STOCHASTIC INFORMATION FRICTION, BUSINESS CYCLES AND ASSET PRICES

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Abstract

In this paper, I offer a structural DSGE framework to analyze the impact of stochastic information friction in explaining business cycles and asset prices. I document a new mechanism to generate time variation in uncertainty from the information channel, where rational agents' beliefs from Bayesian learning features time-varying uncertainty in a stochastic imperfect information environment. Information uncertainty provides considerable explanatory power for business fluctuations and carries a negative price of risk for asset valuations. The interaction between imperfect information and financial market friction provides an important channel to amplify the effect of information uncertainty on asset pricing. Empirical evidence supports the model's prediction of negatively priced information uncertainty risk. Firms with high exposure to information friction shock generate significantly lower returns than firms with low information friction shock exposure. A mimicking portfolio, IFS factor, generates 6% excess return per annum.

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1 Introduction

Uncertainty and expectation play a central role in explaining business cycles and asset price fluctuations. Economic conditions are perturbed by various uncertainties; rational agents form expectations and make optimal decisions accordingly. A majority of macroeconomic and asset pricing researchers focus on the uncertainty part. Economists propose various types of shocks (to technology, investment, policy, etc.) to explain business cycles. However, the expectation part draws less attention from academia. Some progress has been made recently in the literature of imperfect information-driven business cycles, in which expectation errors occur because of imperfect information. In this paper, I explore the idea that time-varying imperfect information induces time variation in rational agents' belief uncertainty – thus generating fluctuations in macroeconomic quantities and asset prices at business cycle frequencies.

Two recent works, Lorenzoni (2009) and Blanchard et al. (2013), have renewed attention to imperfect information and limited information processing as sources of expectational errors in a rational framework. The key story is: fundamental productivity is unobservable; agents learn it from noisy public signals and form rational beliefs. Thus, the presence of noisy signals generates expectation errors. In these papers, the noise level in the public signals is constant. It captures the information quality in the economy. In contrast to previous studies, I ask the questions: what if the information quality is time-varying and subject to uncertainty shock? Does information uncertainty affect business cycles or asset price dynamics? Intuitively, when information quality is bad, the signal extraction problem becomes less precise and agents' beliefs become more uncertain. To better answer these questions, I conduct a structural analysis under a rational DSGE framework, and study the impact of time-varying information uncertainty on macroeconomic dynamics and asset prices.

The analysis is based on a standard real business cycle model with three main ingredients. First, I introduce both permanent and transitory productivity shocks. Agents observe the total productivity, but not the decomposition of the two parts. In addition to total productivity, agents have access to another noisy signal regarding the permanent component of the productivity. Second, I incorporate the time-varying information environment, by introducing Information Friction Shocks (IFS), which affect the noise level of the noisy signal that agents receive. Rational agents update their beliefs with Bayesian learning. The information friction shocks affect agents' inference problems and generate time-varying belief uncertainty. Belief dispersion becomes wider when information friction is more severe. Third, I incorporate a reduced-form financial sector – exhibiting time-variant financial frictions – which is correlated with the information condition. Severe information imperfection induces higher financial friction, resulting in higher capital adjustment cost.

Information friction plays two roles in the model. The first role is to generate time-variant belief uncertainty through Bayesian learning. Deterioration in information quality induces more uncertainty in agents' beliefs. The second role is to affect the allocation efficiency through interaction with financial market friction. Deterioration in information quality induces greater costs associated with information acquisition and greater capital adjustment costs. I calibrate the model to match the moments of key macroeconomic variables and asset returns. A positive information friction shock increases belief uncertainty. In response to this higher belief uncertainty, households consume less, invest more, and work more. Marginal utility of consumption responds positively to the information friction shock through interaction with the financial friction channel. The model predicts a negatively priced risk associated with information friction uncertainty.

To validate the model's prediction, I explore the asset pricing implication empirically. First, I construct an empirical measure to proxy for information friction shocks. In the model, information friction is closely related to belief uncertainty of the current period's productivity. Therefore, I use the Survey of Professional Forecasters data, and focus on the individual-level forecast of nominal GDP of the current quarter to construct the belief dispersion measure. Belief dispersion (BD) is defined as the 75 and 25 percentile differences in the logarithm of nominal GDP. To mitigate the concern that belief dispersion may be driven by fundamental macro uncertainty, I orthogonalize the time series of belief dispersion using estimates from Jurado et al. (2015) to control any effects from the fundamental uncertainty channel. The remaining orthogonalized part is defined as the proxy for information friction shocks (IFS).

