Dynamic Q-Theory with Agency Investment Frictions and Cross-sectional Stock Returns*

Lei Mao
Finance Group, Warwick Business School
University of Warwick
Tel: (44)-2476-524-966
Email: Lei.Mao@wbs.ac.uk

Mike Qinghao Mao
Finance Group, Department of Business Economics
Erasmus University Rotterdam
Tel: (31)-104-081-322
Email: mao@ese.eur.nl

K.C. John Wei
Department of Finance
Hong Kong University of Science and Technology
Tel: (852) 358-7676; Fax: (852) 2358-1749
Email: johnwei@ust.hk

Abstract

We investigate the impact of managerial investment diversion on a firm's investment paths and the investment-return relation in a dynamic q-theory model. When efficiency of investment is not observed by shareholders, the manager may divert investment for private benefits. An agency investment friction emerges from the cost associated with high-powered managerial compensations to prevent the investment diversion. The state-dependency of agency investment frictions predicts cross-sectional variations in the relation between investment and subsequent stock returns. Our empirical results are consistent with the model predictions and suggest that managerial agency costs influence investment levels and stock returns across U.S. firms.

JEL Classification: G12; G14; G34; D82
Keywords: Dynamic agency, Q-theory of investment, Cross-section of stock returns

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Corresponding author: K.C. John Wei, Department of Finance, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Tel: (852)-2358-7676; Fax: (852)-2358-1749. E-mail: johnwei@ust.hk.

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Abstract

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1. Introduction

The recent empirical asset pricing literature has highlighted the role of investment in explaining the cross section of stock returns (e.g., Titman, Wei, and Xie (2004); Cooper, Gulen, and Schill (2008); Fama and French (2015); Hou, Xue, and Zhang (2015)).[1] The negative relation between investment and subsequent stock returns, the so-called "investment effect" or "investment anomaly," is a central implication of the production-based asset pricing models (e.g., Cochrane, 1991, 1996; Liu, Whited, and Zhang, 2009).[2] In these models, a firm optimally adjusts investments according to changes in the cost of capital or expected returns. A general underlying assumption in these models is that there are no agency conflicts that might distort the investment policy. But a growing literature of dynamic corporate finance shows quantitatively that agency conflicts between a firm's manager and its shareholders shape the firm's policies.[3] In this paper, we study the implications of agency costs in a dynamic q-theory model of investment. We are interested in how agency costs affect the investment paths of the firm and how the impacts are reflected in the cross–sectional patterns of stock returns.

In particular, we consider an informational friction in which a shock in the capital stock accumulation process is only observed by the manager but not shareholders. We refer to the shock as the uncertainty in investment efficiency. This informational friction incentivizes the manager to divert investment for private benefits and motivates shareholders to deter such behavior by offering the manager an incentive-compatible contract that elicits truthful

[1] This literature finds that in general firms that make more investment or expand their assets more earn lower subsequent returns.
[2] Cochrane (1991) links investment returns to asset returns. Cochrane (1996) uses investment returns to estimate a stochastic discount factor. Liu, Whited, and Zhang (2009) directly estimate a q-theory model from the investment-expected-return relation. Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004) attribute the negative relation between investment and returns to the exercise of real options. Kogan and Papanikolaou (2013) argue that firms with a higher loading on the investment-specific shock display both high investment levels and low returns.
[3] For example, Morellec, Nikolov, and Schuerhoff (2012) show that managerial agency costs explain leverage decisions, and Nikolov and Whited (2013) infer managerial agency costs from the cash policy of the firm.

investment behavior. Therefore, both investment and compensation are jointly determined in the model. The framework allows us to explore the *limit* of an optimally designed compensation in alleviating the investment inefficiency. This limit, shaped by some firm characteristics, renders cross-sectional implications for a firm's investment level and the investment-return relation.

We follow the dynamic agency literature in formulating the optimal contract problem (DeMarzo and Sannikov (2006); He (2009); DeMarzo, Fishman, He, and Wang. (2012)). For example, in the model of DeMarzo et al. (2012), investors cannot observe the shocks to productivity and thus the manager can divert the firm's *output* for private benefits. Offering an incentive-compatible contract to the manager can prevent this behavior. But the agency problem cannot be fully resolved since the manager might terminate the contract, which would trigger a deadweight cost. As a result, the marginal q of the firm is reduced by the possibility that the manager will depart at some point in the future.

By modeling agency costs in the form of *investment* diversion rather than *output* diversion as in the model of DeMarzo et al. (2012), our model delivers very different implications. More specifically, in our model, the agency cost of investment diversion can be represented by an augmented (increased) capital adjustment cost function. We refer to this agency cost as the agency investment friction. In capital budgeting decisions, the shareholders anticipate that an increase in the investment level would require a higher-powered compensation contract to elicit truthful action from the manager. However, a higher-powered contract has greater volatility, which increases the probability of inefficient contract termination, since the manager can opt for outside options when the compensation falls below some threshold. Therefore, under the incentive-compatible contract for truthful actions, managerial investment diversion implies a necessarily higher cost per unit of new investment, due to the expected deadweight cost when the contract is terminated. Controlling for marginal q, a firm plagued by investment diversion would invest less than a first-best firm.[4]

The magnitude of the agency investment friction is both firm-specific and time-varying. For firms with higher uncertainty in investment efficiency, the agency investment friction increases with the volatility of the incentive-compatible contract. Moreover, incentive compatibility requires the compensation to be dynamically linked to the performance of the firm. When the compensation to the manager is substantially reduced by a series of negative shocks in past performance, the per unit increase in compensation volatility is more costly, as is new investment. Therefore, within the same firm, the agency investment friction is state-dependent.

The model generates testable predictions: a strong investment friction lowers the optimal investment level; it also leads to optimal investment being more inelastic to the risk premium and therefore a stronger negative relation between investment and subsequent stock returns. Since the agency investment friction varies across firms and states, holding the firm's systematic risk and risk premium constant, the model predicts a state-dependent relation between investment and subsequent returns.[5]

Using data from firms listed in the US during 1963-2014, we show strong evidence supporting our model predictions. Empirically, we measure uncertainty in investment efficiency by the idiosyncratic volatility of the firm's stock returns and measure past performance by past stock returns.[6] We find that higher idiosyncratic volatility and lower past stock returns are

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[4] In DeMarzo et al. (2012), agency costs do not influence investment. Controlling for marginal q, firms with or without agency costs would make the same level of investment.

[5] Li and Zhang (2010) show that the magnitude of the investment-return relation increases with investment frictions in a simple one-period model.

[6] These measures are consistent with our model. Strictly speaking, the idiosyncratic volatility of stock returns consists of the idiosyncratic volatility of cash flows and that of investment efficiencies. But cross-sectionally, a firm with higher idiosyncratic volatility of stock returns tends to have noisier investment processes. Our interpretation of the idiosyncratic volatility of stock returns as informational uncertainty in investment technology is similar to that in

associated with a lower level of investment and a stronger investment effect. The effects of idiosyncratic volatility and past stock returns on the investment-return relation are strong across firms when investment is measured by investment-to-asset ratios (Lyandres, Sun, and Zhang (2008)), asset growth (Cooper, Gulen, and Schill (2008); Titman, Wei, and Xie (2013); Watanabe, Xu, Yao, and Yu (2013)), and a simple measure of capital expenditures divided by capital stock (Liu, Whited, and Zhang (2009)), but weaker when investment is measured by investment growth (Xing (2008); Mao and Wei (2015)).

Overall, we make several important contributions to the literature. We contribute to the dynamic corporate finance literature by showing the impact of managerial agency costs on investment. Our model generates implications which are directly testable in an empirical setting. Existing models in the literature face empirical difficulties, for example, due to the measurement errors in observed q and unobserved (and heterogeneous) risk premiums.[7] In a related paper, Nikolov and Schmid (2012) directly estimate the model of DeMarzo et al. (2012) by simulated method of moments. We make an extension of the literature by linking agency costs to the capital market implications of firm investment policy. Such extension is essential to empirically infer the agency costs across firms. By focusing on the investment-return relation, we are able to control for the cross-sectional risk premiums in our empirical tests.[8]

The state-dependency of agency investment frictions in our model enriches the empirical predictions on the investment-return relation, compared with the simple one-period q-theory

Kogan and Papanikolaou (2013). Similarly, we use stock returns as a proxy for the performance of investment technology.

[7] For example, agency costs lead to a reduced marginal q in both our model and that of DeMarzo et al. (2012). But empirically, it is very difficult to identify a reduction in the marginal q, not only because q is measured with noise, but also because we generally do not have a first-best q against which to compare the reduced q. The first-best q is influenced by cross-sectional risk premiums, since a higher risk premium implies that the future cash flow of the firm would be discounted more heavily.

[8] We show that, in the model of DeMarzo et al. (2012), the effect of agency costs cannot be separated from the effect of (unobserved) cross-sectional risk premiums, if the latter is incorporated. Our model specification allows us to infer the agency costs from the investment-return relations in the cross section of firms.

model with investment frictions as the one in Li and Zhang (2010). In addition, we offer a rational explanation for the positive association between idiosyncratic volatility and the investment effect, which is often used as evidence to support a behavioral explanation. For example, Li and Zhang (2010) and Lam and Wei (2011) argue that high idiosyncratic volatility makes it more difficult for arbitrageurs to eliminate the mispricing due to investors' under-reaction to the manager's overinvestment which destroys firm value. By linking idiosyncratic volatility with the agency investment friction, our model shows that such cross-sectional investment effect can be rationalized in a dynamic q-theory model. Furthermore, we document new evidence that the negative investment-return relation is stronger among firms with poorer past stock performance, and provide a rational explanation based on the q-theory of investment with agency costs.[9]

In two recent studies, Panousi and Papanikolaou (2012) and Glover and Levine (2015) attribute the negative relation between investment and idiosyncratic risk to the risk aversion of the under-diversified managers. We show that firm-specific uncertainty can also influence investment through the agency channel. But the interpretation of firm-specific uncertainty is different; we interpret it as a proxy for the information gap between the manager and investors. In line with the data, the model also implies a much higher investment volatility than a model with fixed capital adjustment costs through the counter-cyclical agency investment friction.[10]

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[9] Stambaugh, Yu, and Yuan (2012) show that the cross-sectional return anomalies are stronger in the short leg, following *high* levels of investor sentiment. They interpret the evidence as support for the mispricing explanation of the anomalies.

[10] Barlevy (2004) and Lansing (2011) show that in rational q-theory models with fixed capital adjustment costs, investment growth exhibits the same volatility as output growth. But in the data, the former is much more volatile than the latter. Hirshleifer, Li, and Yu (2015) generate an investment process with excess volatility by introducing extrapolative bias.

The remainder of the paper is organized as follows. Section 2 sets up the model and solves it. Section 3 derives asset pricing implications from the model. Section 4 develops empirical hypotheses and presents the results. Section 5 concludes the paper.

2. The Model

2.1 Model set-up

We set up a neoclassic investment model which incorporates managerial agency costs (e.g., DeMarzo et al. (2012)). There are two types of external shocks: a productivity shock which affects the output and an investment efficiency shock which affects the capital accumulation process. We assume that the investment efficiency shock is only observed by the manager (insider) but not investors. This creates an incentive for the manager to divert investment for private benefits, which reduces the capital stock of the firm. Expecting this behavior from the manager, investors solve the optimal contracting problem and then determine the optimal investment process.

