Diffusing Coordination Risk*

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Abstract

Panic-based bank runs can happen when depositors make their withdrawal decisions simultaneously. By imposing a withdrawal limit, one can prevent depositors from rushing to withdraw all their funds on one date. This partitions each individual's decision, thereby inducing a coordination game with dynamic payoff externalities. The information about whether the bank has survived all withdrawals so far is revealed publicly. We show that if the withdrawal limit is sufficiently small, depositors ignore their private information and coordinate on this public news, once they reach the last node. We use a backward inductive argument to show that depositors who anticipate that no one will withdraw their funds later do not withdraw earlier either. Thus, any solvent bank can be made immune to self-fulfilling runs. By a similar argument, sufficiently asynchronous debt structures could overcome rollover risk. We formalize this policy in a dynamic global game of regime change and show that a sufficiently diffused policy unravels the coordination risk from the end.

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Introduction

Agents failing to coordinate on the right course of action will cause economic turmoil and recessions. Pessimistic investors worrying about the non-participation of other potential investors may decide to walk away from a profitable investment opportunity. Countries trying to attract investments or financial institutions trying to convince their creditors to roll over their debt claims are often faced with such challenges. Is there a way to persuade agents to coordinate on the right course of action?

Consider a demand-deposit contract, which is vulnerable to self-fulfilling bank runs.[1] Depositors will withdraw their funds early if they believe other depositors will do so. This paper proposes a feasible way to make a bank immune to panic-based runs. Instead of allowing depositors to withdraw any amounts of their funds on any date, the bank can restrict individual withdrawal at some interval of time, e.g., depositors can withdraw at most 10% of their deposits every three days in a month. Depositors cannot rush to withdraw all their funds on any date when such policy is in place. A more (or less) diffused policy would be setting a daily (or weekly) limit. After every three days when depositors decide whether to withdraw 10% of their fund or not, they learn if the bank is still viable. In this paper, we show that any solvent bank can avoid self-fulfilling runs if it can impose a small enough limit. Similarly, making the debt structure sufficiently asynchronous could overcome the rollover risk. We call such policy diffusing coordintation risk.

To appreciate the general appeal, we first consider a standard global game of regime change.[2] We formalize the policy of diffusing coordination risk in this general context. Then we apply this policy to specific coordination games like bank runs and debt crises. There are two regimes. Although agents prefer to attack the regime that will fail, the designer (such as a bank) wants agents to coordinate on not attacking her preferred regime. Agents gather noisy private information about the underlying fundamental strength of the designer's preferred regime. If the aggregate attack exceeds this fundamental strength, the preferred regime fails and vice versa. Following Morris and Shin (2003), agents will not attack the preferred regime if and only if they get a sufficiently high signal. Consequently, the preferred regime will materialize if and only if it is sufficiently strong. The

[1] Diamond and Dybvig (1983) shows that a bank run happens because all depositors withdraw expecting others to withdraw is an equilibrium. Goldstein and Pauzner (2005) has extended the bank run model of Diamond and Dybvig (1983) to a global game setting and shows that a bank run happens only when the fundamental is sufficiently weakly.

[2] For examples of a global game of regime change, see Morris and Shin (1998) and Angeletos et al. (2007) for the currency crisis; Goldstein and Pauzner (2005) for self-fulfilling bank runs; Vives (2014) for financial fragility; and Edmond (2013) for riots and political change.

designer's objective is to minimize the probability that agents will fail to coordinate on favoring the preferred regime. We call this probability the *coordination risk*.[3] So, how can the designer reduce this coordination risk?

The designer can influence the game environment in many possible ways. We consider a specific policy that the designer commits to - namely, "diffusing coordination risk" - for its practical appeal. Instead of letting all agents make their decisions at the same time, the designer partitions the mass of agents into finitely many groups and makes them move chronologically. The game proceeds to a group moving later only when the preferred regime is still viable. With a mass of agents spread uniformly over a time interval, an equivalent policy is to select certain dates to check whether the preferred regime is still viable and to disclose this information publicly. If individual action is divisible, the designer can also diffuse the coordination risk by setting a limit to individual attack as in the bank run problem. Under diffused coordination risk, if the preferred regime is still viable, agents learn this information publicly before they take their action. We call this piece of information *the public information of survival*. On the other hand, if the preferred regime has failed, the game stops and the public survival news did not arise. The game reaches group t with some residual fundamental strength. The public information of survival at time t says that the residual fundamental at t is non-negative, which always helps to reduce coordination risk. Upon receiving this information, all agents become more optimistic about the current fundamental and this is publicly known. In a dynamic coordination game, group t agents are concerned about whether the residual fundamental can withstand the attacks from agents moving at t and agents moving afterwards. We call them *intra-group* coordination risk and *inter-group* coordination risk, respectively.

The public information of survival breaks down the uniqueness result we get from the global game. In particular, there can be an equilibrium where agents ignore their private information and never attack the preferred regime. We refer to this as the first best. If the private information is sufficiently noisy, we show that this is indeed the only strategy that survives the iterated elimination of never best responses (Lemma 1). Diffusing coordination risk dilutes private information regarding the *effective fundamental*, or per capita residual fundamental. Suppose after 95% of agents have al-

[3]From the agent's perspective, coordination risk arises from others attacking the regime that she favors. Thus, there are two coordination risks from favoring either regime, which are complement to each other. The coordination risk we defined refers to the one arising from favoring the designer's preferred regime.

ready made their decisions and the preferred regime is still viable and can sustain at most 4% attack more. The last group consisting of 5% agents gather private information about the residual fundamental (4%). The payoff relevant information for them is whether the proportion of agents among them attacking the preferred regime is lower than the effective fundamental ($\frac{4\%}{5\%} = 80\%$). Thus, the private information about the residual fundamental (4%), or the *effective private information*, will become very noisy about the effective fundamental (80%). Therefore, when the last group is sufficiently small, agents will ignore their private information and never attack the preferred regime. Given the last group is sufficiently small, the same argument applies to the second-last group, who face no inter-group coordination risk. Hence, if a designer can choose sufficiently fine partition, we can construct an optimal policy backwards such that all the coordination risk unravels from the end (Theorem 1).

If agents are gathering information right at the instance while making their decisions and such information is sufficient to form their beliefs about the future play, then their decision at time t is independent of how they get there. Consequently, the game can be solved backwards. However, if they gather information when the game starts, group t agents will try to infer how the game reaches time t. Hence, the game cannot be solved backwards. This complicates the role of diffusion considerably. Nonetheless, we show that the result is even stronger (Theorem 2).

In the information cascading models of Banerjee (1992) and Bikhchandani et al. (1992), we also see agents ignore private information when they observe overwhelming evidence from past actions. However, in our model, agents receive very limited information from past actions. They only observe that the preferred regime has survived thus far. Agents are forward-looking because their payoff depends on the future actions of other agents. In fact, the payoff externality amplifies the positive effect of the public information of survival. Only if there are no future attacks and the private information about the effective fundamental is sufficiently noisy, will agents completely ignore their private information and coordinate purely on the public survival news.

Committing to such a diffused policy is feasible in practice. Hedge fund managers can lift investor-level gates to limit investor's redemption within a certain period. A common investor-level gate limits redemptions to 25% of an investors money each quarter over four quarters.[4] Choi et al. (2014) also find that corporate bond issuers are adopting similar policies through the diversification

[4]See "Hedge funds try new way to avoid big redemptions, by Alistair Barr (June 10, 2010), http://www.marketwatch.com/story/hedge-funds-try-new-way-to-avoid-big-redemptions-2010-06-10

of debt rollovers across maturity dates. However, this type of policy is more commonly adopted when banks or fund managers are facing liquidity shortages during times of crisis. For example, in the recent financial crisis in Greece,[5] the government imposed such withdrawal limits for households. Regulators wanted to adopt partial suspension to prevent liquidity from drying up. The recent Securities and Exchange Commission reform of money market funds in the U.S. provides fund managers with the ability to set redemption gates within a certain period when a fund's liquidity position is not favorable.[6] In our model, the designer makes commitment to a non-contingent diffused policy ex-ante, so that there is no signaling effect of the policy.

Our main result shows that the first best outcome can be achieved uniquely through diffusing coordination risk. What if achieving this requires the designer to infinitely diffuse the coordination risk? Our main contribution is to show that this is not the case. There is a critical size for each group such that no matter how small the fundamental strength is, if no group is larger than this critical size, the preferred regime will succeed.

Related Literature

Our benchmark model, when all of the coordination risk is concentrated at a particular time, follows Morris and Shin (1998, 2003, 2007) (henceforth, MS). This strand of literature is commonly referred to as global games. Following the refinement idea of Carlsson and Van Damme (1993), the assumption of the common knowledge of payoffs is relaxed. Agents gather private information, and we have a unique equilibrium that characterizes the coordination risk.

When the coordination risk is diffused, a dynamic global game ensues. Chamley (1999) and Angeletos et al. (2007) both considered a dynamic regime change problem in which agents get repeated opportunities to make decisions. In their models, an agent's payoff is realized period by period, and there is no dynamic incentive to coordinate with future actions. In our model, diffusion forces agents to make their irreversible decision at a pre-specified time, and which regime will materialize depends on the aggregate attack of all agents. Thus, agents have dynamic incentives to coordinate with agents moving later.

Dasgupta (2007), Dasgupta et al. (2012), and Mathevet and Steiner (2013) developed a global

[5] See "Greek Banks Reopen Their Doors, but With Strict Limits Still in Place", by Nektaria Stamouli (July 20, 2015), Wall Street Journal, http://www.wsj.com/articles/greek-banks-reopen-their-doors-1437376273.

[6] See http://www.sec.gov/rules/final/2014/33-9616.pdf.

game with similar dynamic incentives. Chassang (2010) considered a dynamic global game problem in which the fundamental varies over time. The underlying fundamental in our model is drawn only at the beginning. The residual fundamental evolves endogenously depending on past actions. Apart from receiving private information, agents learn publicly that the designer's preferred regime is still viable. Unlike Morris and Shin (2002) and Angeletos and Werning (2004), the public information in our model is binary and truth-telling. We see similar public information in Angeletos et al. (2007). When such public information is available, the precision of private information has quite a different effect in the equilibrium selection. We find that if the private information is sufficiently noisy, agents will ignore their private information and coordinate purely on this positive public news. Diffusion is a way to make the private information about effective fundamental noisier. Dasgupta (2007) considers the case where agents have private noisy information about the past aggregate action. If diffusion induces repeated learning of the past actions, then more diffusion will allow agents to learn the fundemental more precisely. We have not considered such repeated learning from diffusion in this model. However, we show that our main result will hold true through as long as more diffusion could make effective fundamental noiser.

Similar to Bergemann and Morris (2013), we consider a designer who wants to minimize the risk that agents will fail to coordinate on favoring her preferred regime. But our designer may not have full control over disclosing more information. She adpots a particular policy. If the preferred regime survives the previous attacks, this policy reveals this information publicly. This manipulates agents' beliefs and thus their action. Unlike Angeletos et al. (2006), the designer commits to the diffused policy ex-ante, so there is no signaling effect of the policy.

This paper is related also to herding literature (see Banerjee (1992) and Bikhchandani et al. (1992)), emphasizing information externality. However, the reason why agents ignore their private information is very different, as discussed before. Our mechanism works with a continuous state and signal, while Smith and Sørensen (2000) showed that under continuous signal-setting, cascades may not form. Avoiding self-fulfilling runs is an important application of our mechanism. We will discuss how our work relates to this strand of literature in Section 3.

Outline

This paper is organized as follows: In Section 1, we describe the regime change benchmark, where all the risk of coordination is concentrated at one point in time. This section also shows how public information of survival affects the equilibrium selection. In Section 2, we introduce the policy of diffusing coordination risk. We will characterize monotone equilibria recursively when this type of policy is implemented. Our main result shows that when agents are gathering information instantaneously and coordination risk is sufficiently diffused, there is no attack against the preferred regime (Theorem 1). Our main result is even stronger if agents are gathering information ex-ante (Theorem 2). In Section 3, we extend this model to show how diffusing coordination risk can avoid panic-based bank runs (Theorem 3) and self-fulfilling debt crises (Corollary 2). In Section 4 we explain how the inter-group and intra-group coordination risks interact and what the designer should do if she has limited power of diffusion (Theorem 4). In Section 5, we discuss the robustness and extensions of our main results. Section 6 concludes. All the proofs appear in the Appendix.

1 Concentrated Coordination Risk

There are two regimes, indexed by $\mathcal{R} \in \{0, 1\}$ and a continuum of agents, indexed by $i \in [0, 1]$. Agents simultaneously decide whether to take an action in favor of regime 1 or regime 0. We will refer to agent i's action of favoring regime $\mathcal{R}$ as $a_i = \mathcal{R}$. Favoring a regime is equivalent attacking the other one. Let θ be the fundamental strength of regime 1 and $w = \int_i \mathbb{1}(a_i = 0)di = \int_i (1 - a_i)di$ be the aggregate attack against regime 1. Regime 1 successfully materializes if and only if its fundamental strength θ is strong enough to withstand the aggregate attack against it, i.e., $\theta \geq w$[7]. Nature picks θ from the commonly known prior $U[\underline{\theta}, \bar{\theta}]$. Agent i gets noisy private information about θ denoted by $s_i = \theta + \sigma\epsilon_i$, where the error ϵ_is are conditionally independent[8] and identically distributed with zero mean. Let F denote the error distribution. We assume F is continuously differentiable and log-concave[9]. Let f be the density and $supp(f) = [-0.5, 0.5]$. σ denotes the standard deviation of the error and $\tau = \frac{1}{\sigma^2}$ is the precision. Assume that $\underline{\theta} \leq -\sigma$ and $\bar{\theta} \geq 1 + \sigma$.

[7]Equivalently, $1 - \theta$ is the fundamental strength of regime 0. Regime 0 materializes if and only if regime 1 failed, i.e., $1 - \theta > 1 - w$.

[8]See Judd (1985) for the existence of continuum of independent random variables.

[9]Most of the distributions we use in practice satisfy logconcavity e.g. uniform, normal, exponential. The exceptions are students t, lognormal distribution etc.

This assumption guarantees that the common prior is uninformative when agents have private information. If $s < -\frac{\sigma}{2}$ (or $s > 1 + \frac{\sigma}{2}$), agents understand for sure that $\theta < 0$ (or $\theta > 1$), which give us the dominance regions. Given $s \in [-\frac{\sigma}{2}, 1 + \frac{\sigma}{2}]$, θ is distributed according to $1 - F(\frac{s-\theta}{\sigma})$. Similarly, given θ, the probability of receiving private signal s is distributed according to $F(\frac{s-\theta}{\sigma})$.

Agents are ex-ante identical and risk neutral. The payoff $u(a_i, w, \theta)$ is defined as follows.

$$u(1, w, \theta) = \begin{cases} b_1 & \text{if } w \leq \theta \\ c_0 & \text{if } w > \theta \end{cases}, \; u(0, w, \theta) = \begin{cases} c_1 & \text{if } w \leq \theta \\ b_0 & \text{if } w > \theta \end{cases} \tag{1}$$

The payoff structure captures strategic complementarity. If regime $\mathcal{R}$ succeeds then agent i would be better off if he had not attacked it, i.e., $b_{\mathcal{R}} > c_{\mathcal{R}}$, $\mathcal{R} \in \{0, 1\}$.

There is a designer who prefers one regime over the other. We will follow the convention that the regime 1 is the designer's preferred regime and θ is the fundamental strength of the preferred regime. The preferred regime may or may not be the default regime. From private information s_i, agents can learn the fundamental θ, other agent's belief about θ, others' beliefs about others' beliefs and so on. Agents receiving higher signals believe that - the preferred regime can withstand more attacks. They also believe that - other players believe that - the preferred regime can withstand more attacks and so on (higher order beliefs). Therefore, agents with higher signals are more likely not to attack the preferred regime. We will look into the monotone equilibrium where agents do not attack the preferred regime if and only if $s_i \geq s^\star$. In equilibrium, the higher the fundamental θ is, the larger share of agents will receive signal above $s^\star$. Thus, there exists $\theta^\star$ such that the preferred regime materializes if and only if $\theta \geq \theta^\star$. The monotone equilibrium can be summarized by $(\theta^\star, s^\star)$. Following Morris and Shin (2003), we know this monotone equilibrium is indeed the unique equilibrium.

Proposition 1 *There is a unique equilibrium where an agent takes action favoring the preferred regime if and only if she gets a signal $s \geq s^\star$. The preferred regime materializes iff $\theta \geq \theta^\star$, where $\theta^\star = p_0 := \frac{1}{1+\frac{b_1-c_1}{b_0-c_0}}$ and $s^\star = \theta^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\theta^\star)$.*

$p_0 \in (0, 1)$ is such that, if an agent believes that the preferred regime will succeed with probability at least p_0, he will not attack the preferred regime. We will call probability $P(\theta < \theta^\star)$ the *coordination risk*. $\theta^\star$ is a sufficient statistic for the coordination risk. The designer wants to minimize θ^*. $\theta^* = 0$ is the first best outcome for the designer. Even if no one attacks, the preferred regime cannot succeed when $\theta < 0$.

For certain coordination game, agents may naturally have the public information $\theta \geq 0$. The public information $\theta \geq 0$ tells that the preferred regime will materialize if there is no attack against it. Such public information will play a crucial role in our analysis of diffusing coordination risk.

Public Information of Survival: $\theta \geq 0$

When $\theta \geq 0$ is publicly known, there is no lower dominance region. Agents understand that the preferred regime will succeed if nobody attacks it. All agents ignoring their private information and never attacking the preferred regime is always an equilibrium. Unlike standard global game, iterated elimination of never best responses may not lead to unique rationalizable strategy. However, we will show that given uninformative prior, if the private information is sufficiently noisy, then not attacking the preferred regime irrespective of the private signal is the unique rationalizable strategy.

Lemma 1 *Given uninformative prior and the log-concavity of F, when public information $\theta \geq 0$ is available, there exists τ^* such that if $\tau \leq \tau^\star$, then the preferred regime succeeds whenever $\theta \geq 0$.*

Thus, when private information is sufficiently noisy, $\theta^\star = 0$ is the unique equilibrium threshold of fundamental.[10] As long as the realization of θ is non-negative, the preferred regime will materialize. When $\tau > \tau^\star$, we have multiple equilibria. The following Proposition is a static version of Angeletos et al. (2007).

Proposition 2 *With public information of survival, there are multiple equilibria. A monotone equilibrium $(\theta^\star, s^\star)$ is such that either $\theta^\star = 0$, $s^\star = -\frac{1}{2\sqrt{\tau}}$ or it satisfies the following conditions.*

$$G(\theta^\star, \sqrt{\tau}) := \frac{\theta^\star}{F(F^{-1}(\theta^\star) + \sqrt{\tau}\theta^\star)} = p_0. \tag{2}$$

$$s^\star(\theta^\star) = \theta^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\theta^\star). \tag{3}$$

[10] The above result will be exactly the same if we assume improper prior (uniform over the real line) and unbounded support of noise. However, under this assumption, we will not be able to define the coordination risk properly. If we consider the prior approaching the improper prior, we can say the designer will like to minimize θ^*. To avoid this unnecessary complication, we assume a bounded prior and bounded support of noise.

For any possible fundamental threshold $\theta^\star$, let $s^\star(\theta^\star)$ be such that if agents follow a threshold strategy $s^\star(\theta^\star)$, then when $\theta \geq (<)\theta^\star$, the aggregate attack $w \leq (>)\theta^\star$. $G(.)$ captures the belief of the corresponding threshold agent $s^\star(\theta^\star)$ that the preferred regime will succeed. The following Figure 1 shows the threshold agent's belief $G(\theta^\star, \sqrt{\tau})$. Without the public information of survival, for any possible $\theta^\star$, the threshold agent with private information $s^\star(\theta^\star)$ believes the probability that preferred regime will succeed is $P(\theta \geq \theta^*|s^\star(\theta^\star)) = \theta^*$. In equilibrium, the threshold agent believes the preferred regime succeeds with probability p_0. Hence, we have a unique equilibrium $\theta^* = p_0$. When agents have the public survival news, the threshold agent's belief is $P(\theta \geq \theta^*|\theta \geq 0, s^*(\theta^\star)) = G(\theta^\star, \sqrt{\tau})$, which is a non-monotonic function of $\theta^\star$. Multiple equilibria could arise but the coordination risk will be lower no matter which equilibrium is selected since $G(\theta^\star, \sqrt{\tau}) \geq \theta^\star$. As can be seen in Figure 1, if τ decreases, the $G(\theta^\star, \sqrt{\tau})$ shifts upwards. When τ is sufficiently small, there is no positive $\theta^\star$ satisfying Equation (2).

