Volatility and Expected Option Returns*

Guanglian Hu Kris Jacobs
University of Houston

December 8, 2014

Abstract

We provide a theoretical and empirical analysis of the relationship between expected option returns and the volatility of the underlying securities. We show analytically that in a Black-Scholes framework, the expected return from holding a call option is a decreasing function of the volatility of the underlying. The expected return from holding a put option is an increasing function of the volatility of the underlying. These predictions are strongly supported by the data. In the cross-section of stock option returns, returns on call (put) option portfolios decrease (increase) with underlying stock volatility. This strong negative (positive) relation between call (put) option returns and volatility is not due to cross-sectional variation in expected stock returns. It holds in various option samples with different maturities and moneyness, and it is robust to alternative measures of underlying volatility and different weighting methods. Time-series evidence also supports the predictions from option pricing theory. Future returns on S&P 500 index call (put) options are negatively (positively) related to S&P 500 index volatility.

JEL Classification: G12

Keywords: expected option returns; volatility; cross-section of option returns.

*Please send correspondence to Kris Jacobs, C.T. Bauer College of Business, 334 Melcher Hall, University of Houston, Houston, TX 77204-6021, USA; telephone: (713) 743-2826. E-mail: kjacobs@bauer.uh.edu.

1 Introduction

Recent empirical work on equity options has documented several interesting patterns in the cross-section of option returns that are related to the volatility of the underlying securities. Goyal and Saretto (2009) show that straddle returns and delta-hedged call option returns increase as a function of the volatility risk premium, the difference between historical volatility and implied volatility. Vasquez (2012) reports that the slope of the implied volatility term structure is positively related to future option returns. Cao and Han (2013) document a negative relation between the underlying stock's idiosyncratic volatility and delta-hedged equity option returns.

These findings are purely empirical. At a theoretical level, it is of course well understood that option prices increase with underlying volatility, but the existing literature is silent about the theoretical relation between underlying volatility and expected option returns. This paper addresses precisely this issue. Building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), we derive analytical expressions for expected holding period option returns in the context of the Black-Scholes framework. We demonstrate that the expected return on holding a call option is a decreasing function of the underlying volatility, while the expected return on holding a put option is an increasing function of the underlying volatility.

We provide cross-sectional and time-series tests of this theoretical relation between stock volatility and expected option returns. Using the cross-section of stock option returns for 1996-2013, we document that call (put) option portfolio returns exhibit a strong negative (positive) relation with underlying stock volatilities. Sorting all available options into quintiles, we find a a statistically significant difference of -13.8% (7.1%) per month between the average returns of the quintile call (put) option portfolio with the highest underlying stock volatilities and the call (put) quintile portfolio with the lowest underlying volatilities. We show that these findings are not driven by cross-sectional variation in expected stock returns. Our results are robust to using different option maturities and moneyness, as well as alternative measures of underlying volatility and portfolio weighting methods.

We also provide time-series evidence. We find that index call (put) options tend to have lower (higher) returns in the month following high volatility periods. The findings are robust to different index volatility proxies and are not driven by illiquid option contracts. The time-series results complement our cross-sectional findings and provide strong empirical support for our theoretical predictions.

We provide several extensions of the benchmark theoretical analysis. The empirical shortcomings of the Black-Scholes model are well-documented, and we therefore investigate if realistic extensions of the Black-Scholes model lead to different theoretical predictions. We show that if the expected stock return is positively related to the stock's volatility, the theoretical predictions are strengthened. We use realistic parameterizations of the Heston (1993) model to show that, if volatility is time-varying and if the innovations to volatility and returns are correlated, similar theoretical predictions obtain. Finally, we also provide some theoretical results for option straddles.

Interestingly, the notion that call option returns decrease with the underlying volatility has been appreciated and applied in other areas of the finance literature. Galai and Masulis (1976) show, under the joint assumption of the CAPM and the Black-Scholes model, that under certain realistic conditions the expected instantaneous rate of return on firm equity, which is essentially a call option on firm value, decreases with the variance of the rate of return on firm value. Johnson (2004) points out that in a levered firm, the expected equity return should decrease as a function of idiosyncratic asset risk. He uses this insight to explain the puzzling negative relation between stock returns and the dispersion of analysts' earnings forecasts. Lyle (2014) explores the implications of the negative relation between expected call option returns and underlying volatility to study the relation between information quality and future option and stock returns. Broadie, Chernov, and Johannes (2009) use simulations to show that expected put option returns increase with underlying volatility.

We contribute to this literature by analytically demonstrating the negative (positive) relation between expected call (put) option return and underlying volatility, and by providing empirical evidence consistent with these theoretical predictions. To the best of our knowledge these theoretical and empirical findings are new to the literature.

We also contribute to the growing empirical literature on the cross-section of equity option returns. Besides the studies by Goyal and Saretto (2009), Vasquez (2012), and Cao and Han (2013), several recent studies have documented interesting empirical regularities in the cross-section of option returns. Boyer and Vorkink (2014) document a negative relation between ex-ante option total skewness and future option returns. Goodman, Neamtiu, and Zhang (2013) find that fundamental accounting information is related to future option returns. Karakaya (2014) proposes a three-factor model to explain the cross-section of equity option returns. Christoffersen, Goyenko, Jacobs, and Karoui (2014) report illiquidity premia in the cross-section of equity options. Our paper adds to this growing literature by theoretically identifying the relation between expected option returns and stock volatility, perhaps the most intuitively appealing determinant of option prices and returns. Given that this theoretical relationship is validated by the data, our work suggests that new empirical work on option returns may want to control for the effect of volatility when identifying other determinants of option returns.

The paper proceeds as follows. Section 2 provides the theoretical results on the relation between expected option returns and underlying stock volatility in the Black-Scholes model. Section 3 presents our main empirical results, using data on the cross-section of stock option returns. Section 4 discusses extensions of the theoretical and empirical analysis. Section 5 performs an extensive set of robustness checks. Section 6 presents time-series tests using index options, and Section 7 concludes the paper.

2 Volatility and Expected Option Returns: Theory

In this section, we derive the theoretical results on the relationship between option returns and the volatility of the underlying security. We derive these results in the context of the Black-Scholes (1973) model, even though it is well known that the Black-Scholes model has some important empirical shortcomings. Most importantly, more accurate valuation of options is possible by accommodating stochastic volatility as well as jumps in returns and volatility.[1] However, the Black-Scholes model has the important advantage of analytical tractability, and we therefore use it to derive a benchmark set of theoretical results. In Section 4, we investigate if these results continue to hold if other, more realistic, processes are assumed for the underlying securities.

Even within the confines of the Black-Scholes model, the relation between option returns and the volatility of the underlying security can be studied in several ways. Much of the literature on option returns uses expected instantaneous option returns. In the Black-Scholes (1973) model, consider the following notation for the geometric Brownian motion dynamic of the underlying asset:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB \tag{2.1}$$

where S is the price of underlying asset, σ is the volatility parameter, and μ is the drift or the expected return of the underlying asset. It can be shown that in this model, the expected instantaneous option return is linear in the expected instantaneous return on underlying asset:

$$E(\frac{dO}{O}) = rdt + \frac{\partial O}{\partial S}\frac{S}{O}(\mu - r)dt \tag{2.2}$$

where O is the price of the European option, and r is the risk-free rate. This expression provides some valuable intuition regarding the determinants of expected option returns. The expected option return depends on $\frac{\partial O}{\partial S}\frac{S}{O}$, which reflects the leverage embedded in the option. The leverage itself is a function of moneyness, maturity, and the volatility of the underlying security.

While equation (2.2) provides valuable intuition, it has some important limitations for our

[1] Bates (2003) and Garcia, Ghysels, and Renault (2010) provide excellent surveys.

purposes. It describes the relation between expected option returns with underlying volatility at each instant, but these returns are not empirically observable. For empirically observable holding periods, the linear relation between the option returns and the underlying asset returns may not hold because the option price is a convex function of the price of underlying asset.

We therefore analyze the impact of underlying volatility on expected option returns by building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), who point out that expected option returns can be computed analytically within models that allow for analytic expressions for option prices. For our benchmark results, we therefore rely on the classical Black-Scholes option pricing model to obtain an analytical expression for the expected return of holding an option to maturity. We then compute the first order derivative of the expected option return with respect to the volatility of the underlying security. We show that the expected return for holding a call option to maturity is a decreasing function of the underlying volatility, while the expected return for holding a put option to maturity is an increasing function of underlying volatility.

Denote the time t prices of European call and put options with strike price K and maturity T by $C_t(t,T,S_t,\sigma,K,r)$ and $P_t(t,T,S_t,\sigma,K,r)$ respectively. By definition, the expected gross returns for holding the options to expiration are given by:

$$R_{call} = \frac{E_t[\max(S_T-K,0)]}{C_t(t,T,S_t,\sigma,K,r)} \tag{2.3}$$

$$R_{put} = \frac{E_t[\max(K-S_T,0)]}{P_t(t,T,S_t,\sigma,K,r)} \tag{2.4}$$

Propositions 1 and 2 indicate how these expected call and put option returns change with respect to the underlying volatility σ. We provide the detailed proof for the case of the call option in Proposition 1, because the proof provides valuable intuition for the result. The intuition for the case of the put option is similar and the proof is relegated to the appendix.

Proposition 1 *Everything else equal, the expected return of holding a call option to expiration is higher if the underlying asset has lower volatility* ($\frac{\partial R_{call}}{\partial \sigma} < 0$).

Proof. We start by reviewing several well-known facts that are needed to derive the main result. If the underlying asset follows a geometric Brownian motion, the price of a European call option written on the asset is given by the Black-Scholes formula:

$$C_t(t, T, S_t, \sigma, K, r) = S_t N(d_1) - e^{-r\tau} K N(d_2) \quad (\tau = T - t) \tag{2.5}$$

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \tag{2.6}$$

Vega is the first-order derivative of the option price with respect to the underlying volatility. It measures the sensitivity of the option price to small changes in the underlying volatility. The Black-Scholes Vega is the same for call and put options:

$$\nu = \sqrt{\tau} S_t \psi(d_1) \tag{2.7}$$

where ψ is the probability density function of the standard normal distribution. We also have:

$$S_t \psi(d_1) = e^{-r\tau} K \psi(d_2). \tag{2.8}$$

Now we are ready to derive the main result. We first write the expected call option return in (2.3) in a convenient way, so that both the numerator and the denominator exhibit the functional form of the Black-Scholes formula. This allows us to conveniently evaluate the derivative of the expected option return with respect to the underlying volatility, using the Black-Scholes Vega in (2.7).