2.1.1 Capital accumulation and managerial investment diversion

We assume that the evolution of capital stock K_t follows a stochastic process as follows:

$$dK_t = \left(I_t - \delta K_t\right)dt + \varepsilon I_t dB_{I,t} - a_{I,t} I_t dt \tag{1}$$

where $a_{I,t}$ is the manager's actions, I_t is the investment rate, $\delta > 0$ is the depreciation rate, and $\varepsilon > 0$ is the volatility of the shock to investment efficiency. $B_{I,t}$ is a Brownian motion that generates shocks to investment efficiency.

The total cost $C(I_t, K_t)$ of making investment I_t is defined as follows:

$$C(I_t, K_t) = c(i_t)K_t = \left(i_t + \frac{1}{2}\phi i_t^2\right)K_t \tag{2}$$

where $i_t \equiv I_t / K_t$ is the investment ratio, and $\phi i^2 / 2$ with a constant ϕ is a quadratic capital adjustment cost function that is usually assumed in the literature.[11]

We assume that the manager can choose her action $a_{I,t} \in \{0,1\}$.[12] When $a_{I,t} = 0$, the manager puts all of the investment fund into the capital stock. When $a_{I,t} = 1$, the manager diverts all investment into her own account and enjoys an instantaneous benefit $\lambda I_t dt$, where λ measures the efficiency of diversion. We assume that $0 < \lambda < 1$, which indicates that diversion involves a deadweight cost and is inefficient. For simplicity, it is assumed that the diverted investment cannot be saved or invested and thus must be consumed by the manager immediately.

To model agency costs, we assume that the evolution of $B_{I,t}$ is only observed by the manager but not investors, who are unable to distinguish the manager's diversion behavior from a shock to investment efficiency. As a result, they would rely on an incentive contract to induce the truthful behavior from the manager, which will be described in Section 2.2. Moreover, we assume that $B_{I,t}$ is idiosyncratic. That is, the shocks generated by $B_{I,t}$ are not correlated with the pricing kernel ξ_t (which will be specified later), and thus the risk of investment efficiency is not reflected directly in the returns required by investors.

Uncertainty in investment efficiency can be interpreted as uncertainty in the efficiency of transferring external resources to productive capital. It has been studied in the recent literature of technological shocks and asset prices (e.g., Kogan and Papanikolaou (2013, 2014); Li (2014)).

[11] For capital stock K_t to be positive at any time in equation (1), the investment ratio $i_t = I_t / K_t$ must be bounded by some constant a.e., of which we will verify later in the derivation of the optimal investment policy. We also restrict the parameter values so that the investment ratio would be non-negative.

[12] This is without loss of generality in comparison to assuming that $a_{I,t}$ is a non-negative value. The reason is that we are focusing on the equilibrium path along which the managerial compensation ensures $a_{I,t} = 0$ a.e..

But in this literature, the shocks to investment efficiency are systematic and thus are priced in expected returns. In another model in Albuquerque and Wang (2008), the shocks to investment efficiency are also systematic so that the resulting agency costs are priced in returns directly. However, in our model, the investment efficiency shocks are purely idiosyncratic so that the associated agency costs are not directly priced in returns. We aim to study whether idiosyncratic shocks in investment efficiency associated with agency costs can have an impact on optimal investment and firm value, via a dynamic contracting framework. He (2009) also presents a dynamic agency problem in which the manager is able to control the asset growth of a firm, which is otherwise exogenously determined by a geometric Brownian motion. Our model differs in that the optimal investment is determined endogenously, from shareholders' capital budgeting decisions.

2.1.2 Production technology

The firm uses the capital stock to generate output. The output Y_t evolves as

$$dY_t = K_t dA_t, \tag{3}$$

where A_t is the production technology. We assume that A_t evolves as

$$dA_t = \mu dt + \sigma dB_{A,t}, \tag{4}$$

where μ is the expected output per unit of capital, $B_{A,t}$ is a Brownian motion, and σ is the exposure of the technology to this shock. We assume that the shock to production, $B_{A,t}$, is systematic. That is, it correlates with the pricing kernel ξ_t, and generates the excess return of the firm's stock.[13] The pricing kernel ξ_t follows

$$d\ln\xi_t = -rdt - \eta dB_{A,t} - \frac{1}{2}\eta^2 dt, \tag{5}$$

where η is the risk premium for one unit risk in $B_{A,t}$. As a result, the risk-free rate in the economy is r.

We also assume that $B_{A,t}$ and $B_{I,t}$ are not correlated, and investors cannot infer one from the other. The evolution of A_t is fully observable by both the manager and investors and contractible. We therefore eliminate the information friction on productivity uncertainty and possible agency costs on the production side. By assuming that A_t is fully contractible, we deviate from DeMarzo et al. (2012) who generate agency costs in a q-theory model based on information frictions on $B_{A,t}$. We, instead, focus on an information friction on $B_{I,t}$ described in Section 2.1.1, in order to investigate the implications of managerial agency costs originating from the investment process.[14]

2.1.3 First-best benchmark

We now provide a solution for the Tobin's *q* for a benchmark firm where the manager and shareholders are perfectly aligned in their interests, which we denote by q^{FB} (the first-best *q*).

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[13] We assume the shock to production $B_{A,t}$ to be purely systematic to affect cross-sectional stock returns. An idiosyncratic component could be added to A_t, but it will not affect our results. We do not hold a view on whether or not an observable $B_{A,t}$ results from its being systematic.

[14] In reality, the manager's discretion over investment decisions might be larger than earnings management, with the latter being restricted by accounting rules. The agency costs associated with investment might therefore be more significant than those associated with output misreporting. We explain the identification strategy in Section 3 and show that our specification is a better option for empirical tests on agency costs than the original specification by DeMarzo et al. (2012).

In this first-best case, the manager does not divert investment, since the investment diversion is not efficient ($\lambda < 1$). The objective of the manager is to maximize the present value of cash flows:

$$P^{FB}(K_0) = \max_{I} \mathrm{E}\left\{\int_0^{\infty} \xi_t \left[K_t dA_t - C(I_t, K_t)dt\right]\right\}. \tag{6}$$

In the Appendix A1 we show that the solution to (6) is

$$P^{FB} = q^{FB} K_0, \tag{7}$$

with the optimal investment rate i determined by

$$1 + \phi i = q^{FB}. \tag{8}$$

q^{FB} is the solution to the following equation:

$$\left(r+\delta\right) q^{FB} = \mu - \sigma\eta + \frac{\left(q^{FB}-1\right)^2}{2\phi}, \tag{9}$$

which is the same as the standard Tobin's q in the literature (e.g., Hayashi (1982)). The marginal q and the average q are the same in the first-best benchmark. Due to the investment efficiency shock, the capital stock process in our model is stochastic, but the idiosyncratic investment efficiency shock does not influence the firm value or the optimal investment.

In the next sections, we derive the optimal contract and investment paths when the interests of the manager and shareholders are not aligned. Different from the first best case, the manager may divert investment which is not perfectly observed by shareholders.

2.2 The optimal contract

We assume that the manager has no initial wealth and the value of her reservation value is zero. She is essential for operation, and if she leaves the firm, the capital of the firm, K_t, is sold for lK_t. We assume $1 \le l < q^{FB}$, so that liquidation is strictly inefficient.[15] We assume that outside investors can commit to a compensation contract, $\Phi = (I, U, \tau)$, which specifies the

[15] Since the liquidation value of the firm $lK_t < P^{FB} = q^{FB}K_t$, liquidation means a deadweight cost of $P^{FB} - lK_t$ and is thus inefficient. On the other hand, $l \ge 1$ ensures that investment rate is non-negative a.e., which is required by the model.

investment rule I_t, the manager's cumulative monetary compensation U_t, and a stochastic termination time τ. U_t is a weakly increasing process due to limited liability of the manager.

Since the manager cannot divert investment efficiently, i.e., $\lambda < 1$, it is desirable for investors to transfer the monetary payoff to the manager through a contract that prevents diversion in the first place. Therefore, we focus on the incentive-compatible contract under which the manager has no incentive to divert investment ($a_t = 0$ for $0 \le t < \tau$) following the dynamic agency literature. Under such a contract, the expected payoff to the manager is derived solely from the monetary payment U_t:

$$W(\Phi) = \mathrm{E}\left[\int_0^\tau e^{-(\kappa-r)t}\xi_t dU_t\right], \tag{10}$$

subject to Φ being incentive compatible and $W(\Phi) \ge 0$ (individual rationality of the manager). The $\kappa - r \ge 0$ measures the relative impatience of the manager and shareholders.[16] The optimal incentive-compatible contract Φ^* is determined in the following maximization problem of shareholders:

$$P(K_0, W_0) = \max_{\Phi, i} \mathrm{E}\left\{\int_0^\tau \xi_t \left[K_t dA_t - c(i_t)K_t dt\right] + l\xi_\tau K_\tau - \int_0^\tau \xi_t dU_t\right\}. \tag{11}$$

We will now characterize the optimal contract. For any incentive-compatible contract at time t ($0 \le t < \tau$), the manager's expected payoff from staying with the firm adjusted for her impatience relative to that of shareholders under the contract Φ is

$$W_t(\Phi) = \mathrm{E}_t\left[\int_t^\tau e^{-(\kappa-r)(s-t)}\xi_s dU_s\right]. \tag{12}$$

We now show in the following lemma that the optimal contract can be defined by the joint evolution of W_t and U_t.

[16] In the dynamic agency models (e.g., DeMarzo and Sannikov (2004); DeMarzo et al. (2012)), $\kappa > r$ is a technical requirement that ensures the firm pays out to the manager. As in He (2009), we require $\kappa \ge r$.

Lemma 1. *Under the optimal incentive-compatible contract* Φ^*, *the differential form of the manager's compensation follows*

$$dW_t = \kappa W_t dt + \lambda \left[dK_t - \left(I_t - \delta K_t \right) dt \right] - dU_t, \tag{13}$$

for $t \in [0, \tau)$, *where* $\tau = \inf\{t \geq 0 : W_t = 0\}$. dU_t *reflects* W_t *at* $\bar{W}$, *where* $\bar{W}$ *is defined as* $P_W(\bar{W}) = -1$.

Proof. All proofs of lemmas and propositions are given in the Appendix.

Lemma 1 states that the optimal contract accounts for the time preference of the manager ($\kappa W_t dt$) and rewards her for truthful investment ($\lambda I_t \varepsilon dB_{I,t}$). For every unit of shock in $\varepsilon B_{I,t}$, the optimal contract provides a monetary payoff equal to λI_t, which is no less than the private benefit derived from diverting investment I_t and ensures incentive compatibility. Lemma 1 also states that the optimal contract does not load on the temporary shock $B_{A,t}$ in the output Y_t, but it loads on the shock $B_{I,t}$ which will have a long-term impact as it is accumulated in the capital stock. Intuitively, the shock of $B_{A,t}$ is multiplied by K_t to produce the temporary shocks in the output Y_t, which is too noisy at the level of K_t (which is the aggregated level of investment flow I_t) to provide useful inference on each I_t at every split second.[17]

As in DeMarzo and Sannikov (2006), for $W \leq \bar{W}$, investors are better off delaying payment to the manager by setting $dU_t = 0$. This is because when the manager leaves the firm upon W_t hitting zero, inefficient liquidation takes place. The accumulated managerial payoff kept in the firm reduces this probability. However, keeping W_t at a high level has its cost, since the manager is impatient compared to investors and retaining payment inside the firm implies a

[17] Sannikov and Skrzypacz (2007) show a similar effect and provide a detailed proof.

cost to the firm $(\kappa - r)W_t$. Thus there must be an upper boundary $\bar{W}$ beyond which the firm starts to pay out.