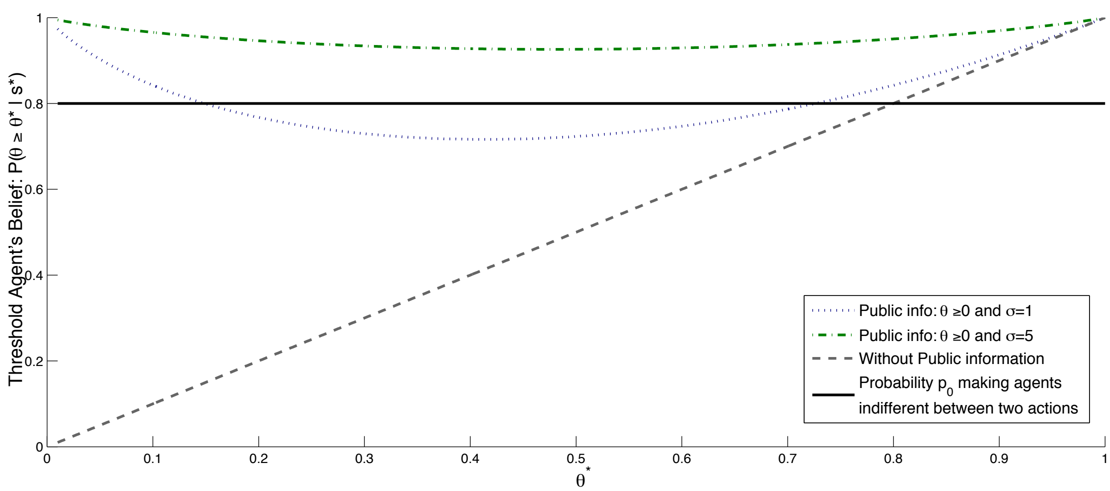

Figure 1: Threshold agent's belief with public information $\theta \geq 0$

Based on Lemma 1, if the designer can control the precision of private information τ, the first best can be implemented as the unique equilibrium outcome. However, she is not likely to have such unlimited manipulation power. In the rest of the paper we assume τ is fixed and agents have uninformative prior. We will show in next section how diffusion could make the *effective* private information noisy.

2 Diffusing Coordination Risk

Section 1 characterizes the coordination risk when agents make their decisions simultaneously. In this section, we will consider the model where the designer diffuses coordination risk.

Diffusion

The designer restricts aggregate attacks in any interval of time by spreading the timing of action. This is done by dividing the agents into finitely many groups and let them make their decisions chronologically. Diffusion is a partition of agents, i.e., $(T, (\alpha))$, where $T \in \mathbb{N}$ and $(\alpha) \equiv (\alpha_1, \alpha_2, \ldots \alpha_T) \in \Delta^{T-1}$.[11] We refer $(T, (\alpha))$ as a policy or a mechanism. Agents are risk neutral and ex-ante identical. They have been assigned to any group randomly. If an agent has been assigned to group t, he will take action only once at time t and his action is irreversible.

The game proceeds further only when the preferred regime is still viable. The design commits to the diffused policy $(T, (\alpha))$ and it is publicly announced. The preferred is said to be still viable if it materializes in absence of any further attack. If this is the case, agents learn this information publicly. We call this *public information of survival.*

Residual strength

Suppose α_t mass of agents are moving at time t and w_t be the proportion of agents moving at t who attack the preferred regime, i.e., $w_t = \frac{1}{\alpha_t} \int (1 - a_{it}) di$, where a_{it} is the action taking by agent i in group t. Diffusion restricts how much attack can happen within one period i.e., $\alpha_t w_t \leq \alpha_t$, but does not change the maximum possible aggregate attack, i.e., $\sum_{t=1}^{T} \alpha_t = 1$. The preferred regime succeeds if and only if it withstands all attack against it. The payoff is exactly the same as in Equation (1), where $w := \sum_{t=1}^{T} \alpha_t w_t$.

Let θ_t be the residual fundamental strength at time t.

$$\theta_t := \theta_{t-1} - \alpha_{t-1} w_{t-1} = \theta_1 - \sum_{u=1}^{t-1} \alpha_u w_u \text{ for } t = 2, 3, ..., T \quad \text{and} \quad \theta_1 = \theta \tag{4}$$

Public information of survival arises naturally when coordination is diffused. The designer continues the game to group t only if residual fundamental is non-negative, i.e., $\theta_t \geq 0$. Agents at group t

[11] $\Delta^{T-1} := \{(\alpha_1, \alpha_2, \ldots, \alpha_T) \in \mathbb{R}^T | \ \alpha_t \geq 0 \ \forall t = 1, 2 \ldots T, \ \sum_{t=1}^{T} \alpha_t = 1\}$ is the $T - 1$ simplex.

publicly learn $\theta_t \geq 0$ before they make their decisions.

Definition 1 *(Monotone Equilibrium) $(\theta_t^\star, s_t^\star)_{t=1}^T$ is said to be a monotone equilibrium if upon reaching group t, for agent i of group t, $a_{it} = 1$ if and only if the private information $s_{it} \geq s_t^\star$. If $\theta_t \geq \theta_t^\star$, the preferred regime will withstand all attacks from t to T. The preferred regime materializes if and only if $\theta \geq \theta_1^\star$.*

2.1 Diffusing Coordination Risk with Instantaneous Information

Agents in group $t \in \{1, 2, ..., T\}$ enter the game right before they are about to make decisions. They share prior $\theta_t \sim U[\underline{\theta}, \bar{\theta}]$ and they gather noisy private information regarding θ_t, $s_{it} = \theta_t + \sigma\epsilon_{it}$, where the distribution of ϵ_{it} is the same as in concentrated coordination case. Under this assumption, prior is uninformative at any time. Thus we neglect the influence of updating prior from diffusion. We refer to this information structure as instantaneous information. The timing and information structure of bifurcated diffusion is summarized in figure 2.

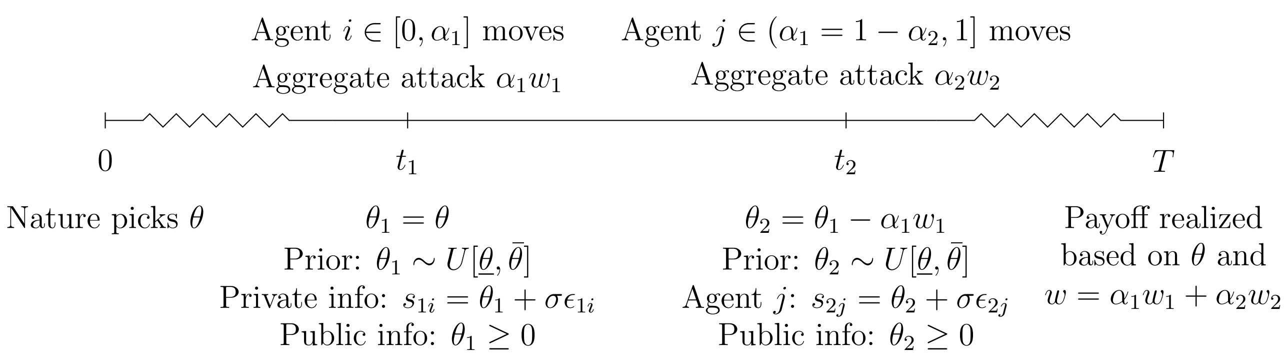

Figure 2: Timeline: Bifurcated Diffusion

Monotone Equilibrium

Agents in group t have public information that $\theta_t \geq 0$ in addition to their private information. Given $\theta_{t+1}^\star$, for any possible threshold of residual fundamental $\theta_t^\star$, let $s_t^\star(\theta_t^\star)$ be such that if group t agents follow the threshold strategy $s_t^\star(\theta_t^\star, \theta_{t+1}^\star)$, then when $\theta_t \geq (<)\theta_t^\star$, the residual fundamental $\theta_{t+1} \geq (<)\theta_{t+1}^\star$. Equivalently, if $\theta_t \geq (<)\theta_t^\star$,

$$\theta_{t+1} = \theta_t - \alpha_t w_t = \theta_t - P(s_t < s_t^\star | \theta_t) \geq (<)\theta_{t+1}^\star, \text{ or } P(s_t < s_t^\star | \theta_t) \geq (<)\frac{\theta_t - \theta_{t+1}^\star}{\alpha_t}.$$

This gives us $s_t^\star(\theta_t^\star, \theta_{t+1}^\star) = \theta_t^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t})$. We call $\frac{\theta_t - \theta_{t+1}^\star}{\alpha_t}$ the effective fundamental strength at time t. The threshold agent with private information $s_t^\star$ will believe the preferred regime materializes with probability $P(\theta_t \geq \theta_t^\star | s_t^\star, \theta_t \geq 0)$. In equilibrium, the threshold agent will be indifferent between attacking either regime.

$$P(\theta_t \geq \theta_t^\star | s_t^\star(\theta_t^\star, \theta_{t+1}^\star), \theta_t \geq 0) = \frac{\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}}{F(\sqrt{\tau}\theta_t^\star + F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}))} = p_0$$

This solves for $\theta_T^\star$ (since $\theta_{T+1}^\star = 0$) and going backwards we can solve for the sequence of $\{\theta_t^\star\}_{t=1}^{T-1}$.

Let us define (abusing notation) $G : [0,1] \times \mathbb{R} \times [0,1] \to [0,1]$ as follows

$$G(x, m, y) := \frac{x}{F(F^{-1}(x) + mx + my)} \tag{5}$$

Lemma 2 *$G(x, m, y)$ has the following properties:*

1. *$G(x, m, y)$ is continuously differentiable with respect to x;*

2. *$x \leq G(x, m, y) \leq 1$, $G(x, m, y)$ is decreasing in y and $\lim_{x \to 1} G(x, m, y) = 1$;*

3. *Given $m > 0$ and $y \in (0, 1)$, for some $p_0 \in (0, 1)$, define $x^\star := \max_x \{x \in [0, 1] | G(x, m, y) \leq p_0\}$, we have the following properties:*

   (a) $x^\star < 1$, and if $x^\star > 0$, $G(x^\star, \alpha) = p_0$;

   (b) For alll x satisfying $G(x, \alpha) < p_0$, $x < x^\star$;

   (c) For all x satisfying $G(x, \alpha) > p_0$, $x > x^\star$ and $\frac{dG}{dx}|_{x=x^\star} > 0$.

Notice that when $y = 0$, we write $G(x, m, y = 0) = G(x, m)$ (as defined in Equation (2)). Given the inter-group coordination risk $\theta_{t+1}^\star$, for any possible $\theta_t^\star$, $G(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}, \alpha_t\sqrt{\tau}, \frac{\theta_{t+1}^\star}{\alpha_t})$ captures the belief of the corresponding threshold agent $s_t^\star(\theta_t^\star, \theta_{t+1}^\star)$ that the preferred regime will succeed.

Proposition 3 *When coordination risk is diffused, there are multiple equilibria. A monotone equilibrium $(\theta_t^\star, s_t^\star)_{t=1}^T$ is such that for some $t' \in \{1, 2, \ldots T+1\}$, $\theta_{t'}^\star = \theta_{t'+1}^* = \ldots \theta_{T+1}^* = 0$, and when $t' > 1$, for all $t = 1, 2, \ldots t' - 1$, $\theta_t^\star$ satisfies the following recursive relation:*

$$G(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}, \alpha_t\sqrt{\tau}, \frac{\theta_{t+1}^\star}{\alpha_t}) = p_0 \tag{6}$$

and $s_t^\star = \theta_t^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t})$.

The above recursive relation may not have unique solution because of the public survival news. The first best, i.e., $\theta_t^\star = 0$ for all $t = 1, 2, \ldots, T$ is always a solution. The public survival news always helps to reduce the strategic uncertainties among agents irrespective of whether the coordination risk is diffused or not. On the other hand, diffusion devoid of public survival news has no influence on coordination risk.

Corollary 1 *If there is public survival information,* $\theta_1^* \leq p_0$*; if not,* $\theta_1^* = p_0$*.*

An Optimal Policy

The following Theorem represents the main result of this paper. The validity of this Theorem is not limited to the specific assumption regarding the designer's preference.

Theorem 1 *Suppose agents have uninformative prior and they gather noisy private information instantaneously. Given* (p_0, τ)*, there exists* $\alpha^* > 0$*, such that if the designer sufficiently diffuses the coordination risk, i.e., takes a policy* $(T, (\alpha))$ *such that* $\alpha_t \leq \alpha^*$ *for all* $t = 1, 2 \ldots T$*, then the unique equilibrium is* $\theta_1^\star(T, (\alpha)) = 0$*.*

This implies, for any non-negative realization of θ(however small), there is a uniform bound on how much the designer needs to diffuse the coordination risk to make sure the preferred regime will succeed.[12] The proof is constructive. The effective fundamental for agents moving at time T is $\frac{\theta_T}{\alpha_T}$. The preferred regime fails only if the share of agents who attack the preferred regime, or w_T, exceeds the effective fundamental. Agents have the public information that $\frac{\theta_T}{\alpha_T} \geq 0$ and the private information regarding the effective fundamental.

$$\frac{s_T}{\alpha_T} = \frac{\theta_T}{\alpha_T} + \frac{1}{\alpha_T \sqrt{\tau}} \epsilon_T. \tag{7}$$

The precision of private information regarding the effective fundamental is $\alpha_T \sqrt{\tau}$. Thus, by adopting a diffused policy, the designer is able to make the private information noisier (regarding the effective fundamental). From Lemma 1 it follows that if the designer can make α_T sufficiently small, the

[12] If the information $\theta \geq 0$ is not publicly known for the first group of players, the designer has to set α_1 arbitrarily small to achieve $\theta_1^{*h} = 0$

private information will be sufficiently noisy so that no rational agents will attack the preferred regime. Applying this argument backwards, the coordination risk unravels from the end.

2.2 Diffusing Coordination Risk with Ex-ante Information

As in the concentrated coordination risk, all agents are gathering private information only at the beginning and diffusion reveals the survival news publicly. Agents receive no additional information about θ_t expect the public survival news $\theta_t \geq 0$. The detailed information structure of a simple bifurcated diffusion is illustrated in the following figure.

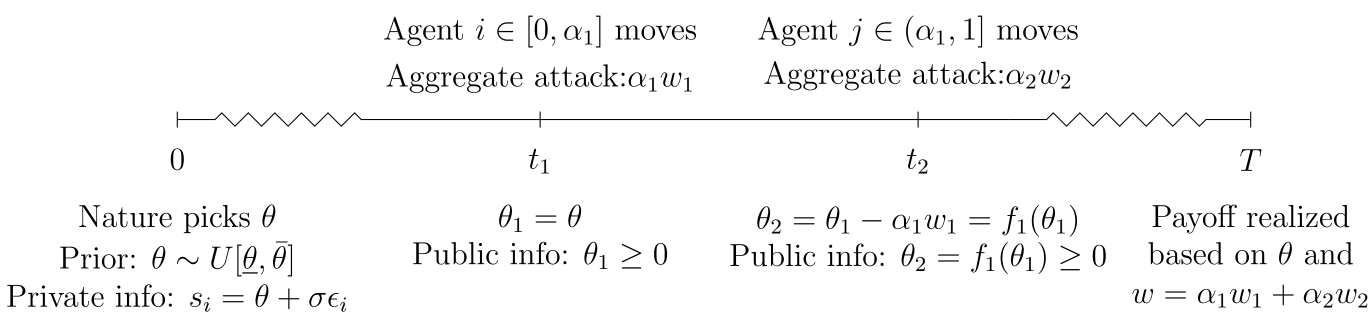

Figure 3: Information structure and Timeline of gathering information at beginning

Let us define the residual fundamental strength at time $t+1$ as

$$\theta_{t+1} = f^t(\theta_1) := f_t \circ f_{t-1} \circ \dots f_1(\theta_1) \tag{8}$$

in which $f_t(\theta) := \theta - \alpha_t P(s_t < s_t^\star|\theta)$ is transformation function from θ_t to θ_{t+1}. Define $h_t(x) := (f^{t-1})^{-1}(x)$ as the fundamental strength that would have resulted in the fundamental strength x at time t. These are increasing functions, which arise from the equilibrium specification. For agents moving at time t, the public survival news is interpreted as $\theta_1 \geq h_t(0)$. Agents update their belief about θ_1 accordingly.

Monotone Equilibrium

Proposition 4 *When coordination risk is diffused, there are multiple equilibria. A monotone equilibrium* $(\theta_t^\star, s_t^\star)_{t=1}^T$ *is such that for some* $t' \in \{1, 2, \dots T+1\}$, $\theta_{t'}^\star = \theta_{t'+1}^* = \dots \theta_{T+1}^* = 0$ *and for*

$t = 1, 2 \ldots t' - 1$, $\theta_t^\star$ *satisfies the following recursive relation:*

$$\frac{\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}}{F\left(F^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}) + \sqrt{\tau}(h_t(\theta_t^*) - h_t(0))\right)} = p_0 \tag{9}$$

and $s_t^\star = h_t(\theta_t^\star) + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t})$.

The above recursive relation does not necessarily have a unique fixed point because of the public survival news. $\theta_t^\star = 0$ for all $t = 1, 2, \ldots, T$ is always an equilibrium. To solve for the sequence of (θ_t^*) from the above recursive relation, we take a candidate solution θ_1^*. If $\theta_1^* = 0$, then $\theta_t^* = 0$ for all t. Suppose $\theta_1^* > 0$. Since $h_1(\theta) = \theta$, we can solve for $\theta_2^*(\theta_1^*)$ from Equation (9). This gives us $s_1^*(\theta_1^*, \theta_2^*(\theta_1^*))$. Given $s_1^\star$, we have $f_1(.|\theta_1^\star)$ and hence $h_2(.|\theta_1^\star)$. From the recursive relation 9 we can then solve for θ_3^* and so on. Finally, if $\theta_{t+1}^* = 0$ for some $t = 1, 2 \ldots T$, then it must be that

$$\frac{\frac{\theta_t^*(\theta_1^*)}{\alpha_t}}{F\left(F^{-1}(\frac{\theta_t^*(\theta_1^*)}{\alpha_t}) + \sqrt{\tau}(h_t(\theta_t^*(\theta_1^*|\theta_1^*)) - h_t(0|\theta_1^*))\right)} = p_0.$$

Difference between Two Information Structures

Suppose the designer change the policy such that $(\alpha_t, \ldots, \alpha_T)$ remains the same, while $(\alpha_1, \ldots, \alpha_{t-1})$ changes, then will the equilibrium behavior change upon reaching group t ? What agents care about at time t is whether the residual strength θ_t can withstand the attack from the agents who have not yet made their decisions. If agents are gathering private information instantaneously about θ_t, then the game will be played exactly the same way as long as $(\alpha_t, \ldots, \alpha_T)$ remains the same. However, if agents have gathered their information only at the beginning, the change in $(\alpha_1, \ldots, \alpha_{t-1})$ will change agent's belief given the same signals. Thus upon reaching group t the game will be played differently. Thus, the equilibrium thresholds cannot be solved backwards when agents gather information ex-ante.

However, the following Lemma shows that the public information of survival is more effective in reducing the coordination risk when agents gather private information ex-ante. Given $\theta_{t+1}^\star$, the effectiveness of public survival news can be measured by $\sqrt{\tau}(\theta_t^\star - 0)$ (or $\sqrt{\tau}(h_t(\theta_t^\star) - h_t(0)$ for ex-ante information).[13] The larger the $\sqrt{\tau}(\theta_t^\star - 0)$, the less effective the public information.

[13]The public information tells that $\theta_t \geq 0$ and the preferred regime will materialize iff $\theta_t \geq \theta_t^\star$. The distance $\theta_t^\star - 0$ can measure the effectiveness of public information when there is no private information. With noisy private

Lemma 3 *For any $t = 1, 2 \dots T$, $h_t(\theta_t^*) - h_t(0) \leq \theta_t^* - 0$.*

Optimal Policy

Instantaneous information provides a lower bound to the effectiveness of the public information of survival when agents gather information ex-ante. Thus, it is easier to achieve the first best uniquely through diffusion when agents gather information ex-ante.