With the empirical proxy for information friction shocks, I use Fama-French 25 size-value portfolios to test whether IFS is a priced risk factor. The results show a significant negative price of risk for these test assets. The classical Fama-French 3 factors model is able to explain the test portfolios' returns with an R^2 of 70%. Adding the IFS factor into the Fama-French 3 factor model significantly improves the explanatory power of test assets' returns, with an R^2 increased from 70% to 80%. I also explore the cross-sectional return predictability using portfolio sorts methodology. My main finding is that exposure to information uncertainty risk strongly predicts future asset returns. Firms with high exposure to information friction shock generate significantly lower returns than firms with low information friction shock exposure. The estimated risk premium associated with information friction shock is negative and statistically significant. A long-short zero investment strategy earns a significant 55 bps excess return per month, or about 6.8% per annum. All of these empirical findings are consistent with the model prediction of a negatively priced information uncertainty risk.

Since the seminal work of Bloom (2009), a large and growing body of literature has studied the effects of uncertainty shock in explaining macroeconomic dynamics. It is important to distinguish information uncertainty, which is the main interest of this paper, from fundamental uncertainty. The previous works on uncertainty shocks are usually studied in a perfect information environment. The uncertainty shock is typically defined as the conditional volatility of a disturbance to economic fundamentals. Agents are sure about the economic condition today, but are not sure about the volatility tomorrow. However, information uncertainty in this paper refers to the agents' beliefs uncertainty about the economic condition today which features time-variant volatility, even when economic fundamentals do not exhibit second moment variations. One important message this article delivers is that time variation in uncertainty could be generated from two mechanisms. One is macro uncertainty from the fundamental channel; the other is information uncertainty from the information channel. To better understand the time-varying uncertainty from these two channels, I also analyze a DSGE model with both information friction shocks and uncertainty shocks. Both contribute to the explanation of macroeconomic quantities and asset prices. Both information uncertainty and fundamental uncertainty carry a negative price of risk. I find information uncertainty and fundamental uncertainty each contributes 50% to the total uncertainty risk in the model.

Through the welfare analysis, I find high information friction harms social welfare. A policy implication is to reduce the information uncertainty. Increasing the quality of public news, increasing the accuracy of public reports, increasing transparency, and reducing policy uncertainty are all effective ways to reduce information uncertainty or costs associated with information acquisition in the economy. Reducing information friction enhances social welfare by increasing allocation efficiency.

Related Literature This paper is mainly related to three strands of the literature: 1) expectational error driven business cycles, 2) the literature that aims to explain the joint behavior of macroeconomic dynamics and asset prices, and 3) uncertainty shocks.

First, this paper contributes to the expectational error driven business cycles literature. The idea that imperfect information can cause sluggish adjustment in economic variables and generate fluctuation driven by expectational errors goes back, at least, to Lucas (1972). More recently, Blanchard et al. (2013) and Boz et al. (2011) have renewed attention to imperfect information and limited information processing as sources of expectational errors in a rational framework. Along this direction, there is also some progress in asset pricing literature featuring imperfect information with rational learning, such as Ai (2010). In all of these models, the economy features a constant level of information imperfection. The noisy signal about the unobservable has a constant noise level. In contrast to these papers, I allow the noise level to be time-varying, thus inducing time-variant beliefs uncertainty. To my knowledge, this paper is the first to incorporate a time-variant information environment with Bayesian learning into the DSGE model to study the effects of time-varying expectational errors.

Second, this paper contributes to a growing body of literature on macroeconomic asset pricing models that aims to jointly explain macroeconomic quantities and asset prices. The starting point of this literature goes back to Jermann (1998) and Tallarini (2000). Some recent progress includes work by Croce (2014) and Papanikolaou (2011). Croce (2014) considers a one-sector stochastic growth model with Epstein-Zin preferences and examines the long-run productivity risk. Papanikolaou (2011) considers a multi-sector model and explores the cross-sectional risk premia from investment-specific technology shocks. In contrast to their studies, I focus on the implications of information friction shocks risk premium. I also incorporate financial friction shocks into the model. The interaction between information friction and financial friction provides a promising and considerable explanation power for both macroeconomic quantities and asset valuation fluctuations.

