Abstract

We explain the clustering of underpricing in initial public offerings (IPOs). The model features an industry with aggregate demand uncertainty and asymmetric information about firms’ quality. In the IPO market, firms can signal quality by underpricing or under-issuing new shares. Expected aggregate demand for the industry’s products increases with the publicity that the industry creates through IPO underpricing. We show that asymmetric information and expectations on aggregate product demand interact with each other to generate multiple equilibria. Underpriced IPOs cluster in one equilibrium but not in the other. We use these results to explain why the clustering often occurs in particular industries, is short-lived, and is sensitive to economic conditions.

Keywords: Initial public offering; Clustering; Signaling; Multiple equilibria.
JEL classifications: E44, D82.
1 Introduction

Shares in initial public offering (IPO) are said to be underpriced when they have large price gains shortly after IPO. Such underpricing clusters sporadically and occurs in particular industries. In this paper, we construct a signaling model to explain the clustering as an equilibrium phenomenon and explore the common features of the clustering.

The clustering of IPO underpricing is an important phenomenon because it generates boom-and-bust fluctuations. Since the IPO market is a key source of capital for young firms, these fluctuations hinder the market’s ability of providing capital to the firms. The extent of the fluctuations can be best illustrated by the Internet “craze” during the period 1999 – 2000. At the height of the craze, the average first-day return on IPOs shot up to a staggering 65 percent. Most of those “hot issues” were concentrated in the dot.com industry, while concurrent IPOs in other industries had lackluster performances.1 The boom soon turned into bust near the end of 2000, when the number of underpriced IPOs and the magnitude of underpricing both dropped. As a result, the overall IPO activity fell dramatically – the total number of IPOs decreased from 496 in 1999 to 91 in 2001 (Peristiani and Hong, 2004, Table 1).

The Internet craze is not the only episode of clustering. Clustering also occurred in biotech IPOs in the early 1990s and other hot-issue markets (see Ritter, 1984). Common in all those episodes, the clustering was concentrated in particular industries and was short lived. Why do firms underprice their IPOs at roughly the same time? Why does such clustering occur in particular industries? And, Why is the clustering short lived? Answering these questions is important for understanding the common features of clustering. Surprisingly, there is very little theoretical modeling on clustering in the IPO literature. Instead, the literature has focused on a single firm’s pricing behavior (see section 1.1 for more discussions).

We construct a theoretical model to explain clustering as a result of the interaction between

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1In fact, there was an increase in cancellations and withdrawals from the IPO market by non-Internet firms in 1999. As the chief executive of a large dry pet food company complained, “If you look at the IPO market, there’s large-capitalization activity and dot.com activity, but little else. I feel sorry for small-cap companies that are nondonet.com, and which need to complete their deals.” (Prial, 1999)
aggregate demand uncertainty in the industry and asymmetric information about the quality of firms going to IPO. The model features an industry facing aggregate uncertainty in the demand for its products. The market’s expectations on such demand increase with the industry’s publicity created in the IPO market. We model the industry’s publicity as the average amount of IPO underpricing in the industry. As a positive externality, this industry-wide publicity yields a higher benefit to high-quality firms than to low-quality firms. A firm’s quality is defined through consumers’ preferences for the firm’s product. When the quality is publicly known, a high-quality firm can attract higher demand for its product than a low-quality firm. Moreover, as the total expenditure on the industry’s products increases, consumers increase the share of expenditure on high-quality products. Thus, the differential in expected earnings between the two types of firms increases with the industry’s publicity.

A firm’s quality is private information before IPO. A high-quality firm likes the market to know its quality because only after the quality is known can the firm attract more customers in the product market. However, this potential benefit also gives low-quality firms the incentive to mimic high-quality firms. To prevent mimicking, a high-quality firm must take costly actions such as under-issuing and underpricing the shares in IPO. Underpricing is more costly than under-issuing because in the event of underpricing, the firm must increase the number of shares in IPO in order to raise the required amount of capital, which reduces the original owner’s claim on the firm’s future earnings. As such, underpricing is also more effective in signaling quality than under-issuing. A high-quality firm chooses to underprice when low-quality firms’ temptation to mimic is strong, and to under-issue when the temptation is weak.

IPO underpricing clusters when individual firms’ incentive to underprice interacts with expectations on the industry’s publicity. Industry-wide underpricing cannot be sustained in the absence of incentive to underprice by individual firms. On the other hand, underpricing will not arise at the firm level without expectations of the industry-wide underpricing and its positive externality on individual firms’ expected earnings. However, when the industry-wide underpricing is expected, high-quality firms will underprice and result in the clustering of IPO underpricing.
To illustrate this mechanism, suppose that all firms expect the industry’s publicity to be high. In this case, the aggregate demand for the industry’s products will be high, and so the differential in expected earnings between the two types of firms will be high. There will be a large benefit to high-quality firms from signaling. At the same time, this benefit will create strong temptation for low-quality firms to mimic. To signal successfully, a high-quality firm will greatly underprice IPO. As all high-quality firms underprice, post-IPO gains in share prices will cluster. This clustering will fulfill the initial expectations that the industry’s publicity will be high. Similarly, low expectations of the industry’s publicity are also self-fulfilling. In that case, low-quality firms’ temptation to mimic is weak and every high-quality firm signals quality by under-issuing rather than underpricing.\(^2\)

The above mechanism explains why underpriced IPOs often cluster in particular industries and why the clustering is short-lived. Underpricing is likely to cluster in industries that have high aggregate uncertainty in product demand and strong asymmetric information regarding firms’ quality. The Internet industry and the Biotech industry are some of the examples. The clustering is short-lived and fragile because even non-fundamental events can induce the equilibrium to switch from underpricing to no-underpricing by changing investors’ beliefs. Moreover, our model suggests that the clustering of underpriced IPOs be more pronounced in economic upturns and easy credit markets than in downturns and tight credit markets.

A main assumption in our analysis is that expected demand for the industry’s products increases with the industry-wide IPO underpricing. This assumption is reasonable for a new industry where it is difficult to predict the product demand. Spectacular price gains in IPO can create publicity for the industry and increase consumer awareness, thus benefiting the industry as a whole. For example, if Internet firms that sell books, auction goods, or provide market information have large price gains in IPOs, their collective publicity can attract businesses away from the traditional firms that provide similar services. Although one can formalize a mechanism

\(^2\)Multiple equilibria arise here at the industry level rather than at the firm level. For each individual firm, the industry’s publicity is given and there is a unique equilibrium in the signaling game. Also, all equilibria in this paper are separating equilibria refined by the intuitive criterion of Cho and Kreps (1987). We are not interested in the pooling equilibrium.
to support our assumption, we remain agnostic about such mechanisms.  

An important (and perhaps obvious) fact is that the above assumption alone does not lead to underpricing. On the contrary, the assumption tends to reduce underpricing. The reason is that the industry’s publicity is an externality to individual firms. All firms like to free-ride on the industry’s publicity. If firms’ quality were public information, each firm would not want to underprice, because doing so is costly. In this case, underpricing would not arise at all and the industry’s publicity would be low. It is the combination of asymmetric information and the structure of the product market that enables the IPO market to overcome the free-rider problem. Competition in the product market makes a high-quality firm’s expected earnings more sensitive to the industry’s publicity than a low-quality firm’s earnings. Asymmetric information creates the temptation for low-quality firms to mimic, and hence forces high-quality firms to underprice in order to capture the benefit from the industry’s publicity.

We use a signaling model to describe IPO decisions and abstract from the institutional features such as underwriters’ reputation. One may ask why firms choose to signal quality by underpricing its IPO, rather than by advertisements. One reason is that advertisements require current resources which a new firm may not have, but underpricing IPO entails only expected future earnings. Another reason is that, when the entire industry is new, advertisements are not as effective or convincing as the hard evidence of IPO price gains.

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3 For a single firm, Stoughton et al. (2001) formalize a positive link between the action of going public and the expected demand for the firm’s product. Their argument can be modified to rationalize our assumption. For example, suppose that a new industry competes against an old industry. All firms in the new industry have a common component that is lacking in the old industry but that is not directly observable by the market. When the firms underprice IPOs, they signal the common component and attract customers away from the old industry.

4 Underwriters’ concern for reputation may have limited explanatory power for the clustering of underpricing. When underpriced IPOs cluster, price gains are prevalent across underwriters, concentrated in a particular industry, and short-lived. In contrast, underwriters who are motivated by reputation should underprice IPOs in all industries, rather than a particular industry, and for a long period of time (since reputation needs time to build) rather than for a short period of time like 1999.

5 Michaely and Shaw (1994) find that post-IPO earnings do not have much explanatory power for underpricing. This evidence against the signaling model is inconclusive. First, the signaling model predicts that underpricing depends positively only on the part of future earnings that is private information prior to IPO. Most empirical tests do not distinguish this part of future earnings from the part that is publicly expected prior to IPO. Second, because post-IPO performances depend on post-IPO investment strategies that may not be foreseen at the time of IPO, such performances may not be good indicators of the firms’ earnings expected at the time of IPO. Most empirical tests do not control for such a diversity in post-IPO investment strategies.
1.1. Comparisons with the Literature

Our paper belongs broadly to the literature of self-fulfilling, multiple equilibria (e.g., Diamond and Dybvig, 1983), but the focus on IPO is specific. Moreover, the actions clustered here are signaling activities. Such signaling or asymmetric information is not important for multiple equilibria in the Diamond-Dybvig model. Private information is important in the herding models (e.g., Banerjee, 1992, and Bikhchandani et al., 1992), but herding occurs there when agents choose to ignore, rather than signal, their private information.

The main contribution of our paper to the IPO literature is to combine asymmetric information in the IPO market and aggregate uncertainty in the product market to generate the clustering of underpriced IPOs. As explained earlier, this combination is useful for explaining the facts that the clustering is industry specific, fragile, and short-lived. By contrast, the literature focuses on a single firm’s decision and does not link the clustering to the structure of the product market. This literature is too vast to be surveyed here.\(^6\) To focus on the clustering, we will deliberately restrict the difference in intrinsic earnings between a high-quality and a low-quality firm to be small so that it would not generate underpricing in a signaling model of a single firm.

Two exceptions in the literature are Hoffmann-Burchardi (2001) and Benveniste et al. (2002), who examine the clustering by emphasizing information acquisition.\(^7\) In these models, there are two firms whose values have a common factor. The firms go to the IPO market sequentially in an exogenous order. After firm 1’s IPO, investors acquire information about the firms’ values. In Hoffmann-Burchardi (2001), this information acquisition changes expectations on firm 2’s value differently for investors and firm 2. The difference can induce firm 2 to underprice after firm 1 underprices. In Benveniste et al. (2002), the common factor in firms’ values induces investors to free-ride across the IPOs. To compensate firm 1 for going public first, an underwriter can

\(^6\)See Michaely and Shaw (1994) and Loughran and Ritter (2000) for references. Well-known models of IPO signaling include Allen and Faulhaber (1989), Welch (1989), and Grinblatt and Hwang (1989). Rock (1986) emphasizes the winner’s curse. Others attribute underpricing to underwriters’ reputation building behavior (e.g., Beatty and Ritter, 1986, and Benveniste and Spindt, 1989), to a firm’s concern for liquidity in the secondary market (e.g., Mauer and Senbet, 1992), or to behaviors that are not Bayesian rational (Loughran and Ritter, 2000).

\(^7\)These papers were written concurrently with our paper and we became aware of these papers after completing the first version of our paper in 1999.
cross-subsidize the firms, which may induce both firms to underprice.