Theorem 2 *When agents gather information only at the beginning and the designer diffuses the coordination risk Theorem 1 holds true.*

2.3 Alternative Interpretation of Diffusion

Suppose a mass of agents is uniformly spread over a time interval $[0, 1]$ and makes their decision sequentially. If they have no information about what other agents are doing, then this is exactly a simultaneous move game. One can also interpret diffusion as the designer deciding to check whether the preferred regime is still viable or not at certain dates. The policy is publicly known, and the results from the checking will be publicly disclosed. If at some date it is revealed that the preferred regime is not viable, then the game stops. The game continues only if the preferred regime is still viable. Agents can either learn the fundamental strength only once at the beginning (ex-ante information-gathering), or they can learn the residual fundamental strength privately on the dates when the preferred regime is being checked (instantaneous information-gathering). A partition $(\alpha_1, \alpha_2, \dots, \alpha_T)$ is equivalent to saying that (after the initial checking at time 0) the designer will check after α_1 time passes by, then after another α_2 time passes by, and so on. Diffusion by checking the availability of the preferred regime is equivalent to making a partition of agents.

When agents are instantaneous information-gatherers, our Theorem 1 (or Theorem 2 for ex-ante information) states that if the designer can check sufficiently frequently (i.e., the time span between any two adjacent checking dates is shorter than $\alpha^\star$), then the preferred regime will materialize for sure as long as the strength is non-negative. Suppose, for some external reasons, that the designer cannot perform the checking at certain intervals of time, and the first best cannot be implemented

information, the more precise the private information, the less effective is the public survival news. Thus, $\sqrt{\tau}(\theta_t^\star - 0)$ can measure the effectiveness of public survival news.

as the unique equilibrium. Based on our Theorem 4, the cautious designer should always check as frequently as she can when agents are gathering information instantaneously.

When an agent's action is divisible, the designer can also diffuse coordination by restricting individual action at any time interval and thus restrict the aggregate action. Consider, for example, investors deciding whether to withdraw their investment. The designer sets a limit on how much they can withdraw in any time interval. Thus, a policy $(\alpha_1, \alpha_2, \ldots, \alpha_T)$ is equivalent to saying that the designer restricts the maximum withdrawal an agent can make in the time interval of $[0, \alpha_1]$ to be α_1 share of the total funds, with $(\alpha_1, \alpha_1 + \alpha_2]$ to be α_2 and so on. In the next section, we will consider this type of diffusion with ex-ante information-gathering in the context of a self-fulfilling bank run model. We will show that diffusing coordination could help to avoid such runs.

3 Avoiding Panic-Based Runs

3.1 Panic-Based Bank Runs

The demand-deposit contract allows a bank to pool depositors' savings together to finance long-term, profitable investments and, at the same time, to fulfill the depositors' liquidity needs. However, the demand-deposit contract is prone to panic-based runs (see Diamond and Dybvig (1983)). Banks may not be able to afford to pay all early withdrawals because of their illiquid investments. Impatient depositors who have liquidity needs will withdraw before the long-term investment matures, while patient depositors may withdraw if they expect the marjority of other depositors will run. Hence, the panic-based bank run is nothing but a coordination failure among patient depositors (see Goldstein and Pauzner (2005)). In this section, based on our theory of diffusing coordination risk, we propose a mechanism to make demand-deposit contracts immune to such panic-based runs. We show that if a bank can make a sufficiently small withdrawal limit of a depositor's decision, then the partially suspended demand-deposit contract could completely avoid panic-based runs.

Following Goldstein and Pauzner (2005), we first present the simplest model of panic-based bank runs, where depositors are making their decisions simultaneously.[14] Depositors are uncertain about their liquidity needs. They may receive the liquidity shock so that they have to make early withdrawals. There is no aggregate uncertainty about the liquidity shock, and depositors know

[14]Equivalently, depositors can make decisions sequentially, but there will be no additional information available about earlier depositor's actions.

that they are equally likely to be (im)patient. We assume there is measure 1 of depositors who will be (im)patient after the liquidity shock realizes. Depositors are risk-neutral, and each made a \$1 deposit in the bank. The bank makes an investment in a profitable, long-term investment, which produces a riskless return R. Before the return from the long-term investment realizes, depositors decide whether to withdraw their deposits early ($a_i = 0$) or not ($a_i = 1$). The long-term investment cannot be reversed, and the bank has limited liquidity $1 + \theta$ to meet the early withdrawals. Depositors learn whether they have liquidity needs before they make their withdrawal decisions. They also receive noisy private information regarding the bank's liquidity. The measure 1 of impatient depositors will withdraw independent of their information about the bank's liquidity, but there are strategic uncertainties among patient depositors. Denote w as the measure of patient depositors who decide to withdraw. If the total withdrawal $(1 + w)$ is not higher than the liquidity of the bank $(1 + \theta)$, the long-term investment will succeed. Those who do not withdraw will earn an interest of r and those who withdraw will receive their principals back. If the total withdrawal is higher than the liquid asset holding, the bank will have to liquidate the long-term investment at a fire sale price, and the liquidation value will be paid out pro rata to depositors. Depositors who didn't withdraw will have nothing left. In our analysis, we worked with the extreme case where the fire sale price is 0.[15] This assumption simplifies the payoff structure, since being insolvent is equivalent to bankrupcty. The details of payoff for patient depositors are as follows.

$$u(1, w, \theta) = \begin{cases} 1 + r & \text{if } w \leq \theta \\ 0 & \text{if } w > \theta \end{cases}, \quad u(0, w, \theta) = \begin{cases} 1 & \text{if } w \leq \theta \\ \frac{\theta+1}{w+1} & \text{if } w > \theta \end{cases} \tag{10}$$

Notice that when $\theta \geq 0$, the bank is financially sound. Its liquidity $1+\theta$ is sufficient to pay the early withdrawals from impatient depositors, and its revenue from the long-term investment is sufficient to pay patient depositors (i.e., $R > 1 + r$). The bank run can happen only if there are sufficiently many patient depositors who decide to withdraw early. Note that, difference to our basic model, the strategic complementarity is weakened in the regime of default as in Goldstein and Pauzner (2005). In the default regime, depositors have a lower incentive to withdraw if they believe more depositors will withdraw. Under a similar setting, Goldstein and Pauzner (2005) showed that a unique equilibrium exists in this coordination game, and the ex-ante probability of panic-based

[15]This assumption is not necessary for our mechanism to work. We prove our results for any non-positive and bounded payoff difference($b_0 - c_0$ and $b_1 - c_1$).

bank runs is non-negative.

Proposition 5 *There exists $\sigma^\star > 0$, such that when $\sigma < \sigma^\star$, a unique equilibrium exists $(\theta^\star, s^\star)$, such that patient depositors will withdraw if and only if $s < s^\star$, and the bank will default if and only if $\theta < \theta^\star$.*

Suppose the bank is able to make a partition of a depositor's withdrawal decisions. At any date $t \in \{1, 2, ..., T\}$ before the long-term investment matures, the bank can restrict that each depositor can, at most, withdraw α_t of their total deposits. Notice that T is not a fixed number. For example, the bank can restrict the total withdrawal on each day, every twelve hours or every hour. Depositors can make a withdrawal at any time but only for a limited amount.

Assume that each impatient depositor's liquidity need is uniformly distributed over time. Since $\sum_{t=1}^{t=T} \alpha_t = 1$, each depositor actually has only one chance to make a decision for every cent of her deposits in the bank. Further assume the withdrawal limit α_t is uniform over t, i.e., $\alpha_t = \frac{1}{T}$. Under this assumption, this diffused deposit contract is still able to fulfill impatient depositors' liquidity needs, and they will withdraw the full amount (i.e., α_t share of their deposits), at any time t.

Define $l = 1+\theta$ as the liquid asset holding of the bank, which could be interpreted as returns from some legacy assets or simply the cash held by the bank. Denote the share of patient depositors withdrawing at time t as w_t. Thus, the residual liquidity at time t is $l_t = 1 + \theta - \sum_{u=1}^{t-1} \alpha_u - \sum_{u=1}^{u=t-1} \alpha_u w_u$. $\sum_{u=1}^{t-1} \alpha_u$ and $\sum_{u=1}^{u=t-1} \alpha_u w_u$ are the withdrawals from the impatient depositors and patient depositors before t, respectively. Define $\theta_t := \theta - \sum_{u=1}^{u=t-1} \alpha_u w_u$ as the liquidity left to withstand the patient depositors' withdrawals from time t to T. If $\theta_t < 0$, or $l_t = \sum_{u=t}^{T} \alpha_u + \theta_t < \sum_{u=t}^{T} \alpha_u$, then the bank will not be able to meet the withdrawals from impatient depositors in the future. Even if all patient depositors decide not to withdraw from t to T, the bank will default. Before paying the withdrawers at time $t-1$, the bank checks whether the current withdrawals from patient depositors ($\alpha_{t-1} w_{t-1}$) have exhausted all liquidity left for future withdrawals from impatient depositors ($\theta_{t-1} = l_{t-1} - \sum_{u=t-1}^{T} \alpha_u$), i.e., $\theta_t < 0$. If it is, the bank defaults for sure at some date before the long-term investment matures, and thus the bank will liquidate all its assets to meet its obligations. The bank will split its liquid assets to pay the time $t-1$ withdrawers up to the principal \$1. Whatever is left will be distributed evenly to each depositor.[16] Based on this

[16] The residual liquidity will be evenly distributed since each depositor has the same amount of deposits that she has not yet made withdrawal decisions on. In the proof of Lemma 4, we show that the policy of distributing the residual liquidity is irrelevant to a depositor's withdrawal decision, as long as it does not depend on the amount the

mechanism, depositors will understand publicly that $\theta_t \geq 0$ whenever depositors need to make their withdrawal decisions at time t.

Depositors are learning the liquidity of bank θ only once at the beginning (ex-ante information-gatherer). They are risk-neutral, and there is no discounting. At any time t, the depositors will get the amount of their withdrawal(s) back, as long as the bank can still withstand future withdrawals from impatient depositors (i.e., $\theta_t \geq \alpha_t w_t$). Only if the bank can meet all withdrawals from time 1 to time T will depositors get paid for the amount depositors decide not to withdraw after the long-term project matures. The details of payoff for depositors moving at time t are as follows.

$$
\begin{aligned}
u(1,\theta_t,\alpha_t w_t, w^t) &= \begin{cases} 1+r & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ 0 & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases} \\
u(0,\theta_t,\alpha_t w_t, w^t) &= \begin{cases} 1 & \text{if } \theta_t \geq \alpha_t w_t \\ \min\{\frac{\sum_{u=t}^{T}\alpha_u + \theta_t}{\alpha_t + \alpha_t w_t}, 1\} & \text{if } \theta_t < \alpha_t w_t \end{cases}
\end{aligned} \tag{11}
$$

where $w^t = \sum_{u=t+1}^{u=T} \alpha_u w_u$ and $w^T = 0$.

There is a continuum of patient depositors, and thus, any individual's choice has no impact on the aggregate outcome. The following Lemma shows that, at time t, depositors are making a binary decision to maximize their expected payoff.

Lemma 4 *If the withdrawal limit at time t is α_t and $\sum_{t=1}^{T} \alpha_t = 1$, then she will withdraw $\beta_t \in \{0, \alpha_t\}$.*

The payoff structure is slightly different from the model of diffused coordination risk presented in Section 2. In that model, the payoffs depend on whether the fundamental strength can sustain the aggregate attack. However, in this dynamic bank-run model, the payoff of withdrawing does not depend on depositors' future actions. This creates a higher incentive for depositors to withdraw. However, depositors still have dynamic concerns, since the payoff of rolling over depends on depositors' future actions. The following theorem shows that by making a sufficiently fine partition of depositors' withdrawal decisions, panic-based bank runs can be avoided.

depositor decides to roll over before the default happens.

Theorem 3 *There exists $T^\star \in \mathbb{N}$, such that $\forall T > T^\star$. If the withdrawal limit at any time t is $\frac{1}{T}$, then the panic-based runs could be avoided as long as the bank has non-negative liquidity for patient depositors, i.e., $\theta \geq 0$.*

The bank does not default at time t if $\theta_t \geq \alpha_t w_t$ and it does not default after time t if $\theta_t \geq \alpha_t w_t + w^t$. When no patient depositors withdraw at any time, these two criteria becomes identical. Thus, the mechanism in Theorem 2 goes through in this context. Our mechanism does not restrict a depositor's ability to withdrawing her deposits before the return of long-term investment realizes. Instead, it restricts depositors from rushing to the bank on any date but allows them to withdraw gradually. Depositors can still take all of their money back within that period. This mechanism is welfare-improving, since it avoids the costly liquidation of a profitable, long-term project.[17]

Green and Lin (2003) and Andolfatto et al. (2014) built on the idea of lining up a finite number of depositors to reduce the uncertainties between them. However, their results rely on the perfect observability and contractability of past actions and depositors' types. Unlikely the mechanism design problem where incentives are provided through payoff, here incentives are provided through information. Our mechanism is also related to the idea of preventing bank runs by the suspension of convertibility. Panic-based bank runs could be avoided when complete suspension of convertibility can be implemented (see Diamond and Dybvig (1983)). Ennis and Keister (2009) showed that complete suspension of convertibility is not ex-post efficient and thus not an ex-ante credible commitment. Based on our theory, suspending withdrawals completely is not necessary to avoid panic-based runs.

Redemption Gates

Money market funds are an important liquidity provider in the financial market (see Brunnermeier (2009)). The 2008 collapse of the \$62.5 billion Reserve Primary Fund caused a widespread run on funds and helped freeze global credit markets. In order to make money market funds safer and more stable, the Securities and Exchange Commission (SEC) allows boards of prime money market funds

[17] Suppose the depositors with liquidity needs are not distributed uniformly over the unit interval. In this case, diffusing coordination risk will be costly for the agents who have liquidity needs. However, based on our optimal mechanism to overcome rollover risk, if the depositors with liquidity needs can write contracts among themselves (e.g., a depositor who needs the money early can write a contract with a depositor who will need the money later), then diffusion will not restrict an impatient depositor from fulfilling his or her liquidity needs.

to impose redemption gates.[18] Under certain conditions, prime money market funds can suspend redemptions temporarily for a period of time. For instance, when such redemption gates have been lifted, investors in a prime money market fund can, at most, withdraw 10% of their investment per day for ten days. Hedge funds could also impose this type of policy at their own discretion. Could this help to reduce the self-fulfilling runs on redemption? Based on our theory, if the money market fund or hedge fund could adopt sufficiently small redemption gates, the coordination risk among investors in their redemption could be avoided.[19]

3.2 Asynchronous Debt Structure

Coordination failure among creditors in rolling over debt could give rise to self-fulfilling debt runs. This coordination risk in maturity mismatch problem was one of the main causes of the recent financial crisis (Brunnermeier, 2009). In this subsection, we will show that a sufficiently asynchronous debt structure could help in avoiding self-fulfilling debt runs. Suppose a borrower finances a profitable, riskless, long-term investment project by issuing short-term debt. There is mass 1 of creditors. Each of them made a loan of \$1 to the borrower. The borrower has only limited liquid assets (θ) to roll over withdrawals before the long-term investment matures.

Instead of issuing short-term debt with the same maturity, suppose the borrower is able to issue debt with different maturity dates.[20] For example, the profit of a long-term investment will be realized in five years, while one-third of the debt matures in two and a half years, three years, and four years, respectively. Let the debt structure be $(\alpha_1, \alpha_2, \ldots, \alpha_T)$, such that the α_t share of the short-term debt will mature at time $t \in \{1, 2, \ldots, T\}$ before the long-term investment matures. The debt structure is such that all debt holders only have one chance to make rollover decisions.[21] The asynchronous debt structure is able to finance the investment, i.e., $\sum_{t=1}^{t=T} \alpha_t = 1$ and it is publicly

[18] See http://www.sec.gov/rules/final/2014/33-9616.pdf.

[19] For the signaling effect of lifting redemption gates (about the liquidity of money market funds or the net present value of their investment), See Section 5 for discussion.

[20] Why the borrower would use short-term financing in the first place is out of the scope of this study. We will analyze how to design the structure of short-term debt to minimize the rollover risk (i.e., the coordination risk among creditors). There could be many reasons for short-term financing to be superior to long-term financing. For example, short-term financing could fulfill the liquidity needs of creditors (see Diamond and Dybvig (1983)); it could work as a signaling tool when creditors have asymmetric information (see Flannery (1986)); it could work as a disciplining device (see Calomiris and Kahn (1991)); or it may arise due to a contractual externality (see Brunnermeier and Oehmke (2013))

[21] We are assuming that the initial debt contract does not specify the maturity date and creditors have been assigned to one of these groups randomly. We are not modeling how credtors choose their maturity dates.

announced. Creditors of debt maturing at time t decide whether to roll over their debt claim on that date. If they decide to make withdrawals ($a_i = 0$), they will get their principals \$1 back immediately if the borrower remains solvent. Otherwise, they will get paid pro rata by splitting the borrower's remaining assets θ_t evenly.[22] If they decide to roll over their debt ($a_i = 1$), they will get paid on the next maturity date (after the long-term investment matures). They will get an interest payment r only if the borrower met all withdrawals before the profit from the long-term investment realizes. Otherwise, the debt holders who roll over will get nothing. The long-term investment has a riskless return of R, which is high enough to pay back all debt claims and interest rates (i.e., $R > 1 + r$).

Denote w_t as the share of group t creditors (holding debt that matures at time t) deciding to withdraw. Then, we can define the residual liquidity as before: $\theta_t = \theta_1 - \sum_{u=1}^{u=t-1} \alpha_t w_t$. Creditors are instantaneous information-gatherers. They have no prior knowledge about the borrower's liquidity position but learn the current standing of the borrower before they need to make their rollover decision. However, if the borrower fails to meet the earlier withdrawals and thus liquidation takes place, it will be publicly known, and all creditors who have not withdrawn their debts will end up receiving nothing. Thus, group t creditors understand that the borrower successfully rolls over all earlier withdrawals ($\theta_t \geq 0$) before they make their withdrawal decisions. The payoff structure is as follows.

$$u(1, \theta_t, \alpha_t w_t, w^t) = \begin{cases} 1 + r & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ 0 & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases}, \quad u(0, \theta_t, \alpha_t w_t, w^t) = \begin{cases} 1 & \text{if } \theta_t \geq \alpha_t w_t \\ \max\{\frac{\theta_t}{\alpha_t w_t}, 0\} & \text{if } \theta_t < \alpha_t w_t \end{cases} \tag{12}$$

where $w^t = \sum_{u=t+1}^{u=T} \alpha_u w_u$ and $w^T = 0$.

There are three differences between this asynchronous debt problem and the bank run model presented in the previous subsection: (1) the borrower is making a partition of creditors instead of restricting individual action; (2) the creditors are instantaneous information-gatherers instead of learning the fundamental at the beginning; and (3) there are no impatient depositors who must make withdrawals to fulfill their liquidity needs. We show in the following corollary that our optimal

[22]As in the bank run example, to simplify the model, we assume the liquidation value of the long-term investment is zero. Based on this assumption, the borrower goes default upon liquidation.

policy of diffusing coordination risk could work to avoid self-fulfilling debt runs.

Corollary 2 *If the borrower can make the debt structure sufficiently asynchronous, the self-fulfilling debt runs can be avoided.*

The dynamic debt-run problem has been investigated by He and Xiong (2012). While they focused on a time-varying fundamental problem with complete information, we investigated the case with incomplete information and a fixed fundamental. Our work provides some rationale for the debt structure with a fixed rollover frequency (i.e., debt is continuously retired at a constant fractional rate), commonly assumed in the finance literature (see Leland (1998); He and Xiong (2012)). This asynchronous debt structure is feasible for avoiding self-fulfilling debt runs in reality. There is empirical evidence that the granularity of corporate debts has a lot of variation across firms and across time (see Choi et al. (2014)).

4 Limited Diffusion

Intra-group and Inter-group Coordination Risk

When coordination is diffused, group $t(1 \leq t \leq T-1)$ agents faces strategic uncertainties from agents moving at t and agents moving afterwards (from $t+1$ to T). We will call them intra-group coordination risk and inter-group coordination risk respectively.