2.3 Optimal investment paths

Once the optimal contract is determined, we can determine the optimal investment process. Since the system is in the scale of K_t, we can reduce the number of state variables by setting a new state variable $w_t = W_t / K_t$. Denote the investors' value function by $P(K_t, W_t) = p(w_t)K_t$, where $P(K_t, W_t)$ is defined in (11). Analogous to $\bar{W}$ in Lemma 1, $\bar{w}$ is a reflective boundary for w_t and is defined by

$$p'(\bar{w}) = -1, \tag{14}$$

which indicates that at $\bar{w}$, outside investors are indifferent between starting to pay the manager and keeping the payoff in the firm, since the value implication of $p(w)$ is the same.

For $t < \tau$ and $dU_t = 0$, the process for $w_t \leq \bar{w}$ is represented by the following equation:

$$dw_t = (\kappa - i_t + \delta)w_t dt + i_t(\lambda - w_t)\varepsilon dB_{I,t}. \tag{15}$$

When $w_t > \bar{w}$, investors pay the manager an instantaneous transfer equal to $dU_t = (w_t - \bar{w})K_t$ which sets w_t back to $\bar{w}$, and $\bar{w}$ is a reflecting barrier for the process w_t.[18] The investors' value function in the form of the discounted cash flow is

$$p(w_t)K_t = P\left(K_t, W_t\right) = \mathrm{E}_t\left\{\int_t^\tau \xi_s\left[K_s dA_s - c(i_s)K_s ds - dU_s\right] + l\xi_\tau K_\tau\right\}, \tag{16}$$

[18] The traded security is the total value of the firm, which is $P(K,W) + W = (p(w) + w)K$. At $\bar{w}$, $(p(w) + w)' = 0$ satisfies the no-arbitrage condition at the reflecting barrier.

and the Hamilton-Jacobi-Bellman (HJB) equation of $p(w)$ for $0 < w_t < \overline{w}$ is as follows (by dropping the subscript t):[19]

$$rp(w) = \sup_i \left[\mu - \sigma\eta - c(i) + p(w)(i-\delta) + p'(w)w(\kappa - i + \delta) + \frac{1}{2} p''(w) i^2 (\lambda - w)^2 \varepsilon^2 \right]. \quad (17)$$

The optimal investment i is thus determined as follows:

$$i^* = \frac{p(w) - wp'(w) - 1}{\phi - p''(w)(\lambda - w)^2 \varepsilon^2}. \quad (18)$$

The marginal q of the firm is

$$q^A(w) = \frac{\partial(P(K,W)+W)}{\partial K} = p(w) - wp'(w). \quad (19)$$

Along the optimal investment path, the model firm with agency costs has a lower marginal q than the first-best marginal q in a firm without agency costs, q^{FB}. Even at $\overline{w}$, where it is maximized and the firm starts to pay the manager, $\partial(P(K,W)+W)/\partial K$ is still less than q^{FB} due to the positive probability that w may reduce to zero and the manager leaves the firm.

The value function $p(w)$ is concave. As w increases, the risk of the manager's leaving the firm decreases, which, in turn, increases outside investors' value. But the value loss due to the higher effective discount rate of the manager is also larger. This result is formally stated in the following lemma:

Lemma 2. $p''(w) < 0$ *for* $w \in (0, \overline{w})$.

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[19] An important adjustment of the process w_t to adapt it into (17) is to eliminate the term $-\varepsilon^2 i^2(\lambda - w_t)$ in the drift from its physical path. This is because the process of w_t in (17) is under a measure which is risk-neutral to $1/K$ (and as a result, under this measure, K can be divided from both sides of the HJB equation). Hence it differs from the physical measure of w_t by that drift. See footnote 14 in He (2009).

Equations (18) and (19) give a simple representation of the model. That is, we are able to summarize the agency costs of investment diversion into a modified marginal q as well as a modified capital adjustment cost function, which is stated in the following proposition:

Proposition 1. *Let* $p(.)$ *be defined as in* (17) *and* w *be the scaled value of the manager's continuing in the firm with its evolution as in* (15). *Define two state variables* $\phi^A(w)$ *and* $q^A(w)$ *as follows:*

$$\phi^A(w) = \phi - p''(w)(\lambda - w)^2 \varepsilon^2, \tag{20}$$

$$q^A(w) = p(w) - wp'(w). \tag{21}$$

Then the agency investment friction $\phi^A(w)$ *and marginal q* $q^A(w)$ *summarize the effects of agency costs on the firm investment decision in the model. Given the corresponding capital adjustment cost function*

$$c^A(i;w) = i + \frac{1}{2}\phi^A i^2, \tag{22}$$

the optimal investment level i^* *is determined by equating the marginal cost of capital to the marginal q:*

$$\frac{\partial c^A(i;w)}{\partial i} = 1 + \phi^A i = q^A. \tag{23}$$

Proposition 1 states that the agency problem in the model is reflected in both the agency investment friction $\phi^A(w)$ and the (reduced) marginal q $q^A(w)$. It implies that, by adjusting both the marginal q and the capital adjustment cost function, we are able to adapt a general agency problem into a simple q-theory of investment.

The term $-p''(w)(\lambda - w)^2 \varepsilon^2$ in equation (20) represents an adjustment due to the agency costs.[20] Since $p''(w) < 0$, this adjustment term is positive. Hence, the manager's diversion of investment increases the adjustment cost of capital. The intuition is that, an additional unit of investment increases the variance of the manager's compensation w by exactly $(\lambda - w)^2 \varepsilon^2$, which effectively prevents the manager from diverting investment in the first place. However, using this high-powered compensation increases the volatility of w and makes it more likely for w to hit the zero boundary which would trigger the inefficient liquidation of the firm. The second-order derivative of the value function, $-p''(w)$, measures the concavity of the value function for investors and how much the value is reduced by the variance of w. Therefore, the product $-p''(w)(\lambda - w)^2 \varepsilon^2$ is the additional cost of each unit of investment given the agency costs as is indicated by the shape of $p(.)$ and the current state variable w.

The implication of Proposition 1 becomes clearer if we take the limiting case where $\kappa = r$ which means that the discounting of investors is close to the discounting of the manager.[21] When $\kappa \to r$, in the limit, the HJB equation (17) can be simplified to

$$rq^A(w) = \mu - \sigma\eta - c^A(i;w) + q^A(w)(i - \delta). \tag{24}$$

This equation has the same form as the investment equation for the first-best benchmark without agency costs, that is,

$$rq^{FB} = \mu - \sigma\eta - c(i) + q^{FB}(i - \delta). \tag{25}$$

Given state w, the firm with managerial investment diversion would invest like a first-best firm as if it had a state-dependent investment friction $\phi^A(w)$ and a marginal q $q^A(w)$. In other words,

[20] In Appendix A7, we argue that $w > \lambda$ is not possible in our model.
[21] $\kappa = r$ is a special case of the model, where all the results of the model will be retained, except that $\overline{w} = \lambda$ will be an absorbing barrier for w. See Section 2.3.2 in He (2009).

the effect of agency costs is reflected in the two functions $\phi^A(w)$ and $q^A(w)$, which vary according to the state variable w.

3. Asset Pricing Implications

In this section we discuss the asset pricing implications of the model. In our discussion, we restrict ourselves to the limiting case where $\kappa \to r$, since the comparison with the first-best case would be much simpler.

3.1 Cross-sectional stock returns

As we have incorporated the pricing kernel in the model, we are able to derive asset pricing implications in a q-theory model. As in DeMarzo et al. (2012), the traded security is the firm value, which is a combination of investors' value $P(K,W)$ and the manager's deferred payment W.

Following Kogan and Papanikolaou (2014), the excess return of holding this security will be determined by the covariance of the change in firm value with the pricing kernel:[22]

$$\mathrm{E}\left[R\right]-r=-\mathrm{cov}\left[\frac{dY-c(i)Kdt+dP+dW}{P+W},\frac{d\xi}{\xi}\right]/\,dt=\frac{\sigma\eta}{p+w}. \tag{26}$$

The total systematic risk of the firm is σK, since it produces a cash flow KdA which has exposure σ to the pricing kernel. Since the price per unit of risk in the pricing kernel is η, the total compensation for risk for the whole firm $(P+W)$ is $\sigma\eta K$ and the per unit risk premium is $\sigma\eta/(p+w)$. The average market-to-book ratio, $(p+w)$, is negatively associated with $\sigma\eta$,

[22] We loosely call $\sigma\eta$ the risk premium (which is actually the risk premium per unit of capital stock) and $\sigma\eta\,/\,(p+w)$ the excess return of the firm (which is the risk premium per unit of firm value).

suggesting a positive value premium (e.g., Fama and French (1992)).[23] This is due to the fact that a higher risk premium $\sigma\eta$ leads to a lower market value per unit of capital K, which is documented by Berk (1995).

More importantly, a higher risk premium $\sigma\eta$ leads to a lower marginal q and thus lower investment levels, which is the key observation in the investment-based asset pricing literature (e.g., Liu, Whited, and Zhang (2009)) and is obvious in the limiting case where $\kappa \to r$. For any w, we have

$$q^A = 1 + \phi^A i^*, \tag{27}$$

where i^* is the optimal investment rate under agency costs solved from (24):[24]

$$i^* = r + \delta - \sqrt{\left(r+\delta\right)^2 - \frac{2\left(\mu - \sigma\eta - r - \delta\right)}{\phi^A}}. \tag{28}$$

Therefore, the optimal investment i^* is monotonically decreasing in $\sigma\eta$.

Furthermore, our model can generate the cross-sectional return-investment relation as in Li and Zhang (2010), based on the cross-sectional variation in agency investment friction. More specifically, when $\kappa \to r$, we show in the Appendix A5 that, analogous to Li and Zhang (2010), we have

$$\frac{\partial \left|\partial i^* / \partial\left(\sigma\eta\right)\right|}{\partial \phi^A} < 0, \tag{29}$$

which literally means that when the agency investment friction ϕ^A is larger, optimal investment i^* becomes inelastic to the risk premium $\sigma\eta$. In a group of firms with a higher agency investment friction, a larger (cross-sectional) difference in the risk premium $\sigma\eta$ would be

[23] Following DeMarzo and Sannikov (2006), it can be shown that $\partial(p+w)/\partial(\sigma\eta) = \mathbb{E}[e^{-r\tau}|w_0 = w] - 1 < 0$.
[24] We restrict the parameter values to ensure that the investment rate is positive, and the marginal q is not infinite. To this end, we assume $\mu - \sigma\eta - r - \delta > 0$ and $(r+\delta)^2 - 2(\mu - \sigma\eta - r - \delta)/\phi > 0$.

required to generate a given difference in the investment rate i. As a result, the observed relation between investment and subsequent stock returns would be more pronounced.

The relation in (29) drives our empirical identification of ϕ^A. We show in the next subsection that empirical tests based on ϕ^A differentiate our model from that of DeMarzo et al. (2012). Our model also leads to better inferences on the effect of managerial agency costs on asset prices given the empirical difficulties that both marginal q and the firm risk premium $\sigma\eta$ are unobservable.

3.2 Comparison to DeMarzo et al.'s (2012) model

Our model is different from that of DeMarzo et al. (2012) in that we focus on the agency costs of *investment* (i.e., *input*), and we aim to generate cross-sectional return patterns. To this end, we adapt the pricing kernel (5) to the q-theory of investment. The same pricing kernel can also be applied to DeMarzo et al. (2012) and the majority of their results will hold. The major difference lies in the specifications on agency costs. In their model, the manager is able to under-report the *productivity* shocks (shocks to A_t) and divert the *output* of the firm for private benefits. While they find a reduction in marginal q, we find an additional effect: an increased capital adjustment cost, i.e., the agency investment friction.

