Forecasting Correlation and Covariance with a Range-Based Dynamic Conditional Correlation Model

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Abstract

This paper proposes a range-based Dynamic Conditional Correlation (DCC) model, which is an extension of Engle's (2002a) DCC model. The efficiency of the range data in volatility estimation is documented in Parkinson (1980), Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2002), and Chou (2004a, b), among others. It is hence natural to consider the implication of this result in the estimation of multivariate GARCH models. In the DCC model, the conditional correlation coefficients are estimated by a dynamic model for the product of the pair-wise return series with each normalized by their conditional standard deviations. The conditional standard deviation is calculated by using a univariate GARCH for the return series.

We use the Conditional Autoregressive Range (CARR) model of Chou (2004a), as an alternative to the univariate GARCH in the DCC first-step estimation. We therefore construct a range-based DCC model. The substantial gain in efficiency in the volatility estimation can induce an efficiency gain in the estimation of the series of the time-varying correlation coefficient and covariance. For comparison we estimate the generalized return-based DCC model as a benchmark to gain insights into the difference of these methods. We use three data sets for empirical analyses: the stock indices of S&P500 and Nasdaq, and the 10-year Treasury bond yield. Both in-sample and out-of-sample results indicate that our argument is supported in terms of the precision in estimating and forecasting the correlation and covariance matrices.

Keywords: DCC, CARR, range, dynamic correlation, covariance, volatility

I. Introduction

It is of primary importance in the practice of portfolio management, asset allocation, and risk management to have an accurate estimate of the covariance matrices for asset prices. When valuing derivatives, forecasts of volatilities and correlations over the whole life of the derivative are usually required. The univariate ARCH/GARCH family of models provides effective tools to estimate the volatilities of individual asset prices. Tailored to the needs of different asset classes, these various models have achieved remarkable success. For a survey of this vast literature, see Bollerslev, Chou, and Kroner (1992), and Engle (2004). It is, however, still an active research issue in estimating the covariance or correlation matrices of multiple, especially large sets of asset prices. Early attempts include the VECH model[1] of Bollerslev, Engle, and Wooldridge (1988), the BEKK model[2] of Engle and Kroner (1995), and the constant correlation model of Bollerslev (1990), among others. The constant correlation model is too restrictive as it imposes the stringent constraint that the dynamic structure of covariance is completely determined by the individual volatilities. The VECH and the BEKK models are more flexible in allowing time-varying correlations. The BEKK parameterization for a bivariate model involves 11 parameters, only two more than the VECH parameterization, but for higher-dimensional systems, the extra number of parameters in the BEKK model increases, and a completely free estimation becomes very difficult indeed.

In a series of papers, Engle and Sheppard (2001), Engle (2002a), and Engle, Cappiello, and Sheppard (2003) provide a solution to this problem by using a model entitled the Dynamic Conditional Correlation Multivariate GARCH (henceforth DCC). The conditional covariance estimation problem is simplified by estimating univariate GARCH models for each asset's variance process. Carrying on by using the transformed standardized residuals from the first stage, and estimating a time-varying conditional correlation estimator in the second stage, the DCC model is not linear, but can be estimated simply with the two-stage methods based on the maximum likelihood method. A meaningful and strong performance of this model is reported in these studies especially considering the ease of implementation of the estimator. Other methods for estimating the time-varying correlation are proposed by Tsay (2002) and

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[1] The n-dimensional VECH model is written as vech(H_t)=A+B vech($\xi_{t-1}\xi_{t-1}'$)+C vech(H_{t-1}), where H_t is the conditional covariance matrix at time t and vech(H_t) is the vector that stacks all the elements of the covariance matrix.

[2] A general parameterization that involves the minimum number of parameters while imposing no cross equation restrictions and ensuring positive definiteness for any parameter value is the BEKK model, named after Baba, Engle, Kraft, and Kroner who wrote the preliminary version of Engle and Kroner (1995).

by Tse and Tsui (2002).

In this paper, we consider a refinement of the DCC model by utilizing the high/low range data of asset prices. In estimating the volatility of asset prices, there is a growing awareness of the fact that the range data of asset prices can provide sharper estimates and forecasts than the return data based on close-to-close prices. Studies of supporting evidence include Parkinson (1980), Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992), and more recently Gallant, Hsu, and Tauchen (1999), Yang and Zhang (2000), Alizadeh, Brandt, and Diebold (2001), Brandt and Jones (2002), Chou (2004a, 2004b) and Chou, Wu, and Liu (2004). Chou (2004a) proposed the Conditional Autoregressive Range (henceforth CARR) model where can capture the dynamical volatility process and obtained some insightful evidence in real data. In other words, a range-based volatility model is an alternative manner out of the return-based volatility model. In light of the success of the range-based univariate volatility models, it is natural to inquire whether this estimation efficiency can be extended to a multivariate framework, in this case of the DCC model.

The remainder of the study proceeds in the following manner. Section 2 introduces the framework of the bivariate models to estimate the correlation and covariance process, especially for the return-based and the range-based DCC models. Section 3 describes the empirical data used and gives a discussion of the empirical results. The conclusion and directions for future studies are given in section 4.

II. Correlation/Covariance Estimation and the DCC Model

Our objective is to estimate the current level of covariance and correlation. Traditionally, the conditional covariance and correlation between two random variables r_1 and r_2 with zero means are defined by:

$$COV_{12,t} = E_{t-1}(r_{1,t} r_{2,t}) , \tag{1}$$

$$\rho_{12,t} = \frac{E_{t-1}(r_{1,t} r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2) E_{t-1}(r_{2,t}^2)}} . \tag{2}$$

In this definition, the conditional covariance and correlation are decided by previous information. This method has two problems, namely, too early data are used and equal weights are assigned for every previous lag. To overcome the first problem, we introduce the moving average type with a 100-week window, MA(100):

$$COV_{12,t} = \frac{1}{100} \sum_{s=t-100}^{t-1} r_{1,s} r_{2,s} , \tag{3}$$

$$\hat{\rho}_t = \frac{\sum_{s=t-100}^{t-1} r_{1,s} r_{2,s}}{\sqrt{(\sum_{s=t-100}^{t-1} r_{1,s}^2)(\sum_{s=t-100}^{t-1} r_{2,s}^2)}} . \tag{4}$$

It makes sense to give more weight to recent data. From this point of view, we introduce an exponentially-weighted moving average (EWMA) model where the weights decrease exponentially as we move back through time. The exponentially-weighted moving average model has the attractive feature that relatively little data need to be stored. Exponential averages assign the most weight to the most recent observation, with weights declining exponentially with time. Hence, the EWMA model for covariance and correlation can be illustrated as follows.

$$COV_{12,t} = (1-\lambda) \sum_{s=1}^{t-1} \lambda^{t-s-1} r_{1,s} r_{2,s} , \tag{5}$$

$$\hat{\rho}_t = \frac{\sum_{s=1}^{t-1} \lambda^{t-s-1} r_{1,s} r_{2,s}}{\sqrt{(\sum_{s=1}^{t-1} \lambda^{t-s-1} r_{1,s}^2)(\sum_{s=1}^{t-1} \lambda^{t-s-1} r_{2,s}^2)}} . \tag{6}$$

The value of λ governs how responsive the estimate of the current volatility is to the most recent period's percentage change. As to the coefficient λ is usually called the exponential smoother[3] in this model by RiskMetrics™. The RiskMetrics™ approach uses exponential moving averages to estimate future volatility, because it believes the method responds rapidly to market shocks.

Bollerslev (1990) proposes the Constant Correlation Coefficient (henceforth CCC) model, which specifies that

$$H_t = D_t R D_t , \tag{7}$$

where R is the sample correlation matrix and D_t is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the i^{th} diagonal, where $\sqrt{h_{i,t}}$ is the square root of the estimated variance. Under such a situation, we can obtain the estimate of conditional covariance by the information of the fixed correlation and the product of the two conditional standard deviations.