Given the equilibrium threshold $(\theta_t^\star)_{t=1}^{t=T}$, $\theta_t^\star$ is a sufficient statistic for the coordination risk from time t to T. Thus, the inter-group coordination risk for agents moving at any $t(t < T)$ is captured by $\theta_{t+1}^\star$. The intra-group coordination risk for agents moving at time t is the residual risk, captured by $\theta_t^\star - \theta_{t+1}^\star$. The intra-group coordination risk captures the risk among group t agents such that, if too much attack happened within time t, the residual fundamental left for agents moving later is less than $\theta_{t+1}^\star$, i.e., $\alpha_t w_t > \theta_t - \theta_{t+1}^\star$. In this case, the preferred regime will fail at sometime between t and T.[23] The preferred regime will materialize if and only if $w_t \leq \frac{\theta_t - \theta_{t+1}^\star}{\alpha_t}$. Thus, given $\theta_{t+1}^\star$, the *effective fundamental strength* for group t is $\frac{\theta_t - \theta_{t+1}^\star}{\alpha_t}$.

Let $P(T, (\alpha))$ be the coordination risk given policy $(T, (\alpha))$. The designer chooses $(T, (\alpha))$ to minimize $P(T, (\alpha))$. Given the potential multiplicity of equilibria, we first need to define the

[23] Notice that only if the current attack $\alpha_t w_t$ is higher than θ_t, the preferred regime fails at time t and the game will not enter into time $t+1$.

objective function for the designer.

Lemma 5 *([Milgrom and Roberts (1990)](#)) When coordination risk is diffused, the worst equilibrium(with highest coordination risk) and the best one (with lowest coordination risk) are both is in monotone strategy.*

Given any policy $(T, (\alpha))$, the *best* equilibrium is the monotone equilibrium with threshold fundamental $(\theta_1^{\star l} = 0 \ldots, \theta_T^{\star l} = 0)$. In this equilibrium, agents never attack preferred regime irrespective of their private signals. Let $\{\theta_t^{*h}(T, (\alpha))\}_{t=1}^{T}$ be the maximum solution to 6. This is the *worst* equilibrium. Let us define $\underline{P}(T, (\alpha)) := P(\theta < \theta_1^{*l}(T, (\alpha)))$ and $\bar{P}(T, (\alpha)) := P(\theta < \theta_1^{*h}(T, (\alpha)))$ corresponding to the *best* and the *worst* equilibrium respectively. Thus, given any policy $(T, (\alpha))$,

$$\bar{P}(T, (\alpha)) \geq P(T, (\alpha)) \geq \underline{P}(T, (\alpha))$$

P is the prior belief of the designer.[24]

We can say that for any policy $(T, (\alpha))$, there is $\theta_1^{*l}(T, (\alpha))$ and $\theta_1^{*h}(T, (\alpha))$ such that : (1) If $\theta \geq \theta_1^{*h}(T, (\alpha))$ the preferred regime materializes irrespective of whatever equilibrium is played. (2) If $\theta < \theta_1^{*l}(T, (\alpha))$ the preferred regime does not materialize irrespective of whatever equilibrium is played and (3) if $\theta \in [\theta_1^{*l}(T, (\alpha)), \theta_1^{*h}(T, (\alpha)))$, there exists some equilibrium such that the preferred regime does not materialize. A *Cautious* designer, or a *max-min* designer, who wants to minimize the coordination risk anticipating the worst can happen (see Gilboa and Schmeidler (1989) for max-min preference), will then minimize $\theta_1^{*h}(T, (\alpha))$.

Interplay of Intra-group and Inter-group coordination risk

Given the inter-group coordination risk for group t, θ_{t+1}^*, agents in group t are essentially facing a static coordination problem with effective fundamental strength $\frac{\theta_t - \theta_{t+1}^*}{\alpha_t}$. If agents in group t have not received the public information survival, the intra-group coordination risk does not depend on the inter-group risk, i.e., $\theta_t^* - \theta_{t+1}^* = \alpha_t p_0$. When agents receive public surivival news $\theta_t \geq 0$, agents understand that the effective fudanmental is above $-\frac{\theta_{t+1}^*}{\alpha_t}$.[25] Thus, the lower the inter-

[24] We assume the designer shares the same prior as agents, or the policy cannot convey any additional information to agents.

[25] In Equation (26), when $\theta_{t+1}^\star > \frac{1}{\sqrt{\tau}}$, the denominator is equal to 1 for any candidate solution $\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}$. Thus, $\theta_t \geq 0$ is effective only when $\theta_{t+1}^\star \leq \frac{1}{\sqrt{\tau}}$. *Ceteris paribus*, decreasing τ, or making the private information noisier,

group coordination risk ($\theta^\star_{t+1}$), the more effective the public information in reducing the intra-group coordination risk.

Corollary 3 *For any group $t \in \{1, 2, \ldots T-1\}$ who has the public information $\theta_t \geq 0$, the intra-group coordination risk $\theta^\star_t - \theta^\star_{t+1}$ (weakly) increases with the inter-group coordination risk $\theta^\star_{t+1}$.* [26]

When agents in group t face no inter-group coordination risk, i.e., $\theta^*_{t+1} = 0$, agents will ignore their private information and never attack the preferred regime if the private information is sufficiently noisy(Lemma 1). What can we say, if there is positive inter-group coordination risk, i.e., $\theta^*_{t+1} > 0$? The following corollary shows that when inter-group coordination risk is positive, so is intra-group coordination risk (no matter how noisy the private information is). Hence, the public survival news can be sufficiently effective to persuade group t agents never to attack the preferred regime only if there is no inter-group risk and the private information is sufficiently noisy.

Corollary 4 *If $\theta^*_t - \theta^*_{t+1} = 0$, then $\theta^*_t = \theta^*_{t+1} \ldots = \theta^*_T = 0$*

Limited Diffusion under Instantaneous Information

Suppose some agents are clumped together and the designer cannot separate them into different groups. Recall that in Theorem 1, the optimal policy places a small enough (less than α^*) mass of agents in each group. What if when such policies are not feasible? We will call such situation limited diffusion. Multiple equilibria will arise and the first best is not the unique equilibrium. The designer is assumed to be max-min or cautious. Does coordination risk decrease as the designer diffuse the coordination risk more?

Lemma 6 *A max-min designer weakly prefers a finer partition over a coarser one.*[27]

Notice the above Lemma is applicable only for partitions that are comparable in the sense whether one is finer than the other. Let $\mathcal{Q}$ be the set of all feasible partitions. Suppose $(0.4, 0.6)$

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will enlarge the set of inter-group risk $\theta^\star_{t+1}$ where the public information can be effective.

[26] In the rest of the paper, we will focus on the worst equilibrium $\theta^{\star h}_t$ whenever there are multiple equilibria. For convenience, we will drop the superscript h from $\theta^{\star h}_t$.

[27] Define a block as a set of mutually exclusive sets in the partition. A partition Q_1 is said to finer than partition Q_2 if every block in Q_1 is contained in some block in Q_2.

and $(0.5, 0.5)$ are two feasible partitions. This implies the lump of agents are not such that the partition $(0.4, 0.1, 0.5)$ is infeasible. Formally, we will assume the following

Assumption 1 *If $Q_1, Q_2 \in \mathcal{Q}$, then $Q_1 \wedge Q_2 \in \mathcal{Q}$.*[28]

Given assumption 1, we can say there is a finest partition in $\mathcal{Q}$, which is $\wedge_{Q \in \mathcal{Q}} Q$. Lemma 6 coupled with assumption 1 implies that the max-min designer will diffuse the coordination risk as much as possible.

Theorem 4 *Suppose there is a max-min designer who has limited diffusion power and the feasible policy set $\mathcal{Q}$ satisfies assumption 1, then the designer's optimal policy is the finest partition in $\mathcal{Q}$.*

Instantaneous information gathering seems an appealing assumption when different groups make their decisions at different dates. However, this also means that the private information structure is different comparing to the model of concentrated coordination risk. Next, we will consider the case when diffusion does not change the private information structure.

Limited Diffusion under Ex-ante Information

We have seen that when the designer can diffuse the coordination risk enough, she can achieve the first best as the unique equilibrium outcome. However, contrary to the conventional wisdom that further diffusion creates public survival news and induces lower coordination risk, we argue that coordination risk is not decreasing in diffusion. When agents are gathering information ex-ante, the interpretation of public survival news ($\theta_1 \geq h_t(0)$) depends on the equilibrium object h_t. Any change in the policy of diffusion will affect the monotone equilibrium $(\theta_t^\star, s_t^\star)_{t=1}^{t=T}$ and thus $\{h_t(T, (\alpha))\}_{t=1}^{T}$.

Let us think about a simple example to see why more diffusion does not always reduce the coordination risk. Suppose there are two alternative policies of diffusion: a coarser partition (named partition A) $(\alpha, 1-\alpha)$ and a finer partition (called partition B) $(\beta, \alpha-\beta, 1-\alpha)$. $0 < \beta < \alpha$. Assume partition B is not sufficiently fine to achieve the first best. Let us divide the first group in partition A into two groups artificially as in B, i.e., $(\beta, \alpha - \beta, 1 - \alpha)$ and make the comparison accordingly. The second group in B will attack less aggressively (compared to that in A) since they receive the public survival news. However, unlike instantaneous information, this will reduce the effectiveness

[28] $Q_1 \wedge Q_2$ is the meet of Q_1 and Q_2, which is the largest partition that refines Q_1 and Q_2, i.e., the blocks in $Q_1 \wedge Q_2$ are all the nonempty intersections of a block from Q_1 with a block from Q_2.

of the public survival news received by the third group in B, which implies the third group in B will attack more aggressively (compared to A). Taking this into consideration, the second group in B facing more inter-group coordination risk will attack more aggressively as well. The aggressiveness of the second group will affect the effectiveness of the public information received by the third group, which will affect the inter-group risk faced by the second group and thus have an impact on how aggressively they attack the preferred regime. This loop will continue. Thus the effect of additional public information of survival on coordination risk can go either way. Although this effect is ambiguous, we know from Lemma 3 that for any policy, the coordination risk will be weekly below the coordination risk when agents are gathering information instantaneously. Note that these upper bounds of the policies are comparable (see Lemma 6). Therefore, we can show that when the coordination risk is sufficiently diffused and agents are gathering information ex-ante, agents will ignore their private information and never attack the preferred regime.

5 Discussions

We have presented a model where coordination risk is diffused over time and show how it helps in reducing coordination risk. We provide a practical application of this idea in preventing self-fulfilling runs. Recall that, our optimal mechanism has been designed for two different environments: instantaneous information (Theorem 1) and ex-ante information (Theorem 2). Here, we will discuss some extensions which maybe relevant to either information structure or both.

5.1 Diffusion without Noisy Private Information

In the benchmark model, if agent had no private information, then all agents favoring the preferred regime and all agents attacking the preferred regime can both occur as equilibrium outcome.[29] If in addition agents learn publicly that $\theta \geq 0$, then all agents attacking the preferred regime can still be an equilibrium only if $P(\theta \geq 1|\theta \geq 0) \leq p_0$, i.e., $(1-p_0)\bar{\theta} \leq 1$. Now suppose the coordination risk has been diffused. Agents in group T with effective per capita fundamental $\frac{\theta_T}{\alpha_T}$ learns publicly that $\theta_T \geq 0$. If α_T is sufficiently small, or $\bar{\theta}(1-p_0) > \alpha_T$, then all agents at time T attacking the preferred regime cannot be supported as an equilibrium. Thus, when coordination is sufficiently diffused, i.e.,

[29] Assume $\bar{\theta}$ in the prior is sufficiently large and $\underline{\theta}$ is sufficiently small.

$\alpha_t < \bar{\theta}(1-p_0)$ for all t, no one will attack the preferred regime. No matter how pessimistic agent's belief (however small $\bar{\theta}$) is or how reluctant agents are willing to favor the preferred regime (however large p_0), as long as the designer can make group size α sufficiently small, diffusion could make the first best the unique equilibrium.[30]

5.2 Informative Policy

In this paper, we have assumed that the designer share the same prior as the agents and she does not have any additional information before she implements the policy. As we proposed, the designer should commit to the policy of diffusing coordination risk ex-ante. Therefore, the policy does not convey any information about the fundamental. However, in reality policy maker may adopt this type of policy only when her preferred regime is relatively weak. In that case, we cannot ignore the signaling effect of the policy itself since the policy could be informative about the fundamental. In the recent sovereign debt crisis in Greece, Greece government imposes very strict withdrawal limit for households. depositors could learn that the bank only has very limited liquid assets or cash to meet its obligations. Can diffusion still help reducing the coordination risk?

Proposition 6 *If agents are gathering information ex-ante and the support of the uniform prior $\underline{\theta}$ and $\bar{\theta}$ depends on the policy $(T, (\alpha))$, the first best can be achieved uniquely if the policy of diffusion could make $\alpha_t < \min\{\alpha^\star, \frac{\bar{\theta}_{(T,(\alpha))}}{1+\sigma}\}$ for all $t \in \{1, 2, \ldots, T\}$, where $\alpha^\star$ is as defined in Theorem 1.*[31]

Thus, even if the policy itself could make agents more pessimistic about the preferred regime's strength, as long as $\bar{\theta}_{(T,(\alpha))}$ is decreasing slower than the rate of decrease in the group size when the designer further diffuses coordination, diffusing coordination risk could still help to avoid the coordination risk.[32]

[30] In the section 2 where coordination risk is diffused and agents have private noisy information, we assume that agents have uninformative prior. Thus, the effect of public survival news in making the prior more optimistic is absent.

[31] We only work with the ex-ante information case to avoid the complication in updating informative prior when agents get information instantaneously.

[32] Notice that when there is no diffusion, or $\alpha_1 = 1$, $\bar{\theta} > 1 + \sigma$. Consider the case where diffusion is uniform, i.e., $\alpha_t = \frac{1}{T}$, if $\bar{\theta}(T, (\alpha))$ decreases at a slower rate than α_t, then there exists $\tilde{\alpha} > 0$ such that when $\alpha_t \leq \tilde{\alpha}$, $\frac{\bar{\theta}}{\alpha_t} > 1 + \sigma$.

5.3 Endogenous Information Gathering

Our mechanism relies on the fact that diffusion dilutes private information. Suppose the precision of private information is endogenous and depends on the policy. If agents get less precise private information when the partition is finer, this makes our mechanism even more effective in reducing the coordination risk to zero.What if a diffused policy design makes the agents access more precise private information?

Suppose, $\tau_t(T,(\alpha))$ be the endogenously chosen precision of group t agents, given the policy $(T,(\alpha))$. Suppose $\tau_t(T,(\alpha))$ is weakly higher for a finer partition. The following Proposition provides sufficient conditions for our mechanism to work even if agents will access more precise information when coordination risk is diffused.

Proposition 7 *If (1)* $\alpha_t\sqrt{\tau_t(T,(\alpha))}$ *falls as the partition becomes finer and (2)* $\lim_{\alpha_t\to 0}\alpha_t\sqrt{\tau_t(T,(\alpha))} = 0$*, then the designer can achieve the first best by adopting a sufficiently diffused policy.*

From Proposition 7, we can see exactly how the diffusion works in avoiding coordination risk. As long as the precision of private information about the effective fundamental $(\alpha_t\sqrt{\tau_t})$ decreases with the group size α_t, our optimal policy is sufficiently effective to avoid coordination risk.

5.4 General Equilibrium Concern

We have shown that sufficiently diffused policy could implement the first best uniquely if the payoffs do not depend on the policy of diffusion. Considering the incentive of agents before joining this coordination game, e.g., depositing their funds in the bank, the diffused policy itself will affect the endogenous risk faced by the agents and thus the payoffs have to be adjusted to satisfy agents' participation constraints. It is worthwhile to consider the incentive of depositors in a general equilibrium framework. We will discuss this in the self-fulfilling bank model and we will show that our theory for avoiding bank runs works with an endogenous price (interest rate).

Consider the diffused structure $(T,(\alpha))$. As defined in Section 2, the probability of default is $P(T,(\alpha)) := P(\theta < \theta_1^*(T,(\alpha)))$. Suppose e is the depositor's outside option, and $r(T,(\alpha))$ is the interest rate. The participation constraint for the depositor can be written in form of

$$r(T,(\alpha)) \geq U(P(T,(\alpha)),e),$$

in which U is increasing in $P(T,(\alpha))$ and e. To make the participation constraint satisfied, the interest must be higher than the outside option, i.e., $r(T,(\alpha)) > e$. In equilibrium, the bank will choose an optimal diffused structure to maximize the expected profit by taking the depositor's participation constraint into consideration, i.e.,

$$\max_{(T,(\alpha))} \Pi(P(T,(\alpha)), r(T,(\alpha))) \quad \text{s.t.} \;\; r(T,(\alpha)) \geq U(P(T,(\alpha)), e))$$

The expected profit function Π is increasing in P and decreasing in r. Note that, given any outside option e, Theorem 3 shows that there exists $T \geq T^\star(e)$, such that the diffused policy $\alpha_t = \frac{1}{T}$ for all $t = 1, 2, \ldots, T$ will make $\theta^\star = 0$. Hence, under this policy, $P(T,(\alpha)) = 0$ and $r = e$ since there is no risk of default. By this argument, for any outside option e, sufficiently diffused policy will make the first best implementable even if we consider the price is endogenously determined.

The intuition for this result is rather simple. The bank's objective is still to minimize the risk of a bank run. A lower probability of a bank run would not only increase the probability of earning a positive profit but also increase the profit because the interest rate payment will be reduced.

5.5 Comparative Statistics: Financial Fragility

Following the discussion of preventing self-fulfilling debt runs, the proposed optimal mechanism is still fragile in the face of some exogenous shocks.[33] Suppose due to some exogenous reason, the creditors start to actively gather more accurate private information about the fundamental. If creditors have better information about the fundamental and about what other creditors will do, it is harder for them to coordinate purely on the public survival news. As shown in the following corollary 5, debt crises may still happen when the borrower implements the pre-designed asynchronous debt structure.

Corollary 5 *$\alpha^\star$ is decreasing in τ.*

This is consistent with the explanation for a financial crisis by Gorton and Ordoñez (2014). They argue that one potential trigger of a financial crisis could be that agents being more skeptical

[33] Suppose there are some negligible costs of diffusion. The borrower will only set the group size into $\alpha^\star$ to avoid any bank runs. For instance, there might be some issuance costs for each type of debt. One can interpret the cost of diffusing the debt structure as the illiquidity discount, since more debt issues with smaller sizes will have a less liquid secondary market than a few debt issues with larger sizes.

about the underlying fundamental of the economy start to gather more precise information. In their case, the crisis takes place because some low-quality projects with a positive net present value have been screened out and cannot be financed. In our case, a crisis can take place because the public survival news cannot overcome the coordination risk when creditors have more accurate private information. Based on our theory, if the borrower can adjust the debt structure, she will adopt a more asynchronous structure in facing such a shock.

5.6 General Dynamics of Fundamental

In certain dynamic coordination games, the dynamics of fundamental strength may not be linear. Consider the case that the residual fundamental is possibly a non-linear function of original fundamental and previous attacks, i.e., $\theta_{t+1} = b(\theta_t, \alpha_t w_t)$. Intuitively, θ_{t+1} will increase with θ_t and decrease with $\alpha_t w_t$. Therefore, the preferred regime can sustain attacks at time t only if $b(\theta_t, \alpha_t w_t) \geq 0$. For example, in the static regime change model of Morris and Shin (1998) and Iachan and Nenov (2014), regime change will not take place if $b(\theta, w) = g(\theta) - w \geq 0$. The following Proposition will show that our mechanism can work to avoid coordination risk with general dynamics of fundamental satisfying assumption 2.

Assumption 2 *The law of motion of fundamental* $\theta_{t+1} = b(\theta_t, \alpha_t w_t)$ *satisfies* $\frac{\partial b}{\partial \theta_t} \geq \underline{k}_1 > 0$, $-\frac{\partial b}{\partial \alpha_t w_t} \geq \bar{k}_2 > 0$ *and* $b(0,0) = 0$

Proposition 8 *Under general dynamics of fundamental satisfying assumption 2, Theorem 1 holds true.*

5.7 Heterogeneity

In this paper we analyze how a designer should partition a mass of identical agents. What will the designer do if agents are heterogeneous in term of payoff incentive or in term of information quality? Sakovics and Steiner (2012) considers a similar heterogeneous problem where the designer decides which group to subsidize. Unlike them the designer does not provide monetary incentive but rather decide the order in which she will let them make their decision.