The denominator of (2.3) is the price of the call option and is therefore given by the Black-Scholes formula in (2.5). The numerator of (2.3), the expected option payoff at expiration, can be transformed into an expression that has the same functional form as the Black-Scholes formula. We get:

$$E_t[\max(S_T - K, 0)] = \int_{z^*} (S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z} - K) \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{2.9}$$

$$= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] \tag{2.10}$$

where

$$d_1^* = \frac{\ln \frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln \frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{2.11}$$

Combining (2.5) and (2.10), the expected return for holding a European call option to maturity is given by:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, \sigma, K, r)} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}. \tag{2.12}$$

Using a different approach, Boyer and Vorkink (2014) derive an equivalent expression for the expected return of holding a call option to maturity.

Taking the derivative of (2.12) with respect to σ gives:

$$\begin{aligned} \frac{\partial R_{call}}{\partial \sigma} &= \frac{e^{\mu\tau}\sqrt{\tau} S_t \psi(d_1^*)[S_t N(d_1) - e^{-r\tau} K N(d_2)] - e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]\sqrt{\tau} S_t \psi(d_1)}{[S_t N(d_1) - e^{-r\tau} K N(d_2)]^2} \\ &= \frac{e^{\mu\tau}\sqrt{\tau} S_t \{\psi(d_1^*)[S_t N(d_1) - e^{-r\tau} K N(d_2)] - \psi(d_1)[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]\}}{[S_t N(d_1) - e^{-r\tau} K N(d_2)]^2}. \end{aligned} \tag{2.13}$$

Note that we use equation (2.7) to derive (2.13). From (2.13) it can be seen that $\frac{\partial R_{call}}{\partial \sigma}$ inherits the sign of $\psi(d_1^*)[S_t N(d_1) - e^{-r\tau} K N(d_2)] - \psi(d_1)[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]$. Denote this expression by EX. We now show that EX is negative. We have:

$$\frac{1}{\psi(d_1^*)\psi(d_1)} EX = \frac{S_t N(d_1) - e^{-r\tau} K N(d_2)}{\psi(d_1)} - \frac{S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)}{\psi(d_1^*)} \tag{2.14}$$

Using equation (2.8), it follows that

$$\frac{1}{\psi(d_1^*)\psi(d_1)} EX = \frac{S_t N(d_1) - \frac{S_t \psi(d_1)}{\psi(d_2)} N(d_2)}{\psi(d_1)} - \frac{S_t N(d_1^*) - \frac{S_t \psi(d_1^*)}{\psi(d_2^*)} N(d_2^*)}{\psi(d_1^*)} \tag{2.15}$$

$$= S_t[(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)})]. \tag{2.16}$$

Because according to economic theory, the expected rate of return on risky assets must exceed the risk-free rate ($\mu > r$), we have $d_1^* > d_1$ and $d_2^* > d_2$. We also have $d_1^* > d_2^*$ and $d_1 > d_2$, from the definition of (2.6) and (2.11). Now consider $\frac{N(d)}{\psi(d)}$. It can be shown that it is an increasing and convex function of d. Evaluating $\frac{N(d)}{\psi(d)}$ at d_1, d_2, d_1^*, and d_2^*, it can be seen that the expression $(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)})$ effectively amounts to the negative of the second difference (derivative) of an increasing and convex function. Therefore:

$$(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)}) < 0. \tag{2.17}$$

This implies $EX < 0$ which in turn implies $\frac{\partial R_{call}}{\partial \sigma} < 0$. ■

Proposition 2 *Everything else equal, the expected return of holding a put option to expiration* *is higher if the underlying asset has higher volatility* ($\frac{\partial R_{put}}{\partial \sigma} > 0$).

Proof. See Appendix A. ■

3 Empirical Results: Volatility and the Cross-Section of Option Returns

In this section, we empirically test Propositions 1 and 2 using the cross-section of options written on individual stocks. For our benchmark empirical analysis, we use the cross-section of stock options that are at-the-money and one month away from expiration. Our motivation is that Propositions 1 and 2 predict a relationship between expected option returns and underlying volatility, everything else equal. It is therefore critical to control for other option characteristics that affect returns when studying the relation between option returns and the underlying volatility. Existing studies have documented that moneyness and maturity also affect option returns, see for example Coval and Shumway (2001).

To address this issue, we therefore use an option sample that is homogeneous in maturity and moneyness. We choose one month at-the-money options because these are the most frequently traded options, and they are subject to fewer data problems (see, among others, Goyal and Saretto, 2009).

3.1 Data

We obtain stock return data from CRSP and relevant accounting information from Compustat. We obtain option data from OptionMetrics through WRDS. OptionMetrics provides historical option closing bid and ask quotes, as well as information on the underlying securities for U.S. listed index options and equity options. Every month, on the first trading day after monthly option expiration, we select equity options with $0.95 \leq K/S \leq 1.05$ that expire over the next month. The expiration day for standard exchanged-traded options is the Saturday immediately following the third Friday of the expiration month, so our sample consists mainly of Mondays. Occasionally it is a Tuesday if Monday is an exchange holiday.

We apply several standard filters to the option data. An option is included in the sample if it meets all of the following requirements: 1) the best bid prices is positive and the best bid price is smaller than the best offer price; 2) it does not violate no-arbitrage bounds; for call options we require that the price of the underlying exceeds the best offer, which is in turn higher than $max(0, S - K)$; for put options we require that the exercise price exceeds the best bid, which is in turn higher than $max(0, K - S)$; 3) for equity options, we require that no dividend is paid over the duration of the option contract; 4) open interest is positive; 5) volume is positive; 6) we require that the bid-ask spread exceeds \$0.05 when the option price is below \$3, and \$0.10 when the option price is higher than \$3; 7) the expiration day is standard, the Saturday following the third Friday of the month; 8) settlement is standard; 9) implied volatility is not missing.

We compute the monthly return from holding the option to expiration using the mid-point of the bid and ask quotes as a proxy for the market price of the option contract.[2] If an option expires in the money, the return to holding the option to maturity is the difference between the terminal payoff and the initial option price divided by the option price. If an option expires out of the money, the option return is −100%. Our equity option sample contains 247,859 call options and 188,046 put options over the time period from January 1996 to July 2013. In our benchmark results, we measure volatility using the realized volatility computed using daily data for the preceding month, and we refer to this as 30-day realized volatility.[3]

Table 1 reports summary statistics for equity options across moneyness categories. Moneyness is defined as the strike price over the underlying stock price. On average the returns to buying call (put) options are positive (negative). Both call and put option returns increase with the strike price. These patterns are consistent with the stylized facts in Coval and Shumway (2001). Also note that option-implied volatility exceeds realized volatility for all moneyness categories, but the differences are often small. Gamma and Vega are highest for at-the-money options and decrease as options move away from the money.

3.2 The Cross-Section of Option Portfolio Returns

Each month, on each portfolio formation date, we sort the options into five quintile portfolios based on their realized volatility, and we compute equal-weighted returns for these option portfolios over the following month. We conduct this exercise for call options and put options separately.

Panel A of Table 2 displays the averages of the resulting time series of returns for the five call option portfolios, as well as the return spread between the two extreme portfolios. Portfolio "Low" contains call options with the lowest realized volatility, and portfolio "High" contains

[2]Stock options are American options. Several studies (see, among others, Boyer and Vorkink 2014) have demonstrated that adjusting for early exercise has very minimal empirical implications. We further minimize the impact of the American feature of the contract by only studying equity options that do not have an ex-dividend date during the life of the option contract.

[3]Because this measure uses data for the previous month, it is effectively based on approximately 22 returns. For convenience, we use calendar days and refer to it as 30-day volatility. The same remark applies to volatility measures for other horizons used throughout the paper.

call options with the highest realized volatility. Proposition 1 states that the expected call option return is a decreasing function of the underlying stock volatility. Consistent with this result, we find that call option portfolio returns decrease monotonically with the underlying stock volatility. The average returns for portfolio High and portfolio Low are 0.9% and 14.7% per month respectively. The resulting return difference between the two extreme portfolios (H-L) is −13.8% per month and highly statistically significant, with a Newey-West (1987) t-statistic of −3.42.

Panel B of Table 2 presents the averages of the resulting time-series of returns for the five put option portfolios. Again, portfolio Low (High) contains put options with the lowest (highest) underlying stock volatilities. For put option portfolios, the average return increases from −14.6% per month for portfolio Low to −7.5% per month for portfolio High, with a positive and significant H-L return difference of 7.1%. This finding confirms Proposition 2, which states that expected put option returns are an increasing function of the underlying stock volatility.

Table 2 also provides results using only options with $0.975 \leq K/S \leq 1.025$. By using a tighter moneyness interval, we reduce the impact of moneyness on expected option returns. The results are very similar. The average option portfolio returns decrease (increase) with the underlying stock volatility for calls (puts). The H-L differences are −13.8% and 7.7% for call and put option portfolios respectively, and are statistically significant. This suggests that our empirical results are not due to differences in option moneyness.

These results are obtained using option returns computed using the mid-point of the bid and ask quotes. To ensure that our results do not depend on this assumption, Panel C of Table 2 computes average option portfolio returns based on the ask price. As expected, average returns are somewhat smaller than in Panels A and B. However, we again find a strong negative (positive) relation between call (put) option portfolio returns and the underlying stock volatility. The H-L differences are both statistically significant and are of a similar order of magnitude than the ones reported in Panels A and B.

4 Discussion and Extensions

In this section, we further extend our theoretical and empirical analysis. An potential concern is that our theoretical results are obtained using the Black-Scholes model. The Black-Scholes model has some well-known empirical shortcomings, and it is possible that adjusting the theoretical model for these empirical shortcomings may affect the results. Most importantly for the purpose of our investigation, it is well-known that Black-Scholes assumption of constant volatility is strongly rejected by the data. We therefore investigate the implications of the Heston (1993) stochastic volatility model for returns.

