Employee Bargaining Power, Inter-Firm Competition, and Equity-Based Compensation

Abstract

We develop a model to illustrate that employee compensation and product market decisions are related. When the product market is competitive and employees have low bargaining power, the unique equilibrium in our setting is for each firm to offer equity-based compensation to their employees. In this setting, equity-based compensation leads to a lower wage rate, which makes each firm more competitive with its rival. However, this unique equilibrium is a Prisoner's Dilemma for the firms' original owners. Our results are consistent with several empirical regularities and provide predictions on when firms will offer equity-based compensation to their employees.

1. Introduction

Hölmstrom (1979) suggests that one way to elicit a high level of effort from an agent when agent effort is both unobservable and costly is to make the agent's compensation contingent on the firm's profits. One practical way to achieve this outcome is for a firm's owners to grant an employee an equity stake in the company. However, as Hölmstrom (1982) notes, when too many employees are incentivized with equity-based compensation, a free-rider problem arises that may diminish an agent's desire to put in a greater effort. In such a setting, as Oyer (2004) eloquently suggests, equity incentives may have no incentive effect. Given this insight, it is puzzling that publicly-traded firms, which usually have many employees, often adopt equity-based compensation plans.

To address this puzzle, several papers have put forth alternate motivations for offering equity grants to employees. For example, Oyer (2004) suggests that a firm's owners might compensate employees with company stock in order to index employee compensation to outside options. In this respect, equity-based compensation acts as a useful tool for employee retention. Separately, Lazear (2004), Arya and Mittendorf (2005), and Bergman and Jenter (2007) posit that a firm's owners might use equity-based compensation for sorting purposes, where employees reveal private information about either the firm or their own abilities by "putting their money where their mouth is."[1]

We propose a complementary and empirically relevant motivation. We posit that a firm's owners may offer equity-based compensation to their employees in order to negotiate low wage rates. A lower wage rate reduces the firm's marginal cost and makes the firm not

[1] While there is evidence that firms provide equity grants for the purpose of incentive alignment (e.g., Core and Guay, 1999), there is also support for the alternate motivations. Oyer and Shaffer (2005) consider three economic justifications for providing equity compensation to employees—incentive alignment, sorting, and employee retention—and find evidence consistent with the latter two motivations. Core and Guay (2001), Ittner, Lambert, and Larker (2003), Rajgopal, Shevlin, and Zamora (2006), and Balsam and Miharjo (2007) also find evidence consistent with firms using equity-based pay for employee retention purposes.

only more profitable, but also more competitive when facing a rival. We generate these inferences from a model of employee bargaining power and inter-firm competition. We implement our idea in a three-period model in which two firms compete in a product market. In the first period, each firm's original owners simultaneously make decisions on contract forms. Specifically, each firm's non-employee owners decide whether to compensate their respective employees with wages only (a wage-based contract) or with wages and an equity stake in the company (an equity-based contract). In the second period, the compensation terms—i.e., the level of wages and, in cases where equity compensation is offered in the first period, the percentage of the firm that will be given to employees—of both firms are simultaneously determined via bargaining games between each firm's respective owners and employees. Note that, by incorporating a bargaining framework, the model allows for an imperfectly competitive market for labor, where employees can extract varying levels of above-market rents from the firm, contingent on their bargaining power.[2] In the third and final period, firms make production decisions and play a Cournot game in the product market.

We find that two types of equilibria emerge in our economy: an "employee ownership equilibrium," in which each firm's owners offer an equity-based contract; or a "wage only equilibrium," in which each firm's owners offer a wage-based contract. These two equilibria arise at the nexus of several competing forces. On the one hand, when bargaining is over both wages and an equity stake, the total surplus of non-employee owners and employees is maximized, which tends to make the employee ownership equilibrium more favorable to non-employee owners. On the other hand, when negotiating over wages only, employees end up with a smaller portion of the ensuing total surplus, because they are aware that the

[2] An imperfectly competitive market for labor is plausible in many industries. Many factors such as unionization, local unemployment rates, and firm-borne employee switching costs can lead to inefficiencies in the labor market that allow employees to extract rents above the competitive market wage (see Bova, Dou, and Hope, 2014; Lindbeck and Snower, 1986, 2001).

firm's production is decreasing in the wage rate and thus do not demand overly exorbitant wages. This force tends to make the wage only equilibrium more favorable to non-employee owners. The relative strength of the two competing forces depends on the product market's competitiveness and employees' bargaining power.

We find that when inter-firm competition is sufficiently intense and employees' bargaining power is sufficiently low, the dominant strategy for each firm's non-employee owners is to offer their employees an equity stake in the game's first period. In this case, the employee ownership equilibrium is the unique equilibrium. In contrast, when inter-firm competitiveness is sufficiently low and employees' bargaining power is sufficiently high, the dominant strategy for each firm's non-employee owners is to offer their employees wages only. In this case, the wage only equilibrium prevails as the unique equilibrium.

Our results contribute to the literature by providing a novel, complementary, and empirically relevant channel for the incidence of employee ownership. For example, alternative existing theories, such as agency models, retention models, and sorting models, all feature uncertainty. By contrast, all parameters in our economy are commonly known, there is no unobservable effort on the part of the agent, and there are no stochastic returns. It is new, then, that equity-based compensation can still arise endogenously as an equilibrium outcome in our deterministic setting. Specifically, this result runs in contrast to the intuition in previous models, where in a first-best scenario without uncertainty, a principal *would not* compensate an agent with equity in the company.[3]

In a broad sense, our paper makes a basic point that employee compensation and product market decisions are related. In our setting, the structure of employee compensation affects the product market equilibrium through its effect on the wage rate, because the wage rate

[3] For example, in the classic Hölmstrom (1979) model, when the agent's actions are observable, the optimal strategy of a self-interested principal is to only pay the agent a fixed wage provided the agent supplies the desired effort.

is the marginal production cost in the product market. Given that labor relations can influence the product market equilibrium, a firm's owners have incentives to design employee compensation so as to influence the product market in their favor.

Our analysis also generates insights on the pros and cons of employee ownership, both from a normative and a positive perspective. On the normative side, we find that total profits are higher under equity-based compensation than under wage-based compensation, because of the lower wage rate negotiated under equity-based compensation. This lower wage rate and more efficient production additionally benefit each firm's consumers in the product market. However, when the employee ownership equilibrium prevails, it is a Prisoner's Dilemma for each firm's non-employee owners as each group would be better off, had they been able to commit to offering their respective employees wages only in the game's first period. Moreover, when competition is sufficiently intense and employee bargaining power is sufficiently low, not only are non-employee owners worse off by not being able to commit to offering wages only, but so are each firm's employees. This result may help explain why equity-based compensation seems to arise endogenously as an equilibrium outcome, even in situations where it appears to make both employees and non-employee owners worse off.

From a positive perspective, our analysis is useful for understanding a wide range of financial phenomena and suggesting new testable predictions. For instance, our model provides an explanation for why (1) equity-based compensation is less commonly observed in highly unionized settings (McCarthy, Voos, Eaton, Kruse, and Blasi, 2011); (2) equity grants often substitute, rather than complement, wages when employees are compensated with larger equity stakes (Kim and Ouimet, 2014); and (3) firm and stakeholder outcomes vary with the size of employee equity stakes (Kim and Ouimet, 2014; Faleye, Mehrotra, and Morck, 2006). Our results are also consistent with the observed outcomes of several recent contract negotiations—for example, the 2009 contract negotiations between the United Auto Workers and two major American automobile manufacturers. Finally, our model generates additional empirical predictions regarding the incidence of equity-based compensation. Some of these predictions are consistent with existing empirical findings, while others offer new opportunities to test our model. For example, our model would predict that equity-based compensation should be more prevalent among firms whose product market is more competitive, whose employees have lower bargaining power, and where labor is a more important factor of production.

The remainder of the paper is organized as follows. Section 2 describes the model, and Section 3 characterizes the equilibrium. In Section 4, we conduct an efficiency analysis to examine the normative implications of equity-based compensation. Section 5 explores the empirical implications of our analysis and Section 6 further discusses several key features and assumptions of our model. Finally, Section 7 summarizes and concludes. The appendix includes all the proofs.

2. The Model

We consider an economy with three periods, $t = 1, 2,$ and 3. The timeline of the economy is described in Fig. 1. Our analysis focuses on a monopolistic sector with two firms, where each firm uses labor as a sole input to produce a differentiated good. Firms are originally owned by their non-employee shareholders. In period 1, each firm's original owners decide whether to offer their respective employees wages only, or wages and an equity stake in the firm. In period 2, each firm's non-employee owners and employees negotiate over the terms of the compensation (i.e., the level of wages and, in cases where equity compensation is offered, the percentage of the firm that will be given to employees). The negotiated outcome will be set through Nash bargaining and in this respect, the model allows for an inefficient labor market where employees can extract varying above-market rents from the firm, contingent on their bargaining power. In period 3, each firm sets production to maximize its profits, consumers purchase firms' products, prices are realized, and profits accrue to non-employee owners and employees, provided employees have an equity stake in the firm.

[INSERT FIG. 1 HERE]

2.1. Production and product markets

The product market operates in period 3, and the two firms play a Cournot game in this market. As in Singh and Vives (1984), the demand for each firm's products is generated by a representative consumer who has a utility function as follows:

$$\begin{aligned} U(q_1, q_2) &= (1-k)\left[-\frac{1}{2}q_1^2 + q_1 - \frac{1}{2}q_2^2 + q_2\right] \\ &\quad + k\left[-\frac{1}{2}(q_1+q_2)^2 + (q_1+q_2)\right] - (P_1 q_1 + P_2 q_2), \qquad (1) \end{aligned}$$

where $k \in (0,1)$ is a constant, q_i is the amount of good i, and P_i is its price.

The first two terms in equation (1) represent the consumer's intrinsic utility from consuming the two goods, while the third term captures the cost of purchasing these goods. The first term is quadratic in q_1 and q_2, respectively, which reflects how good 1 and good 2 separately affects the consumer's utility. By contrast, the second term is quadratic in $(q_1 + q_2)$, and thus the two goods are perfectly substitutable when affecting the consumer's utility through this term. Hence, parameter k captures the degree of substitutability of the two goods in the consumer's preference. The higher the value for k, the more substitutable are the two goods, and the more competitive the two firms are in the product market. Thus, parameter k represents a measure of the intensity of product market competition.

The representative consumer chooses quantities q_1 and q_2 to maximize her preference in

(1) taking prices P_1 and P_2 as given. This utility maximization problem gives rise to the following standard linear inverse demand function for firm i:

$$P_i = 1 - q_i - kq_j, \text{ for } i, j = 1, 2, \text{ and } i \neq j. \tag{2}$$

As $k \rightarrow 0$, equation (2) degenerates to $P_i = 1 - q_i$, which is the inverse demand function for a monopoly firm. As k becomes higher, firm i's product price is affected more by firm j's production quantity, and thus the product market becomes more competitive.

The supply of products in the market arises from each firm's optimal production decisions. The production process utilizes only one input, labor. Production features constant returns to scale so that one unit of labor produces one unit of product. Firm i's cost for one unit of labor is given by a wage rate, w_i, which is endogenously determined through a bargaining game in period 2 (which will be introduced shortly). Thus, firm i's gross profit is $\Pi_i = q_i P_i - q_i w_i$, and its optimal production q_i is determined by[4]

$$Max_{q_i} \left(q_i P_i - q_i w_i \right), \tag{3}$$

where P_i is given by the inverse demand function (2). In the profit maximization problem, firm i takes the wage rate w_i and its rival's production q_j as given. The first-order condition of program (3) will yield the best response function of firm i. As per usual, the intersection of the two best response functions (for $i = 1, 2$) determines the equilibrium quantity produced by each firm.

2.2. Contracts and surplus

At the beginning of the economy, each firm is originally owned by a set of non-employee shareholders. In period 1, the non-employee owners for each firm i, consider offering their

[4]Here, the firm is maximizing total profits, which means that the firm's management works on behalf of all shareholders in period 3. Alternatively, we can assume that the firm continues to operate in the interest of the original non-employee owners in period 3. Then, the optimal production q_i is determined by $Max_{q_i} (1 - z_i) (q_i P_i - q_i w_i)$, which will yield the same solution as (3), provided the orginial non-employee owners continue to own a strictly positive share of the firm.

employees two possible forms of compensation, either wage-based compensation or equity-based compensation. Under a wage-based contract, the firm compensates its employees with wages only, while under an equity-based contract, the firm compensates its employees with both wages and a portion z_i of the firm's gross profits Π_i. The welfare of non-employee owners is particularly relevant in our analysis, since they control which form of contract is chosen in period 1, and offering an equity stake to employees dilutes their stake in the firm.

Once the contract form is set in period 1, the terms of the contract are determined in period 2 according to a Nash bargaining game between non-employee owners and employees of each firm. Note that for the wage-based contract, only the wage rate w_i is negotiable, while for the equity-based contract, both the wage rate w_i and the equity stake proportion z_i are negotiable. Intuitively, starting from a position of non-ownership for employees, offering a wage-based contract suggests that equity ownership is simply not on the bargaining table in period 2. By contrast, if non-employee shareholders do open the door for employee ownership in period 1, then employees can bargain over both wages and the size of their equity stake in period 2.

In the period-2 bargaining game, the equilibrium outcome depends on each party's utility surplus resulting from reaching an agreement. If employees of firm i decide to work for the firm, they will receive an amount of $q_i w_i + z_i \Pi_i$ (with $z_i = 0$ in settings where only the the wage-based contract is offered in period 1). If they decide not to work for the firm, we assume that they can work at an exogenous, lower market wage rate $c \in (0, 1)$;[5] that is, they can work at their opportunity cost of c per unit of labor. Thus, the surplus of firm i's employees is

$$E_i \equiv q_i w_i + z_i \Pi_i - q_i c = q_i (w_i - c) + z_i \Pi_i. \tag{4}$$

[5] We impose the constraint of $c < 1$, which means that the competitive labor market rate c is lower than the maximum product price, 1, in equation (2).

In the Online Appendix, we show that E_i is consistent with the objective function of an employee base in various bargaining settings as described in McDonald and Solow (1981).

Next we define the surplus that accrues to firm i's non-employee owners. The non-employee owners for firm i receive the residual profits that do not accrue to employees, $(1 - z_i)\,\Pi_i$. Additionally, the non-employee owners' outside option is 0. This outcome arises because we assume that each firm faces switching costs (e.g., unionization, unique human capital amongst their employee base) that preclude the owners from replacing current employees with outsider workers. These switching costs create an imperfectly competitive market for labor within the firm which, in turn, generates the bargaining power for each firm's employees. As a result, when calculating the surplus that accrues to firm i's non-employee owners, the competitive wage rate c is irrelevant. Taken together, firm i's non-employee owners receive a surplus of

$$S_i \equiv (1 - z_i)\,\Pi_i. \tag{5}$$

To generate the equilibrium compensation levels in period 2, the non-employee owners and employees set the wage, w_i, and in cases where the equity-based contract was offered in period 1, the portion of equity, z_i, to maximize the generalized Nash product below,

$$E_i^{\beta} S_i^{1-\beta}, \tag{6}$$

where parameter $\beta \in (0, 1)$ represents the strength of employee bargaining power. As our analysis will illustrate, when the firm and its employees can negotiate over the terms of particular forms of compensation (i.e., wages or wages and equity), there exist important interactions between employee compensation and product market decisions, so that bargaining in our setting has not only distributional consequences, but also allocational consequences.

3. The equilibrium

An equilibrium in the economy is defined in a subgame perfect sense. Formally, we define an equilibrium as production quantities $q_i(w_i, w_j, k)$ of each firm in period 3, contract terms $w_i(\beta, k)$ and $z_i(\beta, k)$ between each firm and their respective employees in period 2, and contract form choices of each firm's non-employee owners in period 1, such that (1) the production quantities $q_i(w_i, w_j, k)$ form a Cournot duopoly equilibrium in the period-3 product market, (2) the contract terms, $w_i(\beta, k)$ and $z_i(\beta, k)$ (in case where an equity stake is offered in the first period), for both firms are simultaneously determined as the outcomes of their respective Nash bargaining games in period 2, given the Cournot outcome in period 3, and (3) the contract form choices of each firm form a Nash equilibrium from the perspective of the two groups of non-employee owners in period 1, given the period-2 and period-3 equilibrium outcomes. In this section, we first work out the equilibrium using backward induction and then conduct comparative statics on the equilibrium outcomes.

3.1. Product market equilibrium in period 3

In period 3, each firm simultaneously chooses a production quantity to maximize its profits in (3), given the employee wages negotiated in period 2, the quantity choice of its rival, and the inverse demand function (2). Solving the first-order condition yields the best response function of firm i as follows:

$$q_i = \frac{1 - kq_j - w_i}{2}, \text{ for } i, j = 1, 2, \text{ and } i \neq j.$$

Using the two best response functions, we can compute the Cournot duopoly equilibrium in the period-3 product market, which is summarized in the following lemma.

Lemma 1. *For any given employee wages (w_1, w_2) and product market competitiveness k, the product market equilibrium is unique with the production quantity and profit of firm i*

being given respectively by

$$q_i(w_i, w_j, k) = \frac{2-k-2w_i+kw_j}{4-k^2}, \tag{7}$$

$$\Pi_i(w_i, w_j, k) = \frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}, \tag{8}$$

for $i, j = 1, 2,$ *and* $i \neq j$.

3.2. Bargaining game equilibrium in period 2

In period 2, each firm's non-employee owners and employees negotiate over the terms of the compensation—i.e., the wage rate w_i under a wage-based contract, and the wage rate w_i and equity portion z_i under an equity-based contract—according to a generalized Nash bargaining game. Inserting the expressions of q_i and Π_i in Lemma 1 into equations (4) and (5), we can obtain the employees' total compensation E_i and non-employees' residual profits S_i as follows:

$$S_i(z_i, w_i, w_j, k) = (1-z_i)\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}, \tag{9}$$

$$E_i(z_i, w_i, w_j, k) = \frac{2-k-2w_i+kw_j}{4-k^2}(w_i-c) + z_i\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}, \tag{10}$$

for $i, j = 1, 2,$ and $i \neq j$.

