Evidence indicates that individuals prefer proximate, familiar choices and dislike exchanges from status quo positions. We offer an integrated explanation for these phenomena based upon fear of change and of the unfamiliar. In our model, individuals who face model uncertainty focus on adverse scenarios in evaluating defections from familiar choice options. We derive excessive inertia in individual choices, consistent with the endowment effect, home and local biases, high hurdle rates and escalation bias in capital budgeting, limited investor diversification, and attention effects wherein news on average increases demand for a stock. Empirical simulations indicate that plausible levels of model uncertainty imply severe underdiversification and home bias.
The first time a Fox saw a Lion he was so terrified that he almost died of fright. When he saw him again, he was still afraid, but hid his fear. But when he met him the third time, he was so brave he began to talk to him as though they were old friends. *Familiarity breeds contempt.*

— Aesop’s Fables, “The Fox and the Lion.”

1 Introduction

People fear change and the unknown. This generalization is supported by evidence from markets and the experimental laboratory, as discussed in Section 2 of this paper. To give a brief indication of the range of findings, there is evidence that individuals favor gambles and investments that they are more familiar with, and that are geographically and linguistically proximate (sometimes called the familiarity, local, or home bias); that investors are reluctant to trade away from their current ownership positions (the endowment effect), and are biased in favor of choice options made salient as default choices (*status quo* bias); that investors refuse even small better-than-fair new gambles (loss aversion); and hold strongly to past investment choices (escalation bias, sunk cost effects, inertia). Furthermore, individuals are reluctant to take seemingly risky actions such as vaccination, often preferring to bear the much bigger risks associated with remaining passive (omission/commission bias).

Moving beyond a pure decision context, individuals tend to like stimuli that are more familiar (the mere exposure effect), are loyal to local sports teams and organizations, are suspicious of outsiders and strangers (xenophobia), form friendships with those whom they are located close to (propinquity effects) and prefer similarity to self in choice of friends and mates.

We offer an integrated explanation for a range of observed behaviors based on two phenomena. One is the tendency for individuals to use familiar or salient default choice alternatives as benchmarks for comparison. We refer to such a salient choice option as the
‘status quo’ option. The other is the tendency to evaluate alternatives that deviate from the status quo skeptically. We argue that when individuals contemplate alternative, non-status quo choices in the face of uncertainty about how the world works, they fear change and the unfamiliar.

We model fear of the unfamiliar as arising from egocentrically pessimistic guesses about how the world works. For any given choice alternative, different possible models of the world imply different possible probability distributions over payoff outcomes. Thus, the individual faces a layering of gambles over different possible consumption levels: first a gamble over which model describes the world, and second, for any given model, over the realized outcome. Although the individual’s action has no effect on the state of nature, the individual assesses probabilities of states of nature differently depending on what action choice he is contemplating.

Specifically, in our approach the decisionmaker acts as if he thinks that any choice that deviates from the status quo action choice is likely to be countered by a structure of the world that minimizes his welfare. In other words, we model an inclination of individuals who are faced with model uncertainty to focus on the worst-case (or at least, bad-case) scenarios. This conditional pessimism causes what we call familiarity bias.

Our approach does not require that individuals have a declarative belief that the universe responds inimically to their choices. Rather, our premise is that emotions of fear and suspicion are involuntarily incited by the prospect of an unfamiliar course of action.¹ In Section 2 we suggest speculatively why human minds might be designed to work this way.

Change is often risky, so there can be good reason to fear it to some degree. However, consider an individual who is presented with the opportunity to take a gamble that has an uncertain mean payoff, who can picture plausible scenarios in which the mean is either positive or negative. If the prospect of accepting the gamble causes the individual to increase his probability assessment that the mean is negative, he may reject a good project. Similarly, diversifying a portfolio into unfamiliar stocks may feel too dangerous even though it is in fact risk-reducing.

¹“We fear things in proportion to our ignorance of them,” Livy (Titus Livius), 59 BC-17 AD, Roman author and historian; www.quoteworld.org.
Since the values of goods and securities are always uncertain, such perceptions will make individuals resistant to exchange. For a range of prices, neither buying nor selling is desired, as the prospect of buying makes the good seem less attractive and the prospect of selling makes it seem more attractive. Similarly, managers will tend to be too resistant to adopting new investment projects, and to terminating existing investment projects. Thus, hurdle rates for investment will tend to be too high, yet the implicit hurdle rates for continuation of an existing investment will tend to be too low. More broadly, there is uncertainty in making social commitments of various sorts. Familiar individuals are viewed as comfortable and safe, as in the fable of the fox and the lion at the head of the text. Outsiders are often viewed as being a threat (beyond any rational assessment of the actual risks), leading to xenophobia.

A general distaste for model uncertainty is called ambiguity aversion or uncertainty aversion. An experimental literature documents settings in which individuals dislike layering of gambles (discussed in Section 2). As discussed later, past literature (see footnote 2) has shown how aversion to model uncertainty can address several asset pricing and economic issues (see footnote 4 and Section 2).

In contrast, in our approach aversion to model uncertainty is conditional. Individuals do not penalize the status quo choice option for the model uncertainty associated with its outcomes. Pessimistic beliefs are primed only by contemplation of an action that deviates from the status quo choice (a familiar, endowed, or default choice option). This linkage between contemplated action and pessimism captures fear of change or of the unfamiliar—familiarity bias.

There is a natural connection between fear of the unknown and what has come to be called model uncertainty. For example, purchasing 100 shares of Google involves layered gambles, first over possible structures of the world (will Google be the dominant search engine in the future?), and second, given a structure of the world, over possible return outcomes (conditional on Google remaining or not remaining the dominant search engine, what profits will it generate?).

A Bayesian investor deals with such layering in a straightforward way, using the prior distribution over the different models to derive a single probability distribution over consumption states. Thus, an investor’s preferences are represented by a Savage (subjective)
expected utility function. However, if investors are not Bayesians the layering of gambles can matter. Ellsberg’s and later experiments (Ellsberg, 1961) established that individuals are averse to the layering of gambles.

We consider a preference relation that reflects aspects of the models of Bewley (2002) and Gilboa and Schmeidler (1989), but which emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action. In Bewley (2002), a consumption bundle dominates another if and only if its expected utility is higher than the expected utility of the other for all distributions in a probability set. In our setting, the status quo choice option is privileged. In the basic model, an individual selects a strategy over the status quo option only if the strategy provides higher expected utility for all possible models of the world (within some prespecified set). More generally, we consider a setting in which an individual selects a strategy over the status quo option only if the strategy provides higher expected utility over a sufficiently large probability mass of possible models of the world.

We use this framework to examine how lack of familiarity can induce anomalies relating to the unwillingness to trade or to shift investment policy: the endowment effect, the diversification puzzle, local investment biases, the home bias puzzle, the proximity puzzle in cross-listings, reluctance to engage in new investment, escalation bias, and attention effects in securities trading. (These effects are discussed in more detail in Section 2.)

In a trading context, a natural status quo or reference position is the investor’s current consumption or investment portfolio. Endowment effects arise because an investor evaluate purchases under a probability distribution that is adverse to buying, i.e., one in which the expected from the good or security is low. Similarly, an investor evaluates a possible sale under a distribution that is adverse to selling. We find that given an endowed portfolio, there is an interval of prices within which the investor does not trade. Thus familiarity bias acts like a shadow transaction cost that is proportional to the degree of uncertainty. Under constant absolute risk aversion utility and a normal family of payoff distributions parameterized by an uncertain mean, we find that the gap between the willingness to pay and to accept is proportional to the product of the degree of uncertainty and risk. Thus, our

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approach offers implications for the magnitude as well as the direction of familiarity bias.

If investors are endowed with portfolios that include only a subset of available goods or securities, then this pessimism about trades provides a quantifiable explanation for various puzzles of non-participation in securities markets. For example, our model is consistent not just with poorly chosen portfolio weights, but with investors holding zero quantities of certain securities and asset classes. It is consistent not just with home bias, but with some investors holding a null position in a foreign stock market. This is the case even if there is greater uncertainty about the domestic market (consider a domestic investor in the Georgian Republic) than about the foreign market (for example, the UK).

Familiarity bias can also explain excessive inertia in other types of choices. For example, the model is consistent with managers using excessively high hurdle rates in investment choices, and being subject to escalation bias (reluctance to terminate) in existing investments, and with individuals maintaining a sub-optimal fixity in their retirement fund investments (see Section 2). The analysis also offers an explanation for limited diversification of investors across stocks and asset classes; special cases include the home bias puzzle, the preference of individuals to invest in company stock, and attachment to investing ‘styles’ (such as preferences among industries, value versus growth, high versus low momentum, and high versus low dividend yield).

Portfolio theory predicts that investors will hold diversified portfolios in order to minimize the risk of obtaining any given expected return. In reality, many investors hold poorly diversified portfolios (see Section 2). We define the point at which an investor stops diversifying owing to familiarity bias as the minimum number of stocks needed to make a portfolio undominated by any other portfolio. In calibration analysis we find that with a reasonable degree of uncertainty about the means of normally distributed stock returns, the number of stocks acquired before diversification ceases can be quite small. More broadly, this analysis is consistent with individuals holding relatively few asset classes in their portfolios.

A major puzzle in international financial markets is that investors have a strong bias toward holding domestic assets (see Section 2). We calibrate the model using data on portfolio holdings and stock returns in the international stock markets. We define the set of possible models an individual entertains using two alternative approaches. In the first,
investors consider a set of normal distributions parameterized by mean payoffs on domestic and foreign equity that lie in a rectangle. In the second approach, we consider mean payoffs that lie in an ellipse such that the likelihood function of the model lies above a threshold level.

In calibrations we find that familiarity bias substantially reduces the perceived certainty equivalent diversification gains from holding foreign in addition to domestic equity. Modest levels of model uncertainty induce a home bias comparable to the observed magnitude. Specifically, for period between January 1975 to December 2001, we demonstrate that the perceived certainty equivalent gains from diversifying into foreign equity markets for German investors diminish to zero when the combined uncertainty on both domestic and world equity markets is less than 1.5 times of the sum of the standard deviations of the domestic equity return and the world equity return. For the U.S. investors, the perceived certainty equivalent gains from diversification go down to zero when the combined uncertainty is slightly under two times of the sum of the standard deviations. Even for Japanese investors who benefit the most from diversifying into the world equity market, the perceived certainty equivalent gains diminish to zero when the combined uncertainty is three times of the sum of the standard deviations. Interestingly, for the sample period, the UK investors are better off holding only domestic equity reflecting the relative good performance of UK stock market. In all cases, the corresponding portfolio choices converge to their initial endowment positions.

We also analyze trading and equilibrium price determination. We find that in a frictionless international securities market, in equilibrium investors with familiarity bias may not buy any foreign stocks. Furthermore, in equilibrium stock markets in which investors have strong familiarity bias are overpriced relative to stock markets in which investors have weaker familiarity bias.

Our approach is also consistent with attention effects wherein stocks that receive greater publicity or have greater news arrival tend to be purchased more heavily (even if the news is on average neutral)— see the evidence of Section 2. Stocks whose names are mentioned in the news or by others become more familiar. In our approach this implies that investors will feel greater comfort for such stocks, and will assess them more favorably— the stocks have good “buzz.” This implies greater purchases by otherwise-non-participating investors.
Individuals’ choices in our model depend upon salient benchmarks, but in a fashion different from prospect theory (as introduced by Kahneman and Tversky (1979)). In our approach decisionmakers fear deviations from a salient choice alternative. In contrast, under prospect theory, individuals are averse to deviations from a benchmark payoff level.\textsuperscript{3} We show that dislike of unfamiliar choice alternatives generates the predictions mentioned above, such as inertia and endowment effects. In contrast, prospect theory does not unambiguously do so.\textsuperscript{4}

Our paper builds upon previous research relating ambiguity aversion and model uncertainty to underdiversification and to the home bias puzzle. Our paper differs from this research in considering agents who are influenced by their initial endowments. In our setting, pessimistic beliefs are triggered by trading—the move from the initial endowment to another position—not just by the final position being contemplated. In addition, our paper differs from much of this literature in considering not just portfolio choices, but equilibrium price determination.

Dow and Werlang (1992) examine a setting with a riskfree asset and a risky asset when investors are averse to model uncertainty and demonstrate that when their uncertainty aversion is high investors will, regardless of initial endowment, not hold the risky asset. In contrast, our approach provides a no-trading result, which sometimes but not always implies a no-holding results. When familiarity bias and uncertainty are high, investors do not trade. In this circumstance, an investor who starts out holding a riskfree asset will not buy the risky asset, the no-holding result of Dow and Werlang. However, an individual who is endowed with the risky asset in our setting may continue to hold it.

Uppal and Wang (2003) examine a setting in which investors are averse to uncertainty, as reflected in a preference for robustness (as introduced in Andersen, Hansen, and Sargent (1999)), and in which some assets have greater model uncertainty than others. They find that investors hold less of assets with high model uncertainty, which leads to underdiversification,

\textsuperscript{3}Intuitively, fear of the unfamiliar seems to be best captured as a concern about making a choice that one has not taken before, rather than a concern about whether the payoffs on different action alternatives (treating the alternatives symmetrically) will be above or below some payoff benchmark. In further contrast to prospect theory, in our setting it is pessimistic beliefs rather than the shape of the utility function that induce investor’s conservatism.

