book-to-market sorted portfolios from Fama and French (1992 and 1993). The monthly portfolio returns and the risk-free rate can be downloaded from Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.frenc The first two panels show the failure of the CAPM over the sample period: high average returns are not generally associated with high betas. The last two panels report univariate factor loadings with respect to the short- and long run volatility factors. For both the short-run and the long-run factors, larger and lower book-to-market stocks are generally associated with more negative loadings.

Equation (7) gives an expression for the vector of expected stock returns in equilibrium. Expected returns depend on the covariance matrix of individual stocks with the market portfolio, and the covariance of each stock with the short- and long-run volatility components s and q. In order to estimate the cross-section of expected returns, we derive a beta-representation in section A.5 of the appendix. We show that the expected excess return of stock i at time t is given by:

$$E_t\left[dp_t^i\right] = \beta_t^{iM}\lambda_t^M + \beta_t^{is}\lambda_t^s + \beta_t^{iq}\lambda_t^q \tag{10}$$

where β^{ik} denotes the conditional, partial regression coefficient of asset i on the change of factor k, and λ^k denotes the conditional price of risk of factor k. Note that the price of risk of the three factors is potentially time-varying, as it depends on the variance-covariance matrix of the pricing kernel.

The cross-sectional Fama-MacBeth regressions that we present in the next section allow for time-variation of the prices of risk λ. In our empirical implementation, we implicitly assume that the volatility of each stock's systematic risk is proportional to the variance of the market return, and that the correlation of individual stocks with the market return and the volatility factors are constant, so that betas are constant.

Table 5 reports the summary statistics of the cross-sectional regressions for the 25 size and book-to-market sorted portfolios. The benchmark CAPM and Fama-French 3-factor models are presented in columns (i) and (ii), respectively. The J-statistic is 102.2 for the CAPM, and 89.1 for the Fama-French model. As reported by Fama and French (1992, 1993), we find that both the CAPM and the Fama-French models are rejected in cross-sectional asset pricing tests. Furthermore, the Smb factor has an insignificant risk premium, probably reflecting the fact that the size premium diminished in the 1980's.

Column (iii) reports the results of cross-sectional regressions with the market return and volatility innovations as pricing factors. The total variance factor is significant at the 4% level, and has a negative price of risk. This finding confirms the result presented by Ang, Hodrick, Xing and Zhang (2004) that the market variance is an important pricing factor with a negative price of risk. The J-statistic of model (iii) improves relative to the CAPM, but is larger than the pricing errors of the Fama-French model.

Our preferred model is reported in column (iv). In addition to the market return, the short-run and long-run volatility innovations are included in the cross-sectional regressions. The long-run volatility component is significant at the 1% level, the short-run volatility component is significant at the 1.2% level, and both have a negative price of risk. The price of risk of the long-run volatility factor is 6.5 times higher than the price of risk of the short-run volatility factor, reflecting the fact that long-run volatility risk is more permanent. The J-statistic is slightly lower than the Fama-French model (89.1 for the Fama-French model versus 88.5 for the three-factor volatility), and significantly lower than the CAPM and the model with the market return and total market variance as pricing factors (102.2 and 97.4, respectively). The price of risk of the market factor is close to the average market excess return over the sample period.

Column (v) reports the results for a 4-factor model with the market return, the total market variance, and the long- and short-run volatility components as pricing factors. Only the short-run and long-run volatility factors are significant. The J-test is marginally improved compared to our preferred three factor model with only the short- and long-run volatility components.

In column (vi), we show estimates for our preferred model augmented with the Hml and Smb factors. Hml and Smb are insignificant, whereas s and q stay highly significant. The pricing errors do decrease compared to model (iv), suggesting that Hml and Smb might capture some additional sources of priced risk.

We also analyze the characteristics of realized variance as a factor in column (vii) and realized variance decomposed into short and long run components using an HP-filter in column (viii). As expected, realized variance has a negative price of risk. However, its pricing performance is inferior to conditional variance with a 4% higher J-statistic. When realized variance is decomposed into components, the long run component has a statistically significant negative price of risk. The components model using realized variance has a pricing error 5% lower than when total realized variance is used. This, once again, suggests that the long and short run volatility components are priced differently in the cross-section.

5 The cross section of expected returns 1986-2003

In Table 6, we present summary statistics for the sub-sample 1986-2003, together with summary statistics for option implied volatility using VIX. In order to make the implied volatility from the VIX a comparable to our daily variance measure v, we report summary statistics for $VIX^2/365$.

There are a number of important differences between the implied volatility measure from the VIX and our estimated volatility v. The implied volatility is derived from options on the S&P 100, whereas our stock market portfolio encompasses the whole universe of CRSP stocks. We are measuring the volatility of the market excess return, whereas the VIX is a measure of the volatility of the market return. Finally, the VIX is computed using the Black-Scholes option pricing formula, which only provides unbiased estimates of expected volatility under fairly restrictive assumptions.

Despite all of these differences, the correlation of our daily variance measure v and the $VIX^2/365$ is 82.6% on a monthly basis. However, several significant differences are apparent based on their moments. The mean of the $VIX^2/365$ is slightly lower than the mean of v, and it is less volatile. Stock market implied volatilities are known to be biased predictors of future volatility due, for example, to a "volatility risk premium". This bias is documented in Fleming (1999) and Rosenberg (2000).

There are also some notable differences between the whole sample period 1963-2003 and 1986-2003. In the second sub-sample, excess returns are 50% higher compared to the whole sample.

Furthermore, the average estimated variance is 26% percent higher, excess returns are more skewed and have fatter tails. These differences between the second sub-sample and the first sub-sample can be mainly attributed to the crash of 1987, and the high volatility starting in the late 1990's.

Table 7 reports the summary statistics of the monthly cross-sectional regressions for the sub-sample 1986-2003. The market return is insignificant in the CAPM specification for this sample (column (i)). None of the 3 factors in the Fama-French model are significant (column (ii)). The VIX is a highly significant asset pricing factor with a negative price of risk, but including the VIX in the regression does not reduce the J-statistic by much (column (ix)) compared to the CAPM benchmark (column (i)). Our estimated measure of market variance (v) is insignificant, but produces pricing errors that are marginally smaller than in the case of the VIX model (column (iii) compared to column (xi)).

In our benchmark model reported in column (iv), the J-statistic is substantially reduced compared to both the CAPM and the Fama-French model (by 22% and 15%, respectively). The short-run volatility factor s is insignificant, but the long-run volatility factor q is significant at the 1% level. When we include the Hml and Smb factors along with the market return and the s and q factors in our cross-sectional regressions, we find that the q factor stays significant at the 1% level, and neither Hml nor Smb are significant.

The estimated price of risk of the short- and long-run volatility components is quite different in the whole sample compared to the second sub-sample (compare columns (iv) in Table 5 and Table 7). In particular, the price of risk of s is -0.58 for 1963-2003 and -0.11 for 1986-2003, and the price of risk of q is -3.8 for 1963-2003 and -1.86 for 1986-2003. This time-variation of the price of risk is compatible with the predictions of the ICAPM. We can see from equation (10) that the ICAPM does not predict that the price of risk is constant over time. We also find that the prices of risk of Hml and Smb are changing over time, as is apparent by comparing columns (vi) in Table 4 and Table 6.

The conditional variance, realized variance, and implied variance factors (column (iii), (vii) and (ix)) have negative prices of risk, although only the implied variance is statistically significant. Of these models, conditional variance has the smallest pricing errors.

We decompose implied variance into a short- and long-run factor using the same HP-methodology as for realized variance. Using short- and long-run realized or implied volatility as factors, neither performs as well as short and long-run conditional variance. The HP-filtered long-run realized volatility component in column (viii) is statistically significant and negative, while the HP-filtered short-run volatility component in column (x) is statistically significant and negative.[3]

6 Macroeconomic and financial market measures

We next analyze the relationship of short- and long-run volatility with macroeconomic and financial market variables. While previous papers have identified relationships between macroeconomic factors and financial market volatility (e.g., Schwert, 1989), we are able to use our volatility decomposition to sharpen estimates of these relationships and in some cases draw different inferences.[4]

In Panel A of Table 8, we find that market volatility is significantly correlated with business cycle factors. The market variance is significantly negatively correlated with the industrial production growth rate (-19.2%), and significantly positively with changes in the unemployment rate (17.9%). Interestingly, these correlations are entirely due to the long-run volatility component, the short-run component appears to be unrelated to either of these two measures. The long-run component of market risk is thus countercyclical: uncertainty increases when industrial production growth is low, and when the unemployment rate high.