Under a wage-based contract, we have $z_i = 0$, and firm i's wage rate w_i is determined by

$$\underset{w_i}{Max}\,[E_i(0, w_i, w_j, k)]^{\beta}[S_i(0, w_i, w_j, k)]^{1-\beta}. \tag{11}$$

Under an equity-based contract, firm i's wage rate w_i and the portion of employee equity z_i are determined by

$$\underset{z_i, w_i}{Max}\,[E_i(z_i, w_i, w_j, k)]^{\beta}[S_i(z_i, w_i, w_j, k)]^{1-\beta}. \tag{12}$$

Note that in (11) and (12), firm i's negotiated compensation terms hinge on its rival's wage rate w_j, which in turn depends on firm j's contract choice in period 1. Thus, there are three possible equilibrium paths in period 2, depending on each firm's contract choices in period

1: each firm offers wage-based contracts; each firm offers equity-based contracts; and one firms offers a wage-based contract while the other offers an equity-based contract. Below we use Proposition 1 to characterize the equilibrium contract terms determined in period 2 for these three cases.[6]

Proposition 1. *(1) When both firms offer wages only to their respective employees in the first period, we have*

$$\begin{aligned} w_1^{ww}(\beta,k) &= w_2^{ww}(\beta,k) = c + \frac{\beta(2-k)(1-c)}{4-k\beta}, \\ q_1^{ww}(\beta,k) &= q_2^{ww}(\beta,k) = \frac{2(2-\beta)(1-c)}{(4-k\beta)(k+2)}, \\ S_1^{ww}(\beta,k) &= S_2^{ww}(\beta,k) = \Pi_1^{ww}(\beta,k) = \Pi_2^{ww}(\beta,k) = \frac{4(2-\beta)^2(1-c)^2}{(4-k\beta)^2(k+2)^2}, \\ \text{and } E_1^{ww}(\beta,k) &= E_2^{ww}(\beta,k) = \frac{2\beta(2-\beta)(2-k)(1-c)^2}{(4-k\beta)^2(k+2)}. \end{aligned}$$

(2) When both firms offer wages and equity stakes to their respective employees in the first period, we have

$$\begin{aligned} z_1^{zz}(\beta,k) &= z_2^{zz}(\beta,k) = \beta + \frac{k^2}{2}(1-\beta), \\ w_1^{zz}(\beta,k) &= w_2^{zz}(\beta,k) = c - \frac{k^2(1-c)}{4+2k-k^2}, \\ q_1^{zz}(\beta,k) &= q_2^{zz}(\beta,k) = \frac{2(1-c)}{4+2k-k^2}, \\ S_1^{zz}(\beta,k) &= S_2^{zz}(\beta,k) = \frac{2(1-\beta)(2-k^2)(1-c)^2}{(4+2k-k^2)^2}, \\ \Pi_1^{zz}(\beta,k) &= \Pi_2^{zz}(\beta,k) = \frac{4(1-c)^2}{(4+2k-k^2)^2}, \\ \text{and } E_1^{zz}(\beta,k) &= E_2^{zz}(\beta,k) = \frac{2\beta(2-k^2)(1-c)^2}{(4+2k-k^2)^2}. \end{aligned}$$

(3) When firm 1 offers wages and equity stakes and firm 2 offers wages only to their respective

[6] For each case, we use the first (second) superscript of each variable to indicate firm 1's (firm 2's) contract choice. Letter "w" represents "wage-based compensation" and letter "z" represents "equity-based compensation." For instance, $w_1^{zw}(\beta,k)$ refers to the equilibrium wage rate of firm 1, when firm 1 adopts an equity-based contract and firm 2 adopts a wage-based contract.

employees in the first period, we have

$$
\begin{aligned}
z_1^{zw}(\beta,k) &= \beta+\frac{k^2}{2}(1-\beta), w_1^{zw}(\beta,k)=c-\frac{k^2(2-k)(4+k\beta)(1-c)}{32-16k^2+\beta k^4}, \\
w_2^{zw}(\beta,k) &= c+\frac{\beta(4-k^2)(4-2k-k^2)(1-c)}{32-16k^2+\beta k^4}, \\
q_1^{zw}(\beta,k) &= \frac{2(2-k)(4+k\beta)(1-c)}{32-16k^2+\beta k^4}, q_2^{zw}(\beta,k)=\frac{2(2-\beta)(4-2k-k^2)(1-c)}{32-16k^2+\beta k^4}, \\
S_1^{zw}(\beta,k) &= \frac{2(1-\beta)(2-k^2)(2-k)^2(4+k\beta)^2(1-c)^2}{(32-16k^2+\beta k^4)^2}, \\
\Pi_1^{zw}(\beta,k) &= \frac{4(2-k)^2(4+k\beta)^2(1-c)^2}{(32-16k^2+\beta k^4)^2}, \\
S_2^{zw}(\beta,k) &= \Pi_2^{zw}(\beta,k)=\frac{4(2-\beta)^2(4-2k-k^2)^2(1-c)^2}{(32-16k^2+\beta k^4)^2}, \\
E_1^{zw}(\beta,k) &= \frac{2\beta(2-k^2)(2-k)^2(4+\beta k)^2(1-c)^2}{(32-16k^2+\beta k^4)^2}, \\
\textit{and } E_2^{zw}(\beta,k) &= \frac{2\beta(2-\beta)(1-c)^2(4-k^2)(4-2k-k^2)^2}{(32-16k^2+\beta k^4)^2}.
\end{aligned}
$$

When firm 1 offers wages only and firm 2 offer wages and equity stakes to their respective employees in the first period, the variables $w_1^{wz}(\beta,k)$, $w_2^{zw}(\beta,k)$, $z_2^{wz}(\beta,k)$, $q_i^{wz}(\beta,k)$, $\Pi_i^{wz}(\beta,k)$, $S_i^{wz}(\beta,k)$, and $E_i^{wz}(\beta,k)$ are characterized symmetrically.

3.3. Contract choice equilibrium in period 1

3.3.1. Equilibrium characterization

In period 1, each firm's non-employee owners simultaneously choose the forms of compensation contracts. The payoffs of non-employee owners are determined by the equilibrium outcomes of the bargaining games in period 2. For instance, if both firms compensate their respective employees with wages only, then the non-employee owners of both firms will end up with a payoff of $S_i^{ww}(\beta,k)$, which is given by Part (1) of Proposition 1. Using the expressions of $S_i^{ww}(\beta,k)$, $S_i^{zz}(\beta,k)$, $S_1^{zw}(\beta,k)$, and $S_2^{zw}(\beta,k)$ in Proposition 1, we can construct the payoff matrix of the game played by firms in period 1 and describe it in Fig. 2. Analyzing this payoff matrix, we can compute all the pure strategy equilibria in period 1, which are summarized in the following proposition.

[INSERT FIG. 2 HERE]

Proposition 2. *For any $k \in (0,1)$, there exist two threshold values of β, $\hat{\beta}_1(k)$ and $\hat{\beta}_2(k)$ (with $0 < \hat{\beta}_1(k) < \hat{\beta}_2(k) < 1$), which are functions of parameter k and respectively defined by equations (A11) and (A15) in the Appendix, such that the equilibrium is characterized as follows:*

(1) If $\beta \in \left(0, \hat{\beta}_1\right)$, then there is a unique equilibrium, in which both firms offer wages and equity stakes to their respective employees.

(2) If $\beta \in \left[\hat{\beta}_1, \hat{\beta}_2\right]$, then there are two pure strategy equilibria. In one equilibrium, both firms offer wages and equity stakes to their respective employees. In the other equilibrium, both firms offer wages only to their respective employees.

(3) If $\beta \in \left(\hat{\beta}_2, 1\right)$, then there is a unique equilibrium, in which both firms offer wages only to their respective employees.

Examining Proposition 2, we find that two types of equilibria are supported in period 1: either both firms offer an equity-based contract or both firms offer a wage-based contract. We refer to the first type of equilibrium as the "employee ownership equilibrium" and the second type as the "wage only equilibrium." We use Fig. 3 to plot the regimes of equilibrium types in the parameter space of (k, β). Generally speaking, as k becomes higher and β becomes lower, the employee ownership equilibrium is more likely to prevail.

[INSERT FIG. 3 HERE]

Corollary 1. *Firms are more likely to provide equity based compensation to their respective employees when the product market is more competitive and when employees have lower bargaining power (i.e., when k is higher and when β is lower).*

3.3.2. Special case: $k = 0$

We now examine the special case of $k = 0$, in which case each firm becomes a monopolist in its own product market. This analysis serves two purposes. First, it helps to develop the intuition of Proposition 2 for the general case of $k \in (0, 1)$. Second, it illustrates the importance of competition in generating the employee ownership equilibrium, as the wage only equilibrium always prevails as the unique equilibrium in this monopoly case.

Corollary 2. *Equity-based compensation is never in the interest of non-employee owners in the absence of competition. That is, if $k = 0$, the wage only equilibrium prevails as the unique equilibrium for any $\beta \in (0, 1)$.*

We use Fig. 4 to illustrate the intuition for Corollary 2. Here, we plot the period-2 Nash bargaining outcomes for firm i under a wage-based contract (in blue) and an equity-based contract (in red), respectively. In Fig. 4, we set the parameter values as follows: $\beta = 0.5, c = 0.2$, and $k = 0$.

Under the wage-based contract, the bargaining frontier is:

$$E_i = (1 - c)\sqrt{S_i} - 2S_i. \tag{13}$$

To obtain (13), we first insert $z_i = k = 0$ into (9) and (10) to find the expressions of S_i and E_i (which indicate the surplus of both non-employee owners and employees for a specific wage rate w_i), and then cancel w_i in these two expressions to get a relation between E_i and S_i (which indicates the possible combinations of (E_i, S_i) when w_i is varied under a wage-based contract). The Nash bargaining outcome is determined by the tangency of the bargaining frontier and the indifference curve implied by the objective function (6) in the bargaining game. This outcome is depicted as point N^w in Fig. 4.

[INSERT FIG. 4 HERE]

Under the equity-based contract, the bargaining frontier is:

$$S_i + E_i = \frac{(1-c)^2}{4}. \tag{14}$$

We obtain (14) as follows. Inserting $k = 0$ into equations (9) and (10), we have:

$$S_i = (1 - z_i)\frac{(1-w_i)^2}{4} \text{ and } E_i = \frac{(1-w_i)}{2}(w_i - c) + z_i\frac{(1-w_i)^2}{4}. \tag{15}$$

Using (15), we can get all the achievable bargaining outcomes (E_i, S_i) by varying w_i and z_i. For a given w_i, we first cancel z_i in (15) and link E_i and S_i as follows:

$$S_i + E_i = \frac{(1-w_i)}{2}(w_i - c) + \frac{(1-w_i)^2}{4}, \tag{16}$$

which indicates the total surplus that is available for non-employee owners and employees to divide under a specific wage rate w_i. We then choose w_i to maximize this total surplus in (16), which yields the bargaining frontier (14).

Direct computation shows that the total-surplus-maximizing wage rate is the market wage rate c. This result is intuitive, since each firm is a monopolist in its product market and the true marginal cost of labor is c. To understand this result, we use the definition of E_i, S_i, and Π_i, and compute the total producer surplus T_i for the case of $k = 0$ as follows:

$$T_i \equiv S_i + E_i = q_i (w_i - c) + \Pi_i = q_i (1 - q_i - c).$$

The total surplus is maximized at $q_i = \frac{1-c}{2}$, which is exactly the product quantity implied by equation (7) with $w_i = c$ and $k = 0$. Again, the bargaining outcome, depicted as point N^z in Fig. 4, is determined by the tangency of the bargaining frontier and the indifference curve.

Comparing (13) with (14), which are depicted in Fig. 4, we have the following two observations. First, the frontier under the wage-based contract lies inside the frontier under the equity-based contract. This outcome arises because the wage rate w_i under the wage-based contract is generally different from the market wage rate c, which causes the product quantity to be distorted from the value that maximizes total surplus. The two frontiers (13) and (14) coincide only at the S-intercept, at which point non-employee owners get all of the surplus and employees get none (at this point, under the wage-based contract, the wage rate w_i is also set to be c and thus, the production is total-surplus efficient). Hence, by committing to wage bargaining, the non-employee owners commit to bargaining over a set of inefficient production quantities. This consideration tends to make equity-based compensation more attractive.

Second, wage bargaining causes the frontier in (13) to pivot inward while the frontier in (14) is linear under the equity-based contract. The bargaining frontier in (14) is linear for equity-based compensation, because the bargaining is directly over the maximized total surplus T_i. Given the Cobb-Douglas preference (6), the resulting bargaining outcome is that employees obtain a share β of the total surplus. In contrast, with wage bargaining, the surplus that employees can extract is limited by the fact that every increase in the wage rate also reduces output and therefore the total surplus, and as a result, wage bargaining causes the frontier in (13) to pivot inward. This inward pivot renders the indifference curve to be steeper at the tangency point N^w than at the tangency point N^z, leading to a Nash bargaining outcome that is more favorable to non-employee shareholders, despite wage bargaining leading to less efficient production and lower total surplus. Taken together, in the period-1 equilibrium, self-interested non-employee shareholders will choose to offer a wage-based contract in the monopoly case.

3.3.3. General intuition

Having presented the monopoly case, we now consider the competitive case with $k > 0$. Proposition 2 shows that the equilibrium type depends on employees' bargaining power β. We use Fig. 5 to illustrate the intuition. We first conduct an analysis similar to Fig. 4 and plot the period-2 Nash bargaining outcomes for firm i, when β takes a low value of 0.1 (in Panel a1 of Fig. 5) or when β takes a high value of 0.9 (in Panel a2 of Fig. 5). In both panels, the other parameters are set at $k = 0.8$ and $c = 0.2$, and we assume that firm j offers an equity-based contract to its employees. We then use Panel b of Fig. 5 to present the best response functions of each firm and the resulting Nash equilibrium in period 1.

[INSERT FIG. 5 HERE]

Similar to obtaining equation (13), we can use equations (9) and (10) to compute the frontier under the wage-based contract as follows:

$$E_i = \left[1 - c - \frac{k}{2}(1 - w_j)\right]\sqrt{S_i} - \left(2 - \frac{k^2}{2}\right)S_i. \tag{17}$$

We can also compute the following bargaining frontier under the equity-based contract:

$$E_i + S_i = \frac{(2 - 2c - k + kw_j)^2}{8(2 - k^2)}, \tag{18}$$

which, as in the monopoly case, is obtained by choosing the wage rate w_i to maximize the total surplus,

$$\begin{aligned} T_i(w_i, w_j, k, c) &\equiv E_i(z_i, w_i, w_j, k) + S_i(z_i, w_i, w_j, k) \\ &= \frac{2 - k - 2w_i + kw_j}{4 - k^2}(w_i - c) + \left(\frac{2 - k - 2w_i + kw_j}{4 - k^2}\right)^2, \end{aligned} \tag{19}$$

where the second equality follows from the expressions of $S_i(z_i, w_i, w_j, k)$ and $E_i(z_i, w_i, w_j, k)$ in equations (9) and (10).

Equations (17) and (18) respectively extend equations (13) and (14) to the general case of $k \geq 0$. In equations (17) and (18), w_j is fixed at their corresponding period-2 bargaining game equilibrium values specified by Proposition 1. For instance, if both firms offer an equity-based contract (e.g., the red line in Panels a1 and a2 of Fig. 5), then w_j takes the value of $w_j^{zz}(\beta, k) = c - \frac{k^2(1-c)}{4+2k-k^2}$ by Part (2) of Proposition 1.

In the top panels of Fig. 5, the bargaining frontier is still linear under the equity-based contract, while it pivots inward under the wage-based contract, for the same reason as in Fig.

4. However, unlike Fig. 4, the two frontiers no longer coincide on the S-axis where the non-employee owners get all the surplus. For instance, in Panel a1 of Fig. 5, the frontier (18) now has a higher total surplus than the frontier (17). This is because with wage compensation, the wage rate w_i is equal to c at the S-intercept of equation (17), while at the S-intercept of equation (18), the wage rate w_i is smaller than c, which makes firm i more competitive in the period-3 product market and leads to a higher total surplus, taking the rival's contract as given. The more competitive is the product market, the larger the gap between the two frontiers, because firm i can obtain a larger market share from the product market by committing to a lower wage rate and hence a lower marginal production cost.

The outward shift of the bargaining frontier under the equity-based contract resembles something like an income effect. This income effect favors the choice of the equity-based contract, as it increases the total surplus T_i and hence the non-employee owners' payoff S_i (recall that $S_i = (1 - \beta) T_i$ at point N^z which depicts the Nash bargaining outcomes for the equity-based compensation). By contrast, as we discussed for the monopoly case, the inward bending frontier under the wage-based contracts makes the indifference curve steeper at the tangency point N^w than at the tangency point N^z. This effect resembles a substitution effect, and it favors the choice of the wage-based contract. In Panel a1 of Fig. 5, where β is low, the income effect dominates so that firm i chooses to offer an equity-based contract. In Panel a2 of Fig. 5, where β is high, the opposite is true.

The difference between Panels a1 and a2 can also be understood from the expression of S_i in Proposition 1, the variable determining the contract choice of non-employee owners of firm i. Under equity-based compensation, non-employee owners' payoff S_i linearly decreases with β, while S_i is convex in β under wage-based compensation. Intuitively, under the equity-based contract, employees simply negotiate a share β of the total surplus T_i. Under the wage-based contract, however, an increase in β not only raises employees' share of the pie but also reduces the pie itself by driving up the wage and hence driving down the total surplus. The latter effect mitigates the negative effect of an increase in β on the negotiated payoff S_i. Thus, as we increase β from 0.1 in Panel a1 of Fig. 5 to 0.9 in Panel a2 of Fig. 5, S_i decreases less under the wage-based contract than under the equity-based contract, which explains why non-employee owners start to favor wage-based compensation once β becomes sufficiently high.

Panel b of Fig. 5 presents the best response functions of each firm and the resulting period-1 equilibrium. Specifically, when employees' bargaining power β is sufficiently low, offering the equity-based contract is a dominant strategy for each firm, and thus the employee ownership equilibrium prevails as the unique equilibrium in period 1. By contrast, when β is sufficiently high, offering the wage-based contract becomes the dominant strategy for each firm, which makes the wage only equilibrium the unique equilibrium in period 1. When β takes an intermediate value, each firm's best response depends on the choices of its rival. If firm j adopts an equity-based contract (a wage-based contract, respectively), it is in firm i's interest to adopt an equity-based contract (a wage-based contract, respectively) as well. As a result, both the employee ownership equilibrium and the wage only equilibrium can be supported as a pure strategy equilibrium in period 1.

Note that under the employee ownership equilibrium, the negotiated wage rate w_i is below the market wage rate c, i.e., $w_i^{zz}(\beta, k) = c - \frac{k^2(1-c)}{4+2k-k^2} < c$ as long as $k > 0$. As we mentioned above, this outcome arises because under equity-based compensation, each firm maximizes the total surplus in (19) and chooses a lower wage rate, in order to generate a lower marginal production cost and thus become more competitive in the product market. As a result of the lower negotiated wage rate, the negotiated share z exceeds β (i.e., $z_i^{zz}(\beta, k) =$

$\beta + \frac{k^2}{2}(1-\beta) > \beta$ for $k > 0$), because employees want not only their share β of profits but also need to be reimbursed for accepting a below-market wage rate w_i^{zz}.

This insight is comparable to an insight generated from the "top-dog" strategy (Fudenberg and Tirole, 1984), where a firm commits to be tough in the product market in order to get a rival firm to retreat. Brander and Lewis (1986) and Fershtman and Judd (1987) also propose related ideas. In Brander and Lewis (1986), shareholders strategically take on debt to become more aggressive in the product market. Higher debt levels force equity holders to restrict attention on higher marginal profit states, which leads to more aggressive production decisions. In Fershtman and Judd (1987), shareholders write contracts rewarding managers for maximizing revenues instead of profits, which encourages managers to make more aggressive decisions in the product market.

3.4. Comparative statics

3.4.1. Comparative statics with respect to β

By Proposition 2, either an employee ownership equilibrium or a wage only equilibrium prevails in period 1. Thus, we first conduct comparative statics with respect to parameter β on these two equilibria respectively, and then make predictions on the period-1 outcomes that take into account equilibrium switches.

Proposition 3. *(1) In the wage only equilibrium, if the employees' bargaining power β increases, then: the employee wage $w_i^{ww}(\beta,k)$ and the employee compensation $E_i^{ww}(\beta,k)$ increase; and the firm production $q_i^{ww}(\beta,k)$, the firm profits $\Pi_i^{ww}(\beta,k)$, and the non-employee owners' residual profit $S_i^{ww}(\beta,k)$ decrease. That is, $\frac{\partial w_i^{ww}(\beta,k)}{\partial \beta} > 0$, $\frac{\partial q_i^{ww}(\beta,k)}{\partial \beta} < 0$, $\frac{\partial S_i^{ww}(\beta,k)}{\partial \beta} < 0$, $\frac{\partial \Pi_i^{ww}(\beta,k)}{\partial \beta} < 0$, and $\frac{\partial E_i^{ww}(\beta,k)}{\partial \beta} > 0$, for $i = 1, 2$.*

(2) In the employee ownership equilibrium, if the employees' bargaining power β increases,

then: the equilibrium portion $z_i^{zz}(\beta, k)$ *of equity stakes and the employee compensation* $E_i^{zz}(\beta, k)$ *increase; the employee wage* $w_i^{zz}(\beta, k)$*, the firm production* $q_i^{zz}(\beta, k)$*, and the firm profits* $\Pi_i^{zz}(\beta, k)$ *do not change; and the non-employee owners' residual profit* $S_i^{zz}(\beta, k)$ *decreases. That is,* $\frac{\partial z_i^{zz}(\beta,k)}{\partial \beta} > 0$, $\frac{\partial w_i^{zz}(\beta,k)}{\partial \beta} = \frac{\partial q_i^{zz}(\beta,k)}{\partial \beta} = \frac{\partial \Pi_i^{zz}(\beta,k)}{\partial \beta} = 0$, $\frac{\partial S_i^{zz}(\beta,k)}{\partial \beta} < 0$*, and* $\frac{\partial E_i^{zz}(\beta,k)}{\partial \beta} > 0$*, for* $i = 1, 2$*.*

The intuition for Proposition 3 is as follows. In the wage only equilibrium, employees can bargain over their wages only. Thus, when their bargaining power increases, they end up with a higher wage rate in equilibrium. This increased wage rate in turn increases the production cost of the firm, and in response, the firm produces less and has a lower profit. Higher bargaining power also makes employees better off while making non-employee owners worse off.

In the employee ownership equilibrium, the equilibrium wage rate is not affected by employee bargaining power. Recall that under equity-based compensation, the wage rate is set such that the total surplus is maximized, and thus the equilibrium wage rate is independent of bargaining power. The resulting total surplus is divided proportionally depending on which party has more bargaining power. As a result, the employees' equity share increases with employee bargaining power. Because the wage rate, and hence the firm's cost structure, are independent of the employees' bargaining power, so are the production levels and profits of the firm. As before, it is intuitive that when employees have more bargaining power, they will be better off overall, while the non-employee owners are worse off.