Although the CCC model is meaningful, the setting of constant conditional correlations can be too restrictive. Engle (2002) extends the CCC model to the DCC model. The DCC model is a new form of the multivariate GARCH that is particularly convenient for complex systems and suitable for time-varying conditional correlations. The DCC model differs from the CCC model only in allowing R to be changed over time. Thus, the DCC model can be shown as follows.

$$H_t = D_t R_t D_t , \tag{8}$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} . \tag{9}$$

Here, D_t is defined like equation (7) and

$$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1} Z_{t-1}' + B \circ Q_{t-1} . \tag{10}$$

[3] The RiskMetrics™ database uses the exponentially-weighted moving average model with λ =0.94 for updating daily volatility estimates. J.P. Morgan found that, across variant market variables, this value of λ gives forecasts of the volatility that come closest to the realized volatility. Following J.P. Morgan's suggestion, the variable λ equals 0.94 for the time being in the latter empirical discussion.

In equation (10), A and B are parameters and o denotes the Hadamard matrix product operator, i.e., element-wise multiplication. The symbol ι is a vector of ones and S is the unconditional covariance of the standardized residuals. Finally, $Z_t = D_t^{-1} \times r_t$ are the standardized but correlated residuals. The variable r_t represents the returns of assets. The returns can be either mean zero or the residuals from a filtered time series, i.e.

$$r_t \left| I_{t-1} \sim N(0, H_t) \right. . \tag{11}$$

The conditional variances of the components of Z_t are, in other words, equal to 1, but the conditional correlation matrix is given by the variable of R_t. If A and B are zero, then we obtain the results of the CCC model. It is important to recognize that although the dynamic of the D_t matrix has usually been structured as a standard univariate GARCH model, it can extend to many other types. For instance, one could adopt the EGARCH model to capture the asymmetric effects in the volatility process or the FIGARCH model to allow for the long memory volatility processes. Later on, we shall propose to use the Conditional Autoregressive Range (CARR) model of Chou (2004a) as an alternative. The details will be given in the later part of the section.

As for parameters A and B, it is shown that if A, B, and $(\iota\iota' - A - B)$ are positive semi-definite, then Q_t will be positive semi-definite. If any one of the matrices is positive definite, then Q_t will also be so. For the ij[th] element of R_t, the conditional correlation matrix is given by $q_{ij,t} \big/ \sqrt{q_{ii,t} q_{jj,t}}$. As to the conditional covariance, it can then be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. Engle and Sheppard (2001) show results that simplify finding the necessary conditions for R_t to be positive definite and hence a correlation matrix with a real, symmetric positive semi-definite matrix, with ones on its diagonal line. The log-likelihood of this estimator can be written as:

$$
\begin{aligned}
L &= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+\log\left|H_t\right|+r_t{}'H_t^{-1}r_t\right)\\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+\log\left|D_tR_tD_t\right|+r_t{}'D_t^{-1}R_t^{-1}D_t^{-1}r_t\right)\\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+2\log\left|D_t\right|+\log\left|R_t\right|+Z_t{}'R_t^{-1}Z_t\right)
\end{aligned}
\tag{12}
$$

Here, $Z_t \sim N(0,R_t)$ are the univariate GARCH standardized residuals. Based on Engle (2002a)'s argument, we can perform the estimation in two steps. This estimator will no longer be efficient, but still consistent (also see Hafner and Franses (2003)). Let the parameters in D_t be denoted θ and the additional parameters in R_t will be denoted by ϕ. The log-likelihood function can be split into two respective parts:

$$
L(\theta,\phi)=L_V(\theta)+L_C(\theta,\phi)\,. \tag{13}
$$

The former term expresses the volatility part:

$$
L_V(\theta)=-\frac{1}{2}\sum_t\left(n\log(2\pi)+\log\left|D_t\right|^2+r_t{}'D_t^{-2}r_t\right). \tag{14}
$$

The latter term is the correlation component:

$$
L_C(\theta,\phi)=-\frac{1}{2}\sum_t\left(\log\left|R_t\right|+Z_t{}'R_t^{-1}Z_t-Z_t{}'Z_t\right). \tag{15}
$$

At the first step, equation (14) is maximized with respect to θ. At the second step, equation (15) is maximized with respect to θ and ϕ. We use this two-step estimation procedure in our empirical study.

The volatility part of the likelihood is the sum of the individual GARCH likelihood if D_t is determined by a GARCH specification.

$$
L_V(\theta)=-\frac{1}{2}\sum_t\sum_{i=1}^{k}\left(\log(2\pi)+\log(h_{i,t})+\frac{r_{i,t}^2}{h_{i,t}}\right). \tag{16}
$$

This can be jointly maximized by separately maximizing each term. If D_t is determined by a CARR specification, then the likelihood function of the volatility term is

$$L_V(\theta) = -\frac{1}{2}\sum_t \sum_{i=1}^{k} \left(\log(2\pi) + 2\log(\lambda_{i,t}^{*}) + \frac{r_{i,t}^{2}}{\lambda_{i,t}^{*2}} \right), \tag{17}$$

where $\lambda_{i,t}^{*}$ is the conditional standard deviation as computed from a scaled expected range from the CARR model.

The second part of the likelihood will be used to estimate the correlation parameters. As the squared residuals are not dependent on these parameters, they will not enter the first-order conditions and can be ignored. The two-step approach to maximizing the likelihood is to find

$$\hat{\theta} = \arg\max\{L_V(\theta)\}, \tag{18}$$

and then take this value as given in the second stage,

$$\max_{\phi}\{L_C(\hat{\theta},\phi)\}. \tag{19}$$

It is shown in Engle and Sheppard (2001) that under reasonable regularity conditions, consistency of the first step will ensure consistency of the second step. The maximum of the second step will be a function of the first-step parameter estimates, and so if the first step is consistent, then the second step will be too as long as the function is continuous in a neighborhood of the true parameters. These conditions are similar to those given in White (1994) where the asymptotic normality and the consistency of the two-step QMLE estimator are established.

Another theoretical justification of the above result is appeared in Engle (2002a). He referred to the work of Newey and McFadden (1994) whereby in Theorem 6.1, a formulated two-step GMM problem can be applied to this model. Consider the moment condition corresponding to the first step as $\nabla_\theta\{L_v(\theta)\}=0$. The moment condition corresponding to the second step is $\nabla_\phi\{L_C(\hat{\theta},\phi)\}$. Under standard regularity conditions which are given as conditions i) to v) in Theorem 3.4 of Newey and McFadden, the parameter estimates will be consistent, and asymptotically normal, with a covariance matrix of familiar form. This matrix is the product of two inverted Hessians around an outer product of scores. Details of this proof can be found in Engle and Sheppard (2001)

The DCC model is a new type of multivariate and can fit the GARCH or CARR model in the first stage, which is particularly convenient for complex systems. The DCC method first estimates volatilities for each asset and computes the standardized residuals. For bivariate cases, we use the following GARCH and CARR structures to perform the first step, respectively. The covariances are then estimated between these using a maximum likelihood criterion and one of several models for the correlations.

For the GARCH volatility structure (return-based conditional volatility model):

$$
\begin{aligned}
& r_{k,t} = \varepsilon_{k,t} \qquad \varepsilon_{k,t} \mid I_{t-1} \sim N(0, h_{k,t}),\ k=1,2 \\
& h_{k,t} = \omega_k + \alpha_k \varepsilon^2_{k,t-i} + \beta_k h_{k,t-1}, \\
& z^a_{k,t} = r_{k,t} / \sqrt{h_{k,t}}\ .
\end{aligned}
\tag{20}
$$

If the volatility model is CARR（range-based conditional volatility model）:

$$
\begin{aligned}
& \Re_{k,t} = u_{k,t} \qquad u_{k,t} \mid I_{t-1} \sim \exp(1;\cdot),\ k=1,2 \\
& \lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}, \\
& z^c_{k,t} = r_{k,t} / \lambda^*_{k,t}, \ \text{ where } \ \lambda^*_{k,t} = adj_k \times \lambda_{k,t}, \quad adj_k = \frac{\overline{\sigma}}{\overline{\hat{\lambda}_k}},
\end{aligned}
\tag{21}
$$

where $\Re_{k,t}$ is the high/low range in logarithm, of the k^{th} asset during time interval t, $\overline{\sigma}$ and $\overline{\hat{\lambda}_k}$ are respectively the unconditional variance of the return series and the sample mean of the estimated conditional range of the series k. This is a special case of the multiplicative error model of Engle (2002b). The specification of the exponential distribution of the disturbance term provides a consistent although inefficient estimator for the parameters. For specific discussions also see Chou (2004a).