Both the short- and long-run volatility factors are significantly positively correlated with the CPI. In results not reported here, we also used a number of other measures of inflation such as the producer price index and the core cpi, and they were all positively correlated with the various variance measures.

When we examine volatility correlations with market variables, we see the importance of the volatility decomposition. While total volatility is does not exhibit significant correlation with any of the Treasury yield variables, the short run component is significantly positively correlated with

[3] Vanden (2004) demonstrates that the return the put- and call-options are significant pricing factors for size-and book-to-market sorted portfolios. How much cross-sectional pricing power of the two volatility components might captured by the inclusion of options returns as risk factors is an open question.

[4] Estrella (2005) uses spectral filtering to estimate relationships between macro and financial variables at the business cycle frequency.

three month and ten year yields at the 1% level. In contrast, the long run stock market volatility is negatively correlated with ten year yields. Apparently, short run market volatility tends to increase when yields rise, but long run market volatility either declines or is unaffected.

Our findings using market variables also provide evidence that volatility is countercyclical. The term spread (ten-year minus three-month Treasury yield) is significantly negatively correlated with the short run volatility component, so steeper (flatter) term structures are associated with lower (higher) interest rate volatility. Estrella and Hardouvelis (1991) show that upward (downward) sloping term structures are associated with expansion (recession). We also find that wider credit spreads (Moody's BAA yield minus ten year Treasury) are related to higher stock market volatility, although in this case the long-run volatility component seems to be the driver.

All three measures of market risk are significantly negatively correlated with the Pastor-Stambaugh liquidity measure. The correlation between the short-run component and the Pastor-Stambaugh measure is nearly 50%. For the Acharya-Pederson illiquidity measure, the correlation is nearly 60%.

In Panel B of Table 8, we report correlations of market risk with estimated volatilities of the macroeconomic and financial indicators. We estimate volatilities for each macroeconomic and financial measure using an Arma(1,1)-Garch(1,1) model.

We find that market risk is not significantly correlated with either the volatility of industrial production growth or the volatility of the unemployment rate. However, the long run volatility component is positively correlated with all of the other indicators at the 1% level. The volatility of the inflation rate is a source of fundamental uncertainty that is a natural driver of uncertainty in the stock market. From a theoretical point of view, stock and bond market volatility should be correlated, and finding this strong relationship is not surprising. Previous studies such as Fleming, Kirby, and Ostdiek (1998) report similar results. In addition, we find that the long-run component of market risk is strongly correlated with the volatility of both liquidity measures. Highlighting the importance of the volatility decomposition, in no case is the short-run volatility significantly correlated with an indicator variable at the 5% level.

7 Conclusion

We model stock market return volatility in a setting where there are short- and long-run shocks to the volatility process. We demonstrate within an ICAPM framework that both the short- and long-run component of market volatility should be priced. Our main result comes from cross-sectional regressions: both the short- and long-run volatility components are highly significant pricing factors after controlling for the market factor. By decomposing volatility into two components, we are able to significantly reduce the J-statistic compared to a model based on total volatility. Our estimate of the price of risk of long-run volatility is over eight times higher than that of short-run volatility.

It is worth pointing out that our benchmark three-factor model with the market return and the long- and short-run volatility components compares very favorably to the Fama-French (1993) three-factor model and the Ang, Hodrick, Xing, and Zhang (2004) two-factor model, even though our pricing factors are not returns as is the case in their papers. We conjecture that we can improve our pricing results significantly by forming factor mimicking portfolios for the two volatility components along the lines of Ang, Hodrick, Xing, and Zhang (2004).

A decomposition of a nonparametric measure of market volatility produces a long-run volatility component that is strikingly similar to the one obtained with the Egarch-components model. We also relate our volatility components to macroeconomic and financial variables. We find that our long-run volatility factor is closely linked to business cycle fluctuations such as the growth rate of industrial production, changes in the unemployment rate, the credit spread, and measures of macroeconomic uncertainty. The long-run volatility factor is shown to be countercyclical. The short-run factor is more highly correlated with stock market liquidity measures and the Hml and Smb factors. Correlations of market risk with these macroeconomic and financial market measures can be driven by either the long-run or short-run component that sometimes have opposing effects. This is another confirmation that it is important to study the short- and long-run volatility components separately.

A Appendix

This appendix develops the asset pricing implications of an economy with multiple stocks, multiple investors, and a two-component stochastic volatility process. The set-up is similar to Merton's (1973) ICAPM or Cox, Ingersoll, and Ross' (1985) CIR model. We derive all the equations that are discussed in the text and estimate them directly from the model, which allows us to interpret the empirical findings in later sections within a theoretical context. Using results from Nelson (1990), we derive the continuous time limit of our estimated Egarch-components model. We also derive the beta representation of the cross-sectional pricing restrictions.

A.1 Structure of Financial Markets

The economy is assumed to have $i = 1, ..., N$ stocks, and a riskless bond. Uncertainty is described by a Brownian motion Z_t for $0 \leq t \leq \infty$ with $K > N$ dimensions, defined on a complete probability space $(\Omega, \mathcal{F}, P)$, where $\mathcal{F}$ is the augmented filtration generated by Z_t. K is finite, and Z_t is a column vector of K one-dimensional Brownian motions. The probability space fulfills the usual conditions as described in Karatzas and Shreve (1991, chapter 1). Only processes appropriately adapted to $\mathcal{F}$ are considered. Note that we assume $K > N$ as we want to model the impact of stochastic volatility on equilibrium expected returns. Financial markets are thus generically incomplete (but the probability space is complete).

The price of each stock i is denoted P_t^i. Securities pay an instantaneous dividend δ_t^i. There is a risk-free asset that pays an instantaneous rate r_t^f. The cum-dividend instantaneous excess return is defined as:

$$dp_t^i = \frac{dP_t^i + \delta_t^i dt}{P_t^i} - r_t^f dt \tag{11}$$

The N-dimensional vector of excess returns is denoted by dp_t and is assumed to be represented by the following process:

$$dp_t = \mu_t dt + \sigma_t dZ \tag{12}$$

where μ_t denotes the N-dimensional vector of drifts of the return process, and σ_t is a $N \times K$ dimensional matrix. Both μ_t and σ_t are time-varying and stochastic. The instantaneous variance-covariance matrix of excess returns is denoted by Ψ_t:

$$\Psi_t = E\left[dp_t dp_t' | \mathcal{F}_t\right] \tag{13}$$

Each stock is in fixed supply S^i. The price of the market portfolio is defined as $P_t^M = \sum_i S^i P_t^i$. The vector of value weights are denoted by $\pi_t = \left[P_t^1 S^1/P_t^M, P_t^2 S^2/P_t^M, ..., P_t^N S^N/P_t^M\right]'$. The drift of the market portfolio μ_t^M is defined as $\mu_t^M = \pi_t' \mu_t$, and the value weighted innovations are assumed to be $\sqrt{v_t} dZ_t^M = \pi_t' \sigma_t dZ_t$, where Z_t^M is a Brownian motion that is a weighted average of the vector of K underlying shocks Z_t. The market return is thus:

$$dp_t^M = \mu_t^M dt + \sqrt{v_t} dZ_t^M \tag{14}$$

The instantaneous standard deviation of the market return is assumed to be a function of two state variables s_t and q_t: $\sqrt{v_t} = V(s_t, q_t, t)$. The evolution of these state variables is:

$$ds_t = \kappa^s(s_t, q_t)\, dt + \chi^s(s_t, q_t)\, dZ_t^s \tag{15}$$

$$dq_t = \kappa^q(s_t, q_t)\, dt + \chi^q(s_t, q_t)\, dZ_t^q \tag{16}$$

where Z_t^s and Z_t^q are standard Brownian motions that are weighted averages of the K underlying Brownian motions in Z_t. Market volatility is therefore driven by a two-factor structure. The aim of the model is to determine how μ_t^M depends on the state variables and the market portfolio p_t^M in equilibrium, and to derive the pricing implications for the cross-section of returns (i.e. show how μ_t^i depends on $\left[p_t^M, s_t, q_t\right]$ in equilibrium).