Another potential problem is that in the Black-Scholes model, the expected return on the underlying stock is the constant μ. However, if we model the drift of the underlying stock using equilibrium model, it is possible to obtain a result where the drift is positively related to the volatility of the underlying stock or the stock index. It is therefore important to investigate how a positive relationship between μ and σ could affect our theoretical results.

At a purely empirical level, we document the cross-sectional relation between volatility and option returns after controlling for the return on the underlying stock. We also explore the relation between volatility and straddle returns, because prices of straddles are volatility-sensitive and expected straddle returns do not depend on the drift of the underlying stock.

4.1 Volatility-Dependent Return Drifts

In the simple Black-Scholes set-up, μ and σ are constants and thus by definition independent. The impact of volatility on expected option returns is therefore exclusively due to the diffusive part of the stock return. For more general models, or when considering equilibrium foundations for the Black-Scholes model, the return drift and the volatility are not likely to be independent. For instance, a CAPM-type setup suggests a positive relation between the drift and volatility in the case of index returns. For stock returns the relation with stock volatility is of course not as explicit, but it is reasonable to assume that stock volatility will be positively related to the determinants of the risk premium. It is therefore worth exploring if our empirical findings are due to the indirect effect of volatility on expected option returns through μ.

A positive relation between expected stock returns and volatility means that a higher volatility leads to a higher μ, which further implies a higher expected call option return and a lower expected put option return, as the expected call (put) option return increases (decreases) with μ. This is a fairly intuitive result which can also be shown analytically.[4] This channel therefore has the opposite effect of the relation between volatility and option returns highlighted by Propositions 1 and 2. In other words, if the expected stock return is (partly) determined by stock volatility, this will reduce the strength of the empirical relationship documented in Table 2.

We therefore conclude that the empirical relationship documented in Table 2 is not due to volatility-dependent drifts in the underlying security, and more likely due to the theoretical channels studied in Propositions 1 and 2.

[4] Appendix B shows that $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$.

4.2 Controlling for Expected Stock Returns

Section 4.1 shows analytically that if expected stock returns depend positively on volatility, which is plausible, our cross-sectional findings in Section 3 cannot be explained by the effect of volatility on return drifts. We now further investigate if the underlying stock returns affect our results by empirically controlling for expected stock returns. This is of course challenging because available measures of expected returns are likely to be very noisy.

Expected call option returns increase with the expected return of the underlying asset and expected put option returns decrease with the expected return of underlying asset. Thus, if the high volatility portfolios in Section 3 are primarily composed of stocks that have lower expected returns than those in the low volatility portfolios, the result that average call (put) options in the high volatility portfolios earn lower (higher) returns may not be due to volatility.

In this section, we therefore study the relation between option returns and underlying stock volatility after controlling for expected stock returns. Unlike volatility, expected stock returns are notoriously difficult to measure. According to the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972), the covariance of a stock's return with the market or beta is the sole determinant of the cross-sectional variation in expected stock returns. We therefore first rely on a double sort on beta and volatility to ensure our results are not driven by cross-sectional differences in expected stock returns. We form five quintile portfolios based on underlying stock betas, and then within each beta quintile options are further sorted into five quintile portfolios according to underlying stock volatility. We once again measure underlying stock volatility by 30-day realized volatility.[5] Beta is estimated using the market model over the 30 days preceding the portfolio formation date:

$$r_t^i - r_t^f = \alpha^i + \beta^i (r_t^M - r_t^f) + \epsilon_t^i$$

where r_t^i and r_t^M are the daily returns on stock i and the market respectively, and r_t^f is the risk-free rate.

Table 3 presents the results of this double sort. Consistent with the single sort results, in each beta quintile call (put) option portfolio returns decrease (increase) with underlying volatility. In all beta quintiles, the average return differences between the two extreme call option portfolios are negative, ranging from -17% to -8% per month, and statistically significant. For put options, the high minus low differences are all positive and statistically significant for the third through fifth beta quintiles. These findings strongly suggest that our results are not driven by differences between the expected returns of the underlying stocks.

[5]Results are similar when using other volatility measures.

Next, we run Fama-MacBeth (1973) regressions that allow us to simultaneously control for risk factors and stock characteristics that are related to expected stock returns. Every month we run the following cross-sectional regression

$$R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon \tag{4.1}$$

where R^i_{t+1} is the return on holding option i from month t to month $t+1$, VOL^i_t is the underlying stock volatility for option i, and Z^i_t is a vector of control variables that includes moneyness, stock beta, firm size, and book-to-market. Both VOL^i_t and Z^i_t are observable at time t for option i. We use three proxies for the underlying stock volatility: 30-day, 60-day, and 365-day realized volatility, all computed using daily returns with the relevant windows.

Table 4 reports the time-series averages of the γ and Φ coefficients in equation (4.1), along with Newey-West (1987) t-statistics which adjust for potential autocorrelation and heteroscedasticity. Columns (1) to (3) report regression results for call options. The average slope coefficient is always negative and highly significant for all volatility measures. For example, in column (1) of Table 4 we find the average slope coefficient on 30-day realized volatility is -0.254 with a Newey-West t-statistics of -4.57. Because the difference in average underlying volatility between the two extreme call option portfolios in Table 2 is 0.6, this implies a decline of $-0.254 \times 0.6 = 15.24\%$ per month in average returns if a call option were to move from the bottom volatility portfolio to the top volatility portfolio, other characteristics held constant. Note that this estimate is very similar to the result in Table 2.

In columns (4)-(6), we provide results for put options. As expected, the average slope coefficient on underlying volatility is positive and statistically significant for all volatility proxies, ranging from 10.5% to 22.2% per month. These findings again suggest that our results cannot be attributed to differences in expected stock returns.

The empirical results in Tables 3 and 4 strongly suggest that our benchmark results in Table 2 are not due to differences in the returns on the underlying stocks. Because it is difficult to control for expected returns, we see these results as merely as a confirmation of the analytical argument in Section 4.1. Overall, we conclude that our main result that in the cross-section, call (put) options with high underlying volatilities tend to have lower (higher) returns in the next month is not only consistent with Propositions 1 and 2, but is due to the mechanism by which volatility affects expected option returns in the simple Black-Scholes model.

4.3 Holding-Period Expected Option Returns

Appendix C derives the analytical expression for the expected call option return over any holding period h in the Black-Scholes model:[6]

$$
\begin{aligned}
R^h_{call} &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \qquad (4.2) \\
d_1^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu-r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\
d_2^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu-r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}}
\end{aligned}
$$

where the timeline is shifted to $[0, T]$ from $[t, T]$ to ease notation, h is the holding period ($0 < h < T$), and $HP = h/T$ is the ratio of the holding period to the life of the option contract. In the interest of brevity, we exclusively focus on call options in this section. The extension to put options is straightforward.

Note that the expected holding-to-expiration option return in (2.12) is nested in (4.2), for $HP = 1$. We can use the structure of the proof of Proposition 1 to show $\frac{\partial R^h_{call}}{\partial \sigma} < 0$, by observing $r+(\mu-r)HP > r$. Thus, we conclude that expected call (put) option returns decrease (increase) with underlying volatility for any holding period in the Black-Scholes model.

4.4 Expected Option Returns in the Heston Model

In Section 4.1, we addressed a potential shortcoming of the Black-Scholes model for studying the impact of volatility on expected option returns. We established that extending the model by allowing for a positive relation between the drift of the underlying stock and the volatility cannot explain the empirical relationship documented in Table 2. When considering volatility, the Black-Scholes model has another well-documented disadvantage. A large number of studies have shown that volatility is time varying, and that (the innovations to) volatility and stock returns are correlated.[7] This correlation is often referred to as the leverage effect.

To address the implications of time-varying volatility and the leverage effect, we now analyze expected option returns using the Heston (1993) model instead of the Black-Scholes model. The Heston model has become an important modeling tool in the option literature, because it captures important stylized facts such as time-varying volatility and the leverage effect, while

[6] See Rubinstein (1984) for a similar expression based on a discrete-time model.

[7] See Engle (1982), Bollerslev (1986), Nelson (1991), Glosten, Jagannathan and Runkle (1993), and Engle and Ng (1993), among many others.

also allowing for quasi-closed form European option prices. Appendix D shows that the expected return of holding a call option to expiration in the Heston model is given by:

$$R_{call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*]}{S_t P_1 - e^{-r\tau} K P_2} \tag{4.3}$$

where P_1, P_2, P_1^* and P_2^* are defined in Appendix D. The expected call option return in the Heston model has the same functional form as in the Black-Scholes model. Unlike for the case of the Black-Scholes model, the sign of $\frac{\partial R_{call}^{Heston}(S_t, V_t, \tau)}{V_t}$ cannot be derived analytically. However, the expected option return in equation (4.3) can be easily calculated given a set of parameter values.

In Table 5, we compute expected option returns according to (4.3) for different parameterizations of the expected stock returns and the conditional stock variance. For all other parameters, we use the parameters from Broadie, Chernov, and Johannes (2009). Table 5 indicates that the patterns in expected option returns in a stochastic volatility model are similar to the patterns in Black-Scholes expected option returns. In particular, expected call option returns increase (decrease) with expected stock return (current stock variance), whereas expected put option returns decrease (increase) with expected stock return (current stock variance). In unreported results using other parameterizations, similar conclusions obtain.

4.5 Volatility and Expected Straddle Returns

We now investigate the cross-sectional relation between volatility and expected straddle returns. Most of the existing papers that investigate the cross-sectional relation between option returns and different aspects of volatility focus on straddle returns. See for example Goyal and Saretto (2009) and Vasquez (2012). Those studies do not test a direct implication of a theoretical model, and they focus on straddles to separate the cross-sectional effects of volatility and the underlying stock returns. Our approach is somewhat different because our main focus is to test the theoretical results in Propositions 1 and 2. These results are for expected returns on puts and calls, and within the context of the Black-Scholes model the expected return on the underlying stocks are explicitly taken into account. However, as discussed in Section 4.1, when considering a generalization of the Black-Scholes model that allows for volatility-dependent drifts, controlling for the expected stock return becomes relevant. This motivates us to look at returns on straddles.