Suppose there are two groups of equally informed agents. Group 1 is more reluctant to favor the preferred regime than group 2,[34] i.e., $p_{01} > p_{02}$. The designer decides whether to let a more reluctant group or a less reluctant group first make their decision i.e., chose the order $(1, 2)$ or $(2, 1)$. Or suppose, the two groups have same payoff but group 1 is more informed than group 2. Should the designer let a more informed group or a less informed group first make their decision i.e., choose the order $(1, 2)$ or $(2, 1)$?

Proposition 9 *Suppose the designer is max-min type and agents gather information instantaneously. There are two groups of agents $\{1, 2\}$ of equal mass. For any group $g \in \{1, 2\}$, the precision of private signal is τ_g and the minimum required probability of success to induce agents to favor the preferred regime is p_{0g}. Survival information is publicly available only to the group moving later. Then,*

1. *(Information Heterogeneity) If $\tau_1 > \tau_2$ while $p_{01} = p_{02} = p_0$, then the designer should choose the order $(1, 2)$.*

2. *(Payoff Heterogeneity) If $p_{01} > p_{02}$ while $\tau_1 = \tau_2 = \tau$, then the designer should choose the order $(1, 2)$ if and only if $G_x < 1$.*

Let $\psi(k, m, y)$ be such that

$$G(\psi(k, m, y), m, y) = k \tag{13}$$

$\psi(k, m, y)$ solves for the intra-group coordination risk when the inter-group coordination risk is y, precision of private information is m and the minimum required probability of success to induce agents to favor the preferred regime is k. When group 1 is more informed than group 2, choosing the order $(1, 2)$ means there is less inter-group risk (since less informed group will be affected more by the survival news). Without the public survival news at $t = 1$, the intra-group risk will remain unchanged. Thus, the designer would let the more informed group first make their decision.[35]

When group 1 is more reluctant than group 2, choosing the order $(1, 2)$ means less inter-group risk but higher intra-group risk. When there is no survival news at $t = 1$, the changes in inter-group risk does not affect the intra-group risk faced by agents moving at $t = 1$. Thus, the optimal

[34]For example, some firms benefit more if the new technology is successfully adopted, which makes them less reluctant to attack the new technology(preferred regime).

[35] If the survival news is available at $t = 1$, when picking the order $(1, 2)$, the public survival news at $t = 1$ will be less effective in reducing the intra-group risk although it could reduce the inter-group risk more. Whether the survival news is more effective in reducing the aggregate coordination risk depends on the cross partial derivatives ψ_{my}. Thus, generalization of this result will require the knowledge of cross partial derivatives of the solution.

permutation depends on whether this reduction in inter-group risk outweighs the increase in intra-group risk. $G(.)$ captures the belief of the threshold agent in equilibrium and depends on the distribution of noise $F(.)$. $G_x = \frac{1}{\psi_k}$ captures how this belief changes as the threshold changes. Had there been no survival information (at $t = 1$), this slope would be 1. Thus $G_x < 1$ says that the reduction in inter-group risk will outweigh the increase in intra-group risk. We can say a payoff incentive (reduction in p_0) is more effective in reducing the intra-group risk on later agents than earlier agents if and only if $\psi_k > 1$.

6 Conclusion

To conclude, this paper proposes a simple and feasible policy to avoid coordination risk. In a noisy private information setting, we considered a special type of public information. The public information shows that the designer's preferred regime sustained all previous attacks. It is binary, truth-telling, and generated naturally in the dynamic coordination game. We showed the effect of this public information on social learning. This public survival news always helps to reduce the strategic uncertainties among agents. Diffusing is more than just a way to repeat the public survival news. It strengthens the positive effect from the public information in two ways. First, diffusion dilutes the private information and forces agents to coordinate more on the public good news. Secondly, the intra-group coordination risk will be further reduced when agents moving earlier face lower inter-group coordination risk. Agents will completely ignore their private information and coordinate purely on the public survival news whenever their private information about the effective fundamental is sufficiently noisy and there is no inter-group coordination risk. To focus on how diffusion helps to reduce the coordination risk, we have abstracted some features from a more general model. A more comprehensive analysis is needed to include issues such as reversibility of action, other forms of information, or the informativeness of the policy itself. These are important issues, and we leave them to future research.

Appendix

Proof of Proposition 1 This is an existing result and proof can be found in Morris and Shin (2003). For the shake of completeness we reproduce the proof in the online appendix.

Proof of Lemma 1 We will find the rationalizable strategy by iterated elimination of never best response. The range of private signal is $s \in [\underline{S}, \bar{S}]$, where $\underline{S} := \underline{\theta} - \frac{\sigma}{2}$ and $\bar{S} := \bar{\theta} + \frac{\sigma}{2}$. We start from the worst possible scenario. Suppose agents attack the preferred regime irrespective of their private information, i.e., $s \leq \bar{s}_0 := \bar{S}$ and thus the preferred regime fails whenever $\theta < \bar{\theta}_0 := 1$. For any agent with information s, given the belief of $\bar{s}_0$ and $\bar{\theta}_0$ (others will not attack the preferred regime only if $s > \bar{s}_0$ and the preferred regime will succeed only if $\theta \geq \bar{\theta}_0$), the preferred regime succeeds with probability $P(\theta \geq \bar{\theta}_0 | s, \theta \geq 0)$. Let $\bar{s}_1$ be such that when $s = \bar{s}_1$ the agent believes that the preferred regime succeeds with probability p_0.

$$P(\theta \geq \bar{\theta}_0 | \bar{s}_1, \theta \geq 0) = \frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F(\frac{\bar{s}_1}{\sigma})} = p_0. \tag{14}$$

Thus, attacking the preferred regime when $s > \bar{s}_1$ is never a best response. Anticipating the worst possible case under the belief of $\bar{s}_0$ and $\bar{\theta}_0$, agents take action to favor the preferred regime only if $s > \bar{s}_1$. Thus, the preferred regime succeeds only when $\theta \geq \bar{\theta}_1$ where $\bar{\theta}_1$ is defined as

$$P(s \leq \bar{s}_1 | \bar{\theta}_1) = \bar{\theta}_1 \tag{15}$$

Before proceeding to the next round of eliminating, we can show that $\bar{s}_1 < \bar{s}_0$. This follows from the log-concavity of F.

$$\frac{\partial \frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F(\frac{\bar{s}_1}{\sigma})}}{\partial \bar{s}_1} = \frac{f(\frac{\bar{s}_1}{\sigma}) f(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F^2(\frac{\bar{s}_1}{\sigma})} \left(\frac{F(\frac{\bar{s}_1}{\sigma})}{f(\frac{\bar{s}_1}{\sigma})} - \frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{f(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})} \right) > 0$$

$\frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F(\frac{\bar{s}_1}{\sigma})}$ is increasing in $\bar{s}_1$ with $\lim_{\bar{s}_1 \to \underline{s}} \frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F(\frac{\bar{s}_1}{\sigma})} = 0$ and $\lim_{\bar{s}_1 \to \bar{s}} \frac{F(\frac{\bar{s}_1 - \bar{\theta}_0}{\sigma})}{F(\frac{\bar{s}_1}{\sigma})} = 1$. Thus, there exists a unique solution of $\bar{s}_1 \in (\underline{s}, \bar{s})$. Solving $\bar{s}_1$ from Equation (15), $\bar{s}_1$ is an increasing function of $\bar{\theta}_1$, i.e., $\bar{s}_1 = \bar{\theta}_1 + \sigma F^{-1}(\bar{\theta}_1)$. Since $\bar{s}_1 < \bar{s}_0$ and $\bar{s}_0 = \bar{\theta}_0 + \sigma F^{-1}(\bar{\theta}_0)$, $\bar{\theta}_1 < \bar{\theta}_0 = 1$.

Based on the updated belief of $\bar{s}_1$ and $\bar{\theta}_1$, define $\bar{s}_2$ such that the agent with signal $s = \bar{s}_2$ believes $\theta > \bar{\theta}_1$ with probability p_0. Hence, agents attacking the preferred regime when $s > \bar{s}_2$ is a never best response. For the threshold agent with $\bar{s}_2$, we have

$$P(\theta \geq \bar{\theta}_1 | \bar{s}_2, \theta \geq 0) = \frac{F(\frac{\bar{s}_2 - \bar{\theta}_1}{\sigma})}{F(\frac{\bar{s}_2}{\sigma})} = p_0 \tag{16}$$

Comparing Equation (16) to Equation (14), $\bar{s}_2 < \bar{s}_1$ since $\bar{\theta}_1 < \bar{\theta}_0$ and $\frac{F(\frac{\bar{s}-\bar{\theta}}{\sigma})}{F(\frac{\bar{s}}{\sigma})}$ is increasing in $\bar{s}$. Let $\bar{\theta}_2$ satisfies $P(s \leq \bar{s}_2|\bar{\theta}_2) = \bar{\theta}_2$. $\bar{\theta}_2 < \bar{\theta}_1$ since $\bar{s}_2 < \bar{s}_1$.

Recursively, $\forall n \geq 2$, given the updated belief of $\bar{s}_n$ and $\bar{\theta}_n$, the worst possible case can be characterized by $\bar{s}_{n+1}$ and $\bar{\theta}_{n+1}$, which satisfy the following two equations

$$\frac{F(\frac{\bar{s}_{n+1}-\bar{\theta}_n}{\sigma})}{F(\frac{\bar{s}_{n+1}}{\sigma})} = p_0 \tag{17}$$

$$\bar{\theta}_{n+1} = P(s \leq \bar{s}_{n+1}|\bar{\theta}_{n+1}) \tag{18}$$

Solve for $\bar{s}_{n+1}$ from Equation (18) and plug it into Equation (17), we have[36]

$$\frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))} = p_0 \tag{19}$$

After n rounds of iterated elimination of never best responses, the worst possible case is that the preferred regime succeeds only when $\theta \geq \bar{\theta}_n$. Inductively, the sequence $\{\bar{\theta}_n\}_{n=0}^{\infty}$ is decreasing and bounded below by 0. Hence the limit exists and it is the infimum of the sequence. If we prove that for sufficiently large σ, $\inf \bar{\theta}_n = 0$. The preferred regime will succeed only when $\theta \geq 0$ and the preferred regime will fail only when $\theta < 0$. Suppose, for contradiction, for some sufficiently large σ, $\inf \bar{\theta}_n = b > 0$.

$\forall \zeta > 0$, $\exists N > 0$, $\forall n \geq N$, $\bar{\theta}_n \in [b, b+\zeta]$. Thus, both $\bar{\theta}_n$ and $\bar{\theta}_{n+1}$ are in $[b, b+\zeta]$. The LHS of Equation (19) is increasing in $\bar{\theta}_{n+1}$ and decreasing in $\bar{\theta}_n$ as

$$\begin{aligned}\frac{\partial \frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}}{\partial \bar{\theta}_{n+1}} \quad = \quad & \frac{\frac{1}{\sigma} + \frac{1}{f(F^{-1}(\bar{\theta}_{n+1}))}}{F^2(\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))f(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))f(\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))} \\ & \times \left(\frac{F(\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))}{f(\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))} - \frac{F((\frac{\bar{\theta}_{n+1}}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))}{f(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma} + F^{-1}(\bar{\theta}_{n+1}))} \right) > 0\end{aligned}$$

[36]Notice that $\theta \leq \bar{\theta}$ can be taken as another piece of public information. Taking this piece of public information into consideration, Equation (17) can be written as $P(\theta > \bar{\theta}_n|\bar{s}_{n+1}, 0 \leq \theta \leq \bar{\theta}) = \frac{F(\frac{\bar{s}_{n+1}-\bar{\theta}_n}{\sigma})--F(\frac{\bar{s}_{n+1}-\bar{\theta}}{\sigma})}{F(\frac{\bar{s}_{n+1}}{\sigma})-F(\frac{\bar{s}_{n+1}-\bar{\theta}}{\sigma})} = p_0$. Combine this with Equation (19), we have $\frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))-F(\frac{\bar{\theta}_{n+1}-\bar{\theta}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))-F(\frac{\bar{\theta}_{n+1}-\bar{\theta}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))} = p_0$. Because we assume the prior is uninformative, i.e., $\bar{\theta} > 1+\sigma$ and $\bar{\theta}_{n+1} \in [0,1]$, we have $0 \leq F(\frac{\bar{\theta}_{n+1}-\bar{\theta}}{\sigma} + F^{-1}(\bar{\theta}_{n+1})) \leq F(-1+F^{-1}(1)) = F(-1+\frac{1}{2}) = 0$. Thus, the assumptions on F and $\bar{\theta} > 1+\sigma$ guarantee that we can ignore the public information $\theta \leq \bar{\theta}$.

Thus,

$$\min\left(\frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}\right)=\frac{F(\frac{-\zeta}{\sigma}+F^{-1}(b))}{F(\frac{b}{\sigma}+F^{-1}(b))} \tag{20}$$

Notice that $\frac{F(\frac{-\zeta}{\sigma}+F^{-1}(b))}{F(\frac{b}{\sigma}+F^{-1}(b))}$ is continuously increasing in σ and

$$\lim_{\sigma\to\infty}\frac{F(\frac{-\zeta}{\sigma}+F^{-1}(b))}{F(\frac{b}{\sigma}+F^{-1}(b))}=1$$

Hence, there exists σ^* such that for all $\sigma>\sigma^*$, $\forall\bar{\theta}_n\in[b,b+\zeta]$ and $\forall\bar{\theta}_{n+1}\in[b,b+\zeta]$,

$$\frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}\geq\min\left(\frac{F(\frac{\bar{\theta}_{n+1}-\bar{\theta}_n}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}{F(\frac{\bar{\theta}_{n+1}}{\sigma}+F^{-1}(\bar{\theta}_{n+1}))}\right)>p_0$$

Hence, contradiction and $\theta^\star=0.\square$

Proof of Monotone Equilibrium Proposition 2 and 3 and 4 characterizes the monotone equilibrium under slightly different settings. The proofs are very similar. We will provide the proof for prop 3 here. Proof of Proposition 2 is a special case of this. Proof of 4 can be found in the online appendix.

Let $(\theta_t^*,s_t^*)_{t=1}^T$ be the equilibrium threshold. Let us define the net payoff from favoring the preferred regime to attacking it (when an agent gets a signal s and it is commonly known that $\theta_t\geq 0$) as follows:

$$\psi(s,\theta_t^*)=(b_1-c_1+b_0-c_0)P(\theta_t\geq\theta_t^*|s,\theta_t\geq 0)-(b_0-c_0)$$

Proof of Proposition 2 and 3

If agents gather information instantaneously, $P(\theta_t\geq\theta_t^\star|s,\theta_t\geq 0)=\frac{P(\theta_t\geq\theta_t^*|s)}{P(\theta_t\geq 0|s)}=\frac{F(\sqrt{\tau}(s-\theta_t^*))}{F(\sqrt{\tau}s)}$. Let $s_t^*(\theta_t^*,\theta_{t+1}^*)$ be the threshold signal such that if agents will not attack the preferred regime iff $s_t\geq s_t^*(\theta_t^*,\theta_{t+1}^*)$, then $\theta_t\geq\alpha_t w(\theta_t)+\theta_{t+1}^*$ iff $\theta_t\geq\theta_t^*$. Therefore,

$$s_t^*(\theta_t^*,\theta_{t+1}^*)=\theta_t^*+\frac{1}{\sqrt{\tau}}F^{-1}(\frac{\theta_t^*-\theta_{t+1}^*}{\alpha_t})$$

Define the net payoff from taking an action to favor the preferred regime, for the threshold agent who gets the signal $s_t^*(\theta_t^*, \theta_{t+1}^*)$ and it is publicly known that $\theta_t \geq 0$ as follows: $\Psi(\theta_t^*, \theta_{t+1}^*) = \psi(s_t^*(\theta_t^*, \theta_{t+1}^*), \theta_t^*)$. Notice that by definition $\theta_t^\star \in [\theta_{t+1}^\star, \sum_{u=t}^T \alpha_t]$. In equilibrium, there are three cases: 1. $\exists \theta_t^\star \in (\theta_{t+1}^\star, \sum_{u=t}^T \alpha_t)$, $\Psi(\theta_t^*, \theta_{t+1}^*) = 0$; 2. $\Psi(\theta_t^*, \theta_{t+1}^*) > 0$ for all $\theta_t^\star \in [\theta_{t+1}^*, \sum_{u=t}^T \alpha_t]$, i.e., $\theta_t^* = \theta_{t+1}^*$; 3. $\Psi(\theta_t^*, \theta_{t+1}^*) < 0$ for all $\theta_t^\star \in [\theta_{t+1}^*, \sum_{u=t}^T \alpha_t]$, i.e., $\theta_t^* = \sum_{u=t}^T \alpha_t$.

Case 1 gives us the recursive relation 6 for instantaneous information.

Case 2 can happen only when $\theta_{t+1}^\star = 0$. If $\Psi(\theta_t^*, \theta_{t+1}^*) > 0$ for all $\theta_t^\star \in [\theta_{t+1}^\star, \sum_{u=t}^T \alpha_t]$, $\theta_t^* = \theta_{t+1}^*$ and $s_t^\star = \theta_t^* - \frac{1}{2}\frac{1}{\sqrt{\tau}}$. Under this case, if $\theta_{t+1}^\star > 0$,$P(\theta_t \geq \theta_t^\star | s_t^\star, \theta_t \geq 0) = 0$ and $\psi(s_t^\star, \theta_t^*) = -(b_0 - c_0) < 0$, which contradicts with $\Psi(\theta_t^*, \theta_{t+1}^*) > 0$. Thus, $\theta_t^* = \theta_{t+1}^\star$ only if $\theta_{t+1}^\star = \theta_{t+2}^\star = ... = \theta_T^\star = 0$. When $\theta_{t+1}^\star = 0$, $P(\theta_t \geq \theta_t^\star | s_t^\star, \theta_t \geq 0) = 1$ and $\Psi(\theta_t^*, \theta_{t+1}^*) > 0$.

Case 3 cannot be an equilibrium. If $\Psi(\theta_t^*, \theta_{t+1}^*) < 0$ for all $\theta_t^\star \in [\theta_{t+1}^*, \sum_{u=t}^T \alpha_t]$, $\theta_t^* = \sum_{u=t}^T \alpha_t$. Since $\theta_{t+1}^* \leq \sum_{u=t+1}^T \alpha_u$, $\frac{\sum_{u=t}^T \alpha_t - \theta_{t+1}^*}{\alpha_t} \geq 1$ and $s_t^\star = \theta_t^* + \frac{1}{2}\frac{1}{\sqrt{\tau}}$. This means that $P(\theta_t \geq \theta_t^\star | s_t^\star, \theta_t \geq 0) = 1$, which contradicts with $\Psi(\theta_t^*, \theta_{t+1}^*) < 0$.

Thus, a monotone equilibrium $(\theta_t^\star, s_t^\star)_{t=1}^T$ is such that for some $t' = 1, 2, \dots T+1$, $\theta_{t'}^\star = \theta_{t'+1}^* = \dots \theta_{T+1}^* = 0$ and for $t = 1, 2, \dots t'-1$, $\theta_t^\star$ satisfies $\Psi(\theta_t^*, \theta_{t+1}^*) = 0$ and $s_t^*(\theta_t^*, \theta_{t+1}^*) = \theta_t^* + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t})$ (Equation (6)).

Proof of Lemma 2

1. Since $F(.)$ is continuously differentiable, so is $G(.)$.

2. Since $F(.) \leq 1$, $G(x, \alpha, k) \geq x$. Since $\sqrt{\tau}\alpha(x + k) \geq 0$, we have $G(x, \alpha, k) \leq 1$. Obviously $G(.)$ is decreasing in α. $\lim_{x \to 1} G(x, \alpha) = \frac{1}{F(\frac{1}{2} + \sqrt{\tau} a \times 1)} = 1$.