Adapting the pricing kernel to our setting, the HJB equation that determines the optimal investment and firm value in DeMarzo et al. (2012) is as follows (when $\kappa \to r$):

$$rq^A(w) = \mu - \sigma\eta - c(i) + q^A(w)(i-\delta) + \frac{1}{2}p''(w)\lambda^2\varepsilon^2, \tag{30}$$

where $q^A(w) = p(w) - wp'(w)$ is the marginal q with agency costs, which is the same as that in our model. However, λ is the efficiency of the manager's stealing from the firm's *output* which is the counterpart (*input*) of λ in our model. ε^2 is the variance of the shocks to the *productivity* that are observable to the manager only. In this equation, since $p''(w)$, which measures the agency costs created by a volatile w, is also less than zero, the term $p''(w)\lambda^2\varepsilon^2/2$ actually measures how much the marginal q $q^A(w)$ is reduced from the first-best q (i.e., q^{FB}) which is solved by (25). In DeMarzo et al. (2012), the optimal investment is the same as that in a first-best firm with a marginal q equal to q^A; in other words, controlling for the marginal q, firms with or without agency costs should make the same levels of investment.

The reduction in marginal q due to agency costs derived in DeMarzo et al. (2012) is difficult to test empirically. First, marginal q is not directly observable and/or is measured with noise. It is difficult to compare marginal q in a cross section generated by the degree of agency costs. Second, from (30) we can see that while the agency costs would reduce marginal q (by the term $p''(w)\lambda^2\varepsilon^2/2$), the risk premium $\sigma\eta$ would have the same effect. Even if we do observe a cross-sectional pattern of marginal q, we still cannot attribute the effect purely to agency costs, unless we can identify either the (unobservable) risk premium $\sigma\eta$ or the exact functional shape of $p(w)$.

We propose a new empirical test based on our specification. We do not rely directly on the cross-sectional marginal q or the cross-sectional risk premiums, but on the link between the cross-sectional investment-return relation and the agency investment friction ϕ^A. To illustrate this, in our model, the HJB function that determines optimal investment is (when $\kappa \to r$)

$$rq^A(w) = \mu - \sigma\eta - i - \frac{1}{2}\underbrace{\left(\phi - p''(w)(\lambda - w)^2\varepsilon^2\right)}_{\phi^A} i^2 + q^A(w)(i - \delta). \tag{31}$$

Compared to the first-best case in (25), the agency costs manifest as a component in ϕ^A (note that $c(i) = i + \phi i^2 / 2$). A firm with higher agency costs of investment tends to have a higher ϕ^A which leads to a more negative relation between investment and returns in the data as shown in (29). This is a unique empirical prediction of our model. Note that this prediction is not driven by any cross-sectional variations in the pricing kernel $\sigma\eta$, since ϕ^A is determined by the concavity of the function $p(w)$, state variable w, and parameters λ and ε^2.[25]

3.3 State-dependent agency investment frictions, ϕ^A

We briefly discuss the state-dependency of the agency investment friction ϕ^A which will be useful for our empirical hypotheses in Section 4. Holding the *physical* adjustment cost ϕ constant for all firms, the model generates variations in the agency-related component $-p''(w)(\lambda - w)^2 \varepsilon^2$, due to different investment efficiency shocks ε^2 and shocks to the state variable w.

First, ϕ^A increases with the variance of investment efficiency shock ε^2. When $\varepsilon^2 = 0$ and the manager cannot divert investment, the agency problem does not affect the adjustment cost of investment, and the model is reduced to the first best. By assuming that $\varepsilon^2 > 0$ and that there is managerial investment diversion, we establish a channel through which idiosyncratic volatility in investment efficiency affects optimal investment.[26] Intuitively, as ε^2 increases, to induce truthful investment behavior from the manager would require a higher-powered and more volatile compensation. As a result, each marginal unit of investment would lead to a higher

[25] Strictly speaking, the risk premium $\sigma\eta$ can still affect ϕ^A by influencing the second-order derivative of $p(w)$. But since it is of the second order, the effect would be minimum.

[26] Idiosyncratic shocks generally have no effect on investment in the traditional neo-classical investment literature.

increase in the probability of inefficient liquidation, and the expected cost that is reflected in the agency investment friction would be higher.

In general, ε^2 can be interpreted as the noisiness of information on the investment opportunity which creates an information gap between insiders and outsiders. This information gap is related to the volatility of idiosyncratic returns of the stock. In our model, the variance of idiosyncratic returns is related to ε^2 by our assumption that the shock to productivity is purely systematic and the shock to investment efficiency is idiosyncratic. However, even if we assume that there is an idiosyncratic component in productivity shock, investment efficiency shock would still contribute to the overall idiosyncratic shocks of stock returns. As a result, in reality if we sort the firms based on idiosyncratic volatility of stock returns, we would be able to generate a cross-sectional variation in the investment-return relation across ε^2 groups.

Second, ϕ^A is also related to w, which is the manager's payoff from staying with the firm. In Appendix A6 we show that $\partial\phi^A / \partial w < 0$ when $r \to \kappa$. In general, the agency costs are mitigated by increasing w: a higher w reduces the probability of the manager's leaving the firm (i.e., when w hits zero) which is what triggers inefficient liquidation. Therefore the cost associated with implementing the incentive contract decreases, and the firm has a reduced cost of investment ϕ^A. At the pay-out boundary $w = \bar{w}$, $p''(w) = 0$ and $\phi^A = \phi$, the firm's adjustment cost of capital reduces to the physical adjustment cost.

The manager's payoff from staying with the firm w or W is not observed in the data.[27] The model predicts that the change in w or W is related to the idiosyncratic return of the firm. We prove in Appendix A7 that, while w has a loading $i(\lambda - w)\varepsilon$ on the idiosyncratic shock

[27] Especially, it is not the observed managerial compensation. Instead, it should be the present value of all of the manager's future compensation.

$dB_{I,t}$, the return of the firm as shown in (26) has a loading $\left[p - p'w + (p'+1)\lambda\right] i\varepsilon / (p+w)$ on $dB_{I,t}$. Cross-sectionally, if a firm is hit by a series of negative idiosyncratic returns, w is also reduced accordingly. Therefore we would be able to generate a cross-sectional variation in ϕ^A based on the past performance of the firm's stock returns.

4. Empirical Tests

4.1 Hypothesis development

Since both i^* and the elasticity of i^* with respect to the expected returns decrease in ϕ^A, we hypothesize that

H$_1$. *The optimal investment level is lower for firms with higher agency costs.*

H$_2$. *The negative association between the expected stock returns and the investment level is stronger for firms with higher agency costs.*

To test **H$_1$**, we relate firm-level investment to agency costs in a panel regression while controlling for firm size, book-to-market ratios, and firm and year fixed effects. To test **H$_2$**, we estimate the following Fama and MacBeth (1973) cross-sectional regression of monthly stock returns:

$$R_{it} = \gamma_0 + \gamma_1 Log(MV)_{it-1} + \gamma_2 Log(BM)_{it-1} + \gamma_3 Inv_{it-1} + \gamma_4 \phi^A_{it-1} + \gamma_5 Inv_{it-1} * \phi^A_{it-1} + u_{it}, \qquad (28)$$

where R_{it} is monthly stock returns from July of year t to June of year t+1, and u_{it} is an error term. *Inv* is a measure of firm investment and ϕ^A is a measure of the agency investment friction. *Log*(*MV*) and *Log*(*BM*) are included to account for the well-documented size and value effects (e.g., Fama and French (1992)). According to the hypothesis, we predict a negative coefficient γ_5 on the interaction term between investment and agency costs.

In a different setting, we estimate the Fama and MacBeth (1973) cross-sectional regression separately for the subsamples of firms with high and low agency costs which allows all regression coefficients to vary across the subsamples. We then compare the coefficients on the investment variable in the two subsamples. In the subsample of firms with high agency costs, the coefficient on the investment variable is predicted to be more negative. In addition, we use double sorting to calculate portfolio returns. We test whether the return differentials between the high- and low-investment portfolios are larger in the group of firms with higher agency costs. The portfolio approach allows us to gauge the economic magnitude of the investment effect and unveil the non-monotonic relation between agency costs and the investment effect if any.

As discussed in Section 3.3, ϕ^A increases in ε^2 and decreases in w. Therefore, cross-sectionally, we posit that firms with higher idiosyncratic volatility of stock returns and lower past stock performance tend to have a higher ϕ^A.[28] Thus the empirical proxies for ϕ^A are the firm's idiosyncratic volatility of stock returns and past stock returns.

4.2 Data

We obtain accounting data from Compustat and stock returns data from the Center for Research in Security Prices (CRSP). Following the literature, we restrict the sample to include only common shares and exclude all financial firms (SIC codes between 6000 and 6999). The sample period is from 1963 to 2014.

We use four measures of investment used widely in the literature. They are investment-to-asset ratio, *I/A* (Lyandres, Sun, and Zhang (2008)); asset growth, $\Delta A/A$ (Cooper, Gulen, and

[28] Strictly speaking, a higher ϕ^A is associated with lower past idiosyncratic returns in the model. However, as empirical asset pricing models tend to have low R-squared in explaining returns of individual stocks, past idiosyncratic returns are highly correlated with past returns.

Schill (2008); Titman, Wei, and Xie (2013)); investment growth, $\Delta I/I$ (Xing (2008); Mao and Wei (2015)); and investment-to-capital ratios, *I/K* (Liu, Whited, and Zhang (2009)). All variable definitions are detailed in Appendix A8.

Agency costs are proxied by idiosyncratic volatility (*IVOL*), a measure of the volatility in investment efficiency, and past stock returns ($Rtn_{-1,-12}$), a measure of past performance. We follow Ang, Hodrick, Xing, and Zhang (2009) to calculate the idiosyncratic volatility as the standard deviation of the residuals from a regression of daily stock returns on the Fama-French three-factor model over the previous 12 months as of the end of June.[29] We measure past stock returns by accumulating the returns of a stock over the previous 12 months as of the end of June.

Panel A of Table 1 presents the summary statistics. The average monthly stock returns is 1.03% with a standard deviation of 15.32%. The average idiosyncratic volatility in monthly percentage is 15.42% with a standard deviation of 9.81%. Compared to the other three measures of investment, the distribution of $\Delta I/I$ is more dispersed with a standard deviation of 2.25. This implies that the firm investment level is more persistent than the growth in investment. Panel B shows the pairwise cross-sectional Pearson correlations. All four investment proxies are highly correlated with the correlation coefficients ranging from 33% to 67%. On average, stock returns are significantly negatively correlated with all investment proxies and positively correlated with the book-to-market ratio. Also, the investment proxies are all negatively correlated with the book-to-market ratio.

[Insert Table 1 here]

4.3 Investment-level tests

To test $\mathbf{H_1}$, we use a panel regression setting to relate firm investment to various determinants. Specifically, we regress each of the four investment proxies on firm size, book-to-

[29] We require at least 60 non-missing observations to calculate the idiosyncratic volatility.

market ratio (an inverse proxy for investment opportunities), idiosyncratic volatility, and past returns while controlling for both firm and year fixed effects. Panel A of Table 2 reports the results that lend support to $\mathbf{H_1}$. The strongly negative associations between idiosyncratic volatility and investment suggest that higher firm-specific volatility in investment efficiency leads to a lower optimal investment level. The strongly positive associations between past returns and investment suggest that higher agency costs due to poorer past performance also lead to a lower optimal investment level. In addition, the coefficients on *Ln*(*MV*) and *Ln*(*BM*) indicate that small firms and growth firms have higher investment levels, consistent with the literature.