A straddle consists of the simultaneous purchase of a call option and a put option on the same underlying asset. The call and put options have the same strike price and time to maturity.

The expected gross return on a straddle is given by:

$$R_{straddle} = \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)}$$

where $C_t(\tau, S_t, \sigma, K, r)$ and $P_t(\tau, S_t, \sigma, K, r)$ are the call and put prices that an investor has to pay to build a long position in straddle. Appendix E shows that $d_2 > 0$ is a sufficient condition for a negative relation between straddle returns and underlying volatility. Recall that $d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$. Therefore, the condition $d_2 > 0$ is likely to hold for straddles with strike prices below the current stock price. We therefore investigate if average straddle returns decrease with underlying volatility for such straddles.

Table 6 reports the empirical results. Consistent with our hypothesis, we find a strong negative relation between straddle returns and underlying stock volatilities. Each month, we form straddles using only equity options with $0.95 \leq K/S \leq 1$. These straddles are sorted into five quintile portfolios based on 30-day realized volatility. We then compute equal-weighted and volume-weighted straddle portfolios returns over the following month. The straddle volume is the average of call and put volume.

Panel A of Table 6 reports time-series average returns for the five straddle portfolios. The returns decrease with underlying stock volatility. For example, the equal-weighted average straddle portfolio returns drop from 2.8% per month for the low volatility portfolio to −2.6% per month for the high volatility portfolio. The resulting return difference is −5.4% per month and highly statistically significant with a Newey-West adjusted t-statistic of −2.90. With volume weighting, the return spread is −6.3% per month and also statistically significant.

In Panels B and C of Table 6, we investigate two more straddle samples with $0.875 \leq K/S \leq 0.95$ and $0.8 \leq K/S \leq 0.875$. The results are very similar and reinforce our conclusion that there is a negative relation between straddle returns and underlying volatility provided the condition $d_2 > 0$ holds.

5 Robustness

In this section we investigate the robustness of the results in Table 2 to a number of implementation choices. We investigate the robustness to the measure of realized volatility, the weights used to compute portfolio returns, and the composition of the option sample.

5.1 Robustness With Respect to the Volatility Measure

Table 2 uses realized volatility computed using daily data for the preceding month to proxy for the underlying volatility. This is a standard volatility measure that is often used in the literature. Ang, Hodrick, Xing, and Zhang (2006) and Lewellen and Nagel (2006) argue that 30-day realized volatility strikes a good balance between estimating parameters with a reasonable level of precision and capturing the conditional aspect of volatility. We now consider four alternative estimators of underlying stock volatility. We proxy underlying volatility using realized volatilities computed over the past 14 days, the past 60 days, and the past 365 days, as well as using option-implied volatility.

Panel A of Table 7 presents time series average returns for the five call option portfolios and Panel B reports average returns for put option portfolios. Consistent with our benchmark results in Table 2, we find that for all underlying volatility proxies, the returns on the call option portfolios exhibit a strong negative relation with underlying stock volatilities, while put option portfolio returns display a strong positive relation with underlying stock volatilities. For example, when sorted on 60-day realized volatility, the average returns for call option portfolios with the largest and smallest underlying volatilities are 1.4% and 15.5% per month respectively. The resulting difference between the two extreme portfolios is −14.1% per month and is highly statistically significant with a Newey-West t-statistic of −3.44. In contrast, for put option portfolios, the average returns monotonically increase from −15.7% per month for the lowest volatility portfolio to −5.9% per month for the highest volatility portfolio. The resulting difference is 9.8% per month and is also statistically significant.

When sorted on 14-day and 365-day realized volatility, the returns display a same pattern. The average returns decrease (increase) with underlying volatilities for call (put) portfolios. The return differences between the two extreme call option portfolios are negative and statistically significant with a magnitude of −13% and −8.6% per month, respectively. The corresponding differences for put option portfolios are positive and statistically significant, with a magnitude of 5.9% and 11.7% per month, respectively.

We also sort options based on their implied volatilities. Option-implied volatilities are attractive because they provide genuinely forward-looking estimates, but they are model-dependent and may include volatility risk premiums.[8] Again consistent with our benchmark results, we find that call (put) option portfolios with larger implied volatilities earn lower (higher) returns. Panel

[8] Equity options are American options. We use implied volatilities from OptionMetrics, which are calculated using the Cox, Ross, and Rubinstein binomial tree model. On the volatility risk premium embedded in individual stock options, see Bakshi and Kapadia (2003b), Driessen, Maenhout, and Vilkov (2009), and Carr and Wu (2009) for more details.

A of Table 7 reveals that returns on call option portfolios monotonically decrease with implied volatilities. The return spread is -16.6% per month and is highly statistically significant. The return spread for the two extreme put option portfolios is positive with a magnitude of 5.9% per month, but it is not statistically significant.

5.2 Robustness With Respect to the Option Sample

We now investigate the relationship between expected option returns and underlying volatility using five other option samples with different maturities and moneyness. We examine the following five option samples: two-month at-the-money options, one-month in-the-money options, two-month in-the-money options, one-month out-of-the-money options, and two-month out-of-the-money options. We define at-the-money as having moneyness of $0.95 \leq K/S \leq 1.05$, in-the-money calls as $0.80 \leq K/S < 0.95$, and in-the-money puts as $1.05 < K/S \leq 1.20$. Out-of-the-money calls are defined as $1.05 < K/S \leq 1.20$ and out-of-the-money puts as $0.80 \leq K/S < 0.95$. The results are presented in Table 8.

Panel A of Table 8 provides time-series average returns of call option portfolios sorted on 30-day realized volatility for the five alternative option samples. Consistent with the benchmark results in Table 2, we find that returns on call option portfolios decrease with underlying volatility for all option samples. The return differences between the two extreme portfolios are negative and statistically significant in all cases, with magnitudes ranging from -7.8% to -18.6% per month. For instance, for two-month at-the-money calls, the equal-weighted average option portfolio returns decrease monotonically with underlying volatility. The return spread is -17.1% per month and highly significant with a Newey-West t-statistic of -3.04.

Panel B of Table 8 summarizes average returns of put option portfolios sorted on 30-day realized volatility for the five option samples. Average put option returns exhibit a strong positive relationship with underlying volatilities. The returns spreads are all positive and statistically significant, ranging from 5.7% to 17.8% per month. For instance, for two-month at-the-money puts, average returns monotonically increase from -20.7% per month for the lowest volatility portfolio to -5.6% per month for the highest volatility portfolio. The resulting return spread is 15.1% per month and is both economically and statistically significant.

5.3 Robustness With Respect to the Portfolio Weighting Method

In this subsection, we examine if the negative (positive) relation between call (put) option portfolio returns and underlying volatility persists if different weighting methods are used for computing option portfolio returns. We calculate option volume weighted, option open interest weighted and option value weighted average portfolio returns. Option value is defined as the product of the option's open interest and its price.[9]

Table 9 contains return spreads for option portfolios sorted on 30-day realized volatility, using these alternative weighting methods. Regardless of the weighting method, the return spreads are negative (positive) for call (put) option portfolios, and they are statistically significant in most cases. These results suggest that our empirical findings are not due to the equal-weighting method used in Table 2.

6 Volatility and the Time Series of Index Option Returns

Section 3 uses the cross-section of equity options to provide empirical evidence supporting Propositions 1 and 2. We now turn to the implications of our results for the extensive literature on index option returns.[10] We document that call (put) options with high underlying stock volatilities tend to have lower (higher) returns in the subsequent month. In this section, we explore the time-series implications of Propositions 1 and 2 by studying the relation between monthly S&P 500 index option (SPX) returns and S&P 500 index volatility. Consistent with Proposition 1 and 2, we find that SPX call (put) options tend to have lower (higher) returns in the month following a high volatility month.

6.1 Data

On the first trading day after each month's option expiration date, we collect index options that mature in the next month with $0.9 \leq K/S \leq 1.1$. Table 10 provides summary statistics for SPX option data by moneyness. Index put options (especially out-of-the-money puts) generate large negative returns, consistent with the existing literature (see, among others, Bondarenko, 2003). For example, for the moneyness interval $0.94 < K/S \leq 0.98$, the average return is -40.6% per month in our sample. Table 10 also shows that in our sample, out-of-the-money SPX calls have large negative returns. This is puzzling because expected call option returns should increase as a function of the strike price. This may be due to our sample period.

Comparing Tables 10 and 1 highlights several important differences between index options and individual stock options. First, the volatility skew, the slope of implied volatility against

[9] We also considered portfolio returns weighted by underlying stock capitalization and find similar results. These results are available upon request.

[10] This literature includes the work by Jackwerth (2000), Coval, and Shumway (2001), Bakshi, and Kapadia (2003), Bondarenko (2003), Jones (2006), Driessen, and Maenhout (2007), Driessen, Maenhout, and Vilkov (2009), Santa-Clara, and Saretto (2009), Broadie, Chernov, and Johannes (2009), Constantinides et al. (2009, 2011 and 2013) and Buraschi, Trojani, and Vedolin (2014).

moneyness, is much less pronounced for individual stock options. Second, the average monthly S&P 500 index volatility is 17%, which translates into a volatility risk premium (implied volatility minus realized volatility) as large as 10% for index options. This is in stark contract to the volatility risk premium we observe for individual stock options. Third, index options have much larger Vega and smaller Gamma than stock options.

6.2 Analyzing the Time Series of Index Option Returns

Propositions 1 and 2 characterize a general property of expected option returns: call (put) option returns decrease (increase) with underlying volatility. This property should hold in the time series of option returns as well as in the cross-section. We investigate the time-series implications of Propositions 1 and 2 by using index option returns to estimate the following time-series regression:

$$R^i_{t+1} = constant + \beta_1 VOL_t + \beta_2 Moneyness^i_t + \beta_3 Index\_ret_t + \epsilon \quad (6.1)$$

where R^i_{t+1} is the return on holding index option i from month t to month $t+1$, $Index\_ret_t$ is the return of S&P 500 in month t and VOL_t is the index volatility. We consider four proxies for S&P 500 index volatility: the 14-day realized volatility, the 30-day realized volatility, the 60-day realized volatility, and the implied volatility. These volatilities are defined as in the cross-sectional analysis and are known in month t.