A.2 Portfolio Choice

We assume that there are $j = 1, ..., L$ investors who maximize utility from a flow of consumption C^j. Denote the N-dimensional vector of shares of investor $j's$ wealth invested in each of the N

stocks by x_t^j. The wealth of investor j is denoted by W_t^j. Each investor's budget constraint is:

$$dW_t^j = \left(W_t^j r_t^f - C_t^j\right) dt + W_t^j x_t^{j\prime} dp_t \quad \forall j \tag{17}$$

Investor $j's$ utility over consumption flow at time t is denoted $u^j\left(C_t^j, t\right)$.[5] For simplicity, we assume that $u^j(\cdot)$ is HARA. The value function of investor j is then defined as:

$$J_t^j = \max_{C^j, x^j} E_t\left[\int_t^\infty u^j\left(C_\tau^j, \tau\right) d\tau\right] \quad \forall j \tag{18}$$

Merton (1973) shows that investor j's value function J^j is a function of (W, s, q). The stacked system of first order conditions solves the following system of equations for investor j:

$$\gamma_t^j \Psi_t x_t^j = \mu_t - F_s^j \rho_t^{ps} - F_q^j \rho_t^{pq} \quad \forall j \tag{19}$$

where $\gamma^j = -W^j J_{WW}^j / J_W^j$ and $F_s^j = -W^j J_{Ws}^j / J_W^j$ and $F_q^j = -W^j J_{Wq}^j / J_W^j$ and $\rho_t^{ps} = E\left[dp_t ds_t | \mathcal{F}_t\right]$ and $\rho_t^{pq} = E\left[dp_t dq_t | \mathcal{F}_t\right]$. Merton(1973) shows that HARA preferences imply that F_s^j and F_q^j only depend on s and q, and not on W^j.

Campbell and Viceira (1999) and Lynch (2001) compute the portfolio choice problem of a dynamically optimizing investor in environments with changing investment opportunity set and find that rational investors should time the market substantially. Chacko and Viceira (2004) analyze the portfolio choice problem with a one-factor stochastic volatility process.

A.3 Equilibrium excess returns

We assume that the economy is an endowment economy, so that consumption equals the sum of dividends in equilibrium. Furthermore, we assume that the risk-free asset is in zero net supply.[6]

[5] We could extend the model and specify stochastic differential utility as presented by Duffie et al. (1992) in order to allow for Epstein-Zin-Weil preferences and habit formation. In a companion note, we do just that and solve the model by taking a first-order approximation to the Hamilton-Jacobi-Bellman equation (see Adrian and Rosenberg (2004)). For the empirical findings presented in this paper, the theoretical predictions of the more complicated model do not add significant insights, which is why we opted to present the slightly simpler model with HARA preferences.

[6] The assumptions that consumption equals dividends and that the risk-free rate is in zero net supply is for simplicity only. In our companion paper Adrian and Rosenberg (2004), we do not make either one of these assumptions

Then the aggregate budget constraint is solved if $\sum_j W_t^j = P_t^M$. This implies the following restriction for equilibrium expected returns:

$$\mu = \gamma \Psi \pi + F_s \rho^{ps} + F_q \rho^{pq} \tag{20}$$

where γ denotes the aggregate (wealth weighted) coefficient of relative risk aversion, and F_q and F_s are the aggregate (wealth weighted) hedging demands. We have dropped subscripts t for notational convenience. Pre-multiplying by the vector of value weights π, we obtain the equilibrium expected market returns:

$$\mu^M = \gamma v + F_s \rho^{Ms} + F_q \rho^{Mq} \tag{21}$$

The expected excess return of the market portfolio thus depends on the variance of equilibrium excess returns (v) as well as the covariance of the market portfolio with the two volatility factors s and q. In general, the dependence of μ_M on s and q can be non-linear. In a one-factor stochastic volatility setup, Abel (1988) and March and Genotte (1993) derive closed form solutions to the equilibrium market return when volatility is stochastic. In our two-component setup, we can solve the model in closed form if we make the assumption that the two volatility components follow Ornstein-Uhlenbeck processes. In that case, both F_s and F_q are constants. For more general processes, F_s and F_q are both functions of s and q.

From equations (20) and (21), we can see that the evolution of the equilibrium stochastic discount factor m is:

$$\frac{dm}{m} = -\left(\gamma dp^M + F_s ds + F_q dq\right) \tag{22}$$

The pricing kernel of our ICAPM economy consists therefore of three factors: the market return as well as the long- and short-run components of market volatility. As we assumed that the risk-free asset is in zero net supply, it follows that the risk-free rate is a function of the volatility factors, justifying our implicit assumption that the risk-free rate is not an independent state variable.[7]

---

and derive asset pricing predictions.

[7] In empirical results not reported in this paper, we actually find that the risk-free rate is highly correlated with both the long- and short-run volatility components.

A.4 Diffusion limit of the Egarch-components model

In this section, we show that the continuous time limit of the Egarch-components is the stochastic volatility model defined by that from section (2) subject to a covariance constraint. Consider the partition of time into k increments of length h such that $hk = t$. Then we can write the Egarch-components model as follows:

$$\begin{aligned}
R^M_{t+h} &= \left(\alpha_0 + \alpha_1 e^{(s_t+q_t)} + \alpha_2 q_t + \alpha_3 s_t\right) h + e^{(s_t+q_t)} \varepsilon_{t+h} \\
\varepsilon_{t+h} &\sim N(0,h) \\
s_{t+h} &= \alpha_4 s_t h + \alpha_5 \varepsilon_{t+h} + \alpha_6 \left(|\varepsilon_{t+h}| - \sqrt{2h/\pi}\right) \\
q_{t+h} &= (\alpha_7 + \alpha_8 q_t)\, h + \alpha_9 \varepsilon_{t+h} + \alpha_{10} \left(|\varepsilon_{t+h}| - \sqrt{2h/\pi}\right)
\end{aligned} \tag{23}$$

where R^M_{t+h} denotes the excess return to the market portfolio between time t and $t+h$. Using the results of Nelson (1991), we will show that as $h \to 0$, the Egarch-components model (23) converges weakly to the following system of diffusions:

$$\begin{aligned}
dp^M_t &= (\alpha_0 + \alpha_1 v_t + \alpha_2 q_t + \alpha_3 s_t)\, dt + \sqrt{v_t} dZ^M_t \\
ds_t &= -\kappa^s s_t dt + \chi^s dZ^s_t \\
dq_t &= \kappa^q (\bar{q} - q_t)\, dt + \chi^q dZ^q_t
\end{aligned} \tag{24}$$

where $\ln(v_t) = 2(s_t + q_t)$, $\kappa^s = 1 - \alpha_4$, $\kappa^q = 1 - \alpha_8$, and $\bar{q} = \alpha_7/\kappa^q$. In order to apply Theorem 3.1 of Nelson (1990), we need to show distributional uniqueness of the system of diffusions (24). We start by deriving moment conditions. Define excess returns from t to $t+h$ as:

$$R^M_{t+h} = p^M_{t+h} - p^M_t = \frac{P^M_{t+h} - P^M_t + \delta^M_t h}{P^M_t} - r^f_t h \tag{25}$$

The system of equations (23) can be rewritten as:

$$
\begin{aligned}
p_{t+h}^{M} &= \left(p_t^M + \alpha_0 + \alpha_1 \exp\left(\left(s_t + q_t\right)/2\right) + \alpha_2 q_t + \alpha_3 s_t\right) h + \exp\left(\left(s_t + q_t\right)/2\right) Z_{t+h}^h \\
s_{t+h} &= \left(1 - \kappa^s\right) s_t h + \chi^s \left[\theta^s Z_{t+h}^h + \varphi^s \left(\left|Z_{t+h}^h\right| - \sqrt{2h/\pi}\right)\right] \\
q_{t+h} &= \kappa^q \bar{q} h + \left(1 - \kappa^q\right) q_t h + \chi^q \left[\theta^q Z_{t+h}^h + \varphi^q \left(\left|Z_{t+h}^h\right| - \sqrt{2h/\pi}\right)\right]
\end{aligned}
\tag{26}
$$

where:

$$
Z_t^h \sim N\left(0, h\right) \tag{27}
$$

and $\theta^s, \theta^q, \varphi^s, \varphi^q$ are determined by matching moments:

$$
\theta^s = \rho^{Ms}, \varphi^s = \chi^s \sqrt{\frac{1 - \left(\rho^{Ms}\right)^2}{1 - 2/\pi}} \tag{28}
$$

$$
\theta^q = \rho^{Mq}, \varphi^q = \chi^q \sqrt{\frac{1 - \left(\rho^{Mq}\right)^2}{1 - 2/\pi}} \tag{29}
$$

$$
\rho^{qs} = \rho^{Ms} \rho^{Mq} h + \chi^s \chi^q \sqrt{1 - \left(\rho^{Ms}\right)^2} \sqrt{1 - \left(\rho^{Mq}\right)^2} \tag{30}
$$