Using Proposition 3, we have the following corollary on the period-1 equilibrium outcomes z_i^* and w_i^* that takes into account equilibrium switches as k changes:

Corollary 3. *For any* $k \in (0, 1)$*, as the employees' bargaining power* β *monotonically increases from* 0 *to* 1*, the equilibrium portion* z_i^* *of equity stakes first monotonically increases* *and then jumps down to* 0*, while the equilibrium wage rate* w_i^* *first keeps at a constant and then jumps upward and monotonically increases.*

3.4.2. Comparative statics with respect to parameter k

Proposition 4. *(1) In the wage only equilibrium, an increase in the product market competition parameter* k *will decrease the employee wage* $w_i^{ww}(\beta, k)$*, the firm production* $q_i^{ww}(\beta, k)$*, the firm profits* $\Pi_i^{ww}(\beta, k)$*, the non-employee owners' residual profit* $S_i^{ww}(\beta, k)$*, and the employee compensation* $E_i^{ww}(\beta, k)$*. That is,* $\frac{\partial w_i^{ww}(\beta,k)}{\partial k} < 0$, $\frac{\partial q_i^{ww}(\beta,k)}{\partial k} < 0$, $\frac{\partial \Pi_i^{ww}(\beta,k)}{\partial k} < 0$, $\frac{\partial S_i^{ww}(\beta,k)}{\partial k} < 0$*, and* $\frac{\partial E_i^{ww}(\beta,k)}{\partial k} < 0$*, for* $i = 1, 2$*.*

(2) In the employee ownership equilibrium, an increase in the product market competition parameter k *will increase the equilibrium portion* $z_i^{zz}(\beta, k)$ *of equity stakes, but decrease the employee wage* $w_i^{zz}(\beta, k)$*, the firm production* $q_i^{zz}(\beta, k)$*, the firm profits* $\Pi_i^{zz}(\beta, k)$*, the non-employee owners' residual profit* $S_i^{zz}(\beta, k)$*, and the employee compensation* $E_i^{zz}(\beta, k)$*. That is,* $\frac{\partial z_i^{zz}(\beta,k)}{\partial k} > 0$, $\frac{\partial w_i^{zz}(\beta,k)}{\partial k} < 0$, $\frac{\partial q_i^{zz}(\beta,k)}{\partial k} < 0$, $\frac{\partial \Pi_i^{zz}(\beta,k)}{\partial k} < 0$, $\frac{\partial S_i^{zz}(\beta,k)}{\partial k} < 0$*, and* $\frac{\partial E_i^{zz}(\beta,k)}{\partial k} < 0$*, for* $i = 1, 2$*.*

In Proposition 4, all variables except z_i decreases with product market competition k in both types of equilibrium. The intuition is as follows. It is natural that competitive pressure tends to reduce firms' profits Π_i and hence non-employee owners' welfare S_i. As a result, in a more competitive product market, firms can only afford to pay lower wage rates w_i. Employees are worse off because of the lower total wages and in case of equity compensation, lower profits. In the employee ownership equilibrium, as the product market becomes more competitive, firms have a greater incentive to lower their wage rates by offering larger equity stakes to their employees, which explains why the equilibrium equity stake $z_i^{zz}(\beta, k)$ increases with k.

Proposition 4 implies the following corollary on the period-1 equilibrium outcomes z_i^* and w_i^* that takes into account equilibrium switches as β changes:

Corollary 4. *When β is high, as the product market competition parameter k monotonically increases from 0 to 1, the equilibrium portion z_i^* of equity stakes is kept constant at 0, while the equilibrium wage rate w_i^* monotonically decreases. When β is low, as k monotonically increases from 0 to 1, z_i^* first stays constant at 0 and then jumps upward and monotonically increases, while w_i^* first monotonically decreases and then jumps downward and monotonically decreases.*

4. Efficiency analysis

In this section, we explore the normative implications of offering employees an equity stake. Given that our focus is on the choice of compensation contracts, which occurs in period 1, we will keep the period-2 and period-3 subgames at their equilibria, respectively. This approach is standard in the literature. For instance, in the efficiency analysis of Goldstein, Ozdenoren, and Yuan (2013), the authors vary the level of coordination in investors' trading decisions, which is the focus of their paper, while keeping all the other equilibrium features of the economy. We compare settings where either both sets of owners choose a wage-based contract or both sets of owners choose an equity-based contract. This treatment is also appealing in the sense that these two options are the only possible equilibrium outcomes characterized in Proposition 2.

The variables of interest are: q_i, the production of each firm; Π_i, the profits of each firm; S_i, the residual profits of the non-employee owners; and E_i, the compensation of employees. It is clear that variables S_i and E_i capture the welfare of non-employee owners and employees, respectively. Variables q_i and Π_i serve two purposes. First, they measure the production efficiency associated with each firm. Note that by Lemma 1, we have $\Pi_i = q_i^2$ in equilibrium (which is implied by the first-order condition of each firm's production decisions in period 3), and so these two production efficiency measures are essentially the same. Second, Π_i and q_i also measure consumers' welfare. Specifically, by inserting the consumers' demand function (2) into their objective function (1) and by noting $q_1 = q_2$, we can compute the consumers' utility evaluated at the period-3 product market equilibrium as follows:

$$U(q_1, q_2) = (1+k)\,\Pi_i = (1+k)\,q_i^2. \tag{20}$$

Therefore, more efficient production ultimately improves consumers' welfare.

Using Proposition 1, we can characterize the efficiency implications of offering equity compensation as follows.

Proposition 5. *Relative to both firms offering the wage-based contract, when both firms offer the equity-based contract, we have:*

(1) Firm production and profits that are higher, i.e., $q_i^{zz}(\beta, w) > q_i^{ww}(\beta, w)$ *and* $\Pi_i^{zz}(\beta, w) > \Pi_i^{ww}(\beta, w)$ *for* $i = 1, 2$*;*

(2) Consumers that are better off, i.e., $U\left(q_1^{zz}(\beta, w), q_2^{zz}(\beta, w)\right) > U\left(q_1^{ww}(\beta, w), q_2^{ww}(\beta, w)\right)$*;*

(3) Non-employee owners that are worse off, i.e., $S_i^{zz}(\beta, w) < S_i^{ww}(\beta, w)$ *for* $i = 1, 2$*; and*

(4) Employees that are better off if and only if their bargaining power is sufficiently high, i.e.,

$$E_i^{zz}(\beta, w) > E_i^{ww}(\beta, w) \ \ (\textit{for } i = 1, 2) \Longleftrightarrow \beta > Max\left\{0, \hat{\beta}_3(k)\right\},$$

where $\hat{\beta}_3(k) \equiv \frac{2\left(-k^3+6k^2+4k-8\right)}{k^2(2+k)}$.

As we discuss above, equity-based contracts enable firms to lower their negotiated wage rates. These lower wage rates lead to a lower marginal cost of production, a greater volume of production, and more profits for each firm. From equation (20), consumers also benefit from the resulting greater production. This explains Parts (1) and (2) of Proposition 5.

Part (3) of Proposition 5 shows that both groups of non-employee owners are strictly worse off under equity-based contracts than under wage-based contracts. This welfare result has two implications for the properties of the equilibrium characterized by Proposition 2.

First, according to Part (1) of Proposition 2, when $\beta \in \left(0, \hat{\beta}_1\right)$, each firm's non-employee owners offering equity compensation is a dominant strategy and thus, the employee ownership equilibrium constitutes the unique equilibrium in period 1. However, by Part (3) of Proposition 5, both sets of non-employee owners would be better off had they been able to commit to compensating employees with wages only. Thus, while the prevailing employee ownership equilibrium is a unique pure strategy equilibrium when $\beta < \hat{\beta}_1$, it is also a Prisoner's Dilemma for each set of non-employee owners.

The intuition for this result is as follows. Although the focal firm can negotiate a lower wage rate w_i and make itself more profitable by adopting the equity-based contract, the competing firm's best response to this decision is to *also* offer equity-based compensation when $\beta < \hat{\beta}_1$. When both firms adopt the equity-based contract, there is parity in wages across both firms, ex post, resulting in neither firm having a competitive cost advantage over the other. However, each firm's non-employee owners still retain a diluted position in the firm as a result of providing an equity stake to their employees. Taken together, each firm's owners would have been better off, had they both been able to commit to providing wages only in the game's first period.[7]

Second, by Part (2) of Proposition 2, when $\beta \in \left[\hat{\beta}_1, \hat{\beta}_2\right]$, both the employee ownership equilibrium and the wage only equilibrium are supported. Nonetheless, according to Part (3) of Proposition 5, the latter equilibrium Pareto dominates the former in terms of non-

[7] Firms can also end up with a Prisoner's Dilemma in the settings derived in Brander and Lewis (1986) and Fershtman and Judd (1987), where each firm tries to become more aggressive in the product market by committing to taking on more debt or by rewarding managers with revenues more than profits. The commitment seems to be more credible in our setting, because wages and employee ownership are more visible and recontracting is more costly than it is when dealing with individual creditors or managers.

employee owners' payoff.

Corollary 5. *(1) When $\beta \in \left(0, \hat{\beta}_1\right)$, the employee ownership equilibrium prevails as the unique equilibrium in period 1 and it is a Prisoner's Dilemma from non-employee owners' perspective.*
(2) When $\beta \in \left[\hat{\beta}_1, \hat{\beta}_2\right]$, among the two pure strategy equilibria in period 1, the wage only equilibrium Pareto dominates the employee ownership equilibrium, in terms of non-employee owners' welfare.

Part (4) of Proposition 5 examines the welfare of employees for each firm. Relative to wage-based contracts, offering equity-based contracts makes employees face a trade-off. First, under equity-based contracts, their wage rate is lower, and so they suffer in terms of total wages. Second, equity-based compensation provides employees with a benefit by allowing them to get a portion of the firm's profits. When employee bargaining power β is relatively high, employees can bargain for a larger equity stake, and thus their benefit from profit sharing is larger than the cost to reduced wages. This outcome makes employees better off overall. By contrast, when β is sufficiently low, employees are given a smaller equity stake, ex post, and in this case lower wages outweigh the profit sharing benefits, so that employees of each firm are worse off than if their respective owners had offered them wages only.

[INSERT FIG. 6 HERE]

This result, together with Part (1) of Corollary 5, implies that when the employee ownership equilibrium is the unique supported equilibrium in period 1, all the supply-side stakeholders of each firm—i.e., employees and non-employee owners—can be worse off than if both firms' non-employee owners could commit to offering wages only. When this outcome happens, we label the prevailing equilibrium as a "firm inefficient equilibrium," since the welfare of all stakeholders within the firm could be improved. We use Fig. 6 to plot the region of parameters (k, β) that supports firm inefficient equilibria. The formal characterization of this region is provided in the following corollary.

Corollary 6. *When* $\beta < Min\left\{\hat{\beta}_1, Max\left\{0, \hat{\beta}_3\right\}\right\}$, *the employee ownership equilibrium is the unique pure strategy equilibrium and it is firm inefficient.*

Notably, insights from Kim, and Ouimet (2014) and Faleye, Mehrotra, and Morck (2006) suggest that, when employees are granted large equity stakes in the company, it is possible that all parties become worse off. It seems puzzling that a decision to grant large equity stakes could arise endogenously if those equity stakes make both non-employee owners and employees worse off. Corollary 6 provides a potential explanation for this puzzle. That is, when $\beta < Min\left\{\hat{\beta}_1, Max\left\{0, \hat{\beta}_3\right\}\right\}$, the firm inefficient equilibrium arises endogenously as the unique equilibrium. In this equilibrium, each firm's owners offer their employees equity stakes and, at the same time, both non-employee owners and employees are worse off than if each firm's owners had offered their employees wages only.

5. Empirical implications

In this section, we first use our analysis to explain several existing empirical findings in the literature and then make testable predictions on the incidence of equity compensation.

5.1. The UAW example

We use the analysis in Section 3.4.1. to generate insights regarding the 2009 contract negotiations between the United Auto Workers (UAW) and General Motors and Chrysler. The auto industry is characterized by intense inter-industry competition and direct labor hours are a significant input in the production process for the industry. These two features fit well with our model setting. As Cody (2015) pointed out, the UAW entered this particular contract negotiation with reduced bargaining power due to the Financial Crisis of 2008. Thus, this experiment corresponds to a decrease in parameter β.

[INSERT FIG. 7 HERE]

We use Fig. 7 to graphically illustrate the comparative statics with respect to parameter β. Here, we set $k = 0.8$ and $c = 0.2$, and plot the following equilibrium variables against employees' bargaining power β in Panels a – f, respectively: the equilibrium portion of employee equity ownership, z_i^*; the equilibrium employee wage rate, w_i^*; the equilibrium production of firms, q_i^*; the equilibrium profit of firms, Π_i^*; the equilibrium payoff of non-employee owners, S_i^*; and the equilibrium payoff of employees, E_i^*. Solid curves indicate that an employee ownership equilibrium prevails (i.e., $z_i^* > 0$ in Panel a), while dashed curves indicate that a wage only equilibrium is supported (i.e., $z_i^* = 0$ in Panel a). All panels are fully consistent with Proposition 3 and Corollary 3 in Section 3.4.1..

Several outcomes that followed the 2009 negotiations are consistent with Fig. 7. First, following negotiations, both firms agreed to provide increased equity stakes and profit-sharing arrangements to their respective employees. As a result, UAW members became some of the largest employee-owners of publicly-traded stock in the U.S.[8] This outcome can be qualitatively explained by what we might expect to occur in Panel a of Fig. 7, following a negative shock to employee bargaining power, β. For instance, suppose that the UAW's initial bargaining parameter β is relatively high, say, $\beta = 0.5$, so that the wage only equilibrium initially prevailed and the equilibrium employee ownership z_i^* is zero. Next, lets assume that, consistent with the evidence, prior to the 2009 contract negotiation, β drops, say, to a level of 0.1. Following this drop, the unique equilibrium switches to the employee ownership

[8] Employees own roughly 17.5% of G.M.'s shares and 55% of Chrysler's shares; see http://www.heritage.org/research/reports/2012/06/auto-bailout-or-uaw-bailout-taxpayer-losses-came-from-subsidizing-union-compensation.

equilibrium, with each firm's employees garnering an equity stake of $z_i^* \approx 0.4$.

Second, in return for large equity stakes and profit sharing arrangements, UAW employees made concessions to their cash wages (e.g., entry position hourly wages were reduced to \$14/hour).[9] This result is consistent with Panel b of Fig. 7: for example, as β decreases from 0.5 to 0.1, the equilibrium wage rate w_i^* decreases from 0.3 to 0.1. Finally, consistent with Panels c and d of Fig. 7, the wage concessions appear to have increased both production and profits at both companies in the period following negotiations.

5.2. An application to equity stake sizes

The literature has documented mixed evidence regarding the firm and stakeholder benefits to adopting employee ownership plans in publicly-traded companies (e.g., Blasi, Conte, Jampani, and Kruse, 1996). In particular, there have been several puzzling findings related to the *size* of employee equity stakes and various stakeholder outcomes. For example, Kim and Ouimet (2014) suggest that when a firm compensates employees with small equity stakes in the company, not only is the firm more profitable, but employees are better off following the equity grant. Conversely, findings in Kim and Ouimet (2014) and Faleye, Mehrotra, and Morck (2006) suggest that when employees are granted a large equity stake in the company, negative outcomes can often ensue for all parties. In addition, Kim and Ouimet (2014) document that wage concessions are often made when employees are compensated with large equity stakes.

The analysis in Section 3.4.2. may be useful for understanding these various observations regarding the size of equity stakes in employee compensation. As in Fig. 7, we use Fig. 8 to plot the equilibrium values of z_i^*, w_i^*, q_i^*, Π_i^*, S_i^*, and E_i^* against product market competitiveness k, when the other parameters are set as $\beta = 0.1$ and $c = 0.2$. All panels in

[9] http://labornotes.org/2011/05/unequal-pay-equal-work.

Fig. 8 are consistent with Proposition 4 and Corollary 4.

[INSERT FIG. 8 HERE]

Consider the following thought experiment. Suppose that k initially takes a value of 0.4 in Fig. 8. According to Panel a, the unique equilibrium is the wage only equilibrium, i.e., $z_i^* = 0$, because the product market is not competitive enough to support an employee ownership equilibrium. Now suppose that k increases *slightly*, say, to a value of 0.45. Then, the employee ownership equilibrium can be supported in Panel a, yielding a relatively small employee equity portion, $z_i^* \approx 0.2$. Examining Panels d and f, there will be upward jumps in firm profits Π_i^* and employee compensations E_i^* accordingly. Additionally, employees are better off because wages are still relatively high and they get to enjoy a small share of the firm's profits when the firm's profits are also relatively large due to lower competition intensity at $k = 0.45$.

By contrast, suppose that k increases *significantly*. For example, suppose that k doubles, so that $k = 0.8$. In this case, Panel a shows that the unique equilibrium is the employee ownership equilibrium, with a relatively large value for the equilibrium employee equity stake, $z_i^* \approx 0.4$. That is, as k increases from 0.4 to 0.8, the employee equity stake z_i^* increases from 0 to 0.4. Meanwhile, accompanying this increase in z_i^*, the equilibrium wage rate w_i^* experiences a large drop from 0.22 to 0.08 in Panel b. This outcome maps well into the Kim and Ouimet (2014) finding that wages often decrease—i.e., equity grants and wages are substitutes—when employees are compensated with very large equity stakes. In addition, in Panel d, when k increases from 0.4 to 0.8, the equilibrium firm profit Π_i^* is almost flat.

Finally, in Panels e and f, when k increases from 0.4 to 0.8, both non-employee owners and employees are worse off. Non-employee owners are always worse off with the employee ownership equilibrium due to the Prisoner's Dilemma we describe before. Employees are worse off because employees make steep concession in their wage rate, and although they also now enjoy a larger share of the firm's profits, the firm's profits are also comparatively low because competitive pressures are high at $k = 0.8$. Our results highlight the intensity of product market competition as a primary determinant of owners' decision to grant large equity stakes to their employees. Our results also illustrate why the decision to grant large equity stakes might arise endogenously, even when such a decision might make both non-employee owners and employees worse off.

5.3. Empirical predictions on incidence of employee ownership

In this subsection, we make three testable predictions regarding when firms are likely to grant equity-based compensation to their employees. The first two predictions concern parameters k and β respectively, which are derived from Corollary 1. Some of these predictions are consistent with the available evidence, while others need to be tested. Along the discussion, we also highlight the unique features of our predictions and make suggestions on how to differentiate our theory from alternative ones.

Prediction 1 *Equity-based compensation is more common among firms whose product markets are more competitive.*

This prediction comes directly from Corollary 1. We might expect to see the employee ownership equilibrium arise following positive shocks to industry competitiveness (i.e., positive shocks to k that lead to $\beta < \hat{\beta}_1(k)$). For example, the Airline Deregulation Act in 1978 led to more heated price competition in the airline industry (Cappelli, 1985). Consistent with Prediction 1, by the mid-90s at least 11 major airline carriers compensated employees with significant equity stakes in their respective companies, with United Airlines becoming the first majority employee-owned, publicly-traded American airline in 1994.[10]

Broader supporting evidence comes from Oyer and Schaefer's (2005) analysis. They find that industry volatility is positively and significantly related to option-based plans. Oyer and Schaefer (2005) argue that this finding is consistent with retention theory (Oyer, 2004) and sorting theory (Lazear, 2004). However, this result may also be consistent with Prediction 1. Specifically, Peress (2010) documents that firms with more market power have less volatile profits, because market power acts as a hedge that allows firms to pass shocks on to their customers. Therefore, industry volatility can be viewed as a proxy for product market competitiveness, which is positively associated with the existence of equity-based plans in Oyer and Schaefer (2005), and also consistent with our theory.

Prediction 2 *Equity-based compensation is more common among firms whose employees have a lower bargaining leverage.*

This prediction is implied by Corollary 1. Prediction 2 is broadly consistent with the observation that equity-based compensation is *less* prominent in highly unionized settings than it is in non-unionized settings, ceteris paribus (McCarthy, Voos, Eaton, Kruse, and Blasi, 2011). Notably, unionized firms tend to have more negotiation leverage over their employers than their non-unionized counterparts, in part, because their ability to strike and bargain collectively allows them to hold up the firm. Consistent with this evidence, our model would predict that owners are better off paying their unionized employees (i.e., high-β employees) higher wages but also retaining all of the profits, than offering their employees an equity stake in the company, paying much lower wages, but also retaining a much smaller portion of the profits.

Prediction 2 can also potentially differentiate our story from various models of retention

[10] http://dept.kent.edu/oeoc/publicationsresearch/winter1999-2000/employeeownershipintheairlines.htm.

theory (Oyer, 2004). The retention model predicts that employee ownership will be more common when firms compete for the same set of workers. Note that our model may predict a competing tension, because when firms compete for the same workforce, the bargaining power of that labor force should be higher. In turn, according to Prediction 2, owners should be less likely to offer equity-based compensation when firms are competing for the same workforce.

Prediction 3 *Other things being equal, Predictions 1 and 2 are more significant among labor intensive firms/industries than capital intensive firms/industries.*

Our theory relies on the assumption that labor is an important variable input in the production process. Thus, we expect that Predictions 1 and 2 are stronger among those firms or industries where labor is a more important factor of production. This prediction is unique to our theory, as it is unclear how labor intensity affects the incidence of employee ownership through other channels.