The slope coefficient estimate on volatility β_1 is the main object of interest. According to Propositions 1 and 2, we expect β_1 to be negative for SPX call options and positive for SPX put options. Moneyness (K/S) is also included in the regression because previous studies (e.g., Coval and Shumway, 2001) have shown that moneyness is an important determinant of option returns.

Table 11 presents the coefficient estimates, t-statistics, and adjusted R-squares for the regressions in equation (6.1). Consistent with Propositions 1 and 2, the slope coefficient on index volatility is always negative (positive) for SPX call (put) options, regardless of the index volatility proxy. For example, column (2) of Panel A of Table 11 shows that when using 30-day realized volatility as the volatility proxy, the slope coefficient on index volatility is -0.92 for SPX calls and is highly significant with a t-statistic of -3.79. For a 1% increase in S&P 500 volatility, the return to holding an SPX call option over the next month is expected to decrease by 0.92%. In contrast, in column (2) of Panel B of Table 11, the slope coefficient on index volatility for SPX puts is 1.39 and it is also highly statistically significant.

Table 10 indicates that in-the-money SPX options are much less traded than their at-the-money and out-of–the-money counterparts. The regressions so far, however, are based on the full sample that also contains in-the-money SPX options. To ensure our results are not driven by illiquid in-the-money options, we repeat the regressions in (6.1) using only liquid options. Specifically, we only consider SPX calls with $0.98 \leq K/S \leq 1.10$ and SPX puts with $0.90 \leq K/S \leq 1.02$.

The regression results using only liquid options are presented in column (5) to (8) in Table 11. Consistent with the results using the full sample, we find that the slope coefficient estimate on index volatility is always negative (positive) and statistically significant for SPX calls (puts) regardless of the volatility proxy. For example, when using 60-day realized volatility as a proxy, we find a slope coefficient of -1.62 for SPX calls and 1.58 for SPX puts, and both are highly significant with t-statistics of -3.77 and 2.98 respectively. These results confirm that our findings are not due to illiquid index options.

7 Conclusion

This paper analyzes the relation between expected option returns and underlying volatility. We demonstrate analytically that in the simple Black-Scholes framework, the expected call option return is a decreasing function of underlying volatility and the expected put option return is an increasing function of underlying volatility.

Our empirical results confirm this theoretical prediction. We conduct a cross-sectional test using stock options. We find that call (put) options on high volatility stocks tend to have lower (higher) returns over the next month. We also conduct a time-series test using index option returns. Following high volatility periods, index call (put) options tend to have lower (higher) returns over the next month. Our empirical findings are robust to different empirical implementation choices, such as different option samples, weighting methods, and volatility proxies..

Our findings have important implications for other areas of finance research. Many financial instruments, such as credit default swaps, callable bonds, and levered equity, to name just a few, have embedded option features. Our theoretical results are also applicable to these assets. We plan to address this in future research.

Appendix A: Proof of Proposition 2

In this appendix, we present the proof of Proposition 2. The expected gross return of holding a put option to expiration in (2.4) can be rewritten using the Black-Scholes formula.

$$\begin{aligned} R_{put} &= \frac{E_t[\max(K - S_T, 0)]}{P_t(\tau, S_t, \sigma, K, r)} \\ &= \frac{\int^{z^*}(K - S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z})\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz}{P_t(\tau, S_t, \sigma, K, r)} \\ &= \frac{e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]}{e^{-r\tau}KN(-d_2) - S_tN(-d_1)} \end{aligned} \tag{A.1}$$

$$d_1^* = \frac{\ln\frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln\frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln\frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Taking the derivative with respect to σ in (A.1) yields:

$$\begin{aligned} \frac{\partial R_{put}}{\partial\sigma} &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\sqrt{\tau}S_t\psi(-d_1)}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \\ &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\{\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\}}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \end{aligned}$$

where we use the fact that the Vega of a put option is $\sqrt{\tau}S_t\psi(-d_1)$. Clearly, the sign of $\frac{\partial R_{put}}{\partial\sigma}$ depends on $\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]$, which we denote by B. Next we show B is positive. To see this,

$$\begin{aligned} B &= \psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)] \\ \frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{e^{-r\tau}KN(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)}. \end{aligned}$$

Using the fact that $e^{-r\tau}K\psi(-d_2) = S_t\psi(-d_1)$,

$$\begin{aligned}
\frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{\frac{S_t\psi(-d_1)}{\psi(-d_2)}N(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{\frac{S_t\psi(-d_1^*)}{\psi(-d_2^*)}N(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)} \\
&= S_t\{[\frac{N(-d_2)}{\psi(-d_2)} - \frac{N(-d_1)}{\psi(-d_1)}] - [\frac{N(-d_2^*)}{\psi(-d_2^*)} - \frac{N(-d_1^*)}{\psi(-d_1^*)}]\} \\
&= S_t\{[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]\}.
\end{aligned}$$

Because the expected rate of return on a risky asset exceeds the risk-free rate ($\mu > r$), we have $d_1^* > d_1$ and $d_2^* > d_2$. One can easily verify that $\frac{N(-d)}{\psi(-d)}$ is a decreasing and convex function in d. It follows that[11]

$$[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}] > 0.$$

Therefore,

$$B > 0 \Rightarrow \frac{\partial R_{put}}{\partial \sigma} > 0$$

Appendix B: Expected Stock Returns and Expected Option Returns

In this appendix, we show that expected call (put) option returns increase (decrease) with expected stock returns: $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$. First, recall from (2.12):

$$R_{call} = \frac{e^{\mu\tau}[S_tN(d_1^*) - e^{-\mu\tau}KN(d_2^*)]}{S_tN(d_1) - e^{-r\tau}KN(d_2)}$$

$$d_1^* = \frac{\ln\frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln\frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln\frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

[11] The second-order derivative of a decreasing and convex function is positive. Effectively $[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]$ is the second order derivative of $\frac{N(-d)}{\psi(-d)}$ with respect to d and therefore it is positive.

Taking the derivative with respect to μ leads to

$$\frac{\partial R_{call}}{\partial \mu} = \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + e^{\mu\tau}[\tau e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

where ψ is the probability density function of standard normal distribution. Note that we apply the fact that the Rho of a call option is $\tau e^{-\mu\tau} K N(d_2^*)$ in deriving the above equation. $\frac{\partial R_{call}}{\partial \mu}$ can be further simplified:

$$\begin{aligned}
\frac{\partial R_{call}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + \tau K N(d_2^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} \\
&= \frac{\tau e^{\mu\tau} S_t N(d_1^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} > 0.
\end{aligned}$$

To see that the derivative is positive, notice that the denominator is just the price of call option which is always positive, and the numerator is obviously greater than zero.

Next we show that the expected put option return is a decreasing function of the expected stock return. Recall that the expected put option return is:

$$R_{put} = \frac{e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)}$$

where d_1^*, d_2^*, d_1, and d_2 are defined the same as the above. Taking the derivative with respect to μ yields:

$$\begin{aligned}
\frac{\partial R_{put}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)] + e^{\mu\tau}[-\tau e^{-\mu\tau} K N(-d_2^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} \\
&= \frac{-\tau e^{\mu\tau} S_t N(-d_1^*)}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} < 0.
\end{aligned}$$

Note the denominator is the price of put option which is always positive, and therefore the ratio itself is negative.

Appendix C: Holding-Period Expected Option Returns

In this appendix, we derive expected holding-period option returns in the Black-Scholes model. To save space, we only focus on call options. The analysis of put options proceeds along the same lines. To facilitate the notation, we consider an European call option at time 0 that matures at time T. By definition, the expected return of holding the call option from time 0 to time h

$(h < T)$ is:

$$R^h_{call} = \frac{E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\}}{S_0 N(d_1) - e^{-rT} K N(d_2)}$$

where $S_h N(d_1') - e^{-r(T-h)} K N(d_2')$ is the future value of the option at time h, and

$$d_1' = \frac{\ln \frac{S_h}{K} + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}} \qquad d_2' = \frac{\ln \frac{S_h}{K} + (r - \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}$$

$$d_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln \frac{S_0}{K} + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

The expected future value of the option at time h can be split into two pieces:

$$\begin{aligned}
E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\} &= \int_{-\infty}^{\infty} [S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') - e^{-r(T-h)} K N(d_2')] \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
&= \int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
&\quad + \int_{-\infty}^{\infty} -e^{-r(T-h)} K N(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz.
\end{aligned}$$

For the first integral, it can be shown that

$$\begin{aligned}
&\int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
= \; & S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-\sigma\sqrt{h})^2}{2}} N\left(\frac{\ln \frac{S_0}{K} + \mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}\right) dz. \quad \text{(C.1)}
\end{aligned}$$

Define a new variable $z^* = z - \sigma\sqrt{h}$ and (C.1) becomes

$$S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N\left(\frac{\ln \frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T-h}} + \sqrt{\frac{h}{T-h}} z^*\right) dz^*. \qquad \text{(C.2)}$$

Now, using the fact that (see Rubinstein 1984)

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(A + Bz^*) = N\left(\frac{A}{\sqrt{1+B^2}}\right)$$

(C.2) can be further simplified as

$$S_0 e^{\mu h} N\left(\frac{\ln \frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right). \qquad \text{(C.3)}$$

Following the same steps, the second integral is rewritten as

$$\int_{-\infty}^{\infty} -e^{-r(T-h)}KN(d_2')\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz = -e^{-r(T-h)}KN(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}). \quad \text{(C.4)}$$

Putting (C.3) and (C.4) together, we obtain

$$R^h_{call} = \frac{S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}) - e^{-r(T-h)}KN(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}})}{S_0 N(d_1) - e^{-rT}KN(d_2)}.$$

This can be further simplified to

$$\begin{aligned} R^h_{call} &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T}KN(d_2^*)]}{S_0 N(d_1) - e^{-rT}KN(d_2)} \quad \text{(C.5)} \\ d_1^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu - r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\ d_2^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu - r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \end{aligned}$$

where $HP = h/T$.