Our model is related to these models in the sense that the clustering in our model also relies on a common factor in firms’ values – the aggregate demand for the industry’s products. However, our model illustrates a different mechanism of the clustering, i.e., the interaction between the industry’s publicity and individual firms’ private information. By generating multiple equilibria at the industry level, this mechanism helps to explain why all hot-issue markets are short-lived and why they are sensitive to economic conditions. These questions cannot be answered clearly by the mechanisms in the above models. In addition, our model differs from the above models in other aspects. First, we explicitly link the clustering to the features in the industry’s product market. Second, our model generates the clustering regardless of whether firms go to the IPO market simultaneously or sequentially (and we examine both). By contrast, a sequential structure is essential for the clustering in the above models. Although sequential moves may be realistic, the time interval between some underpriced IPOs is too short to allow for investors to acquire sufficient information during the interval.8

2. The Model

The key ingredients of the model are industry-wide uncertainty and asymmetric information regrading individual firms’ quality. In this section we describe these ingredients.

2.1. Industry Uncertainty and Private Information

Consider an industry with uncertainty in the aggregate demand for its products. The expected demand is susceptible to the industry-wide IPO underpricing. To be specific, let $Y$ be consumers’ aggregate expenditure on the industry’s products and $\bar{D}$ the industry-wide average price gain per new share immediately after IPOs. The expected aggregate demand is:

$$E(Y|\bar{D}) = Y_0 + \rho \bar{D}, \quad 0 < \rho < 1, \quad (2.1)$$

8Another comparison of our model with Benveniste et al. (2002) is that our analysis does not rely on the role of underwriters. A model that places underwriters at the center of the story has certain realism, but it can only explain some aspects of the clustering. For example, such a model predicts that the clustering and underpricing of IPOs are both more pronounced with a large underwriter than with a small underwriter. This is not obviously the case in a hot-issue market like the Internet IPO market in 1999.
where $Y_0 > 0$ is a constant and the expectations are investors’ expectations at the time of IPOs. We refer to $\bar{D}$ as the industry’s publicity. This is a positive externality to individual firms, because firms take $\bar{D}$ as given. As explained in the introduction, the assumption $\rho > 0$ is reasonable for a new industry, but the assumption does not automatically lead to the clustering of underpriced IPOs. The auxiliary assumption $\rho < 1$ ensures that the industry’s publicity be not overwhelming in determining the aggregate demand.

The industry consists of two types of firms. A fraction $\alpha$ are $H$ firms and a fraction $1 - \alpha$ are $L$ firms. $H$ firms produce high-quality products while $L$ firms produce low-quality products. A firm’s quality is private information prior to IPO. In order to produce, a firm must have enough funds or capital. Let a firm’s total amount of required funds be $k_0 > 1$, and the firm’s internal funds be $(k_0 - 1)$. Thus, a firm must raise one unit of external fund. This can be done through issuing shares in IPO or seeking alternative financing such as loans and venture capital. Of course, a firm can combine the two financing methods, as described later. Let $k_i$ be a type $i$ firm’s total cost of capital (including the firm’s internal funds), which will be determined in the equilibrium. Because external capital is at least as costly as internal funds, $k_i \geq k_0 > 1$ for $i = H, L$.

Once the external funds are obtained, firms produce and compete in the product market. A firm’s quality is modeled through consumers’ preferences, where a product of publicly known high quality yields a higher marginal utility than a low-quality product. There is perfect competition in low-quality goods, and so net profit of an $L$ firm is zero. By contrast, if an $H$ firm’s quality is publicly known, then the firm enjoys a higher demand for its product and higher earnings. However, consumers do not know each firm’s quality unless the firm signals its quality successfully in the financing stage. Thus, a firm’s expected earnings depend on the outcomes in IPO.

In Appendix A, we explicitly model the competition in the product market and derive firms’ earnings. To summarize the results, suppose that the IPO market separates the two types of firms, an outcome which we will focus on in this paper. Let $r_i$ be a type $i$ firm’s earnings that can be distributed to its lenders and shareholders, where $i = H, L$. Once firms’ qualities become
public information, competition in the product market implies:

$$r_L = k_L,$$

(2.2)

$$r_H - r_L = \frac{1}{\alpha}(Y - k_L).$$

(2.3)

The important feature of the above results is that an $H$ firm’s earnings increase with the aggregate demand for the industry’s products, $Y$, while an $L$ firm’s earnings are independent of $Y$. This is intuitive. When the aggregate expenditure on the industry’s products rises, consumers will increase the share of expenditure on high-quality products and reduce the share on low-quality products. All $H$ firms benefit from this increased demand, and so each $H$ firm’s earnings increase. For an $L$ firm, however, competition drives the earnings down to the cost of capital.

This feature is important for IPO because it implies that an $H$ firm’s expected earnings are more sensitive to the industry-wide underpricing than an $L$ firm’s earnings. To see this, continue to suppose that the IPO market successfully separates the two types of firms. Anticipating the result that only $H$ firms will possibly underprice their IPOs in such separating equilibria, we have $\bar{D} = \alpha D$, where $D$ denotes the average amount of underpricing per share by $H$ firms. Let $R_i$ denote expected earnings of a type $i$ firm conditional on all firms’ IPO activities, i.e., $R_i = E(r_i|D)$ for $i = H, L$. Then, (2.1), (2.2) and (2.3) imply

$$R_L = k_L, \quad R_H = xR_L + \rho D,$$

(2.4)

where $x \equiv 1 + \frac{1}{\alpha}(Y_0/k_L - 1)$. Indeed, an $H$ firm’s expected earnings increase in the industry-wide underpricing while an $L$ firm’s expected earnings are independent of such aggregate underpricing. To ensure $R_H > R_L$ for all $D \geq 0$, we assume $x > 1$ (i.e., $Y_0 > k_L$) throughout this paper.

It is worth emphasizing that the earnings differential between the two types of firms is endogenous in the equilibrium since it depends on the industry-wide underpricing. By contrast, most previous models of IPO underpricing assume that the earnings differential is exogenous. To emphasize the endogenous component in expected earnings, we separate it from the exogenous component and call the latter the intrinsic earnings of a firm. Thus, the intrinsic earnings are $R_L$
for an $L$ firm and $xR_L$ for an $H$ firm. Denote $x_H = x > 1$ and $x_L = 1$. We simply refer to $x_i$ as the quality of a type $i$ firm, where $i = H, L$.

### 2.2. Financing Methods and the Timing

A firm can obtain external funds by combining IPO and alternative financing. In the IPO market, a firm chooses the offer price $s$ and the number of shares $f$ to be offered.\(^9\) Normalize the total number of a firm’s shares to 1, so that $f \in (0, 1]$. The firm’s original owners keep $1 - f$ shares after IPO. The market price of shares is $p$. The gain to IPO investors is $d \equiv p - s$ per share. The firm is said to underprice IPO if $d > 0$. The IPO revenue is $q \equiv sf$. If $q < 1$, then the firm either underpriced or under-issued shares in IPO. In either case, the firm raises the remaining required capital through alternative methods.

The cost of alternative financing is important for a firm’s IPO decision because it is the opportunity cost of raising capital through IPO. As explained later, an $H$ firm’s ability to signal quality depends critically on its lower unit cost of alternative funds. To be specific, let us suppose that firms obtain alternative funds through the following realistic mechanism. The financiers of alternative funds do not know a firm’s quality and only have an imperfect technology to screen the firm. The technology gives a noisy signal that is positively correlated with the firm’s quality. That is, the signal is not always a correct indication of the firm’s true quality, but it is more likely to be correct than wrong. When the signal indicates high quality, the financiers provide loans to the firm at a low rate. Otherwise, the financiers charge a high loan rate.

The above mechanism has two important features. First, because the screening signal is positively correlated with the firm’s true quality, an $H$ firm will more likely get a low loan rate than an $L$ firm. That is, the expected cost of alternative funds will be lower for an $H$ firm than for an $L$ firm. To capture this feature in a simple way, we assume that the expected unit cost of alternative funds for a type $i$ firm is $(1 + bx_i^{-1})$, where $b > 0$ is a constant and $x_i$ is the true

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\(^9\)By allowing a firm to choose $f$ as well as $s$, we achieve some realism and conform with the literature. More importantly, we can distinguish a separating equilibrium with underpricing from a separating equilibrium with under-issuing. Since $f$ responds to the industry’s publicity quite differently in these two equilibria (see Figure 2 later), fixing $f$ may artificially constrain a firm’s incentive to underprice.
quality of the firm. Thus, if a type $i$ firm raises only a revenue $q < 1$ in IPO, the total cost of alternative funds is $(1 + bx_i^{-1})(1 - q).^{10}$

Second, because the screening technology is imperfect, the financiers may not know a firm’s true quality even after screening the firm. In particular, an $H$ firm may be wrongly labeled as an $L$ firm. Thus, seeking alternative financing does not resolve the problem of asymmetric information regarding the firms’ quality.\textsuperscript{11}

The second feature implies that the order in which a firm uses the two financing methods is not critical for the analysis. Even if a firm seeks alternative financing before IPO, its loan rate does not indicate the firm’s quality perfectly. In the case where an $H$ firm is wrongly labeled as an $L$ firm, the firm may still have incentive to signal its quality in IPO. Nevertheless, it is convenient to assume that firms go to IPO first before seeking alternative funds.

With this clarification, we specify the timing of events explicitly as follows. First, all firms issue IPOs at the same time (see Section 6 for sequential decisions). Second, if a firm’s IPO revenue falls short of the required amount (1 unit), the firm seeks alternative financing. Third, firms produce. They repay the alternative funds first and then the shareholders.

\textbf{2.3. Payoffs to IPO Firms and the Investors}

Consider an individual IPO firm. For convenience, we describe the firm’s choices as the IPO revenue $q$ (rather than the issuing price $s$) and the number of IPO shares $f$. Denote $a = (f, q)$. The payoff to the firm is defined as the total expected return to the firm’s original owners. To calculate this payoff, let $I \in [0, 1]$ be the posterior probability with which investors believe that the firm is of high quality, after observing all firms’ financing activities. Given this belief, the firm’s expected earnings are:

$$R_I = R_H I + R_L (1 - I).$$

\textsuperscript{10}All analytical results in this paper continue to hold for a more general cost function $(1 + b/x)C(1 - q)$ that satisfies $C(0) = 0$, $C'(0) \geq 1$ and $C'' > 0$.\textsuperscript{11}Empirical evidence seems to support this feature. For example, James and Wier (1990) and Slovin and Young (1990) find that firms with previously established borrowing relationships can still experience IPO underpricing, although they may underprice by less than other IPOs.
From such earnings, the firm repays the alternative financiers and then distributes the rest to the shareholders. Since the original owners hold onto \((1 - f)\) shares after IPO, the firm’s payoff is:

\[
V(f, q; R_I, x_i) \equiv (1 - f) \left[ R_I - \left( 1 + bx_i^{-1} \right) (1 - q) \right].
\] (2.6)

The firm chooses \(a\) to maximize \(V\), taking \(D\) (and hence \(R_H\)) as given.

Notice that we use the market’s expected earnings of the firm to calculate the firm’s payoff. This is because consumers have the same beliefs about the firm’s quality as investors. If the firm does not signal its quality successfully in IPO, the firm cannot be sure that its high quality will attract a high demand for its product. The purpose of signaling in IPO is to affect the expected earnings, \(R_I\), by affecting the market’s belief of the firm’s type.

Investors care about the expected rate of return per share. To simplify the analysis, we assume that investors are risk neutral and that the risk-free (gross) rate of return is one. To eliminate profitable arbitrage in the post-IPO market, the market price of the shares after IPO must be equal to the expected return to the shares. Otherwise, the rate of return to the shares would exceed the risk-free rate. Let \(p_I\) be the market price of the shares conditional on \(I\). Then,

\[
p_I = R_I - \left( 1 + bx_I^{-1} \right) (1 - q),
\] (2.7)

where \(x_I^{-1} \equiv x^{-1} I + (1 - I)\). The rate of return to buying an IPO share is \(p_I/s\), where \(s = q/f\).

For investors to participate in IPO, this rate must be at least equal to the risk-free rate (one). (It can be greater than one since the firm controls the offer price and rations the number of shares in IPO.) Thus, \(0 \leq s \leq p_I\).