6. Discussions

6.1. Interactions between employee compensation and product market decisions

In our analysis, the structure of employee compensation can affect product market decisions. This outcome arises because institutionally, firms and employees bargain over primarily wages and possibly equity stakes, and it is the negotiated wages (as variable costs in the product market) that link bargaining outcomes and production decisions. Specifically, the crux of our analysis is that the negotiation outcomes can drive both the size and the split of the total surplus, mediated through wages that may be either above (in the wage only equilibrium) or below (in the employee ownership equilibrium) the exogenous market wage rate c. Given that employees' outside option is always fixed at c, the exogenous market rate c is the true marginal cost in determining the total surplus $T_i = q_i(P_i - c)$. Thus, the negotiated wage rate w_i under both contracts has no direct effect on the total surplus T_i, but does have an indirect effect on T_i through its impact on the firm's product decisions q_i, because q_i is a function of w_i.

To illustrate that it is the ability to bargain over the wage rate that leads to variation in production outcomes across the equilibria, we attempt to separate employee compensation and product market decisions by considering an alternative hypothetical contract. Under this hypothetical contract, the parties only bargain over the equity stake z_i, and not over the wage rate w_i. Instead, the wage rate w_i is set exogenously, say, at the market rate c. In this setting, the period-3 production q_i and the total surplus T_i in the period-2 bargaining game are completely determined by the period-3 product market equilibrium. For instance, if $w_1 = w_2 = c$, then by equation (7), the period-3 production quantity is $q_i = \frac{1-c}{2+k}$, and by equation (19), the total surplus is $T_i = \left(\frac{1-c}{2+k}\right)^2$, for $i = 1, 2$. The bargaining outcome is that non-employee owners and employees divide the predetermined total surplus according to their respective bargaining power, i.e., $S_i = (1 - \beta) T_i$ and $E_i = \beta T_i$. Thus, the equilibrium under this hypothetical contract delinks the interactions between the period-2 bargaining outcomes and the period-3 production outcomes.

However, this hypothetical contract is useful mostly for theoretical considerations. It has little to say about how employees are paid, since their wages are set exogenously. In addition, when employees have the ability to negotiate compensation above the competitive market price (i.e., $\beta > 0$), both the equity stake and the wage rate should be negotiable, and in turn endogenous. In this sense, the two contracts with an endogenous wage rate, which are considered in our model, are more empirically relevant than the hypothetical contract with an exogenous wage rate.

6.2. Discussions on model assumptions

Risk aversion and uncertainty. To convey the idea most effectively, our model does not feature uncertainty. Thus the risk preferences of the employees play no role in equilibrium. Specifically, absent uncertainty, the value of one dollar of wages is equivalent to one dollar of firm profit, and employees will negotiate the same contracts irrespective of whether they are risk neutral, risk averse, or risk seeking. As we mention in the Introduction, this setting highlights the novelty of our results, because uncertainty and unobservability are crucial to generate an equity compensation equilibrium in alternative theories (e.g., Hölmstrom, 1979; Oyer, 2004). We expect that our results continue to hold in a setting with uncertainty and risk-averse agents. Note that, in the presence of uncertainty, a risk-averse agent should value \$1 of expected equity returns less than \$1 of guaranteed wages (Guay, 1999). To capture this idea, we have analyzed a variation in which employees impose a discount on company profits (i.e., the returns on their equity stakes) relative to their wages in the employees' objective function given by equation (4). We find that the general tenor of our results continues to hold, i.e., both firms' non-employee owners will offer wages and an equity stake to their employees when the employees' bargaining power is sufficiently low and the product market is sufficiently competitive.

Correlated β and k. The UAW example in Section 5.1. suggests that in some empirical settings, product market competition k may be negatively correlated with employee bargaining power β. In the Online Appendix, we have analyzed a setting in which the product market competition and labor power are perfectly negatively correlated (i.e., $k = 1 - \beta$). We show that our results are robust to this specification. There exists a threshold of β under which the employee ownership equilibrium prevails as the unique equilibrium. The β-threshold is no longer a function of k as in the baseline model, since we have set $k = 1 - \beta$. Instead, the

β-threshold is a real number, $\bar{\beta}_1 \approx 0.23712$. In short, the main theme of our findings remains the same when running a specification where β and k are perfectly negatively correlated.

Bertrand product market. In our analysis, we have assumed that the product market features Cournot competition. We choose the Cournot framework because Singh and Vives (1984) show that if firms can precommit to compete in outputs or prices in product markets, then competing in outputs is a dominant strategy, provided that the two products are substitutes, which is true in our setup (i.e., $k > 0$).

In the Online Appendix, we have also examined a variation in which firms play a Bertrand game in the product market. That is, firms use prices instead of outputs as strategic variables to maximize profits given the demand functions for their products. As in our original Cournot setting, when β is low and k is high—specifically, when $\beta < \frac{k^2}{2}$—each firm's non-employee owners have an incentive to offer their respective employees an equity stake in the firm. However, in these settings the equilibrium equity stake z_i^* is negative, resulting in a wage rate w_i^* that is even *higher* than it would have been in the wages only equilibrium.

The way to interpret this outcome is that, in a Bertrand setting, the firm's non-employee owners would be willing to offer a negative equity stake in order to force employees to ask for even higher wages. Specifically, prices are strategic complements under Bertrand competition—i.e., a firm's best response to a competing firm's decision to decrease prices is to also decrease prices—and thus, Bertrand firms will make the choice that softens competition (i.e., that leads to higher prices), and higher wages help firms to soften competition. In economic terms, a negative equity stake can be interpreted as the employees taking a portion of their wages equal to $|z_i^*|$ percent of the firm's profits, and giving it back to the non-employee owners. However, as offering employees a negative equity stake seems implausible in practice, we restrict our attention to the parameter space where $z_i^* \geq 0$. Over this parameter space, the wages only equilibrium is the unique equilibrium.

The combined results suggest that, in practice, firms engaged in Bertrand competition should not offer an equity stake to their employees to lower wage rates, although these firms may still offer equity stakes to employees for other reasons (e.g., retention, mitigating the moral hazard problem, etc.). The combined results also yield another insight. Specifically, both *competition intensity* and *the very nature of competition (price or quantity)* are important factors in explaining variation in non-employee owners' incentives to provide employees with equity-based compensation.

7. Conclusion

While equity compensation is frequently lauded as a means to align incentives between owners and employees, it is not clear whether the incentive effect remains when the firm employs a large number of workers. Given this point, it is surprising to observe that numerous publicly-traded firms, which employ many workers, offer equity-based compensation to their employees. The literature has posited several explanations, such as retention and sorting, for this outcome. The preceding analysis provides another motivation. In settings where employee bargaining power is sufficiently low and inter-firm competition is sufficiently intense, we may expect a firm to compensate its employees with company stock in order to lower wage rates. Our paper makes a basic point that employee compensation and product market decisions are related. Because labor relations can influence the product market equilibrium, a firm's owners have incentives to use equity compensation to influence the product market in their favor.

Our analysis has both normative and positive implications. On the normative side, we show that when the employee ownership equilibrium prevails, it is a Prisoner's Dilemma for the firms' original owners because they, and in some cases their employees, would be better off had the firms been able to commit to compensating employees with wages only. This result provides a potential explanation for how the decision to grant large equity stakes arises endogenously, even when large equity stakes appear to make both non-employee owners and employees worse off.

On the positive side, the model provides a possible explanation for several empirical regularities, such as the positive correlation between employee equity stakes and firm production and the substitutionary relationship between employee ownership and unionization. Finally, the model provides a set of testable predictions regarding the incidence of equity-based compensation plans. For example, we might expect negative shocks to employee bargaining power or positive shocks to competition intensity to precede the adoption of employee ownership plans not only for specific firms, but also for entire industries.

Appendix

Proof of Proposition 1

By equations (7), (8), (9), and (10), we can express

$$S_i(z_i, w_i, w_j, k) = (1 - z_i)\frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}, \quad \text{(A1)}$$

$$E_i(z_i, w_i, w_j, k) = \frac{2 - k - 2w_i + kw_j}{4 - k^2}(w_i - c) + z_i\frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}, \quad \text{(A2)}$$

for $i, j = 1, 2$, and $i \neq j$.

If firm i offers wages only, then $z_i = 0$, and by (A1) and (A2), the Nash bargaining problem in (11) can be expressed as

$$\underset{w_i}{Max}\left[\frac{2 - k - 2w_i + kw_j}{4 - k^2}(w_i - c)\right]^{\beta}\left[\frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}\right]^{1-\beta}.$$

The first-order condition to the above problem is

$$4w_i = \beta(2 - k) + 2(2 - \beta)c + k\beta w_j. \quad \text{(A3)}$$

If firm i offers wages and equity stakes, then by (A1) and (A2), the Nash bargaining problem in (12) can be expressed as

$$\underset{z_i,w_i}{Max}\left[\frac{2 - k - 2w_i + kw_j}{4 - k^2}(w_i - c) + z_i\frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}\right]^{\beta}\left[(1 - z_i)\frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}\right]^{1-\beta}.$$

The first-order conditions are

$$\beta\frac{\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}}{\frac{2-k-2w_i+kw_j}{4-k^2}(w_i - c) + z_i\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}} = \frac{1 - \beta}{1 - z_i}, \quad \text{(A4)}$$

$$\beta\frac{\frac{-2}{4-k^2}(w_i - c) + \frac{2-k-2w_i+kw_j}{4-k^2} + 2z_i\frac{(2-k-2w_i+kw_j)}{(4-k^2)^2}(-2)}{\frac{2-k-2w_i+kw_j}{4-k^2}(w_i - c) + z_i\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}} = (1 - \beta)\frac{\frac{4}{4-k^2}}{\frac{2-k-2w_i+kw_j}{4-k^2}}. \quad \text{(A5)}$$

In Part (1), both firms offer wages only, and we can use equation (A3) for $i = 1, 2$ to form a system of two unknowns w_1 and w_2. Solving this system yields the expressions of w_i. The expressions of other variables follow from inserting the expressions of w_i and $z_i = 0$ into equations (7), (8), (A1), and (A2).

In Part (2), both firms offer wages and equity stakes, and we can use equations (A4) and (A5) for $i = 1, 2$ to form a system of four unknowns, w_1, w_2, z_1, and z_2. Solving this system yields the expressions of w_i and z_i. The expressions of other variables follow from inserting the expressions of w_i and z_i into equations (7), (8), (A1), and (A2).

In Part (3), we have $z_1 > 0$ and $z_2 = 0$. Then, we use equation (A3) for $i = 1$ and equations (A4) and (A5) for $i = 2$ to form a system of three unknowns, w_1, w_2, and z_1. Solving this system yields the expressions of w_1, w_2, and z_1. The expressions of other variables follow from inserting the expressions of w_1, w_2, and z_1 and $z_2 = 0$ into equations (7), (8), (A1), and (A2). □

Proof of Proposition 2

We prove this proposition in two steps. First, we compute the best response functions. Second, we discuss pure strategy equilibria case by case.

Step 1: Best response functions

Since the game is symmetric, we focus on the best response function of firm 1.

Response to firm 2 playing "w":

Suppose that firm 2 offers wages only. Firm 1 needs to compare $S_1^{ww}(\beta, k)$ and $S_1^{zw}(\beta, k)$, which correspond to the payoff of firm 1 when it plays "w" or "z" respectively. By the expressions of $S_1^{ww}(\beta, k)$ and $S_1^{zw}(\beta, k)$ in Proposition 2, we can compute

$$S_1^{zw}(\beta, k) - S_1^{ww}(\beta, k) = \frac{(1-c)^2\, 2\left(2\beta + k^2(1-\beta)\right)}{(k+2)^2(4-k\beta)^2\left(k^4\beta - 16k^2 + 32\right)^2} Q_1(\beta, k),$$

where

$$\begin{aligned} Q_1(\beta, k) \equiv\ & -k^4\left(4-k^2\right)^2\beta^4 + 16k^4\beta^3 + 8k^2\left(64 - 40k^2 + 3k^4\right)\beta^2 \\ & -256\left(4-k^4\right)\beta + 256k^2\left(2-k^2\right). \end{aligned} \tag{A6}$$

Given $\frac{(1-c)^2 2\left(2\beta+k^2(1-\beta)\right)}{(k+2)^2(4-k\beta)^2\left(k^4\beta-16k^2+32\right)^2} > 0$, we have

$$S_1^{zw}(\beta, k) > S_1^{ww}(\beta, k) \Longleftrightarrow Q_1(\beta, k) > 0. \tag{A7}$$

Next we prove that $Q_1(\beta, k)$ is decreasing in β and crosses zero once. The idea is to show that $\frac{\partial Q_1(\beta,k)}{\partial\beta} < 0$ for all $\beta, k \in (0, 1)$. We will first establish that $\frac{\partial Q_1(\beta,k)}{\partial\beta}$ is increasing in β and thus an upper bound for $\frac{\partial Q_1(\beta,k)}{\partial\beta}$ is $\left.\frac{\partial Q_1(\beta,k)}{\partial\beta}\right|_{\beta=1}$. Then we can compute $\left.\frac{\partial Q_1(\beta,k)}{\partial\beta}\right|_{\beta=1} < 0$, so that $\frac{\partial Q_1(\beta,k)}{\partial\beta} < 0$ for any $\beta \in (0, 1)$.

Specifically, direct computation shows

$$\frac{\partial Q_1(\beta, k)}{\partial\beta} = -4k^4\left(4-k^2\right)^2\beta^3 + 48k^4\beta^2 + 16k^2\left(64 - 40k^2 + 3k^4\right)\beta - 256\left(4-k^4\right), \tag{A8}$$

$$\frac{\partial^2 Q_1(\beta, k)}{\partial\beta^2} = 4k^2\left(256 + 24k^4\beta^2 + 24k^2\beta + 12k^4 - 48k^2\beta^2 - 3k^6\beta^2 - 160k^2\right). \tag{A9}$$

By (A9) and the fact $\beta, k \in (0, 1)$, we have

$$\frac{\partial^2 Q_1(\beta, k)}{\partial\beta^2} > 4k^2(256 - 48 - 3 - 160) = 180k^2 > 0.$$

That is, $\frac{\partial Q_1(\beta,k)}{\partial\beta}$ is increasing in β. Thus, for any $\beta \in (0,1)$, we have

$$\begin{aligned}\frac{\partial Q_1(\beta,k)}{\partial\beta} &< \left.\frac{\partial Q_1(\beta,k)}{\partial\beta}\right|_{\beta=1} = 4\left(2-k^2\right)\left(64k^2-18k^4+k^6-128\right)\\ &< 4\left(2-k^2\right)(64+1-128) < 0.\end{aligned}$$

As a result, $Q_1(\beta,k)$ decreases with β at $[0,1]$.

Note that $Q_1(0,k) = 256k^2(2-k^2) > 0$ and $Q_1(1,k) = -(32-16k^2+k^4)^2 < 0$. Thus, for any $k \in (0,1)$, there exists a unique $\hat{\beta}_1 \equiv \hat{\beta}_1(k)$, such that

$$Q_1(\beta,k) > 0 \Longleftrightarrow \beta < \hat{\beta}_1, \tag{A10}$$

where $\hat{\beta}_1$ is defined by

$$Q_1\left(\hat{\beta}_1,k\right) = 0. \tag{A11}$$

By conditions (A7) and (A10), firm 1's best response to firm 2 playing "w" is to play "z" if and only if $\beta < \hat{\beta}_1$.

Response to firm 2 playing "z":

Suppose that firm 2 offers wages and equity stakes to its employees. Now the relevant payoffs of firm 1 are $S_1^{wz}(\beta,k)$ and $S_1^{zz}(\beta,k)$, which result from firm 1 playing "w" or "z" respectively. By the expressions of $S_1^{wz}(\beta,k)$ and $S_1^{zz}(\beta,k)$ in Proposition 2, we can compute

$$S_1^{zz}(\beta,k) - S_1^{wz}(\beta,k) = \frac{2\left(2\beta+k^2(1-\beta)\right)(1-c)^2}{\left(4+2k-k^2\right)^2\left(32-16k^2+k^4\beta\right)^2}Q_2(\beta,k),$$

where

$$Q_2(\beta,k) \equiv -k^8\beta^2 - 16\left(16-16k^2+7k^4-2k^6\right)\beta + 8k^2\left(16-8k^2-k^4\right). \tag{A12}$$

Given $\frac{2\left(2\beta+k^2(1-\beta)\right)(1-c)^2}{(4+2k-k^2)^2(32-16k^2+k^4\beta)^2} > 0$, we have

$$S_1^{zz}(\beta,k) > S_1^{wz}(\beta,k) \Longleftrightarrow Q_2(\beta,k) > 0. \tag{A13}$$

Direct computation shows

$$\frac{\partial Q_2(\beta,k)}{\partial\beta} = -2k^8\beta - 16\left(\left(16+2k^4\right)\left(1-k^2\right)+5k^4\right) < 0.$$

Thus, $Q_2(\beta,k)$ is decreasing in β; that is, for $\beta \in (0,1)$, only the right branch of the quadratic function $Q_2(\cdot,k)$ is relevant. Noting that $Q_2(0,k) = 8k^2(16-8k^2-k^4) > 0$ and $Q_2(1,k) = -(4-2k-k^2)^2(4+2k-k^2)^2 < 0$, we know that the larger root of $Q_2(\beta,k)=0$ lies between 0 and 1. Therefore, we have

$$Q_2(\beta,k) > 0 \Longleftrightarrow \beta < \hat{\beta}_2, \tag{A14}$$

where $\hat{\beta}_2 \equiv \hat{\beta}_2(k)$ is the larger root of $Q_2(\beta, k) = 0$, i.e.,

$$\begin{aligned}\hat{\beta}_2 &= \frac{2(4-k^2)\sqrt{2(2-k^2)(64-64k^2+36k^4-14k^6+k^8)}}{k^8} \\ &\quad -\frac{8(16-16k^2+7k^4-2k^6)}{k^8}. \qquad \text{(A15)}\end{aligned}$$

By conditions (A13) and (A14), firm 1's best response to firm 2 playing "z" is to play "z" if and only if $\beta < \hat{\beta}_2$.

Step 2: Pure strategy equilibrium characterization

We first show $\hat{\beta}_1(k) < \hat{\beta}_2(k)$ for any $k \in (0,1)$ and then use these two threshold values to divide the parameter space into three cases.

Proof of $\hat{\beta}_1 < \hat{\beta}_2$:

It suffices to show $Q_1\left(\hat{\beta}_2, k\right) < 0$, because by the definition of $\hat{\beta}_1$ (given by $Q_1\left(\hat{\beta}_1, k\right) = 0$), we have

$$Q_1\left(\hat{\beta}_2, k\right) < 0 = Q_1\left(\hat{\beta}_1, k\right) \Rightarrow Q_1\left(\hat{\beta}_2, k\right) < Q_1\left(\hat{\beta}_1, k\right) \Rightarrow \hat{\beta}_1 < \hat{\beta}_2,$$

where the last arrow follows from the fact that $Q_1(\beta, k)$ is decreasing in β. Now we prove $Q_1\left(\hat{\beta}_2, k\right) < 0$.

Note that

$$\begin{aligned}Q_2\left(\hat{\beta}_2, k\right) &= 0 \Rightarrow Q_2\left(\hat{\beta}_2, k\right)\frac{(4-k^2)^2}{k^4}\hat{\beta}_2^2 = 0 \\ &\Rightarrow Q_2\left(\hat{\beta}_2, k\right)\frac{(4-k^2)^2}{k^4}\hat{\beta}_2^2 - Q_1\left(\hat{\beta}_2, k\right) = -Q_1\left(\hat{\beta}_2, k\right),\end{aligned}$$

and thus,

$$Q_1\left(\hat{\beta}_2, k\right) < 0 \Longleftrightarrow Q_2\left(\hat{\beta}_2, k\right)\frac{(4-k^2)^2}{k^4}\hat{\beta}_2^2 - Q_1\left(\hat{\beta}_2, k\right) > 0.$$

By the definitions of Q_1 and Q_2 in (A6) and (A12), we have

$$\begin{aligned}&Q_2\left(\hat{\beta}_2, k\right)\frac{(4-k^2)^2}{k^4}\hat{\beta}_2^2 - Q_1\left(\hat{\beta}_2, k\right) \\ &= \frac{32(2-k^2)\left(2\hat{\beta}_2 + k^2\left(1-\hat{\beta}_2\right)\right)}{k^4}\left[\left(k^6 - 8k^4 + 16k^2 - 32\right)\hat{\beta}_2^2 + \left(32k^2\right)\hat{\beta}_2 - 8k^4\right].\end{aligned}$$

As a result,

$$\begin{aligned}Q_2\left(\hat{\beta}_2, k\right)\frac{(4-k^2)^2}{k^4}\hat{\beta}_2^2 - Q_1\left(\hat{\beta}_2, k\right) &> 0 \Longleftrightarrow \\ \left(k^6 - 8k^4 + 16k^2 - 32\right)\hat{\beta}_2^2 + \left(32k^2\right)\hat{\beta}_2 - 8k^4 &> 0.\end{aligned}$$

Using (A15), we can show that the above condition is equivalent to

$$\begin{aligned}&(13k^8 - 2k^{10} - 52k^6 + 112k^4 - 160k^2 + 128)\,2(4-k^2)\sqrt{2(2-k^2)(64-64k^2+36k^4-14k^6+k^8)} \\ &> 8(16-16k^2+7k^4-2k^6)(13k^8 - 2k^{10} - 52k^6 + 112k^4 - 160k^2 + 128)\end{aligned}$$

$-k^8 \left(k^{10} + 4k^8 - 16k^6 + 48k^4 - 64k^2\right).$

Note that $13k^8 - 2k^{10} - 52k^6 + 112k^4 - 160k^2 + 128 > 0$ for $k \in (0, 1)$. So, the left-hand side of the above equation is always positive. When the right-hand side of the above condition is negative, then the above condition always holds. If the right-hand side is positive, then by taking square on both sides and taking difference, we can show that the above condition is equivalent to $k^{22}\left(2 - k^2\right)\left(64 - 64k^2 + 36k^4 - 14k^6 + k^8\right)\left(4 - k^2\right)^2 > 0$, which is true.