We now need to verify that condition B of the appendix A of Nelson (1990) is satisfied. Define the following auxiliary vectors:

$$
b\left(y_1, y_2, y_3\right) = \begin{bmatrix} y_1 + \alpha_0 + \alpha_1 \exp\left(\left(y_2 + y_3\right)/2\right) + \alpha_2 y_3 + \alpha_3 y_2 \\ \left(1 - \kappa^s\right) y_2 \\ \kappa^q \bar{q} + \left(1 - \kappa^q\right) y_3 \end{bmatrix} \tag{31}
$$

$$
a\left(y_1, y_2, y_3\right) = \begin{bmatrix} e^{y_2 + y_3} & \chi^s \rho^{sM} e^{y_2 + y_3} & \chi^q \rho^{qM} e^{y_2 + y_3} \\ \chi^s \rho^{sM} e^{y_2 + y_3} & \left(\chi^s\right)^2 & \chi^q \chi^s \rho^{qs} \\ \chi^q \rho^{qM} e^{y_2 + y_3} & \chi^q \chi^s \rho^{qs} & \left(\chi^s\right)^2 \end{bmatrix} \tag{32}
$$

Condition B of Appendix A of Nelson (1990) requires 1) that a and b are locally bounded (which is clearly satisfied) and 2) that $\forall R > 0$ and $\forall T > 0$ there exists a number $\Lambda_{R,T} > 0$ such that $\forall \left(y_1, y_2, y_3, t\right)$ satisfying $0 \leq t \leq T$ and $\|y_1, y_2, y_3\| \leq R$, and $a\left(y_1, y_2, y_3\right) - \Lambda_{R,T} \cdot I_3$ is positive definite. The second condition is satisfied as a is a symmetric matrix, so that we can choose the first eigenvalue as $\Lambda_{R,T}$.

In order to show that the non-explosion condition holds, we define:

$$\begin{aligned}\phi(y_1, y_2, y_3) &= K + f(y_1)|y_1| + f(y_2)\exp(|y_2|) + f(y_3)\exp(|y_3|) \\ \text{where } f(y_i) &= \begin{cases} \exp(-1/|y_i|) \text{ if } y_i \neq 0 \\ 0 \text{ if } y_i = 0 \end{cases}\end{aligned} \tag{33}$$

Note that $\phi(\cdot)$ is nonnegative, arbitrarily differentiable, and satisfies $\underset{\|y\|\to\infty}{lim} \inf_{0\leq t\leq T} \phi(y_1, y_2, y_3) = \infty$.

Then, for any $T > 0$, we can choose K and λ_T such that

$$\sum_{i=1}^{3} b_i(y) \frac{\partial\phi(y)}{\partial y_i} + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}(y) \frac{\partial^2\phi(y)}{\partial y_i \partial y_j} \leq \lambda_T \phi(y) \tag{34}$$

as the derivatives of $\phi(\cdot)$ are locally bounded.

All of the conditions for Theorem 3.1. of Nelson (1990) hold, and the system of equations (26) converges weakly to the system of diffusions:

$$\begin{aligned}dp_t^M &= (\alpha_0 + \alpha_1 \exp((s_t + q_t)/2) + \alpha_2 q_t + \alpha_3 s_t)\,dt + \exp((s_t + q_t)/2)\,dZ_t \\ ds_t &= -\kappa^s s_t dt + \exp((s_t + q_t)/2)\,\chi^s dZ_t^s \\ dq_t &= \kappa^q(\bar{q} - \kappa^q q_t)\,dt + \exp((s_t + q_t)/2)\,\chi^q dZ_t^q\end{aligned} \tag{35}$$

with $E_t[Z_t^q Z_t^s] = \rho^{Ms}\rho^{Mq} + \chi^s\chi^q\sqrt{1-(\rho^{Ms})^2}\sqrt{1-(\rho^{Mq})^2}$. Finally, choosing $h = 1$ in (23) gives the system of diffusions (24).

A.5 Beta representation of the equilibrium expected returns

In this section, we derive the beta representation of (20). The equilibrium stock pricing condition (20) can be written for individual assets as:

$$\mu_t^i = \gamma_t E_t\left[dp_t^i dp_t^M\right] + E_t\left[dp_t^i ds_t\right] F_s + E_t\left[dp_t^i dq_t\right] F_q \tag{36}$$

Introduce the following notation:

$$dS_t = \left[dp_t^M, ds_t, dq_t\right]' \qquad \Sigma_t = E_t\left[dS_t dS_t'\right] \qquad \Upsilon_t = \left[\gamma_t, F_s, F_q\right]' \tag{37}$$

Then the pricing equation can be written as:

$$\mu_t^i = \Upsilon_t' E_t\left[dS_t dp_t^i\right] \tag{38}$$

This is a pricing equation that has to hold for any asset. Denote the price of risk of factor k by λ_t^k and the vector of the prices of risk by λ_t. Then:

$$\lambda_t = \Upsilon_t' E_t\left[dS_t dS_t'\right] \tag{39}$$

Solving for $\Upsilon_t' = \Sigma_t^{-1}\lambda_t$ and replacing back into the pricing equation gives:

$$\mu^i = \Sigma_t^{-1} E_t\left[dS_t dp_t^i\right]\lambda_t \tag{40}$$

The last step is to realize that $\Sigma_t^{-1} E_t\left[dS_t dp_t^i\right]$ denotes the population moment of a regression of asset returns dp^i on the vector of risk factors dS, we can write:

$$\mu_i = \beta_t^{iM}\lambda_t^M + \beta_t^{is}\lambda_t^s + \beta_t^{iq}\lambda_t^q \tag{41}$$

where $\beta_t^{i\eta}$ are partial regression coefficients for $\eta = M, s, q$.

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Table 1:
Stochastic Volatility Model
Maximum Likelihood Estimation
1963/5/1 – 2003/12/31 (daily)

This table reports the maximum likelihood estimates of the stochastic volatility model. The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a daily frequency. The standardized error term ε is assumed to be distributed normally with mean zero and variance one; however, our parameter estimates are consistent even when the distribution is non-normal (Bollerslev and Wooldridge, 1992). The variance of the market excess return v is defined as $v=exp(2(s+q))$, where q denotes the long-run component of the market variance and s denotes the short-run component of market variance.

Market excess returns: $R^M_{t+1} = \alpha_0 + \alpha_1 v_t + \alpha_2 s_t + \alpha_3 q_t + \sqrt{v_t}\varepsilon_{t+1}$

	α_0	α_1	α_2	α_3
Coef.	-0.010	0.049	-0.116	-0.021
Std. err.	(0.027)	(0.025)	(0.055)	(0.032)
p-value	[0.969]	[0.045]	[0.036]	[0.506]

Short-run component: $s_{t+1} = \alpha_4 s_t + \alpha_5 \varepsilon_{t+1} + \alpha_6 \left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right)$

	α_4	α_5	α_6
Coef.	0.867	-0.058	0.034
Std. err.	(0.014)	(0.003)	(0.005)
p-value	[0.000]	[0.000]	[0.000]

Long-run component: $q_{t+1} = \alpha_7 + \alpha_8 q_t + \alpha_9 \varepsilon_{t+1} + \alpha_{10} \left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right)$

	α_7	α_8	α_9	α_{10}
Coef.	0.000	0.996	-0.011	0.039
Std. err.	(0.0003)	(0.001)	(0.002)	(0.003)
p-value	[0.855]	[0.000]	[0.000]	[0.000]

p-value of test α_8=1: [0.000]

	10 lags	20 lags
Ljung-Box Q-statistic of ε^2	8.372	11.779
p-value	[0.593]	[0.923]

Table 2:
Comparison of Different Volatility Models
Maximum Likelihood Estimation
1963/5/1 – 2003/12/31 (daily)

This table compares four different volatility model specifications. All models are estimated via maximum likelihood. The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a daily frequency. The standardized error term ε is assumed to be distributed normally with mean zero and variance one; however, our parameter estimates are consistent even when the distribution is non-normal (Bollerslev and Wooldridge, 1992). Standard errors are reported in parenthesis, p-values in brackets. The indicator function is denoted by $I(\cdot)$. The Akaike criterion and Schwarz criterion are information criteria that allow model comparison, taking the number of parameters into account.