We say that an IPO action \(a = (f, q)\) is feasible under the belief \(I\) if \(f, q \in [0, 1]\) and \(0 \leq s \leq p_I\). For convenience, let us use (2.7) to rewrite the requirements \(p_I \geq 0\) and \(p_I \geq s\) as

\[
q \geq M_I \equiv 1 - \frac{R_I}{1 + bx_I^{-1}} \quad \text{and} \quad f \geq S_I(q),
\] (2.8)

where the function \(S_I(.)\) is defined as follows:

\[
S_I(q) = q \left/ \left[ R_I - \left( 1 + bx_I^{-1} \right) (1 - q) \right] \right..
\] (2.9)

For all \(q \geq M_I\), IPO is sold at the full price if \(f = S_I(q)\) and underpriced if \(f > S_I(q)\).
3. Public Information versus Private Information

In this section, we first analyze the case where the quality of firms is publicly known and show that underpricing does not occur in this case. This result establishes the earlier claim that the assumption (2.1) itself does not generate the clustering of underpriced IPOs. Then, we discuss firms’ incentive to signal in the presence of private information and lay out some assumptions.

Consider a type $i$ firm, where $i$ is either $H$ or $L$, and suppose that the type is publicly known. Then, the firm’s expected earnings are equal to $R_i$, regardless of what the firm does in the IPO market. In this case, underpricing only wastes resources without any gain. So, it is optimal for the firm to issue its IPO at the full price, i.e., at $s = p_i$. Moreover, it is optimal to raise all external funds through IPO, i.e., to set $q = 1$. If the firm uses alternative financing to raise a positive amount of funds, $\delta$, the benefit is $\delta$ and the cost is $(1 + b/x_i)\delta$, and so the firm’s payoff is reduced.\footnote{Formally, under public information we have $p_i = R_i - (1 + b/x_i)(1 - q)$ from (2.7) and $f = q/p_i$ from (2.8). Then, the firm’s payoff is $V = R_i - (1 + b/x_i)(1 - q) - q$, which is maximized at $q = 1$.} Thus, we have established the following proposition:

**Proposition 3.1.** If quality is public information, no firm underprices IPO in equilibrium, despite the influence of the industry’s publicity on the aggregate product demand. Moreover, every firm raises all external capital through IPO. Thus, a type $i$ firm’s optimal choice is $(f_i, q_i) = (1/R_i, 1)$ and the payoff is $(R_i - 1)$, where $i = H, L$.

Industry-wide underpricing cannot be sustained without asymmetric information. This result arises from the free-rider problem. Although the industry’s publicity benefits all $H$ firms, it is a public good. Every firm likes to enjoy this public good, but no firm wants to contribute to the public good by underpricing its own IPO. Instead, every firm waits for other firms to underprice. As a result, underpricing does not occur and the industry’s publicity is low. This outcome can be a collective loss to the $H$ firms. When the industry’s publicity has a strong influence on the aggregate demand, all $H$ firms would gain if all of them could somehow be coerced to underprice their IPOs.\footnote{To verify this, suppose that all $H$ firms underprice by $\delta > 0$ and choose $q = 1$. Then, $D = \delta$, $1/f = p_H - \delta$ and $p_H = R_H = xR_L + \rho\delta$. An $H$ firm’s payoff is $p_H \left[1 - (p_H - \delta)^{-1}\right]$, which increases in $\delta$ when $\rho$ is close to 1.} Of course, such coercion is not a market outcome.
Let us now return to the case of private information. When the quality is not publicly known, an $H$ firm may have incentive to signal its quality even if other $H$ firms do not signal. The main benefit to a firm from signaling is that if it can induce the market to treat it as a high-quality firm, there will be higher demand for its product and hence higher expected earnings, $R_H$. By contrast, if the firm does not signal and if other $H$ firms have not signaled, the market’s belief on the firm’s type will be $I = \alpha$ and the particular firm’s expected earnings will be reduced to $[\alpha R_H + (1 - \alpha) R_L]$. If all other $H$ firms have signaled, the market’s belief of the particular firm’s type will be $I = 0$ and its expected earnings will be reduced further to $R_L$. The benefit from signaling may be stronger enough to overcome the free-rider problem.\textsuperscript{14}

The benefit from signaling increases with the industry’s publicity, because the earnings differential increases in $D$. Thus, if all other $H$ firms underprice IPOs by more, an $H$ firm is also likely to underprice its own IPO by more. This dependence of individual firms’ incentive to underprice on the industry-wide underpricing will be key to the clustering of IPO underpricing. Ironically, it is the same dependence of the earnings differential on $D$ that induces free-riding.

An $H$ firm can signal by either underpricing or under-issuing IPO. Both actions reduce the IPO revenue. By refraining from raising as much IPO revenue as it desires, the firm forces itself to go through costly alternative financing. If investors are convinced that only $H$ firms are willing to take such costly actions, then the firm succeeds in signaling its quality. The more the firm refrains from the IPO revenue, the larger the total expected cost of alternative funds will be, and the more credible the signal will be.

Underpricing and under-issuing differ in how they change the claim of the original owners of the firm on the firm’s future earnings. By under-issuing, the firm increases the fraction of shares, $(1 - f)$, that the original owners retain after IPO. If the firm underprices, however, the firm must increase the number of shares in IPO in order to raise the same amount of IPO revenue. This reduces the original owners’ claim on the firm’s future earnings. Thus, underpricing is more

\textsuperscript{14}Signaling also reduces the unit cost of alternative funds that IPO investors expect the firm to incur, and hence increases the market price of the firm’s IPO. This can be an additional benefit to the firm, because the higher market price allows the firm to keep a larger fraction of the shares to itself.
costly than under-issuing to the original owners. As such, underpricing is also more credible in
signaling the firm’s quality.

For signaling to be successful, a necessary condition is that an $H$ firm has stronger incentive
to signal than an $L$ firm. This is known as the single-crossing property (see Fudenberg and Tirole
1993, p.259). The firm’s payoff function has this property, as the following features show:

$$\frac{\partial}{\partial x} \left( - \frac{\partial V}{\partial R} \right) < 0; \quad \frac{\partial}{\partial x} \left( - \frac{\partial V}{\partial f} \right) < 0. \quad (3.1)$$

The first feature states that, for a fixed number of shares issued in IPO, an $H$ firm is willing to
reduce the IPO revenue by more than does an $L$ firm in order to receive the market’s “reward”
of an increase in the expectations on earnings. The second feature states that, for a fixed IPO
revenue, an $H$ firm increases the number of shares issued in IPO by less than does an $L$ firm in
the event of an increase in expected earnings. Both features arise from the fact that the expected
unit cost of alternative funds is lower for an $H$ firm than for an $L$ firm. The cost differential
enables an $H$ firm to refrain more from the IPO revenue than an $L$ firm can.

To emphasize the interplay between the industry-wide underpricing and individual firms’
incentive to underprice, we impose the following assumption:

**Assumption 1.** (1A): The difference in intrinsic earnings between the two types of firms is not
too large, i.e., $R_L(x - 1) < b$. (1B): When the quality is publicly known and the external capital
comes entirely from alternative funds, an $H$ firm can make a positive return but an $L$ firm cannot;
i.e., $xR_L > 1 + b/x$ and $R_L < 1 + b$.

Assumption 1A is opposite to the ones made in signaling models of a single firm’s IPO (see
the references in the introduction). For those models to generate underpricing, the intrinsic
difference between the two types of firms must be large. This assumption seems realistic for the
firms in a new industry, because the intrinsic difference between those firms does not seem large
at the beginning. Moreover, by deliberately restricting the intrinsic difference to be small, we
sharpen the focus on how the industry-wide publicity can induce individual firms to underprice.
Nevertheless, a small difference in intrinsic earnings does not always imply a small difference in expected earnings, since the later depends on the industry’s publicity which is endogenous.

In order to signal, $H$ firms must have some (albeit small) advantage over $L$ firms in intrinsic earnings. Assumption 1B specifies this advantage in the case of public information. This is a weak assumption: Because the quality is private information, this assumption alone implies neither that signaling will necessarily occur nor that signaling entails underpricing.

4. Signaling Equilibrium and Separation

In this section we characterize an individual firm’s strategy under private information, taking the industry’s publicity $D$ (and hence $R_H$) as given. This strategy is the firm’s best response to the industry’s publicity. We refer to this best response of a single firm, together with the market belief, as a signaling equilibrium. Of course, the industry’s publicity must also be determined in a market equilibrium, which we will do in the next section.

For any given $R_H$, a Bayesian signaling equilibrium consists of market beliefs $I$ and the firm’s decisions $(f, q)$ that satisfy the following conditions: (i) Given the beliefs, the firm’s decisions maximize the payoff; (ii) With the firm’s choices, the beliefs are rational according to Bayes updating. We will focus on those equilibria in which $H$ firms successfully signal their quality and hence separate themselves from $L$ firms.$^{15}$

To examine when and how an $H$ firm can signal successfully, let us first describe a reference point – the case where IPO actions do not separate the two types of firms. This “pooling” outcome happens when the two types of firms take the same action in IPO. Consider any such pooling action $a_0 \equiv (f_0, q_0)$. After observing this action, the market does not gain any new information about a firm’s quality. Thus, the market’s belief of every firm is $I = \alpha$. The market price of a share after IPO is $p_\alpha$, given by (2.7) with $I = \alpha$. A type $i$ firm’s payoff from pooling is $V^0(x_i) \equiv V(f_0, q_0; R_\alpha, x_i)$, where the function $V$ is defined in (2.6).

Now we analyze possible actions that an $H$ firm can take to signal its high quality. Consider

$^{15}$More precisely, we use the intuitive criterion of Cho and Kreps (1987) to restrict the beliefs off the equilibrium path. Thus, the term “equilibrium” in this paper stands for an equilibrium that satisfies this criterion.
such an action \( a = (f, q) \), where \( a \neq a_0 \). For this action to signal quality successfully, it should satisfy the following intuitive requirements (see Cho and Kreps, 1987). First, the action must be feasible for an \( H \) firm, i.e., \( f, q \in [0, 1], q \geq M_H \) and \( f \geq S_H(q) \) (see the end of Section 2.3). Second, the action must be unattractive to an \( L \) firm; Otherwise, \( L \) firms would mimic the action. Since the payoff to an \( L \) firm from mimicking the action \( a \) is \( V(f, q; R_H, 1) \), the action is unattractive to an \( L \) firm if \( V(f, q; R_H, 1) \leq V^0(1) \). Third, once an \( H \) firm’s action induces the market to believe its high quality, the action should generate a higher payoff to the firm than with pooling. That is, \( V(f, q; R_H, x) > V^0(x) \). Any action that satisfies these requirements is a credible signal for high quality. Because an \( L \) firm never gains from such an action but an \( H \) firm does gain, investors should naturally interpret any firm taking the action as an \( H \) firm.

To depict the set of credible signals in a diagram, let us rewrite the second requirement as:

\[
f \geq \text{IND}_L(q) \equiv 1 - V^0(1) / \left[ R_H - (1 + b)(1 - q) \right]. 
\] (4.1)

Define a critical level, \( Q_1 \), as follows:

\[
Q_1 \equiv 1 - \frac{R_H - V^0(1)}{1 + b}. 
\] (4.2)

There are two cases to consider: \( Q_1 \leq 0 \) and \( Q_1 > 0 \), which are depicted in Figures 1a and 1b.

Figures 1a and 1b here.

The curve \( f = S_H(q) \) in these figures is the full-price curve for an \( H \) firm when separation occurs. Underpricing occurs if an \( H \) firm chooses an action above this curve. For a reference, we also depict the full-price curve under pooling, \( f = S_\alpha(q) \). The curve \( f = \text{IND}_L(q) \) is the set of actions which generate the same payoff to a mimicking \( L \) firm as the pooling action \((f_0, q_0)\) does. Actions above this curve generate strictly lower payoffs to an \( L \) firm even if the firm is viewed as an \( H \) firm as a result of mimicking. Thus, the shaded area in each diagram is the set of actions that are feasible to an \( H \) firm and that will never be taken by an \( L \) firm. Credible signals are actions in this shaded area that give an \( H \) firm a higher payoff than under pooling.