In the following analysis, we use two alternative versions of DCC. The first one is the standard DCC with mean reversion (henceforth MR_DCC), discussed in Engle (2002). The second one is the integrated DCC (henceforth I_DCC). Both of these two models are simplified versions of the general expression in equation (8).

For the bivariate case, the MR_DCC is constructed by the following equation.

$$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1}Z_{t-1}' + B \circ Q_{t-1} \text{, or}$$

$$\begin{bmatrix} q_{1,t} & q_{12,t} \\ q_{12,t} & q_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} \circ \begin{bmatrix} 1-a_1-b_1 & 1-a_3-b_3 \\ 1-a_3-b_3 & 1-a_2-b_2 \end{bmatrix} + \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix} \circ \begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & z_{2,t-1}^2 \end{bmatrix}$$

$$+ \begin{bmatrix} b_1 & b_3 \\ b_3 & b_2 \end{bmatrix} \circ \begin{bmatrix} q_{1,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{2,t-1} \end{bmatrix}, \tag{22}$$

where $\bar{q}_{12} = \frac{1}{T}\sum_{t=1}^{T} z_{1,t}z_{2,t}$.

For I_DCC, the dynamic structure simplifies to:

$$Q_t = A \circ Z_{t-1}Z'_{t-1} + (1-A) \circ Q_{t-1} \text{, or}$$

$$\begin{bmatrix} q_{1,t} & q_{12,t} \\ q_{12,t} & q_{2,t} \end{bmatrix} = \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix} \circ \begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & z_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} 1-a_1 & 1-a_3 \\ 1-a_3 & 1-a_2 \end{bmatrix} \circ \begin{bmatrix} q_{1,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{2,t-1} \end{bmatrix}. \tag{23}$$

Like the specific property of volatilities, the correlation and covariance matrices are also unobservable. We use daily data to construct the proxies for the weekly-realized covariance/correlation observations. The purpose of such doing is to extract these so-called “measured covariance/correlation”, denoted MCOV/MCORR respectively, as one kind of benchmark in determining the relative performance of the return-based DCC model and the range-based DCC model for the time being. On the other side, we perform the tailor-made regression framework proposed by Mincer and Zarnowitz (1969) for the in-sample comparison. We demonstrate the regression expression below:

$$MCORR_t = \gamma_0 + \gamma_1 \hat{\rho}_t^{return} + \varepsilon_{1,t}$$

$$MCORR_t = \gamma_0 + \gamma_2 \hat{\rho}_t^{range} + \varepsilon_{2,t} \tag{24}$$

$$MCORR_t = \gamma_0 + \gamma_1 \hat{\rho}_t^{return} + \gamma_2 \hat{\rho}_t^{range} + \varepsilon_{3,t} \,.$$

The major focus here is to check the significance of coefficients γ_1 and γ_2. The statistical intuition here is similar to the conventional OLS framework. Similarly, we construct the system of covariance in (25):

$$\begin{aligned}
MCOV_t &= \phi_0 + \phi_1 COV_t^{return} + \varepsilon_{1,t} \\
MCOV_t &= \phi_0 + \phi_2 COV_t^{range} + \varepsilon_{2,t} \\
MCOV_t &= \phi_0 + \phi_1 COV_t^{return} + \phi_2 COV_t^{range} + \varepsilon_{3,t} \,,
\end{aligned} \tag{25}$$

where $COV_t = \hat{\rho}_t \times \hat{\sigma}_{1,t} \times \hat{\sigma}_{2,t}$, $\hat{\sigma}_{k,t}$ are standard deviations estimated from the return-based DCC model or the range-based DCC model.

In constructing the comparison of in-sample data in our subsequent empirical analysis about correlation and covariance, several related models are included, such as MA(100), EWMA with $\lambda = 0.94$, and the CCC models. However, we exclude the correlation coefficient shown for the CCC model, due to the constant restriction in nature[4]. Here one just uses the estimated correlation regression on the realized correlation, and the correlation is similar in the same manner. For a simple regression, the R-squared can be used as a rough proxy for the model's performance.

For completeness, we also perform out-of-sample forecast comparisons. It is very straightforward to derive the formulation in computing the out-of-sample conditional correlation for a MR_DCC specification. Given T as the sample size, the

[4] From expression (24), any one of the explanatory variables is significantly different from zero in statistics, which will reject the null hypothesis for the correlation being constant.

T+1st observation is obtained by:

$$\begin{bmatrix} q_{1,T+1} & q_{12,T+1} \\ q_{12,T+1} & q_{2,T+1} \end{bmatrix} = \begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} \circ \begin{bmatrix} 1-a_1-b_1 & 1-a_3-b_3 \\ 1-a_3-b_3 & 1-a_2-b_2 \end{bmatrix} + \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix} \circ \begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix}$$

$$+ \begin{bmatrix} b_1 & b_3 \\ b_3 & b_2 \end{bmatrix} \circ \begin{bmatrix} q_{1,T} & q_{12,T} \\ q_{12,T} & q_{2,T} \end{bmatrix}, \tag{26}$$

where $\rho_{T+1} = q_{12,T+1} / \sqrt{q_{1,T+1} q_{2,T+1}}$.

For the period of t+h, with $h \geq 2$, the correlation is:

$$\begin{bmatrix} q_{1,T+h} & q_{12,T+h} \\ q_{12,T+h} & q_{2,T+h} \end{bmatrix} = \begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} \circ \begin{bmatrix} 1-a_1-b_1 & 1-a_3-b_3 \\ 1-a_3-b_3 & 1-a_2-b_2 \end{bmatrix}$$

$$+ \begin{bmatrix} a_1+b_1 & a_3+b_3 \\ a_3+b_3 & a_2+b_2 \end{bmatrix} \circ \begin{bmatrix} q_{1,T+h-1} & q_{12,T+h-1} \\ q_{12,T+h-1} & q_{2,T+h-1} \end{bmatrix}. \tag{27}$$

Since the values for the out-of-sample correlation forecast derived from the I_DCC model are constants, we abridge the redundant explanation. In addition to the range-based and return-based DCC models, the MA(100) and the CCC models are introduced for an out-of-sample predictive comparison[5]. It is also worth noting that the correlation is constant for the out-of-sample forecast based on the CCC model, and so we focus on the performance of the models in forecasting the conditional covariance. Empirically speaking, we still take the value of R-squared as an indication for the comparison of preciseness.

[5] It is also intuitively clear that the out-of-sample forecasts for the correlation and covariance are both constant in the EWMA model. Thus, we ignore the related discussion here.

III. Evaluation of Conditional Correlation and Covariance Forecasts

The data employed in this study comprises 835 weekly observations on the S&P500 Composite (henceforth S&P500), the Nasdaq stock market index, and the yield for 10-year treasury bond (Tbond) spanning the period January 4, 1988 to January 2, 2004. In addition, daily observations are used to construct the series of measured or so-called the realized covariance and correlation in the related literature. We retrieve the ranges and returns data for the entire period from Yahoo's database.

It is worth taking a look at some descriptive statistics. Panels A, B, and C in Figure 1 demonstrate the weekly data patterns for the time-series of the S&P500 stock market index, the Nasdaq index and the yield to maturity for the 10-year Tbond over the sample period. Additionally, Table 1 provides summary statistics for weekly continuously compounded returns and weekly ranges for these indices.