Appendix D: Expected Option Returns in the Heston Model

In this appendix, we derive the expected return of holding a call option to expiration in the Heston (1993) stochastic volatility model. The Heston (1993) model assumes that the asset price and its spot variance obey the following dynamics under the physical measure P

$$\begin{aligned} dS_t &= \mu S_t dt + S_t\sqrt{V_t}dZ_1^P \\ dV_t &= \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dZ_2^P \end{aligned}$$

where μ is the drift of the stock price, θ is the long run mean of the stock variance, κ is the rate of mean reversion, σ is the volatility of volatility, and Z_1 and Z_2 are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$. The dynamics under the risk-neutral measure Q are

$$\begin{aligned} dS_t &= rS_t dt + S_t\sqrt{V_t}dZ_1^Q \\ dV_t &= [\kappa(\theta - V_t) - \lambda V_t]dt + \sigma\sqrt{V_t}dZ_2^Q \end{aligned}$$

where r is the risk-free rate and λ is the market price of volatility risk. Again we consider the expected return of holding a call option to expiration:

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, V_t))} = \frac{E_t^P[\max(S_T - K, 0)]}{E_t^Q[e^{-r\tau}\max(S_T - K, 0)]}.$$

Heston (1993) provides a closed-form solution to an European call option, up to a univariate numerical integral:

$$C(t, T, S_t, V_t) = E_t^Q[e^{-r\tau}\max(S_T - K, 0)] = S_t P_1 - e^{-r\tau} K P_2 \tag{D.1}$$

where P_1 and P_2 are given by[12]

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi \ln K} f_j(x, V, \tau; \phi)}{i\phi})d\phi \tag{D.2}$$

$$f_j(x, V, \tau; \phi) = e^{C(\tau;\phi) + D(\tau;\phi)V + i\phi x}$$

$$C(\tau; \phi) = r\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1 - ge^{d\tau}}{1 - g}]\}$$

$$D(\tau; \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa + \lambda - \rho\sigma, b_2 = \kappa + \lambda.$$

By analogy, it can be shown that expected call option payoff at expiration is

$$E_t^P[\max(S_T - K), 0] = e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*] \tag{D.3}$$

where

$$P_j^* = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi \ln K} f_j^*(x, V, \tau; \phi)}{i\phi})d\phi \tag{D.4}$$

$$f_j^*(x, V, \tau; \phi) = e^{C(\tau;\phi) + D(\tau;\phi)V + i\phi x}$$

[12] Note that $x = \ln S$.

$$C(\tau;\phi) = \mu\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1-ge^{d\tau}}{1-g}]\}$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d\tau}}{1-ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa - \rho\sigma, b_2 = \kappa.$$

Putting (D.1) and (D.3) together, the analytical expected holding-to-maturity call option return in Heston model is

$$R^{Heston}_{Call}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P^*_1 - e^{-\mu\tau}KP^*_2]}{S_t P_1 - e^{-r\tau}KP_2}. \quad \text{(D.5)}$$

Appendix E: Expected Straddle Returns

In this appendix we study the relation between expected straddle returns and the underlying volatility. The expected gross return on a straddle is defined as

$$\begin{aligned} R_{straddle} &= \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)} \\ &= \frac{[S_t N(d^*_1) - e^{-\mu\tau}KN(d^*_2)]e^{\mu\tau} + [e^{-\mu\tau}KN(-d^*_2) - S_t N(-d^*_1)]e^{\mu\tau}}{S_t N(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_t N(-d_1)}. \end{aligned}$$

We investigate the impact of volatility on expected straddle returns by taking the derivative of $R_{straddle}$ with respect to σ. It follows that

$$\begin{aligned} \frac{\partial R_{straddle}}{\partial\sigma} &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\psi(d^*_1)A - 2e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1)B}{[S_t N(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_t N(-d_1)]^2} \\ &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\{\psi(d^*_1)A - \psi(d_1)B\}}{[S_t N(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_t N(-d_1)]^2} \end{aligned}$$

where $A = S_t N(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_t N(-d_1)$ and $B = S_t N(d^*_1) - e^{-\mu\tau}KN(d^*_2) + e^{-\mu\tau}KN(-d^*_2) - S_t N(-d^*_1)$. It is clear that the sign of $\frac{\partial R_{straddle}}{\partial\sigma}$ is determined by $\psi(d^*_1)A - \psi(d_1)B$. This term can be positive or negative depending on underlying parameters.

It can be shown that $d_2 > 0$ is a sufficient condition for $\frac{\partial R_{straddle}}{\partial\sigma} < 0$. We now prove that $d_2 > 0$ implies $\psi(d^*_1)A - \psi(d_1)B < 0$. First recall from previous analysis $d^*_1 > d_1 > d_2$. We then have

$$d_2 > 0 \Rightarrow 0 < \psi(d_1^*) < \psi(d_1). \tag{E.1}$$

Moreover, note that

$$\frac{\partial A}{\partial r} = \tau e^{-r\tau} K[N(d_2) - N(-d_2)]$$

and therefore,

$$d_2 > 0 \Rightarrow \frac{\partial A}{\partial r} > 0$$

which further implies

$$0 < A < B \tag{E.2}$$

by noting that B is obtained by replacing r with μ in A. Putting together (E.1) and (E.2),

$$d_2 > 0 \Rightarrow \psi(d_1^*)A - \psi(d_1)B < 0 \Rightarrow \frac{\partial R_{straddle}}{\partial \sigma} < 0$$

References

[1] Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. Journal of Finance, 61(1), 259-299.

[2] Bakshi, G., and Kapadia, N. (2003a). Delta-hedged gains and the negative market volatility risk premium. Review of Financial Studies, 16(2), 527-566.

[3] Bakshi, G., and Kapadia, N. (2003b). Volatility risk premiums embedded in individual equity options: Some new insights. Journal of Derivatives, 11(1), 45-54.

[4] Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options. Review of Financial Studies, 9(1), 69-107.

[5] Bates, D. S. (2003). Empirical option pricing: A retrospection. Journal of Econometrics, 116(1), 387-404.

[6] Black, F. (1972). Capital market equilibrium with restricted borrowing. Journal of Business, 444-455.

[7] Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 637-654.

[8] Bollen, N. P., and Whaley, R. E. (2004). Does net buying pressure affect the shape of implied volatility functions? Journal of Finance, 59(2), 711-753.

[9] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

[10] Bondarenko, O. (2003). Why are put options so expensive? Working paper, University of Illinois at Chicago.

[11] Boyer, B. H., and Vorkink, K. (2014). Stock options as lotteries. Journal of Finance.

[12] Broadie, M., Chernov, M., and Johannes, M. (2009). Understanding index option returns. Review of Financial Studies, 22(11), 4493-4529.

[13] Buraschi, A., Trojani, F., and Vedolin, A. (2014). When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia. Journal of Finance, 69(1), 101-137.

[14] Cao, J., and Han, B. (2013). Cross section of option returns and idiosyncratic stock volatility. Journal of Financial Economics, 108(1), 231-249.

[15] Carr, P., and Wu, L. (2009). Variance risk premiums. Review of Financial Studies, 22(3), 1311-1341.

[16] Christoffersen, P., Goyenko, R., Jacobs, K., and Karoui, M. (2014). Illiquidity premia in the equity options market. Working paper.

[17] Constantinides, G. M., Czerwonko, M., Carsten Jackwerth, J., and Perrakis, S. (2011). Are options on index futures profitable for risk-averse investors? Empirical Evidence. Journal of Finance, 66(4), 1407-1437.

[18] Constantinides, G. M., Jackwerth, J. C., and Perrakis, S. (2009). Mispricing of S&P 500 index options. Review of Financial Studies, 22(3), 1247-1277.

[19] Constantinides, G. M., Jackwerth, J. C., and Savov, A. (2013). The puzzle of index option returns. Review of Asset Pricing Studies.

[20] Coval, J. D., and Shumway, T. (2001). Expected option returns. Journal of Finance, 56(3), 983-1009.

[21] Cox, J. C., Ross, S. A., and Rubinstein, M. (1979). Option pricing: A simplified approach. Journal of Financial Economics, 7(3), 229-263.

[22] Driessen, J., and Maenhout, P. (2007). An empirical portfolio perspective on option pricing anomalies. Review of Finance, 11(4), 561-603.

[23] Driessen, J., Maenhout, P. J., and Vilkov, G. (2009). The price of correlation risk: Evidence from equity options. Journal of Finance, 64(3), 1377-1406.

[24] Duarte, J., and Jones, C. S. (2007). The price of market volatility risk. Working paper, University of Southern California.

[25] Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 987-1007.

[26] Engle, R., and Ng, V. (1993). Measuring and Testing the Impact of News on Volatility. Journal of Finance, 48(5), 1749-1778.

[27] Fama, E. F., and French, K. R. (1992). The cross-section of expected stock returns. Journal of Finance, 47(2), 427-465.

[28] Fama, E. F., and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1), 3-56.

[29] Fama, E. F., and MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. Journal of Political Economy, 607-636.

[30] Galai, D. (1978). On the Boness and Black-Scholes models for valuation of call options. Journal of Financial and Quantitative Analysis, 13(01), 15-27.

[31] Galai, D., and Masulis, R. W. (1976). The option pricing model and the risk factor of stock. Journal of Financial Economics, 3(1), 53-81.

[32] Garcia, R., Ghysels, E., and Renault, E. (2010). The econometrics of option pricing. Handbook of Financial Econometrics, 1, 479-552.

[33] Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance, 48(5), 1779-1801.

[34] Goodman, T. H., Neamtiu, M., and Zhang, F. (2013). Fundamental analysis and option returns. Working paper.

[35] Goyal, A., and Saretto, A. (2009). Cross-section of option returns and volatility. Journal of Financial Economics, 94(2), 310-326. .

[36] Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies, 6(2), 327-343.