\textsuperscript{4}For example, investors under prospect theory will sometimes prefer new untried gambles owing to the convexity of the value function in the loss region.
and to home bias if domestic investors are more uncertain about foreign assets than about domestic assets. In Uppal and Wang, these findings are endowment-independent, so that even if an investor is endowed with a diversified portfolio that includes foreign stocks, the individual will trade to a home-biased portfolio. In our approach, under appropriate parameter values individuals who are endowed with domestic assets retain home-biased portfolios, but if they are endowed with diversified portfolios they retain these instead. Furthermore, in Uppal and Wang’s setting individuals generically participate in all asset markets. In our analysis, owing to trade-induced pessimism, individuals may choose complete non-participation in certain markets.

Cao, Wang and Zhang (2004) examine how limited participation may arise in equilibrium with heterogeneous uncertainty averse agents in the presence of model uncertainty and its implications for asset prices. They find that in equilibrium investors with low uncertainty participate the risky asset market while investors with high uncertainty optimally choose to stay sidelined. Their study differs from Dow and Werlang (1992), Uppal and Wang (2003) in that it investigates the limited participation in a general equilibrium setting. However, their limited participation result is endowment-independent.

Epstein and Miao (2003) apply the approach introduced in Chen and Epstein (2001) to the home bias puzzle. In their model, agents exhibit extreme pessimism with respect to their multiple priors. As in Uppal and Wang (2003), home bias is endowment-independent and results from differences in uncertainty across different agents about different assets. Domestic investors have high uncertainty about the returns of foreign assets, causing them to favor domestic assets. In contrast, in our model investors have the same degree of uncertainty about the returns of different assets. Home bias in our model results because the pessimism induced by a given degree of uncertainty is greater in an unfamiliar asset or portfolio than in a familiar one that the individual is endowed with.

The remainder of the paper is organized as follows. Section 2 review evidence relating to human attitudes toward the familiar, and toward deviations from salient benchmark choice alternatives. Section 3 describes the model. Section 4 analyzes the endowment effect. Section 5 considers a security market with one stock and one bond; Section 6 extends the analysis to multiple stocks to perform calibration analysis of levels of uncertainty and home bias. Section 7 provides an equilibrium model of investment with familiarity bias. Section 8
explores the under-diversification puzzle. Section 9 concludes. Some technical details are in the appendix; proofs are in the appendix unless otherwise indicated in the text.

2 Motivating Evidence

We begin by summarizing more specifically the evidence relating to human attitudes toward the familiar, and toward deviations from salient benchmark choice alternatives. Starting with Zajonc (1968), psychologists have documented a strong and robust mere exposure effect wherein exposure to an unreinforced stimulus tends to make people like it more (see, e.g., Bornstein and Dagostino (1992), Moreland and Beach (1992)). Advertisers try to take advantage of this by repeatedly exposing consumers to the name of a brand. Relatedly, there are propinquity effects in the formation of friendships (see e.g., Aronson, Wilson and Akert (1999)), and arguably, for a liking of acquaintances as contrasted with prejudice toward outsiders.

Of course, greater understanding of or experience with a familiar phenomenon may, ceteris paribus, resolve uncertainty, reducing risk. However, the preference for familiarity goes beyond this. For example, individuals prefer to bet on a matter about which they know more than on another equivalent gamble (Heath and Tversky (1991)). People also like similarity in choice of friends and mates (Berscheid and Reis (1998)).

It has been well documented that people often demand a higher price to give up an object than they would be willing to pay to acquire it. This difference between the willingness to pay (WTP) and the willingness to accept (WTA) is called the endowment effect (Thaler (1980)). Kahneman, Knetsch and Thaler (1991) show that students who were randomly chosen to receive mugs demand higher prices to give them up than students who do not have mugs were willing to pay to obtain them. An agent values an object higher simply because it is his object. An early laboratory demonstration of the endowment effect was offered by Knetsch and Sinden (1984). The participants in this study were given either a lottery ticket or $2.00. Some time later, each subject was offered an opportunity to trade the lottery ticket

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5 According to evolutionary psychology, people prefer familiar and similar individuals because in our past environment of evolutionary adaptation, these were indicators of genetic relatedness (e.g., Trivers (1985)).
for the money, or vice versa. Very few subjects chose to switch.

A reluctance by a manager to invest or to terminate investment is much like the endowment effect. Investment is the exchange of cash for a project, and termination is the opposite exchange. Dixit (1992) discusses evidence that firms commonly use hurdle rates that exceed the cost of capital, thereby discouraging new projects. More recently, Poterba and Summers (1995), in a survey of Fortune 500 firms, report that large firms use hurdle rates that are about 5 percent higher than what would be implied by standard asset pricing models. In another survey, Graham and Harvey (1999) find that small firms were significantly less likely to apply the Capital Asset Pricing Model in setting hurdle rates. Conventional descriptions assert that venture capitalist firms use hurdles on the order of 40% and higher for their investments.

Several authors have also argued that managers are reluctant to terminate ongoing projects. Psychologists have described a tendency for individuals to escalate commitment to a previously selected strategy (Staw (1976), Staw and Ross (1987)). This can be referred to as escalation bias, or inertia. A related phenomenon is the sunk-cost effect (Arkes and Blumer (1985)), wherein an initial investment in a project creates reluctance to terminate it.

Sharing some of the flavor of the endowment effect is status quo bias. This is a preference for either the current state or some choice alternative that has been made salient as the default option that will apply should no alternative be selected explicitly. In a set of experiments on portfolio choices following a hypothetical inheritance, Samuelson and Zeckhauser (1988) find that an option becomes significantly more popular when it is designated as the status quo while others are designated as alternatives. This indicates that investors’ preferences are anchored by designated alternatives. Samuelson and Zeckhauser also find some evidence in field data on health plan choices and portfolio compositions in TIAA-CREF plans among Harvard employees. Fox and Tversky (1995) find evidence supporting a status quo bias in initial probability assessments in an experimental setting.

Madrian and Shea (2000) provide evidence that people stick for long periods of the time to the default offered by their firm in deciding on 401(K) participation and saving, an illustration of status quo bias as well as inertia. Ameriks and Zeldes (2001) describe
data from TIAA-CREF and Surveys of Consumer Finance. Almost half of the individuals in their sample made no change in portfolio composition over the course of the nine year sample period, while the same period saw drastic changes in the returns to bonds and stocks. Camerer and Lowenstein (2003) provides a review of further evidence in this regard.

Individuals tend to dislike risks that derive from active choices more than risks that result from remaining passive. Psychologists have referred to this as the omission/comission bias (Ritov and Baron 1990, Josephs et al. (1996)). A possible example is a reluctance to buy additional stocks to diversify one's portfolio.

In contrast with the recommendation of portfolio theory, many investors hold poorly diversified portfolios (which was especially the case prior to the rise of mutual funds and defined contribution pension funds). For example, Blume and Friend (1975) found that investors held highly undiversified portfolios. Using more recent data from a major discount brokerage firm, Barber and Odean (2000) found that investors on average held 4.3 stocks at this brokerage with the median being only 2.6 stocks.

There is a great deal of evidence suggesting that individuals prefer familiar stocks in their investment choices. Experimentally, Ackert et al. (2003) find that investors prefer to invest in stocks that have more familiar names. Coval and Moskowitz (1999, 2001) find that U.S. investment managers invest disproportionately in locally headquartered firms. Although their evidence is consistent with an informational explanation, a psychological bias in favor of the familiar could also play a role. There is also evidence that both institutional and individual investors tend to hold the shares of firms that have nearby headquarters, that investor's culture, and that communicate in the investor's native tongue (Grinblatt and Keloharju, 2001). In a study of 14 countries, Portes and Rey (2003) find that common language is a significant predictor of bilateral cross-border equity flows. Zhu (2003) and Kumar (2004) find that much of the local bias of U.S. individual investors does not come from access to superior information about firms prospects.

Benartzi (2001) finds that employees allocate 401(k) retirement savings to their own firm's stock based on past return performance (apparently based upon naive extrapolation of past returns). Huberman (2001) documents that customers of a given U.S. Regional Bell
Operating Companies (RBOC) tend to hold more of its shares and invest more money in it than in other RBOCs.

Corporate employees often hold on to the stocks they receive as part of their compensation plans long after the stocks are vested. This is exemplified by the heavy investment by some employees of now-defunct Enron of retirement savings in Enron stock. Malmendier and Tate (2002) document that CEOs do not exercise options efficiently and do not diversify their holdings sufficiently beyond ownership of their own firm.

In international financial markets, investors tend to hold domestic assets instead of diversifying across countries, a puzzle known as home bias (see French and Poterba, 1991, Cooper and Kaplanis, 1994, Tesar and Werner 1995, Lewis 1999, and Obstfeld and Rogoff 2000). Although various explanations such as transaction costs, differential taxes, political risk, exchange rate risk, asymmetric information, purchasing power parity and non-tradable assets have been offered, none has been shown to explain the magnitude of observed home bias. 6

The evidence of Shiller, Konya and Tsutsui (1996), Strong and Xu (1999), and Kilka and Weber (2000) in surveys of U.K., U.S., Continental Europe and Japanese investors indicates that both individuals and portfolio managers have relatively pessimistic expectations about foreign stocks than about domestic stocks. This is consistent with our beliefs-based approach to the home bias puzzle. In international cross listings, firms also tend to cross list their stocks in countries where investors are more familiar with the firms to be listed (see footnote 6). Taken together, this evidence suggests a preference for the familiar that goes above and beyond any motivation based upon lower true risk or higher returns.

Several recent papers have provided evidence of attention effects in which stocks that have had heavy recent news arrival tend to be purchased more heavily by investors, which in turn tends to drive up the price. Gervais, Kaniel and Minelgrin (2001) find that stocks with high recent volume (presumably associated with high recent arrival of either public or

6For example, Cooper and Kaplanis (1994) estimate that the required transaction cost based on a standard international CAPM with an assumed risk aversion consistent with domestic equity premium, is a few percent per annum higher than observed transaction cost such as withholding taxes. Coval (2003) provides an asymmetric approach to home bias and other risk-sharing anomalies, and Stulz (1999) reviews the literature on home bias. A related phenomenon is the familiarity bias in international cross-listings documented by Pagano et al (2002) and Sarkissian and Schill (2003).
private information) tend to be priced relatively high, and therefore earn low subsequent returns. Barber and Odean (2003) find that after news announcements, individual investors tend to purchase the stock. For example, this tendency applies more specifically in a general sample of earnings announcements, for which the news is on average neutral (Hirshleifer et al. 2002).

Overall, the evidence described in this section raises the question of why a preference for familiarity may have evolved. We now discuss some speculative explanations. However, the acceptance of these speculations is not necessary for a reader who accepts the basic point that individuals tend to be averse to the unfamiliar.

Speculatively, familiarity preference may have arisen because of limited information processing power and the challenges of exchange with potentially exploitive partners. (The term exchange may be interpreted broadly to include friendships and mating attachments.) In any potential exchange, there are many possible dimensions for cheating. Natural selection presumably favored both the ability to devise ways to cheat, and the ability to deal with potential cheaters.

One crude defense against cheaters is simply to refuse to trade. This approach protects at the expense of the potential gains from the trade. In some cases it these gains turn out to be large, the terms of the trade are easy to understand, and the trading partner clearly reliable. But in others it is hard to tell whether the trade is likely to be profitable. It is cognitively costly to foresee, monitor and protect against different possible means of exploitation. As a second best measure, it may be profitable to have a generalized skepticism about exchange and proposed moves from the status quo. Even if the deal looks good in every characteristic the individual can identify, it will be rejected if it fails to clear the hurdle by a sufficiently large margin. This may be a meta-optimal adjustment for the possibility of hard-to-detect problems.7 In those cases where in fact concealed exploitation is not occurring, the individual foregoes desirable alternatives. Hence the phenomenon of biases toward dysfunctional behaviors such as holding undiversified portfolios (especially company stock), and holding only domestic securities.

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7Even in modern times, some individuals may as a blanket policy simply refuse to consider buying a used car, making stock trades based on a broker’s cold calls, or buying insurance from door-to-door salesmen. Such a policy can be cheaper than trying to dissect in detail exactly wherein the professional’s deceptions lie.
The emotional and cognitive mechanisms that deter the individual from accepting what are apparently good deals are presumably feelings such as fear and suspicion of the unknown, skepticism about proposed shifts from saliently identified status quo options; and correspondingly warm feelings towards the known and familiar.

3 The Model

To highlight the intuition of the model, we first consider a portfolio choice decision for an investor who is subject to familiarity bias in an economy with one stock and a bond, making several simplifying assumptions about the payoff distribution. We later extend our model to multiple stocks.

3.1 The Basic Setting

We consider a two-date economy in which investment decisions are made at date 0 and consumption takes place at date 1. The riskfree rate of interest is set to zero. The payoff of the risky asset (stock) is denoted $r$.