Let $MCORR_t$ and $MCOV_t$ represent the measured or realized correlation coefficient and measured covariance, respectively. The $MCORR_t$ is defined as

$$MCORR_t = \frac{1}{\tau}\sum_{i=1}^{\tau} \rho_t^i \,, \tag{28}$$

where τ denotes the trading days during the week t and ρ_t^i is the correlation coefficient at the i^{th} trading day of the week t. This series is obtained from using the daily returns data and fitting them with a MR_DCC model. By the same way, the measured covariance ($MCOV_t$) can be expressed as follows.

$$MCOV_t = \sum_{i=1}^{\tau} (r_{1t}^i \times r_{2t}^i)\,, \tag{29}$$

where r_{kt}^i represents the daily returns of index k at the i^{th} trading day during week t. This expression is a direct extension of the concept of the realized volatility of Andersen, Bollerslev, Diebold, and Labys (2000).

Figure 2 shows the graphs of $MCORR_t$ and $MCOV_t$ for the S&P500 and Nasdaq indices. We allocate them at Panels A and B in Figure 2 separately. It is interesting to note that for the last two years, the two series were highly positively correlated, and moved more smoothly. Taking Tbond returns to be minus the changes in the 10-year benchmark yield to maturity as in Engle (2002), the correlation and covariance between the Tbond market and stock market are traced. As to Panels A and B in Figure 3, we report the time series of $MCORR_t$ and $MCOV_t$ for S&P500 and Tbond yields. On the same picture we also show the $MCORR_t$ and $MCOV_t$ series between the Nasdaq index and S&P500 in Figure 4. Judging from Figures 3 and 4, we find that the correlation patterns between the bond market and stock markets appear to show a reverting phenomenon around approximately the year 2000. It can also be determined that the covariance processes are more volatile after the year 2000.

A. In-sample forecast comparison

In this section we present the results of using the in-sample data; that is, the forecast performances are constructed and measured using the same database. Two variant DCC forms are discussed, namely, the mean reverting DCC form and the integrated DCC form (i.e., MR_DCC and I_DCC models for shorthand).

Panel A in Table 2 describes the in-sample forecasting performance for MCORR. It is very consistent to recognize that the interactive regression model fitting to the relationship between different class markets is more suitable than the same class of stock markets judged from the R-squared index. With the exception of the correlation between the S&P500 and Nasdaq indices, all of the estimates of R-squared for the range DCC model are statistically preferred to the return DCC model. As to the in-sample forecast performance, there are not clear dominant advantages either for the MR_DCC or I_DCC model. Whatever DCC model is chosen and whether the return-based approach or range-based approach or the moving average model and exponential smoothing model are used, there are sufficient reasons to infer that the correlation is a time-varying variable from the significant coefficient in the t-value. Here, the t-value in our regression model is shown after adjustment of the White heteroskedasticity-consistent standard errors. Surprisingly, the exponential smoothing method for the correlation fitting under the in-sample forecasting scenario is better than the others, regardless of which two markets' interaction is dissected, especially in the S&P500 and Nasdaq indices.

The exponential smoothing model seems to perform well in the sample for correlation forecasting, but poorly for the out-of-sample forecasting. Technically, based on the exponential smoothing model, it obtains a constant value when we predict the out-of-sample correlation between any two variables. In other words, we cannot capture the term structure of correlation for the out-of-sample period. Finally, no matter what market data is used, the MA100 is the worst model in our in-sample correlation forecast performance comparison. Each analysis and inference herein is consulted after the introduction of the realized correlation proxy in expression (28).As to other observations from Panel A in Table 2, the range-based parameter $\hat{\gamma}_2$ under the MR_DCC model is not significantly different from zero in statistics when both of the two independent variables (i.e. $\hat{\rho}_t^{return}$ and $\hat{\rho}_t^{range}$) are simultaneously fitted.

From Panel B in Table 2, we find that the $\hat{\gamma}_1$ coefficient appears negative under the MR_DCC model when the two correlation estimates are independent variables at the same time. However, the inference is indifferent from zero by the viewpoint of conventional statistics. Due to the value of R-squared almost staying at the same level, it seems that when the in-sample MCORR forecasting is evaluated, the return-based proxy derived by the GARCH type volatility is dominated by the range-based proxy derived by the CARR type volatility based on the S&P500 index and Tbond yield data.

As to the in-sample forecasting of MCORR for the Nasdaq stock index and Tbond yield for Panel C in Table 2, we find that the ability to explain the realized correlation variable's changes is better and can be shown when the significance of the return-based parameter estimate of $\hat{\gamma}_1$ is reduced. Contrasting to both independent variables that are incorporated at the same time, we achieve the analogous result illustrated in Panel B. It is apparently that the range proxy fitting to the in-sample forecasts of correlation is again better on the setting of the DCC model.

From Panel A in Table 3 we arrange the in-sample MCOV forecasting performance comparison for several useful models. Regardless of which DCC model is selected, the values of R-squared for the range-based models are all larger than those of the return-based models. Nonetheless, model fitting to the covariance pattern is a slightly poorer than to corresponding correlation pattern. Clues come from the overall information of R-squared adopted by Tables 2 and 3. It is interested that the value of R-squared drops by a significant amount when we discuss the covariance behavior. To our knowledge, the change for the covariance pattern might be more volatile than the corresponding correlation series and cannot easily capture the movement, and we can shown the R-squared value is lower than the corresponding correlation that evidence from Table 2.

It is also worth seeing that when the two independent variables are incorporated simultaneously in the regression model, then the return-based coefficient estimate $\hat{\phi}_1$ is no more significantly different from zero under the MR_DCC or I_DCC model. Evidently, the proxy variable of the range-based model for volatility is more powerful than the return-based one in explaining the variant of the realized covariance variable once more.

The focus on Panel B in Table 3 mentions about the covariance relationship between the S&P500 and Tbond yield series. Judging from the R-squared value and the significance of the coefficient, we can see the fact that the range-based proxy produces superior in-sample forecasts for MCOV relative to the return-based one when the two independent variables are fitted in the same regression equation. No matter if the MR_DCC model or the I_DCC model is extracted, the conclusion is consistent.

As to Panel C in Table 3 compares the in-sample covariance forecasting between the Nasdaq stock index and Tbond yield for alternative models. The inference is analogous to the Nasdaq stock index and Tbond yield when the issue of correlation behavior is explored. Namely, it is highly possible for the range-based variable for volatility to replace and dominate the return-based variable for volatility when the patterns of MCOV and MCORR are captured.

From Table 3, we obtain other information for covariance when the lower part in each panel is checked. When using traditional and conventional manners to describe the covariance activities between variables, for instance, moving average approach, exponential smoothing approach, or constant conditional correlation model proposed by Bollerslev (1990), the forecasting ability for the in-sample covariance has no clear advantage direction among them. However, the only inference is that the DCC-based family is better than the traditional methodologies in the performance of the in-sample forecasting for the covariance variable.

B. Out-of-sample forecast comparison

To assess the relative performances for the out-of-sample correlation and covariance forecasting, we adopt the procedure of a rolling sample to estimate the out-of-sample forecasts using MR_DCC and I_DCC specifications for both the return-based and range-based models. For each individual model, we compute the out-of-sample forecasts for the horizons of 1, 2, 3, and 4 weeks. In all cases, we re-estimated the estimates 100 times. We then use a simple regression to compare the explanatory power of these various forecasts on the realized covariances or correlations.

Table 4 reports the value of R^2 from a linear regression of MCORR on each of these out-of-sample forecast series. The overall result is consistent whatever out-of-sample horizon is chosen. We achieve a confirmation that the DCC-range model is more powerful then DCC-return model when the forecasting intervals of correlation are during one-month period. With the exception of the relationship between the Nasdaq index and the Tbond yield for the four weeks out of sample prediction, all of the estimates of R-squared for any other market data witness the inference.