Third, this paper also contributes to the vast literature on uncertainty shocks. The effects of uncertainty shocks have been widely studied in business cycles and asset pricing, e.g. Bloom (2009) and Bansal et al. (2014). These works are usually studied in a perfect information environment. The uncertainty shocks from these works come from the perturbation of economic fundamentals. One novelty of this paper is that I am able to distinguish between information uncertainty and fundamental uncertainty in an economy featuring imperfect information. The information uncertainty comes from the imperfect information channel via Bayesian learning; it doesn't depend on economic fundamentals. One important message this article delivers is that time variation in uncertainty could be generated from two mechanisms. Both contribute to explain macroeconomic quantities and asset prices.

The reminder of the paper is organized as follows: Section 2 presents the model; Section 3 solves the Bayesian learning problem; Section 4 presents the model solutions and illustrates the main mechanism; Section 5 investigates the asset pricing implication from the model, followed by the empirical evidence in Section 6; Section 7 presents the analysis of a model with both information and fundamental uncertainty shocks; Section 8 offers some concluding remarks.

2 A Model with Stochastic Information Friction

Production

There is one representative firm in the economy. The production takes a standard Cobb-Douglas form, with capital K_t and labor L_t as inputs

$$Y_t = (A_t L_t)^{1-\alpha} K_t^{\alpha} \tag{1}$$

where Y_t is the output and $\alpha \in (0,1)$ is the capital's share of output. Notice that this specification ensures a balanced growth path, and $A_t^{1-\alpha}$ is the total factor productivity (TFP). There is a deterministic growth component Γ^t in the productivity A_t.

$$A_t = \Gamma^t e^{a_t} \tag{2}$$

The term $\mu \equiv log(\Gamma)$ represents the deterministic long run growth rate. The a_t (in logs) represents the business cycle component of the productivity. Productivity a_t has two components: the permanent component x_t and the transitory component z_t.

$$a_t = x_t + z_t \tag{3}$$

The permanent component x_t follows the unit root process. It builds up gradually with a series of "growth" shocks $g_t \equiv \Delta x_t$. In particular, g_t follows a stationary

AR(1) process.

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_t^g \tag{4}$$

The transitory component z_t also follows a stationary AR(1) process.

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t^z \tag{5}$$

The coefficients ρ_g and ρ_z are in [0, 1), ϵ_t^g and ϵ_t^z are i.i.d. standard normal shocks.

Stochastic Information Friction

The economy features an imperfect information environment. Productivity is driven by two shocks: a permanent shock and a transitory shock. The agent does not observe the two shocks separately, but only the realized level of productivity. This creates a signal extraction problem for the agent. On top of observing the realized a_t each period, the agent receives an additional noisy signal regarding the permanent component of the productivity. This captures the idea that agents in the economy process public information, such as macro quantities reports, financial news, etc., and form exceptions regarding the economic fundamentals.

$$s_t = x_t + \sigma_{st} \epsilon_t^s \tag{6}$$

This third source of information is also noisy. The signal s_t is driven by i.i.d. standard normal shocks ϵ_t^s, which I call ”noise” shocks. The σ_{st} is time-varying. This captures the idea that the information environment of the economy is stochastic. I interpret this as *stochastic information friction*. The σ_{st} controls the degree of information imperfection. A high σ_{st} means severe information friction. I assume the logarithm of σ_{st} follows a stationary AR(1) process, and it is perturbed by i.i.d.

standard normal shocks η_t^s.

$$\log(\sigma_{st}) = (1-\kappa_s)\log(\bar{\sigma}_s) + \kappa_s \log(\sigma_{st-1}) + \omega_s \eta_t^s \tag{7}$$

This way of modeling the logarithm of σ_{st} ensures that the standard deviation of the shocks remains positive at all times. The σ_{st} is perturbed by i.i.d. innovations η_t^s, which I call "*Information Friction Shock*" (IFS). A bad IFS increases σ_{st}, and the information friction becomes more severe. The signal s_t becomes more noisy and less informative. It becomes more difficult for agents to extract the true economic fundamentals. As $\sigma_{st} \to \infty$, s_t does not provide any additional information compared to the realizations of a_t. As $\sigma_{st} \to 0$, households perfectly infer the permanent and transitory components to productivity; thus, the economy features a perfect information environment in that case.