3. Notice that $G(x = 1, a) = 1 > p_0$. Thus, $x^\star < 1$. If $x^\star > 0$, $\exists x_1 \in (0, x^\star)$ such that $G(x_1, a) < p_0$. Based on the continuity of $G(x, a)$ in x and $G(1, a) = 1 > p_0$, $G(x^\star, a) = p_0$. Suppose $\frac{dG}{dx}|_{x=x^\star} > 0$ is not true, then $\frac{dG}{dx}|_{x=x^\star} \leq 0$. Since $\frac{dG}{dx}$ is continuous, $\frac{dG}{dx}|_{x=x^\star} \leq 0$ obviously contradicts with the fact that $x^\star = \max_x \{x \in [0, 1] | G(x, a) \leq p_0\}$. The left is based on the definition of $x^\star$.

□

Proof of Corollary 1 Suppose the diffusion is $(T, (\alpha))$. At the last period T, in equilibrium, $(\theta_T^\star, s_T^\star)$ satisfies $P(\theta_T \geq \theta_T^* | s_T^*) = p_0$, $\alpha_T P(s_T \leq s_T^* | \theta_T^*) = \theta_T^*$. Hence, the unique equilibrium is $\theta_T^* = \alpha_T p_0$. Similarly, for $t < T$, we have $\theta_{t+1}^\star - \theta_t^* = \alpha_t p_0$. By induction, we have $\theta_1^* = p_0$.

Suppose $T = 1$ and there is public information $\theta_t \geq 0$ in the model of concentrated coordination risk, then the monotone equilibrium $(\theta^\star, s^\star)$ satisfies $G(\theta^*, 1) = p_0$ or $\theta^\star = 0$. Since $G(\theta^*, 1) \geq \theta^*$ (see Lemma 2), we have $\theta^\star \leq p_0$.

In the dynamic coordination model with $T > 1$. Starting from the last period T, it is easy to see that $\frac{\theta_T^\star}{\alpha_T} \leq p_0$, since $G(\frac{\theta_T^*}{\alpha_T}, \alpha_T) \geq \frac{\theta_T^*}{\alpha_T} = p_0$ or $\theta_T^\star = 0$. Similarly, for $t < T$, we have $G(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}, \frac{\theta_{t+1}^*}{\alpha_t}, \alpha_t) \geq \frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t} = p_0$. Or, we have $\theta_t^\star = \theta_{t+1}^\star = ... = \theta_T^\star = 0$. Hence, $\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t} \leq p_0$. Recursively, we have $\theta_t^\star \leq p_0 \sum_{u \geq t} \alpha_t$. Hence, we proved that $\theta_1^\star \leq p_0$, no matter which equilibrium is selected. $\square$

Proof of Theorem 1 Recall G as defined in Equation (2). Let us define

$$x(\alpha) := \arg\min_{x \in [0,1]} G(x, \alpha\sqrt{\tau})$$

$$y(\alpha) := G(x(\alpha), \alpha\sqrt{\tau}) \tag{21}$$

By theorem of maximum we know $y(\alpha)$ is well defined and it is continuous in α. $\forall \alpha_1 < \alpha_2$, $y(\alpha_1) = G(x(\alpha_1), \alpha_1\sqrt{\tau}) > G(x(\alpha_1), \alpha_2\sqrt{\tau}) > G(x(\alpha_2), \alpha_2\sqrt{\tau}) = y(\alpha_2)$. The first inequality comes from the fact that $G(x, \alpha\sqrt{\tau})$ is decreasing in α and the second one follows the definition of $x(\alpha)$. Hence, $y(\alpha)$ is decreasing with α. $y(0) = \lim_{\alpha \to 0} y(\alpha) = 1 > p_0$. If $y(1) \leq p_0$ then $\exists \alpha^* > 0$ such that $\forall \alpha < \alpha^\star, \forall x \in [0,1], G(x, \alpha\sqrt{\tau}) > p_0$. If $y(1) > p_0$ then $\alpha^* = 1$.

Based on Proposition 3, in equilibrium for group T either $G(\frac{\theta_T^\star}{\alpha_T}, \alpha_T\sqrt{\tau}) = p_0$ or $\theta_T^* = 0$. If $\alpha_T < \alpha^*$ then $\min_{\theta_T^*} G(\frac{\theta_T^\star}{\alpha_T}, \alpha_T\sqrt{\tau}) > p_0$. Hence $\theta_T^\star = 0$ is the unique equilibrium.

Based on recursive relation in Equation (6) and $\theta_T^\star = 0$, group $T-1$ agents are facing exactly the same problem as agents in group T. Hence, $G(\frac{\theta_{T-1}^\star}{\alpha_{T-1}}, \alpha_{T-1}\sqrt{\tau}) = p_0$ or $\theta_{T-1}^\star = 0$. Thus, if $\alpha_{T-1} < \alpha^*$ then $\theta_{T-1}^* = 0$ is the unique equilibrium. Proceeding the same way if $\alpha_t < \alpha^*$ for all $t = 1, 2, \ldots T$ then we have the unique equilibrium $\theta_1^* = 0$. $\square$

Proof of Lemma 3 By definition $f^{t-1}(h_t(\theta_t^*)) = h_t(\theta_t^*) - \sum_{u=1}^{t-1} \alpha_u F(\sqrt{\tau}(s_u^* - h_t(\theta_t^*))) = \theta_t^*$ and $f^{t-1}(h_t(0)) = h_t(0) - \sum_{u=1}^{t-1} \alpha_u F(\sqrt{\tau}(s_u^* - h_t(0))) = 0$. Therefore, $h_t(\theta_t^*) - h_t(0) - (\sum_{u=1}^{t-1} \alpha_u (F(\sqrt{\tau}(s_u^* - h_t(\theta_t^*))) - F(\sqrt{\tau}(s_u^* - h_t(0))))) = \theta_t^* - 0$. Notice that $h_t(\theta_t^*) \geq h_t(0)$ and thus $F(\sqrt{\tau}(s_u^* - h_t(\theta_t^*))) - F(\sqrt{\tau}(s_u^* - h_t(0))) < 0$ for all $u \in \{1, 2, ..., t-1\}$. Hence, $h_t(\theta_t^*) - h_t(0) \leq \theta_t^* - 0$. $\square$

Proof of Theorem 2

$$
\begin{aligned}
P(\theta_T \geq 0 | s_T^*) &= F\left(F^{-1}(\frac{\theta_T^*}{\alpha_T}) + \sqrt{\tau}(h_T(\theta_T^*) - h_T(0))\right) \\
&\leq F\left(F^{-1}(\frac{\theta_T^*}{\alpha_T}) + \alpha_T\sqrt{\tau}\frac{\theta_T^*}{\alpha_T}\right) \text{ (using Lemma 3)}
\end{aligned}
$$

The threshold agent believes that the preferred regime will succeed with probability

$$
\begin{aligned}
P(\frac{\theta_T}{\alpha_T} \geq \frac{\theta_T^*}{\alpha_T} | s_T^*, \frac{\theta_T}{\alpha_T} \geq 0) &= \frac{\frac{\theta_T^*}{\alpha_T}}{F\left(F^{-1}(\frac{\theta_T^*}{\alpha_T}) + \sqrt{\tau}(h_T(\theta_T^*) - h_T(0))\right)} \\
&\geq \frac{\frac{\theta_T^*}{\alpha_T}}{F\left(F^{-1}(\frac{\theta_T^*}{\alpha_T}) + \alpha_T\sqrt{\tau}\frac{\theta_T^*}{\alpha_T}\right)} \\
&= G(\frac{\theta_T^*}{\alpha_T}, \alpha_T\sqrt{\tau})
\end{aligned}
$$

The rest of the proof is exactly same as the proof of Theorem 1. □

Proof of Lemma 4 This follows from the linearity of the value function on the fraction an agent withdraws. The detailed proof can be found in the online appendix. □

Proof of Theorem 3 We will prove this Proposition in three steps. We will first prove our Theorem 1 holds true with more general payoff structure with instantaneous information gathering. Then we turn to the model with ex-ante information gathering, we will show Theorem 2 holds true with more general payoff structure as well. In the end, we will show that our Theorem 2 holds when agents with ex-ante information get paid immediately when they attack the preferred regime and the payoff only depends on the whether the preferred regime can withstand the current attack. This is exactly our Theorem 3.

Step 1: Under general payoff structure satisfying assumption 3.1 and 3.3, Theorem 1 holds true.

At time $t \in \{1, 2 \ldots T\}$ of the diffused coordination risk model, suppose the player i's payoff u depends on the her action a_{it}, the current non-coordination $\alpha_t w_t$, the future aggregate non-coordination $w^t \equiv \sum_{u=t+1}^{T} \alpha_u w_u$ and the current fundamental strength θ_t.

$$u(a_{it} = 1, \theta_t, \alpha_t w_t, w^t) = \begin{cases} b_1(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ c_0(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases}$$

$$u(a_{it} = 0, \theta_t, \alpha_t w_t, w^t) = \begin{cases} c_1(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ b_0(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases}$$

in which $w_{T+1} = w^T = 0$.

Assumption 3 *The payoff satisfies the following properties*

1. $\bar{u}(\theta_t, \alpha_t w_t, w^t) := b_1(\theta_t, \alpha_t w_t, w^t) - c_1(\theta_t, \alpha_t w_t, w^t) > 0$ *and* $\underline{u}(\theta_t, \alpha_t w_t, w^t) := c_0(\theta_t, \alpha_t w_t, w^t) - b_0(\theta_t, \alpha_t w_t, w^t) < 0$

2. *(Monotonicity)* $b_1(\theta_t, \alpha_t w_t, w^t), c_1(\theta_t, \alpha_t w_t, w^t), b_0(\theta_t, \alpha_t w_t, w^t), c_0(\theta_t, \alpha_t w_t, w^t)$ *are all non-decreasing in* θ_t *and non-increasing in* $\alpha_t w_t$ *and* w^t*.*

3. *(Boundedness)* $0 < \underline{n} < \bar{u}(.) < \bar{n}$ *and* $0 < \underline{m} < -\underline{u}(.) < \bar{m}$ *in which* $\underline{m}, \bar{m}, \underline{n}, \bar{n}$ *are constants.*

Proof. Let $\{\theta_t^\star, s_t^\star\}_{t=1}^T$ be the equilibrium thresholds. We begin our proof from the last period and then complete it by backward induction. Let $H(s_T, s_T^*)$ be the payoff difference when the agent receiving s_T and other agents follow the threshold rule s_T^*, as defined in Equation (32).

The distribution of w_T is

$$\begin{aligned} P(w_T &\leq k | s_T^\star, \theta_T \geq 0) = P(F(\frac{s_T^\star - \theta_T}{\sigma}) \leq k | s_T^\star, \theta_T \geq 0) \\ &= \frac{P(\theta_T \geq s_T^\star - \sigma F^{-1}(k))}{P(\theta_T \geq 0)} = \frac{k}{F(\frac{s_T^\star}{\sigma})} \end{aligned}$$

Based on Equation (34) in the proof of Proposition 1, we can write $H(s_T^\star, s_T^*)$ as follows

$$\begin{aligned} H(s_T^\star, s_T^\star) = & \frac{1}{F(\frac{s_T^\star}{\sigma})} \int_0^{\theta_T^\star} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T \\ & + \frac{1}{F(\frac{s_T^\star}{\sigma})} \int_{\theta_T^\star}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T \end{aligned} \tag{22}$$

in which $v(w_T, s_T^\star) \equiv s_T^\star - \sigma F^{-1}(w_T)$.

We need to show that if α_T in the last period is small enough, then the only rationalizable action for agents at $t = T$ is not to attack the preferred regime. In other words, for any possible $s_T^\star$ and $\theta_T^\star$ satisfying $\alpha_T P(s_T < s_T^\star | \theta_T^\star) = \theta_T^\star$, the net expected payoff $H(s_T^\star, s_T^\star) \geq 0$. Recall the net payoff can be written as Equation (22). Hence, it is sufficient to show that

$$\int_0^{F(\frac{s_T^\star - \theta_T^\star}{\sigma})} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T + \int_{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T \geq 0$$

for any $s_T^\star$ and $\theta_T^\star$ satisfying $\alpha_T P(s_T < s_T^\star | \theta_T^\star) = \theta_T^\star$. We know that

$$\begin{aligned}
& \textstyle\int_0^{F(\frac{s_T^\star - \theta_T^\star}{\sigma})} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T + \int_{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T \\
> \quad & F(\tfrac{s_T^\star - \theta_T^\star}{\sigma}) \times \underline{n} + \Big(F(\tfrac{s_T^\star}{\sigma}) - F(\tfrac{s_T^\star - \theta_T^\star}{\sigma})\Big) \times (-\bar{m}) \\
= \quad & F(\tfrac{s_T^\star - \theta_T^\star}{\sigma}) \times (\underline{n} + \bar{m}) - F(\tfrac{s_T^\star}{\sigma}) \times \bar{m}
\end{aligned}$$

Thus, it is sufficient to show that there exist $\alpha^\star \in (0, 1)$ such that for any $\alpha_T \leq \alpha^\star$, the following inequality always hold for any $\theta_T^\star \in [0, \alpha_T]$.

$$\frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} = \frac{\frac{\theta_T^\star}{\alpha_T}}{F(\frac{1}{\sigma}\theta_T^\star + F^{-1}(\frac{\theta_T^\star}{\alpha_T}))} \geq \bar{p} = \frac{\bar{m}}{\underline{n} + \bar{m}} \tag{23}$$

Equivalently, we need to show that $\forall x \in [0, 1]$

$$G(x, \alpha_T) = \frac{x}{F(\sqrt{\tau}\alpha_T x + F^{-1}(x))} \geq \bar{p}$$

Since $w^{T-1} = w_T = 0$ is the unique equilibrium, agents moving at $T - 1$ are facing the same problem as the last group. The rest of the argument follows from Theorem 1. Our payoff structure in (11) satisfies assumption 3.1 and 3.3. All we need for this result is the boundedness of the payoff and not monotonicity. ■

Step 2: Under general payoff structure satisfying assumption 3.1 and 3.3, Theorem 2 holds true.

Proof. With ex-ante information, we can still define the monotone equilibrium as $\{\theta_t^\star, s_t^\star\}_{t=1}^T$.

For any monotone equilibrium, we can define $h_t(.) = (f^{t-1})^{-1}(.)$ (f^{t-1} is defined in Equation (8)). As in step 1, We first look at the last period problem and prove this by backward induction. Similar to Equation (23), for $\theta^\star_T = 0$ to be the unique equilibrium, it is sufficient to show that there exist $\alpha^\star \in (0,1)$ such that for any $\alpha_T < \alpha^\star$, the following inequality always hold for any $\theta^\star_T \in [0, \alpha_T]$.

$$\frac{F(\frac{s^\star_T - h_T(\theta^\star_T)}{\sigma})}{F(\frac{s^\star_T - h_T(0)}{\sigma})} = \frac{\frac{\theta^\star_T}{\alpha_T}}{F(\frac{1}{\sigma}(h_T(\theta^\star_T) - h_T(0)) + F^{-1}(\frac{\theta^\star_T}{\alpha_T}))} \geq \bar{p} \tag{24}$$

From Lemma 3, we know that $h_t(\theta^*_T) - h_t(0) \leq \theta^*_t$. Thus, we have

$$\frac{\frac{\theta^\star_T}{\alpha_T}}{F(\frac{1}{\sigma}(h_T(\theta^\star_T) - h_T(0)) + F^{-1}(\frac{\theta^\star_T}{\alpha_T}))} \geq \frac{\frac{\theta^\star_T}{\alpha_T}}{F(\frac{1}{\sigma}\theta^\star_T + F^{-1}(\frac{\theta^\star_T}{\alpha_T}))} \tag{25}$$

The proof of step 1 shows that RHS of 25 is larger than $\bar{p}$ when α_T is sufficiently small. This completes the proof of step 2. ■

Step 3: Theorem 3

Proof. It is a more complicated game comparing to the game in step 2. The criteria of success in attacking($a_{it} = 0$): $\theta_t \geq \alpha_t w_t + w^t$ is different from the that in not attacking($a_{it} = 1$): $\theta_t \geq \alpha_t w_t + w^t$. We focus on the monotone equilibrium $\{\theta^\star_t, s^\star_t\}_{t=1}^T$. However, the subgame in period T is exactly the same that in step 2 since $w^T = 0$. We know that when α_T is sufficiently small, $\theta^\star_T = w^\star_T = 0$ is the unique equilibrium. Now since $w^{T-1} = w_T = 0$, the game in $T-1$ is exactly as that in the step 2. Thus, by backward induction, the unique equilibrium is $\theta_1 = 0$ by restricting $\alpha_t < \alpha^\star$ for all $t \in \{1, 2, ..., T\}$. ■

Proof of Corollary 2 First, the payoff structure in (12) satisfies assumption 3.1 and 3.3. The last period problem is exactly the same as the that in step 1 of Theorem 3. There exists $\alpha^\star(p_0, \tau)$ such that when $\alpha_T < \alpha^\star$, $\theta^\star_T = w_T = 0$ is the unique equilibrium. Notice that $w^{T-1} = w_T = 0$. For group $T-1$ creditors, if the borrower can remain liquid at time $T-1$, she can be liquid in the end. Thus, they are facing exactly the same game as group $T-1$ agents in the step 1 proof of Theorem 3. The same logic applies. By backward induction, $\theta^\star_1 = 0$ is the unique equilibrium if $\alpha_t < \alpha^\star$ for all $t \in \{1, 2, ..., T\}$. □

Proof of Corollary 5 As defined in proof of Theorem 1, $y(\alpha_t) := \min_{x\in[0,1]} G(x, \alpha_t)$ is decreasing with α_t. Given α_t, $y(\alpha_t)$ is decreasing with τ. As $\alpha^\star := \max\{\alpha_t | y(\alpha) \geq \frac{\bar{m}}{\underline{n}+\bar{m}}\}$, $\alpha^\star$ is decreasing in τ. □

Proof of Lemma 5 This is a natural extension of Milgrom and Roberts (1990). The detailed proof is in the online appendix. □

Proof of Corollary 3 If public information of survival is available at time t, the equilibrium threshold $\theta_t^\star$ (given $\theta_{t+1}^\star$) is the largest possible solution to the following equation

$$G(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}, \alpha_t\sqrt{\tau}, \frac{\theta_{t+1}^\star}{\alpha_t}) = \frac{\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}}{F(F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}) + \alpha_t\sqrt{\tau}\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t} + \sqrt{\tau}\theta_{t+1}^\star)} = p_0 \tag{26}$$

According to Lemma 2, G is decreasing with $\theta_{t+1}^\star$ and increasing with $\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}$. Thus, $\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}$ increases with $\theta_{t+1}^\star$. □

Proof of Corollary 4 This follows from Proposition 3 (see case 2 in the proof of Proposition 3). □

Proof of Lemma 6 Consider two policies: a more diffused one $(T, (\alpha_1, ..., \alpha_t, \alpha_{t+1}, \alpha_{t+2}, ..., \alpha_T))$ and a less diffused one $(T-1, (\alpha_1, ..., \alpha_t + \alpha_{t+1}, \alpha_{t+2}, ..., \alpha_T))$. Notice that the former one has T groups while the later one has $T-1$ groups. We work with the case there the first best is not the unique equilibrium with the coarser partition. If it is, according to Theorem 1, it is the unique equilibrium with the finer partition as well. To prove that the designer weakly prefers the finer partition, it is sufficient to show that the former one is associated with (weakly) lower coordination risk (for the worst possible case).