Panousi and Papanikolaou (2012) also document a negative association between idiosyncratic volatility and investment. However, in their model, the reduced investment is a result of the manager's not wanting to be exposed to idiosyncratic risk from her sizable equity ownership of the firm. They also find that the effect is mitigated when the compensation is mainly composed of options rather than shares. Since the inclusion of equity options in compensation structures has become popular after 1990 (e.g., Hall and Liebman (1998)), the prediction would be that the negative effects of managerial risk aversion should be weakened after 1990. To see if this is indeed the case, we include an interaction term between idiosyncratic volatility and a dummy variable that equals one for years after 1990. Panel B of Table 2 shows that the evidence is mixed. The effect of *IVOL* on investment is weaker in the latter period when *I/A* and Δ*A/A* are used as proxies, while stronger in the latter period for the case of *I/K*. Therefore, we posit that the managerial risk aversion story may not be able to fully account for decreasing investment levels in idiosyncratic volatility.

[Insert Table 2 here]

It is worth noting that the results from the investment-level tests are consistent with our model, but they are plagued by errors in the measurement of investment opportunities (e.g., Erickson and Whited (2012)). Although we have controlled for the book-to-market ratio, idiosyncratic volatility and past returns may contain information about hidden investment opportunities. In particular, the investment-level tests cannot distinguish our model from that of DeMarzo et al. (2012). As in their model, the agency costs (proxied by idiosyncratic volatility and past returns) might drive a larger difference between an average q and a marginal q. In this sense, their model would also predict investment levels in the same way as ours. The lack of a good empirical measure for marginal q suggests that our next test, which relies on the agency investment frictions and cross-sectional return patterns, would be a better option for detecting agency costs.

4.4 Fama and MacBeth cross-sectional regressions of returns

Next, we explore the relation between investment and subsequent stock returns and the effect of agency costs. Table 3 presents the estimated coefficients from the Fama-MacBeth cross-sectional regressions of monthly returns on firm size, book-to-market ratios, and investment. The significantly positive coefficients on the book-to-market ratio and the significantly negative coefficients on the investment proxies in regressions (1), (3), (5), and (7) confirm the stylized cross-sectional return patterns, namely, the value effect and the investment effect. When we include idiosyncratic volatility (*IVOL*) and its interaction with the investment proxies in regressions (2), (4), (6), and (8) shown in Panel A, the coefficients on the investment proxies become insignificant. To emphasize, the coefficients on the interaction terms are all significantly negative, manifesting the effect of idiosyncratic volatility on the investment-return relations as predicted by our model.

Panel B shows the coefficient estimates when we include past-one-year returns and its interaction with the investment proxies. The significantly positive coefficients on the interaction term in regressions (2), (4), and (8) underline the effect of past performance ($\text{Return}_{-1,-12}$) on the investment-return relations, which also lends support to the model prediction. The corresponding coefficient is insignificant when investment is proxied by the investment growth variable, which is consistent with the model implications that cross-sectional returns should respond to investment levels, rather than investment growth rates. In addition, the fact that all investment proxies remain statistically significant after controlling for past stock performance indicates that the investment effect is not subsumed by the momentum effect (Jegadeesh and Titman (1993)).

[Insert Table 3 here]

The pooled cross-sectional regressions implicitly assume that the regression coefficients are the same across firms with different levels of agency costs or agency investment frictions. To relax this assumption, we replicate the tests using the subsamples split by idiosyncratic volatility or past stock performance. More specifically, firms are split into terciles at the end of June of each year based on *IVOL* (in Panel A) or $Rtn_{-1,-12}$ (in Panel B) in Table 4.

Table 4 reports the coefficient estimates for the subsamples of the highest tercile and the lowest tercile, and a statistical comparison of the coefficient on the investment proxy across the subsamples. Panel A shows that except for the case of $\Delta I/I$, the magnitude of the coefficient on the investment proxy is significantly higher for the subsample of firms with high idiosyncratic volatility than for the one with low idiosyncratic volatility. On average, the magnitude of the coefficient in the high volatility subsample is about three times that in the low volatility subsample. Besides, the value effect is generally stronger for firms with high idiosyncratic volatility, as demonstrated by the coefficients on the book-to-market ratio. Similarly, Panel B of Table 4 shows that the investment-return relation is significantly stronger for the subsample of firms with low past stock performance than that with high past stock performance, except for the case of $\Delta I/I$. The magnitude of the coefficient in the low past stock performance subsample is about twice that in the high past stock performance subsample.

To summarize, both Tables 3 and 4 provide evidence lending support to Hypothesis 2 that the negative association between investment and expected returns is stronger for firms with higher agency investment frictions as reflected in higher idiosyncratic volatility and poorer past stock performance.

[Insert Table 4 here]

4.5 Portfolio tests

The cross-sectional regressions examine the linear relation between investment and subsequent stock returns and its interaction with agency costs. Next we use the portfolio approach to gauge the economic magnitudes of the effect of agency investment frictions on the investment anomaly defined as the return differentials between the stocks with high and low investment. We first sort stocks into quintiles at the end of June based on the measure of agency investment frictions. Within each quintile, we further sort stocks into quintiles based on the investment proxy. The investment hedge portfolio returns are calculated as the differences between the average stock returns in the highest and lowest investment quintiles. Stocks are rebalanced at the end of every June. We then compare the investment effect between the highest and lowest investment friction groups.

Table 5 presents the results. Panel A shows that the average monthly returns of the investment hedge portfolios are approximately -1.0% in the highest idiosyncratic volatility group and approximately -0.1% in the lowest idiosyncratic volatility group. The magnitudes increase almost monotonically from the lowest to the highest *IVOL* groups. The significant spreads between the two extreme *IVOL* groups demonstrate that higher firm-specific volatility in investment efficiency amplifies the negative association between investment and expected returns. Similarly, Panel B of Table 5 shows that the investment effect is stronger in the groups with lower past stock returns. The average returns of the hedge portfolios are approximately -1.0% in the lowest past return group and approximately -0.4% in the highest past return group. The differentials between the two are all statistically significant.

Note also that the patterns are more pronounced when the variables of investment levels rather than investment growth serve as the proxy for investment. This is in line with the model implication that agency costs are mainly imposed on the optimal level of investment and its sensitivity to the expected returns rather than the time-series changes in investment level and the corresponding return sensitivity. Panels C and D of Table 5 report abnormal returns of the investment hedge portfolio corresponding to those in Panels A and B. The abnormal returns are calculated using the Fama-French three-factor model. The results are similar to the case of raw returns and therefore the implications remain robust after controlling for the risk factors.

[Insert Table 5 here]

4.6 Summary of empirical findings and discussions

We find that higher idiosyncratic volatility and poorer past stock performance lead to a lower level of investment and a stronger negative association between corporate investment and subsequent stock returns. The evidence supports the predictions of our model which generates agency investment frictions. Higher idiosyncratic volatility and poorer past stock performance increase investment frictions through the channel of agency costs. As a result, a higher agency investment friction implies a lower optimal investment level and a higher investment-return inelasticity.

Similar empirical results are documented in the literature as evidence supporting the mispricing explanation for the investment effect. For example, Li and Zhang (2010) and Lam and Wei (2011) also find that the negative relation between stock returns and investment anomalous variables is more pronounced in firms with higher idiosyncratic volatility. They interpret the results to be consistent with the argument of mispricing with limits to arbitrage, since higher idiosyncratic volatility can imply a higher arbitrage risk and thus a stronger return anomaly. Zhang (2006) uses the evidence of a positive relation between stock volatility and short-term price continuation to support the behavioral interpretation that a greater information uncertainty leads to a greater price drift.

Although we cannot fully exclude the mispricing explanations, our model—to a certain extent—rationalizes the relation between idiosyncratic volatility and the investment effect. From the corporate perspective, in firms with higher idiosyncratic volatility or noisier information, it is costly to induce truthful investment behavior from the manager, which translates into a higher capital adjustment cost and a lower investment. Therefore, in our model, the effect of idiosyncratic volatility on the investment effect is interpreted as the fundamental risk of the firm's business rather than the arbitrage risk from the perspective of investors.

The evidence regarding the effect of past stock performance on the investment effect is new to the literature. Although both the momentum effect and the investment effect are well documented, few studies connect the insights of the two. Our model and the empirical results highlight a fundamental mechanism of how past stock performance can affect the investment-return relation in a rational framework. Stambaugh, Yu, and Yuan (2012) show that the cross-sectional return anomalies are stronger in the short leg. They interpret the evidence as support for the mispricing due to investor sentiment and short-sell impediments. Our tests are different in that we compare the returns of the long-short strategies across subsamples sorted by past stock performance instead of separating the long side from the short side.

5. Conclusion

We set up a dynamic q-theory of investment model with managerial investment diversion to evaluate the impact of agency costs on a firm's investment policy. We find that the possibility of investment diversion due to high idiosyncratic volatility in investment efficiency and poor past stock performance increases the investment friction of the firm. The implication of the model is that the investment effect or the negative association between firm investment and expected returns is more pronounced for firms with higher fundamental uncertainty and more negative shocks to the wealth of the manager. We use idiosyncratic volatility to proxy for the fundamental uncertainty of the firm and past stock performance to proxy for negative shocks to the wealth of the manager to test the implications of our model.

The model offers a rational explanation for the effect of idiosyncratic stock volatility on the negative investment-return relation, which was previously used as evidence to support the mispricing or behavioral explanation. We view idiosyncratic volatility as a measure of information noisiness and the degree of managerial agency problems. Besides, the model also offers the novel prediction that in firms hit by poor stock performance, the investment friction is amplified by agency costs, which leads to a more negative return-investment relation. Both predictions on the variations in the cross-sectional return-investment relation are supported by the data.

Overall, our study extends the rational explanation for the investment effect based on a modified q-theory of investment by taking into account the degree of agency costs.[30] We interpret agency costs as a major source of investment frictions. The cross-sectional relations between investment and subsequent returns can shed light on the variations in the degree of agency costs across firms. We show that connecting a corporate finance model to cross-sectional stock returns can provide rich implications, and potentially circumvents some major difficulties (e.g., errors in the measurement of Tobin's *q*) in empirical corporate finance.

[30] Titman, Wei, and Xie (2004) also provide an explanation for the investment effect based on a pure agency cost argument. In contrast, our model provides a solution to the agency problem by offering the manager an incentive contract to prevent investment diversion.

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Appendix

We define the change of measure according to the pricing kernel (5). Given the filtered probability space $\{\Omega, \mathcal{F}, \mathcal{F}_t\}$, under measure P, and with $\{B_{A,t}\}$ being a Brownian motion, we can define the risk-neutral measure Q such that

$$\frac{d\mathrm{Q}}{d\mathrm{P}} = \exp(-\eta B_{A,t} - \frac{1}{2}\eta^2 t). \tag{32}$$

Then under the risk-neutral measure Q, $\{B^Q_{A,t}\}$ is a Brownian motion such that

$$dB^Q_{A,t} = dB_{A,t} + \eta_t dt. \tag{33}$$

And the productivity process $\{\mathrm{A}_t\}$ under measure Q is

$$dA_t = (\mu - \sigma\eta)dt + \sigma dB^Q_{A,t}, \tag{34}$$

where $(\mu - \sigma\eta)$ is the risk-adjusted productivity.

In all parts of this appendix, we will work under the risk-neutral measure Q, and the expectation operator $\mathrm{E}[.]$ means that the expectation is taken under the measure Q.