Preference

The representative household has Epstein and Zin (1989) and Weil (1989) recursive preferences over streams of consumption C_t and leisure $1 - L_t$.

$$V_t = \left\{ (1-\beta)\left(C_t^{\theta}(1-L_t)^{1-\theta}\right)^{1-\frac{1}{\psi}} + \beta E_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{8}$$

The preference parameters are the discount factor β, risk aversion γ, and the elasticity of intertemporal substitution (EIS) ψ. The parameter θ controls the leisure share. In this economy, the stochastic discount factor (SDF) can be written as:

$$M_{t+1} = \beta \left(\frac{C_{t+1}^{\theta}(1-L_{t+1})^{1-\theta}}{C_t^{\theta}(1-L_t)^{1-\theta}} \right)^{1-\frac{1}{\psi}} \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \tag{9}$$

The EZ framework features the timing preferences of the resolution of uncertainty. If $\gamma > 1/\psi$, the agent prefers an early resolution of uncertainty, and if $\gamma < 1/\psi$, a later resolution. When $\gamma = 1/\psi$, the representative agent is indifferent to the resolution of uncertainty, and the recursive preferences collapse to the CRRA case.

The representative household maximizes lifetime utility, subject to the resource constraint.

$$Y_t = C_t + I_t \tag{10}$$

Capital Market and Financial Friction

The household can convert consumption into capital by investing in capital markets. The firm accumulates capital according to the following inter-temporal law of motion:

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t \tag{11}$$

Capital depreciates at the rate δ. $\phi(.)$ is a positive concave function, capturing capital market friction. I follow Jermann (1998) and specify the $\phi(.)$ in a similar form.

$$\phi\left(\frac{I_t}{K_t}\right) = \tau_1 + \frac{\tau_2}{1-\frac{1}{\xi_t}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\xi_t}} \tag{12}$$

The τ_1 and τ_2 are constants. They are set to ensure that adjustment costs do not affect the steady state of the model [1]. The only difference from Jermann (1998) is the time-varying ξ_t. The ξ_t captures the elasticity of the investment-capital ratio with respect to Tobin's Q. Higher ξ_t represents smaller capital adjustment costs. If $\xi_t = +\infty$, the capital market becomes frictionless. The ξ_t evolves according to the

[1] In particular, I set $\tau_1 = (e^{\mu} + \delta - 1)/(1-\bar{\xi})$ and $\tau_2 = (e^{\mu} + \delta - 1)^{1/\bar{\xi}}$

following equations:

$$\xi_t = (\bar{\xi} - 1)e^{f_t} + 1 \tag{13}$$

$$f_t = \rho_f f_{t-1} + \sigma_f \epsilon_t^f \tag{14}$$

Note the specification of (13) ensures the ξ_t is always greater than 1 if the steady state value $\bar{\xi}$ is greater than 1. In a reduced form, f_t captures the financial friction in the market. It follows a stationary AR(1) process, and it is perturbed by i.i.d. standard normal shock ϵ_t^f. A positive ϵ_t^f shock will increase the f_t and ξ_t, and firms will pay less capital adjustment cost. In the model, I allow financial friction shock ϵ_t^f and information friction shock η_t^s to be correlated, and use ϱ^{sf} to denote the correlation coefficient between the two shocks. This is motivated by a vast body of literature that investigates the relationship between capital markets and information asymmetry (Greenwald et al. (1984) and Ivashina (2009)).

Labor Market

The representative household also supplies labor service L_t to the production firm. Wage rate w_t is set at the marginal product of labor. But the firm pays an extra wage adjustment cost as shown in the quadratic form below.

$$\phi_t^L = \xi_w \left(1 - \frac{A_{t-1}}{A_{t-2}} \frac{w_{t-1}}{w_t}\right)^2 w_t L_t \tag{15}$$

Asset Prices

The firm dividend D_t is specified as

$$D_t = Y_t - \iota K_t - I_t - w_t L_t - \phi_t^L \tag{16}$$

where ι denotes operating cost per unit of capital. $Y_t - \iota K_t$ represents the operating profit of the firm. The operating cost ιK_t and wage adjustment cost ϕ_t^L provide some operating leverage effects for the dividend claim. I assume these costs paid by the firm go to the household as a form of household income, so that the resource constraint in equation (10) still holds. The firm maximizes firm value, which is equal to the present discounted value of all current and future expected dividend flows. The firm's equity return is

$$R_t^E \equiv \frac{P_t + D_t}{P_{t-1}} \tag{17}$$

where P_t denotes the price of a claim on all future dividends.

Equilibrium Conditions

The economy is non-stationary. To derive a stationary equilibrium, I de-trend all the non-stationary variables by A_{t-1}. A variable with a tilde represents its re-scaled counterpart. Note that the choice of A_{t-1} as the normalization factor ensures the information consistency of the model, if var_t is in the agent's information set at time $t-1$, so is the $\widetilde{var}_t$.