Among credible signals, an \( H \) firm chooses the one that maximizes its payoff. To find the best choice, notice the following features (stated formally in Lemma B.2 in Appendix B). First,
for any action in the interior of the shaded area in Figures 1a and 1b, there is an action on
the lower boundary of the area that raises the same IPO revenue but with a fewer number of
shares. Thus, an $H$ firm prefers the actions on the lower boundary of the shaded area to the
actions in the interior of the area. Second, an $H$ firm’s payoff is an increasing function of $q$
along the full-price curve $f = S_H(q)$. The reason is that, as the firm raises a higher IPO revenue
without underpricing, the firm economizes on the cost of alternative funds and so expected profit
increases. Third, an $H$ firm’s payoff is a decreasing function of $q$ along the curve $f = IND_L(q)$.
To explain this, recall that an $H$ firm’s desire to increase the number of shares issued in IPO is
weaker than an $L$ firm’s (see (3.1)). As actions move upward along the curve $f = IND_L(q)$, the
IPO revenue increases and such actions are increasingly more enticing to an $L$ firm. To keep an
$L$ firm indifferent between these actions and the pooling action, the number of shares issued in
IPO must increase more sharply than it is desirable to an $H$ firm.

These features imply that the optimal signal an $H$ firm can give is to choose point $A$ depicted
in Figures 1a and 1b. Denote the choices at point $A$ as $a_b = (f_b, q_b)$. Let $Q_A$ be the IPO revenue
at the intersection between the curve $f = S_H(q)$ and the curve $f = IND_L(q)$. Then,

\[
\begin{align*}
  f_b &= 1 - \frac{V^0(1)}{R_H - 1 - b}, \quad q_b = 0, \quad \text{if } Q_1 \leq 0 \\
  f_b &= S_H(Q_A), \quad q_b = Q_A, \quad \text{if } Q_1 > 0.
\end{align*}
\]

(4.3)

When the action $a_b$ yields a higher payoff than the one under pooling, an $H$ firm will choose
the action to separate itself from $L$ firms. Otherwise, an $H$ firm will choose not to signal its
quality. When separation does occur, investors can perfectly infer every firm’s quality by their
IPO actions. In this case, the best action for an $L$ firm is $(f, q) = (1/R_L, 1)$ and the payoff is
$V^0(1) = R_L - 1$ (see Proposition 3.1). With this value of $V^0(1)$, the condition $Q_1 \leq 0$ becomes
$R_H \geq R_L + b$. Denote the values of $a_b$ under this value of $V^0(1)$ as $a^* = (f^*, q^*)$. Then, for
$R_H \geq R_L + b$, Figure 1a applies and

\[
  f^* = 1 - \frac{R_L - 1}{R_H - (1 + b)} \quad \text{and} \quad q^* = 0.
\]

(4.4)

For $R_H < R_L + b$, Figure 1b applies, in which case the action $(f^*, q^*)$ solves:

\[
  f^* = S_H(q^*) \quad \text{and} \quad \frac{q^*}{R_H - (1 + b)(1 - q^*)} = 1 - \frac{R_L - 1}{R_H - (1 + b)(1 - q^*)}
\]

(4.5)
We have the following proposition (see Appendix B for a proof):

**Proposition 4.1.** There exists $\alpha \in (0,1]$ such that, if $\alpha < \alpha$, there exists a unique signaling equilibrium. This equilibrium is a separating equilibrium. When $R_H - R_L \geq b$, an $H$ firm’s action is described by (4.4) which entails underpricing. When $R_H - R_L < b$, an $H$ firm’s action is described by (4.5), which entails under-issuing but not underpricing. In both cases, an $L$ firm’s action is $f = 1/R_L$ and $q = 1$. If $\alpha \geq \alpha$ and $R_H - R_L < b$, the separating equilibrium is still the unique equilibrium. However, if $\alpha > \alpha$ and $R_H - R_L \geq b$, a pooling equilibrium can exist.

An $H$ firm can signal its quality successfully when either the differential in expected earnings or the fraction of $H$ firms in the market is small. However, signaling can fail when both the differential in expected earnings and the fraction of $H$ firms are large. A large differential in expected earnings makes signaling difficult by increasing the temptation for $L$ firms to mimic. A large fraction of $H$ firms in the market makes signaling difficult by reducing the difference in the expected unit cost of alternative funds between an $H$ firm and a pooling firm. As discussed earlier, this difference is the key to an $H$ firm’s ability to signal. When both difficulties are present, successful signals are so costly that an $H$ firm prefers pooling.

From now on, we assume $\alpha < \alpha$ so that the separating equilibrium is the only equilibrium. Thus, our focus will not be on the difference between a separating equilibrium and a pooling equilibrium. Rather, the focus will be on the difference between the separating equilibrium with underpricing and the separating equilibrium with under-issuing.

Whether an $H$ firm underprices or under-issues IPO depends on the differential in expected earnings between the two types of firms, $(R_H - R_L)$. As explained in section 3, underpricing is more costly to the firm’s original owners than under-issuing, but more effective in signaling quality. When $R_H - R_L < b$, the benefit to an $L$ firm from mimicking is small. In this case, under-issuing is sufficient for deterring mimicking. When $R_H - R_L \geq b$, however, an $L$ firm has strong temptation to mimic, which can no longer be deterred by under-issuing.\[^{16}\]

---

\[^{16}\]To verify this, suppose that an $H$ firm sets $f = \varepsilon$ and $q = 0$, where $\varepsilon > 0$ is arbitrarily close to 0. This is the most credible signal given by under-issuing alone. The amount of expected earnings signaled by this action is the
$H$ firm underprices in order to signal quality.

Because an individual firm’s underpricing depends on the differential in expected earnings, it depends on the industry’s publicity. To express this result explicitly, denote the amount of underpricing by an individual $H$ firm as $d(D)$. Then, $d(D) = p_H - q^*/f^*$. Denote $D_0 \equiv [b - R_L(x - 1)]/\rho$. Note that $D_0 > 0$ (by Assumption 1A). Since $R_H - R_L \geq b$ if and only if $D \geq D_0$, we can write the two cases of the separating equilibrium in Proposition 4.1 as follows:

$$d(D) = \begin{cases} 0, & \text{if } D < D_0 \\ p_H = \rho D + xR_L - 1 - bx^{-1}, & \text{if } D \geq D_0. \end{cases}$$

(4.6)

The higher the industry’s publicity, the more likely individual $H$ firms will underprice and, when they underprice, the large the amount of underpricing will be. This interplay between individual firms’ incentive to underprice and the industry-wide underpricing is a powerful mechanism that can generate the clustering of large underpricing, as we will show in the next section.

The number of an $H$ firm’s shares in IPO also depends on $R_H$. This dependence has a V shape, as depicted in Figure 2. When an $H$ firm’s expected earnings are low, it under-issues shares to signal quality. As expected earnings increase, the number of IPO shares decreases toward a minimum, which is zero in this version of the model. Afterward the firm switches into underpricing. As expected earnings increase further, the magnitude of underpricing increases, which requires the number of IPO shares to increase (see Section 6.1 for more discussions).

Figure 2 here.

5. Market Equilibrium and Clustering

Although the signaling equilibrium is unique for each firm under any given amount of the industry’s publicity, the industry’s publicity is endogenous. In this section we show that multiple levels of the industry’s publicity can be self-fulfilling.

solution for $R_H$ from the equality form of (4.1) under $(f,q) = (\epsilon,0)$ and $V^0(1) = R_L - 1$. This solution is lower than (and arbitrarily close to) $R_L + b$. Thus, under-issuing cannot succeed in signaling if $R_H \geq R_L + b$. 

19
5.1. Market Equilibrium and Self-Fulfilling Expectations

A **market equilibrium** is defined as a pair \((d(D), D)\) such that \(d(D)\) is the best response of each \(H\) firm to \(D\), given by (4.6), and that \(d(D) = D\). The condition \(d = D\) is required because all \(H\) firms are symmetric. We say that a market equilibrium is an underpricing equilibrium if \(D > 0\) and a no-underpricing equilibrium if \(D = 0\). In both equilibria, \(H\) firms separate from \(L\) firms successfully, as shown in the previous section.

Substituting \(d(D)\) from (4.6) and invoking the requirement \(d(D) = D\), we can solve for \(D\). Then, the following proposition can be proven (the proof is straightforward and omitted).

**Proposition 5.1.** Define \(\rho \in (0, 1)\) as follows:

\[
\rho = \frac{b - R_L(x - 1)}{b(1 - x^{-1}) + R_L - 1}.
\] (5.1)

When \(0 \leq \rho < \rho_\), only the no-underpricing equilibrium exists. When \(\rho \leq \rho < 1\), both the underpricing equilibrium and the no-underpricing equilibrium exist. In the underpricing equilibrium, the amount of underpricing increases with \(\rho\).

Figure 3 depicts the case \(\rho < \rho < 1\), where the two market equilibria co-exist. In the diagram, the underpricing “curve” depicts the best response (4.6). Point \(EN\) is no-underpricing equilibrium, in which \(H\) firms separate from \(L\) firms by under-issuing. Point \(EU\) is the underpricing equilibrium, in which \(H\) firms separate by underpricing.

Figure 3 here.

The above proposition contains the following interesting features. First, no underpricing occurs in the equilibrium when the industry’s publicity has only a weak effect on the industry’s expected product demand, i.e., when \(0 \leq \rho < \rho_\). This result can be seen in Figure 3. When \(\rho\) is small, the level \(D_0\) is large and the underpricing curve lies below the 45-degree line for all \(D > 0\). It is easy to explain this result. When \(\rho\) is small, the differential in expected earnings between the two types of firms is small, and hence it does not pay for an \(H\) firm to underprice. At the
same time, the temptation for \( L \) firms to mimic is weak. For an \( H \) firm to signal its quality successfully, it is sufficient to under-issue rather than to underprice.\(^{17}\)

Second, underpricing can occur when the industry’s publicity has a strong effect on the aggregate demand for the industry’s goods, i.e., when \( \rho \geq \rho_* \). This result is intuitive. When firms believe that the industry’s publicity will be high, the expected aggregate demand for the industry’s products will be high. Because \( H \) firms will share this higher demand, expected earnings for each \( H \) firm will be high and the differential in expected earnings between the two types of firms will be large. To obtain this large benefit, an \( H \) firm must signal its quality to the market. This cannot be achieved by only under-issuing IPO, because the large differential in expected earnings creates strong temptation for \( L \) firms to mimic. To deter this mimicking behavior, an \( H \) firm must resort to the more costly action – underpricing. As every \( H \) firm underprices, underpricing clusters in the industry. This generates a large amount of aggregate underpricing which supports the belief that the industry’s publicity will be high.

Third, even when \( \rho \geq \rho_* \), underpricing is not an inevitable outcome. On the contrary, no-underpricing is also a market equilibrium in this case. All that is needed to generate no-underpricing is that firms believe that the industry’s publicity will be low. Under this belief, the differential in expected earnings between the two types of firms will be small. This small benefit does not justify the costly action of underpricing. At the same time, \( L \) firms’ temptation to mimic will be weak, and so under-issuing will be sufficient for signaling quality. The absence of underpricing supports the belief that the industry’s publicity will be low.