	Log likelihood	Akaike criterion	Schwarz criterion	Ljung-Box Q of ε^2	
				10 lags	20 lags
(i) Garch-GJR, Glosten, Jagannathan and Runkle (1993)					
$R^M_{t+1} = \underset{(0.009)}{0.035} - \underset{(0.016)}{0.007}\, v_t + \sqrt{v_t}\,\varepsilon_{t+1}$ $v_{t+1} = \underset{(0.001)}{0.007} + \underset{(0.003)}{0.913}\, v_t + \underset{(0.005)}{0.091}\, v_t\varepsilon^2_{t+1} I(\varepsilon_{t+1} < 0) + \underset{(0.004)}{0.034}\, v_t\varepsilon^2_{t+1}$	-11,806	2.26	2.27	6.21 [0.80]	12.11 [0.91]
(ii) Egarch, Nelson (1991)					
$R^M_{t+1} = \underset{(0.009)}{0.027} - \underset{(0.016)}{0.001}\, v_t + \sqrt{v_t}\,\varepsilon_{t+1}$ $\ln(v_{t+1}) = -\underset{(0.005)}{0.126} + \underset{(0.001)}{0.984}\, \ln(v_t) - \underset{(0.003)}{0.071}\, \varepsilon_{t+1} + \underset{(0.005)}{0.148}\, \lvert\varepsilon_{t+1}\rvert$	-11,761	2.25	2.26	7.54 [0.67]	12.34 [0.90]
(iii) Garch components, Engle and Lee (1999)					
$R^M_{t+1} = \underset{(0.006)}{0.046} + \sqrt{v_t}\,\varepsilon_{t+1}$ $v_{t+1} = q_{t+1} + \underset{(0.010)}{0.865}\, (v_t - q_{t+1}) + \underset{(0.007)}{0.119}\, v_t\varepsilon^2_{t+1} I(\varepsilon_{t+1} < 0) - \underset{(0.007)}{0.017}\, v_t\varepsilon^2_{t+1}$ $q_{t+1} = \underset{(0.083)}{0.579} + \underset{(0.0007)}{0.997}\, (q_t - \alpha_7) + \underset{(0.003)}{0.037}\, v_t\varepsilon^2_{t+1}$	-11,824	2.26	2.27	5.76 [0.84]	7.73 [0.99]
(iv) Egarch components in mean model					
$R^M_{t+1} = -\underset{(0.027)}{0.010} + \underset{(0.025)}{0.049}\, \exp(2(s_t + q_t)) - \underset{(0.055)}{0.116}\, s_t - \underset{(0.032)}{0.021}\, q_t + \exp(s_t + q_t)\varepsilon_{t+1}$ $s_{t+1} = \underset{(0.013)}{0.867}\, s_t - \underset{(0.003)}{0.058}\, \varepsilon_{t+1} + \underset{(0.005)}{0.034}\, \lvert\varepsilon_{t+1} - \sqrt{2/\pi}\rvert$ $q_{t+1} = \underset{(0.0003)}{0.000} + \underset{(0.0007)}{0.996}\, q_t - \underset{(0.002)}{0.009}\, \varepsilon_{t+1} + \underset{(0.003)}{0.039}\, \lvert\varepsilon_{t+1} - \sqrt{2/\pi}\rvert$	-11,664	2.24	2.24	8.37 [0.59]	11.78 [0.92]

Table 3:
Summary Statistics
Market Excess Return and Variance Measures
1963/5/1 – 2003/12/31 (daily and monthly)

The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a daily frequency. The daily conditional variance of the market excess return (v), the short-run volatility component (s), and the long-run component (q) are taken from the maximum likelihood estimates reported in Table 1 with $v=exp(2(s+q))$. Monthly estimates of v, q, and s are obtained by averaging the daily estimates over each month and multiplying by 21 (the average number of trading days in the sample). The realized market variance is the squared daily excess return. The HP-filtered long-run component is the trend component of Hodrick-Prescott (1997) filtered realized log market variance (log daily squared excess returns) with smoothing parameter 6,812,100. The HP-filtered short-run component is the difference between the realized log market variance and the HP-filtered long-run component. Monthly estimates of the realized market variance and the HP-filtered short-run and the HP-filtered long-run components are obtained by averaging the daily series within each month and multiplying by 21.

Panel A: Daily (10,237 observations)

	Market excess return	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Mean	0.021	0.780	0.802	-0.001	-0.298	0.000	-1.013
Median	0.044	0.526	0.190	-0.017	-0.320	0.212	-1.064
Maximum	8.638	34.058	294.630	1.228	0.680	3.313	-0.065
Minimum	-17.165	0.057	0.000	-0.366	-1.263	-9.781	-1.862
Std. Dev.	0.895	0.899	3.608	0.144	0.347	1.145	0.378
Skewness	-0.761	9.796	55.495	0.726	0.117	-1.349	0.114
Kurtosis	21.334	252.411	4334.353	4.386	2.882	6.754	2.534

Correlations

	Market excess return	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Market excess return	1.000	0.017	-0.159	-0.061	0.008	0.004	-0.022
Conditional market variance v		1.000	0.289	0.501	0.696	0.071	0.616
Realized market variance			1.000	0.166	0.182	0.251	0.175
Short-run component s				1.000	0.237	0.132	0.171
Long-run component q					1.000	0.027	0.922
HP-filtered short-run component						1.000	0.017
HP-filtered long-run component							1.000

Table 3 Panel B: Monthly (488 observations)

	Market excess return	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Mean	0.483	16.341	16.774	-0.026	-6.273	0.022	-21.269
Median	0.735	11.329	9.508	-0.020	-6.749	-0.138	-22.376
Maximum	16.010	157.485	511.090	8.748	12.900	23.792	-1.396
Minimum	-23.000	1.686	0.747	-4.295	-24.613	-22.350	-39.027
Std. Dev.	4.489	15.558	28.471	2.083	7.212	5.860	7.941
Skewness	-0.478	3.192	11.407	0.552	0.113	0.222	0.116
Kurtosis	4.912	19.897	188.858	3.566	2.858	3.881	2.530

Correlations

	Market excess return	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Market excess return	1.000	-0.294	-0.300	-0.735	-0.044	-0.191	-0.092
Conditional market variance v		1.000	0.825	0.493	0.799	0.317	0.747
Realized market variance			1.000	0.445	0.482	0.372	0.463
Short-run component s				1.000	0.260	0.496	0.250
Long-run component q					1.000	0.163	0.933
HP-filtered short-run component						1.000	0.070
HP-filtered long-run component							1.000

Table 4:
Summary Statistics of the 25 Size and Book-to-Market sorted Portfolios
1963/7 – 2003/12 (monthly)

This table reports summary statistics for the 25 size and book-to-market sorted value weighted portfolio excess returns of Fama and French (1992 and 1993), downloadable at the website of Kenneth French. We report Newey-West (1987) standard errors.

Summary Statistics of Portfolio Excess Returns

		Small	Size 2	Size 3	Size 4	Large
Growth	Mean	0.254	0.388	0.414	0.519	0.425
	Std. dev.	(8.302)	(7.520)	(6.908	(6.136)	(4.856)
B/M 2	Mean	0.828	0.648	0.702	0.497	0.467
	Std. dev.	(7.103)	(6.084)	(5.502	(5.194)	(4.609)
B/M 3	Mean	0.871	0.897	0.713	0.722	0.496
	Std. dev.	(6.067)	(5.399)	(4.952	(4.892)	(4.362)
B/M 4	Mean	1.178	0.953	0.864	0.839	0.571
	Std. dev.	(5.956)	(5.168)	(4.782	(4.681)	(4.292)
Value	Mean	1.092	1.034	1.013	0.884	0.564
	Std. dev.	(5.663)	(5.750)	(5.396	(5.388)	(4.852)

Loadings on the Market Factor

		Small	Size 2	Size 3	Size 4	Large
Growth	Coeff.	1.437	1.429	1.356	1.251	1.013
	Std. Err.	(0.056)	(0.044)	(0.036	(0.028)	(0.023)
B/M 2	Coeff.	1.218	1.161	1.102	1.068	0.954
	Std. Err.	(0.055)	(0.045)	(0.034	(0.033)	(0.022)
B/M 3	Coeff.	1.067	1.023	0.965	0.973	0.855
	Std. Err.	(0.051)	(0.046)	(0.040	(0.036)	(0.029)
B/M 4	Coeff.	1.008	0.962	0.905	0.907	0.789
	Std. Err.	(0.056)	(0.044)	(0.040	(0.037)	(0.033)
Value	Coeff.	0.981	1.038	0.985	0.990	0.832
	Std. Err.	(0.051)	(0.054)	(0.052	(0.048)	(0.043)