Table 5 shows the results for the comparison of out-of-sample forecasts for the covariance variable. A clear blueprint emerges immediately from the implication of this table. It is important to note that the DCC model with the range-based framework is significantly dominant than the return-based DCC model. Judging from the R-squared index, no matter what markets' trading data are shown in this period. The range-based DCC model outperforms the others in all of the 12 cases for out-of-sample covariance forecasting, too. There are some differences from the correlation prediction we obtained from Table 4. We obtain that the return-based DCC model no longer significantly better than the conventional moving average approach in forecasting for the covariance variable. At the extreme, the returned-based DCC is worse than the moving average approach when the Nasdaq stock index and Tbond yield are discussed. As to the CCC model, it cannot deal with the property of the time-varying correlation naturally and its performance for the out-of-sample forecasting about the covariance variable is poor for one-month period. In fact, as to the performance for out-of-sample forecasting in correlation and covariance, the latter does not prevail against the former. With the exception of the one-week time horizon for covariance forecasting based on the range-DCC model, other substitute models hardly capture the outline for the out-of-sample covariance variance. It is possible to infer that the covariance pattern is not easily captured and the characteristic of the market trading data is another suitable reason. Due to the volatility of correlation being flatter than the covariance, the R-squared being higher than the corresponding covariance model is quite recognizable.

IV. Conclusions

In this paper, a new estimator of the time-varying correlation/covariance matrices is proposed utilizing the range data by combining the CARR model proposed by Chou (2004a) and the framework of Engle (2002a)'s DCC model. The advantage of this range-based DCC model outperforming the standard return-based DCC model hinges on the relative efficiency of the range over the return data in estimating volatilities. Using weekly returns of S&P500, Nasdaq and 10-year treasury bond rates, we find consistent results that the range-based DCC model outperforms the return-based models in estimating and forecasting covariance and correlation matrices, both in-sample and out-of-sample.

Although we apply this estimator to the bivariate systems, it can be applied to larger systems in a manner similar to the application of the return-based DCC model structures that is demonstrated in Engle and Sheppard (2001). Future research will be useful in adopting more diagnostic statistics or tests based on value at risk calculations as is proposed by Engle and Manganelli (1999). Applications to the estimation of optimal portfolio weighting matrices and the calculation of the dynamic hedge ratio in the futures market will also be fruitful.

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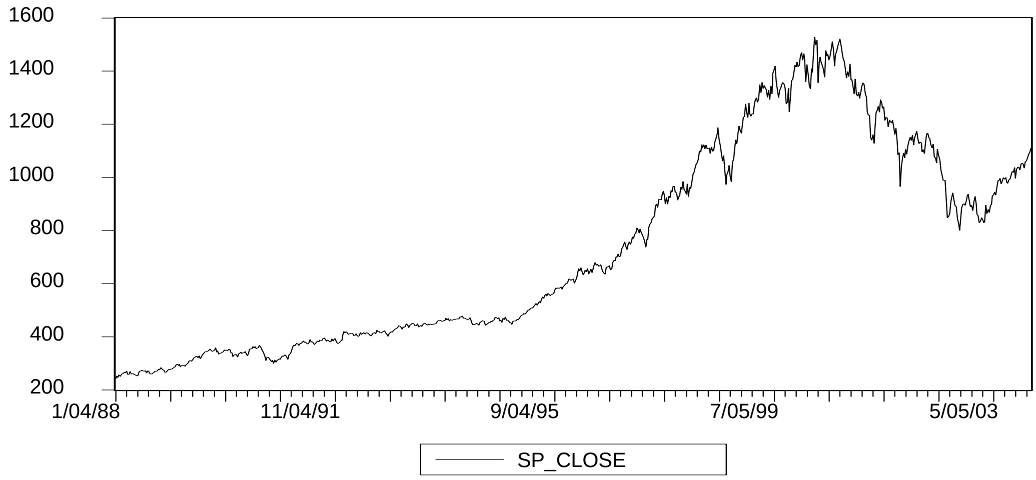
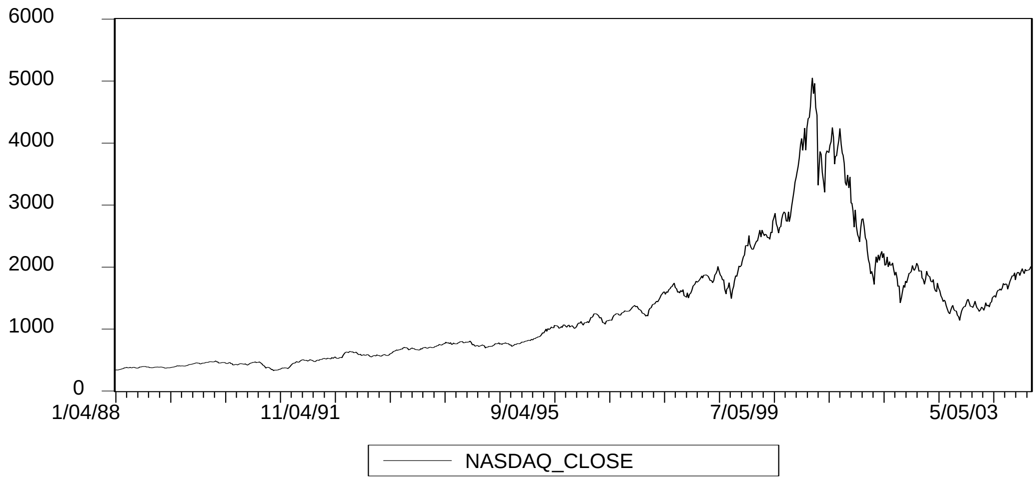
Figure 1: S&P500, Nasdaq Indices, and Tbond Yield Weekly Prices, 1/4/1988-1/2/2004