The above exposition shows that multiple, self-fulfilling equilibria arise from the interplay between individual firms’ incentive to signal and the industry-wide underpricing. Since each firm is small in the industry, it has very little effect on the industry’s publicity. However, the industry’s publicity affects an individual firm’s expected earnings and underpricing decisions. If all other \( H \) firms choose not to underprice, an individual \( H \) firm also finds it optimal not to underprice,

\(^{17}\)Assumption 1A is important for this result. When the intrinsic earning difference between a high-quality and a low-quality firm is large enough to violate Assumption 1A, then \( \rho < 0 \) and there is a need for an \( H \) firm to underprice anyway. In fact, only the underpricing equilibrium exists in this case.
even when all $H$ firms collectively prefer underpricing to no-underpricing. This interplay cannot
be appreciated in a model with only one firm, which has been the focus of most models in the
literature on IPO underpricing.\footnote{This literature has examined multiple equilibria in a single firm's signaling game. We have deliberately eliminated this type of multiplicity by imposing Assumption 1A and the restriction $\alpha < \alpha_l$. In our model, the signaling equilibrium is unique for any given $D \geq 0$.} When a firm is the only one in the industry, it will either
underprice or not underprice, whichever gives the higher payoff. The two outcomes cannot both
be an equilibrium for given parameters, in contrast to the multiple equilibria in our model.

Finally, the interplay between individual firms' underpricing and the industry’s publicity can
produce large underpricing. In the underpricing equilibrium, $H$ firms offer their shares free of
charge! When expected industry’s publicity passes over the critical level $D_0$, the offer price of
and $H$ firm’s IPO drops to 0 and the percentage of discount to IPO investors jumps from 0 to
100%.\footnote{Of course, zero offer price is unrealistic. In section 6.1 we will extend the model to generate a positive offer price in the underpricing equilibrium.} This large underpricing is a reminiscent of the phenomenal price gains observed in some
Internet IPOs in 1999. Considering that the intrinsic difference between high-quality and $L$ firms
is small (Assumption 1A), the large magnitude of underpricing is remarkable.

Another way to understand the large underpricing is to notice that the market price of an
individual $H$ firm’s IPO varies with the industry’s publicity, even when the firm’s type becomes
publicly known. Despite the small intrinsic difference between the two types of firms, the industry’s publicity can magnify it into a large difference in the market price of IPO, and hence into
large underpricing. By contrast, previous signaling models deal with only one firm and assume
that the market price of shares is exogenous once the firm’s type is known. Those models cannot
generate underpricing when the difference between firms’ intrinsic earnings is small.

5.2. Further Implications

Our model explains why underpriced IPOs cluster in particular industries. The clustering is most
likely to occur in a new industry that has three features – highly uncertain aggregate product
demand, a strong externality in the product demand and very asymmetric information regarding
firms’ quality. Uncertainty in the aggregate demand and demand externality make individual
high-quality firms’ expected earnings more sensitive to the industry’s publicity than low-quality firm’s expected earnings. Asymmetric information forces high-quality firms to signal quality in order to benefit from the industry’s publicity. Together, these features lead to the clustering of large underpricing. These features seem to describe well the Internet industry in 1999 and the Biotech industry in the early 1990s, where phenomenal IPO price gains clustered.

The model can also explain why the clustering of underpriced IPOs is fragile and short-lived. First, because multiple equilibria exist, even non-fundamental events can change investors' beliefs and switch the equilibrium from underpricing to no-underpricing. Second, as investors acquire information about the industry, forecasts about earnings will become more reliable and less susceptible to the influence of the industry’s publicity. Third, as the industry matures, competition among firms in the same industry becomes more important than against firms in traditional sectors. In this case one firm’s underpricing may hurt rather than benefit other firms in the same industry, and so IPOs with large underpricing are less likely to cluster.\(^\text{20}\)

The clustering may vary with business cycles and credit market conditions. This is because underpricing depends on the cost of alternative funds, which is likely to change with economic conditions. In the upturn of business cycles or an easy credit market, the unit cost of alternative funds is low. In this case, \(L\) firms are more tempting to mimic \(H\) firms, because it can easily find alternative funds to make up for the shortfall in the IPO revenue. To separate successfully, an \(H\) firm is more likely to underprice IPO and do so in a large magnitude. Thus, underpricing is more likely to cluster in economic upturns and easy credit markets than in economic downturns and tight credit markets. In this sense, the hot IPO market in 1999 and the cooling off in later 2000 can be viewed as events with some regularity, rather than random events.

The above argument can be made precise by noticing that the critical level for the existence of the underpricing equilibrium, \(\rho\), increases in the parameter \(b\). Since \(b\) is positively associated with the unit cost of alternative funds, an economic upturn or an easy credit market reduces \(b\) and

\(^{20}\) An example of this negative externality is the case \(\rho < 0\), which we did not allow. If \(\rho < 0\) is allowed, then the condition for underpricing in (4.6), \(D \geq D_0\), should be written as \(\rho D > b - R_L(x - 1)\). When \(\rho < 0\), an underpricing equilibrium does not exist under Assumption 1A.
increases the likelihood for underpricing. Moreover, for any given $\rho$, the amount of underpricing is higher when $b$ is lower.

6. Extensions and Robustness

In this section we illustrate the robustness of the qualitative results and improve the quantitative implications by incorporating the following elements: a positive lower bound on the IPO revenue, individual firms’ own publicity, and sequential IPO decisions. We omit the proofs here and refer interested readers to the working paper (Cao and Shi, 1999) for these proofs.

6.1. A Lower Bound on the IPO Revenue

A firm may face a limit on alternative funds it can obtain, and so it may have to raise at least a minimum revenue from IPO. Let this minimum be $Q_b s/p$, where $Q_b \in (0, 1)$ is a constant. This specification incorporates the idea that alternative financiers are more willing to supply funds to a firm whose IPO had a larger price gain. Substituting the market price of shares in a separating equilibrium, we can rewrite the constraint $q \geq Q_b s/p$ for an $H$ firm as:

$$f \geq \frac{Q_b}{R_H - (1 + bx - 1)(1 - q)} \equiv LB(q). \quad (6.1)$$

This constraint does not alter the most important features of the model, such as the co-existence of the two separating equilibria. However, the separating action may no longer be described by point $A$ in Figures 1a and 1b. For example, in the case $Q_1 > 0$ depicted in Figure 4, the best separating action is given by point $B$. Because point $B$ lies above the full-price curve $f = S_H(q)$, IPO is underpriced. Such an underpricing equilibrium exists if and only if the curve $f = LB(q)$ crosses the curve $f = IND_L(q)$ before crossing $f = S_H(q)$. Equivalently, this requires $IND_L(Q_b) > S_H(Q_b)$, which is satisfied when $Q_b$ is sufficiently close to 1.

This extension improves the quantitative predictions of the basic model in two ways. First, an underpricing firm’s offer price can be positive, as point $B$ in Figure 4 illustrates. Second,
the number of shares issued in IPO does not necessarily increase with expected earnings in the underpricing equilibrium. When \( R_H \) increases in Figure 4, for example, the curve \( f = \text{IND}_L(q) \) shifts up but the curve \( f = \text{LB}(q) \) shifts down. The two forces change \( f \) in opposite ways, and so the effect of \( R_H \) on \( f \) is ambiguous. However, when the externality is sufficiently strong, \( f \) is likely to increase with \( R_H \) and firms are likely to underprice greatly, as in the basic model.

6.2. A Firm’s Own Publicity

A firm may directly benefit from its underpricing, as well as from the industry’s publicity. To allow for this benefit, let us return to the basic model and modify

\[
R_H = xR_L + \rho(\gamma d + D),
\]

(6.2)

where \( d \) is the firm’s own underpricing and \( \gamma > 0 \) measures the impact of the firm’s own underpricing on its expected earnings. The basic model corresponds to the case \( \gamma = 0 \).

Now a firm cannot take \( R_H \) as given, because its own decision directly affects \( R_H \). Denote the part that the firm takes as given as \( W = xR_L + \rho D \). Using (6.2) to compute \( R_I \) and substituting into (2.7), we obtain the market price of the firm’s shares under the belief \( I \) as

\[
p_I = \frac{1}{1 - I\rho\gamma} \left[ IW + (1 - I)R_L - I\rho\gamma q/f - (1 + bx^{-1})(1 - q) \right].
\]

(6.3)

Restrict \( 0 \leq \rho < 1/\gamma \) to ensure \( p_I > 0 \). The constraint \( s \leq p_H \) can be written as

\[
f \geq q / \left[ W - (1 + bx^{-1})(1 - q) \right].
\]

(6.4)

**Proposition 6.1.** There exist \( \gamma_1 > 0 \) and \( \rho_1 \in (0, 1/(1 + \gamma)) \) such that an underpricing equilibrium exists if \( \rho \in (\rho_1, 1/(1 + \gamma)) \) and \( \gamma \leq \gamma_1 \). There exist \( \gamma_2 > 0 \) and \( \rho_2 \in (0, 1/\gamma) \) such that a no-underpricing market equilibrium exists if \( 0 \leq \rho < \rho_2 \) and \( \gamma \leq \gamma_2 \). Moreover, \( \rho_1 < \rho_2 \) and so the two market equilibria coexist when \( \rho \in (\rho_1, \rho_2) \) and \( \gamma \leq \min \{\gamma_1, \gamma_2\} \).

This proposition shows that the qualitative results are similar to those in the basic model. In particular, IPO underpricing can cluster when the industry’s publicity has a strong effect on the expected demand for the industry’s products. The clustering can occur even when \( \gamma > 1 \),
i.e., even when a firm benefits more from its own publicity than from the industry’s publicity. Nevertheless, for the two market equilibria to co-exist, $\gamma$ cannot be too large. If $\gamma$ is very large, the industry’s publicity plays a very small role, in which case every firm’s IPO decisions will be determined by its own intrinsic features.

### 6.3. Sequential Decisions

In the basic model, firms go to IPO at the same time. By this we do not mean literally that firms in reality have the same date for IPO, but rather that some firms’ IPO dates are so close to each other that one firm cannot change the IPO decision after observing other firms’ IPO actions. Although this interpretation is appealing, one may still want to know what happens if firms can modify their IPO decisions after observing other firms’ actions. We analyze this sequential game and show that underpricing still tends to cluster.

Consider only two firms, firm 1 and firm 2. As it is common in the literature, the firms go public in an exogenous order, with firm 1 going at date 1 and firm 2 at date 2 (e.g., Hoffmann-Burchardi, 2001, and Benveniste et al., 2002). To simplify matters, we assume that both firms have earnings only at date 2 and there is no time discounting. Let $d_i$ be the amount of underpricing by firm $i = 1, 2$. If firm $i$ is perceived as an $H$ firm, expected earnings are given by $R_H$ in (2.4) with $D$ being replaced by $d_{i'}$ ($i' \neq i$). This specification keeps the gist of the interaction between firms in (2.4) and simplifies the algebra in this two-firm setup.

Given $d_1$, firm 2’s pricing decision is analogous to that analyzed in the basic model. That is, if firm 2 is an $L$ firm, then $d_2 = 0$; if firm 2 is an $H$ firm, then

$$d_2 = \begin{cases} 
0, & \text{if } d_1 < D_0 \\
\rho d_1 + xR_L - 1 - bx^{-1}, & \text{if } d_1 \geq D_0,
\end{cases}$$

where $D_0 = [b - R_L (x - 1)] / \rho$. Note that firm 2 responds to firm 1’s underpricing positively.

Firm 1 anticipates this influence of its IPO pricing decision on firm 2’s. To make its own decision, firm 1 calculates the expected amount of underpricing by firm 2 as follows:

$$\alpha \chi(d_1 > D_0)(\rho d_1 + xR_L - 1 - bx^{-1}),$$
where $\chi(d_1 > D_0) = 1$ if $d_1 > D_0$ and 0 otherwise. Suppose that firm 1 chooses $d_1 < D_0$. Then $d_2 = 0$ and there is no publicity from which firm 1 can benefit. In this case firm 1’s best decision is $d_1 = 0$ and the payoff to both firms is identical to that in the no-underpricing equilibrium in the basic model. This can be a market equilibrium in the current case if and only if the payoff to firm 1 is not lower than that generated by the choice $d_1 \geq D_0$.