3.2 Familiarity, Preferences and Beliefs

We replace the unique subjective probability distribution used in standard expected utility calculation with a set of probability distributions. When there is uncertainty, an individual is assumed to stick to the status quo action (and resulting wealth distribution) unless it is dominated by another action in the sense that the alternative is preferred for all beliefs in the relevant set.

Each individual has a twice differentiable and concave utility function $U(W)$ defined over the end-of-period wealth, $W$. Let $W(x)$ denote the wealth random variable for an investor following a given strategy $x$. Let $\mathcal{P}$ denote a given set of probability distributions over states of the economy, and let $Q$ be an element of $\mathcal{P}$. We consider a preference relation that reflects aspects of the preferences described by Bewley (2002) and Gilboa-Schmeidler
(1989), but which emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action.

The intuition behind these preference characterizations stems from uncertainty about the probability distribution over states of the economy. Investors’ uncertainty about the distribution of asset payoffs presents them with two-layered gambles. Based on their prior experience or the results of econometric analysis, decision makers have in mind a family of distributions from which they believe the true distribution is drawn. We call this set of distributions over payoff outcomes $P$. To take a concrete example, investors may believe that asset payoffs are distributed normally with given variance, but do not know the value of the mean. In this example, each possible mean and variance corresponds to one distribution in $P$. The range of possible means in this example provides a measure of uncertainty.

**Definition 1 Status Quo Deviation Aversion**

Let $x$ be a feasible strategy and $s$ be the status quo strategy. Then $x$ is strictly preferred over $s$ if and only if $x$ gives higher expected utility than $s$ under any probability distribution $Q$ in $P$.

$$x \succ s \iff \min_{Q \in P} \{E^Q[U(W(x))] - E^Q[U(W(s))]\} > 0.$$  \(1\)

Status Quo Deviation Aversion (SQDA) gives a privileged position to the status quo strategy. A strategy is preferred to the status quo strategy only if it provides higher expected utilities under all scenarios in $P$.

Status quo deviation aversion is an incomplete preference relation, as it does not specify how to compare two non-status-quo alternatives. The following definition gives one way to complete the preference ordering:

**Definition 2 Strong Status Quo Deviation Aversion**

Let $x$ and $y$ be any two strategies and $s$ be the status quo strategy. Then

$$x \succ y \iff \min_{Q \in P} \{E^Q[U(W(x))] - E^Q[U(W(s))]\} > \min_{Q \in P} \{E^Q[U(W(y))] - E^Q[U(W(s))]\}.$$  \(2\)
Under strong status quo deviation aversion (SSQDA), each strategy is compared to other
different strategies by evaluating its utility gains over the status quo strategy under the most adverse
scenario for the utility of that strategy relative to the utility provided by the status quo.
(An immediate consequence is that the status quo choice is privileged.) This reflects fear of
change and uncertainty. It is easy to show that SSQDA implies SQDA. Most of our results
require only the milder SQDA. However, in Section 7, the analysis of portfolio choices in
international markets does apply SSQDA.

Status Quo Deviation Aversion, both in its basic form given by (1) and its strong form
as given by (2), assigns a privileged role to a single status quo alternative. This familiar
option is chosen unless there exists an alternative that is preferred for all possible beliefs
within the set $\mathcal{P}$. Thus, a familiar choice option acts as an anchor from which deviations are
pessimistically considered. When there is uncertainty, deviations from more familiar choices
will be scrutinized with skepticism and suspicion. This results in a tendency to prefer more
familiar choices, or choices that seem to preserve the status quo (see Section 2).

SSQDA implies that when there are choices that dominate the status quo option, the
investor chooses among them according to a procedure similar to that of Gilboa and Schmei-
dler (1989), i.e., the investor evaluates each strategy under the scenario that is most adverse
to that strategy. Thus, if the status quo action is dominated by an alternative strategy $x$,
then strategy $x$ is evaluated according to the minimum gains in expected utility and the
alternative strategy with the highest minimum gains in expected utility is selected.

4 The Endowment Effect

4.1 The Basic Model

In this section we analyze the endowment effect induced by preference for the familiar. We
consider the effect of acquiring a gamble, which for specificity we will call a stock. We
assume that the individual perceives making no trade as the default or status quo choice
option. Let $W_0$ denote the initial wealth in the risk-free bond, $e$ denote the endowment in
the stock, $c$ denote the dollar amount the individual pays for the additional shares of the
stock under measure \( Q \), and \( d \) denote the dollar amount the individual receives for giving up the additional shares of the stock under measure \( Q \). For small additional shares in the stock \( \Delta e \) we let \( \Delta C \) denote the greatest amount an investor would be willing to give up in exchange for the additional quantity of the asset,

\[
\Delta C \equiv \sup_{c} \{ c | \min_{Q \in P} E^{Q}[U(W_{0} + (e + \Delta e)r - c)] - E^{Q}[U(W_{0} + er)] > 0 \}. 
\]  

(3)

Similarly, we can let \( \Delta D \) denote the least amount of cash required to induce an individual to give up a small amount of the stock,

\[
\Delta D \equiv \inf_{d} \{ d | \min_{Q \in P} E^{Q}[U(W_{0} + (e - \Delta e)r + d)] - E^{Q}[U(W_{0} + er)] > 0 \}. 
\]  

(4)

Assuming that the limits of \( \Delta D/\Delta e \) and \( \Delta C/\Delta e \) exist, we define the willingness to pay (WTP) and willingness to accept (WTA) as

\[
WTA = \lim_{\Delta e \to 0} \frac{\Delta D}{\Delta e}; 
\]  

(5)

\[
WTP = \lim_{\Delta e \to 0} \frac{\Delta C}{\Delta e}. 
\]  

(6)

The following results describe how an investor’s willingness to pay and willingness to accept relate to his initial endowment of the stock.

**Proposition 1** Under Status Quo Deviation Aversion, there is an endowment effect, i.e., \( WTA \) is always greater than or equal to \( WTP \). Specifically,

\[
WTA - WTP = \frac{\max_{Q \in P}[A(Q) + B(Q)]}{\max_{Q \in P} E^{Q}[U']}.
\]
where

\[
A(Q) \equiv F - F(Q)
\]
\[
F(Q) \equiv E^Q[(WTA - WTP)U']
\]
\[
F = \max_{Q \in \mathcal{P}} F(Q)
\]
\[
B(Q) \equiv G - G(Q)
\]
\[
G(Q) = E^Q[(WTP - r)U'] \geq 0
\]
\[
G = \max_{Q \in \mathcal{P}} G(Q), \quad \text{and}
\]
\[
A \geq 0, \quad B \geq 0,
\]

which implies that

\[\text{WTA} \geq \text{WTP}.\]

Proposition 1 indicates that there is a kink around the endowed stock and cash position. When determining his willingness to pay, an individual considers the scenario most adverse to buying the stock. Similarly, when determining his willingness to accept, he contemplates the best case scenario for holding on to the stock. Thus our model is consistent with the endowment effect as documented in previous literature.

To derive a closed-form solution for WTA and WTP, we now assume that investors have CARA utility with risk aversion \(\gamma\), and that the stock payoff, \(r\), is normally distributed with uncertainty about the mean. We assume that investors have precise knowledge of the variance of stock payoff but do not know the mean. This is motivated by analytic tractability, the empirical evidence on the predictability of the volatility of stock returns (Bollerslev, Chou, and Kroner (1992)), and the difficulty in estimating precisely the expected stock returns. Thus, in our model fear of the unfamiliar derives from aversion to model uncertainty about the mean payoffs of unfamiliar choice alternatives. Investors will consider a set of probability distributions with different mean payoffs when making their investment decisions.

Following the notation above, we use \(\mathcal{P}\) to denote the set of probability distributions over the stock payoff. When specifying the set \(\mathcal{P}\), we consider a reference distribution and form the set around this reference distribution based on the log likelihood ratio. Specifically, let \(P\) be
a reference probability distribution obtained, say, from econometric analysis. We defined the set of probability distribution \( \mathcal{P} \) as the collection of all probability distributions \( Q \) satisfying 
\[ E^Q[-\ln(dQ/dP)] < \eta \]
for a preselected value \( \eta \). Intuitively, since \( E^Q[-\ln(dQ/dP)] \) is the log likelihood ratio under \( Q \), \( \mathcal{P} \) can be viewed as a confidence region around \( P \) and \( \eta \) can be viewed as the critical value for, say, 95% confidence. For a fixed level of confidence, the size of \( \mathcal{P} \) captures an investor’s uncertainty about the reference probability distribution \( P \). If the investor has more information about \( P \), he would be able to estimate the true probability law more precisely. Then for the same level of confidence, \( \eta \) will be small. Conversely, if the investor has little information about \( P \), then \( \eta \) will be large. In this sense, \( \eta \) can be viewed as a measure of uncertainty.\(^8\)

If \( \mathcal{P} \) is chosen to be the set of normal distributions with a common known variance, Kogan and Wang (2002) show that the confidence region can always be described by a set of quadratic inequalities. In our case, it takes the form of \( \mu + v \) where \( v \) measures the adjustment made to the estimated mean \( \mu \) under probability measure \( Q \) and satisfies
\[ Tv^2\sigma^{-2} \leq \phi^2, \] (8)
where \( \phi \) is a parameter that captures the investor’s uncertainty about the mean of the payoff of the stock, and \( T \) is the number of periods for which data on the stock are available. We therefore define the set \( \mathcal{P} \) as the collection of all normal distributions with mean \( \mu + v \) and variance \( \sigma^2 \) such that \( v \) satisfies (8). The higher is \( \phi \), the wider is the range for the expectation of \( r \). We refer to \( \phi \) the level of uncertainty of the investor about the mean of \( r \). Condition (8) also implies that the deviation from the sample estimate is less than the standard error of the sample estimate times a constant, \( \phi \), the degree of uncertainty, i.e.,
\[ |v| \leq \phi\sigma/\sqrt{T}. \]

Consider an investor with endowment \( e \) of the stock. Under the assumption of CARA utility and the normal distributions for stock payoff, the amount that the investor is willing to pay for \( \Delta e \) additional units of the stock is the gain in certainty equivalence associated with the change of stock holding \( e \) to stock holding \( (e + \Delta e) \) under the worst case scenario
\footnote{As Kogan and Wang (2002) argue, the investor’s aversion to uncertainty can also be embedded in \( \eta \). For ease of exposition, we assume that all investors have the same level of uncertainty aversion.}
for holding additional shares of stock, i.e.,

\[ \Delta C = \min_v \left\{ \left[ W_0 + (e + \Delta e)(\mu + v) - \frac{\gamma \sigma^2}{2} (e + \Delta e)^2 \right] - \left[ W_0 + e(\mu + v) - \frac{\gamma \sigma^2}{2} e^2 \right] \right\} . \]

Under the assumption of normal distributions for the set \( P \), the worst distribution for holding additional shares of stock is a normal distribution with mean payoff \( (\mu - \phi \sigma / \sqrt{T}) \) (the mean stock payoff adjusted downward by \( -\phi \sigma / \sqrt{T} \)). Substituting \( v = -\phi \sigma / \sqrt{T} \) into above equation and combining terms, we arrive at the following expression for the amount that an investor is willing to pay for \( \Delta e \) additional units of stock

\[ \Delta C = \Delta e(\mu - \phi \sigma / \sqrt{T}) - \frac{\gamma \sigma^2}{2} [(e + \Delta e)^2 - e^2] . \]

Letting \( \Delta e \) approach zero, we obtain the marginal willingness to pay

\[ WTP = \mu - \phi \sigma / \sqrt{T} - \gamma e \sigma^2 . \]

Similarly, we derive the amount that the investor requires to give up \( \Delta e \) units of the stock as the gain in certainty equivalence associated with the change of stock holding \( e \) to stock holding \( e - \Delta e \) under the worst case scenario for selling stock. Since the worst case for selling stocks is a normal distribution with a mean payoff \( (\mu + \phi \sigma / \sqrt{T}) \) (the mean stock payoff adjusted upward by \( \phi \sigma / \sqrt{T} \)), we arrive at the following expression for the amount that an investor requires to give up \( \Delta e \) units of stock

\[ \Delta D = \Delta e(\mu + \phi \sigma / \sqrt{T}) - \frac{\gamma \sigma^2}{2} [(e - \Delta e)^2 - e^2] . \]

The marginal willingness to accept is given by

\[ WTA = \mu + \phi \sigma / \sqrt{T} - \gamma e \sigma^2 . \]

The difference between WTP and WTA is

\[ WTA - WTP = 2\phi \sigma / \sqrt{T} . \] (9)

This gap is proportional to the product of the degree of uncertainty, measured by \( \phi \), and the degree of risk, measured by \( \sigma \). When the degree of model uncertainty \( \phi = 0 \), or when
the amount of available data $T$ goes to infinity, the gap approaches zero. The gap occurs because when an investor purchases a share of stock, he considers the scenario that is most adverse to buying, and when he sells a share of stock, he considers the scenario that is most adverse to selling. The disparity in WTA and WTP comes from the difference in perceived outcome distribution.