Proof of (1)–(3) of Proposition 2:

From the above results, we know the following:

(1) When $\beta \in \left(0, \hat{\beta}_1\right)$, each firm's dominant strategy is to offer both wages and equity stakes. So, the only equilibrium is the employee ownership equilibrium, in which both firms offer their employes wages and equity stakes.

(2) When $\beta \in \left[\hat{\beta}_1, \hat{\beta}_2\right]$, if firm 2 plays "$w$" then firm 1's best response is to play "w;" and if firm 2 plays "z" then firm 1's best response is to play "z." Similarly, if firm 1 plays "w" then firm 2's best response is to play "w;" and if firm 1 plays "z" then firm 2's best response is to play "z." So, there are two pure strategy equilibria: either both firms play "w," or both of them play "z."

(3) When $\beta \in \left(\hat{\beta}_2, 1\right)$, each firm's dominant strategy is to offer wages only. So, the only equilibrium is that both firms offer only wages to their respective employees. That is, the wage only equilibrium prevails. □

Proof of Corollary 2

Since $\hat{\beta}_1$ and $\hat{\beta}_2$ are defined by $Q_1\left(\hat{\beta}_1, k\right) = 0$ and $Q_2\left(\hat{\beta}_2, k\right) = 0$, then by the definitions of Q_1 and Q_2 in (A6) and (A12), we have

$$\lim_{k \to 0} \hat{\beta}_1(k) = \lim_{k \to 0} \hat{\beta}_2(k) = 0.$$

Thus, by Proposition 2, the wage only equilibrium prevails as long as $\beta > 0$. □

Proof of Proposition 3

(1) Using Part (1) of Proposition 1, we can compute

$$\begin{aligned}
\frac{\partial w_i^{ww}(\beta,k)}{\partial\beta} &= \frac{4(2-k)(1-c)}{(4-\beta k)^2} > 0,\\
\frac{\partial q_i^{ww}(\beta,k)}{\partial\beta} &= -\frac{4(1-c)(2-k)}{(k+2)(4-k\beta)^2} < 0,\\
\frac{\partial S_i^{ww}(\beta,k)}{\partial\beta} &= \frac{\partial\Pi_i^{ww}(\beta,k)}{\partial\beta} = 2q_i^{ww}(\beta,k)\frac{\partial q_i^{ww}(\beta,k)}{\partial\beta} < 0,\\
\frac{\partial E_i^{ww}(\beta,k)}{\partial\beta} &= \frac{4(2-k)(1-c)^2(4(1-\beta)+k\beta)}{(k+2)(4-k\beta)^3} > 0,
\end{aligned}$$

for $i = 1, 2$, and $\beta, k \in (0,1)$.

(2) Using Part (2) of Proposition 1, we can compute,

$$\begin{aligned}
\frac{\partial z_i^{zz}(\beta,k)}{\partial\beta} &= 1-\frac{k^2}{2} > 0,\\
\frac{\partial w_i^{zz}(\beta,k)}{\partial\beta} &= \frac{\partial q_i^{zz}(\beta,k)}{\partial\beta} = \frac{\partial\Pi_i^{zz}(\beta,k)}{\partial\beta} = 0,\\
\frac{\partial S_i^{zz}(\beta,k)}{\partial\beta} &= -\frac{2(2-k^2)(1-c)^2}{(4+2k-k^2)^2} < 0,\\
\frac{\partial E_i^{zz}(\beta,k)}{\partial\beta} &= \frac{2(2-k^2)(1-c)^2}{(4+2k-k^2)^2} > 0,
\end{aligned}$$

for $i = 1, 2$, and $\beta, k \in (0,1)$. □

Proof of Corollary 3

Examining Fig. 3, we know that for any given $k \in (0,1)$, as β monotonically increases from 0 to 1, the economy starts with an employee ownership equilibrium, and eventually ends up with a wage only equilibrium. Thus, by Proposition 3, z_i^* first increases with β when it is equal to $z_i^{zz}(\beta,k)$, and finally it jumps down to 0. Similarly, by Proposition 3, w_i^* first keeps at a constant $w_i^{zz}(\beta,k) = c - \frac{k^2(1-c)}{4+2k-k^2}$, which is smaller than c, and then w_i^* jumps to $w_i^{ww}(\beta,k) = c + \frac{\beta(2-k)(1-c)}{4-k\beta}$, which is larger than c and increasing in β. □

Proof of Proposition 4

(1) Using Part (1) of Proposition 1, we can compute

$$\begin{aligned}
\frac{\partial w_i^{ww}(\beta,k)}{\partial k} &= -\frac{2\beta(2-\beta)(1-c)}{(4-k\beta)^2} < 0, \\
\frac{\partial q_i^{ww}(\beta,k)}{\partial k} &= -\frac{4(2-\beta)(1-c)(2-\beta(1+k))}{((4-k\beta)(k+2))^2} < 0, \\
\frac{\partial S_i^{ww}(\beta,k)}{\partial k} &= \frac{\partial \Pi_i^{ww}(\beta,k)}{\partial k} = 2q_i^{ww}(\beta,k)\frac{\partial q_i^{ww}(\beta,k)}{\partial k} < 0, \\
\frac{\partial E_i^{ww}(\beta,k)}{\partial k} &= -\frac{4\beta(2-\beta)(1-c)^2(2+k^2\beta+4(1-\beta)+2(1-k\beta))}{(4-k\beta)^3(k+2)^2},
\end{aligned}$$

for $i = 1, 2$, and $\beta, k \in (0,1)$.

(2) Using Part (2) of Proposition 1, we can compute,

$$\begin{aligned}
\frac{\partial z_i^{zz}(\beta,k)}{\partial k} &= k(1-\beta) > 0, \\
\frac{\partial w_i^{zz}(\beta,k)}{\partial k} &= -\frac{2k(k+4)(1-c)}{(4+2k-k^2)^2} < 0, \\
\frac{\partial q_i^{zz}(\beta,k)}{\partial k} &= -\frac{4(1-c)(1-k)}{(4+2k-k^2)^2} < 0, \\
\frac{\partial \Pi_i^{zz}(\beta,k)}{\partial k} &= 2q_i^{zz}\frac{\partial q_i^{zz}(\beta,k)}{\partial k} < 0, \\
\frac{\partial S_i^{zz}(\beta,k)}{\partial k} &= -\frac{4(1-\beta)(1-c)^2(k^3+4)}{(4+2k-k^2)^3} < 0, \\
\frac{\partial E_i^{zz}(\beta,k)}{\partial k} &= -\frac{4\beta(1-c)^2(k^3+4)}{(4+2k-k^2)^3} < 0.
\end{aligned}$$

for $i = 1, 2$, and $\beta, k \in (0,1)$. $\square$

Proof of Corollary 4

Examining Fig. 3, we see that when β is high, only the wage only equilibrium prevails for any $k \in (0,1)$, that is, $z_i^* = 0$ and $w_i^* = w_i^{ww}(\beta,k)$. By Part (1) of Proposition 4, w_i^* decreases with k.

When β is low, as k increases from 0 to 1, the economy starts with a wage only equilibrium and eventually it ends up with an employee ownership equilibrium. Thus, z_i^* first stays at 0 and then, by Proposition 4, it increases with k when $z_i^* = z_i^{zz}(\beta,k)$. For w_i^*, as k is low, $w_i^* = w_i^{ww}(\beta,k) = c + \frac{\beta(2-k)(1-c)}{4-k\beta}$, which is larger than c and decreases with k, and as k becomes high, w_i^* jumps downward to $w_i^{zz}(\beta,k) = c - \frac{k^2(1-c)}{4+2k-k^2}$, which is smaller than c and continues to decrease with k. $\square$

Proof of Proposition 5

(1) By the expressions of $q_i^{ww}(\beta,k)$ and $q_i^{zz}(\beta,k)$ in Proposition 2,

$$\begin{aligned} q_i^{zz}(\beta,k) - q_i^{ww}(\beta,k) &= \frac{4(1-c)(2\beta+(1-\beta)k^2)}{(8+8k-k^3)(4-k\beta)} > 0 \\ &\Rightarrow q_i^{zz}(\beta,k) > q_i^{ww}(\beta,k). \end{aligned}$$

Note that $\Pi_i^{zz}(\beta,k) = [q_i^{zz}(\beta,k)]^2$ and $\Pi_i^{ww}(\beta,k) = [q_i^{ww}(\beta,k)]^2$, and so we also have $\Pi_i^{zz}(\beta,k) > \Pi_i^{ww}(\beta,k)$.

(2) Part (2) follows from Part (1) and equation (20).

(3) By the expressions of $S_i^{ww}(\beta,k)$ and $S_i^{zz}(\beta,k)$ in Proposition 2, we can compute

$$\begin{aligned} & S_i^{ww}(\beta,k) - S_i^{zz}(\beta,k) \\ = & \frac{2(2\beta+k^2(1-\beta))(1-c)^2}{(k+2)^2(k^2-2k-4)^2(k\beta-4)^2} \\ & \times \left[\beta^2k^2(2+k)^2 + 16\beta(1-k) + 8k(4(1-\beta k)+k(3-\beta k))\right]. \end{aligned}$$

By the fact of $k,\beta \in (0,1)$, we have $\frac{2(2\beta+k^2(1-\beta))(1-c)^2}{(k+2)^2(k^2-2k-4)^2(k\beta-4)^2} > 0$ and $\beta^2k^2(2+k)^2+16\beta(1-k)+8k(4(1-\beta k)+k(3-\beta k)) > 0$. Thus, $S_i^{ww}(\beta,k) > S_i^{zz}(\beta,k)$.

(4) By the expressions of $E_i^{ww}(\beta,k)$ and $E_i^{zz}(\beta,k)$ in Proposition 2, we have

$$\begin{aligned} & E_i^{zz}(\beta,k) - E_i^{ww}(\beta,k) \\ = & \frac{2(1-c)^2\beta(2\beta+k^2(1-\beta))(k^3+2k^2)}{(k+2)(4+2k-k^2)^2(4-k\beta)^2}\left(\beta + \frac{2(8-4k-6k^2+k^3)}{k^2(2+k)}\right) \Rightarrow \\ & E_i^{zz}(\beta,k) > E_i^{ww}(\beta,k) \\ & \iff \beta > \frac{2(-k^3+6k^2+4k-8)}{k^2(2+k)}, \end{aligned}$$

where the threshold $\frac{2(-k^3+6k^2+4k-8)}{k^2(2+k)}$ is the expression of $\hat{\beta}_3(k)$ in Proposition 5. Given that $\beta \in (0,1)$, we have $E_i^{zz}(\beta,k) > E_i^{ww}(\beta,k) \iff \beta > Max\left\{0, \hat{\beta}_3(k)\right\}$.

We can establish that $\hat{\beta}_3(k)$ can be positive or negative over $k \in [0,1]$. Specifically, let us define

$$f(k) \equiv -k^3 + 6k^2 + 4k - 8.$$

We can compute

$$f'(k) = -3k^2 + 12k + 4,$$

which has two roots of $k \approx 4.31$ and $k \approx -0.31$, and so $f'(k) > 0$ for $k \in [0,1]$, which in turn implies that $f(k)$ is increasing in $k \in [0,1]$. We can check $f(0) < 0$ and $f(1) > 0$. Thus, $\hat{\beta}_3(k) = \frac{f(k)}{k^2(2+k)}$ first is negative and then turns to be positive. We can check that the turning point is around $k \approx 0.92$. □

Proof of Corollary 5

(1) From the proof of Part (1) of Proposition 2, offering both wages and equity stakes is a dominant strategy when $\beta \in \left(0, \hat{\beta}_1\right)$. Combining with the fact of $S_i^{ww}(\beta, k) > S_i^{zz}(\beta, k)$ in Proposition 5, we know that the supported employee ownership equilibrium is a Prisoner's Dilemma.

(2) Part (2) follows immediately from $S_i^{ww}(\beta, k) > S_i^{zz}(\beta, k)$ in Proposition 5. □

Proof of Corollary 6

By Part (1) of Proposition 2, when $\beta < \hat{\beta}_1$, the employee ownership equilibrium prevails. By Parts (3) and (4) of Proposition 5, when $\beta < Max\left\{0, \hat{\beta}_3(k)\right\}$, we have $S_i^{ww}(\beta, k) > S_i^{zz}(\beta, k)$ and $E_i^{ww}(\beta, k) > E_i^{zz}(\beta, k)$. □

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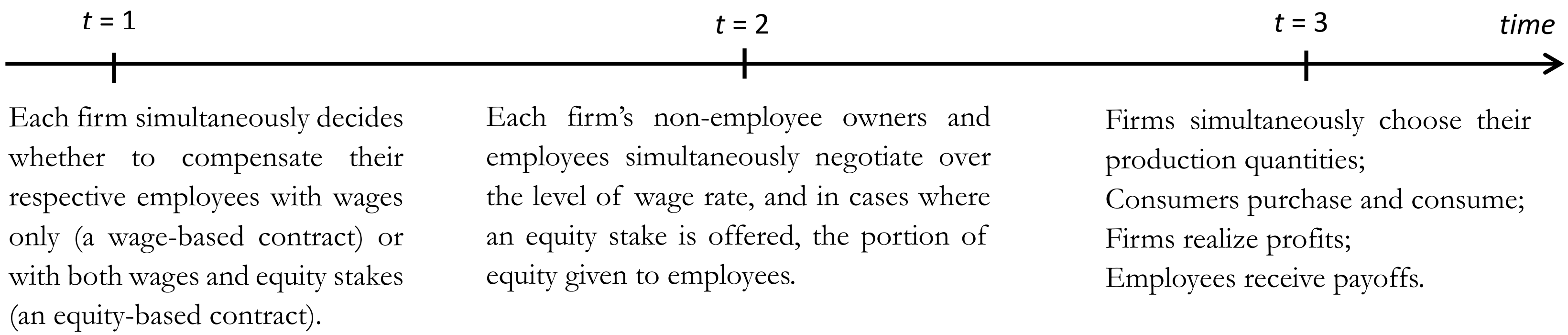

Fig. 1. Timeline. This figure plots the timeline of the model.

		Firm 2: Wage only (w)	Firm 2: Wage & equity (z)
Firm 1	Wage only (w)	$S_1^{ww}=\frac{4(2-\beta)^2(1-c)^2}{(4-k\beta)^2(2+k)^2}$, $S_2^{ww}=\frac{4(2-\beta)^2(1-c)^2}{(4-k\beta)^2(2+k)^2}$	$S_1^{wz}=\frac{4(1-c)^2\left(4-2k-k^2\right)^2(2-\beta)^2}{(32-16k^2+\beta k^4)^2}$, $S_2^{wz}=\frac{2(1-\beta)(1-c)^2(2-k)^2(4+k\beta)^2(2-k^2)}{(32-16k^2+\beta k^4)^2}$
	Wage & equity (z)	$S_1^{zw}=\frac{2(1-\beta)(1-c)^2(2-k)^2(4+k\beta)^2(2-k^2)}{(32-16k^2+\beta k^4)^2}$, $S_2^{zw}=\frac{4(1-c)^2\left(4-2k-k^2\right)^2(2-\beta)^2}{(32-16k^2+\beta k^4)^2}$	$S_1^{zz}=\frac{2(1-\beta)(1-c)^2(2-k^2)}{(4+2k-k^2)^2}$, $S_2^{zz}=\frac{2(1-\beta)(1-c)^2(2-k^2)}{(4+2k-k^2)^2}$

Fig. 2. Payoff matrix in period 1. This figure plots the payoff matrix in period 1. Each firm can offer wages only or offer both wages and equity stakes to their respective employees. These two strategies are denoted by "w" and "z" respectively.

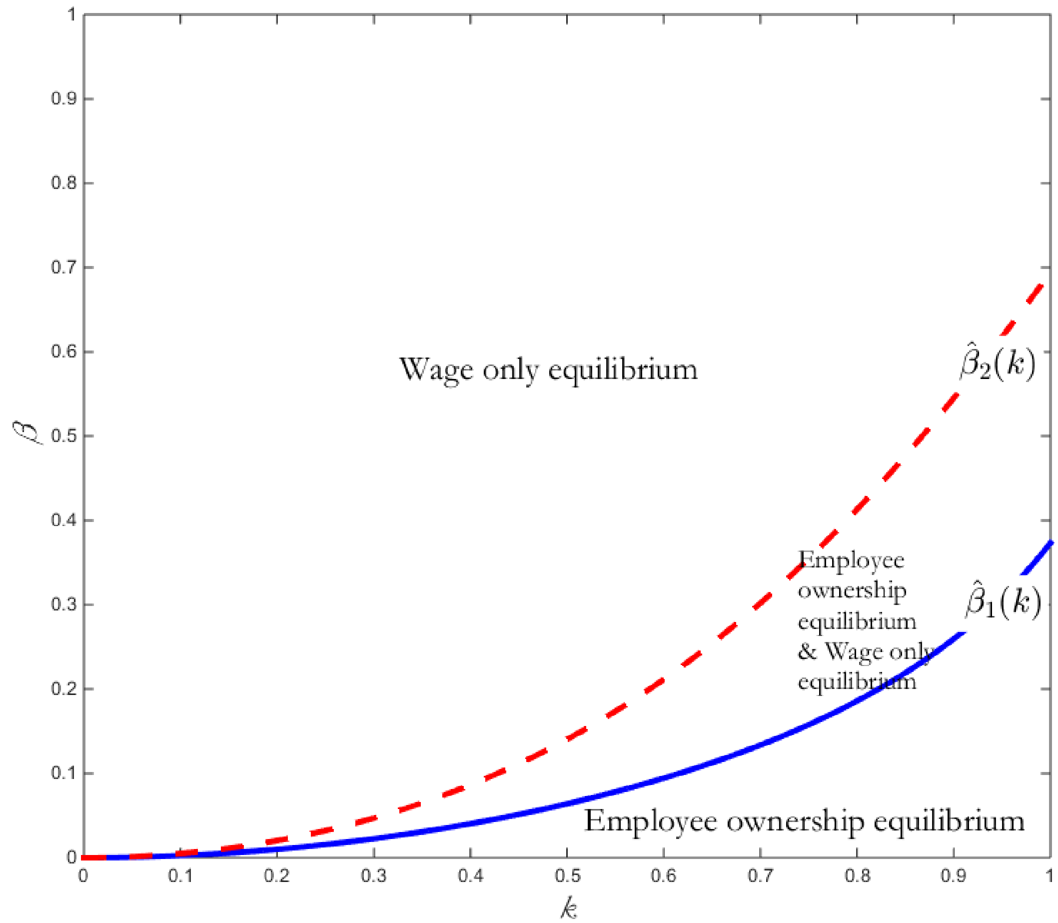

Fig. 3. Parameter space for equilibrium types. This figure plots the equilibrium types in the space of (k, β), where parameter $k \in (0,1)$ denotes the competitiveness of the product market, and parameter $\beta \in (0,1)$ is the employees' bargaining power.