[37] Jackwerth, J. C. (2000). Recovering risk aversion from option prices and realized returns. Review of Financial Studies, 13(2), 433-451.

[38] Johnson, T. C. (2004). Forecast dispersion and the cross section of expected returns. Journal of Finance, 59(5), 1957-1978.

[39] Jones, C. S. (2006). A nonlinear factor analysis of S&P 500 index option returns. Journal of Finance, 61(5), 2325-2363.

[40] Karakaya, M. (2014). Characteristics and expected returns in individual equity options. Working paper, University of Chicago.

[41] Lewellen, J., and Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies. Journal of Financial Economics, 82(2), 289-314.

[42] Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics, 13-37.

[43] Merton, R. C. (1973). Theory of rational option pricing. The Bell Journal of Economics and Management Science, 141-183.

[44] Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. Journal of Finance, 29(2), 449-470.

[45] Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59, 347-370.

[46] Newey, W. K., and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica, 55(3), 703-08.

[47] Rubinstein, M. (1984). A simple formula for the expected rate of return of an option over a finite holding period. Journal of Finance, 39(5), 1503-1509.

[48] Santa-Clara, P., and Saretto, A. (2009). Option strategies: Good deals and margin calls. Journal of Financial Markets, 12(3), 391-417.

[49] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19(3), 425-442.

[50] Vasquez, A. (2012). Equity volatility term structures and the cross-section of option returns. Working paper, ITAM.

Table 1: Summary Statistics for Equity Options

We present averages by moneyness category of monthly equity option returns (return), the underlying stock's realized volatility over the preceding month (30-day realized vol), option implied volatility (implied vol), option volume (volume) and the option Greeks. Panel A reports on call options and Panel B on put options. We compute monthly option returns using the midpoint of bid and ask quotes. Realized volatility is calculated as the standard deviation of the logarithms of daily returns over the preceding month. The sample consists of options that are at-the-money ($0.95 \leq K/S \leq 1.05$) and approximately one month from expiration. The sample period is from January 1996 to July 2013.

Moneyness K/S	$[0.95-0.97]$	$(0.97-0.99]$	$(0.99-1.01]$	$(1.01-1.03]$	$(1.03-1.05]$
Panel A: Call Options					
Return	0.054	0.080	0.111	0.119	0.100
30-day realized vol	47.06%	45.57%	44.70%	44.23%	44.97%
Implied vol	49.03%	46.94%	45.49%	44.90%	45.44%
Volume	232	306	385	430	396
Open interest	1846	1855	1798	1897	1885
Delta	0.68	0.61	0.53	0.45	0.38
Gamma	0.11	0.12	0.14	0.13	0.12
Vega	4.41	4.81	4.95	4.89	4.52
Panel B: Put Options					
Return	-0.137	-0.121	-0.100	-0.104	-0.087
30-day realized vol	45.86%	44.88%	45.51%	46.19%	47.62%
Implied vol	48.97%	47.29%	47.01%	47.24%	48.25%
Volume	318	359	340	278	207
Open interest	1875	1841	1672	1670	1563
Delta	-0.33	-0.39	-0.47	-0.55	-0.61
Gamma	0.10	0.11	0.13	0.12	0.11
Vega	4.69	5.15	5.27	5.25	4.87

Table 2: Cross-Sectional Option Returns Sorted on 30-day Realized Volatility

We report average returns for the time series of equal-weighted option portfolios sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B on put options. Panel C reports results for option returns based on ask prices rather than the midpoint of bid and ask quotes. Every month, all available one-month at-the-money options are sorted into five quintile portfolios according to their 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. Newey-West t-statistics using four lags are reported in parentheses. The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	0.147	0.128	0.111	0.084	0.009	-0.138***
						(-3.42)
$0.975 \leq K/S \leq 1.025$	0.155	0.145	0.120	0.094	0.017	-0.138***
						(-3.50)
Panel B: Put Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	-0.146	-0.153	-0.109	-0.077	-0.075	0.071**
						(2.00)
$0.975 \leq K/S \leq 1.025$	-0.145	-0.157	-0.101	-0.065	-0.068	0.077**
						(2.08)
Panel C: Using Ask Prices						
	Low	2	3	4	High	H-L
Call Option Portfolios	0.048	0.045	0.033	0.012	-0.060	-0.108***
						(-2.94)
Put Option Portfolios	-0.209	-0.209	-0.165	-0.133	-0.133	0.076**
						(2.30)

Table 3: Portfolios Double-Sorted on Beta and Underlying Volatility

We report average equal-weighted returns on option portfolios sorted on beta and 30-day realized volatility. Panel A reports on call options and Panel B on put options. Every month, all available one-month at-the-money options are first ranked into five quintile portfolios according to the underlying stocks' CAPM betas. Then, within each beta quintile, options are further sorted into five portfolios based on 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatility. CAPM beta is estimated using daily returns over the past 30 days preceding the portfolio formation date. Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5% and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Options							
		L	2	3	4	H	H-L
	1	0.16	0.13	0.10	0.07	-0.01	-0.17***
							(-3.36)
	2	0.17	0.15	0.15	0.14	0.06	-0.11**
Beta Quintiles							(-2.20)
	3	0.15	0.19	0.15	0.11	0.05	-0.10*
							(-1.90)
	4	0.11	0.13	0.11	0.11	0.03	-0.08**
							(-2.03)
	5	0.09	0.07	0.10	0.02	0.01	-0.09**
							(-2.18)
Panel B: Put Options							
		L	2	3	4	H	H-L
	1	-0.15	-0.11	-0.12	-0.12	-0.11	0.04
							(0.90)
	2	-0.14	-0.16	-0.16	-0.11	-0.10	0.04
							(0.90)
Beta Quintiles	3	-0.15	-0.20	-0.12	-0.07	-0.06	0.09**
							(2.32)
	4	-0.13	-0.12	-0.13	-0.07	-0.06	0.08*
							(1.81)
	5	-0.13	-0.11	-0.09	-0.06	-0.07	0.06*
							(1.95)

Table 4: Fama-MacBeth Regressions

We report results for the following Fama-MacBeth regression

$$R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon$$

where R^i_{t+1} is the return on option i from month t to month $t+1$, VOL^i_t is the underlying stock volatility, and Z^i_t is a vector of control variables that includes moneyness, the stock's CAPM beta (CAPM beta), firm size (size) and book-to-market (btm). We consider three underlying volatility measures: 30-day realized volatility, 60-day realized volatility, and 365-day realized volatility. CAPM beta is estimated using daily returns over the 30 days preceding the portfolio formation date. Size and btm are computed according to Fama and French (1992). Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013. The sample consists of one-month at-the-money options. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Calls			Puts		
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-0.37	-0.374	-0.412	-0.643	-0.656	-0.690*
	(-1.35)	(-1.37)	(-1.49)	(-1.58)	(-1.60)	(-1.68)
30-day realized vol	**-0.254*** **			**0.105** **		
	(-4.57)			(2.28)		
60-day realized vol		**-0.248*** **			**0.174*** **	
		(-3.88)			(3.36)	
365-day realized vol			**-0.183** **			**0.222*** **
			(-2.39)			(3.35)
CAPM beta	0.011	0.009	0.003	-0.004	-0.01	-0.011
	(0.81)	(0.72)	(0.27)	(-0.29)	(-0.73)	(-0.86)
Size	-0.000	-0.000	-0.000	-0.001*	-0.001	-0.000
	(-0.46)	(-0.40)	(-0.31)	(-1.96)	(-1.64)	(-1.24)
Btm	0.002	0.002	0.002	0.006	0.007	0.01
	(0.82)	(0.84)	(0.75)	(0.97)	(1.11)	(1.52)
Adjusted R-square	2.30%	2.40%	2.70%	2.80%	3.00%	3.30%

Table 5: Expected Option Returns in the Heston Model

We report expected option returns in the Heston (1993) stochastic volatility model. The computations are based on the model parameters reported in Broadie, Chernov, and Johannes (2009), which are calibrated from historical S&P 500 index return data. These parameters are reported in Panel A. For simplicity, the dividend yield is set to zero. Expected option returns are computed for different combinations of the expected stock return (μ), the current stock variance (ν), and current stock prices (S). Panel B reports on moneyness $K/S = 95/100$ and Panel C on $K/S = 105/100$.

Panel A: Parameters

r	θ	κ	σ	ρ	t	λ	K
4.50%	0.15	5.33	0.14	-0.52	0.8333	0	100

Panel B: $S = 95$

			ν								
			0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Calls	μ	8%	0.260	0.171	0.132	0.110	0.096	0.086	0.079	0.073	0.068
		12%	0.618	0.387	0.294	0.242	0.209	0.186	0.169	0.155	0.145
		16%	1.055	0.635	0.473	0.386	0.331	0.293	0.265	0.243	0.225
Puts	μ	8%	-0.051	-0.046	-0.042	-0.039	-0.036	-0.034	-0.032	-0.030	-0.029
		12%	-0.113	-0.101	-0.093	-0.086	-0.081	-0.076	-0.072	-0.069	-0.066
		16%	-0.173	-0.156	-0.143	-0.132	-0.124	-0.117	-0.112	-0.107	-0.102

Panel C: $S = 105$

			ν								
			0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Calls	μ	8%	0.057	0.053	0.050	0.047	0.045	0.043	0.042	0.040	0.039
		12%	0.119	0.111	0.104	0.098	0.094	0.090	0.086	0.083	0.080
		16%	0.181	0.169	0.159	0.150	0.143	0.137	0.131	0.126	0.122
Puts	μ	8%	-0.160	-0.119	-0.097	-0.083	-0.074	-0.066	-0.061	-0.056	-0.052
		12%	-0.318	-0.244	-0.203	-0.175	-0.156	-0.142	-0.130	-0.121	-0.113
		16%	-0.448	-0.353	-0.297	-0.260	-0.233	-0.212	-0.195	-0.182	-0.171