Thus, in this setting when model uncertainty is large there is an autarkic outcome; the individual neither buys nor sells. For example, if he starts with none of the stock, he will not buy any of it. In Dow and Werlang (1992), there is a set of prices such that the individual, regardless of his endowment, trades so as to hold none of the uncertain asset and instead holds only the riskfree asset. Thus, in Dow and Werlang there is a zero-holding outcome, but generically not an autarkic outcome. In Uppal and Wang (2003) and Epstein and Miao (2003), for any given endowments, individuals hold less of assets that are more uncertain, but do not reduce these holding to zero. Thus, in these models there is neither an autarkic nor a zero-holding outcome. However, in these models if uncertainty is large a close-to-zero-holding outcome can occur.

In a trading context, our finding describes an endowment effect: individuals are reluctant to trade from their initial positions. In a capital budgeting context, this finding indicates that managers are reluctant to terminate ongoing investment projects and to adopt new investment projects. The willingness to pay can be interpreted as the amount the manager values the payoff distribution from a new investment project. A lower willingness to pay implies a higher implicit discount rate, and therefore an excessively high hurdle rate for new investment.

For an ongoing project, the willingness to accept can be interpreted as the level of the liquidation value that would make the manager just willing to terminate the project. A WTA that is too high indicates that an implicit hurdle rate for continuing the project is too low. Thus, familiarity preference is consistent both with a resistance to undertaking new projects and with escalation bias (see Staw and Ross (1987); an excessive willingness to continue, or even extend, existing projects).

Furthermore, our approach also helps to explain the proximity bias in international cross-listings (Pagano et al. (2002) and Sarkissian and Schill (2003)). Firms that contemplate
cross-listing in foreign markets may find that problems of investor unfamiliarity are milder for investors who are nearby, and who live in countries that share a common culture, language or legal system. Thus, our approach suggests that the cost of capital will be lower for firms that cross-list in countries that are proximate in these respects.

4.2 Alternative Specification of Familiarity Bias and the Endowment Effect

In the basic approach, investors who exhibit familiarity bias focus on the worst case scenarios associated with contemplated deviations from status quo choices. In this section we show that similar results can be obtained under a less extreme assumption, that investors focus on bad cases instead of worst cases.

In order to define ‘bad cases’ specifically, we consider an investor who is uncertain about which model of the world is valid. Formally, the individual has a probability distribution function, denoted by $P^M$, over the different possible payoff probability distributions in $\mathcal{P}$. For any given contemplated action, the probability distributions in $\mathcal{P}$ are ordered by the expected utilities they provide. An individual who is maximally pessimistic about an action would use the probability distribution in $\mathcal{P}$ with the lowest expected utility relative to that provided by the status quo. These beliefs generate SQDA (and SSQDA) as in Section 3.

More generally, however, the individual may pessimistically select a probability distribution at the $1 - \delta$ quantile for expected utility ($\delta > 0.5$) under probability distribution $P^M$. For example, if $\delta = 0.99$, then for a given action, he considers the level of expected utility to be at the 1% lower tail of expected utilities that can be generated from distributions in $\mathcal{P}$. Thus, the individual prefers the status quo unless the alternative action generates higher expected utility than the status quo under a large set of probability distributions under $P^M$—a set that has probability mass of at least 99%.

To state this idea formally, let $s$ be the status quo strategy and $x$ be an alternative strategy. For any subset $Q \subset \mathcal{P}$ of probability distributions, consider the worst distribution in that subset for an investor contemplating the choice of $x$ instead of the status quo. We can then define the lower bound of expected utility gains associated with strategy $x$ among
the set \( Q \), denoted \( L(Q, x; s) \) as

\[
L(Q, x; s) = \inf_{Q \in Q} \{ E^Q[U(W(x))] - E^Q[U(W(s))] \}.
\]

We use these lower bound benefits to define quantiles ordering the distributions according to their incremental expected utilities relative to the status quo.

Let \( Q^\delta \) be defined as the set of payoff distributions corresponding to the \( \delta \) quantile of the \( P^M \) distribution, where the payoff distributions are ordered according to the increments each provides under strategy \( x \) over the expected utility under the status quo action. Formally, \( Q^\delta \) is the set such that \( \{ P^M(Q) \geq \delta \} \) and \( L(Q^\delta, x; s) > L(R, x; s) \) for any other set \( R \) that contains \( Q^\delta \). We define the quantile utility gain (QUG) as \( L(Q^\delta, x; s) \). Thus, for a given status quo \( s \), the quantile utility gain for an alternative strategy \( x \) at the \( \delta \) quantile of the \( P^M \) distribution is defined as

\[
\text{QUG}^\delta(x; s) = L(Q^\delta, x; s).
\] (10)

Here, the lower bound reflects the pessimistic beliefs of the investors, and the quantile utility gain describes the gain for a given degree of pessimism \( \delta \).

Specifically, under Status Quo Deviation Aversion Based on QUG, a strategy is preferred to the status quo strategy if it provides higher expected utility for \( \delta \) quantile of the probability distributions with \( \delta \) strictly greater than 50%. For instance, if \( \delta = 0.75 \), strategy \( x \) is preferred to the status quo strategy \( s \) if it yields higher expected utility for at least a mass of 75% of the probability distributions in \( P \).\footnote{This is a much milder condition than SQDA, which requires that strategy \( x \) provides higher expected utility under all possible scenarios in \( P \).}

We therefore introduce the following preference:

**Definition 3 Status Quo Deviation Aversion Based on QUG** Let \( x \) be a feasible strategy and \( s \) be the status quo strategy, and let \( \delta \) be a given quantile level. Then \( x \) is strictly preferred to \( s \) if and only if the quantile utility gain for strategy \( x \) at quantile level \( \delta \) is positive:

\[
x \succ s \text{ if and only if } \text{QUG}^\delta(x; s) > 0.
\] (11)
We can define the strong status quo deviation aversion based on the QUG in a fashion
analogous to SSQDA in Section 3.2.

**Definition 4 Strong Status Quo Deviation Aversion Based on QUG** Let \( x \) and \( y \) be two strategies and \( s \) be the status quo strategy. Then

\[
x \succ y \text{ if and only if } QUG^\delta(x; s) > QUG^\delta(y; s) \text{ for a given quantile } \delta.
\]

Intuitively, a decision maker would weakly prefer choice \( x \) to choice \( y \) if, for most of the
probability distributions in the pre-selected set \( \mathcal{P} \) \((0.5 < \delta \leq 1)\), the expected utility of
choice \( x \) is at least as high as that of choice \( y \).

The worst case scenario SQDA and SSQDA preferences of the preceding subsection are
the special cases of (11) and (12) in which \( \delta = 1 \). Similar results of autarky on the part
of individuals (the endowment effect) still obtain under the more moderate familiarity bias
described by the QUG approach. Using similar definitions as in (3)-(6),\(^{10}\) we have:

**Proposition 2** Under Status Quo Deviation Aversion Based on QUG, there is an endow-
ment effect, i.e., WTA is always greater than or equal to WTP.

In the special case of normally distributed stock payoffs and CARA utility, we can derive
the WTA and WTP in closed form as follows. Let \( v^\delta \) be the adjustment to the mean stock
payoff under the quantile utility gain for a given quantile \( \delta \), i.e., \( L(Q^\delta, e + \Delta e; e) > 0 \). The
amount that the investor is willing to pay for \( \Delta e \) additional shares of stock is again given by
the gain in certainty equivalence for stock holding \( (e + \Delta e) \) over stock holding \( e \) under the
QUG at the quantile level \( \delta \), i.e.,

\[
\Delta C = \min_{v^\delta} \left\{ \left[ W_0 + (e + \Delta e)(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} (e + \Delta e)^2 \right] - \left[ W_0 + e(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} e^2 \right] \right\}.
\]

\(^{10}\)For example, we can define the willingness to pay for \( \Delta e \) units of stock as the set of \( c \) such that,

\[
\Delta C \equiv \sup_{c} \{c | QUG^\delta[(e + \Delta e)r - c; er] \geq 0\}.
\]
Let $F$ be the cumulative distribution function (CDF) for $v^\delta$, the adjustment to the estimated mean payoff under quantile utility gain. The worst case mean stock payoff corresponding to the $\delta$ quantile under the ordered probability distribution based on QUG is $\mu + F^{-1}(1 - \delta)$, where $F^{-1}(\cdot)$ is the inverse function of the CDF for the mean stock payoff adjustment $v^\delta$. $F^{-1}(1 - \delta)$ thus is the mean stock payoff adjustment that the investor considers when he exchanges $\Delta e$ units of stock for $\Delta C$ units of cash under the QUG at quantile $\delta$.

Therefore the maximum amount that the investor is willing to give up for $\Delta e$ additional shares of the asset is

$$\Delta C = \Delta e[\mu + F^{-1}(1 - \delta)] - \frac{\gamma \sigma^2}{2}[(e + \Delta e)^2 - e^2].$$

Letting $\Delta e$ approach zero, we obtain the marginal willingness to pay under the QUG at quantile level $\delta$,

$$WTP = \mu + F^{-1}(1 - \delta) - \gamma e \sigma^2.$$

Similarly, we can find the amount that the investor requires to give up $\Delta e$ shares of stock as the certainty equivalent gain for the trade from stock holding $e$ to $(e - \Delta e)$ under the QUG at quantile level $\delta$,

$$\Delta D = \min_{v^\delta} \left\{ W_0 + (e - \Delta e)(\mu + v^\delta) - \frac{\gamma \sigma^2}{2}(e - \Delta e)^2 \right\} - \left\{ W_0 + e(\mu + v^\delta) - \frac{\gamma \sigma^2}{2}e^2 \right\}.$$

The case corresponding to the trade from stock holding $e$ to $e - \Delta e$ is the opposite of that in which the investor acquires additional shares. The adjusted mean stock payoff is thus $\mu + F^{-1}(\delta)$. Substituting the adjusted mean stock payoff into above yields

$$\Delta D = \Delta e[\mu + F^{-1}(\delta)] - \frac{\gamma \sigma^2}{2}[(e - \Delta e)^2 - e^2].$$

Letting $\Delta e$ approach zero, we obtain the marginal willingness to accept under the QUG at quantile level $\delta$,

$$WTA = \mu + F^{-1}(\delta) - \gamma e \sigma^2.$$

The gap between WTP and WTA is $F^{-1}(\delta) - F^{-1}(1 - \delta)$ which is positive when the distribution of $F$ is strictly monotone and $\delta > 0.5$. When $\delta = 1$, the analysis reduces to the Status
Quo Deviation Aversion case, as shown in equation (9). When uncertainty decreases to zero, the gap between WTA and WTP also shrinks to zero. In particular, when $P^M$ is uniform, the gap between WTP and WTA is $(4\delta - 2)\phi \sigma / \sqrt{T}$. The gap depends on the degree of familiarity bias, characterized by $\delta$. As $\delta$ increases between 0.5 and 1, the gap between WTP and WTA also increases. Intuitively, if the decision maker chooses the alternative strategy $x$ over the status quo strategy $s$ only under the condition that the alternative is preferable to the status quo under a wider set of probability distribution, he must have valued his endowment highly relative to the alternative. This leads to a large gap between WTP and WTA.

### 4.3 No-Trade Condition under Familiarity Bias

We now consider the portfolio choice with one risky asset (stocks) and one riskfree asset (bonds) using the basic model described in Subsection 4.1. In this setting a rational Bayesian investor always trades. However, an investor subject to familiarity bias may not trade. We provide conditions under which no trade is perceived to be optimal for an investor who is subject to Status Quo Deviation Aversion. As in Subsection 4.1, we assume that investors have CARA utility, and that the stock payoff, $r$, is normally distributed with uncertainty about the mean.

**Proposition 3** When

$$\left| \frac{\mu - P}{\gamma \sigma^2} - e \right| \leq \frac{\phi}{\gamma \sigma \sqrt{T}},$$

the investor will not deviate from his status quo position.

Proposition 3 implies that for every initial endowment, $e$, there is a price interval $[\mu - \phi \sigma - \gamma \sigma^2 e, \mu - \phi \sigma - \gamma \sigma^2 e]$ such that within the interval the investor does not trade. The effect of familiarity bias on an investor’s portfolio choice resembles that of transaction cost. In the case with a single stock with 10% sample standard deviation based on 100 observations and an uncertainty parameter $\phi = 1$, the effect of familiarity bias on the investor’s portfolio choice is similar to a setting in which there is a 1% proportional transaction cost without familiarity bias.
5 Calibration Analysis of the Home Bias Puzzle

A well known puzzle in international finance is that investors in aggregate tend to hold mostly the assets of the country they reside in, rather than diversifying internationally. Here we calibrate familiarity bias under Status Quo Deviation Aversion to determine how much uncertainty is needed for investors to hold mostly local assets. We first derive the perceived certainty equivalent gains from diversification by allowing investors to hold both the domestic stock market portfolio and the world stock market portfolio. We then turn to investors’ portfolio choices.