Panel A: S&P500 Stock index weekly closing prices



Panel B: Nasdaq stock index weekly closing prices



Panel C: Yield to maturity for 10-year Tbond weekly closing yields

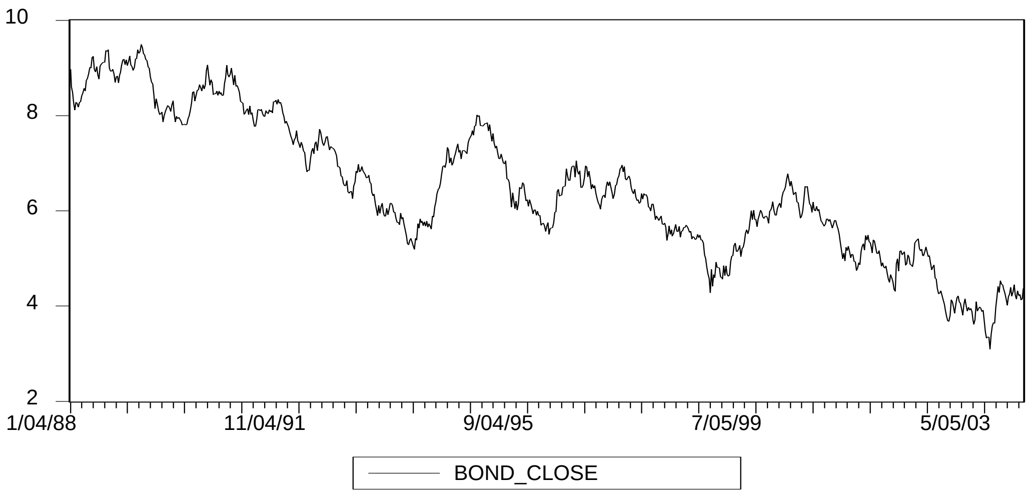
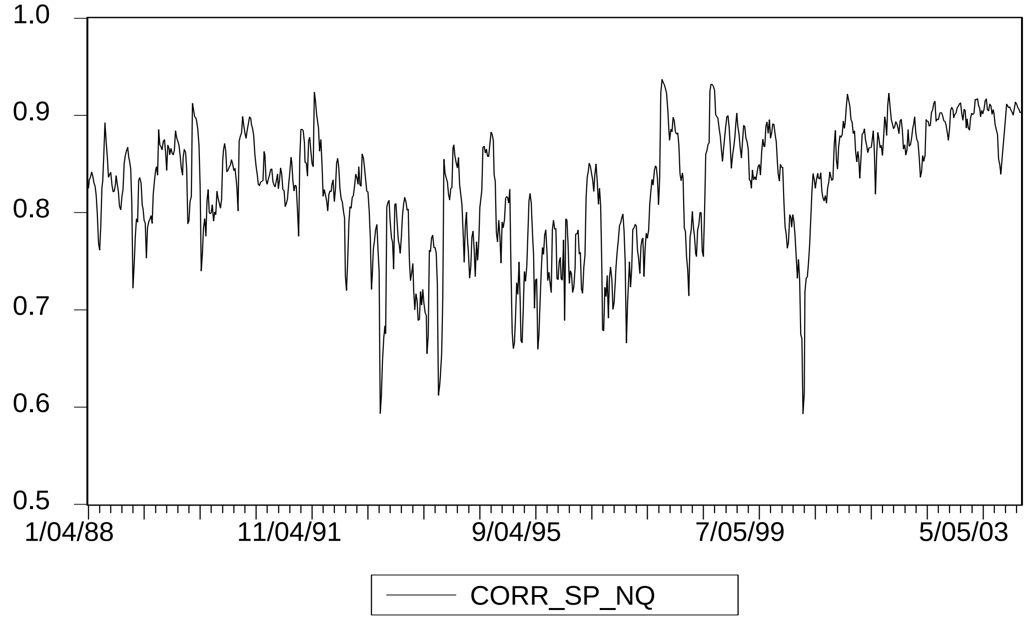
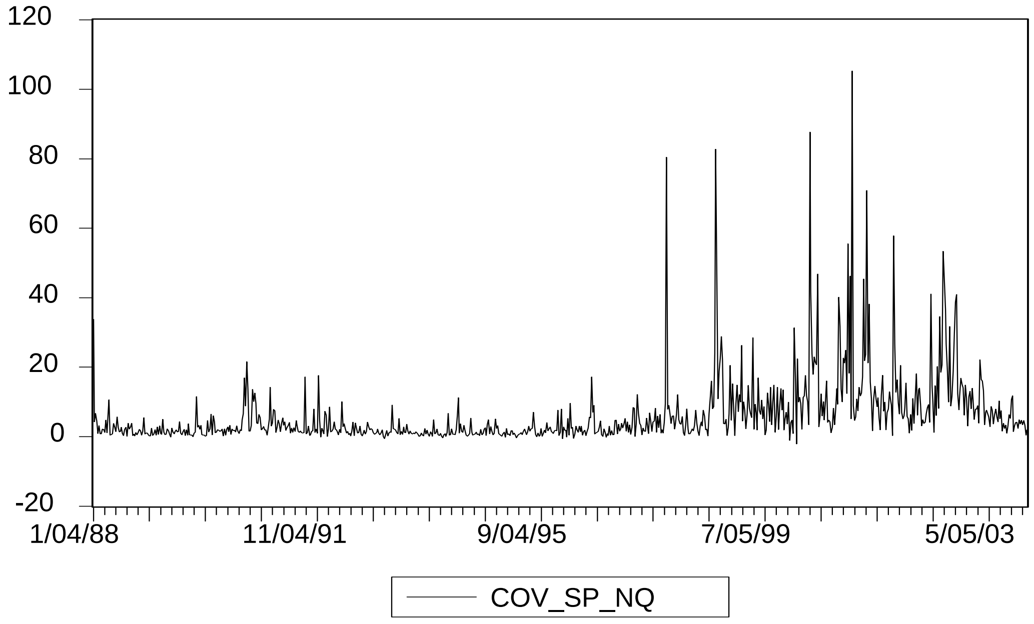


Figure 2: MCORR and MCOV for S&P500 and Nasdaq Indices, 1/4/1988-1/2/2004

Panel A: Correlation series between S&P500 and Nasdaq Indices



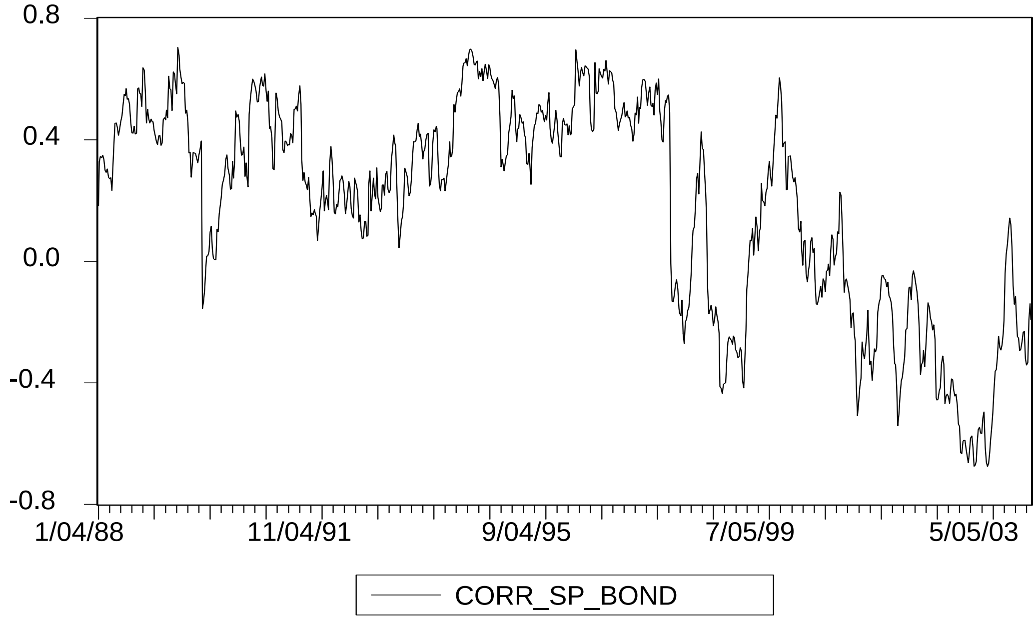
Panel B: Covariance series between S&P500 and Nasdaq Indices