We require the following regulatory condition for the capital accumulation of the firm:

$$\mathrm{E}\left[\int_0^T \left(e^{-rt}K_t\right)^2 dt\right] < \infty \text{ for any } T > 0, \tag{35}$$

and

$$\lim_{T\to\infty}\left[e^{-rT}K_T\right] = 0. \tag{36}$$

We also require the following regulatory condition for the contract:

$$\mathrm{E}\left[\left(\int_0^\tau e^{-\kappa t} dU_t\right)^2\right] < \infty. \tag{37}$$

A1: First-best benchmark

We use the techniques in Hennessy (2004) to derive the first-best firm value with investment efficiency shocks. The HJB equation to solve (6) is

$$rP^{FB} = \max_{I} \, (\mu - \sigma\eta)K - C(I,K) + P_K^{FB}(I - \delta K) + \frac{1}{2} P_{KK}^{FB} \varepsilon^2 I^2 \tag{38}$$

If we define an infinitesimal generator $\mathrm{A}(h)$ such that for an arbitrary second order differentiable function h,

$$\mathrm{A}(h) = h_K (I - \delta K) + \frac{1}{2} h_{KK} \varepsilon^2 I^2, \tag{39}$$

at the optimum, equation (38) could be written as

$$rP^{FB} = (\mu - \sigma\eta)K - C(I,K) + \mathrm{A}(P^{FB}). \tag{40}$$

Differentiating both sides of (38) with respect to K, we have

$$rP_K^{FB} = (\mu - \sigma\eta) - C_K + P_{KK}^{FB}(I - \delta K) - P_K^{FB}\delta + \frac{1}{2} P_{KKK}^{FB} \varepsilon^2 I^2. \tag{41}$$

Multiplying K on both sides of (41), we have

$$rP_K^{FB}K = (\mu - \sigma\eta)K - C_K K - P_K^{FB}\delta K - P_K^{FB}(I - \delta K) - P_{KK}^{FB}\varepsilon^2 I^2 + \mathrm{A}(P_K^{FB}K). \tag{42}$$

Since the optimal investment I satisfies the following first order condition,

$$C_I = P_K^{FB} + P_{KK}^{FB}\varepsilon^2 I, \tag{43}$$

we must have the following equation:

$$C(I,K) = C_I I + C_K K = P_K^{FB} I + P_{KK}^{FB}\varepsilon^2 I^2 + C_K K. \tag{44}$$

Combining (42) and (44) together we have

$$rP_K^{FB}K = (\mu - \sigma\eta)K - C(I,K) + \mathrm{A}(P_K^{FB}K). \tag{45}$$

And it can be seen that

$$r(P^{FB} - P_K^{FB}K) = \mathrm{A}(P^{FB} - P_K^{FB}K). \tag{46}$$

Define $\varsigma_t = e^{-rt}(P_t^{FB} - P_K^{FB}K_t)$, we have

$$d\varsigma_t = e^{-rt}\sigma(\varsigma_t)dB_{I,t} \tag{47}$$

where $\sigma(\varsigma_t) = \varepsilon I P_{KK}^{FB} K$. Therefore, ς_t is a martingale under measure Q. Integrating ς_t up to time T, we have

$$\mathrm{E}\left[e^{-rT}(P_T^{FB}-P_K^{FB}K_T)\right]=P^{FB}-P_K^{FB}K_0+\mathrm{E}\left[\int_0^T e^{-rt}\sigma(\varsigma_t)dB_{I,t}\right]. \tag{48}$$

Let $T\to\infty$ and since the transversality condition (36) holds, the left hand side of (48) is zero, so we have

$$P^{FB}-P_K^{FB}K_0=0. \tag{49}$$

Define $q^{FB}=P_K^{FB}$ as the marginal q for the first-best firm, (49) shows q^{FB} is also equal to the average q ($=P^{FB}/K_0$), so $dq^{FB}/dK=0$. Plugging it back to (38), we have

$$rq^{FB}=\max_i\,(\mu-\sigma\eta)-c(i)+q^{FB}(i-\delta). \tag{50}$$

Therefore, the optimal investment is determined by

$$1+\phi i=q^{FB} \tag{51}$$

and q^{FB} is solved from

$$\left(r+\delta\right)q^{FB}=\mu-\sigma\eta+\frac{\left(q^{FB}-1\right)^2}{2\phi}\ . \tag{52}$$

A2: Proof of Lemma 1.

The process $V_t\equiv\mathrm{E}_t\left[\int_0^\tau e^{-\kappa s}dU_s\right]=\int_0^t e^{-\kappa s}dU_s+e^{-\kappa t}W_t$ for $t\in(0,\tau)$ is a square-integrable martingale. According to the Martingale Representation Theorem, there exists progressively measurable processes $\beta_{I,t}$ and $\beta_{A,t}$ such that $V_t=V_0+\int_0^t e^{-\kappa s}\beta_{I,s}I_s\varepsilon dB_{I,s}+\int_0^t e^{-\kappa s}\beta_{A,s}K_s\sigma dB_{A,s}$. Therefore on the path of an incentive-compatible contract, we have

$$dU_t+dW_t=\kappa W_t dt+\beta_{I,t}I_t\varepsilon dB_{I,t}+\beta_{A,t}K_t\sigma dB_{A,t}. \tag{53}$$

First, we show that to make the contract incentive compatible, with any given $\beta_{A,t}$, we need $\beta_{I,t}\geq\lambda$. Suppose that the manager deviates and diverts investment at the magnitude ΔI for a period from t' to t. Let $\Delta t=t-t'$ converge to zero. The capital stock K_t will be reduced to

$K_t - \Delta I \Delta t$. The output $Y_t - Y_{t'}$ will be reduced from $\sigma K_t \left(B_{A,t} - B_{A,t'} \right)$ to $\sigma \left(K_t - \Delta I \Delta t \right) \left(B_{A,t} - B_{A,t'} \right)$. As a result, the reduction in compensation would converge to $\beta_{I,t} \Delta I \Delta t + \beta_{A,t} \left[\sigma \Delta I \Delta t (B_{A,t} - B_{A,t'}) \right]$. The first term $\beta_{I,t} \Delta I \Delta t$ is an infinitesimal on the same order as $\beta_{I,t} \Delta I O(\Delta t)$ and the second is an infinitesimal on the same order as $\beta_{A,t} \Delta I O(\Delta t^{3/2})$. On the other hand, the manager's private benefit of diversion is $\lambda \Delta I \Delta t$, which is an infinitesimal on the same order as $\lambda \Delta I O(\Delta t)$. Since $\beta_{A,t}$ is a bounded function, $\beta_{A,t} \Delta I \Delta t (z_{t+\Delta t} - z_t)$ is infinitely small relative to $\lambda \Delta I \Delta t$ and $\beta_{I,t} \Delta I O(\Delta t)$ when $\Delta t \to 0$, which means that the choice of $\beta_{A,t}$ has no effect on the manager's action of diverting investment. It also follows that to make the contract incentive compatible, it is necessary to set $\beta_{I,t} \geq \lambda$ a.e..

Next, we show that the optimal compensation satisfies $\beta_{I,t} = \lambda$ and $\beta_{A,t} = 0$. We also show the optimality of the payment boundary $\overline{w}$ as defined in (14). Notice that this part of the proof is the same as DeMarzo et al. (2012). For any incentive-compatible contract $\Phi = (I, U, \tau)$, define its auxiliary gain process $\{G\}$ under the risk-neutral measure as (for $t < \tau$) under the risk-neutral measure

$$G_t\left(\Phi\right) = \int_0^t e^{-rs} \left(K_s dA_s^Q - I_s ds - \frac{\phi I_s^{\ 2}}{2K_s} ds - dU_s \right) + e^{-rt} P_t\left(K_t, W_t\right). \tag{54}$$

At optimum, the drift of dG_t is maximized at zero. Using Ito's Lemma, the auxiliary process can be expressed as

$$
\begin{aligned}
&e^{rt} dG_t(\Phi) \\
&= \left(K_t dA_t^Q - I_t dt - \frac{\phi I_t^{\,2}}{2K_t} dt \right) - rP_t(K_t, W_t)\, dt + P_K\left(\left(i-\delta\right)K\right) dt + P_W \kappa W dt \\
&+ \frac{1}{2}\left(P_{KK} + P_{WW}\beta_{I,t}^{\;2} + 2P_{KW}\beta_{I,t}\right)\varepsilon^2 i^2 K^2 dt + \frac{1}{2} P_{WW}\beta_{A,t}^{\;2} K_t^{\,2}\sigma^2 dt \\
&+ \left(-1 - P_W\right) dU_t + P_W\left(\beta_{I,t}\varepsilon I_t dB_{I,t} + \beta_{A,t} K_t \sigma dB_{A,t}^Q\right) + P_K i_t K_t \varepsilon dB_{I,t} \\
&= K_t \left\{ -rp + \left(\mu - \sigma\eta\right) - i - \frac{\phi}{2} i^2 + \left(i-\delta\right)\left(p - p'w\right) + \kappa p' w + \frac{1}{2}\left(\beta_{I,t} - w\right)^2 p'' \varepsilon^2 i^2 + \frac{1}{2} p'' \beta_{A,t}^{\;2}\sigma^2 \right\} dt \\
&+ \left(-1 - p'\right) dU_t + K_t \left\{ p' \beta_{A,t} \sigma dB_{A,t}^Q + i_t\left(p - p'w\right)\varepsilon dB_{I,t} + p'\beta_{I,t}\varepsilon i_t dB_{I,t} \right\}
\end{aligned}
\tag{55}
$$

Equation (55) is derived from the fact that if $p(w)K = P$ where $w = W/K$, then $P_K = p - p'w$, $P_W = p'$, $P_{KK} = p''w^2/K$, $P_{WW} = p''/K$, and $P_{KW} = -p''w/K$. From the above equation, if $p'' \le 0$ and $p' \ge -1$, the drift of dG_t is maximized by setting $\beta_{I,t} = \lambda$ (which is constrained by the incentive compatibility) and setting $dU_t > 0$ only when $p' = -1$. $p' \ge -1$ is ensured by the simple intuition that the total firm value $p + w$ cannot decrease through a positive value transfer from the shareholders to the manager. We prove $p'' \le 0$ in the proof of Lemma 2.

The proof for setting the optimal $\tau = \inf\{t \ge 0 : W_t = 0\}$ is the same as the proof of Theorem 1 in He (2009). Also from (55), at optimum, the drift of dG_t is zero, and the HJB equation in (17) follows.

A3: Proof of Lemma 2.

Combining the HJB function (17) and the optimal investment (18), we have

$$
\left(r+\delta\right)p = \left(\mu - \sigma\eta\right) + \frac{1}{2}\frac{\left(p - wp' - 1\right)^2}{\phi - p''(\lambda - w)^2\varepsilon^2} + wp'\left(\kappa + \delta\right). \tag{56}
$$

Differentiating p with respect to w in the equation gives

$$\left(r+\delta\right)p' = \frac{-2\left(p-p'w-1\right)wp''}{\phi - p''(\lambda-w)^2\varepsilon^2} + \frac{\left(p-p'w-1\right)^2\left(p'''(\lambda-w)^2\varepsilon^2 - 2p''(\lambda-w)\varepsilon^2\right)}{\left(\phi-p''(\lambda-w)^2\varepsilon^2\right)} \tag{57}$$
$$+\left(wp''+p'\right)\left(\kappa+\delta\right)$$

At boundary $\overline{w}$, $p'\left(\overline{w}\right) = -1$ and $p''\left(\overline{w}\right) = 0$, so the above equation at $w = \overline{w}$ becomes

$$\left(r-\kappa\right)p' = \frac{\left(p-\overline{w}p'-1\right)^2 p'''(\lambda-\overline{w})^2\varepsilon^2}{2\phi}. \tag{58}$$

Therefore, $p'''\left(\overline{w}\right) > 0$.