5.1 International Diversification under Familiarity Bias

Consider the perceived certainty equivalent gains of an investor who contemplates a portfolio choice that deviates from his endowment position. For notational convenience, we use subscript \(d\) and \(w\) to denote the domestic equity portfolio and the world equity portfolio, respectively. Let \(\Sigma\) be the covariance matrix of the payoffs to the domestic and world equity portfolios. Let \(e_d\) be the investor’s position in the domestic market portfolio. \((1 - e_d)\) is thus the investor’s position in the world market portfolio. Let \(e \equiv (e_d, 1 - e_d)\) denote the investor’s total portfolio holdings and \(D\) denote the change in the domestic portfolio. Under the SSQDA preference, the perceived certainty equivalent gains of moving to a portfolio \((e_d + D, 1 - e_d - D)\) is

\[
G(D, e) \equiv \min_v \{-e^{-\gamma[(Du+e)^\top(\mu+v) - \frac{1}{\gamma}(Du+e)^\top \Sigma(Du+e)]} + e^{-\gamma(\mu+v) + \frac{\gamma^2}{2} e^\top \Sigma e}\},
\]

where \(\gamma\) is the risk aversion coefficient, \(u \equiv (1, -1)^\top\), and \(v\) represents the adjustments to the mean domestic stock market return and the mean world market return to account for investors’ uncertainty on the domestic and world stock payoffs.

we assume that investors make adjustments to the mean stock returns estimated based on historical data. These adjustments form the probability set \(P\) which the investors utilize to evaluate their investment strategies. Specifically, in our analysis, we consider two different specifications for the probability set \(P\). In the baseline specification, we assume that the
adjustments to the expected domestic stock returns and expected world stock returns are in an elliptical set, which implies that the log likelihood function from the distributions in $\mathcal{P}$ lies above some threshold level. In the second specification, the adjustments to the expected returns are uniformly distributed in a rectangular set, i.e., the deviations of mean returns from their respective sample estimates are uncorrelated and bounded by a multiple of their respective standard deviations.\footnote{All results reported in the paper are based on the set of probability distributions determined by the likelihood function except for the home bias ratio calculation which is based on the uniform distribution on a rectangular set.}

Using a first order Taylor expansion, we arrive at the following approximation to the certainty equivalent gain

$$G(D, e) \approx \gamma C(D, e),$$

$$C(D, e) \equiv \min_v \{Du^\top (\mu + v) - \frac{\gamma}{2}[D^2u^\top \Sigma u + 2Du^\top \Sigma e]\}.$$  

We define the adjustment to the mean return of portfolio $u$ due to uncertainty over the probability set $\mathcal{P}$ as

$$v_m \equiv -\min_{Q \in \mathcal{P}} u'v.$$  

Under the log likelihood ratio specification for the set $\mathcal{P}$, we identify the mean return adjustment $v_m$ as the solution to

$$v^\top \Sigma^{-1}v \leq \beta^2/T, \tag{15}$$

where $\beta$ represents the uncertainty and $T$ is the number of periods for which data are available. It is straightforward to show that in this case

$$v_m = \max_{Q \in \mathcal{P}} (v_d - v_w) = \beta \sqrt{u^\top \Sigma u}/T, \tag{16}$$

where $v_d$ and $v_w$ represent the adjustments made to the mean domestic stock market return and the mean world portfolio stock market return.

We solve for the optimal asset holdings by maximizing the certainty equivalent gain $C(D, e)$.\footnote{We have also done the numerical analysis using the gains in expected utility function $G(D, e)$. Under the parameter values we considered, the differences were negligible.} The trading strategy under the familiarity bias is given below.
Proposition 4 The net trade in domestic equity perceived to maximize the certainty equivalent gain is given by

\[ D = \begin{cases} 
\frac{\mu_d - \mu_w - v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}, & \text{if } \mu_d - \mu_w - \gamma u^\top \Sigma e > v_m \\
0, & \text{if } |\mu_d - \mu_w - \gamma u^\top \Sigma e| \leq v_m \\
\frac{\mu_d - \mu_w + v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}, & \text{if } \mu_d - \mu_w - \gamma u^\top \Sigma e < -v_m.
\end{cases} \] (17)

We calibrate the model to the data for four countries including Germany, Japan, United Kingdom, and the United States. For each country we quantify the perceived gains of holding a combination of the domestic portfolio and the world equity portfolio.

Table 1 shows the summary statistics of annual stock market returns for the four countries and the world market portfolio.\(^{13}\) To facilitate comparison, we use value-weighted dollar returns for all four countries and the world market portfolio. For the period January 1975 through December 2001, investment in the US stock market yields the lowest average return of 7.3% while the investment in the UK stock market turned in the highest average return of 15.9%. Investment in the US stock market, however, has the lowest annual standard deviation of 13.7%. The Japan stock market is the most volatile, with an annual standard deviation of 28.4%. The value-weighted dollar return for the world market portfolio is 12.0%, with an annual standard deviation of 18.8%. The Japan market return has the highest correlation, 87.9%, with the world market portfolio return, while the US stock market has the lowest correlation, 49.6%, with the world market portfolio.

Figure 1 shows the perceived certainty equivalent gains of moving from the endowment position for investors in Germany, Japan, the UK, and the US assuming that their respective initial asset holdings are 100% domestic stocks.\(^ {14}\) Because it is well costly to sell short in international markets, we assume no short sale in our calculation. The perceived certainty equivalent gain from deviating the initial all domestic equity position with no familiarity bias is approximately 1.6% for Germany, 7.1% for Japan, and 3.0% for the U.S. Interestingly,

\(^{13}\)Data used in our analysis is downloaded from Kenneth French’s website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.

\(^{14}\)Our assumption of the initial endowment of 100% domestic equity offers the highest level of certainty equivalent gains for diversifying into the world equity market. It therefore creates the most challenging situation for home bias.
because the UK stock market performed much better than the world equity portfolio for the sample period, the initial holding of 100% in domestic equity is optimal for the UK investors.

The perceived certainty equivalent gain decreases to zero for Germany, Japan, and the U.S. as the model uncertainty increases. Specifically, for German investors, the perceived certainty equivalent gain from the option to diversify into foreign equity markets vanishes when the combined model uncertainty of domestic and world equity returns is less than 1.5 times of the sum of the standard deviations of the domestic and world equity returns. For the U.S. investors, the perceived certainty equivalent gain goes down to zero when the combined model uncertainty is less than 2 times of the sum of the standard deviations of the domestic and world equity returns. Even for the Japanese investors who benefit the most from diversifying into the world equity portfolio because of the poor performance of domestic equity market during the sample period, the perceived gain disappears when the combined model uncertainty is 3 times of the sum of the standard deviations of domestic and world equity returns. However, the UK investors are better off holding their initial domestic equity portfolio for all levels of model uncertainty.

The portfolio chosen by investors in each country reflects this fear of the high model uncertainty (though low risk) associated with defecting from the endowment in order to invest more globally. Figure 2 plots the domestic equity proportion perceived as optimal in investors’ total portfolio as a function of the model uncertainty for the four countries. At low levels of model uncertainty, the perceived optimal portfolio for investors in Germany, Japan, and the US fall below their respective initial domestic endowments, suggesting that they shift from domestic equity to the world market portfolio.

In fact, in Figure 2 we see that for some levels of uncertainty Japanese investors prefer to hold only the world market portfolio, and none of the domestic equity market portfolio. This can be attributed to the prolonged underperformance of the stock market in Japan during this time period. For investors in the UK, however, the domestic equity proportion perceived as optimal is their initial endowment of 100%. This is consistent with the fact that stocks in the UK performed well relative to the world market portfolio for the sample period.

At high levels of model uncertainty about stock returns, the domestic equity proportion
perceived as optimal coincides with the endowed domestic equity proportion in investors’ portfolio composition. This is consistent with home bias as documented in several empirical studies.

The investor’s aversion to risk also interacts with the model uncertainty in determining the effect on the perceived certainty equivalent gain and the investor’s portfolio decision. As an investor becomes more risk averse, the benefit of holding a balanced portfolio increases. Consequently, when investors contemplate deviating from their initial endowment positions by adding more domestic equity in their portfolios, the level of uncertainty at which the perceived certainty equivalent gain reaches zero declines (figure not shown). When investors consider holding a more balanced portfolio by increasing their exposure to the world market portfolio, the level of uncertainty at which the perceived certainty equivalent gain reaches zero rises. This reflects the tradeoff between the benefit of risk reduction and the fear of the model uncertainty associated with deviating from the initial endowment.

As risk aversion increases, the level of uncertainty required to induce investors to remain at their initial endowments increases. This is because the gains from diversification are higher when investors are more risk averse.

Our results indicate that for plausible parameter values individuals may reach an autarkic outcome. Assuming that domestic investors are endowed with domestic stock, this implies a home bias. In the model of Dow and Werlang (1992), owing to ambiguity aversion, individuals may choose to hold only a riskfree asset and none of an uncertain risky asset. However, this finding is endowment independent, whereas in our approach it is the fact that individuals start out with portfolios tilted toward domestic assets (and in the extreme, hold only domestic assets) that leads to a home bias. Furthermore, in Dow and Werlang, the zero holding result occurs because one asset is much more uncertain than the other. In contrast, in our analysis the domestic and foreign assets can be comparably uncertain. Nevertheless, the uncertainty of the foreign asset (which is not part of the domestic investor’s endowment) makes the investor pessimistic about trading to obtain it.

In Uppal and Wang (2003) and Epstein and Miao (2003), domestic investors hold less of foreign assets because to them foreign assets are highly uncertain. However, in contrast with our model, in their settings generically investors always trade and always hold non-zero
amounts of every foreign asset. Also, in our model a crucial source of home bias is that investors are endowed disproportionately in foreign stock. In contrast, in Uppal and Wang and in Epstein and Miao, if a domestic investor were endowed with foreign stock (perhaps received as an inheritance), he would still sell it to trade to position heavy in domestic stock.

5.2 Gains in the Sharpe Ratio When There is a Riskfree Asset

When there is a riskfree asset, investors can expand their investment opportunity set by combining the riskfree asset and risky securities. We describe the return and risk trade-off perceived by investors, taking into account their pessimism about defecting from the status quo, and the risk reduction achievable by diversifying into international equity markets. Under CARA utility, the perceived certainty equivalent gain is proportional to changes in the square of the Sharpe ratio.

Let $r_f$ denote the risk free rate, $S(Du + e)$ denote the square of the Sharpe ratio of holding portfolio $(e_d + D, 1 - e_d - D)$, and $\sigma \equiv (\sigma_d, \sigma_w)^T$. Under Status Quo Deviation Aversion, the investor compares the Sharpe ratio of the status quo portfolio with that of an alternative portfolio. If an alternative has a higher Sharpe ratio under all distributions than the Sharpe ratio of the status quo, then investor switches away from the status quo. The investor’s decision is described by the optimization problem

$$\max_D \min_v \frac{[(Du + e)^T(\mu + v)]^2}{(Du + e)^T\Sigma(Du + e)} - \frac{[e^T(\mu + v)]^2}{e^T\Sigma e}.$$  \hspace{1cm} (18)

Figure 3 plots the increase in the Sharpe ratio as a function of the degree of model uncertainty when investors in the four countries are presented with the opportunity of investing in both the world equity portfolio and their respective domestic stock market portfolios. In this analysis, we use the US T-bill rate during this period as the risk free rate. We also impose no short sale constraint. All countries are assumed to begin with 100 percent domestic equity holding $(e_d = 1).^{15}$ Consistent with our analysis based upon certainty equivalent gains,

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15We choose the endowment of the domestic investors to be $e_d = 1$. This offers the highest improvement in the Sharpe ratio by diversifying into the international equity markets. Alternative specifications using the data in Tesar and Werner (1995) as the initial endowment do not change the results.
investors in Japan and the United States have the largest potential gains from diversification without the model uncertainty. Investors in the UK have no gain in the Sharpe ratio. Once again, all potential gains from diversification become zero when the model uncertainty is about half of the standard error from respective sample mean return estimate.

Figure 4 shows the investor’s domestic equity proportion in total equity investment as a function of the model uncertainty. Consistent with the finding with no riskfree asset, for Germany, Japan, and the US, the preferred portfolio composition calls for reducing domestic equity holding and increasing investment in the world equity market portfolio. For UK investors, the preferred portfolio strategy with no model uncertainty is to hold their initial composition of 100 percent domestic equity. As the model uncertainty increases to one standard error of the sample mean return, the perceived optimal portfolio choices approaches the investors’ initial endowment position.

5.3 An Alternative Specification of Expected Returns

The calibration analysis above uses sample means and variance-covariance matrix between domestic equity and world equity returns calculated using historical data for each country. Alternatively, investors may believe that returns are generated by a process akin to the international CAPM. We next perform the calibration analysis using expected returns and conditional variance-covariance matrix estimated from an international CAPM model.\textsuperscript{16}

We find that when the expected returns are estimated from the CAPM, the potential increase in the Sharpe ratio is substantially reduced for investors in Japan and the United States. For the same time period, there is no drastic change in the increase in the Sharpe ratio for Germany and the UK. The potential gain of diversification approaches zero more rapidly than in the case when the sample mean and variance-covariance matrix are used. The increase in the Sharpe ratio reaches to zero at a point where the model uncertainty is less than a half of the standard error of the sample mean return.

The investor’s perceived optimal portfolio choices reinforce the finding on the Sharpe ratio. Consistent with the findings on the increase in the Sharpe ratio achievable from the

\textsuperscript{16}For brevity we omit figures for this analysis (available upon request).
option to diversify, the degree of model uncertainty for which the investors choose to hold
the initial endowment is slightly under a half of the standard error of the sample mean return
for nearly all four countries.