Now suppose firm 1 chooses $d_1 \geq D_0$. If the market believes that the firm is of high-quality with probability $I$, expected earnings of the firm are:

$$R_I = (1 - I)R_L + I \left[ xR_L + \rho \alpha(\rho d_1 + xR_L - 1 - bx^{-1}) \right].$$

Slightly change the earlier notation to denote $W = (1 + \rho \alpha)xR_L - \rho \alpha(1 + bx^{-1})$. The market price of such a firm under the belief $I$ is

$$p_I = \frac{1}{1 - I\alpha \rho^2} \left[ IW + (1 - I)R_L - I\alpha \rho^2 q/f - (1 + bx^{-1})(1 - q) \right].$$

This is similar in form to the market price in the last subsection, with $\alpha \rho^2$ replacing $\rho \gamma$, and so firm 1’s decision on the offer price can be analyzed analogously. To ensure a positive market price of shares, we restrict $0 \leq \rho < \alpha^{-1}/2$. The following proposition holds:

**Proposition 6.2.** There exist $\rho_3, \rho_4 \in (0, \alpha^{-1}/2)$ such that firm 1 underprices IPO if and only if it is of high-quality and $\rho \in (\rho_3, \rho_4)$. Firm 2 underprices IPO if and only if it is of high-quality and if firm 1 underprices IPO.

**Example 6.3.** Let $\alpha = 0.1$, $b = 0.2$, $x = 1.18$ and $R_L = 1.02$. Then, $\rho_3 = 1.75 < \rho_4 = 1.91$. Thus, the interval $(\rho_3, \rho_4)$ is non-empty.

As the example shows, both firms underprice IPOs in some cases. More importantly, when firm 2 is of high quality, it underprices IPO if and only if firm 1 also underprices. Thus, the interaction between firms through expectations is important for the clustering, just as in the case of simultaneous decisions. It is not surprising then that the underpricing equilibrium here also requires the externality to be strong (i.e., $\rho > \rho_3$). In contrast to the case of simultaneous
moves, however, too strong an externality (i.e., $\rho > \rho_4$) will destroy the underpricing equilibrium. This is because underpricing is costly and, when the externality is very strong, the amount of underpricing is too large to be desirable for firm 1 as the first mover in the game.

Multiplicity of equilibria disappears with sequential moves, but this disappearance is an artifact of the exogenously fixed order of moves by the two firms. Being a first mover is costly in the current setup, because it must underprice sufficiently in order to entice the other firm to underprice. If firms can choose when to go to the IPO market, they have incentive to go to the market at dates that are very close to each other in order to explore the great externality. Then, the multiplicity analyzed in the basic model will reappear.\footnote{Tambanis and Bernhardt (1999) explicitly model the possibility that firms can delay the timing of their equity issue. However, they do not analyze IPO underpricing.}

7. Conclusion

We construct a theoretical model to explain the clustering of underpriced IPOs. The model integrates aggregate uncertainty in an industry with asymmetric information regarding the quality of individual firms. Expected earnings of each firm increase with the publicity that the industry generates in IPO underpricing. By signaling quality in the IPO market, a high-quality firm can benefit more from this publicity of the industry than a low-quality firm can. We show that two self-fulfilling equilibria exist. In one equilibrium, underpricing clusters. In the other, there is no underpricing. The equilibrium with underpricing is supported by high expectations of the industry’s publicity. Such expectations increase both the benefit of signaling high quality and the temptation to mimic by low-quality firms, thus inducing individual high-quality firms to underprice IPO as a signal of quality. As individual firms underprice, there is a clustering of large post-IPO gains in share prices which fulfills the high expectations of the industry’s publicity. Likewise, the equilibrium without underpricing is supported by self-fulfilling expectations that the industry’s publicity will be low.

The interplay between individual firms and the industry’s performance is a powerful mechanism to generate IPO underpricing. Even when the intrinsic difference between the two types of
firms is small, high expectations of the industry’s publicity can magnify it into a large difference in expected earnings and induce large underpricing by high-quality firms.

Our emphasis on the clustering is a marked shift from the literature’s emphasis on a single firm’s underpricing. The analysis is able to explain three common features of hot-issue markets. First, the clustering of underpricing is industry specific. It occurs more often in industries that are uncertain in product demand, susceptible to the influence of publicity, and with severe private information regarding firms’ qualities. Second, the clustering is fragile and short-lived. Even adverse news about a single firm in the industry can greatly affect all IPO performances by switching expectations from underpricing to no-underpricing. An example is the Biotech industry at the beginning of the 1990s. The heat over biotech stocks cooled down considerably when the US Food and Drug Administration rejected several promising drugs such as Centocor Inc.’s Centoxin, a medicine meant to fight a deadly bacteria infection common in surgery patients. Finally, the clustering is more likely to occur in economic upturns than in downturns, because the opportunity cost of underpricing is lower in economic upturns.

These features suggest that a “hot-issue” market, such as the one in the dot.com industry in 1999, and the subsequent cooling-off could both be outcomes of rational expectations about the new industry’s performance. However, the clustering will become rare as the industry matures, because forecasts about earnings will become more reliable and less susceptible to the influence of the industry’s publicity. Finally, a tight monetary policy can reduce the exuberance in the IPO market by increasing the cost of loanable funds and the opportunity cost of underpricing.
Appendix

A. Competition in the Product Market

In this appendix, we explicitly model the competition in the product market and derive firms’ earnings functions. We model a publicly known high quality as a high demand for the product, which is derived from consumers’ preferences. If a firm does not signal its quality successfully through the IPO market, then consumers do not know the firm’s quality. However, to maintain the focus of the paper, we analyze only the case where the IPO market has already separated the two types of firms before the product market opens.

Firms produce different varieties of a product. Let $j \in [0, 1]$ be the index of varieties. The varieties in $[0, \alpha)$ are produced by $H$ firms and the varieties in $[\alpha, 1]$ by $L$ firms. A representative consumer has the following utility function on the varieties:

$$U = \theta \int_{H} \ln (c_{H}(j) + \psi) dj + \int_{L} \ln (c_{L}(j)) dj, \quad \theta > 1, \psi > 0.$$

Here the subscript of each integral indicates the subset of varieties over which the integral is computed, with $H$ indicating high-quality varieties and $L$ low-quality varieties.

We define a high-quality variety by two features: $\theta > 1$ and $\psi > 0$. The feature $\theta > 1$ requires a high-quality variety to be more desirable than a low-quality variety. The feature $\psi > 0$ implies that as the total expenditure on the varieties increases, a consumer will increase the share of expenditure on high-quality varieties. For convenience, we assume $\psi < Y_{0}$, where $Y_{0}$ is the expected aggregate demand when $D = 0$.

Let $\pi(j)$ be the price of variety $j$. The consumer’s maximization problem is

$$\max U \quad \text{s.t.} \quad \int_{H} \pi(j)c_{H}(j) dj + \int_{L} \pi(j)c_{L}(j) dj \leq Y.$$

Let $\lambda$ be the shadow cost of the constraint. We have:

$$c_{L}(j) = \frac{1}{\lambda \pi(j)}, \quad c_{H}(j) = \frac{\theta}{\lambda \pi(j)} - \psi.$$

Because all varieties of the same quality are symmetric, they must have the same price. That is, $\pi(j) = \pi_{L}$ for all $j \in [\alpha, 1]$ and $\pi(j) = \pi_{H}$ for all $j \in [0, \alpha)$. Moreover, we will explain later
that \( \pi_H = \pi_L = \pi \). Substituting this result and the above demand functions into the consumer’s budget constraint, we can solve for \( \lambda \) and rewrite the demands as follows:

\[
c_L(j) = \frac{\alpha \psi + Y/\pi}{\alpha \theta + 1 - \alpha}, \text{ all } j \in [\alpha, 1]
\]

\[
c_H(j) = \theta \frac{\alpha \psi + Y/\pi}{\alpha \theta + 1 - \alpha} - \psi, \text{ all } j \in [0, \alpha).
\]

The share of expenditure on \( H \) varieties is \( \alpha \pi c_H / Y \). Indeed, this share is increasing in \( Y \) (if and only if \( \psi > 0 \)). The share of expenditure on \( L \) goods is decreasing in \( Y \).

As in a typical quality-ladder model (see Grossman and Helpman, 1991), there is perfect competition in the production of low-quality varieties. Moreover, for each high-quality variety, a low-quality counterpart is also available for production. However, the high-quality firm will prevent the entry of this low-quality counterpart by limit pricing. That is, the firm will set \( \pi_H = \pi_L - \varepsilon \) (where \( \varepsilon > 0 \) is arbitrarily small) at which producing the low-quality counterpart yields negative profit. Since \( \varepsilon \) can be arbitrarily small, we take the limit \( \varepsilon \to 0 \), which yields the result \( \pi_H = \pi_L \) that was used above.

To simplify matters, we assume that the only cost of production is the capital cost. In particular, the labor cost of production is normalized to zero. Because a low-quality firm earns zero profit, its earnings will exactly cover the capital cost. That is, \( r_L = k_L \), as (2.2) states. Since a low-quality firm’s earnings are equal to \( \pi c_L \), the above equation and the earlier expression for \( c_L \) solve for the equilibrium price as follows:

\[
\pi = \frac{1}{\alpha \psi} [k_L (\alpha \theta + 1 - \alpha) - Y].
\]

An \( H \) firm’s earnings are: \( r_H = \pi c_H = \theta k_L - \psi \pi \). Substituting \( \pi \), we obtain (2.3).

**B. Proof of Proposition 4.1**

Before establishing Proposition 4.1, let us state the following three lemmas, whose proofs are omitted here but can be found in Cao and Shi (1999).
Lemma B.1. Let $S$ be defined in (2.9) and $M$ in (2.8). The pooling action $a_0 = (f_0, q_0)$ and the payoff must satisfy the following requirements: (i) $f_0, q_0 \in [0,1]$, $q_0 \geq M$; (ii) $f_0 \geq S_\alpha(q_0)$; (iii) $V^0(1) \geq R_L - 1$; and (iv) $V^0(x), V^0(1) \leq R_\alpha - 1$.

Lemma B.2. Let $S_H(q)$ be defined by (2.9) with $I = H$ and $IND_L(q)$ by (4.1). These functions have the following features. (i) $IND_L'(q) > 0$ and $IND_L''(q) < 0$ for all $q > Q_1$; $S_H'(q) > 0$ and $S_H''(q) < 0$ for all $q > 0$. (ii) If $Q_1 < 0$, then $IND_L(q) > S_H(q)$ for all $q \geq 0$; If $Q_1 \geq 0$, then there is a unique solution to $IND_L(q) = S_H(q)$ in the range $q \geq Q_1$, denoted $Q_A$, such that $IND_L(q) > S_H(q)$ if and only if $q > Q_A$. (iii) A high-quality firm’s payoff is an increasing function of $q$ along $f = S_H(q)$ and a decreasing function of $q$ along $f = IND_L(q)$.

Lemma B.3. Define $a_b = (f_b, q_b)$ by (4.3). Assume that the market views any firm that takes the action $a_b$ as an $H$ firm. Then, an $H$ firm prefers the action $a_b$ to a pooling action $a_0$ iff

$$(1 - f_b)R_H > (1 - f_0)R_\alpha.$$  \hfill (B.1)

Now we prove Proposition 4.1. To start, we locate the position of a pooling action $(f_0, q_0)$ in Figures 1a and 1b. Since a pooling action must satisfy $f \geq S_\alpha(q)$, it must lie on or above the curve $f = S_\alpha(q)$. Also, we can verify that $IND_L(q_0) > f_0$ and so the point $(f_0, q_0)$ must lie below the curve $f = IND(q)$. This implies $f_0 > f_b$ in the case $Q_1 > 0$.

Consider first the case $Q_1 > 0$ (Figure 1b). Since $f_b < f_0$ in this case and $R_H > R_\alpha$, then (B.1) is satisfied, and an $H$ firm will choose $a_b$ to separate itself from $L$ firms. An $L$ firm will take the action $(f, q) = (1/R_L, 1)$. As a result, $V^0(1) = R_L - 1$, and so $a_b = a^*$, where $a^* = (f^*, q^*)$ is given by (4.5). The condition required for this case, $Q_1 > 0$, becomes $R_H - R_L < b$.