Denote the worst possible equilibrium for the former one $\theta_{F,t}^\star$ and the later one $\theta_{C,t}^\star$. If $\theta_{F,1}^\star < \theta_{C,1}^\star$, we can say more diffusion always reduces the maximum possible coordination risk. We solve the game by backward induction and thus it is easy to see that $\theta_{F,t+2}^\star = \theta_{C,t+1}^\star$. Recall $G(.)$ as defined in Lemma 2. The equilibrium conditions are

$$G(\frac{\theta_{C,t}^\star - \theta_{C,t+1}^\star}{\alpha_t + \alpha_{t+1}}, \theta_{C,t+1}^\star, \alpha_t + \alpha_{t+1}) = G(\frac{\theta_{F,t+1}^\star - \theta_{F,t+2}^\star}{\alpha_{t+1}}, \theta_{F,t+2}^\star, \alpha_{t+1}) = p_0$$

Since G decreases with α, we have

$$G(\frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\theta^\star_{C,t+1},\alpha_{t+1}) > G_2(\frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\theta^\star_{C,t+1},\alpha_t+\alpha_{t+1}) = G(\frac{\theta^\star_{F,t+1}-\theta^\star_{F,t+2}}{\alpha_{t+1}},\theta^\star_{F,t+2},\alpha_{t+1})$$

Based on Lemma 2 and $\theta^\star_{F,t+2}=\theta^\star_{C,t+1}$, we have $\frac{\theta^\star_{F,t+1}-\theta^\star_{F,t+2}}{\alpha_{t+1}} < \frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}}$ and thus $\theta^\star_{F,t+1} < \frac{\alpha_{t+1}\theta^\star_{C,t}-\alpha_t\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}}$. Similarly, we have

$$\begin{aligned}G(\frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\frac{\alpha_{t+1}\theta^\star_{C,t}+\alpha_t\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\alpha_t) &= G(\frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\theta^\star_{C,t+1},\alpha_t+\alpha_{t+1}) \\ = G(\frac{\theta^\star_{F,t}-\theta^\star_{F,t+1}}{\alpha_t},\theta^\star_{F,t+1},\alpha_t) &> G(\frac{\theta^\star_{F,t}}{\alpha_t}-\frac{\alpha_{t+1}\theta^\star_{C,t}+\alpha_t\theta^\star_{C,t+1}}{\alpha_t(\alpha_t+\alpha_{t+1})},\frac{\alpha_{t+1}\theta^\star_{C,t}+\alpha_t\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}},\alpha_t)\end{aligned}$$

Hence, $\frac{\theta^\star_{F,t}}{\alpha_t}-\frac{\alpha_{t+1}\theta^\star_{C,t}+\alpha_t\theta^\star_{C,t+1}}{\alpha_t(\alpha_t+\alpha_{t+1})} < \frac{\theta^\star_{C,t}-\theta^\star_{C,t+1}}{\alpha_t+\alpha_{t+1}}$ and we proved that $\theta^\star_{F,t} < \theta^\star_{L,t}$. Obviously, $\theta^\star_{F,1} < \theta^\star_{L,1}$. □

Proof of Theorem 4 Given assumption 4, this directly follows from Lemma 6. □

Proof of Proposition 6 If $\alpha_t < \frac{\bar{\theta}_{(T,(\alpha))}}{1+\sigma}$, the prior of effective fundamental is uninformative and the proof follows the proof of Theorem 1 and 2. □

Proof of Proposition 7 Following the proof of Theorem 1, $x=\frac{\theta^\star_T}{\alpha_T}$ is the solution to the following equation

$$G(x,\alpha)=\frac{x}{F(\alpha\sqrt{\tau}x+F^{-1}(x))}=p_0$$

Recall $y(\alpha)$ as defined in Equation (21). We can reduce α_T by choosing a finer partition. The condition that $\alpha_T\sqrt{\tau_T(\alpha,T)}$ falls as the partition becomes finer means $y(\alpha_T)$ decreases with α_T, and the condition $\lim_{\alpha_T\to 0}\alpha_T\sqrt{\tau_T(\alpha,T)}=0$ guarantees $\lim_{\alpha_T\to 0}y(\alpha_T)=1$. Since $y(\alpha)$ is well defined and continuous in α, we can find $\alpha^\star_T(p_0)$ such that $\forall\alpha_T\leq\alpha^\star_T$, $\forall x\in(0,1)$, $G(x,\alpha_T)>p_0$. Hence, the first best equilibrium is the unique equilibrium of the subgame at time T. By backward induction, it is easy to see that $\alpha_t\sqrt{\tau_t(T,(\alpha))}$ falls as the partition becomes finer and $\lim_{\alpha_t\to 0}\alpha_t\sqrt{\tau_t(T,(\alpha))}=0$ are sufficient conditions to guarantee Theorem 1 works. □

Proof of Proposition 8 Similar to Proposition 3, we know that a monotone equilibrium $(\theta^\star_t,s^\star_t)_{t=1}^T$ is such that for some $t'=1,2,\dots T+1$, $\theta^\star_{t'}=\theta^*_{t'+1}=\dots\theta^*_{T+1}=0$ and for $t=1,2,\dots t'-1$,

$\theta_t^\star$ satisfies the following recursive equations:

$$\theta_{t+1}^\star = b\left(\theta_t^\star, \alpha_t P(s_t < s_t^\star|\theta_t^\star)\right) \tag{27}$$

$$P(\theta_t \geq \theta_t^\star|\theta_t \geq 0, s_t^\star) = p_0 \tag{28}$$

and $s_t^\star = \theta_t^\star + \frac{1}{\sqrt{\tau}}F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t})$. At time T, from Equation (28), we have

$$F(\frac{s_T^\star - \theta_T^\star}{\sigma}) = p_0 F(\frac{s_T^\star}{\sigma})$$

It is easy to show that $s_T^\star$ increases with $\theta_T^\star$ and $(s_T^\star - \theta_T^\star)$ decreases with $\theta_T^\star$ since

$$\frac{ds_T^\star}{d\theta_T^\star} = \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - p_0 f(\frac{s_T^\star}{\sigma})} = \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} f(\frac{s_T^\star}{\sigma})} > 0$$

$$\frac{d\left(s_T^\star - \theta_T^\star\right)}{d\theta_T^\star} = \frac{-p_0 f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - p_0 f(\frac{s_T^\star}{\sigma})} = \frac{-p_0 f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} f(\frac{s_T^\star}{\sigma})} < 0$$

Plug Equation (28) into Equation (27), we have

$$b\left(\theta_T^\star, \alpha_T F(\frac{s_T^\star - \theta_T^\star}{\sigma})\right) = b(\theta_T^\star, \alpha_T p_0 F(\frac{s_T^\star}{\sigma})) = 0 \tag{29}$$

Differentiate $b(\theta_T^\star, \alpha_T p_0 F(\frac{s_T^\star}{\sigma}))$ with respect to $\theta_T^\star$, we have

$$\begin{aligned}
\frac{db}{d\theta_T^\star} &= \frac{\partial b}{\partial \theta} - (-\frac{\partial b}{\partial w}) \times \alpha_T p_0 \times \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \times \frac{ds_T^\star}{d\theta_T^\star} \\
&= \frac{\partial b}{\partial \theta} - (-\frac{\partial b}{\partial w}) \times \alpha_T p_0 \frac{1}{\sigma} \times \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - p_0 f(\frac{s_T^\star}{\sigma})} \\
&= \frac{\partial b}{\partial \theta} - (-\frac{\partial b}{\partial w}) \times \alpha_T p_0 \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}}
\end{aligned}$$

Define $L(\theta_T^\star)$ as

$$L(\theta_T^\star) \equiv \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}}$$

From the log-concavity of F, and the monotonicity of $s_T^\star$ and $s_T^\star - \theta_T^\star$ on $\theta_T^\star$, we know $L(\theta_T^\star)$ is decreasing in $\theta_T^\star$. Hence, $\forall \theta_T^\star > 0$, $L(\theta_T^\star) < \lim_{\theta_T^\star \to 0} L(\theta_T^\star) = \lim_{\theta_T^\star \to 0} \frac{f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma}) - p_0 f(\frac{s_T^\star}{\sigma})} = \frac{1}{1-p_0}$. We have

$$
\begin{aligned}
\frac{db}{d\theta_T^\star}|_{\theta_T^\star \geq 0} &= \frac{\partial b}{\partial \theta}|_{\theta > 0} - (-\frac{\partial b}{\partial w})|_{w>0} \times \alpha_T p_0 \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}} \\
&\geq \underline{k}_1 - \bar{k}_2 \times \alpha_T p_0 \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \lim_{\theta_T^\star \to 0} L(\theta_T^\star) \\
&\geq \underline{k}_1 - \bar{k}_2 \times \alpha_T p_0 \frac{1}{\sigma} \times \max_{\theta_T^\star} f(\frac{s_T^\star(\theta_T^\star)}{\sigma}) \frac{1}{1-p_0}
\end{aligned}
$$

f is bounded because F is continuously differentiable,. There exists $\bar{f} > 0$ such that $\max_{x \in [-\frac{1}{2}, \frac{1}{2}]} f(x) \leq \bar{f}$. Thus, if $\alpha_T < \alpha^\star = \frac{\underline{k}_1 \sigma}{\bar{k}_2 \bar{f}} \frac{1-p_0}{p_0}$, $\frac{db\left(\theta_T^\star, \alpha_T P(s_T < s_T^\star | \theta_T^\star)\right)}{d\theta_T^\star} > 0$ holds for all $\theta_T^\star > 0$. When $\theta_T^\star = 0$, $b(0,0) = 0$. Thus, the only solution to Equation (29) is $\theta_T^\star = 0$. Given the unique Bayesian Nash Equilibrium at T is the $\theta_T^\star = 0$, $s_T^\star = -\frac{1}{2}\sigma$, the problem at time $T-1$ is exactly the same problem as in time T. The argument will go backwards to $t = 1$. This completes this generalization.$\square$

Proof of Proposition 9 The designer's problem is to choose permutation $(1,2)$ or $(2,1)$ to minimize the highest possible threshold θ_1^{*h}. Recall $G(x,m,y)$ as defined in Equation (2) and $\psi(k,m,y)$ as defined in Equation (13).One can easily check that: $\psi_k = \frac{1}{G_x} > 0$ (since we are considering θ^{*h}, see Lemma 2), $\psi_y = -\frac{G_y}{G_x} > 0$ (since $G_y < 0$) and $\psi_m = -\frac{G_m}{G_x} > 0$ (since $G_m < 0$).

Let (t) denote the group moving at $t = 1,2$. In equilibrium

$$
\theta_1^* = \alpha_{(1)} p_{0(1)} + \alpha_{(2)} \psi(p_{0(1)}, 0, \alpha_{(2)} \sqrt{\tau_{(2)}}) \tag{30}
$$

Notice that when there is no survival news at $t = 1$, the intra-group risk facing the group moving at $t = 1$, does not get affected by the inter-group risk they are facing. The designer optimally chooses permutation $(1,2)$ if $\theta_1^*(1,2) < \theta_1^*(2,1)$ and vice versa.

Information Heterogeneity Given $\tau_1 > \tau_2$. From Equation (30)

$$
\theta_1^*(1,2) - \theta_1^*(2,1) = \frac{p_0}{2} + \frac{1}{2}\psi(k, 0, \frac{\sqrt{\tau_2}}{2}) - \frac{p_0}{2} - \frac{1}{2}\psi(k, 0, \frac{\sqrt{\tau_1}}{2}) < 0
$$

The last inequality holds since $\psi_m > 0$.

Payoff Heterogeneity Given $p_{01} > p_{02}$. From equation 30 we see

$$\begin{aligned}\theta_1^*(1,2) - \theta_1^*(2,1) &= \frac{1}{2}[\{p_{01} + \psi(k_2, 0, \frac{1}{2}\sqrt{\tau})\} - \{p_{02} + \psi(p_{01}, 0, \frac{1}{2}\sqrt{\tau})\}] \\ &= \frac{1}{2}[\{p_{01} - \psi(k_1, 0, \frac{1}{2}\sqrt{\tau})\} - \{p_{02} - \psi(p_{02}, 0, \frac{1}{2}\sqrt{\tau}\})]\end{aligned}$$

Define $L(k) := \frac{1}{2}[k - \psi(k, 0, \frac{1}{2}\sqrt{\tau})]$. We have $L'(k) = 1 - \psi_k$. Note that $\psi_k = \frac{1}{G_x}$ captures how θ_2^* changes as p_0 changes. When there is no public information of survival this effect is 1. $L'(k) < 0$ iff $\psi_k = \frac{1}{G_x} > 1$, i.e., $G_x < 1$. Hence, $\theta_1^*(1,2) - \theta_1^*(2,1) < 0$ iff $G_x < 1$. $\square$

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Online Appendix

Proof of Proposition 1 (Morris and Shin (2003)) We are reproducing the MS result here. Readers familiar with global game literature can skip this proof. We will consider a more general payoff structure for this proof. The payoff specification we assume in the paper is a special case of this. We will assume

Suppose the player i's payoff is as follows

$$u(a_i = 1, \theta, w) = \begin{cases} b_1(\theta, w) & \text{if } \theta \geq w \\ c_0(\theta, w) & \text{if } \theta < w \end{cases}, \quad u(a_i = 0, \theta, w) = \begin{cases} c_1(\theta, w) & \text{if } \theta \geq w \\ b_0(\theta, w) & \text{if } \theta < w \end{cases}$$

Assumption 4 *The payoff satisfies the following properties*

1. $b_1(\theta, w) - c_1(\theta, w) = \bar{u}(\theta, w) > 0, \;\; c_0(\theta, w) - b_0(\theta, w) = \underline{u}(\theta, w) < 0$
2. *(Monotonicity)* $\bar{u}(\theta, w)$ *and* $\underline{u}(\theta, w)$ *are non-decreasing in* θ *and non-increasing in* w*.*
3. *(Boundedness)* $0 < \underline{n} < \bar{u}(.) < \bar{n}, \;\; 0 < \underline{m} < -\underline{u}(.) < \bar{m}$ *in which* $\underline{m}, \bar{m}, \underline{n}, \bar{n}$ *are constants.*

Assumption 4.1 and 4.2 capture the (monotonic) strategic complementarity. The incentive to cooperate is non-decreasing in w, while the incentive to take attack is non-increasing in w. Based on Assumption 3, the payoff differences are bounded. The previous assumed payoff structure is a special example of this more general one.

Given the private information s, the player will choose not to attack the preferred regime if she believes the probability of success is sufficiently high, i.e.,

$$P(\theta \geq w|s) \geq \frac{b_0(\theta, w) - c_0(\theta, w)}{b_1(\theta, w) - c_1(\theta, w) + b_0(\theta, w) - c_0(\theta, w)} = \frac{1}{1 + \frac{\bar{u}(\theta, w)}{-\underline{u}(\theta, w)}}$$

Define $\bar{p} = \max(\frac{-\underline{u}}{\bar{u} - \underline{u}}) = \frac{\bar{m}}{\underline{n} + \bar{m}}$ and $\underline{p} = \min(\frac{-\underline{u}}{\bar{u} - \underline{u}}) = \frac{\underline{m}}{\bar{n} + \underline{m}}$.

Suppose now (after n rounds of iterated elimination of dominated strategy) the upper dominance region is $s \geq \bar{s}_n$. Notice that we can start the elimination since $\bar{\theta} > 1 + \sigma$. Agents will not take the attack if $s \geq \bar{s}_n$. Under the worst possible scenario (agents will not attack the preferred regime only if $s \geq \bar{s}_n$), we can derive the next threshold $\bar{s}_{n+1}$ as a function of $\bar{s}_n$. If the fundamental is higher than $\theta^\star(\bar{s}_n) = P(s < \bar{s}_n|\theta^\star)$, the measure of attacks will be smaller than the fundamental $\theta^\star(\bar{s}_n)$

and thus preferred regime will succeed. Based on the belief about $\bar{s}_n$, agents will never attack the preferred regime if $s \geq \bar{s}_{n+1}$, in which $\bar{s}_{n+1}$ solves the $P(\theta \geq \theta^\star(\bar{s}_n)|\bar{s}_{n+1}) = \bar{p}$. From the definition of $\theta^\star(\bar{s}_n)$ and the above Equation for $\bar{s}_{n+1}$, we have

$$\sigma F^{-1}(\bar{p}) + \theta^\star(\bar{s}_n) = \bar{s}_{n+1},\ \sigma F^{-1}(\theta^\star(\bar{s}_n)) + \theta^\star(\bar{s}_n) = \bar{s}_n \tag{31}$$

When $n = 0$, $\theta^\star(\bar{s}_0) \equiv 1$. Since $\theta^\star(\bar{s}_0) > \bar{p}$, we have $\bar{s}_1 < \bar{s}_0$ and thus $\theta^\star(\bar{s}_1) < \theta^\star(\bar{s}_0)$. Recursively, if $\theta^\star(\bar{s}_n) < \theta^\star(\bar{s}_{n-1})$, then $\bar{s}_{n+1} < \bar{s}_n$ and $\theta^\star(\bar{s}_{n+1}) < \theta^\star(\bar{s}_n)$. Given the decreasing sequence $(\bar{s}_j)_{j=1}^\infty$ in the compact set $[\underline{\theta} - \frac{\sigma}{2}, \bar{\theta} + \frac{\sigma}{2}]$, there exist a limit for this sequence, $\bar{s} \equiv \lim_{j\to\infty} \bar{s}_j$. Similarly, there is a upper bound for the lower dominance region, $\underline{s} \equiv \lim_{j\to\infty} \underline{s}_j$.

By iterated elimination of dominated strategies, there exist $\bar{s}$ and $\underline{s}$ such that the only rationalizable action for any agent is never to favor the preferred regime when $s \leq \bar{s}$ and never to attack when $s > \underline{s}$. Thus, the worst possible equilibrium could be that agents choose not to attack the preferred regime if and only if $s \geq \bar{s}$. Agents choose not to attack the preferred regime if and only if $s \geq \underline{s}$ could constitute another equilibrium, or the best possible equilibrium. Thus, if $\underline{s} < \bar{s}$, there are at least two monotone equilibria. We will proceed to prove that there is actually one unique monotone equilibrium and it is the unique equilibrium for the coordination game.

Consider any possible monotone equilibrium $(\theta^\star, s^\star)$. For any agent $i \in [0,1]$, given other agents attack the preferred regime if and only if $s_j < s^\star (j \neq i)$, it is an equilibrium if she will take the same strategy. We will construct equilibrium in this way and prove the uniqueness by showing there is a unique solution to $s^\star$.