The welfare theorems hold in the model. The equilibrium can be characterized by the solution of the social planner's problem. The value function is homogeneous of degree θ. Taking advantage of this homotheticity property, the normalized stationary model is formulated in the recursive form as follows,

$$\widetilde{V}_t(x_{t|t}, g_{t|t}, z_{t|t}, \sigma_{st}, \widetilde{K}_t) = \max_{\widetilde{C}_t, L_t} \left\{ (1-\beta) \left(\widetilde{C}_t^{\theta} (1-L_t)^{1-\theta} \right)^{1-\frac{1}{\psi}} + \widetilde{A}_t^{\theta(1-\frac{1}{\psi})} \beta E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{18}$$

subject to resource constraint. The variables $x_{t|t}$, $g_{t|t}$, and $z_{t|t}$ are the agent's rational beliefs regarding each component of the productivity from Kalman learning. The optimal condition for consumption yields the Euler equation:

$$1 = E_t\left[M_{t+1}R^I_{t+1}\right] \tag{19}$$

The stochastic discount factor M_{t+1} and investment return R^I_{t+1} take the following form:

$$M_{t+1} = \widetilde{A}_t^{\theta(1-\frac{1}{\psi})-1}\beta\left(\frac{\widetilde{C}^{\theta}_{t+1}(1-L_{t+1})^{1-\theta}}{\widetilde{C}^{\theta}_{t}(1-L_{t})^{1-\theta}}\right)^{1-\frac{1}{\psi}}\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}^{1-\gamma}_{t+1}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} \tag{20}$$

$$R^I_t = \tau_2\left(\frac{\widetilde{I}_t}{\widetilde{K}_t}\right)^{-\frac{1}{\xi_t}}\left\{\frac{\alpha\widetilde{Y}_{t+1}}{\widetilde{K}_{t+1}} + \frac{1}{\tau_2\left(\frac{\widetilde{I}_{t+1}}{\widetilde{K}_{t+1}}\right)^{-\frac{1}{\xi_{t+1}}}}\left[1-\delta+\tau_1+\frac{\tau_2}{\xi_{t+1}-1}\left(\frac{\widetilde{I}_{t+1}}{\widetilde{K}_{t+1}}\right)^{1-\frac{1}{\xi_{t+1}}}\right]\right\} \tag{21}$$

The risk-free rate is just the reciprocal of expected SDF.

$$R^f_t = \frac{1}{E_t\left[M_{t+1}\right]} \tag{22}$$

The optimal condition for labor choice satisfies the marginal rate of substitution between consumption and leisure, equal to the marginal product of labor.

$$\frac{1-\theta}{\theta}\frac{\widetilde{C}_t}{1-L_t} = \frac{(1-\alpha)\,\widetilde{Y}_t}{L_t} \tag{23}$$

3 Bayesian Learning and Kalman Filter

3.1 Derivation of Kalman Filter

In an environment in which agents have imperfect information regarding the true decomposition of the productivity shock into its permanent and transitory components, the rational agent forms expectations regarding the decomposition using the Kalman filter. This filter is a commonly used method to estimate the values of state variables of a dynamic system that is excited by stochastic disturbances and measurement noise. To formulate the signal extraction problem with the Kalman filter, I express the filtering problem in a general state space form, which consists of a transition equation (24) and a measurement equation (25).

$$\mathbb{x}_t = \mathbb{A}_t \mathbb{x}_{t-1} + \mathbb{B}_t \mathbb{v}_t \qquad \mathbb{v}_t \sim \mathcal{N}(0, \mathbb{Q}_t) \tag{24}$$

$$\mathbb{s}_t = \mathbb{C}_t \mathbb{x}_t + \mathbb{D}_t \mathbb{w}_t \qquad \mathbb{w}_t \sim \mathcal{N}(0, \mathbb{R}_t) \tag{25}$$