Table 6: Straddle Portfolio Returns Sorted on Volatility

We report average returns for five straddle portfolios sorted on the volatility of the underlying stock. We use three samples of straddles based on moneyness: $0.95 \leq K/S \leq 1$, $0.875 \leq K/S < 0.95$, and $0.80 \leq K/S < 0.875$. Each month, we select call and put options on the same stock with the same strike price and maturity to form straddles. These straddles are then sorted into five quintile portfolio based on the realized volatility over the preceding month. Portfolio Low (High) contains straddles with the lowest (highest) underlying volatility. We report equal-weighted and volume-weighed portfolio returns. Straddle volume is computed as the average volume for the call and put options that form the straddle. The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Low	2	3	4	High	H-L
Panel A: $0.95 \leq K/S \leq 1$						
Equal-weighted	0.028	0.010	0.022	0.014	-0.026	-0.054***
						(-2.90)
Volume-weighted	0.026	-0.006	0.005	0.014	-0.037	-0.063**
						(-2.14)
Panel B: $0.875 \leq K/S < 0.95$						
Equal-weighted	0.022	0.034	0.025	0.015	-0.033	-0.055***
						(-3.25)
Volume-weighted	0.013	0.044	-0.01	0.003	-0.044	-0.057**
						(-2.52)
Panel C: $0.80 \leq K/S < 0.875$						
Equal-weighted	0.020	0.013	0.014	0.000	-0.065	-0.085***
						(-4.97)
Volume-weighted	0.021	0.001	-0.005	-0.021	-0.046	-0.067**
						(-2.33)

Table 7: Option Portfolio Returns Based on Alternative Volatility Measures

We report equal-weighted option portfolio returns sorted on different measures of underlying volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B reports on put options. We consider four volatility measures: realized volatility over the previous 14 days, realized volatility over the previous 60 days, realized volatility over the previous year, and option-implied volatility. Every month, all available options are ranked into five quintile portfolios based on underlying volatilities. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013. The sample consists of one-month at-the-money options. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Low	2	3	4	High	H-L
Panel A: Calls						
14-day realized vol	0.146	0.122	0.114	0.081	0.016	-0.130***
						(-3.54)
60-day realized vol	0.155	0.109	0.115	0.086	0.014	-0.141***
						(-3.44)
365-day realized vol	0.130	0.104	0.117	0.084	0.044	-0.086*
						(-1.80)
Implied vol	0.156	0.117	0.134	0.081	-0.010	-0.166***
						(-3.60)
Panel B: Puts						
14-day realized vol	-0.146	-0.139	-0.103	-0.086	-0.087	0.059*
						(1.77)
60-day realized vol	-0.157	-0.151	-0.109	-0.084	-0.059	0.098**
						(2.49)
365-day realized vol	-0.170	-0.144	-0.120	-0.071	-0.053	0.117***
						(2.82)
Implied vol	-0.130	-0.143	-0.118	-0.087	-0.083	0.047
						(1.13)

Table 8: Option Portfolio Returns for Alternative Option Samples

We report equal-weighted option portfolio returns sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Different option samples are used: two-month at-the-money (ATM) options, one-month in-the-money (ITM) options, two-month ITM options, one-month out-of-the-money (OTM) options, and two-month OTM options. ATM options are defined by moneyness $0.95 \leq K/S \leq 1.05$, ITM options are defined by moneyness $0.80 \leq K/S < 0.95$ for calls and $1.05 < K/S \leq 1.20$ for puts, and OTM options are defined by moneyness $1.05 < K/S \leq 1.20$ for calls and $0.80 \leq K/S < 0.95$ for puts. Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Low	2	3	4	High	H-L
Panel A: Calls						
Two-month ATM	0.144	0.135	0.112	0.035	-0.027	-0.171***
						(-3.04)
One-month ITM	0.053	0.060	0.042	0.026	-0.025	-0.078***
						(-3.68)
Two-month ITM	0.089	0.084	0.068	0.027	-0.067	-0.156***
						(-5.22)
One-month OTM	0.055	0.049	0.077	0.048	-0.066	-0.121**
						(-2.21)
Two-month OTM	0.132	0.088	0.098	0.022	-0.054	-0.186**
						(-2.36)
Panel B: Puts						
Two-month ATM	-0.207	-0.149	-0.118	-0.079	-0.056	0.151***
						(3.20)
One-month ITM	-0.091	-0.069	-0.052	-0.043	-0.034	0.057***
						(2.93)
Two-month ITM	-0.127	-0.090	-0.055	-0.048	-0.023	0.105***
						(3.63)
One-month OTM	-0.309	-0.217	-0.193	-0.090	-0.131	0.178***
						(2.76)
Two-month OTM	-0.276	-0.197	-0.188	-0.118	-0.099	0.177**
						(2.04)

Table 9: Option Portfolio Returns Using Different Weighting Methods

We report differences between extreme portfolio option returns for portfolios sorted on 30-day realized volatility, for different option samples. Alternative weighting methods are used: volume weighted, open interest weighted, and option value weighted. Option value is defined as the product of option price and option open interest Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Volume Weighted	Open Interest Weighted	Option Value Weighted
Panel A: Calls			
One-month ATM	-0.182***	-0.133***	-0.107**
	(-3.56)	(-3.09)	(-2.35)
Two-month ATM	-0.204**	-0.235***	-0.216***
	(-2.40)	(-3.64)	(-2.89)
One-month ITM	-0.113***	-0.060**	-0.066***
	(-3.98)	(-2.51)	(-2.65)
Two-month ITM	-0.210***	-0.188***	-0.191***
	(-3.93)	(-4.33)	(-4.46)
One-month OTM	-0.171**	-0.059	-0.137*
	(-2.14)	(-0.90)	(-1.75)
Two-month OTM	-0.242**	-0.292**	-0.438***
	(-1.99)	(-2.42)	(-2.98)
Panel B: Puts			
One-month ATM	0.073	0.089*	0.052
	(1.43)	(1.87)	(1.06)
Two-month ATM	0.081	0.187***	0.170**
	(0.97)	(2.97)	(2.50)
One-month ITM	0.035	0.099***	0.090***
	(1.09)	(3.68)	(2.93)
Two-month ITM	0.154***	0.134***	0.134***
	(3.45)	(2.86)	(2.90)
One-month OTM	0.268***	0.278***	0.274***
	(3.45)	(4.14)	(3.48)
Two-month OTM	0.310***	0.307***	0.349***
	(3.05)	(2.93)	(3.71)

Table 10: Summary Statistics for S&P 500 Index Options

We report the means of monthly S&P 500 index option returns (return), implied volatility (implied vol), option volume (volume), and option Greeks by moneyness. Panel A reports on calls and Panel B reports on puts. We compute the monthly option return using the midpoint of the bid and ask quotes. The sample consists of S&P 500 index options (SPX) with moneyness $0.90 \leq K/S \leq 1.10$ and one-month maturity. The sample period is from January 1996 to July 2013.

Moneyness K/S	[0.90−0.94]	(0.94−0.98]	(0.98−1.02]	(1.02−1.06]	(1.06−1.10]
Panel A: SPX calls					
Return	0.027	0.057	0.060	-0.112	-0.617
Implied vol	27.30%	22.75%	19.68%	17.42%	17.28%
Volume	251	306	2029	2867	2156
Open interest	9679	11770	15236	15388	14807
Delta	0.88	0.76	0.51	0.20	0.06
Gamma	0.002	0.005	0.007	0.005	0.002
Vega	60.32	93.12	119.86	80.66	32.99
Panel B: SPX puts					
Return	-0.540	-0.406	-0.224	-0.133	-0.171
Implied vol	26.56%	22.87%	19.66%	18.20%	22.68%
Volume	3699	2662	2619	391	338
Open interest	19604	18649	14674	8992	12322
Delta	-0.11	-0.23	-0.48	-0.75	-0.88
Gamma	0.002	0.005	0.007	0.006	0.003
Vega	55.13	90.56	119.80	93.61	53.04

Table 11: Regressions of Index Option Returns on Index Volatility

Using a pooled sample of S&P 500 index options (SPX) with $0.9 \leq K/S \leq 1$ and one-month maturity, we provide results for the regressions of monthly SPX option returns on index volatility:

$$R_{t+1}^{i} = constant + \beta_1 VOL_t^i + \beta_2 Moneyness_t^i + \beta_3 Index\_ret_t + \epsilon$$

where R_{t+1}^{i} is the option return from month t to month $t+1$, $Index\_ret_t$ is the S&P 500 index return in month t and VOL_t is the index volatility. We consider four index volatility measures: realized volatility over the previous 14 days, realized volatility over the preceding month, realized volatility over the previous 60 days, and option-implied volatility. In addition, we run the same regressions using only liquid SPX options, consisting of calls with $0.98 \leq K/S \leq 1.1$ and puts with $0.90 \leq K/S \leq 1.02$. Newey-West t-statistics with four lags are reported in parentheses. The sample period is from January 1996 to July 2013.

Panel A: SPX calls	Full sample				Only liquid options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	4.09	4.18	4.28	7.11	9.27	9.21	9.13	10.85
	(6.01)	(6.37)	(6.69)	(14.00)	(9.44)	(9.31)	(9.15)	(11.93)
14 day realized vol	**-0.46**				-0.19			
	(-1.92)				(-0.55)			
30 day realized vol		**-0.92**				**-0.86**		
		(-3.79)				(-2.52)		
60 day realized vol			**-1.46**				**-1.62**	
			(-4.80)				(-3.77)	
implied vol				**-3.70**				**-4.44**
				(-5.62)				(-4.47)
Adjusted R-square	1.13%	1.22%	1.37%	2.01%	2.54%	1.73%	1.37%	2.01%
Panel B: SPX puts	Full sample				Only liquid options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	-4.24	-4.22	-4.22	-4.50	-6.32	-6.14	-6.07	-6.77
	(-8.80)	(-8.65)	(-8.57)	(-9.33)	(-8.97)	(-8.62)	(-8.44)	(-8.70)
14 day realized vol	**2.11**				**2.66**			
	(3.99)				(3.88)			
30 day realized vol		**1.39**				**1.89**		
		(2.99)				(3.14)		
60-day realized vol			**1.070**				**1.580**	
			(2.64)				(2.98)	
implied vol				0.26				**0.92**
				(0.66)				(1.73)
Adjusted R-square	2.70%	1.73%	1.46%	1.13%	3.18%	2.06%	1.76%	1.27%