For some $x > 0$ and for $w \in (\overline{w}-x, \overline{w})$, $p''(w) < 0$. Now choose the largest $\tilde{w} < \overline{w}$ such that $p''(\tilde{w}) = 0$ and for $w \in (\tilde{w}, \overline{w})$, $p''(w) < 0$. We obtain

$$\left(r+\delta\right)\left(p-\tilde{w}p'(\tilde{w})\right) = \left(\mu-\sigma\eta\right) + \frac{\left[p-\tilde{w}p'(\tilde{w})-1\right]^2}{2\phi} + \left(\kappa-r\right)\tilde{w}p'(\tilde{w}). \tag{59}$$

Since $p-\tilde{w}p' < q^{FB}$ and $\left(r+\delta\right)q^{FB} = \left(\mu-\sigma\eta\right) + \frac{\left[q^{FB}-1\right]^2}{2\phi}$, $p'(\tilde{w}) < 0$. At $\tilde{w}$

$$\left(r-\kappa\right)p'(\tilde{w}) = \frac{\left[p(\tilde{w})-\tilde{w}p'(\tilde{w})-1\right]^2 p'''(\tilde{w})(\lambda-\tilde{w})^2\varepsilon^2}{2\phi^2} \tag{60}$$

and $p'''(\tilde{w}) > 0$. But this contradicts the selection of $\tilde{w}$ as the largest w such that $p''(w) = 0$, since following the selection, it must be true that for $w \in (\tilde{w}, \overline{w})$, $p''(w) < 0$ and $p'''(\tilde{w}) < 0$. Therefore, such $\tilde{w}$ does not exist for $[0, \overline{w}]$, and we have $p''(w) < 0$ for the whole region.

A4: Proof of Proposition 1.

The proof is straightforward, as it follows directly from (18), (19), and (23).

A5: Proof of $\frac{\partial\left|\partial i^* / \partial\left(\sigma\eta\right)\right|}{\partial\phi^A} < 0.$

It can be shown from (28) that

$$\left|\frac{\partial i^*}{\partial(\sigma\eta)}\right| = \frac{1}{\sqrt{\left[(r+\delta)\phi^A\right]^2 - 2(\mu-\sigma\eta-r-\delta)\phi^A}}, \tag{61}$$

and from (61), if $\sqrt{\left[(r+\delta)\phi^A\right]^2 - 2(\mu-\sigma\eta-r-\delta)\phi^A}$ increases in ϕ^A then the proof is complete. This is obvious, since for i^* to be meaningful, we have restricted the parameter values so that

$$\mu-\sigma\eta-r-\delta>0 \tag{62}$$

$$(r+\delta)^2 - 2(\mu-\sigma\eta-r-\delta)/\phi>0 \ . \tag{63}$$

And since $\phi^A>\phi$, $\sqrt{\left[(r+\delta)\phi^A\right]^2 - 2(\mu-\sigma\eta-r-\delta)\phi^A}$ increases in ϕ^A , and $\left|\frac{\partial i^*}{\partial(\sigma\eta)}\right|$ decreases in ϕ^A.

A6: Proof of $\partial\phi^A/\partial w<0$ when $r\to\kappa$:

From (20), we have

$$\frac{\partial\phi^A}{\partial w} = -p'''(\lambda-w)^2\varepsilon^2 + 2(\lambda-w)p''\varepsilon^2 \tag{64}$$

And from (57) we have

$$(r-\kappa)\,p' = (\kappa+\delta-i)\,wp'' + \frac{(p-p'w-1)^2}{2\left[\phi-p''(\lambda-w)^2\varepsilon^2\right]^2}\left(p'''(\lambda-w)^2\varepsilon^2 - 2p''(\lambda-w)\varepsilon^2\right). \tag{65}$$

Since in our model with agency costs the investment level i would be less than the first-best investment i^{FB},

$$i<i^{FB} = r+\delta-\sqrt{(r+\delta)^2 - 2\frac{(\mu-\sigma\eta)-(r+\delta)}{\phi}} < \kappa+\delta. \tag{66}$$

And when $r\to\kappa$, the LHS of (65) is zero. Since $p''<0$, the only way for (65) to be true is that

$$p'''(\lambda-w)^2\varepsilon^2 - 2p''(\lambda-w)\varepsilon^2>0. \tag{67}$$

Thus we have proved that

$$\frac{\partial \phi^A}{\partial w} = -p'''(\lambda - w)^2 \varepsilon^2 + 2(\lambda - w) p'' \varepsilon^2 < 0. \tag{68}$$

A7: The loadings on the idiosyncratic shock dB_I

Equation (15) shows that w has a loading equal to $i(\lambda - w)\varepsilon$ on dB_I. The instantaneous rate of return on the firm's stock is

$$\frac{dY - c(i)Kdt + dP + dW}{P + W}. \tag{69}$$

It can be seen from (55) that while $dY - c(i)Kdt$ has no loading on dB_I, dP has a loading of $K_t\left\{\left(p - p'w\right) + p'\lambda\right\}\varepsilon i$ and dW has a loading equal to $iK\lambda\varepsilon$. Then the instantaneous return would have a loading on dB_I equal to

$$\frac{\left(p - p'w\right) + \left(p' + 1\right)\lambda}{p + w}\varepsilon i \tag{70}$$

Note that in the model, $w > \lambda$ is not possible, since whenever $w = \lambda$, w no longer diffuses, and results in an absorption at $w = \lambda$.[31] Therefore, the shocks of dB_I always hit the instantaneous return and w in the same direction.

[31] We can show that $w = \lambda$ is only possible when $\kappa = r$, and if $\kappa > r$, we have $w \le \overline{w} < \lambda$. The proof is the same as the proofs of Propositions 2 and 3 in He (2009).

A8: Variable definitions

I/A	Investment-to-asset, calculated as the annual change in gross property, plant, and equipment (Compustat item PPEGT) plus the annual change in inventories (item INVT) divided by the lagged book value of assets.
ΔA/A	Asset growth, calculated as the change in total assets (item AT) divided by lagged total assets.
ΔI/I	Investment growth, calculated as the growth rate of capital expenditure (item CAPX).
I/K	Investment-to-capital, calculated as capital expenditure (item CAPX) divided by lagged gross property, plant, and equipment (item PPEGT).
IVOL	Idiosyncratic volatility, calculated as the standard deviation of the residuals in the regression of daily stock returns on Fam-French three factors (MKTRF, SMB, HML) over the previous 12 months ending at the end of June.
$Rtn_{-1,-12}$	Past returns, calculated as cumulative monthly returns over the previous 12 months ending at the end of June.
Ln(MV)	Size, calculated as the log of market value of equity at the end of June.
Ln(BM)	Book-to-market, calculated as the log of the book value of equity divided by the market value of equity as of the end of December of the fiscal year. The book value of equity is calculated following Daniel and Titman (2006).

Table 1: Summary statistics and correlations

Panel A reports the summary statistics for monthly stock returns in percentage (Return), idiosyncratic volatility in monthly percentage (*IVOL*), investment-to-asset ratios (*I/A*), asset growth (Δ*A/A*), investment growth (Δ*I/I*), investment-to-capital ratios (*I/K*), size (*Ln(MV*)), and book-to-market ratios (*Ln*(*BM*)). Variable definitions are detailed in Appendix A8. Variables are winsorized at the 1st and 99th percentiles. Panel B reports the correlations which are the time-series means of the pairwise cross-sectional Pearson correlations for each month. The significance of a given correlation is calculated based on the time-series standard errors. The sample period is from 1963-2014. *** and ** indicate significance levels of 1% and 5%, respectively.

Panel A: Summary statistics

	Mean	Std Dev	1%	25%	Median	75%	99%
Return (%)	1.03	15.32	-39.13	-6.98	0.00	7.64	58.96
IVOL (%)	15.42	9.81	3.62	8.50	12.76	19.35	54.63
I/A	0.11	0.24	-0.36	0.01	0.06	0.15	1.29
ΔA/A	0.26	0.78	-0.49	-0.01	0.08	0.23	4.93
ΔI/I	0.63	2.25	-0.93	-0.25	0.11	0.65	13.11
I/K	0.24	0.40	0.01	0.07	0.12	0.23	2.61
Ln(MV)	4.55	2.16	0.25	2.93	4.42	6.05	10.02
Ln(BM)	-0.53	0.91	-3.24	-1.08	-0.47	0.08	1.55

Panel B: Correlations

	Return (%)	IVOL (%)	I/A	ΔA/A	ΔI/I	I/K	Ln(MV)
IVOL (%)	-0.02***						
I/A	-0.02***	-0.02***					
ΔA/A	-0.02***	0.03***	0.67***				
ΔI/I	-0.01***	0.08***	0.35***	0.33***			
I/K	-0.02***	0.11***	0.52***	0.48***	0.53***		
Ln(MV)	0.00	-0.61***	0.07***	0.04***	-0.07***	-0.02***	
Ln(BM)	0.03***	0.01*	-0.18***	-0.22***	-0.10***	-0.24***	-0.29***

Table 2: Determinants of investment

This table reports the coefficients estimated from panel regressions of investment on firm size (*Ln(MV)*), book-to-market ratios (*Ln*(*BM)*), idiosyncratic volatility (*IVOL*), and past stock returns ($Rtn_{-1,-12}$). Firm and year dummies are included in all regressions. The dependent variable is investment (*Inv*) proxied by investment-to-asset ratios (*I/A*), asset growth (Δ*A/A*), investment growth (Δ*I/I*), and investment-to-capital ratios (*I/K*). *After* is a dummy that equals 1 if the year is after 1990 and zero otherwise. Variable definitions are detailed in Appendix A8. The sample period is from 1963-2014. The *t*-statistics are in parentheses. Standard errors are clustered by firm. ***, ** and * indicate significance levels of 1%, 5% and 10%, respectively.

Panel A: Investment regressions without the interaction term

Dependent variable with Inv =	I/A	ΔA/A	ΔI/I	I/K
	(1)	(2)	(3)	(4)
Ln(MV)	-0.003**	-0.050***	-0.104***	0.017***
	(-2.24)	(-15.80)	(-10.51)	(7.75)
Ln(BM)	-0.066***	-0.231***	-0.341***	-0.076***
	(-39.39)	(-48.09)	(-24.48)	(-28.63)
IVOL	-0.233***	-0.220***	-0.594***	-0.150***
	(-18.33)	(-6.72)	(-4.46)	(-6.98)
$Rtn_{-1,-12}$	0.044***	0.171***	0.411***	0.039***
	(31.99)	(40.64)	(28.13)	(18.72)
Firm Dummy	Yes	Yes	Yes	Yes
Year Dummy	Yes	Yes	Yes	Yes
R-squared	0.11	0.15	0.04	0.10
N	14,302	14,412	14,272	14,286

Panel B: Investment regressions with the interaction term (IVOL×After)

Dependent variable with Inv =	I/A	ΔA/A	ΔI/I	I/K
	(1)	(2)	(3)	(4)
Ln(MV)	-0.003**	-0.050***	-0.104***	0.017***
	(-2.20)	(-15.79)	(-10.51)	(7.74)
Ln(BM)	-0.066***	-0.231***	-0.341***	-0.076***
	(-39.51)	(-48.21)	(-24.50)	(-28.60)
IVOL	-0.330***	-0.461***	-0.680***	-0.048
	(-14.46)	(-10.24)	(-3.20)	(-1.49)
IVOL×After	0.128***	0.317***	0.113	-0.135***
	(5.46)	(6.52)	(0.53)	(-4.10)
$Rtn_{-1,-12}$	0.043***	0.171***	0.411***	0.040***
	(32.00)	(40.65)	(28.12)	(18.74)
Firm Dummy	Yes	Yes	Yes	Yes
Year Dummy	Yes	Yes	Yes	Yes
R-squared	0.11	0.15	0.04	0.10
N	14,302	14,412	14,272	14,286

Table 3: Fama and MacBeth regressions on future returns: Whole-sample analysis

This table reports the coefficients estimated from the Fama and MacBeth (1973) cross-sectional regressions of monthly returns on firm size (*Ln*(*MV*)), book-to-market ratios (*Ln*(*BM*)), and investment (*Inv*). In Panel A, idiosyncratic volatility (*IVOL*) and its interaction with investment are included. In Panel B, past stock returns ($Rtn_{-1,-12}$) and its interaction with investment are included. Investment (*Inv*) is proxied by investment-to-asset ratios (*I/A*), asset growth (Δ*A/A*), investment growth (Δ*I/I*), and investment-to-capital ratios (*I/K*). Variable definitions are detailed in Appendix A8. The sample period is from 1963-2014. The *t*-statistics are in parentheses. ***, ** and * indicate significance levels of 1%, 5% and 10%, respectively.