5.4 Familiarity Bias, Participation, and Investment Styles

We have focused on the international home bias puzzle in our analysis of multiple securities.
However, investing domestically instead of internationally is just a special case of having a
style bias, wherein individuals invest in a certain style to the exclusion of others. Examples
of styles can also include choices amongst major asset categories (stock versus bonds versus
commodities), choices amongst industries, value versus growth, high versus low momentum,
and high versus low dividend yield.

Our approach also offers insight, even among domestic firms, about how news arrival
and publicity will affect stock prices. As discussed in Section 2, there is evidence that stocks
that receive greater publicity or have greater news arrival tend to be purchased more heavily
(even if the news is on average neutral). Our approach is consistent with such attention
effects. Stocks whose names are prominently mentioned in the media or by other individuals
become more familiar. In our approach, investors therefore perceive the uncertainty as
smaller in highly-publicized firms. Thus, investors become less reluctant to deviate from
their endowment position. This can, of course, encourage either sale or purchase of the
stock. However, as discussed in section 2, individuals tend to be undiversified in their stock
holdings (apart from any holdings of stock mutual funds), so that any given stock in the
U.S. is typically not directly held by the vast majority of investors.

In our approach, publicity increases investors’ comfort in contemplating the purchase
of the stock. Less familiarity bias with respect to a stock that is not currently part of the
portfolio will therefore encourage many individuals to switch to holding a positive quantity.
Increased publicity about a stock expands breadth of ownership, increases net demand for
the stock, and thereby induces a positive stock price reaction. Thus, our approach is consis-
tent with the fact that firms sometimes make non-substantive advertisements prominently
emphasizing the name of their firm, apparently directly to potential stockholders. ‘Celebrity
stocks’ ought to be able to win disproportionate purchases on the part of otherwise-non-
We have analyzed the strategies perceived to be optimal by investors who have familiarity bias. We now turn to the question of how familiarity affects trading and prices as part of an endogenously determined market equilibrium. We allow for the possibility that a subset of investors are immune to familiarity bias, and act as rational Bayesians.

We consider price determination in a setting with four groups of investors: domestic and foreign rational investors, and domestic and foreign investors who are subject to familiarity bias. The population size of each country is normalized to one, and the proportion of rational investors in each country is denoted $m$. All investors have mean-variance utility functions with risk aversion coefficient $\gamma$. The payoffs of the stocks in the two countries are $\mu_d$ and $\mu_f$, respectively, which are assumed to be multi-variate normally distributed. The variance-covariance matrix of the payoffs, $\Sigma$, is assumed to be diagonal with diagonal elements of $\sigma_d^2$ and $\sigma_f^2$. The per capita supplies of the domestic and foreign stocks are denoted $x_d$ and $x_f$, respectively.

We assume that domestic investors are initially endowed with the entire supply of domestic stocks while foreign investors are endowed with the entire supply of foreign stocks. For the rational Bayesian investors, the expected payoff is denoted by a $2 \times 1$ vector $\mu$. For investors with familiarity bias, the expected payoff is denoted by a set $\mu + v$, where the adjustments to the mean returns are uniformly distributed on a rectangular set given below

$$v \in [-\alpha \sigma_d / \sqrt{T}, \alpha \sigma_d / \sqrt{T}] \times [-\alpha \sigma_f / \sqrt{T}, \alpha \sigma_f / \sqrt{T}],$$

where $\alpha$ represents the investor’s degree of uncertainty on the expected stock payoff.

Since investors have CARA utility and stock payoffs are independent, trading in the foreign stock does not affect trading in the domestic stock market. Without loss of generality, we focus on the equilibrium in the domestic stock. Let $P_d$ be the equilibrium market price of the domestic stock. The optimal trade in domestic stock by a domestic rational investor

6 Capital Market Equilibrium with Familiarity Bias
\( T_{dr} \) is 
\[
T_{dr} = (\gamma \sigma_d^2)^{-1}(\mu_d - P_d) - x_d.
\]

Similarly, the optimal trade in the domestic stock by a foreign rational investor \((fr)\), \( T_{fr} \) is
\[
T_{fr} = (\gamma \sigma_d^2)^{-1}(\mu_d - P_d).
\]

Assuming that a domestic familiarity-biased investor \((db)\) sells, the optimal trade on the domestic stock of a domestic familiarity-biased investor, \( T_{db} \), is
\[
T_{db} = (\gamma \sigma_d^2)^{-1}(\mu_d + \alpha \sigma_d - P_d) - x_d.
\]

Assuming that a foreign familiarity-biased investor \((fb)\) buys, the optimal trade on the domestic stock of a foreign familiarity-biased investor, \( T_{fb} \), is
\[
T_{fb} = (\gamma \sigma_d^2)^{-1}(\mu_d - \alpha \sigma_d - P_d).
\]

For market clearing, we must have
\[
mT_{dr} + (1 - m)T_{db} + mT_{fr} + (1 - m)T_{fb} = 0. \tag{19}
\]

Solving for equilibrium price for the domestic stock yields
\[
P_d = \mu_d - \frac{1}{2} \gamma \sigma_d^2 x_d. \tag{20}
\]

It remains to be verified whether domestic familiarity-biased investors will sell and foreign familiarity-biased investors will buy. It can be shown that the relevant condition is that the degree of model uncertainty is lower than a threshold given by
\[
\alpha < \gamma \sigma_d x_d / 2.
\]

The equilibrium with \( \alpha \geq \gamma \sigma_d x_d / 2 \) can be determined similarly. In the latter case, investors with familiarity bias will not trade while the rational investors will always trade. To summarize, we have the following results:
Proposition 5 When $\alpha < \gamma \sigma_d x_d / 2$, both rational Bayesian investors and investors who are subject to familiarity bias will trade in both domestic and foreign markets. When $\alpha \geq \gamma \sigma_d x_d / 2$, rational Bayesian investors continue to trade internationally, whereas investors subject to familiarity bias remain at their endowment positions.

In the absence of the familiarity bias, domestic investors should hold $x_d / 2$ of the domestic stock. Thus the ratio of the perceived optimal domestic holdings under familiarity bias to $x_d / 2$ can be used as a measure of home bias:

$$H \equiv \frac{m(T^d_{dr} + x_d) + (1 - m)(T^d_{db} + x_d)}{x_d / 2} = \begin{cases} 1 + \frac{2\alpha(1-m)}{\gamma \sigma_d x_d} & \text{if } \alpha < \gamma \sigma_d x_d / 2 \\ 2 - m & \text{otherwise} \end{cases}$$

(21)

More generally, the relative prices of different stock markets will be influenced by the proportion of familiarity biased investors and investors’ risk tolerance in different markets. When familiarity bias is asymmetric, in equilibrium stock markets in which investors have strong familiarity bias will be overpriced relative to stock markets in which investors have weaker familiarity bias.

In figure 5, we plot the home bias ratio as a function of the model uncertainty ($\alpha$) for Germany, Japan, United Kingdom, and the United States. In all four panels, the home bias ratio linearly increases in the model uncertainty as long as the participation condition for the uncertainty averse investors are satisfied. At very high levels of model uncertainty, with familiarity bias choose not to trade. In this case, only Bayesian investors trade the risky assets and the home bias ratio stays at the peak level.

7 Equilibrium with Wealth Dependent Risk Tolerance

In this section, we extend the analysis in the previous section by allowing the investor’s risk tolerance to depend upon his initial wealth level. We then investigate how the equilibrium equity premium and asset holding change in response to a wealth shock to some investors.

As in the previous section, we consider investors in two countries. For ease of exposition, we call them country $A$ and country $B$. In both countries, the population size is normalized...
to one. In country A, there is a proportion $m$ of rational investors while the rest $(1 - m)$ are familiarity bias investors. With loss of generality, we assume that all investors in country B are rational investors. All investors have mean-variance utility functions. The risk aversion coefficient is however a decreasing function of the investor’s initial wealth denoted by $W_{0j}, \ j = A, B$. Thus, when a country experiences a positive economic shock which increases the investor’s wealth in this country, the investor becomes more risk tolerant (less risk averse), i.e., $\gamma(W_{0j})$ decreases in $W_{0j}, \ j = A, B$.

As in the previous section, we assume that the payoffs of the portfolios in the two countries are independent and are assumed to be multi-variate normally distributed with mean vector $\mu = (\mu_A, \mu_B)\top$ and a diagonal covariance matrix with variances $\sigma_A^2$ and $\sigma_B$, respectively. For the familiarity bias investors in country A, we also assume that the adjustments to the mean returns are uniformly distributed on a rectangular set with $\alpha$ representing the degree of uncertainty on country A portfolio payoff.

We assume that the initial shares of stocks of each country, denoted $x_j, \ j = A, B$ are evenly distributed among investors in respective countries and there is no initial cross country holding of stocks. We further assume that the following condition is satisfied:

$$\alpha \sigma_A(\gamma(W_{0A}) + m\gamma(W_{0B})) < x_A \sigma_A^2 \gamma(W_{0A})^2.$$ (22)

As we demonstrate later, this condition is required in equilibrium for investors with familiarity bias to sell shares of risky stock.

For the purpose of our investigation, we focus on the equilibrium in country A stock market. Let $P_A$ be the equilibrium stock price in country A. The optimal demand of a rational investor in country A, $D^r_A$, and country B, $D^r_B$ are as follow, respectively

$$D^r_A = \frac{\mu_A - P_A}{\gamma(W_{0A})\sigma_A^2},$$

$$D^r_B = \frac{\mu_A - P_A}{\gamma(W_{0B})\sigma_A'}. $$

Assuming that the familiarity bias investors in country A sells shares, the optimal demand
of an familiarity bias investor in country $A$, denoted by $D_A^b$ is given by

$$D_A^b = \frac{\mu_A - P_A + \alpha \sigma_A}{\gamma(W_{0A})\sigma_A^2}.$$  

For country $A$ stock market to clear, we must have the optimal demand equal to the supply

$$mD_A^r + (1 - m)D_A^b + D_B^r = x_A.$$  

The equilibrium price is

$$P_A = \mu_A + \frac{(1 - m)\alpha \sigma_A \gamma(W_{0B})}{\gamma(W_{0A}) + \gamma(W_{0B})} - \frac{x_A \sigma_A^2 \gamma(W_{0A}) \gamma(W_{0B})}{\gamma(W_{0A}) + \gamma(W_{0B})}.$$  

(23)

It can be shown that when condition 22 is satisfied, the familiarity bias investors sell their shares of country $A$ stocks.\(^{17}\)

To examine how equilibrium equity premium is affected by the change in country $A$ investors’ risk tolerance due to a positive wealth shock, we rewrite the equilibrium price for country $A$ stock (23) as follows

$$\mu_A - P_A = \frac{x_A \sigma_A^2 \gamma(W_{0A}) \gamma(W_{0B})}{\gamma(W_{0A}) + \gamma(W_{0B})} - \frac{m \alpha \sigma_A \gamma(W_{0B})}{\gamma(W_{0A}) + \gamma(W_{0B})}.$$  

(24)

The equity premium can be decomposed into two components. The first component is due to investors’ risk aversion (risk premium) and is proportional to the risky asset supply and variance of country $A$ portfolio payoff. The second component is due to investors’ uncertainty aversion (uncertainty premium) and is negatively related to degree of uncertainty of familiarity bias investors on country $A$ portfolio payoff. The negative effect of the uncertainty premium is attributed to reduced selling by the familiarity bias investors when the uncertainty is high.

When investors in country $A$ experience a positive wealth shock, they become more risk tolerant. The risk aversion coefficient for investors in country $A$ increases. It is straightforward to verify that both the risk premium and the uncertainty premium decrease, leading to a lower equity premium.

\(^{17}\)The proof is identical to the proof of proposition 5, thus omitted from the paper.
Next, we examine country A investors’ total demand for country A risky portfolio. Let $TD_A$ denote the total demand for country A risky portfolio from investors in country A. Then, $TD_A$ consists of two components: the country A rational investors’ demand and the country A familiarity bias investors’ demand, i.e., $TD_A \equiv mD^r_A + (1 - m)D^b_A$. Substituting the equity premium (24) into the demand $D^r_A$ and $D^b_A$ yields

$$TD_A = \frac{x_A \gamma(W_{0B})}{\gamma(W_{0A}) + \gamma(W_{0B})} + \frac{(1 - m)\alpha}{(\gamma(W_{0A}) + \gamma(W_{0B}))\sigma_A}.$$ (25)

Therefore, as investors in country A experience a positive wealth shock, their risk avers coefficient ($\gamma(W_{0A})$) decreases and their total demand for domestic risky portfolio increases. Intuitively, as the investor becomes less risk averse, the benefit of diversification is reduced. Investors in country A thus hold more domestic risky stock and a less diversified portfolio.

One testable implication is that investors in wealthier country may hold more domestic assets, leading to a higher degree of home bias than investors in a less wealthier country. Furthermore, home bias in a given country becomes is reduced when investors in the country experience an unfavorable wealth shock.

Country A investors’ holding of domestic risky portfolio (25) also indicates that as the volatility of domestic portfolio payoff increases their domestic risky portfolio holding decreases. This implies that there is an inverse relationship between home bias and the volatility of risky portfolio payoff. Higher degree of home bias is associated with a lower volatility of payoff and a lower degree of home bias is associated with a higher volatility of payoff.