Loadings on the Short-run Volatility Factor s

		Small	Size 2	Size 3	Size 4	Large
Growth	Coeff.	-2.809	-2.666	-2.467	-2.174	-1.603
	Std. Err.	(0.149)	(0.129)	(0.117	(0.102)	(0.087)
B/M 2	Coeff.	-2.452	-2.274	-2.058	-1.908	-1.597
	Std. Err.	(0.124)	(0.116)	(0.105	(0.103)	(0.089)
B/M 3	Coeff.	-2.186	-2.053	-1.855	-1.775	-1.432
	Std. Err.	(0.111)	(0.105)	(0.094	(0.093)	(0.084)
B/M 4	Coeff.	-2.142	-1.908	-1.744	-1.688	-1.352
	Std. Err.	(0.117)	(0.101)	(0.092	(0.081)	(0.081)
Value	Coeff.	-2.033	-2.102	-1.937	-1.814	-1.429
	Std. Err.	(0.112)	(0.116)	(0.115	(0.108)	(0.099)

Loadings on the Long-run Volatility Factor q

		Small	Size 2	Size 3	Size 4	Large
Growth	Coeff.	-1.332	-1.196	-1.094	-0.892	-0.621
	Std. Err.	(0.193)	(0.189)	(0.170	(0.151)	(0.127)
B/M 2	Coeff.	-1.243	-1.092	-0.916	-0.836	-0.657
	Std. Err.	(0.166)	(0.157)	(0.148	(0.143)	(0.130)
B/M 3	Coeff.	-1.075	-1.052	-0.890	-0.810	-0.593
	Std. Err.	(0.147)	(0.139)	(0.133	(0.135)	(0.119)
B/M 4	Coeff.	-1.129	-0.981	-0.833	-0.775	-0.577
	Std. Err.	(0.143)	(0.133)	(0.123	(0.116)	(0.106)
Value	Coeff.	-1.037	-1.082	-0.945	-0.808	-0.624
	Std. Err.	(0.140)	(0.145)	(0.140	(0.138)	(0.127)

Table 5:
Pricing the Cross-Section of 25 Size and Book-to-Market sorted Portfolios
1963/7 – 2003/12 (monthly)

This table reports summary statistics of the cross-sectional Fama-MacBeth (1973) regressions for the 25 Size and Book-to-Market sorted portfolios of Fama and French (1993) for the time period 1963/7–2003/12. In the first stage, portfolio returns are regressed on the pricing factors to obtain factor loadings. In the second stage, for each month, portfolio returns are regressed on the loadings, giving an estimate of the price of risk for each factor. The standard errors (reported in parentheses) and p-values (reported in brackets) are adjusted for autocorrelation and heteroskedasticity using the Newey-West (1987) procedure, and adjusted for the estimation error using the Shanken (1992) correction. The regression coefficients and standard errors thus correspond to first stage GMM estimates with an identity weighting matrix (see Cochrane 2001). The J-statistic corresponds to the joint test that the pricing errors for all 25 portfolios are zero (Hansen and Singleton 1982). The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a monthly frequency. The Hml and Smb factors are taken from Kenneth French's website. The short-run component (s), the long-run component (q), and the variance of excess returns (v) are from the maximum likelihood estimation of the Egarch-components model reported in Table 1. The daily estimates of v, q, and s are aggregated to a monthly frequency by averaging daily observations and multiplying by 21. The pricing factors are residuals of a monthly AR(2) process for q and v, and a monthly AR(1) process for s. The realized variance is the daily squared excess market return averaged over each month and multiplied by 21, and the HP-filtered short-run and long-run components are obtained applying a Hodrick-Prescott (1997) filter to the log of daily squared returns, the pricing factors are the residuals of an AR(1) process.

		(i)	(ii)	(iii)	**(iv)**	(v)	(vi)	(vii)	(viii)
Excess market return	coef.	0.676	0.413	0.522	**0.549**	0.439	0.432	0.514	0.401
	std. err.	(0.312)	(0.292)	(0.295)	**(0.292)**	(0.291)	(0.292)	(0.295)	(0.293)
	p-value	[0.038]	[0.158]	[0.077]	**[0.060]**	[0.130]	[0.139]	[0.082]	[0.172]
Short-run volatility component s	coef.				**-0.584**	-0.966	-1.586		
	std. err.				**(0.231)**	(0.261)	(0.447)		
	p-value				**[0.012]**	[0.000]	[0.000]		
Long-run volatility component q	coef.				**-3.785**	-3.470	-2.218		
	std. err.				**(0.901)**	(0.802)	(0.490)		
	p-value				**[0.000]**	[0.000]	[0.000]		
Conditional variance of excess market return v	coef.			-3.453		-1.013			
	std. err.			(1.646)		(1.413)			
	p-value			[0.036]		[0.473]			
Realized variance of excess market return	coef.							-12.781	
	std. err.							(5.818)	
	p-value							[0.028]	
HP-filtered short-run volatility component	coef.								2.212
	std. err.								(1.583)
	p-value								[0.162]
HP-filtered long-run volatility component	coef.								-1.104
	std. err.								(0.310)
	p-value								[0.000]
Hml	coef.		0.653				0.321		
	std. err.		(0.259)				(0.249)		
	p-value		[0.012]				[0.197]		
Smb	coef.		0.290				0.313		
	std. err.		(0.222)				(0.223)		
	p-value		[0.192]				[0.160]		
	J-statistic	102.2	89.1	97.4	**88.5**	88.3	79.3	101.3	92.9

Table 6:
Summary Statistics
Market Excess Returns and Variance Measures
1986/1/2 – 2003/12/31 (daily and monthly)

The VIX is a measure of the implied volatility for the S&P100 index. In order to compare it to the model variance v, the VIX is squared and divided by 365. The sample 1986/1/2 – 2003/10/31 corresponds to the availability of the VIX on the CBOE website. The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a daily frequency. The daily conditional variance of the market excess return (v), the short-run volatility component (s), and the long-run component (q) are taken from the maximum likelihood estimates reported in Table 1 with $v=exp(2(s+q))$. Monthly estimates of v, q, and s are obtained by averaging the daily estimates over each month and multiplying by 21 (the average number of trading days in the sample). The realized market variance is the squared daily excess return. The HP-filtered long-run component is the trend component of Hodrick-Prescott (1997) filtered realized log market variance (log daily squared excess returns) with smoothing parameter 6,812,100. The HP-filtered short-run component is the difference between the realized log market variance and the HP-filtered long-run component. Monthly estimates of the realized market variance and the HP-filtered short-run and the HP-filtered long-run components are obtained by averaging the daily series within each month and multiplying by 21.

Panel A: Daily (4535 observations)

	Market excess return	$VIX^2/365$	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Mean	0.032	0.980	1.076	1.475	0.000	-0.196	0.000	-0.909
Median	0.067	0.618	0.228	1.171	-0.017	-0.232	0.189	-0.974
Maximum	8.638	34.058	294.630	61.800	1.228	0.680	3.313	-0.065
Minimum	-17.165	0.087	0.000	0.224	-0.366	-0.970	-9.502	-1.576
Std. Dev.	1.037	1.186	5.216	1.819	0.143	0.346	1.142	0.389
Skewness	-1.196	8.949	41.211	17.246	1.037	0.286	-1.181	0.208
Kurtosis	24.699	181.265	2232.627	472.477	6.388	2.355	5.815	1.910

Correlations

	Market excess return	$VIX^2/365$	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Market excess return	1.000	0.035	-0.234	-0.215	-0.009	0.001	-0.001	-0.030
$VIX^2/365$		1.000	0.274	0.786	0.576	0.690	0.085	0.589
Conditional market variance v			1.000	0.609	0.203	0.169	0.251	0.160
Realized market variance				1.000	0.393	0.513	0.098	0.443
Short-run component s					1.000	0.315	0.144	0.214
Long-run component q						1.000	0.025	0.916
HP-filtered short-run component							1.000	0.014
HP-filtered long-run component								1.000

Table 6 Panel B: Monthly (216 observations)