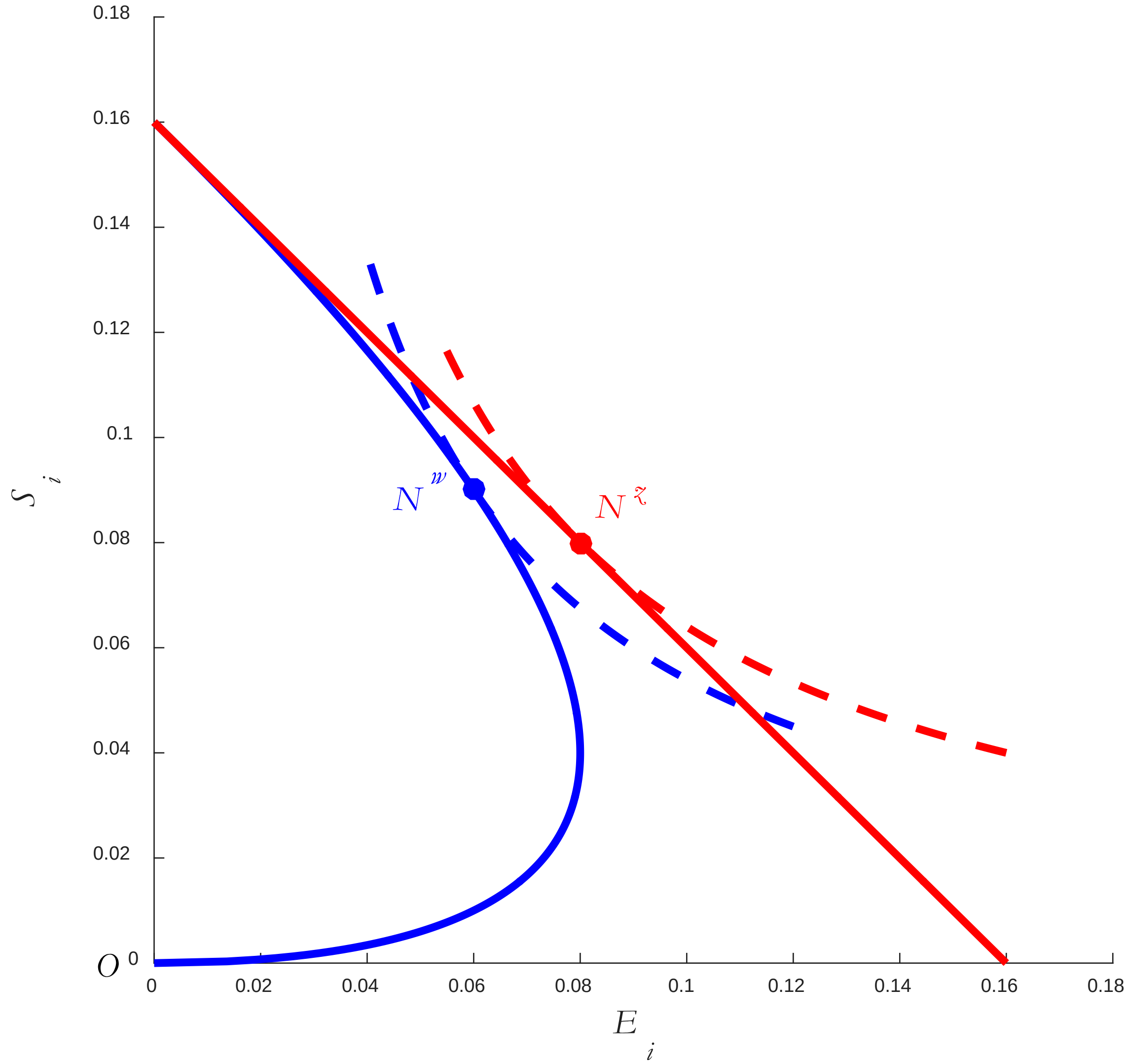

Fig. 4. Nash Bargaining outcomes for the monoploy case. This figure plots the Nash bargaining outcome for firm i in period 2, when both firms are monopolists in their own product market (i.e., $k = 0$). Other parameters are set as $\beta = 0.5$ and $c = 0.2$.

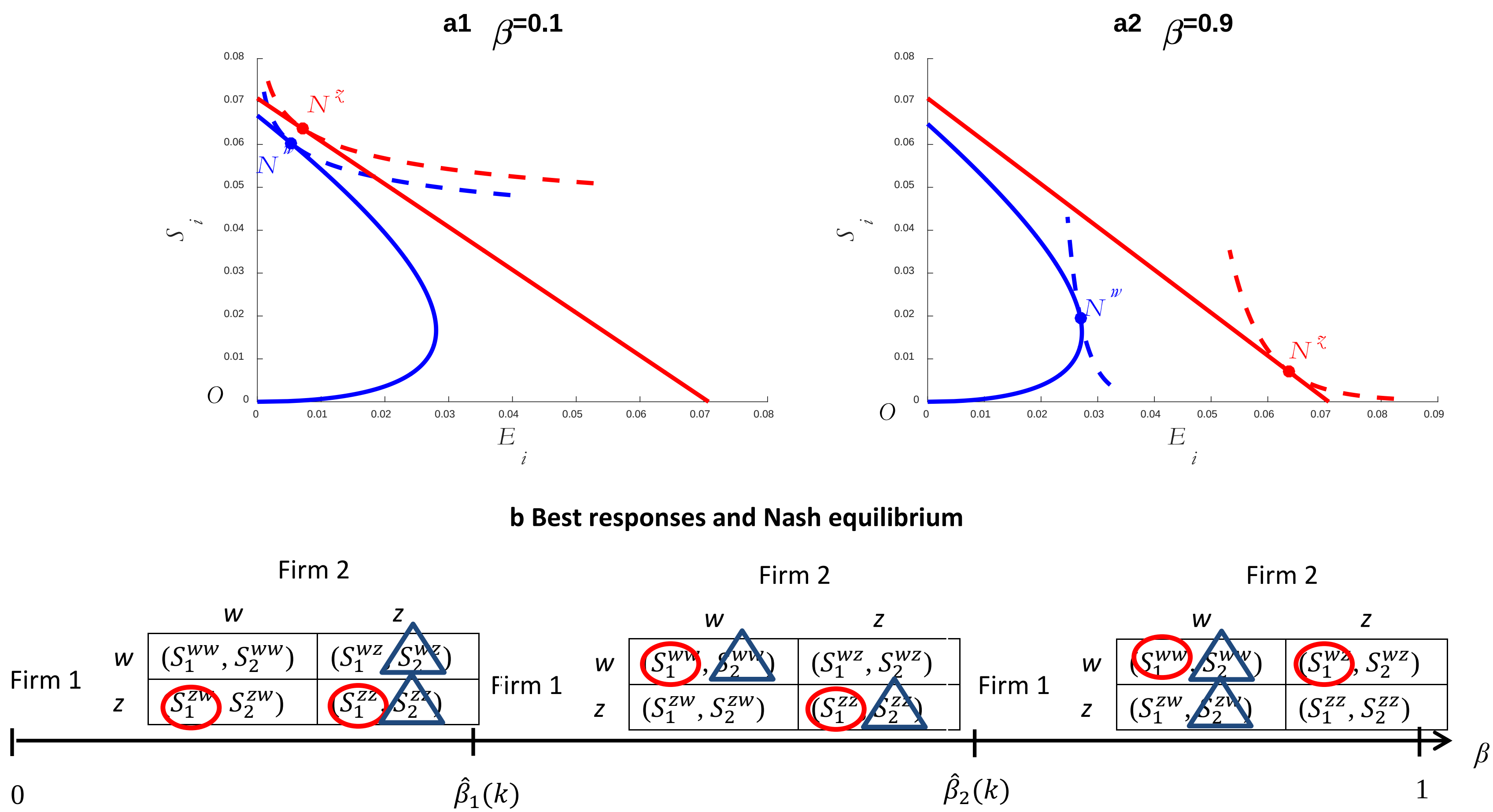

Fig. 5. Equilibrium determination. Panels a1 and a2 plot the period-2 Nash bargaining outcomes for firm i when $\beta = 0.1$ and $\beta = 0.9$, respectively. In both panels, we have set $k = 0.8$ and $c = 0.2$ and assume that firm j offers an equity–based contract. Panel (b) plots best response functions of each firm and the resulting Nash equlibrium in period 1. In the payoff matrix in period 1, each firm can offer wages only or offer both wages and equity stakes to their respective employees. These two strategies are denoted by "w" and "z" respectively. Firm 1's best response is labeled by circles, while firm 2's best response is labeled by triangles. The pure strategy equilibrium is determined by the intersection of best responses, i.e., those cells with both a circle and a triangle.

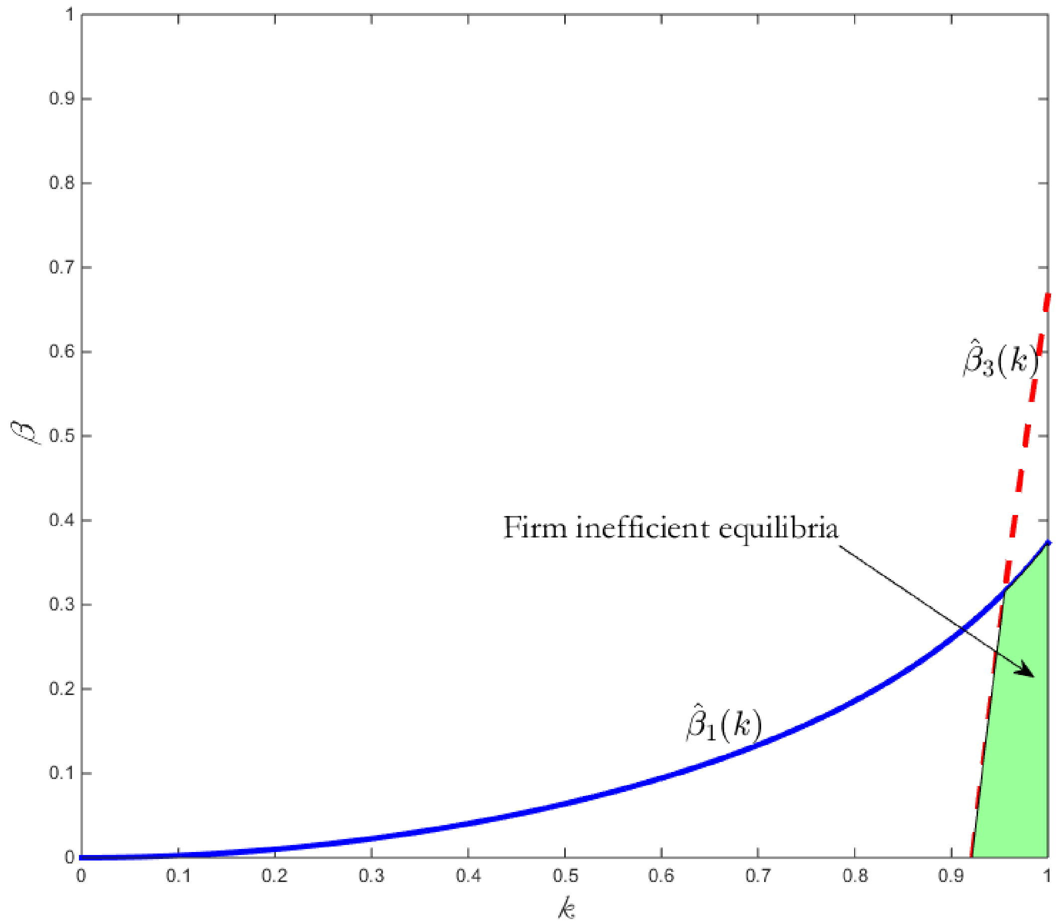

Fig. 6. Firm inefficient equilibria. This figure plots the firm inefficient equilibria in the space of (k, β), where parameter $k \in (0,1)$ denotes product market competitiveness, and parameter $\beta \in (0,1)$ is employee bargaining power. In a firm inefficient equilibrium, firms offer both wages and equity stakes to their employees, but both employees and non-employee owners are worse off than if firms had offered wages only. Those inefficient equilibria are indicated using an shaded area.

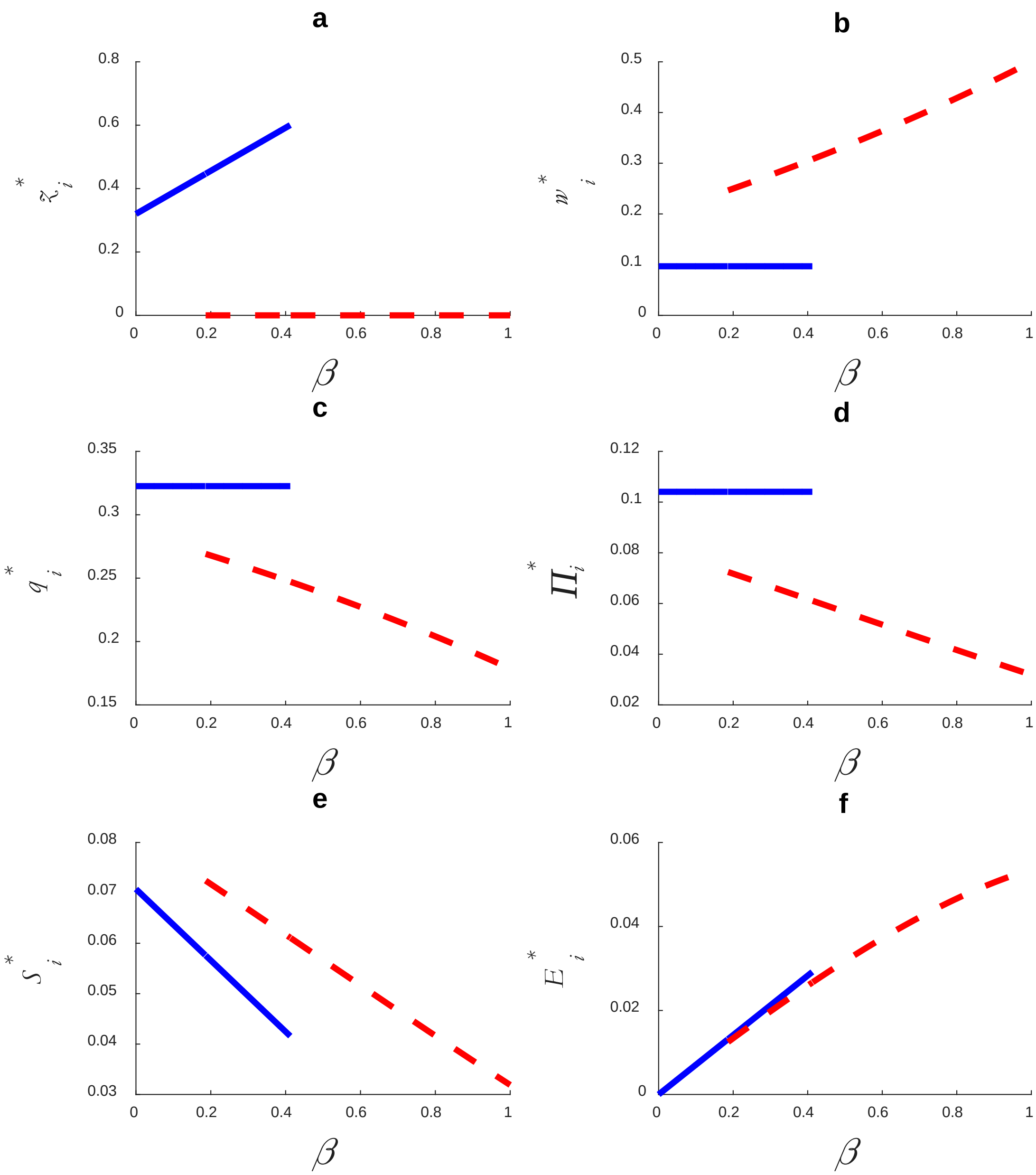

Fig. 7. Comparative statics with respect to parameter β. This figure plots the equilibrium outcomes against parameter β, which captures employee bargaining power. The product market competitiveness parameter is set as $k = 0.8$, and the competitve labor wage rate is $c = 0.2$. The wage only equlibrium is ploted in dashed red curves, while the employee ownership equilibrium is ploted in solid blue curves.

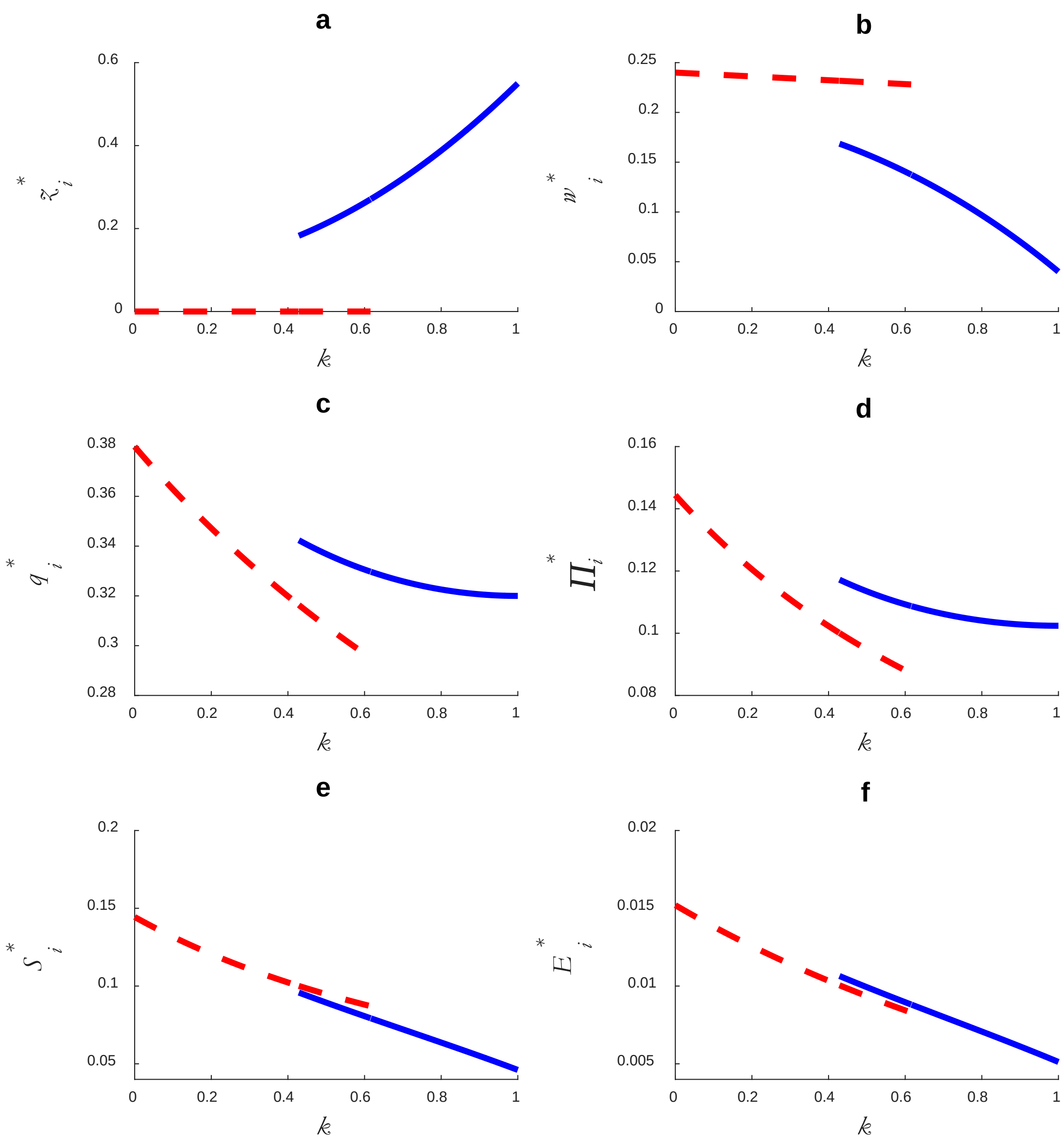

Fig. 8. Comparative statics with respect to parameter *k*. This figure plots the equilibrium outcomes against parameter k, which captures product market competiveness. The employees' bargaining power is set as $\beta = 0.1$, and the competitve labor wage rate is $c = 0.2$. The wage only equlibrium is ploted in dashed red curves, while the employee ownership equilibrium is ploted in solid blue curves.

Online Appendix (Not for Publication)

A. Interpretations of E_i in Terms of McDonald and Solow (1981)

We illustrate that the employees' objective function E_i, given by (4) in our setup, is consistent with two union models described in McDonald and Solow (1981): the simple monopoly union and the commune union.

The Simple Monopoly Union

In their Section I, McDonald and Solow (1981, p. 898) propose a model of monopoly union, which maps to our setting as follows. Suppose that the union of firm i has N_i members, all alike, where N_i is a large fixed constant. Each member can provide one unit of labor, which shares the same spirit as McDonald and Solow's (1981, footnote 2) assumption of ignoring the possibility that workers are free to choose the hours and intensity of work. Under this assumption, when firm i produces q_i units of output and hence demands q_i units of labor, q_i members will be employed by the firm. Therefore, each member has probability $\frac{q_i}{N_i}$ of having a job with the firm and probability $1 - \frac{q_i}{N_i}$ of not being employed by the firm.