Given other agents are playing the threshold strategy $s^\star$, $\theta^\star$ can be defined independent of agent i's strategy, i.e., $\theta^\star = w(\theta^\star) = F(\frac{s^\star - \theta^\star}{\sigma})$. The preferred regime succeeds if and only if the total attack is less than the fundamental, i.e., $w(\theta, s^\star) = F(\frac{s^\star - \theta}{\sigma}) \leq \theta$ or equivalently $\theta \geq \theta^\star$. Define the function $H(s_i, s^\star)$ as the expected payoff difference(from not attacking the preferred regime to attacking) for agent i with private noisy signal s_i

$$H(s_i, s^\star) \equiv \int_{\theta \geq \theta^\star} \bar{u}(\theta, w(\theta)) dF(\theta|s_i) + \int_{\theta < \theta^\star} \underline{u}(\theta, w(\theta)) dF(\theta|s_i) \tag{32}$$

Agent i will take the threshold strategy $s^\star$ if $H(s^\star, s^\star) = 0$. We want to show that there exists a unique solution $s^\star$ to $H(s^\star, s^\star) = 0$. Given $s^\star$ and the realization of fundamental θ, the total attack is $w(\theta, s^\star) = F(\frac{s^\star - \theta}{\sigma})$. Thus, $\theta \equiv v(w, s^\star) \equiv s^\star - \sigma F^{-1}(w)$. w is uniformly distributed for the threshold player as

$$P(w \leq W) = P(F(\frac{s^\star - \theta}{\sigma}) \leq W) = P(\theta \geq s^\star - \sigma F^{-1}(W)) = W$$

Transforming the integral from θ to w,

$$\begin{aligned} H(s^\star, s^\star) &= \int_{\theta \geq \theta^\star(s^\star)} \bar{u}(\theta, w(\theta)) dF(\theta|s^\star) + \int_{\theta < \theta^\star(s^\star)} \underline{u}(\theta, w(\theta)) dF(\theta|s^\star) & (33) \\ &= \int_0^{w(\theta^\star, s^\star)} \bar{u}(v(w, s^\star), w) dw + \int_{w(\theta^\star, s^\star)}^1 \underline{u}(v(w, s^\star), w) dw & (34) \end{aligned}$$

If there are two different solutions to $H(s^\star, s^\star) = 0$, i.e., $H(s_1^\star, s_1^\star) = H(s_2^\star, s_2^\star) = 0$. W.L.O.G, assume that $s_1^\star > s_2^\star$. For any given w, $v(w, s_1^\star) > v(w, s_2^\star)$, thus

$$\bar{u}(v(w, s_1^\star), w) > \bar{u}(v(w, s_2^\star), w) > 0 > \underline{u}(v(w, s_1^\star), w) > \underline{u}(v(w, s_2^\star), w)$$

We know that in equilibrium, $w(\theta^\star, s^\star) = F(\frac{s^\star - \theta^\star}{\sigma}) = \theta^\star$. $w(\theta_1^\star, s_1^\star) = \theta_1^\star > w(\theta_2^\star, s_2^\star) = \theta_2^\star$ since $s_1^\star > s_2^\star$. Thus,

$$\begin{aligned} H(s_1^\star, s_1^\star) - H(s_2^\star, s_2^\star) &= \int_0^{\theta_2^\star} \left(\bar{u}(v(w, s_1^\star), w) - \bar{u}(v(w, s_2^\star), w)\right) dw \\ &+ \int_{\theta_1^\star}^1 \left(\underline{u}(v(w, s_1^\star), w) - \underline{u}(v(w, s_2^\star), w)\right) dw \\ &+ \int_{\theta_2^\star}^{\theta_1^\star} \left(\bar{u}(v(w, s_1^\star), w) - \underline{u}((v(w, s_2^\star), w))\right) dw > 0 \end{aligned}$$

which contradicts with $H(s_1^\star, s_1^\star) = H(s_2^\star, s_2^\star) = 0$. From the proof, it is easy to see $H(s^\star, s^\star)$ is increasing in $s^\star$ and thus there is a unique threshold equilibrium for this coordination game. We have $\bar{s} = \underline{s} = s^\star$. Not attacking the preferred regime if and only $s \geq s^\star$ is the unique rationalizable strategy based on iterated elimination of dominated strategy and this monotone equilibrium is the unique equilibrium. The general payoff structure nests the case all payoffs are constant only depending on $w \lesseqgtr \theta$. Thus, the proof of Proposition 1 is a special case for this general proof. $\square$

Proof of Proposition 4

If agents gather information ex-ante then $P(\theta_t \geq \theta_t^\star | s, \theta_t \geq 0) = P(\theta_1 \geq h_t(\theta_t^\star) | s, \theta_1 \geq h_t(0)) =$

$\frac{F(\sqrt{\tau}(s-h_t(\theta_t^*)))}{F(\sqrt{\tau}(s-h_t(0)))}$. Notice that for instantaneous information this depends only on θ_t^* but for ex-ante information this depends on $(\theta_1^*, \theta_2^*, \ldots \theta_t^*) \equiv \theta^{t*}$. When agents gather information ex-ante, let $s_t^*(\theta^{t*}, \theta_{t+1}^*)$ be the threshold such that if agents will take not attack the preferred regime iff $s \geq s_t^*(\theta^{t*}, \theta_{t+1}^*)$. $\theta_t \geq \alpha_t w(\theta_t) + \theta_{t+1}^*$ iff $\theta_1 \geq h_t(\theta_t^*)$. Therefore,

$$s_t^*(\theta^{t*}, \theta_{t+1}^*) = h_t(\theta_t^*) + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t})$$

We can define $\Psi(\theta^{t*}, \theta_{t+1}^*) = \psi(s_t^*(\theta^{t*}, \theta_{t+1}^*), \theta^{t*})$ for the ex-ante information case. Notice that in equilibrium $h_t(\theta_t^*) = \theta_1^\star$. Similar to the proof of instantaneous information gathering, a monotone equilibrium $(\theta_t^\star, s_t^\star)_{t=1}^T$ is such that for some $t' = 1, 2, \ldots T+1$, $\theta_{t'}^\star = \theta_{t'+1}^* = \ldots \theta_{T+1}^* = 0$ and for $t = 1, 2, \ldots t'-1$, $\theta_t^\star$ satisfies $\Psi(\theta_t^*, \theta_{t+1}^*) = 0$ and $s_t^*(\theta_t^*, \theta_{t+1}^*) = \theta_1^* + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t})$ (Equation 9).

□

Proof of Lemma 5 We prove this in a bifurcated diffusion with group size α_1 and α_2. This result can be extended straightforwardly to any finite horizon dynamic coordination game.

The last group of agents are playing a static coordination game. As in the proof in Proposition 1, by iterated elimination of never best response, there exist $s_2^{\star h}$ and $s_2^{\star l}$ such that any rational agent in group 2 will not take action 0 whenever $s_2 > s_2^{\star h}$ and will not take action 1 whenever $s_2 < s_2^{\star l}$. Accordingly, we can define the worst possible equilibrium $(\theta_2^{\star h}, s_2^{\star h})$ and best possible equilibrium $(\theta_2^{\star l}, s_2^{\star l})$. Notice that both equilibrium are in monotone strategy and $0 \leq \theta_2^{\star l} \leq \theta_2^{\star h} \leq \alpha_2$.

Suppose equilibrium at period 2 tells that the preferred regime will materialize only if $\theta_2 \geq \bar{\theta}_2^\star$ and will fail only if $\theta_2 < \underline{\theta}_2^\star$. This equilibrium is not necessary in monotone strategy. We know that $0 \leq \theta_2^{\star l} \leq \underline{\theta}_2^\star \leq \bar{\theta}_2^\star \leq \theta_2^{\star h} \leq \alpha_2$. For group 1 agents, the preferred regime will materialize for sure if $\theta_1 \geq \alpha_1 + \bar{\theta}_2^\star$ and the preferred regime will fail for sure if $\theta_1 < \underline{\theta}_2^\star$. We have dominance regions for group 1 agent's action since $0 \leq \underline{\theta}_2^\star \leq \bar{\theta}_2^\star \leq 1 - \alpha_1$ and the prior of θ_1 is $U[\underline{\theta}, \bar{\theta}](\underline{\theta} < -\sigma,\ \bar{\theta} > 1+\sigma)$ is uninformative. The rest of proof follows the static case (Proposition 1) where the upper dominance region starts from $\theta \geq 1$ and lower dominance region $\theta < 0$.

There exit $\bar{s}_1^\star(\bar{\theta}_2^\star)$ and $\underline{s}_1^\star(\underline{\theta}_2^\star)$ such that any rational agent will not take action 0 if $s_1 > \bar{s}_1^\star(\bar{\theta}_2^\star)$ and will not take action 1 if $s_1 < \underline{s}_1^\star(\underline{\theta}_2^\star)$. Given $\bar{\theta}_2^\star$ and $\underline{\theta}_2^\star$, we can define the best possible equilibrium $(\bar{\theta}_1^\star, \bar{s}_1^\star)$ and worst possible equilibrium $(\underline{\theta}_1^\star, \underline{s}_1^\star)$ accordingly. They are both monotone equilibria. We can define $\bar{s}_{1n}(\bar{\theta}_2^\star)$ as the maximum private information enables rational agents to take action 0 after n rounds of iterated elimination. From Equation (31), if $\bar{\theta}_2^{\star'} > \bar{\theta}_2^\star$, we have $\bar{s}_{11}(\bar{\theta}_2^{\star'}) > \bar{s}_{11}(\bar{\theta}_2^\star)$ and $\bar{s}_{1n}$ is a non-decreasing function of $\bar{s}_{1,n-1}$. Thus, $\bar{s}_{1n}(\bar{\theta}_2^{\star'}) \geq \bar{s}_{1n}(\bar{\theta}_2^\star)$ for all n. Hence, $\bar{s}_1^\star(\bar{\theta}_2^\star) = \inf_{n\to\infty} \bar{s}_{1n}(\bar{\theta}_2^\star)$ is non-decreasing in $\bar{\theta}_2^\star$. Similarly, we can prove that $\underline{s}_1^\star(\underline{\theta}_2^\star)$ is a non-decreasing in $\underline{\theta}_2^\star$.

Consequently, $(\theta_1^{\star h}, \theta_2^{\star h})$ is worst possible equilibrium where agents in group 1(2) will favor the preferred regime if and only if $s_{1(2)} \geq \bar{s}_{1(2)}^\star(\theta_2^{\star h})$ and $(\theta_1^{\star l}, \theta_2^{\star l})$ is best possible equilibrium where agents in group 1(2) will favor the preferred regime if and only if $s_{1(2)} \geq \underline{s}_{1(2)}^\star(\theta_2^{\star l})$. $\square$

Proof of Proposition 5 (Goldstein and Pauzner (2005)) The proof of existence and uniqueness is similar to the proof of Proposition 1. We will omit the iterated elimination of never best responses and proceed to prove that if $(\theta^\star, s^\star)$ is a monotone equilibrium, then it is unique.

Given other agents are playing the threshold strategy $s^\star$, bank run will not happen if and only if the total attack is less than the fundamental, i.e., $w(\theta, s^\star) = F(\frac{s^\star-\theta}{\sigma}) \leq \theta$ or equivalently $\theta \geq \theta^\star = F(\frac{s^\star-\theta^\star}{\sigma})$. Define the function $H(s_i, s^\star)$ as the expected net payoff from rolling over ($a_i = 1$) for agent i with private noisy signal s_i

$$H(s_i, s^\star) \equiv \int_{\theta\geq\theta^\star} r dF(\theta|s_i) + \int_{\theta<\theta^\star} -\frac{1+\theta}{1+w} dF(\theta|s_i)$$

Given $s_i = s^\star$, the agent believes w is uniformly distributed. Agent i will take the threshold strategy $s^\star$ if $H(s^\star, s^\star) = 0$. We want to show that there exists a unique solution $s^\star$ to $H(s^\star, s^\star) = 0$. Since $w(\theta, s^\star) = F(\frac{s^\star-\theta}{\sigma})$, we have $\theta = s^\star - \sigma F^{-1}(w)$ and $w^\star = \theta^\star$ satisfies $\theta^\star + \sigma F^{-1}(\theta^\star) = s^\star$.

$$H(s^\star, s^\star) = \int_0^{w^\star} r dw + \int_{w^\star}^1 -\frac{1+s^\star-\sigma F^{-1}(w)}{1+w} dw = rw^\star - \int_{w^\star}^1 \frac{1+s^\star-\sigma F^{-1}(w)}{1+w} dw$$

It is sufficient to show that $H(s^\star, s^\star)$ is increasing in $s^\star$. We want to show that

$$\begin{aligned}
\frac{dH(s^\star, s^\star)}{ds^\star} &= (1+r)\frac{dw^\star}{ds^\star} - \ln\frac{2}{1+w^\star} \\
&= (1+r)\frac{1}{1+\frac{\sigma}{f(F^{-1}(w^\star))}} - \ln\frac{2}{1+w^\star} > 0
\end{aligned}$$

This is always true when $\sigma < (\frac{1+r}{\ln\frac{2}{1+w^\star}} - 1) f(F^{-1}(w^\star))$. Suppose $f(x)$ is bounded away from 0 in $[-\frac{1}{2}, \frac{1}{2}]$, i.e., $\exists \eta > 0$, $f(.) \geq \eta$. Thus, we have $H(s^\star, s^\star)$ is increasing in $s^\star$whenever $\sigma < \sigma^\star = (\frac{1+r}{\ln 2} - 1)\eta$. It is easy to see that when $s^\star = \bar{s} = 1 + \frac{\sigma}{2}$, $w^\star = 1$ and $H(\bar{s}, \bar{s}) = r > 0$ and when

$s^\star = \underline{s} = -\frac{\sigma}{2}$, $w^\star = 0$ and $H(\underline{s}, \underline{s}) < 0$. Thus, when $\sigma < \sigma^\star$, there exists a unique equilibrium where $s^\star \in (\underline{s}, \bar{s})$ and $\theta^\star \in (0, 1)$. $\square$

Proof of lemma 4 At time T, if the bank did not default yet, depositors can make decision for α_T share of their deposits. When the private information about θ_T is s_T, in case agent decide to withdraw $\beta_t \in [0, \alpha_t]$ amount, the value of of α_t share deposit can be defined as following.

$$\begin{aligned} V(\beta_T, s_T, T) &= P(\theta_T \geq \alpha_T w_T | s_T, \theta_T \geq 0)(\beta_T \times 1 + (1 - \beta_T)(1 + r)) \\ &\quad + P(\theta_T < \alpha_T w_T | s_T, \theta_T \geq 0)\beta_T \min\{\frac{\alpha_T + \theta_T}{\alpha_T w_T + \alpha_T}, 1\} \end{aligned} \tag{35}$$

Similarly, given the private information s_t, the value of withdrawing β_t from α_t share of deposit at time $t \in \{1, 2, ..., T-1\}$ is

$$\begin{aligned} V(\beta_t, s_t, t) &= \max_{\beta_t \in [0, \alpha_t]} (1 - \beta_t) P(\theta_t \geq \sum_{u=t}^{T} \alpha_u w_u | s_t, \theta_t \geq 0)(1 + r) + \beta_t P(\theta_t \geq \alpha_t w_t | s_t, \theta_t \geq 0) \\ &\quad + \beta_t P(\theta_t < \alpha_t w_t | s_t, \theta_t \geq 0) \min\{\frac{\sum_{u=t}^{T} \alpha_t + \theta_T}{\alpha_t w_t + \alpha_t}, 1\} \end{aligned} \tag{36}$$

We can write the problem faced by group t agent with private information s_t is as following.

$$\max_{\beta_s \in [0, \alpha_s]} \left\{ \mathbb{E} \left[\sum_{u=t}^{T+1} \left(P_u(s_t) (\sum_{s=t}^{u} V(\beta_s, s_s, s) + o_u(\theta_u, w_u)) \right) | s_t, \theta_t \geq 0 \right] \right\} \tag{37}$$

in which default happens at time t with probability $P_u(s_t) := P\left(\theta_t \in [\sum_{s=t}^{u-1} \alpha_s w_s, \sum_{s=t}^{u} \alpha_s w_s) | s_t, \theta_t \geq 0\right)$ for $u \in \{t+1, t+2, ..., T\}$.[37] The probability of no default is $P_{T+1}(s_t) := P\left(\theta_t \in [\sum_{s=t}^{T} \alpha_s w_s, 1] | s_t, \theta_t \geq 0\right)$. $o_t(\theta_t, w_t)$ is the value of deposits that depositors have no chance yet to make withdrawal decisions if bank defaults at time t. If the bank will distribute the residual liquidity evenly to all depositors, $o_t(\theta_t, w_t) = \max\{\frac{\theta_t + \sum_{u=t+1}^{T} \alpha_u - \alpha_t w_t}{2}, 0\}$. By definition, $o_T(\theta_T, w_T) = o_{T+1}(\theta_{T+1}, w_{T+1}) = 0$.

Notice that $\sum_{u=t}^{T+1} P_u(s_t) = 1$. We can rewrite 37 as

$$\max_{\beta_s \in [0, \alpha_s]} \left\{ \mathbb{E} \left[V_t(\beta_t, s_t, t) + P_t(s_t) o_t(\theta_t, w_t) + \sum_{u=t+1}^{T+1} \left(P_u(s_t) (\sum_{s=t}^{u} V(\beta_s, s_s, s) + o_u(\theta_u, w_u)) \right) | s_t, \theta_t \geq 0 \right] \right\} \tag{38}$$

[37]The probability of having default at time t is $P_t(s_t) := P(\theta_t \in [0, \alpha_t w_t) | s_t, \theta_t \geq 0)$.

Since there are a continuum of patient depositors to make their withdrawal decisions, for any individual, the choice of β_t does not affect any aggregate outcome for time t, or w_t, and thus it has no influence on the dynamics of fundamental or the realization of information. Hence, the optimization problem in Equation (37) is separable. Notice that $P_t(s_t)o_t(\theta_t, w_t)$ is independent of any individual's choice β_t as well. Thus, the time t patient depositors will try to solve $\max_{\beta_t \in [0,\alpha_t]} V_t(\beta_t, s_t, t)$. From 36, $V(\beta_t, s_t, t)$ is a linear function of β_t since the choice of β_t has no influence on the probabilities. Assuming agents withdraw everything when they are indifferent between withdrawing and not withdrawing, β_t is either α_t or 0. $\square$

Non-uniform Prior

We work with uninformative prior and general distribution of error. In applied work, it is conventional to use Gaussian prior and error distribution for convenience. With instantaneous information, relaxing the uninformative prior will introduce endogeneity and thus equilibrium cannot be solved backwards. In the following Proposition, we will show that in a Gaussian information structure with sufficiently uninformative prior, when agents gather information ex-ante, diffusion could completely avoid the coordination risk.

Proposition 10 *Suppose $\theta \sim N(\theta_0, {\sigma_0}^2)$ and $s|\theta \sim N(\theta, \sigma^2)$, Theorem 2 holds true when σ_0 is sufficiently high and some regularity conditions are satisfied.*

Proof. Given prior $\theta \sim N(\theta_0, \sigma_0^2)$ and $s \sim N(\theta, \sigma^2)$, $\theta|s \sim N(\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}s + \frac{\sigma^2}{\sigma^2+\sigma_0^2}\theta_0, \frac{\sigma_0^2\sigma^2}{\sigma^2+\sigma_0^2})$. The monotone equilibrium is defined as in Proposition 4. Similar to Equation (9), in the last period, we have

$$\frac{\Phi\left(\frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(\frac{\theta_T^\star}{\alpha_T})+\frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0-h_T(\theta_T^\star))}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}\right)}{\Phi\left(\frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(\frac{\theta_T^\star}{\alpha_T})+\frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0-h_T(\theta_T^\star))+(h_T(\theta_T^\star)-h_T(0))}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}\right)} = p_0,\ \theta_T^\star \in (0, \alpha_T) \tag{39}$$

or $\theta^\star_T = 0$. Because of the log-concavity of Φ[38] and Lemma 3, i.e., $h_T(\theta^\star_T) - h_T(0) \le \theta^\star_T$, the LHS of Equation (39) is greater than $L(\alpha_T, \frac{\theta^\star_T}{\alpha_T}, h_T(0))$, where L is defined as

$$L(\alpha, x, h) := \frac{\Phi\Big(\frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(x) + \frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0 - \alpha x - h(0))}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}\Big)}{\Phi\Big(\frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(x) + \frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0 - \alpha x - h(0)) + \alpha x)}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}\Big)}$$

Notice that $L(\alpha_T, \frac{\theta^\star_T}{\alpha_T} = 0, h_T(0)) = L(\alpha_T, \frac{\theta^\star_T}{\alpha_T} = 1, h_T(0)) = 1 > p_0$ and $L(\alpha_T = 0, \frac{\theta^\star_T}{\alpha_T}, h_T(0)) = 1$. To prove that $\theta^\star_T = 0$ is the unique equilibrium, we only need to show that $L(\alpha, x, h)$ is decreasing in α. Assume that $\frac{dh_T(0)}{d\alpha_T}$ is well defined and bounded.

$$\frac{dL(\alpha, x, h)}{d\alpha} = \frac{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}{\Phi^2(A)}\left[\phi(B)\Phi(A)k(-x - \frac{dh}{d\alpha}) - \phi(A)\Phi(B)(x - kx - k\frac{dh}{d\alpha})\right]$$

in which $A = \frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(x) + \frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0 - \alpha x - h(0)) + \alpha x)}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}$, $B = \frac{\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}\sigma\Phi^{-1}(x) + \frac{\sigma^2}{\sigma^2+\sigma_0^2}(\theta_0 - \alpha x - h(0))}{\frac{\sigma\sigma_0}{\sqrt{\sigma^2+\sigma_0^2}}}$ and $k = \frac{\sigma^2}{\sigma^2+\sigma_0^2}$. $\frac{dL(\alpha,x,h)}{d\alpha} < 0$ is equivalent to have the following inequality for all $x \in (0, 1)$.

$$\frac{dh_T(0)}{d\alpha} > \left(\frac{1}{k}\frac{1}{\left[1 - \frac{\phi(B)}{\Phi(B)}/\frac{\phi(A)}{\Phi(A)})\right]} - 1\right)x$$

Notice that because of log-concavity, $\frac{1}{\left[1 - \frac{\phi(B)}{\Phi(B)}/\frac{\phi(A)}{\Phi(A)})\right]} < 0$. When $\sigma_0 \to \infty$, $k \to 0$ and $\lim_{\sigma_0\to\infty} A = \lim_{\sigma_0\to\infty} B$. Thus for any bounded $\frac{dh_T(0)}{d\alpha}$ the above inequality is true. If $\frac{dh_t(0)}{d\alpha_t}$ is well defined bounded for all t, then Theorem 2 holds true when σ_0 is sufficiently large. ■

[38]Given F is log-concave and $y \le z$, $\frac{F(x+y)}{F(x+z)}$ is increasing in x since $\frac{d\frac{F(x+y)}{F(x+z)}}{dx} = \frac{F(x+y)}{F(x+z)}(\frac{f(x+y)}{F(x+y)} - \frac{f(x+z)}{F(x+z)}) > 0$.