	Market excess return	$VIX^2/365$	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Mean	0.670	20.602	22.538	30.966	-0.002	-4.104	0.036	-19.057
Median	1.225	13.178	12.672	25.420	-0.065	-5.022	0.034	-20.406
Maximum	12.500	157.485	511.090	293.355	8.748	12.900	17.679	-1.396
Minimum	-23.000	3.154	1.662	6.476	-4.174	-18.340	-17.296	-33.020
Std. Dev.	4.668	19.752	39.859	27.320	2.011	7.181	5.430	8.182
Skewness	-0.987	2.705	8.927	4.900	0.756	0.276	0.127	0.203
Kurtosis	5.859	14.206	106.221	42.331	4.379	2.305	3.360	1.904

Correlations

	Market excess return	$VIX^2/365$	Conditional market variance v	Realized market variance	Short-run component s	Long-run component q	HP-filtered short-run component	HP-filtered long-run component
Market excess return	1.000	-0.375	-0.434	-0.348	-0.733	-0.116	-0.267	-0.144
$VIX^2/365$		1.000	0.826	0.901	0.604	0.809	0.394	0.742
Conditional market variance v			1.000	0.869	0.560	0.457	0.438	0.437
Realized market variance				1.000	0.511	0.675	0.361	0.617
Short-run component s					1.000	0.361	0.542	0.323
Long-run component q						1.000	0.172	0.929
HP-filtered short-run component							1.000	0.063
HP-filtered long-run component								1.000

Table 7:
Pricing the Cross-Section of 25 Size and Book-to-Market sorted Portfolios
1986/3 – 2003/12 (monthly)

This table reports summary statistics of the cross-sectional Fama-MacBeth (1973) regressions for the 25 Size and Book-to-Market sorted portfolios of Fama and French (1993) for the time period 1986/3–2003/12. In the first stage, portfolio returns are regressed on the pricing factors to obtain factor loadings. In the second stage, for each month, portfolio returns are regressed on the loadings, giving an estimate of the price of risk for each factor. The standard errors (reported in parentheses) and p-values (reported in brackets) are adjusted for autocorrelation and heteroskedasticity using the Newey-West (1987) procedure, and adjusted for the estimation error using the Shanken (1992) correction. The regression coefficients and standard errors thus correspond to first stage GMM estimates with an identity weighting matrix (see Cochrane 2001). The J-statistic corresponds to the joint test that the pricing errors for all 25 portfolios are zero (Hansen and Singleton 1982). The market excess return is measured as the cum-dividend return of the value weighted CRSP portfolio in excess of the three-month Treasury bill rate appropriately converted to a monthly frequency. The Hml and Smb factors are taken from Kenneth French's website. The short-run component (*s*), the long-run component (*q*), and the variance of excess returns (*v)* are from the maximum likelihood estimation of the Egarch-components model reported in Table 1. The daily estimates of *v*, *q*, and *s* are aggregated to a monthly frequency by summing the daily observations. The VIX is taken from the website of the CBOE, the squared VIX is divided by 365 and summed over each month to obtain a monthly measure of market variance. The pricing factors are residuals of a monthly AR(2) process for *q, s, v*, and *VIX* as it is the covariance with innovations of the factors that is priced (see proposition 2). The realized variance is the daily squared excess market return summed over each month, and the HP-filtered short-run and long-run components are obtained applying a Hodrick-Prescott (1997) filter to the log of daily squared returns.

								realized		implied	
		(i)	(ii)	(iii)	**(iv)**	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Excess market return	coef.	0.723	0.560	0.627	**0.633**	0.736	0.619	0.570	0.598	0.491	.541
	std. err.	(0.457)	(0.419)	(0.424)	**(0.419)**	(0.416)	(0.419)	(0.426)	(0.419)	(0.431)	(.424)
	p-value	[0.114]	[0.181]	[0.140]	**[0.131]**	[0.077]	[0.141]	[0.181]	[0.154]	[0.254]	[0.202]
Short-run volatility component *s*	coef.				**-0.106**	0.949	-1.146				
	std. err.				**(0.378)**	(0.473)	(0.403)				
	p-value				**[0.780]**	[0.045]	[0.004]				
Long-run volatility component *q*	coef.				**-1.864**	-1.261	-2.016				
	std. err.				**(0.664)**	(0.060)	(0.510)				
	p-value				**[0.005]**	[0.036]	[0.000]				
Conditional variance of excess market return *v*	coef.			-2.866		-9.043					
	std. err.			(2.516)		(2.703)					
	p-value			[0.255]		[0.001]					
Realized variance / implied variance VIX^2	coef.							-12.285		-8.801	
	std. err.							(8.251)		(4.518)	
	p-value							[0.137]		[0.051]	
HP-filtered short-run volatility component	coef.								1.601		-1.098
	std. err.								(1.664)		(.675)
	p-value								[0.336]		[0.005]
HP-filtered long-run volatility component	coef.								-0.804		.298
	std. err.								(0.488)		(.234)
	p-value								[0.100]		[0.203]
Hml	coef.		0.664				0.108				
	std. err.		0.517				(0.509)				
	p-value		[0.200]				[0.831]				
Smb	coef.		0.142				0.069				
	std. err.		0.238				(0.327)				
	p-value		[0.664]				[0.831]				
	J-statistic	195.1	173.8	187.4	**148.1**	147.8	129.9	189.2	188.1	191.7	188.9

Table 8:
Correlation of Volatility Components with Macroeconomic and Financial Market Measures
1963/7-2003/12 (monthly)

Panel A reports the correlations of the conditional market excess return variance *v*, the short-run volatility component *s*, and the long-run volatility component *q* with macroeconomic and financial market measures at a monthly frequency. The conditional variance measure and short- and long-run volatility factors are from the estimated stochastic volatility model reported in Table 1. For the industrial production growth rate, changes in the unemployment rate, the CPI inflation rate, the three-month yield, the ten-year yield, the ten-year / three-month spread, the credit spread, the market variance *v* and the short- and long-run components *s* and *q*, Dickey-Fuller tests reject the null of a unit root (not reported). The liquidity measure from Pástor and Stambaugh (2003) is for 1963/09 – 1999/12 and the illiquidity measure from Acharya and Pedersen (2004) for 1964/03-1999/12. Both are calculated using residuals from AR(2) models. Panel B of this table reports correlations with estimated volatilities. For each of the variables from Panel A, we estimate an Arma(1,1)-Garch(1,1) model to obtain conditional volatilities. We then calculate the correlations of the model volatilities with v, s, and q. In both panels, three stars denote significance at the 1% level, two stars denote significance at the 5% level, and one star denotes significance at the 10% level.

Panel A: Correlations

	Conditional market variance *v*		Short-run component *s*		Long-run component *q*	
Industrial production growth rate	-0.192	***	0.028		-0.290	***
Changes in unemployment rate	0.179	***	0.047		0.228	***
CPI inflation rate	0.040		0.180	***	0.090	*
Three-month yield	-0.037		0.236	***	-0.053	
Ten-year yield	-0.056		0.130	***	-0.089	*
Ten-year / three-month spread	-0.037		-0.109	**	0.011	
Credit spread	0.203	***	0.023		0.139	***
Hml	0.114	**	0.194	***	0.022	
Smb	-0.164		-0.451	***	-0.004	
Pástor-Stambaugh liquidity residuals	-0.471	***	-0.497	***	-0.224	***
Acharya-Pedersen illiquidity residuals	0.321	***	0.572	***	0.108	**

Panel B: Correlations with Volatilities

	Conditional market variance *v*		Short-run component *s*		Long-run component *q*	
Industrial production growth volatility	0.004		-0.063		0.026	
Unemployment rate volatility	-0.043		-0.056		0.038	
CPI inflation rate volatility	0.098	**	0.045		0.207	***
Three-month yield volatility	0.126	***	-0.001		0.254	***
Ten-year yield volatility	0.077	*	-0.007		0.189	***
Ten-year / three-month spread volatility	0.089	*	0.007		0.186	***
Credit spread volatility	0.137	***	-0.022		0.286	***
Hml volatility	0.357	***	0.082	*	0.354	***
Smb volatility	0.391	***	0.057		0.411	***
Pástor-Stambaugh liquidity volatility	0.335	***	0.017		0.371	***
Acharya-Pedersen illiquidity volatility	0.268	***	0.021		0.344	***

Figure 1: Market Volatility (Annualized)

This figure plots three measures of the annualized standard deviation of the market return at a daily frequency for 1963/5/1-2003/12/31. The first measure is the implied volatility of the S&P100 stock index from the VIX. The second measure is the volatility estimated with our stochastic volatility model, presented in Table 1. The third measure is the 252-day moving average of realized volatility (daily squared returns).