Suppose that union members are risk neutral so that their utility is linear in wealth (i.e., $U(w) = w$ in terms of McDonald and Solow's (1981) notation). Thus, if employed by firm i, a member achieves a level of utility of $w_i + \frac{z_i \Pi_i}{q_i}$.[11] The first term w_i is the wage associated with the unit labor provided by the member. The second term $\frac{z_i \Pi_i}{q_i}$ captures the wealth generated from equity stakes: the employed members as a group get profits $z_i \Pi_i$ from their equity shares, and so each of the q_i employed members gets profits of $\frac{z_i \Pi_i}{q_i}$. If not employed by firm i, a member achieves an exogenous level of utility $c \in (0, 1)$. McDonald and Solow (1981) interpret c as an unemployment compensation benefit, which can broadly include all other contributions to the standard of living that would not be received if the member were employed by firm i, including walking away to work at a fixed lower market rate, as in our main text.

Under this interpretation, when the union forms an agreement with the firm in the

[11] McDonald and Solow (1981) also assume that working with the firm incurs a fixed disutility D. For simplicity, we here assume $D = 0$.

bargaining game, the expected utility of a union member is

$$\frac{q_i}{N_i} \times \left(w_i + \frac{z_i \Pi_i}{q_i} \right) + \left(1 - \frac{q_i}{N_i} \right) \times c = c + \frac{q_i}{N_i} \times \left(w_i + \frac{z_i \Pi_i}{q_i} - c \right),$$

and so the the union's aggregate expected utility from working with firm i is

$$N_i \times \left[c + \frac{q_i}{N_i} \times \left(w_i + \frac{z_i \Pi_i}{q_i} - c \right) \right] = N_i c + q_i \times \left(w_i + \frac{z_i \Pi_i}{q_i} - c \right).$$

If the union does not reach an agreement with the firm in the bargaining game, then all members receive a fixed utility of c, and so the walk-away utility of the union is simply $N_i c$.

Recall that the bargaining outcome in a bargaining game depends on each agent's utility in the events of agreement vs. no agreement (the latter is called the "outside option"). Thus, for the union (and employees), in McDonald and Solow's (1981. p. 898) terminology, its "aggregate gain" from working with the firm is

$$\begin{aligned} \underbrace{\left[N_i c + q_i \left(w_i + \frac{z_i \Pi_i}{q_i} - c \right) \right]}_{\text{payoff from working with the firm}} - \underbrace{N_i c}_{\text{outside option payoff}} &= q_i \left(w_i + \frac{z_i \Pi_i}{q_i} - c \right) \\ &= q_i \left(w_i - c \right) + z_i \Pi_i, \end{aligned}$$

which is exactly our specification of E_i in (4).

The Union as a Commune

Using Section III, McDonald and Solow (1981, ps. 901–902) propose another model of union, which is labeled as the "union as a commune." The objective function of this union is also consistent with our specification of E_i in (4).

The commune union pools all earnings from its members and redistribute it to everyone, so that all members have equal utility. Let us continue to assume that firm i has N_i members and that each member can provide one unit of labor. Let y be the amount that the union pays out to its member. The union can only pay out what its members pay in, so the resource constraint is

$$N_i y = \underbrace{q_i \times \left(w_i + \frac{z_i \Pi_i}{q_i} \right)}_{\text{revenue from those members working in firm } i} + \underbrace{(N_i - q_i) \times c}_{\text{revenue from those members working in the labor market}}.$$

The left-hand side of the above equation is the total payout. The first term of the right-hand side is the revenue earned by q_i members employed by firm i, while the second term of the right-hand side is the revenue earned by the remaining $(N_i - q_i)$ members who work at a lower market rate c.

Note that under the assumption that members have linear utility, $N_i y$ also represents the total utility of all members and hence the utility of the union when it reaches an agreement with firm i. Again, if the union fails to reach an agreement with the firm, then all members work at a lower rate c, and so the outside option utility is simply $N_i c$. Thus, the union's gain for reaching an agreement with the firm is

$$\begin{aligned} N_i y - N_i c &= q_i \times \left(w_i + \frac{z_i \Pi_i}{q_i} \right) + (N_i - q_i) \times c - N_i c \\ &= q_i (w_i - c) + z_i \Pi_i, \end{aligned}$$

which again coincides with our specification of E_i in (4).

B. Correlated β and k

In this section, we assume $\beta + k = 1$, so that β and k are perfectly negatively correlated. We will show that our results continue to hold. In particular, we will show that offering equity-based compensation is a dominant strategy for firms as long as β is smaller than a threshold value, i.e., a variation of Proposition 2 in the main text. All our results on efficiency in Section 4. and on comparative statics in Section 5. are also valid, because their validity either does not depend on the assumption on β and k (as the results hold for any combination of (β, k)), or can be proved similarly.

Formally, when $\beta + k = 1$, the equilibrium is characterized by the following proposition.

Proposition OA1. *Suppose $\beta + k = 1$. There are two threshold values, $\bar{\beta}_1 \approx 0.237\,12$ and $\bar{\beta}_2 \approx 0.300\,98$, such that:*
(1) If $\beta \in (0, \bar{\beta}_1)$, then there is a unique equilibrium, in which both firms offer wages and equity stakes to their respective employees.
(2) If $\beta \in [\bar{\beta}_1, \bar{\beta}_2]$, then there are two pure strategy equilibria. In one equilibrium, both firms offer wages and equity stakes to their respective employees. In the other equilibrium, both firms offer wages only to their respective employees.
(3) If $\beta \in (\bar{\beta}_2, 1)$, then there is a unique equilibrium, in which both firms offer wages only to their respective employees.

Proof. We check the best responses of firm 1. Once we have the best responses, Parts (1)–(3) follow immediately.

Suppose that firm 2 offers wages only. In the proof of Proposition 2, we know that

$$S_1^{zw}(\beta,k) > S_1^{ww}(\beta,k) \Longleftrightarrow Q_1(\beta,k) > 0, \text{ for any } \beta,k \in (0,1),$$

where $Q_1(\beta,k)$ is defined by equation (A6). Inserting $k = 1-\beta$ into $Q_1(\beta,k)$ and defining $G_1(\beta) \equiv Q_1(\beta, 1-\beta)$, we have

$$\begin{aligned} G_1(\beta) &= -\beta^{12} + 8\beta^{11} - 20\beta^{10} + 8\beta^9 + 58\beta^8 - 168\beta^7 \\ &\quad -28\beta^6 + 1176\beta^5 - 2401\beta^4 + 2688\beta^3 - 1832\beta^2 - 768\beta + 256. \end{aligned}$$

We can compute $G_1(0) = 256 > 0$ and $G_1(1) = -1024 < 0$. In addition, there is a unique root $\bar{\beta}_1 \approx 0.237\,12$ to $G_1(\beta) = 0$ in the range of $\beta \in (0,1)$. Thus, $G_1(\beta) > 0$ if and only if $\beta < \bar{\beta}_1$, implying that $S_1^{zw}(\beta,k) > S_1^{ww}(\beta,k)$ if and only if $\beta < \bar{\beta}_1$. So, when firm 2 offers wages only, firm 1's best response is to offer wages and equity if and only if $\beta < \bar{\beta}_1$.

Suppose that firm 2 offers wages and equity. In the proof of Proposition 2, we have

$$S_1^{zz}(\beta,k) > S_1^{wz}(\beta,k) \Longleftrightarrow Q_2(\beta,k) > 0, \text{ for any } \beta,k \in (0,1),$$

where $Q_2(\beta,k)$ is defined by equation (A12). Again, we insert $k = 1-\beta$ into $Q_2(\beta,k)$, and define $G_2(\beta) \equiv Q_2(\beta, 1-\beta)$, yielding

$$G_2(\beta) = -\beta^{10} + 8\beta^9 - 28\beta^8 + 88\beta^7 - 270\beta^6 + 472\beta^5 - 404\beta^4 + 488\beta^3 - 633\beta^2 - 32\beta + 56.$$

We can compute $G_2(0) = 56 > 0$ and $G_2(1) = -256 < 0$. Moreover, in the range of $\beta \in (0,1)$, there is a unique root $\bar{\beta}_2 \approx 0.300\,98$ to $G_2(\beta) = 0$. As a result, $G_2(\beta) > 0$ and $S_1^{zz}(\beta,k) > S_1^{wz}(\beta,k)$ if and only if $\beta < \bar{\beta}_2$. That is, when firm 2 offers wages and equity, firm 1's best response is to offer wages and equity if and only if $\beta < \bar{\beta}_2$. ■

Comparing with Proposition 2, we see that the main theme is still the same, i.e., when employees have low bargaining power, firms are more likely to issue equity shares to employees. In Proposition 2, variable $\hat{\beta}_1(k)$, which is the threshold value of β for the employee ownership equilibrium to prevail as the pure equilibrium, depends on k. As Fig. 3 shows, $\hat{\beta}_1(k)$ monotonically increases from 0 to 0.373405. In this section, because β and k are negatively correlated such that $\beta + k = 1$, the threshold value $\bar{\beta}_1$ degenerates to a constant $\bar{\beta}_1 \approx 0.237\,12$.

C. Bertrand Competition in Product Market

In this section, we assume that the product market features Bertrand competition so that firms use prices as strategy variables. We keep unchanged all the other features of the baseline model in the main text. We will show that for a Bertrand product market, firms still offer equity-based compensation if and only if the market competition is intense and the employees' bargaining power is low—specifically, if and only if $\beta < \frac{k^2}{2}$—which is consistent with Corollary 1 in the main text. However, when the employee ownership equilibrium prevails, the equilibrium portion z_i^* of employee equity ownership is negative, i.e., employees return wages to the non-employee owners in an amount equal to $|z_i^*|$ percent of the firm's profits.

Using equation (2), we can compute the demand function for firm i as follows:

$$q_i = \frac{1}{1+k} - \frac{1}{1-k^2}P_i + \frac{k}{1-k^2}P_j, \text{ for } i,j = 1,2, \text{ and } i \neq j, \tag{OA1}$$

where parameter $k \in (0,1)$ still controls the degree of product market competition. At the period-3 product market, firm i chooses P_i to maximize its profit $\Pi_i = q_i (P_i - w_i)$, taking as given the demand function (OA1), the other firm's price P_j, and its own wage cost w_i. That is,

$$\underset{P_i}{Max} \left(\frac{1}{1+k} - \frac{1}{1-k^2}P_i + \frac{k}{1-k^2}P_j \right) (P_i - w_i).$$

The first-order condition delivers the following best response function of firm i:

$$P_i = \frac{1-k+w_i+kP_j}{2}, \text{ for } i,j = 1,2, \text{ and } i \neq j. \tag{OA2}$$

Using (OA2) (for $i = 1,2$), we can compute the period-3 equilibrium product price,

$$P_i(w_i, w_j, k) = \frac{(1-k)(2+k) + 2w_i + kw_j}{4-k^2}, \tag{OA3}$$

which is inserted into (OA1) and $\Pi_i = q_i (P_i - w_i)$, yielding the equilibrium quantity and profit at the period-3 product market as follows:

$$q_i(w_i, w_j, k) = \frac{(1-k)(2+k) - (2-k^2)w_i + kw_j}{(1-k^2)(4-k^2)}, \tag{OA4}$$

$$\Pi_i(w_i, w_j, k) = \frac{\left((1-k)(2+k) - (2-k^2)w_i + kw_j\right)^2}{(1-k^2)(4-k^2)^2}, \tag{OA5}$$

for $i,j = 1,2$, and $i \neq j$.

Plugging equations (OA4) and (OA5) into equations (9) and (10), we obtain the following expressions of residual profits $S_i(z_i, w_i, w_j, k)$ and employee compensation $E_i(z_i, w_i, w_j, k)$:

$$S_i(z_i, w_i, w_j, k) = (1 - z_i)\frac{\left((1-k)(2+k) - (2-k^2)w_i + kw_j\right)^2}{(1-k^2)(4-k^2)^2}, \tag{OA6}$$

$$\begin{aligned} E_i(z_i, w_i, w_j, k) &= \frac{(1-k)(2+k) - (2-k^2)w_i + kw_j}{(1-k^2)(4-k^2)}(w_i - c) \\ &+ z_i\frac{\left((1-k)(2+k) - (2-k^2)w_i + kw_j\right)^2}{(1-k^2)(4-k^2)^2}. \end{aligned} \tag{OA7}$$

In period 2, non-employee owners and employees bargain over the wage w_i and equity portion z_i (in the case in which the firm offers equity stakes) to maximize the Nash product, $[E_i(z_i, w_i, w_j, k)]^{\beta}[S_i(z_i, w_i, w_j, k)]^{1-\beta}$, where $E_i(z_i, w_i, w_j, k)$ and $S_i(z_i, w_i, w_j, k)$ are given by (OA6) and (OA7), respectively. The period-2 equilibrium bargaining outcomes again depend on what contracts the two firms offer in the previous period. We summarize the results in the following proposition, which is a counterpart of Proposition 1 in the main text.

Proposition OA2. *(1) When both firms offer wages only to their respective employees, we have, for $i = 1, 2$,*

$$\begin{aligned} w_i^{ww}(\beta, k) &= c + \frac{\beta(k+2)(1-k)(1-c)}{4 - k\beta - 2k^2}, \\ S_i^{ww}(\beta, k) &= \frac{(2-\beta)^2(1-k)(2-k^2)^2(1-c)^2}{(k+1)(2-k)^2(4-k\beta-2k^2)^2}, \\ E_i^{ww}(\beta, k) &= \frac{\beta(2-\beta)(k+2)(1-k)(2-k^2)(1-c)^2}{(k+1)(2-k)(4-k\beta-2k^2)^2}. \end{aligned}$$

(2) When both firms offer wages and equity stakes to their respective employees, we have, for $i = 1, 2$,

$$\begin{aligned} z_i^{zz}(\beta, k) &= \frac{2\beta - k^2}{2 - k^2}, \\ w_i^{zz}(\beta, k) &= c + \frac{k^2(1-k)(1-c)}{4 - 2k - k^2}, \\ S_i^{zz}(\beta, k) &= \frac{2(1-\beta)(1-k)(2-k^2)(1-c)^2}{(k+1)(4-2k-k^2)^2}, \\ E_i^{zz}(\beta, k) &= \frac{2\beta(1-k)(2-k^2)(1-c)^2}{(k+1)(4-2k-k^2)^2}. \end{aligned}$$

(3) When firm 1 offer wages and equity stakes and firm 2 offers wages only to their respective

employees, we have

$$\begin{aligned} z_1^{zw}(\beta,k) &= \frac{2\beta-k^2}{2-k^2}, \\ w_1^{zw}(\beta,k) &= c+\frac{k^2(k+2)(1-k)(4+k\beta-2k^2)(1-c)}{32(1-k^2)+(8-\beta)k^4}, \\ w_2^{zw}(\beta,k) &= c+\frac{\beta(1-k)(4-k^2)(4+2k-k^2)(1-c)}{32(1-k^2)+(8-\beta)k^4}, \\ S_1^{zw}(\beta,k) &= \frac{2(1-\beta)(1-k)(2-k^2)(k+2)^2(4+k\beta-2k^2)^2(1-c)^2}{(k+1)(32(1-k^2)+(8-\beta)k^4)^2}, \\ E_1^{zw}(\beta,k) &= \frac{2\beta(1-k)(2-k^2)(k+2)^2(4+k\beta-2k^2)^2(1-c)^2}{(k+1)(32(1-k^2)+(8-\beta)k^4)^2}, \\ S_2^{zw}(\beta,k) &= \frac{(2-\beta)^2(1-k)(2-k^2)^2(4+2k-k^2)^2(1-c)^2}{(k+1)(32(1-k^2)+(8-\beta)k^4)^2}, \\ E_2^{zw}(\beta,k) &= \frac{\beta(2-\beta)(1-k)(2-k^2)(4-k^2)(4+2k-k^2)^2(1-c)^2}{(k+1)(32(1-k^2)+(8-\beta)k^4)^2}. \end{aligned}$$

When firm 1 offers wages only and firm 2 offer wages and equity stakes to their respective employees, the variables $w_1^{wz}(\beta,k), w_2^{zw}(\beta,k), z_2^{wz}(\beta,k), S_i^{wz}(\beta,k)$, and $E_i^{wz}(\beta,k)$ are characterized symmetrically.

Proof. Suppose that both firms offer only wages in the first period, so that we have $z_1 = z_2 = 0$. By (OA6) and (OA7), firm i's Nash bargaining problem is

$$\underset{w_i}{Max}\left[\frac{(1-k)(2+k)-(2-k^2)w_i+kw_j}{(1-k^2)(4-k^2)}(w_i-c)\right]^{\beta}\left[\frac{((1-k)(2+k)-(2-k^2)w_i+kw_j)^2}{(1-k^2)(4-k^2)^2}\right]^{1-\beta}.$$

The first-order condition is

$$\frac{\beta}{w_i-c}=\frac{(2-\beta)(2-k^2)}{(1-k)(2+k)-(2-k^2)w_i+kw_j}. \tag{OA8}$$

Using (OA8) for $i = 1, 2$, we can compute the expression of $w_i^{ww}(\beta,k)$. Inserting $w_i^{ww}(\beta,k)$ and $z_i = 0$ into (OA6) and (OA7) delivers the expression of $S_i^{ww}(\beta,k)$ and $E_i^{ww}(\beta,k)$.

Suppose that both firms offer both wages and equity stakes. Then, we have $z_1 \neq 0$ and $z_2 \neq 0$. So, by (OA6) and (OA7), firm i's Nash bargaining problem is

$$\underset{z_i,w_i}{Max}\left[\begin{array}{c}\frac{(1-k)(2+k)-\left(2-k^2\right)w_i+kw_j}{(1-k^2)(4-k^2)}(w_i-c)\\ +z_i\frac{\left((1-k)(2+k)-\left(2-k^2\right)w_i+kw_j\right)^2}{(1-k^2)(4-k^2)^2}\end{array}\right]^{\beta}\left[(1-z_i)\frac{((1-k)(2+k)-(2-k^2)w_i+kw_j)^2}{(1-k^2)(4-k^2)^2}\right]^{1-\beta}.$$

The first-order conditions are

$$\frac{\beta\frac{\left((1-k)(2+k)-\left(2-k^2\right)w_i+kw_j\right)^2}{(1-k^2)(4-k^2)^2}}{\frac{(1-k)(2+k)-(2-k^2)w_i+kw_j}{(1-k^2)(4-k^2)}(w_i-c)+z_i\frac{((1-k)(2+k)-(2-k^2)w_i+kw_j)^2}{(1-k^2)(4-k^2)^2}}-\frac{1-\beta}{1-z_i}=0, \tag{OA9}$$

$$\frac{\beta\begin{pmatrix}\frac{-\left(2-k^2\right)(w_i-c)+(1-k)(2+k)-\left(2-k^2\right)w_i+kw_j}{(1-k^2)(4-k^2)}\\+2z_i\frac{\left((1-k)(2+k)-\left(2-k^2\right)w_i+kw_j\right)}{(1-k^2)(4-k^2)^2}\left(-\left(2-k^2\right)\right)\end{pmatrix}}{\frac{(1-k)(2+k)-(2-k^2)w_i+kw_j}{(1-k^2)(4-k^2)}(w_i-c)+z_i\frac{((1-k)(2+k)-(2-k^2)w_i+kw_j)^2}{(1-k^2)(4-k^2)^2}}$$

$$+2(1-\beta)\frac{-\left(2-k^2\right)}{\left((1-k)(2+k)-\left(2-k^2\right)w_i+kw_j\right)}$$

$$= 0. \tag{OA10}$$

Using the above two equations for $i = 1, 2$, we can solve the expressions for $w_i^{zz}(\beta, k)$ and $z_i^{zz}(\beta, k)$. Inserting $w_i^{zz}(\beta, k)$ and $z_i^{zz}(\beta, k)$ into (OA6) and (OA7) delivers the expression of $S_i^{zz}(\beta, k)$ and $E_i^{zz}(\beta, k)$.

Suppose that firm 1 offers equity-based packages while firm 2 offers wages only. Then, for firm 1, equations (OA9) and (OA10) hold, while for firm 2, equation (OA8) holds. Then solving the system of these three equations yields the three unknowns, $w_1^{zw}(\beta, k)$, $w_2^{zw}(\beta, k)$, and $z_1^{zw}(\beta, k)$. Inserting $w_1^{wz}(\beta, k)$, $w_2^{zw}(\beta, k)$, $z_1^{wz}(\beta, k)$, and $z_2^{wz} = 0$ into (OA6) and (OA7) delivers the expression of $S_i^{wz}(\beta, k)$ and $E_i^{wz}(\beta, k)$. ■

Going back to period 1, firms simultaneously choose compensation contracts to maximize their residual profits, S_i. Similar to the baseline model in the main text, we can use the expressions of S_i in Proposition OA2 to construct the payoff matrix of the two sets of non-employee owners. Analyzing the payoff matrix yields the following proposition that characterizes the period 1 equilibrium, as a counterpart of Proposition 2 in the main text.