<u>Figure 3: MCORR and MCOV for S&P500 Index and Tbond Yield,</u>

<u>1/4/1988-1/2/2004</u>

Panel A: Correlation series between S&P500 index and Tbond yield



Panel B: Covariance series between S&P500 index and Tbond yield

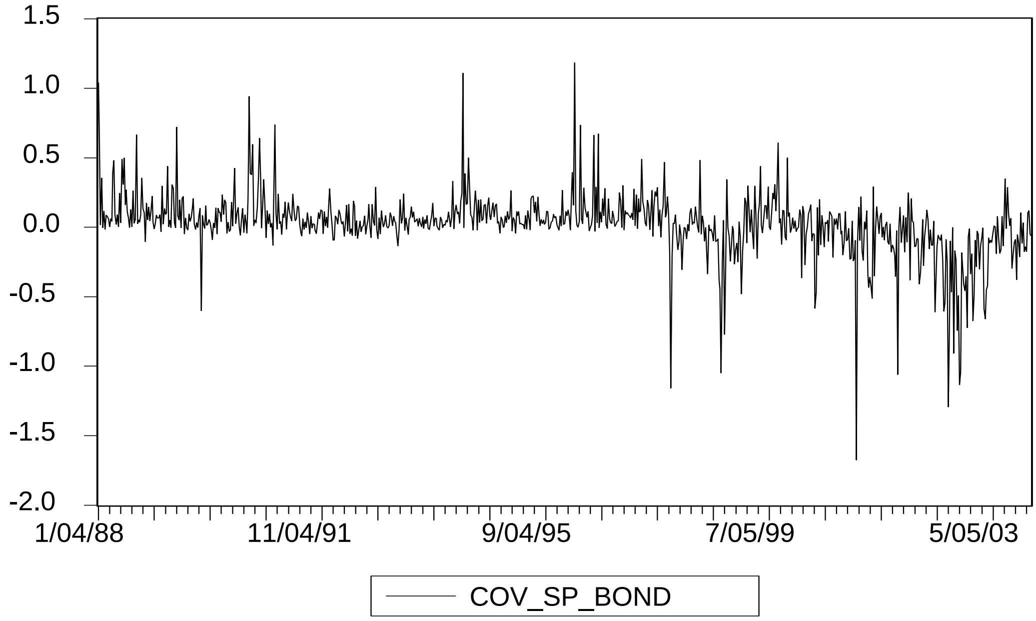
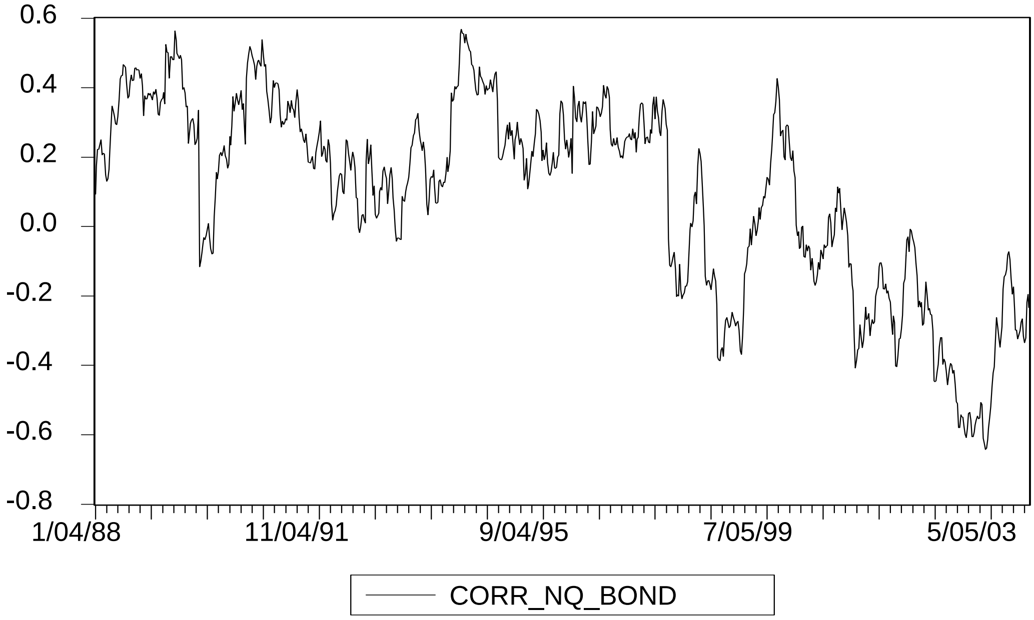


Figure 4: MCORR and MCOV for Nasdaq Index and Tbond Yield, 1/4/1988-1/2/2004

Panel A: Correlation series between Nasdaq index and Tbond yield



Panel B: Covariance series between Nasdaq index and Tbond yield

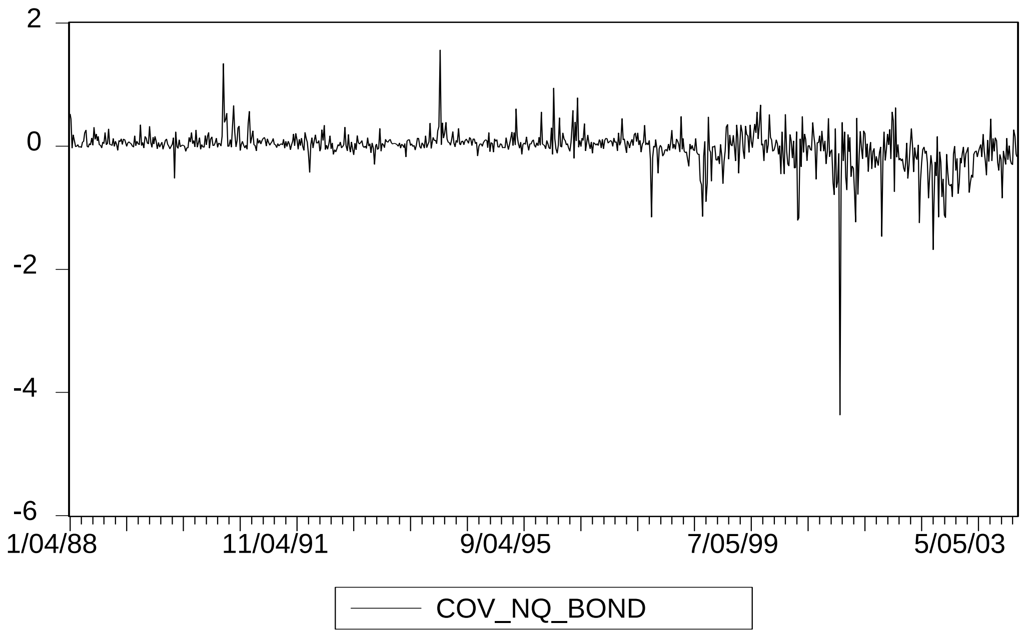


Table 1: Summary Statistics for the Returns and Ranges of Weekly S&P500, Nasdaq Indices, and Tbond Yield, 1/4/1988-1/2/2004

Ranges and returns for stock indices are computed by $100 \times \log(p^{high} / p^{low})$ and $100 \times \log(p_t^{close} / p_{t-1}^{close})$, respectively. Ranges and returns for the 10-year T(reasury)bond are inferred by $100 \times \log(p^{high} / p^{low})$ and $-100 \times \log(p_t^{close} / p_{t-1}^{close})$, respectively. Jarque-Bera is the statistic for normality. There are 835 weekly sample observations. All data are taken from Yahoo! Finance. The computation of the returns of the bond yield follows Engle (2002a).

Index	S&P500		Nasdaq		10-year Tbond	
Type	range	return	range	return	range	return
Mean	3.136	0.182	4.204	0.213	3.053	0.086
Median	2.657	0.339	3.225	0.396	2.646	0.127
Maximum	14.534	7.492	31.499	17.377	16.593	7.756
Minimum	0.707	-12.330	0.514	-29.175	0.112	-12.625
Std. Dev.	1.730	2.153	3.224	3.265	2.007	2.356
Skewness	1.768	-0.458	2.411	-1.124	1.736	-0.694
Kurtosis	8.025	5.733	12.782	13.112	8.222	5.437
Jarque-Bera	1312.227	288.695	4132.758	3728.608	1366.220	273.344

Table 2: In-sample Forecasting for Correlations between the S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1/4/1988-1/2/2004

$$MCORR_t = \gamma_0 + \gamma_1 \hat{\rho}_t^{return} + \varepsilon_{1,t}$$
$$MCORR_t = \gamma_0 + \gamma_2 \hat{\rho}_t^{range} + \varepsilon_{2,t}$$
$$MCORR_t = \gamma_0 + \gamma_1 \hat{\rho}_t^{return} + \gamma_2 \hat{\rho}_t^{range} + \varepsilon_{3,t}$$

This table reports the R^2 from a linear regression of the measured correlation (MCORR) on the correlation forecasts of the return-based DCC model, the range-based DCC model, models based on moving-average methods (MA100) and on exponential smoothing methods (Exp Smoothing). The t values with Heteroskedasticity-Autocorrelation-Consistent standard errors for the regression coefficients are in parentheses. There are 835 weekly sample observations.