Now consider the case $Q_1 \leq 0$. Using (4.3), we can compute:

$$(1 - f_b)R_H = (1 - f_0)R_\alpha = \frac{V^0(1)}{R_H - (1 + b)} R_H - (1 - f_0)R_\alpha = \frac{1 - f_0}{R_H - (1 + b)} [R_\alpha - R_H(1 - q_0)].$$

By Lemma B.3, a separating equilibrium exists if and only if the above expression is positive, i.e., if and only if $q_0 > q_p \equiv 1 - R_\alpha/R_H$. When there is separation between the two types, an
$H$ firm takes the action $a^* = (f^*, q^*)$, which is given by (4.4), and an $L$ firm takes the action $(f, q) = (1/R_L, 1)$. The condition $Q_1 \leq 0$ becomes $R_H - R_L \geq b$.

Even when $R_H - R_L \geq b$, the separating equilibrium is still the unique equilibrium if all pooling actions entail $q_0 > q_p = 1 - R_\alpha/R_H$. To see when the later condition is satisfied, recall that all pooling actions require $V^0(1) \geq R_L - 1$ (see Lemma B.1). To satisfy this requirement, a necessary condition is $R_\alpha > (1 + b)(1 - q_0)$. Otherwise $V^0(1) \leq 0 < R_L - 1$. Suppose that this necessary condition holds. Define

$$\Delta(q_0) = [1 - S_\alpha(q_0)] [R_\alpha - (1 + b)(1 - q_0)] - (R_L - 1).$$

The feasibility condition $f_0 \geq S_\alpha(q_0)$ implies:

$$V^0(1) = (1 - f_0) [R_\alpha - (1 + b)(1 - q_0)] \leq \Delta(q_0) + (R_L - 1).$$

Thus, $V^0(1) \geq R_L - 1$ only if $\Delta(q_0) \geq 0$. Substitute $S_\alpha$ from (2.9). If $R_\alpha \leq 1 + bx_\alpha^{-1}$, then it can be verified that $\Delta'(q_0) > 0$ for all feasible $q_0$. If $R_\alpha > 1 + bx_\alpha^{-1}$, then it can be verified that $\Delta''(q_0) < 0$ for all feasible $q_0$ and that $\Delta'(q_p) > 0$. In both cases, $\Delta'(q_0) > 0$ for all $q_0 \in [0, q_p]$. If $\Delta(q_p) < 0$, then $\Delta(q_0) < 0$ for all $q_0 \in [0, q_p]$. In this case, all pooling actions entail $q_0 > q_p$.

Computing $\Delta(q_p)$, we have:

$$\Delta(q_p) = 1 - R_L \frac{1 + b}{R_H} + (R_H - R_L) \left(1 - \frac{1 + b}{R_H}\right) \left(\alpha - \frac{1 - \alpha}{R_H - 1 - bx_\alpha^{-1}}\right).$$

Treat this expression as a function of $\alpha$. This function is increasing in $\alpha$. When $\alpha = 0$, $\Delta(q_p) < 0$. When $\alpha = 1$, $\Delta(q_p) = R_H - R_L - b$. As said earlier, if $R_H < R_L + b$, then $\Delta(q_p) < 0$ for all $\alpha$, and so the separating equilibrium is the unique equilibrium. When $R_H \geq R_L + b$, there exists $\tilde{\alpha} \in (0, 1]$ such that $\Delta(q_p) < 0$ if and only if $\alpha < \tilde{\alpha}$. In this case, the separating equilibrium is the only equilibrium when $\alpha < \tilde{\alpha}$.

On the other hand, a pooling equilibrium can exist when $\alpha > \tilde{\alpha}$ and $R_H \geq R_L + b$. To see this, set $q_0 = q_p - \varepsilon$ and $f_0 = S_\alpha(q_0)$, where $\varepsilon > 0$ is sufficiently small number. Since $\alpha > \tilde{\alpha}$ and $R_H \geq R_L + b$, then $V^0(1) > R_L - 1$ and $(1 - f_0)R_H < (1 - f_0)R_\alpha$. QED.
References


Figure 1a Deviations by an H firm: $Q_1 < 0$

Figure 1b Deviations by an H firm: $Q_1 > 0$
Figure 2 Dependence of $(f, q)$ on the earnings difference between a high-quality and an $L$ firm in the separating equilibrium

Figure 3 Market equilibria
Figure 4 A separating equilibrium when there is a lower bound on the amount of equity financing.
C. Proofs of Some Lemmas

Proof of Lemma B.1

The requirements (i) and (ii) are needed for the pooling action to be feasible (see section 2.3). To see why (iii) is needed for pooling, suppose \( V^0(1) < R_L - 1 \), to the contrary. Then, an \( L \) firm gets a higher payoff from revealing its type rather than pooling. In this case, all \( L \) firm voluntarily separate from \( H \) firms. For (iv), we show only that \( V^0(x) \leq R_\alpha - 1 \), because the proof for \( V^0(1) \leq R_\alpha - 1 \) is similar. The definition (2.6) implies:

\[
V^0(x) = (1 - f_0) \left[ R_\alpha - (1 + bx^{-1}) (1 - q_0) \right].
\]

Notice that \( R_\alpha \geq R_L > 1 \) for all \( \alpha \in [0, 1] \). If \( R_\alpha \leq (1 + bx^{-1})(1 - q_0) \), then clearly \( V^0(x) \leq 0 < R_\alpha - 1 \). If \( R_\alpha > (1 + bx^{-1})(1 - q_0) \), then

\[
V^0(x) \leq [1 - S_\alpha(q_0)] [R_\alpha - (1 + bx^{-1}) (1 - q_0)]
\]

\[
\leq [1 - S_1(q_0)] [R_\alpha - (1 + bx^{-1}) (1 - q_0)]
\]

\[
= R_\alpha - (1 + bx^{-1})(1 - q_0) - q_0 \leq R_\alpha - 1.
\]

The first inequality follows from (ii) in the lemma; the second inequality follows from the fact that \( S_\alpha(q) \) is decreasing in \( \alpha \); and the last inequality follows from the fact that the preceding expression is increasing in \( q_0 \).

QED

Proof of Lemma B.2

The features in (i) can be verified directly under Assumptions 1A and 1B. Before proving (ii), note that

\[
S_H(1) = 1/R_H < 1/R_\alpha < 1 - (R_\alpha - 1)/R_H < 1 - V^0(1)/R_H = IND_L(1).
\]

The third inequality follows from (iv) in Lemma B.1.

Consider first the case \( Q_1 < 0 \) (see Figure 1a). In this case the relevant range of \( q \) is \( q \in [0, 1] \). Since \( Q_1 < 0 \), we have

\[
S_H(0) = 0 < 1 - V^0(1)/(R_H - 1 - b) = IND_L(0).
\]

Because \( S_H(1) < IND_L(1) \), as shown above, \( IND_L(q) > S_H(q) \) for both \( q = 0 \) and \( 1 \). To show \( S_H(q) < IND_L(q) \) for all \( q \in [0, 1] \), it suffices to show that \( IND_L(q) \) crosses \( S_H(q) \) from below if they ever cross each other in the positive quadrant. To show this crossing property, suppose that the two curves cross each other at \( q_c \in [0, 1] \), i.e.,

\[
1 - V^0(1)/[R_H - (1 + b)(1 - q_c)] = q_c/[R_H - (1 + bx^{-1})(1 - q_c)].
\]

(C.1)
Computing the derivatives $IND'_L(q)$ and $S'_H(q)$ and substituting $V^0(1)$ from (C.1), we can show that $[IND'_L(q_c) - S'_H(q_c)]$ has the same sign as the following expression:

$$\left[ R_H - (1 + b)(1 - q_c) \right] q_c b x^{-1} + \left[ R_H - (1 + b x^{-1})(1 - q_c) - q_c \right] \times \left\{ (1 + b) \left[ R_H - (1 + b x^{-1})(1 - q_c) - [R_H - (1 + b)(1 - q_c)] \right] \right\}.$$ 

The expression in $\{,\}$ is clearly positive. Also, Assumption 1B implies

$$R_H - (1 + b x^{-1})(1 - q_c) - q_c > R_H - (1 + b x^{-1}) > 0.$$ 

Since $Q_1 < 0$, then $R_H - (1 + b)(1 - q_c) > V^0(1) > 0$. So, $IND'_L(q_c) > S'_H(q_c)$, as desired.

Consider now the case $Q_1 > 0$. In this case, the two curves $IND_L(q)$ and $S_H(q)$ do not cross each other in the range $q \in [0, Q_1]$. Thus, consider only the range $q \geq Q_1$. In this range, the above proof for the crossing property between $IND_L(q)$ and $S_H(q)$ is still valid. Moreover, $IND_L(Q_1) = 0 < S_H(Q_1)$. Therefore, there is a unique crossing between the two curves.

Finally, we establish (iii). Along $f = IND_L(q)$, an $H$ firm’s payoff is

$$[1 - IND_L(q)][R_H - (1 + b x^{-1})(1 - q)] = V^0(1) \times \frac{R_H - (1 + b x^{-1})(1 - q)}{R_H - (1 + b)(1 - q)},$$

which is a decreasing function of $q$. Along $f = S_H(q)$, an $H$ firm’s payoff is

$$[1 - S_H(q)] \left[ R_H - (1 + b x^{-1})(1 - q) \right] = R_H - (1 + b x^{-1})(1 - q) - q,$$

which is an increasing function of $q$. QED

**Proof Lemma B.3**

When the market views any firm that takes the action $a_b$ as an $H$ firm, the payoff to an $H$ firm that takes $a_b$ is $V(f_b, q_b; R_H, x)$. The payoff to an $H$ firm from taking a pooling action is $V^0(x) = V(f_0, q_0; R_\alpha, x)$. The difference between the two is as follows:

$$(1 - f_b) \left[ R_H - (1 + b x^{-1})(1 - q_b) \right] - V^0(x) = (1 - f_b) \left[ R_H - (1 + b x^{-1})(1 - q_b) \right] - (1 - f_b) \left[ R_H - (1 + b)(1 - q_b) \right] + \left\{ (1 - f_b) \left[ R_H - (1 + b)(1 - q_b) - V^0(1) \right] + [V^0(1) - V^0(x)] \right\} = b(1 - x^{-1})(1 - f_b)(1 - q_b) - b(1 - x^{-1})(1 - f_0)(1 - q_0) = b(1 - x^{-1}) \left\{ (1 - f_b)R_H - (1 - f_0)R_\alpha \right\}.$$ 

The first equality follows from adding and subtracting the same terms; the second equality follows from the fact that the term in $\{,\}$ is zero by the definitions of $(f_b, q_b)$; the third equality follows from substituting the definitions of $q_b$ and $q_0$. Then, the lemma is evident. QED

**D. Proof of Proposition 6.1**

Let $V^0_L$ be the payoff to a low-quality from a pooling action $(f_0, q_0)$. As in the simple model, we find separating actions that generate lower payoffs to an $L$ firm than in a pooling equilibrium. Then we choose the best among these actions as a candidate for the action of an $H$ firm in a
separating equilibrium. If an \(L\) firm deviates from the pooling action to an action \((f, q)\) and is perceived as an \(H\) firm, the payoff is

\[
(1 - f) [W + \rho \gamma (p_H - s) - (1 + b)(1 - q)] = \frac{1 - f}{1 - \rho \gamma} [W - \rho \gamma q / f - (1 + z)(1 - q)],
\]

where \(W = xR_L + \rho D\) and \(z = \rho \gamma b / x + (1 - \rho \gamma) b\). This payoff is less than that in the pooling equilibrium if and only if

\[
q < G(f) \equiv \frac{1 + z - W + (1 - \rho \gamma) V_L^0 / (1 + z - \rho \gamma)}{1 + z / \rho \gamma f}, \quad \text{for } f > \frac{\rho \gamma}{1 + z};
\]

\[
q > G(f), \quad \text{for } f < \frac{\rho \gamma}{1 + z}.
\]

Figures 5a and 5b here.