The existence of familiarity bias investors in country A also affects the magnitude of price change in country A risky portfolio when the investors in country A incur the wealth shock. To demonstrate how the price change in country A risky portfolio is affected by the familiarity bias investors, we perform the experiment on wealth shocks of the same size of $\epsilon$ but in opposite directions: one negative and one positive. We then examine the price change corresponding to the wealth shocks with and without the familiarity bias investors.

Let $P^r_A$ be the equilibrium risky portfolio price in country A when all investors in country A are rational investors ($m = 1$). Given the equilibrium price equation (23) the price change corresponding to the wealth change from $(W_{0A} - \epsilon)$ to $(W_{0A} + \epsilon)$ in our baseline economy...
with familiarity bias investors is given by

$$P_A(W_{0A} + \epsilon) - P_A(W_{0A} - \epsilon) = x_A \gamma(W_B) \sigma_A^2 \left[ \frac{\gamma(W_{0A} - \epsilon)}{\gamma(W_{0A} + \epsilon) + \gamma(W_{0B})} - \frac{\gamma(W_{0A} + \epsilon)}{\gamma(W_{0A} - \epsilon) + \gamma(W_{0B})} \right]$$  \hspace{1cm} (26)$$

$$+ (1 - m) \alpha \gamma_A(W_{0B}) \sigma_A \left[ \frac{1}{\gamma(W_{0A} + \epsilon) + \gamma(W_{0B})} - \frac{1}{\gamma(W_{0A} - \epsilon) + \gamma(W_{0B})} \right].$$

Similarly, we can derive the price change corresponding to the same magnitude of wealth shocks for the economy without familiarity bias investors as follows

$$P_A'(W_{0A} + \epsilon) - P_A'(W_{0A} - \epsilon) = x_A \gamma(W_B) \sigma_A^2 \left[ \frac{\gamma(W_{0A} - \epsilon)}{\gamma(W_{0A} + \epsilon) + \gamma(W_{0B})} - \frac{\gamma(W_{0A} + \epsilon)}{\gamma(W_{0A} - \epsilon) + \gamma(W_{0B})} \right].$$  \hspace{1cm} (27)$$

Comparing the price changes with and without the familiarity bias investors, we observe that the presence of familiarity bias investors introduces a second component in addition to the price change when all investors are rational. Further, because investors in country $A$ become more risk averse with a negative wealth shock and more risk tolerant with a positive wealth shock, it is straightforward to show that both components in the price change given by (26) are positive. Therefore, the price change in the economy with the familiarity bias investors is larger than in the economy with all rational investors. Intuitively, the existence of familiarity bias investors implies that favorable home country shocks will cause a larger home country price reaction, because risk aversion is reduced and domestic investors plunge into domestic stocks. The reverse occurs on the down side when investors incur negative wealth shocks. Consequently, familiarity bias increases the magnitude of price changes.

8 **Underdiversification**

As discussed in section 2, most investors directly hold only a few stocks in their portfolios, a phenomenon that was especially puzzling prior to the rise of mutual funds in recent decades. We show here that when deviating from the status quo choice triggers investor aversion to model uncertainty, an imperfectly diversified portfolio may not be dominated by a more diversified portfolio. Hence, investors will remain at poorly diversified endowment positions.

Consider the case of $N$ stocks with the same expected payoff $\mu$ and the same variance $\sigma^2$. 41
Assume that asset returns are jointly normally distributed and that the correlation between any two assets is \( \rho \).

We define a portfolio \( p_e \) as undominated if a familiarity-biased investor who starts with \( p_e \) as his status quo prefers to hold \( p_e \). (The subscript ‘e’ stands for ‘efficient,’ though such a portfolio need not be efficient from the perspective of a rational investor.) Thus, a portfolio \( p_e \) is undominated if, for any arbitrary portfolio \( p \),

\[
\min_{Q \in P} [CE(p) - CE(p_c)] \leq 0
\]

We define \( E \) for a given investor as the set of portfolios that are not dominated by any other portfolio. In other words, \( E \) is the set of portfolios \( p_e \) such that, for any arbitrary portfolios \( p \),

\[
E = \{ \min_{Q \in P} [CE(p) - CE(p_e)] \leq 0 \}
\]  

where \( CE(p) \) represents the certainty equivalence for portfolio \( p \).

Once an investor reaches the level of diversification represented by an undominated portfolio \( p_e \) in \( E \), further diversification is not perceived to be beneficial. We will now characterize the minimum number of stocks required for a portfolio to be included in the set \( E \). Let \( p_k \) be a risky portfolio consisting of \( k \) stocks with positive weights. We define \( K \) as the minimum number of stocks required in a portfolio such that it is not dominated by any other portfolio, i.e., \( K \) is the minimum number such that there exists a portfolio \( p_K \in E \).

Since all assets are symmetrically distributed, we can focus on a subset of portfolios. Let \( e_K \) denote an equally weighted portfolio of \( K \) securities and \( e_{N-K} \) the equally weighted portfolio of the remaining securities. Due to the assumption of symmetry in our model, we need only to focus on whether a mixture of \( e_K \) and \( e_{N-K} \) will dominate \( e_K \) or not.

Let \( v = [v_K \ v_{N-K}]^T \) be the adjustments to the mean returns of portfolios \( e_K \) and \( e_{N-K} \), respectively. As before, we assume that the uncertainty adjustments satisfy equation (15). Let \( u = [1 \ -1]^T \). The adjustment to the mean return of portfolio \( u \), \( v_m = -\min_{Q \in P} u'v \), is given by the following lemma.
Lemma 1

\[ v_m \equiv \max_{Q \in \mathcal{P}} (v_K - v_{N-K}) = \beta \sigma \sqrt{\frac{N(1-\rho)}{K(N-K)T}}. \]

For the equally-weighted portfolio not to be dominated by a mix between \(e_K\) and \(e_{N-K}\), following Proposition 4, it must be the true that

\[ v_m - \gamma u^\top \Sigma_1 (1, 0)^\top \leq 0, \]

where \(\Sigma_1\) is the variance-covariance matrix between the returns of portfolios \(e_K\) and \(e_{N-K}\).

\[ \Sigma_1 \equiv \left( \begin{array}{cc} \frac{1+(K-1)\rho}{K} & \frac{\rho}{N-K} \\ \rho & \frac{1+(N-K-1)\rho}{N-K} \end{array} \right) \sigma^2. \]

With some algebra this reduces to

\[ K \geq K^* \equiv \frac{\gamma^2 \sigma^2 (1-\rho)N}{N \beta^2 / T + \gamma^2 \sigma^2 (1-\rho)}. \]

Since \(K\) is the minimum number of stocks in a portfolio that satisfies such a constraint, \(K\) must be the smallest integer that is strictly larger than \(K^*\). Thus,

\[ K = N - \text{Int}[N - K^*], \]

where \(\text{Int}[N - K^*]\) represents the largest integer below \(N - K^*\).

Therefore, for a portfolio not to be dominated, one need to have at least \(K\) stocks. For example, with the model uncertainty associated with deviation from the status quo at \(\beta = 2\), and when \(T = 100, N = 500, \rho = 0.5, \gamma = 1, \sigma = 0.5\), an equally weighted portfolio with only four stocks will not be dominated by any other portfolio.

When \(\beta = 0\), the investor acts as a Bayesian and invests in all \(N\) stocks. However, when \(\beta > 0\), the number of stocks needed for a portfolio to be undominated will be less than \(N\). The equally weighted portfolio will always be an undominated portfolio, but so will less diversified endowments. Figure 6 shows the minimum number of stocks needed to construct an undominated portfolio. The minimum number of stocks is plotted against the
model uncertainty for different levels of risk aversion ranging from 1 to 5. Overall, as model uncertainty increases, the minimum number of stocks needed to construct an undominated portfolio decreases monotonically with $\beta$, reflecting the investor’s desire to reduce the overall uncertainty in his portfolio. For the parameter values used in our numerical analysis, the number of stocks an investor chooses to hold in his portfolio decreases drastically from several hundreds to a few stocks at very low level of model uncertainty ($\beta < 1$). Furthermore, as the investor’s risk aversion increases, the investor increasingly desires to hold a well diversified portfolio. The number of stocks in the investor’s portfolio is uniformly larger for higher values of risk aversion.

Our finding that limited diversification can derive from a fear of unfamiliar choice options suggests that there is a social benefit to mutual funds (and especially index funds) different from that identified in standard explanations. A common argument is that index funds reduce the transaction cost needed for investors to diversify. However, for a long-term buy-and-hold investor, it is not really all that costly to form a diversified portfolio on individual account. In our model, investors stop adding stocks to their portfolios because a large diversification gain is needed to offset the aversion to buying an unfamiliar stock. An index fund can address this issue in two ways. First, the individual needs to add just a single new asset to his portfolio, the mutual fund. Second, by focusing on marketing to investors, mutual funds can make their product more familiar to investors. In other words, where corporations specialize in making profits, mutual funds can specialize in being invested in. Our approach suggests that there is a socially valuable complementary between being good at marketing that assuages investor fears about stocks, and providing a diversified portfolio of securities for individuals to invest in.

9 Conclusion

Experimental and capital market evidence indicates that individuals favor geographically and linguistically proximate and more familiar investments; are biased in favor of sticking to current consumption/investment positions or strategies and in favor of choice alternatives made salient as default options; and are averse even to small gambles when presented as increments relative to a salient reference point. More generally, individuals are more reluctant
to take action that imposes risk than to bear risk associated with remaining passive; tend
to like stimuli they have been exposed to more, tend to like people they are located close to,
and are prone to malice toward outsiders.

These effects have on the whole been discussed separately, as reflected by a variety of
labels: familiarity, local or home bias; the endowment effect; status quo bias; loss aversion;
escalation bias, sunk cost effects, inertia; omission/commission bias; the mere exposure effect;
xenophobia; proximity bias in international cross listings, and propinquity effects. We offer
an integrated explanation for these effects based upon fear of change and of the unfamiliar.

Building upon previous research, we model an inclination of individuals who are faced
with model uncertainty to focus on worst-case scenarios in relation to contemplated choices.
However, in contrast with most past literature, in our approach pessimism about the uncer-
tainty associated with a choice is triggered by the deviation of the individual’s contemplated
decision from some familiar default or status quo choice alternative. Thus, in our setting
the aversion to model uncertainty is not general, but rather is triggered by change or lack of
familiarity.

In our setting individuals evaluate a purchase of a security or product under a probability
distribution that is adverse to buying, and a sale under a distribution that is adverse to
selling. The gap between willingness to pay and willingness to accept increases with both
the degree of uncertainty and risk. Similar reasoning leads to a general tendency to adhere to
status quo options, and to excessive inertia in individual choices. The model thereby offers
an explanation for limited diversification of investors across stocks and asset classes; special
cases include the home bias puzzle and the preference of individuals to invest in company
stock. In calibrations we find that individuals may hold relatively few asset classes in their
portfolios; and that the observed magnitude of home bias can be consistent with a reasonable
level of model uncertainty.

In addition, our approach offers an explanation for attention effects wherein high pub-
licity or information arrival about a stock tends to increase demand for it, even if the news
is on average neutral. The explanation is that greater publicity about a stock increases fa-
miliarity, increasing comfort about the stock. This in turn reduces the pessimism investors
have about securities they are unfamiliar.
The endowment effect arises endogenously in our setting. Contemplated deviations from endowed positions are assumed to trigger aversion to uncertainties associated with the change. Our approach implies that the endowment effect will be stronger in situations where there is more uncertainty about which model to use. In particular, when investors have CARA utility, we show that the gap between WTA and WTP on the margin is twice the product of the measure of uncertainty ($\phi$) and risk ($\sigma$).

We further analyze investors’ portfolio choices when deviating from the status quo induces pessimism about the structure of the world. We show that, for each investor, and for every endowment, there exists a set of prices such that investors will not trade. Thus, unfamiliarity-induced aversion to model uncertainty acts like a shadow transaction cost.\(^{18}\)

In a trading setting, the model therefore implies a bias against trading away from endowed or familiar investment positions. In a managerial setting, the analysis implies that managers will tend to be too resistant to adopting new investment projects, so that the implicit hurdle rate is too high. However, at the same time the analysis also implies that managers will be too reluctant to terminate ongoing projects, implying an implicit hurdle rate for continuing the project that is too low. Thus, familiarity preference is consistent both with a resistance to undertaking new projects, and with escalation bias.

The model offers quantitative predictions as to the importance of the endowment effect. We apply the model to explain the home bias puzzle. We find that home bias can be obtained in equilibrium. We calibrate the model using observed distributions of stock returns in four countries. The observed home bias in these four developed countries is consistent with a moderate level of uncertainty using probability sets with mean payoffs lie in a rectangular set (based on $t$ ratios) or a elliptical set (based on log likelihood ratios). In an international securities market equilibrium, investors with familiarity bias may not buy foreign stocks. More generally, the relative prices of different stock markets will be influenced by the frequency of familiarity biases and risk tolerances of investors in the different markets.