MCORR		$\hat{\gamma}_0$		$\hat{\gamma}_1$		$\hat{\gamma}_2$		R-squared
Panel A: S&P500 and Nasdaq								
MR _DCC	Return	0.251	(10.425)	0.707	(24.691)			0.409
	Range	0.347	(16.937)			0.597	(24.539)	0.389
	BOTH	0.257	(10.524)	0.445	(7.435)	0.258	(5.277)	0.425
I_DCC	Return	0.250	(7.806)	0.705	(18.221)			0.316
	Range	0.280	(10.676)			0.672	(21.554)	0.350
	BOTH	0.171	(5.689)	0.363	(6.545)	0.440	(10.122)	0.392
MA100		0.259	(9.421)	0.699	(21.22)			0.330
Exp. Smoothing		0.375	(20.745)	0.555	(25.732)			0.489
Panel B: S&P500 and Tbond								
MR _DCC	Return	0.030	(4.524)	0.848	(57.328)			0.777
	Range	0.023	(3.789)			0.860	(70.081)	0.800
	BOTH	0.021	(3.621)	-0.399	(-3.513)	1.256	(11.066)	0.803
I_DCC	Return	0.023	(3.539)	0.965	(59.105)			0.794
	Range	0.007	(1.164)			0.945	(72.435)	0.811
	BOTH	0.008	(1.458)	0.128	(1.161)	0.822	(7.730)	0.811
MA100		-0.017	(-1.704)	0.890	(36.625)			0.587
Exp. Smoothing		0.025	(3.947)	0.846	(67.370)			0.792
Panel C: Nasdaq and Tbond								
MR _DCC	Return	0.026	(5.551)	0.851	(57.019)			0.777
	Range	0.029	(6.574)			0.818	(67.500)	0.816
	BOTH	0.031	(7.113)	-0.437	(-5.573)	1.222	(16.445)	0.821
I_DCC	Return	0.032	(6.888)	0.955	(57.549)			0.792
	Range	0.018	(3.987)			0.890	(60.008)	0.802
	BOTH	0.022	(4.970)	0.351	(3.944)	0.572	(6.777)	0.807
MA100		-0.017	(-2.707)	0.871	(42.423)			0.658
Exp. Smoothing		0.030	(6.283)	0.758	(62.599)			0.776

Table 3: In-Sample Forecasting for Covariances between S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1/4/1988-1/2/2004

$$MCOV_t = \phi_0 + \phi_1 COV_t^{return} + \varepsilon_{1,t}$$
$$MCOV_t = \phi_0 + \phi_2 COV_t^{range} + \varepsilon_{2,t}$$
$$MCOV_t = \phi_0 + \phi_1 COV_t^{return} + \phi_2 COV_t^{range} + \varepsilon_{3,t}$$

This table reports the R^2 from a linear regression of the measured covariance (MCOV) on the covariance forecasts of the return-based DCC model, the range-based DCC model, models based on moving-average methods (MA100), on exponential smoothing methods (Exp Smoothing) and the constant conditional correlation (CCC) model. The t values with Heteroskedasticity-Autocorrelation-Consistent standard errors for the regression coefficients are in parentheses. There are 835 weekly sample observations.

MCOV		ϕ_0		ϕ_1		ϕ_2		R-squared
Panel A: S&P500 and Nasdaq								
MR _DCC	Return	0.753	(1.968)	0.939	(10.178)			0.223
	Range	0.339	(0.750)			0.867	(9.438)	0.354
	BOTH	0.677	(1.829)	-0.212	(-1.209)	0.998	(5.553)	0.357
I_DCC	Return	1.148	(2.962)	0.863	(9.571)			0.202
	Range	0.384	(0.837)			0.848	(9.237)	0.351
	BOTH	0.693	(1.836)	-0.186	(-1.206)	0.961	(5.782)	0.354
MA100		0.788	(1.906)	1.011	(10.562)			0.210
Exp. Smooth		1.303	(3.577)	0.862	(9.855)			0.200
CCC		0.669	(1.690)	0.972	(9.994)			0.225
Panel B: S&P500 and Tbond								
MR _DCC	Return	-0.239	(-1.654)	0.960	(10.411)			0.230
	Range	-0.086	(-0.676)			0.860	(10.212)	0.309
	BOTH	0.007	(0.056)	-0.559	(-2.025)	1.262	(4.864)	0.320
I_DCC	Return	-0.298	(-2.014)	1.102	(10.850)			0.231
	Range	-0.199	(-1.489)			0.964	(10.312)	0.309
	BOTH	-0.148	(-1.139)	-0.428	(-1.517)	1.262	(4.775)	0.315
MA100		-0.485	(-2.843)	1.070	(8.285)			0.177
Exp. Smooth		-0.203	(-1.431)	0.945	(10.040)			0.227
CCC		2.527	(9.138)	-3.982	(0.103)			0.103
Panel C: Nasdaq and Tbond								
MR _DCC	Return	-0.280	(-1.633)	0.934	(9.682)			0.163
	Range	-0.025	(-0.149)			0.702	(9.749)	0.220
	BOTH	0.007	(0.043)	-0.301	(-1.157)	0.879	(4.418)	0.222
I_DCC	Return	-0.264	(-1.556)	1.064	(9.921)			0.168
	Range	-0.171	(-1.004)			0.782	(9.847)	0.218
	BOTH	-0.174	(-1.030)	-0.107	(-0.412)	0.844	(4.259)	0.218
MA100		-0.477	(-2.317)	0.955	(8.723)			0.152
Exp. Smooth		-0.280	(-1.661)	0.814	(9.338)			0.161
CCC		1.696	(5.711)	-14.546	(-5.789)			0.078

Table 4: Out-of-Sample Forecasting for the Correlations between S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1/4/1988-1/2/2004

$$MCORR_t = \gamma_0 + \gamma_1 \hat{\rho}_t + \varepsilon_t$$

This table reports the R^2 from a linear regression of the measured correlation (MCORR) on the correlation forecasts of the return-based DCC model, the range-based DCC model, and the moving-average methods (MA100). The regressions are conducted using 100 observations based on the rolling sample method. For each model, 700 observations are used for in-sample estimation and out-of-sample forecasts of horizons 1,2,3 and 4 weeks are made.

Panel A: S&P500 and Nasdaq			
R-squared	MR_DCC		
Forecast horizon	return-based	range-based	MA100
1	0.310	0.548	0.044
2	0.279	0.474	0.076
3	0.248	0.407	0.109
4	0.199	0.325	0.153

Panel B: S&P500 and Tbond			
R-squared	MR_DCC		
Forecast horizon	return-based	range-based	MA100
1	0.491	0.631	0.165
2	0.441	0.547	0.134
3	0.306	0.463	0.098
4	0.229	0.388	0.065

Panel C: Nasdaq and Tbond			
R-squared	MR_DCC		
Forecast horizon	return-based	range-based	MA100
1	0.593	0.786	0.443
2	0.673	0.756	0.397
3	0.705	0.706	0.348
4	0.719	0.654	0.299

Table 5: Out-of-Sample Forecasting, for the MCOV between S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1/4/1988-1/2/2004

$$MCOV_t = \phi_0 + \phi_1 COV_t + \varepsilon_t$$

This table reports the R^2 from a linear regression of the measured covariance (MCOV) on the covariance forecasts of the return-based DCC model, the range-based DCC model, the moving-average method (MA100), and constant conditional correlation (CCC) model. The regressions are conducted using 100 observations based on the rolling sample method. For each model, 700 observations are used for in-sample estimation and out-of-sample forecasts of horizons 1,2,3 and 4 weeks are made.

Panel A: S&P500 and Nasdaq				
R-squared	MR_DCC		MA(100)	CCC
Forecast horizon	return-based	range-based		
1	0.000	0.115	0.000	0.004
2	0.003	0.081	0.004	0.000
3	0.008	0.043	0.008	0.003
4	0.018	0.015	0.018	0.011
Panel B: S&P500 and Tbond				
R-squared	MR_DCC		MA(100)	CCC
Forecast horizon	return-based	range-based		
1	0.001	0.136	0.001	0.006
2	0.000	0.070	0.000	0.010
3	0.000	0.079	0.002	0.004
4	0.007	0.046	0.006	0.006
Panel C: Nasdaq and Tbond				
R-squared	MR_DCC		MA(100)	CCC
Forecast horizon	return-based	range-based		
1	0.009	0.114	0.053	0.003
2	0.002	0.083	0.032	0.007
3	0.004	0.066	0.032	0.000
4	0.004	0.046	0.025	0.000