Let us divide the proof into two cases.

Case 1: \(W > (1 + z)(1 + (1 - \rho \gamma)V_L^0 / (1 + z - \rho \gamma))\). This case is depicted in Figure 5a. Let \(S_H(q)\) now denote the right-hand side of (6.4) and let its inverse be \(S_H^{-1}\). It can be shown that there exists \(\gamma_1 > 0\) such that \(G(f) > S_H^{-1}(f)\) in the region \(f < \rho \gamma / (1 + z)\) if \(\gamma \leq \gamma_1\), as depicted in Figure 5a. Restrict attention to \(\gamma \leq \gamma_1\). In this case the relevant region is \(f > \rho \gamma / (1 + z)\) and the shaded area is the set of actions that yield lower payoff to an \(L\) firm but may yield higher payoff to an \(H\) firm than in the pooling equilibrium. We can verify the following properties for the segment of \(G(f)\) with \(f > \rho \gamma / (1 + z)\):

(1a) \(G(f) > 0\) iff \(f > 1 - (1 - \rho \gamma)V_L^0 / (W - 1 - z)\) (i.e., iff \(f\) is higher than point \(A\)).

(1b) \(G'(f) > 0\) for all \(f > 1 - (1 - \rho \gamma)V_L^0 / (W - 1 - z)\).

(1c) The payoff to an \(H\) firm from taking actions along \(q = G(f)\) is decreasing in \(f\).

These properties imply that, if \(\gamma \leq \gamma_1\), the best deviation for an \(H\) firm from a pooling equilibrium is point \(A\) in Figure 5a. In this case, \(q = s = 0\) and there is underpricing as in the corresponding case in the simple model.

Case 2: \(W < (1 + z)(1 + (1 - \rho \gamma)V_L^0 / (1 + z - \rho \gamma))\). In this case, the best deviations for an \(H\) firm in the region \(f < \rho \gamma / (1 + z)\) lie on the curve \(f = S_H(q)\) and, by property (2c) below, they are strictly dominated by the action at point \(A\) in Figure 5b. Thus, it suffices to consider only the region \(f > \rho \gamma / (1 + z)\). The curve \(q = G(f)\) for \(f > \rho \gamma / (1 + z)\) is depicted by Figure 5b, where the shaded area is the set of deviations that are feasible to a firm (when perceived as an \(H\) firm as a result of deviation) and that generate lower payoffs to an \(L\) firm than in the pooling equilibrium. A lengthy exercise can establish the following properties, some of which are depicted in Figure 5b:

(2a) There exists a level \(f_c \in (\rho \gamma / (1 + z), 1)\) such that the curve \(q = G(f)\) is decreasing in \(f\) for \(f \in (\rho \gamma / (1 + z), f_c)\) and increasing in \(f\) for \(f \in (f_c, 1)\).

(2b) \(S_H(1) = 1 / W > \rho \gamma / (1 + z)\) and \(G(1/W) < 1\). That is, the intersection between the curve \(f = S_H(q)\) and \(q = 1\) lies in the region \(q > G(f)\) and \(f > \rho \gamma / (1 + z)\). Since the curve \(f = S_H(q)\) starts outside this region when \(q\) is small, there is at least one intersection between \(f = S_H(q)\) and \(q = G(f)\), as depicted by point \(A\) in Figure 5b.

(2c) A high-quality firm’s payoff from actions along the curve \(f = S_H(q)\) increases in \(q\).
(2d) A high-quality firm’s payoff from actions along the curve \( q = G(f) \) (for \( f > \rho \gamma / (1 + z) \)) decreases in \( f \) for all \( f \geq (\rho \gamma / W)^{1/2} \).

(2e) There exists \( \gamma_2 > 0 \) such that, if \( \gamma \leq \gamma_2 \), the intersection (point \( A \)) has \( f \geq (\rho \gamma / W)^{1/2} \).

These properties imply that, if \( \gamma \leq \gamma_2 \), the payoff to an \( H \) firm from deviating from the pooling action is maximized at the intersection between the curve \( f = S_H(q) \) and \( q = G(f) \), such as point \( A \) in Figure 5b. There is no underpricing in this case.

When \( \alpha \) is sufficiently small, in both case 1 and case 2 one can also show that the payoff at point \( A \) to an \( H \) firm is higher than the payoff in the pooling equilibrium, provided that the market views such deviation as coming from an \( H \) firm. Thus, the action given by point \( A \) is the separating equilibrium that satisfies the Cho-Kreps criterion. Substituting \( W = xR_L + \rho D \) and noting that the payoff to an \( L \) firm is \( R_L - 1 \) in the absence of pooling (thus \( V_L^0 \) in the above analysis is replaced by \( R_L - 1 \)), we have,

\[
\begin{align*}
  d = p_H &= \frac{1}{1-\rho \gamma} [\rho D + xR_L - 1 - b/x], \\
  &\text{if } xR_L + \rho D > (1 + z) \left[ 1 + \frac{(1-\rho)(R_L-1)}{1+z-\rho \gamma} \right], \quad (D.1) \\
  d = 0 &\quad \text{if } xR_L + \rho D < (1 + z) \left[ 1 + \frac{(1-\rho)(R_L-1)}{1+z-\rho \gamma} \right]. \quad (D.2)
\end{align*}
\]

To solve for market equilibria, impose symmetry \( d = D \). Doing so for case 1 we get:

\[
d = D = \frac{xR_L - 1 - b/x}{1 - \rho(1 + \gamma)}.
\]

Thus, \( d > 0 \) only if \( \rho < 1/(1 + \gamma) \). Also, (D.1) must be satisfied in order to have \( D > 0 \), i.e.,

\[
xR_L + \rho \frac{xR_L - 1 - b/x}{1 - \rho(1 + \gamma)} > (1 + z) \left[ 1 + \frac{(1-\rho)(R_L-1)}{1+z-\rho \gamma} \right]. \quad (D.3)
\]

Note that \( z \) and \( (1 - \rho \gamma)/(1 + z - \rho \gamma) \) are decreasing functions of \( \rho \) and so is the right-hand side of the above inequality. The left-hand side is an increasing function of \( \rho \). Since the inequality is satisfied for \( \rho = 1/(1 + \gamma) \) and violated for \( \rho \rightarrow 0 \), there exists a critical level \( \rho_1 \in (0, 1/(1 + \gamma)) \) such that the above inequality is satisfied if and only if \( \rho > \rho_1 \). Therefore, an underpricing equilibrium exists if \( \rho_1 < \rho < 1/(1 + \gamma) \) and \( \gamma \leq \gamma_1 \).

For the no-underpricing equilibrium, impose \( d = D = 0 \) in case 2. The equilibrium exists if

\[
xR_L < (1 + z) \left[ 1 + \frac{(1-\rho)(R_L-1)}{1+z-\rho \gamma} \right]. \quad (D.4)
\]

The right-hand side of this inequality is a decreasing function of \( \rho \). The inequality is satisfied when \( \rho \rightarrow 0 \) and violated when \( \rho \rightarrow 1/\gamma \). Thus, there exists \( \rho_2 \in (0, 1/\gamma) \) such that the inequality is satisfied for \( 0 < \rho < \rho_2 \). If \( \gamma \leq \gamma_2 \), in addition, the no-underpricing equilibrium exists.

Comparing (D.3) and (D.4) we can immediately show \( \rho_1 < \rho_2 \). Therefore, the underpricing equilibrium and the no-underpricing equilibrium coexist if \( \rho \in (\rho_1, \rho_2) \) and \( \gamma \leq \min\{\gamma_1, \gamma_2\} \). This completes the proof of Proposition 6.1. QED
E. Proof of Proposition 6.2

We have already argued in the text that firm 2 underprices only if firm 1 underprices sufficiently (i.e., if $d_1 \geq D_0$). Analogous to the derivation of (D.1) in Appendix D, we have:

\[ d_1 = \frac{1}{1 - \alpha \rho^2} (W - 1 - b/x), \quad (E.1) \]

if $W > (1 + \alpha \rho^2 b x^{-1} + (1 - \alpha \rho^2) b) \left[ 1 + \frac{(1 - \alpha \rho^2)(R_L - 1)}{1 + \alpha \rho^2 b x^{-1} + (1 - \alpha \rho^2) b - \alpha \rho^2} \right], \quad (E.2)$

where $W = (1 + \rho \alpha) x R_L - \rho \alpha (1 + b x^{-1})$. The underpricing equilibrium has $q = G(f) = 0$. With $V_L^0$ being set to $R_L - 1$, $G(f) = 0$ implies:

\[ f = 1 - \frac{(1 - \alpha \rho^2)(R_L - 1)}{W - [1 + \alpha \rho^2 b x^{-1} + (1 - \alpha \rho^2) b].} \quad (E.3) \]

For firm 1 to underprice, $d_2$ must also be positive and so we need $d_1 \geq D_0$, i.e.

\[ W - 1 - b/x \geq \frac{1 - \alpha \rho^2}{\rho} [b - R_L (x - 1)]. \quad (E.4) \]

Note that $W$ increases in $\rho$ and the right-hand side of (E.2) decreases in $\rho$. Moreover, (E.2) is satisfied when $\rho \to \alpha^{-1/2}$ and is violated when $\rho \to 0$. Then, there exists $\rho_a \in (0, \alpha^{-1/2})$ such that (E.2) is satisfied if and only if $\rho \in (\rho_a, \alpha^{-1/2})$. Similarly, there exists $\rho_b \in (0, \alpha^{-1/2})$ such that (E.4) is satisfied if and only if $\rho \in [\rho_b, \alpha^{-1/2})$. Let $\rho_3 = \max\{\rho_a, \rho_b\}$. Then both (E.2) and (E.4) are satisfied if and only if $\rho \in (\rho_3, \alpha^{-1/2})$.

In addition to the requirement $\rho \in (\rho_3, \alpha^{-1/2})$, the payoff to firm 1 (when it is high-quality) must be higher with $d_1 > 0$ than with $d_1 = 0$ in order for the firm to underprice. With $d_1 = 0$, the payoff to high-quality firm 1 is

\[
(1 - f^*) \left[ x R_L - \left( 1 + \frac{b}{x} \right) (1 - q^*) \right] = x R_L - \left( 1 + \frac{b}{x} \right) (1 - q^*) - q^* \\
= (R_L - 1) \left[ x R_L - (1 + b/x)(1 - q^*) \right] / [x R_L - (1 + b)(1 - q^*)] 
\]

where the inequalities come from substituting the definitions of $(f^*, q^*)$ in (4.5). When $d_1 > 0$ in (E.1), $q = 0$ and $f$ is given by (E.3). The total return to shareholders is $(W - 1 - bx^{-1})/(1 - \alpha \rho^2)$ and the payoff to high-quality firm 1 from underpricing is

\[
\frac{(R_L - 1)(W - 1 - bx^{-1})}{W - [1 + \alpha \rho^2 b x^{-1} + (1 - \alpha \rho^2) b]}.
\]

Substituting $W$ and simplifying, we can show that the firm’s payoff is higher with underpricing than without if and only if

\[
\frac{1 - \alpha \rho^2}{1 + \alpha \rho} > \frac{(1 - q^*)(x R_L - 1 - b/x)}{x R_L - (1 + b/x)(1 - q^*)}.
\]

There exists $\rho_4 \in (0, \alpha^{-1/2})$ such that the above condition is satisfied if and only if $0 \leq \rho < \rho_4$. The level $\rho_4$ is not necessarily greater than $\rho_3$. Only when $\rho_4 > \rho_3$ and $\rho \in (\rho_3, \rho_4)$ does high-quality firm 1 underprice IPO.

QED
Figure 5a When an $H$ firm has its own influence on publicity: Case 1 (large $W$)

Figure 5b When an $H$ firm has its own influence on publicity: Case 2 (small $W$)