Panel A: IVOL as a proxy for agency costs

	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ln(MV)	0.010	-0.057*	0.006	-0.053*	0.000	-0.063**	0.001	-0.059*
	(0.24)	(-1.78)	(0.12)	(-1.70)	(0.01)	(-2.03)	(0.03)	(-1.88)
Ln(BM)	0.347***	0.291***	0.341***	0.298***	0.357***	0.298***	0.330***	0.281***
	(4.84)	(4.93)	(4.91)	(5.14)	(4.73)	(4.73)	(4.75)	(4.80)
Inv	-0.964***	-0.190	-0.395***	0.069	-0.071***	-0.021	-0.525***	0.268
	(-7.29)	(-0.92)	(-6.79)	(0.45)	(-6.83)	(-1.09)	(-5.60)	(1.14)
IVOL		-0.025*		-0.022*		-0.025*		-0.017
		(-1.96)		(-1.66)		(-1.83)		(-1.24)
Inv×IVOL		-0.045***		-0.028**		-0.004***		-0.056***
		(-3.79)		(-2.49)		(-2.82)		(-3.44)
R-squared	0.02	0.04	0.03	0.04	0.02	0.04	0.03	0.04
N(groups)	612	612	612	612	612	612	612	612

Panel B: Past stock performance as a proxy for agency costs

	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ln(MV)	0.010	0.006	0.006	0.004	0.000	-0.003	0.001	-0.002
	(0.24)	(0.13)	(0.12)	(0.08)	(0.01)	(-0.07)	(0.03)	(-0.05)
Ln(BM)	0.347***	0.354***	0.341***	0.350***	0.357***	0.363***	0.330***	0.337***
	(4.84)	(5.35)	(4.91)	(5.51)	(4.73)	(5.22)	(4.75)	(5.30)
Inv	-0.964***	-0.967***	-0.395***	-0.389***	-0.071***	-0.067***	-0.525***	-0.505***
	(-7.29)	(-7.20)	(-6.79)	(-5.79)	(-6.83)	(-5.88)	(-5.60)	(-4.71)
$Rtn_{-1,-12}$		0.013		0.015		0.025*		0.012
		(0.87)		(0.97)		(1.65)		(0.82)
Inv×$Rtn_{-1,-12}$		0.091***		0.030***		0.002		0.046**
		(4.84)		(2.92)		(0.86)		(2.23)
R-squared	0.02	0.03	0.03	0.03	0.02	0.03	0.03	0.03
N(groups)	612	612	612	612	612	612	612	612

Table 4: Fama and MacBeth regressions of future returns: Subsample analyses

This table reports the coefficients estimated from the Fama and MacBeth (1973) cross-sectional regressions of monthly returns on firm size (*Ln(MV)*), book-to-market ratios (*Ln(BM)*) and investment (*Inv*) in the subsamples. Firms are split into terciles at the end of June each year based on idiosyncratic volatility (*IVOL*) in Panel A and on past stock returns ($Rtn_{-1,-12}$) in Panel B. "High" indicates the subsample with firms in the top tercile, while "Low" indicates the one with firms in the bottom tercile. t(High-Low) is the t-statistic comparing the coefficients on the investment variable across the two subsamples. Investment (*Inv*) is proxied by investment-to-asset ratio (*I/A*)s, asset growth ($\Delta A/A$), investment growth ($\Delta I/I$), and investment-to-capital ratios (*I/K*). Variable definitions are detailed in Appendix A8. The sample period is from 1963-2014. The *t*-statistics are in parentheses. ***, ** and * indicate significance levels of 1%, 5% and 10%, respectively.

Panel A: Idiosyncratic volatility (IVOL) as a proxy for agency costs

	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
IVOL Group	Low	High	Low	High	Low	High	Low	High
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ln(MV)	-0.038	-0.053	-0.036	-0.047	-0.042	-0.092*	-0.040	-0.069
	(-1.30)	(-1.06)	(-1.30)	(-0.96)	(-1.45)	(-1.81)	(-1.41)	(-1.34)
Ln(BM)	0.064	0.494***	0.086	0.491***	0.067	0.492***	0.060	0.470***
	(1.04)	(7.91)	(1.45)	(7.63)	(1.06)	(7.25)	(1.04)	(7.18)
Inv	-0.368**	-1.173***	-0.019	-0.488***	-0.051***	-0.072***	-0.166	-0.674***
	(-2.56)	(-8.27)	(-0.14)	(-7.12)	(-3.56)	(-6.62)	(-0.90)	(-7.41)
t(High-Low)	(-5.19)		(-4.24)		(-1.19)		(-3.02)	
R-squared	0.03	0.02	0.03	0.02	0.03	0.02	0.03	0.02
N(groups)	612	612	612	612	612	612	612	612

Panel B: Past stock performance ($Rtn_{-1,-12}$) as a proxy for agency costs

	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
$Rtn_{-1,-12}$ group	Low	High	Low	High	Low	High	Low	High
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ln(MV)	0.015	0.006	0.012	0.005	-0.002	-0.004	0.002	0.004
	(0.31)	(0.14)	(0.26)	(0.11)	(-0.04)	(-0.11)	(0.05)	(0.09)
Ln(BM)	0.447***	0.250***	0.440***	0.238***	0.472***	0.240***	0.432***	0.240***
	(6.43)	(3.43)	(6.52)	(3.44)	(6.45)	(3.14)	(6.36)	(3.45)
Inv	-1.288***	-0.524***	-0.514***	-0.297***	-0.076***	-0.061***	-0.685***	-0.297***
	(-7.85)	(-3.44)	(-6.28)	(-5.26)	(-5.26)	(-4.90)	(-6.46)	(-2.69)
t (High-Low)	(4.53)		(2.60)		(0.87)		(2.79)	
R-squared	0.03	0.03	0.03	0.03	0.03	0.02	0.03	0.03
N(groups)	612	612	612	612	612	612	612	612

Table 5: Portfolio tests

This table reports the average monthly returns in percent for the investment hedge portfolio across different groups sorted by idiosyncratic volatility (*IVOL*) in Panel A and past return ($Rtn_{-1,-12}$) in Panel B. Each year at the end of June, stocks are first sorted into quintiles based on the sorting variable. Stocks are further sorted into quintiles based on investment (*Inv*). The returns of the investment hedge portfolio are calculated as the return differentials between the highest and lowest investment quintiles. Portfolios are rebalanced every June. (5-1) indicates the return differences across the investment hedge portfolios in the highest and lowest sorting variable quintiles. Panels C and Panel D report the excess returns of the portfolios corresponding to those in Panel A and Panel B, respectively. Abnormal returns are calculated using the Fama-French three-factor model. Investment (*Inv*) is proxied by investment-to-asset ratios (*I/A*), asset growth (Δ*A/A*), investment growth (Δ*I/I*), and investment-to-capital ratios (*I/K*). Variable definitions are detailed in Appendix A8. The sample period is from 1963-2014. The *t*-statistics are in parentheses. ***, ** and * indicate significance levels of 1%, 5% and 10%, respectively.

Panel A: Raw returns and using IVOL as a proxy for agency costs

IVOL	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
Rank	Return	t-stat	Return	t-stat	Return	t-stat	Return	t-stat
1	-0.180	(-3.14)	-0.106	(-1.68)	-0.097	(-2.00)	-0.103	(-1.26)
2	-0.356	(-4.33)	-0.313	(-3.50)	-0.108	(-1.74)	-0.351	(-3.72)
3	-0.631	(-6.22)	-0.774	(-6.78)	-0.530	(-6.33)	-0.547	(-4.46)
4	-1.157	(-9.80)	-1.099	(-8.76)	-0.647	(-6.07)	-0.932	(-6.76)
5	-1.153	(-8.79)	-1.103	(-8.23)	-0.634	(-5.12)	-0.958	(-6.46)
(5-1)	-0.973	(-7.42)	-0.997	(-7.26)	-0.537	(-4.14)	-0.856	(-5.76)

Panel B: Raw returns and using past stock performance as a proxy for agency costs

$Rtn_{-1,-12}$	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
Rank	Return	t-stat	Return	t-stat	Return	t-stat	Return	t-stat
1	-1.196	(-8.58)	-1.115	(-7.61)	-0.644	(-5.28)	-0.963	(-6.37)
2	-0.658	(-6.18)	-0.664	(-5.54)	-0.398	(-4.49)	-0.618	(-5.37)
3	-0.632	(-6.94)	-0.650	(-6.61)	-0.403	(-5.11)	-0.512	(-4.53)
4	-0.427	(-4.54)	-0.500	(-5.03)	-0.251	(-3.17)	-0.425	(-3.75)
5	-0.432	(-3.91)	-0.434	(-3.71)	-0.340	(-3.73)	-0.230	(-1.83)
(5-1)	0.764	(4.94)	0.681	(4.40)	0.304	(2.17)	0.733	(4.58)

Panel C: Abnormal returns using IVOL as a proxy for agency costs

IVOL	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
rank	return	t-stat	Return	t-stat	Return	t-stat	Return	t-stat
1	-0.176	(-3.13)	-0.048	(-0.88)	-0.073	(-1.51)	-0.052	(-0.81)
2	-0.345	(-4.42)	-0.212	(-2.79)	-0.083	(-1.39)	-0.293	(-3.87)
3	-0.605	(-6.43)	-0.657	(-6.82)	-0.464	(-5.91)	-0.425	(-4.53)
4	-1.155	(-10.56)	-1.034	(-9.55)	-0.572	(-5.77)	-0.923	(-8.25)
5	-1.075	(-8.60)	-1.012	(-7.88)	-0.567	(-4.60)	-0.902	(-6.67)
(5-1)	-0.900	(-7.04)	-0.964	(-7.12)	-0.494	(-3.76)	-0.850	(-5.57)

Panel D: Abnormal returns using past stock performance as a proxy for agency costs

$Rtn_{-1,-12}$	Inv = I/A		Inv = ΔA/A		Inv = ΔI/I		Inv = I/K	
rank	Return	t-stat	Return	t-stat	Return	t-stat	Return	t-stat
1	-1.105	(-8.37)	-1.040	(-7.85)	-0.593	(-4.97)	-0.902	(-6.92)
2	-0.608	(-5.85)	-0.589	(-5.38)	-0.331	(-3.77)	-0.541	(-5.81)
3	-0.556	(-6.74)	-0.548	(-6.21)	-0.343	(-4.44)	-0.436	(-5.28)
4	-0.354	(-3.95)	-0.351	(-4.19)	-0.145	(-1.95)	-0.355	(-4.04)
5	-0.384	(-3.74)	-0.300	(-2.81)	-0.271	(-3.04)	-0.125	(-1.23)
(5-1)	0.721	(4.63)	0.740	(4.81)	0.322	(2.26)	0.776	(4.82)