We further apply the model to the question of why many investors hold poorly diversified portfolios. If the purchase of an unfamiliar stock or asset class triggers pessimistic evaluations of model uncertainty, then an investor may perceive greater diversification as unattractive

\(^{18}\)This analogy is exact for CARA utility functions and normal payoff distributions in a single stock model.
even if it is in fact risk-decreasing. We calculate the minimum number of stocks in a portfolio such that defection-induced fear of uncertainty deters individuals from diversifying further. We find that for plausible calibrations investors may settle for on the order of just four stocks.

We have argued that the emotions of fear and suspicion are directed to the unfamiliar and toward potential change, and that this phenomenon explains several biases in individual psychology as well as economic and financial decisions. One issue we have not addressed is the effect of these feelings on decisions made in response to the arrival of new information. Such news will occasionally stimulate new uncertainty about the economic environment, thereby making individuals reluctant to trade. For example, it seems likely that extreme economic news could raise doubts among investors about whether their beliefs about how the world is structured are correct. In such circumstances of heightened uncertainty, familiarity bias effects could become especially strong, leading to reduction in trade.¹⁹

Recent empirical work suggests that there are strong and systematic variations in stock market liquidity (see, e.g., Chordia, Roll and Subrahmanyam (2001)), and that the dynamics of investor participation are important for asset prices (see, e.g., Diether, Malloy and Scherbina (2002) and Chen, Hong and Stein (2002)). Fear of the unfamiliar deserves further study as a possible explanation for the dynamics of market participation, liquidity, and prices.

¹⁹See Routledge and Zin (2003) on how ambiguity aversion can lead to fluctuations in liquidity, such as the extreme illiquidity and “flight to quality” that occurred in international bond markets during the Russian debt crisis of August 1998.
References


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[58] Zhu, N., 2003, “The local bias of individual investors”, working paper, Yale University
APPENDIX

Proof of Proposition 1: Since $U$ is a strictly monotonic continuous function, ignoring higher order terms, the first order condition implies that:

$$\min_{Q \in P} E \left[ \left( r - \frac{\Delta C}{\Delta e} \right) U' \right] = 0, \quad (29)$$

$$\min_{Q \in P} E \left[ \left( \frac{\Delta D}{\Delta e} - r \right) U' \right] = 0. \quad (30)$$

Letting $\Delta e \to 0$, we get

$$\min_{Q \in P} E[(r - WTP)U'] = 0 \quad (31)$$

$$\min_{Q \in P} E[(WTA - r)U'] = 0. \quad (32)$$

We must have $WTP \leq WTA$. To show this, suppose that $WTP > WTA$. Then

$$0 = \min_{Q \in P} E[(r - WTP)U'] = \max_{Q \in P} E[(WTP - r)U'] > \min_{Q \in P} E[(WTA - r)U'] = 0,$$

a contradiction. Note that

$$A + B = \{ \max_{Q \in P} E^Q[U'(WTA - WTP)] \} - E^Q[(WTA - r)U'] \geq 0 + \{ \max_{Q \in P} E^Q[(WTP - r)U'] \}. \quad (33)$$

By (31) and (32), we obtain

$$\max_{Q \in P} E^Q[(WTA - WTP)U'] = \max_{Q \in P} E^Q[A + B] - \min_{Q \in P} E^Q[(r - WTP)U']$$

$$+ \min_{Q \in P} E^Q[(WTA - r)U']$$

$$= \max_{Q \in P} E^Q[A + B]. \quad (33)$$

Q.E.D.

Proof of Proposition 2: For any probability set such that $P^M(Q) \geq \delta$, By the definition of the WTP and the WTA, there exist sets of probability distributuions $Q, Q'$ such that

$$\min_{Q \in \mathcal{Q}} E^Q[(r - WTP)U'] \geq 0 \quad (34)$$

$$\min_{Q \in \mathcal{Q}'} E^Q[(WTA - r)U'] \geq 0 \quad (35)$$

We now show that $WTP \leq WTA$. Suppose the opposite, that $WTP > WTA$. Then

$$0 \geq \max_{Q \in \mathcal{Q}} E^Q[(WTP - r)U'] > \max_{Q \in \mathcal{Q}} E^Q[(WTA - r)U'] \geq \min_{Q \in \mathcal{Q}'} E^Q[(WTA - r)U'] \geq 0,$$
a contradiction. Notice that the third inequality holds as \( \delta > 0.5 \) which implies that \( Q, Q' \) cannot be exclusive sets in \( P \). Q.E.D.

**Proof of Proposition 3:** Let \( D \) denote the investor’s net trade. Under the assumption of CARA utility and normality of payoffs, comparison of utility functions reduced to the perceived certainty equivalent comparison between two choices. Let \( G(D, e) \) denote the perceived certainty equivalent gains of a trade \( D \) relative to endowment \( e \) in the most adverse scenario. The certainty equivalent gains \( G \) can be written as:

\[
G(D, e) = \min_{\|v\| \leq \frac{\phi \sigma}{\gamma \sigma^2}} D(\mu - v - P) - \frac{\gamma \sigma^2}{2}[(D + e)^2 - e^2]
\]

\[
= D \left[ \mu - \frac{\text{sign}(D)\phi \sigma}{\sqrt{T}} - P \right] - \frac{\gamma \sigma^2}{2}[(D + e)^2 - e^2]
\]

\[
= -\frac{\gamma \sigma^2}{2} \left\{ \left[ D - \left( \mu - \frac{\text{sign}(D)\phi \sigma}{\gamma \sigma^2} - P \right) \right]^2 - \left[ \frac{\mu - \frac{\phi \sigma}{\sqrt{T}} - P}{\gamma \sigma^2} - e \right] \right\},
\]

where \( \text{sign}(\cdot) \) denotes the sign function. For no trade to be perceived as an optimal choice, the status quo position cannot to be dominated by any other strategy. So we must have \( G(D, e) \leq 0 \), for all \( D \). Given condition (13), when \( D > 0 \), we have

\[
\left| D - \left( \frac{\mu - \text{sign}(D)\phi \sigma}{\gamma \sigma^2} - P \right) - e \right| = \left| D - \left( \frac{\mu - \phi \sigma}{\sqrt{T}} - P \right) - e \right| > \left| \left( \frac{\mu - \phi \sigma}{\gamma \sigma^2} - P \right) - e \right|.
\]

Similarly, when \( D < 0 \), we have

\[
\left| D - \left( \frac{\mu - \text{sign}(D)\phi \sigma}{\gamma \sigma^2} - P \right) - e \right| = \left| D - \left( \frac{\mu + \phi \sigma}{\sqrt{T}} - P \right) - e \right| > \left| \left( \frac{\mu + \phi \sigma}{\gamma \sigma^2} - P \right) - e \right|.
\]

Thus \( G(D, e) \leq 0 \) for all \( D \) and the investor will not deviate from the status quo position. Q.E.D.

**Proof of Proposition 4:** We consider two scenarios: (1) No trading is percevied to be optimal, i.e., \( D = 0 \); 2) Trading is perceived to be optimal, i.e., \( D \neq 0 \).

Case (1): No trading is perceived to be optimal. In this case, for \( D = 0 \) to be perceived as optimal, we must have

\[
\mu_d - \mu_w - v_m - \gamma u^T \Sigma e \leq 0 \quad \text{and} \quad \mu_d - \mu_w + v_m - \gamma u^T \Sigma e \geq 0.
\]

In this case, any deviation from the endowment will make investors worse off.
Case (2): In this case, investors will trade and each choice is evaluated according to the worst case scenario. The maximization problem is

\[ C(D, e) = \min_v \{(Du)^\top(\mu + v) - \frac{\gamma}{2}[(Du + e)^\top\Sigma(Du + e) - e^\top\Sigma e]\}. \]

Thus,

\[ C(D, e) = \{Du^\top(\mu - \text{sign}(D)\alpha\sigma/\sqrt{T}) - \frac{\gamma}{2}[(Du + e)^\top\Sigma(Du + e) - e^\top\Sigma e]\}. \]

When \( D \neq 0 \), the unconstrained first order condition is:

\[ u^\top(\mu - \text{sign}(D)\alpha\sigma/\sqrt{T}) - \gamma u^\top\Sigma(Du + e) = 0, \]

and

\[ D = \left( \frac{u^\top(\mu - \text{sign}(D)\alpha\sigma/\sqrt{T}) - \gamma u^\top\Sigma e}{\gamma \mu^\top\Sigma u} \right). \]

which conforms with the expression in Proposition 4. Q.E.D.

**Proof of Proposition 5:** Because of the symmetry in the distribution of domestic and foreign stocks and the Bayesian and familiarity-biased investors, we only to show the result for domestic stock market.

Substituting the equilibrium price for domestic stock \( P_d = \mu_d - \gamma \sigma_d^2 x_d / 2 \) into the perceived optimal domestic stock holding of domestic familiarity-biased investor, \( T_{db}^d \), and the foreign familiarity-biased investor, \( T_{fb}^d \), we arrive at

\[ T_{db}^d + x_d = \frac{\alpha}{\gamma \sigma_d} + \frac{x_d}{2}, \text{ and } T_{fb}^d = \frac{x_d}{2} - \frac{\alpha}{\gamma \sigma_d}. \]

When \( \alpha < \gamma \sigma_d x_d / 2 \), we have

\[ T_{db}^d + x_d < x_d \text{ or } T_{db}^d < 0 \text{ and } T_{fb}^d > 0. \]

Q.E.D.

**Proof of Lemma 1:** Since the adjustments for the \( M \) securities are the same and the adjustments for the other \( N - M \) securities are the same,

\[ (v_M, v_{N-M})A\Sigma^{-1}A^\top(v_M, v_{N-M})^\top \leq \beta^2 / T, \]

where

\[ A = \left( \begin{array}{c} 1, 1, \ldots, 1 \quad 0, 0, \ldots, 0 \\\n0, 0, \ldots, 0 \quad 1, 1, \ldots, 1 \end{array} \right). \]
Solving the maximization problem yields

\[ v_m = \sqrt{u^\top \Sigma_2 u / T}, \]

where

\[ \Sigma_2 \equiv (A \Sigma^{-1} A^\top)^{-1}. \]

It is easy to verify by direct algebra that the expression of \( v_m \) reduces to the expression in Lemma 1. Q.E.D.
Table 1. Summary statistics of annual stock market returns for various countries.

<table>
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<th>Mean</th>
<th>Std. Dev.</th>
<th>Correlation</th>
</tr>
</thead>
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<tr>
<td>Germany</td>
<td>0.1164</td>
<td>0.2168</td>
<td>0.6864</td>
</tr>
<tr>
<td>Japan</td>
<td>0.0944</td>
<td>0.2837</td>
<td>0.8786</td>
</tr>
<tr>
<td>UK</td>
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<td>US</td>
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<td>0.1371</td>
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<td>World</td>
<td>0.1198</td>
<td>0.1881</td>
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</table>

The reported statistics are annual value-weighted dollar returns using monthly data from January 1975 to December 2001. Correlation measures the correlation between the stock market return in each country and the return on the world market portfolio. The original datasets are obtained from Kenneth French’s website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.
Perceived certainty equivalent gains as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States when the uncertainty on domestic stock returns and the uncertainty on the returns of the world portfolio are correlated. The correlation is set at the sample correlation estimate between the world market return and the stock market returns of respective countries. The perceived certainty equivalent gains are measured by the perceived certainty equivalent of wealth. Investors are allowed to hold their domestic market portfolio and the world market portfolio. The risk aversion coefficient is set to $\gamma = 2$ for all four panels.
Figure 2: Perceived optimal domestic equity proportion as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States when the uncertainty on domestic stock returns and the uncertainty on the returns of the world market portfolio are correlated. The correlation is set at the sample correlation estimate between the world market return and the stock market returns of respective countries. Investors are allowed to hold their domestic market portfolio and the world market portfolio. The dashed line represents the initial endowment for each country. The risk aversion coefficient is set to $\gamma = 2$ for all four panels.
Increases in the Sharpe ratio as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States when the uncertainty on domestic stock returns and the uncertainty on the returns of the world market portfolio are correlated. The correlation is set at the sample correlation estimate between the world market return and the stock market returns of respective countries. Investors initially hold 100 percent of their investment in the domestic market portfolio.
Perceived optimal domestic equity proportion based on the Sharpe ratio as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States when the uncertainty on domestic stock returns and the uncertainty on the returns of the world market portfolio are correlated. The correlation is set at the sample correlation estimate between the world market return and the stock market returns of respective countries. Investors initially hold 100 percent of their investment in the domestic market portfolio.
Figure 5: Home bias ratio

Home bias ratio as a function of model uncertainty ($\alpha$) for Germany, Japan, United Kingdom, and the United States in a general equilibrium model with familiarity bias and both Bayesian and uncertainty averse investors. The home bias ratio is defined as the ratio of actual domestic holdings to $x_d/2$. The proportion of Bayesian investors is set at 20 percent. The risk aversion is set at $\gamma = 2$. 
Figure 6: Under-diversification and uncertainty

The minimum number of stocks in an investor’s portfolio when defection from the endowment induces aversion to model uncertainty, for various risk aversion coefficients. The total number of available stocks is 500, the standard deviation of stock return is set to $\sigma = 0.3$, and the pairwise correlation is set to 0.5.