Proposition OA3. *(1) If $\beta < \frac{k^2}{2}$, the employee ownership equilibrium is the unique equilibrium, where both firms offer wages and equity stakes to their respective employees.*
(2) If $\beta > \frac{k^2}{2}$, the wage only equilibrium is the unique equilibrium, where firms offer wages only to their respective employees.

Proof. Since the game is symmetric, we consider the best response function of firm 1.

Suppose that firm 2 offers wages only (i.e., firm 2 plays "w").

Firm 1 then needs to compare $S_1^{ww}(\beta, k)$ and $S_1^{zw}(\beta, k)$, which correspond respectively to the payoff of firm 1's non-employee owners when they offer a wage-based contract or an equity-based contract in the first period. From the expressions of $S_1^{ww}(\beta, k)$ and $S_1^{zw}(\beta, k)$

in Proposition OA2, we can compute

$$\begin{aligned}
& S_1^{zw}(\beta,k) - S_1^{ww}(\beta,k) \\
= \; & \frac{(1-k)(2-k^2)(1-c)^2}{(k+1)(2-k)^2(32(1-k^2)+(8-\beta)k^4)^2(4-k\beta-2k^2)^2}\left(2\beta-k^2\right)T_1(\beta,k),
\end{aligned}$$

where

$$\begin{aligned}
T_1(\beta,k) \;=\; & -k^4\left(4-k^2\right)^2\beta^4+4k^4\left(2-k^2\right)^2\beta^3+4k^2\left(2-k^2\right)\left(64-56k^2+13k^4\right)\beta^2 \\
& -32\left(4-2k^2+k^4\right)\left(2-k^2\right)^3\beta-32k^2\left(2-k^2\right)^4. \qquad \text{(OA11)}
\end{aligned}$$

We will show that $T_1(\beta,k)<0$ for $\beta,k\in(0,1)$, and thus,

$$S_1^{zw}(\beta,k) > S_1^{ww}(\beta,k) \Longleftrightarrow \beta < \frac{k^2}{2}. \qquad \text{(OA12)}$$

We now show $T_1(\beta,k)<0$. Direct computation shows $\frac{\partial^2 T_1(\beta,k)}{\partial\beta^2} = -12k^4(4-k^2)^2\beta^2 + 24k^4(2-k^2)^2\beta + 8k^2(2-k^2)(64-56k^2+13k^4) > 0$, for $\beta,k\in(0,1)$, and thus $T_1(\beta,k)$ is convex in β. The convexity of $T_1(\beta,k)$ implies that $T_1(\beta,k)$ must hit its maximum at either $\beta=0$ or $\beta=1$. Inserting $\beta=0$ and $\beta=1$ into (OA11), we obtain

$$T_1(0,k) = -32k^2\left(2-k^2\right)^4 < 0 \text{ and } T_1(1,k) = -\left(32-32k^2+7k^4\right)^2 < 0$$

for $k\in(0,1)$. Thus, $T_1(\beta,k)<0$ for all $\beta,k\in(0,1)$.

Suppose that firm 2 offers wages and equity (i.e., firm 2 plays "z").

Firm 1 then needs to compare $S_1^{wz}(\beta,k)$ and $S_1^{zz}(\beta,k)$, which correspond respectively to the payoff of firm 1's non-employee owners when they offer a wage-based contract or an equity-based contract in the first period. By Proposition OA2, we can compute

$$\begin{aligned}
& S_1^{zz}(\beta,w) - S_1^{wz}(\beta,k) \\
= \; & \frac{(1-k)(2-k^2)(1-c)^2}{(k+1)(4-2k-k^2)^2(32(1-k^2)+(8-\beta)k^4)^2}\left(k^2-2\beta\right)T_2(\beta,k),
\end{aligned}$$

where

$$T_2(\beta,k) = k^8\beta^2 + 4\left(16-16k^2-k^4\right)\left(2-k^2\right)^2\beta + 4k^2\left(2-k^2\right)\left(16-8k^2-k^4\right). \quad \text{(OA13)}$$

We will show that $T_2(\beta,k)>0$ for all $\beta,k\in(0,1)$, and thus,

$$S_1^{zz}(\beta,k) > S_1^{wz}(\beta,k) \Longleftrightarrow \beta < \frac{k^2}{2}. \qquad \text{(OA14)}$$

Now let us show $T_2(\beta,k)>0$. Suppose $16-16k^2-k^4\geq 0$. Then $T_2(\beta,k)>0$ for all $\beta,k\in(0,1)$, since all the three terms of (OA13) are positive.

Now suppose $16 - 16k^2 - k^4 < 0$. Define the sum of the last two terms of (OA13) as

$$L(\beta, k) \equiv 4\left(16 - 16k^2 - k^4\right)\left(2 - k^2\right)^2 \beta + 4k^2\left(2 - k^2\right)\left(16 - 8k^2 - k^4\right).$$

Note that $L(\beta, k)$ is decreasing in β when $16 - 16k^2 - k^4 < 0$, and so $L(\beta, k)$ achieves its minimum at $\beta = 1$. We can compute

$$L(1, k) = 8\left(2 - k^2\right)\left(4 - k^2\right)\left(4 - 3k^2\right) > 0.$$

Thus, $L(\beta, k) > 0$ for all $\beta, k \in (0, 1)$.

Taken together, irrespective of the sign of $16 - 16k^2 - k^4$, we have $T_2(\beta, k) = k^8\beta^2 + L(\beta, k) > 0$ for all $\beta, k \in (0, 1)$.

By conditions (OA12) and (OA14), offering equity-based compensation is a dominant strategy if and only if $\beta < \frac{k^2}{2}$. Similarly, offering wage-based compensation is a dominant strategy if and only if $\beta > \frac{k^2}{2}$. Therefore, Parts (1) and (2) follow immediately. ■

The following proposition describes the properties of the equity ownership equilibrium when it is supported.

Proposition OA4. *Suppose $\beta < \frac{k^2}{2}$. Then, in the supported equity ownership equilibrium, the equilibrium employee equity stake z_i^* is negative, and the equilibrium wage rate w_i^* is higher than it would have been in the wages only equilibrium.*

Proof. By Proposition OA3, when $\beta < \frac{k^2}{2}$, the unique equilibrium is the equity ownership equilibrium. Thus, by Proposition OA2, $z_i^* = z_i^{zz}(\beta, k) = \frac{2\beta - k^2}{2 - k^2} < 0$ by $\beta < \frac{k^2}{2}$. Again, by Proposition OA2, we have

$$\begin{aligned} w_i^* - w_i^{ww}(\beta, k) &= w_i^{zz}(\beta, k) - w_i^{ww}(\beta, k) \\ &= \frac{2(1-k)(2-k^2)(1-c)}{(4-2k-k^2)(4-k\beta-2k^2)}\left(k^2 - 2\beta\right) > 0, \end{aligned}$$

and thus $w_i^* > w_i^{ww}(\beta, k)$. ■

D. A Setup with Employees Maximizing $\frac{E_i}{q_i}$

In this section, we consider a variation in which the employees' objective function becomes

$$A_i \equiv \frac{E_i}{q_i} = w_i - c + \frac{z_i \Pi_i}{q_i}, \tag{OA15}$$

where the second equality follows from the definition of E_i in (4) in the main text. The other features of the model are unchanged. The idea of this variation is that in some settings, employees may maximize the average compensation of those employed as opposed to the total compensation of the employed workforce. It turns out that under this alternative specification, the employee ownership equilibrium is the unique equilibrium. Thus, our result is strengthened. Specifically, even in a deterministic setting, equity compensation can arise endogenously as an equilibrium outcome.

We now characterize the equilibrium. In period 3, the product market equilibrium does not change. That is, firms still take their labor costs w_i as given and play a Cournot game, which therefore yields the labor demand $q_i(w_i, w_j, k)$ in (7) and the profit function $\Pi_i(w_i, w_j, k)$ in (8).

In period 2, each firm's non-employee owners and employees choose the wage w_i, and the equity stake z_i, in cases in which the firm offers equity compensation, to maximize the generalized Nash product

$$[A_i(z_i, w_i, w_j, k)]^{\beta} [S_i(z_i, w_i, w_j, k)]^{1-\beta}. \tag{OA16}$$

By equations (9), (OA15), (7) and (8), we can express

$$S_i(z_i, w_i, w_j, k) = (1 - z_i) \frac{(2 - k - 2w_i + kw_j)^2}{(4 - k^2)^2}, \tag{OA17}$$

$$A_i(z_i, w_i, w_j, k) = w_i - c + z_i \frac{2 - k - 2w_i + kw_j}{4 - k^2}, \tag{OA18}$$

for $i, j = 1, 2$, and $i \neq j$.

The following proposition characterizes the bargaining game equilibrium in period 2, which is the counterpart of Proposition 2 in the main text.

Proposition OA5. *(1) When both firms offer wages only to their respective employees, we have, for $i = 1, 2$,*

$$\begin{aligned} w_i^{ww}(\beta, k) &= c + \frac{\beta(2-k)(1-c)}{4 - \beta(2+k)}, \\ S_i^{ww}(\beta, k) &= \left(\frac{4(1-\beta)(1-c)}{(2+k)(4 - 2\beta - k\beta)}\right)^2, \\ A_i^{ww}(\beta, k) &= \frac{\beta(2-k)(1-c)}{4 - \beta(2+k)}. \end{aligned}$$

(2) When both firms offer wages and equity stakes to their respective employees, we have, for

$i = 1, 2,$

$$\begin{aligned} z_i^{zz}(\beta,k) &= \frac{k^2}{2}, \\ w_i^{zz}(\beta,k) &= c + \frac{(1-c)(2\beta-k^2)}{4+2k-2\beta-2k\beta-k^2}, \\ S_i^{zz}(\beta,k) &= \frac{2(1-\beta)^2(2-k^2)(1-c)^2}{(4+2k-2\beta-2k\beta-k^2)^2}, \\ A_i^{zz}(\beta,k) &= \frac{\beta(2-k^2)(1-c)}{4+2k-2\beta-2k\beta-k^2}. \end{aligned}$$

(3) When firm 1 offer wages and equity stakes and firm 2 offers wages only to their respective employees, we have

$$\begin{aligned} z_1^{zw}(\beta,k) &= \frac{k^2}{2}, \\ w_1^{zw}(\beta,k) &= c + \frac{(2-k)(2\beta-k^2)(4+k\beta-2\beta)(1-c)}{16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4)}, \\ w_2^{zw}(\beta,k) &= c + \frac{\beta(4-k^2)(4+2k\beta-2k-2\beta-k^2)(1-c)}{16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4)}, \\ S_1^{zw}(\beta,k) &= \frac{2(1-\beta)^2(2-k^2)(k-2)^2(4+k\beta-2\beta)^2(1-c)^2}{(16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4))^2}, \\ A_1^{zw}(\beta,k) &= \frac{\beta(2-k)(2-k^2)(4+k\beta-2\beta)(1-c)}{16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4)}, \\ S_2^{zw}(\beta,k) &= \frac{16(1-\beta)^2(4+2k\beta-2k-2\beta-k^2)^2(1-c)^2}{(16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4))^2}, \\ A_2^{zw}(\beta,k) &= \frac{\beta(4-k^2)(4+2k\beta-2k-2\beta-k^2)(1-c)}{16(1-\beta)(2-k^2)+\beta(8\beta-6k^2\beta+k^4)}. \end{aligned}$$

When firm 1 offers wages only and firm 2 offer wages and equity stakes to their respective employees, the variables $w_1^{wz}(\beta,k), w_2^{zw}(\beta,k), z_2^{wz}(\beta,k), S_i^{wz}(\beta,k),$ *and* $E_i^{wz}(\beta,k)$ *are characterized symmetrically.*

Proof. Suppose that both firms offer wages only in period 1, so that we have $z_1 = z_2 = 0$. Then, using (OA16), (OA17), and (OA18), firm i's Nash bargaining problem changes to

$$\underset{w_i}{Max}\,(w_i-c)^\beta \left[\frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2}\right]^{1-\beta}.$$

The first-order condition is

$$\frac{\beta}{w_i-c} = \frac{4(1-\beta)}{2-k-2w_i+kw_j}. \tag{OA19}$$

Using (OA19) for $i = 1, 2$, we can compute the expressions of $w_i^{ww}(\beta,k)$. Inserting $w_i^{ww}(\beta,k)$ and $z_i = 0$ into (OA17) and (OA18) yields the expressions of $S_i^{ww}(\beta,k)$ and $A_i^{ww}(\beta,k)$.

Suppose that both firms offer equity-based compensation, such that $z_1 \neq 0$ and $z_2 \neq 0$. By (OA16), (OA17), and (OA18), firm i's Nash bargaining problem changes to

$$\underset{z_i, w_i}{Max} \left[w_i - c + z_i \frac{2-k-2w_i+kw_j}{4-k^2} \right]^{\beta} \left[(1-z_i) \frac{(2-k-2w_i+kw_j)^2}{(4-k^2)^2} \right]^{(1-\beta)}.$$

The two first-order conditions are

$$\frac{\beta}{w_i - c + z_i \frac{2-k-2w_i+kw_j}{4-k^2}} \frac{2-k-2w_i+kw_j}{4-k^2} - \frac{1-\beta}{1-z_i} = 0, \qquad \text{(OA20)}$$

$$\frac{\beta}{w_i - c + z_i \frac{2-k-2w_i+kw_j}{4-k^2}} \left(1 + z_i \frac{-2}{4-k^2} \right) + \frac{2(1-\beta)(-2)}{2-k-2w_i+kw_j} = 0. \qquad \text{(OA21)}$$

Using the above two equations for $i = 1, 2$, we can compute the expressions for $w_i^{zz}(\beta, k)$ and $z_i^{zz}(\beta, k)$. Inserting $w_i^{zz}(\beta, k)$ and $z_i^{zz}(\beta, k)$ into (OA17) and (OA18) yields the expressions of $S_i^{zz}(\beta, k)$ and $A_i^{zz}(\beta, k)$.

Suppose that firm 1 offers equity-based compensation and firm 2 offers wages only. In this case, for firm 1, equations (OA20) and (OA21) hold, while for firm 2, equation (OA19) holds. We can then solve the system of these three equations in terms of three unknowns, $w_1^{zw}(\beta, k)$, $z_1^{zw}(\beta, k)$, and $w_2^{zw}(\beta, k)$. Inserting these solved expressions for w_i and z_i into (OA17) and (OA18) yields the expressions of $S_i^{ww}(\beta, k)$ and $A_i^{ww}(\beta, k)$. ■

Using Proposition OA5, we can form the payoff matrix in period 1 in terms of the payoff of the two groups of non-employee owners. Analyzing this payoff matrix yields the following proposition, as a counterpart of Proposition 2 in the main text.

Proposition OA6. *If employees maximize average compensation A_i, then the employee ownership equilibrium prevails as the unique equilibrium, i.e., both sets of non-employee owners offer wages and equity in period 1.*

Proof. Let us consider the best response of firm 1.

Suppose that firm 2 offers wages only.

By Proposition OA5, we can compute

$$\begin{aligned} & S_1^{zw}(\beta, k) - S_1^{ww}(\beta, k) \\ = & -\frac{2k^4(1-\beta)^2(1-c)^2}{(k+2)^2(2\beta+k\beta-4)^2(32-32\beta-6k^2\beta^2+8\beta^2+16k^2\beta+k^4\beta-16k^2)^2} H_1(\beta, k) \end{aligned}$$

where

$$\begin{aligned} H_1(\beta,k) &= \beta^4 k^6 - 2\beta^2\left(12 - 16\beta + 9\beta^2\right)k^4 \\ &\quad +32\left(8 - 24\beta + 30\beta^2 - 17\beta^3 + 4\beta^4\right)k^2 - 32\left(4 - 8\beta + 5\beta^2\right)(2-\beta)^2 . \end{aligned}$$

We can show that $H_1(\beta,k) < 0$ for all $\beta, k \in (0,1)$, and thus,

$$S_1^{zw}(\beta,k) > S_1^{ww}(\beta,k) \text{ for all } \beta, k \in (0,1). \tag{OA22}$$

Next we establish the statement of $H_1(\beta,k) < 0$. We can compute

$$H_1(\beta,1) = -256 + 768\beta - 856\beta^2 + 384\beta^3 - 49\beta^4,$$

which is negative over the range of $\beta \in [0,1]$ (as its first positive root is $\beta \approx 1.309$). In addition,

$$\lim_{k\to\infty} H_1(\beta,k) = \infty,$$

and thus $H_1(\beta,k)$ crosses zero from below at least once in the range of $k \in (1,\infty)$.

Viewing $H_1(\beta,k)$ as a cubic of k^2, we can compute its discriminant as follows:

$$D_{H_1} = -16\,384\beta^4\left(448 - 1984\beta + 3616\beta^2 - 3408\beta^3 + 1728\beta^4 - 452\beta^5 + 53\beta^6\right)(2-\beta)^2(1-\beta)^4,$$

which is negative because $\left(448 - 1984\beta + 3616\beta^2 - 3408\beta^3 + 1728\beta^4 - 452\beta^5 + 53\beta^6\right) > 0$. Therefore, $H_1(\beta,k) = 0$ has one real root and two complex roots. From the discussion in the previous paragraph, we know that $H_1(\beta,k)$ crosses zero from below and that this crossing occurs at the range of $(1,\infty)$. Thus, $H_1(\beta,k) < 0$ for all $\beta, k \in (0,1)$.

Suppose that firm 2 offers wages and a equity stake.

By Proposition OA5, we can compute

$$\begin{aligned} & S_1^{zz}(\beta,k) - S_1^{wz}(\beta,k) \\ = & -\frac{2k^4(1-\beta)^2(1-c)^2}{\left(-2k + 2\beta + 2k\beta + k^2 - 4\right)^2\left(-32\beta - 6k^2\beta^2 + 8\beta^2 + 16k^2\beta + k^4\beta - 16k^2 + 32\right)^2} H_2(\beta,k), \end{aligned}$$

where

$$\begin{aligned} H_2(\beta,k) &= \beta^2 k^6 + \left(8 - 32\beta + 30\beta^2 - 12\beta^3\right)k^4 \\ &\quad + \left(64 - 192\beta + 256\beta^2 - 152\beta^3 + 36\beta^4\right)k^2 - 8\left(4 - 8\beta + 5\beta^2\right)(\beta-2)^2 \end{aligned} \tag{OA23}$$

We can show that $H_2(\beta,k) < 0$ for all $\beta, k \in (0,1)$, and thus,

$$S_1^{zz}(\beta,k) > S_1^{wz}(\beta,k) \text{ for all } \beta, k \in (0,1). \tag{OA24}$$

We now establish the statement of $H_2(\beta,k) < 0$. We can show

$$H_2(\beta,1) = -56 + 160\beta - 161\beta^2 + 60\beta^3 - 4\beta^4 < 0$$

as the smallest positive root to $-56+160\beta-161\beta^2+60\beta^3-4\beta^4=0$ is $\beta \approx 1.2676$. We also have

$$\lim_{k\to\infty} H_2(\beta,k)=\infty$$

for any $\beta \in (0,1)$. Thus, $H_2(\beta,k)$ crosses zero from below at least once in the range of $k \in (1,\infty)$.

Note that in expression (OA23), only the coefficient on k^4 is undetermined. Specifically,

$$8-32\beta+30\beta^2-12\beta^3>0 \Leftrightarrow \beta<0.34744.$$

The other coefficients have the following signs:

$$\begin{aligned} \beta^2 &> 0, \\ \left(64-192\beta+256\beta^2-152\beta^3+36\beta^4\right) &> 0, \\ -8\left(4-8\beta+5\beta^2\right)(\beta-2)^2 &< 0. \end{aligned}$$

Thus, by the Rule of Signs, if $\beta<0.34744$, there exists a unique positive root to $H_2(\beta,k)=0$, which occurs in the range of $k \in (1,\infty)$. As a result, $H_2(\beta,k)<0$ for all $\beta, k \in (0,1)$, provided $\beta<0.34744$.

Now suppose $\beta \geq 0.34744$ such that we cannot apply the Rule of Signs to pin down the exact number of positive roots. Viewing $H_2(\beta,k)$ as a cubic of k^2, we then can compute its discriminant as

$$D_{H_2}=4096(2-\beta)^2(1-\beta)^7\left(32-208\beta+292\beta^2-144\beta^3+27\beta^4\right).$$

We can show that $32-208\beta+292\beta^2-144\beta^3+27\beta^4<0$ if and only if $\beta>0.20919$. Thus, if $\beta \geq 0.34744>0.20919$, we must have $D_{H_2}<0$, and thus there exists a unique positive root to $H_2(\beta,k)=0$, which occurs at the range of $k^2 \in (1,\infty)$. As a result, we also have $H_2(\beta,k)<0$ for all $\beta,k \in (0,1)$ in the case of $\beta \geq 0.34744$.

By conditions (OA22) and (OA24), offering equity-based compensation is a dominant strategy for both sets of non-employee owners. The employee ownership equilibrium is then the unique equilibrium. ■