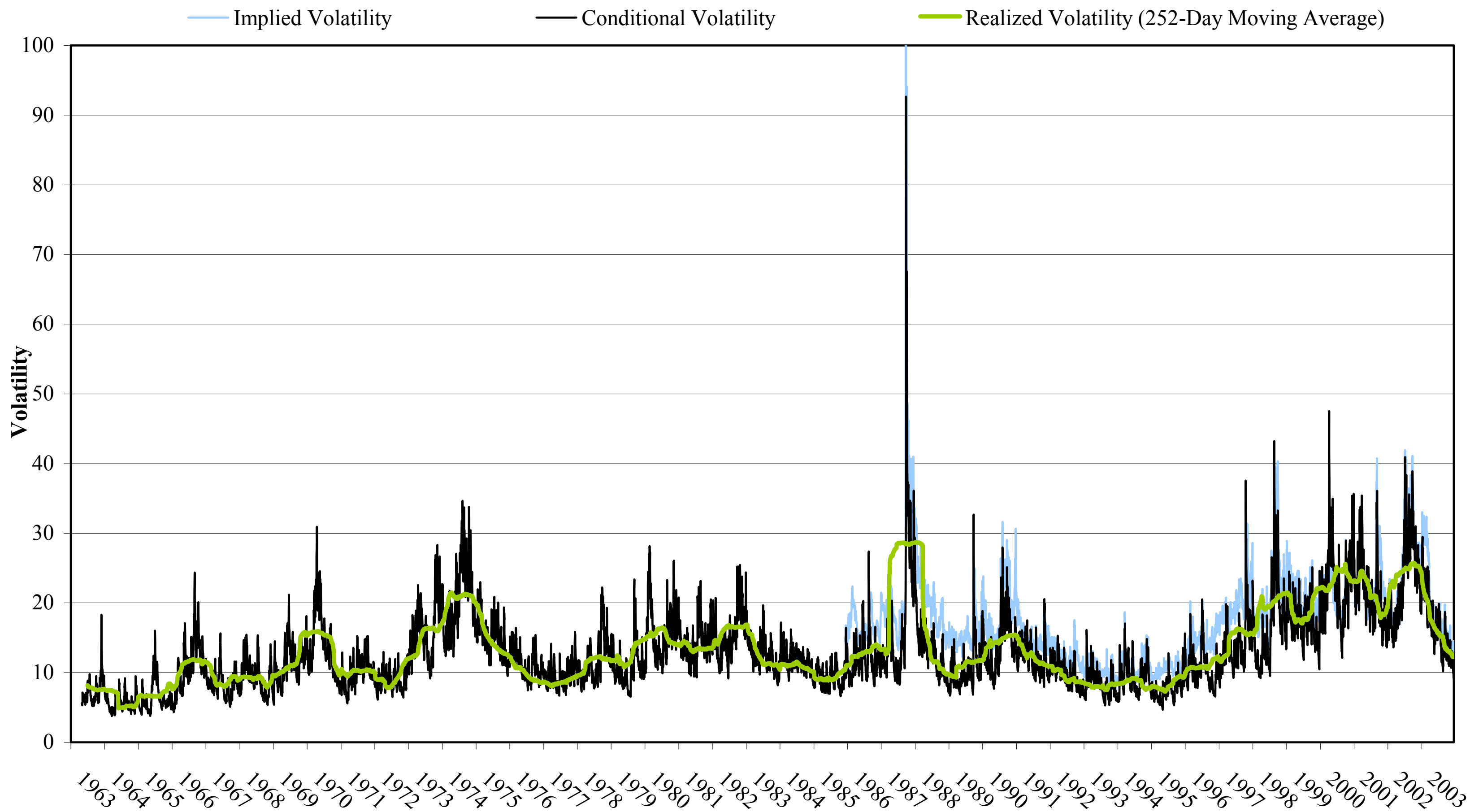

Figure 2: The Long-Run Volatility Component

This figure plots the estimate of the long-run volatility component at a daily frequency for 1963/5/1 - 2003/12/31. The variance of the excess market return is defined as *v=exp(2(s+q))* , where *s* is the short-run component of volatility (Figure 3). The estimate of *q* results from the stochastic volatility model that is reported in Table 1 (left scale). The HP-filtered long-run component are obtained by applying the Hodrick-Prescott (1997) filter to the log of daily squared returns (right scale).

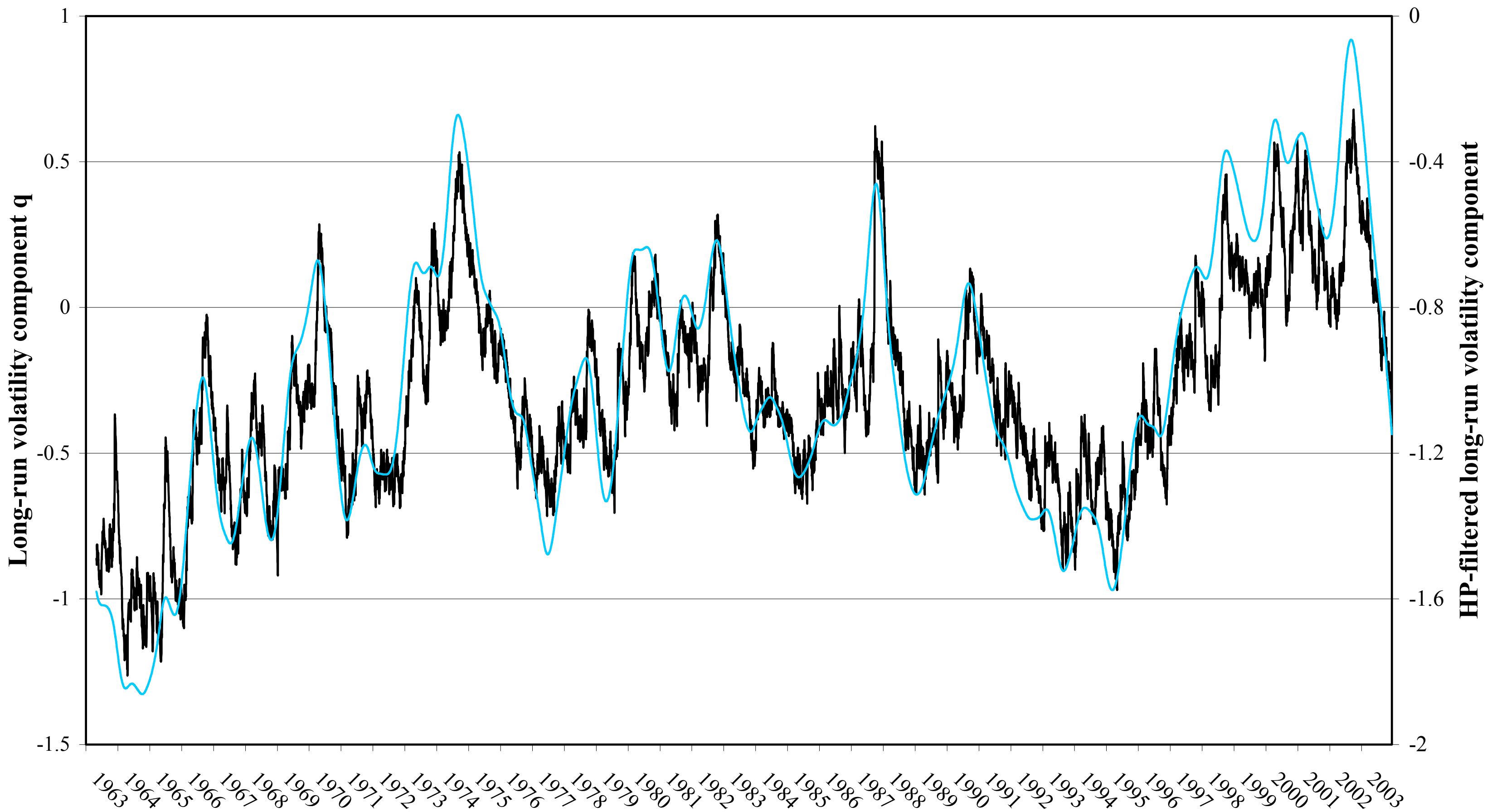

Figure 3: The Short-Run Volatility Component

This figure plots the estimate of the short-run volatility component *s* at a daily frequency for 1963/5/1 - 2003/12/31. The variance of the excess market return is defined as *v=exp(2(s+q))* , where *q* is the long-run component of volatility (Figure 2). The estimate of *s* results from the stochastic volatility model that is reported in Table 1.