Uncertainty is captured by the first transition equation of the exogenous state vector $\mathbb{x}_\mathbb{t}$. The agent observes the vector $\mathbb{s}_t$ expressed in the second measurement equation, which contains noise to the true signal. The $\mathbb{A}_t$, $\mathbb{B}_t$, $\mathbb{C}_t$, $\mathbb{D}_t$, $\mathbb{Q}_t$ and $\mathbb{R}_t$ are system matrices; $\mathbb{v}_t$ and $\mathbb{w}_t$ are vectors of mutually independent i.i.d. shocks. More specifically, for this model, the state vector is $\mathbb{x}_t = (x_t, g_t, z_t)^\intercal$, the measurement vector is $\mathbb{s}_t = (a_t, s_t)^\intercal$, the shock vectors are $\mathbb{v}_t = \mathbb{w}_t = (\epsilon_t^g, \epsilon_t^z, \epsilon_t^s)^\intercal$, and the system matrices $\mathbb{A}_t$, $\mathbb{B}_t$, $\mathbb{C}_t$, $\mathbb{D}_t$, $\mathbb{Q}_t$ and $\mathbb{R}_t$ are

$$\mathbb{A}_t = \begin{bmatrix} 1 & \rho_g & 0 \\ 0 & \rho_g & 0 \\ 0 & 0 & \rho_z \end{bmatrix} \mathbb{B}_t = \begin{bmatrix} \sigma_g & 0 & 0 \\ \sigma_g & 0 & 0 \\ 0 & \sigma_z & 0 \end{bmatrix}$$

$$\mathbb{C}_t = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \mathbb{D}_t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma_{st} \end{bmatrix}$$

$$\mathbb{Q}_t = \mathbb{R}_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note that, in this setting, I express the standard deviation parameters of the shocks in the $\mathbb{B}_t$ and $\mathbb{D}_t$ matrices. $\mathbb{Q}_t$ and $\mathbb{R}_t$ are time-invariant 3×3 identity matrices. In the baseline model, the parameters in matrix $\mathbb{A}_t$ and $\mathbb{C}_t$ are also time-invariant, so I simply drop the subscript t, and use $\mathbb{A}$, $\mathbb{C}$, $\mathbb{Q}$ and $\mathbb{R}$ from now on. The time-varying matrix $\mathbb{D}_t$ contains the parameter σ_{st}, which controls the degree of information friction in the economy. When σ_{st} = 0, s_t and a_t together fully reveal the components of productivity. This leads to an economy with a perfect information environment. The matrix $\mathbb{B}_t$ contains σ_g and σ_z, which control the volatility of fundamental productivity. In this baseline model, fundamental uncertainty is constant, so I simply use $\mathbb{B}$ without the subscript t. In the later section, I also consider an economy with fundamental stochastic volatility. In that case, matrices $\mathbb{B}_t$ and $\mathbb{D}_t$ are both time-varying.

To derive the recursive form of the Kalman filter, let me first define the following variables. I use $\mathbb{x}_{t|t}$ to denote the agent's expectation regarding $\mathbb{x}_t$ based on all information at time t, and $\mathbb{P}_{t|t}$ for the a posteriori error covariance matrix:

$$\mathbb{x}_{t|t} \equiv \mathbb{E}_t[\mathbb{x}_t]$$

$$\mathbb{P}_{t|t} \equiv \mathbb{E}_t[(\mathbb{x}_t - \mathbb{x}_{t|t})(\mathbb{x}_t - \mathbb{x}_{t|t})^\intercal]$$

Let $\mathbb{x}_{t|t-1}$ denote the agent's expectation prior the new measurement $\mathbb{s}_t$, and $\mathbb{P}_{t|t-1}$ denote the a priori error covariance matrix:

$$\mathbb{x}_{t|t-1} \equiv \mathbb{E}_{t-1}[\mathbb{x}_t]$$

$$\mathbb{P}_{t|t-1} \equiv \mathbb{E}_t[(\mathbb{x}_t - \mathbb{x}_{t|t-1})(\mathbb{x}_t - \mathbb{x}_{t|t-1})^\intercal]$$

The essence of the Kalman filter algorithm is estimating the state process by implementing a linear form feedback control. The filter estimates the process state through transition equation, and then obtains feedback from the measurement equation. As such, the recursive form of the Kalman filter after the a posteriori error minimization problem falls into two groups: *updating* equations (26, 27, 28) and *projecting* equations (29, 30).

$$\mathbb{x}_{t|t} = (\mathbb{I} - \mathbb{K}_t\mathbb{C})\mathbb{x}_{t|t-1} + \mathbb{K}_t\mathbb{s}_t \tag{26}$$

$$\mathbb{K}_t = \mathbb{P}_{t|t-1}\mathbb{C}^\intercal(\mathbb{C}\mathbb{P}_{t|t-1}\mathbb{C}^\intercal + \mathbb{D}_t\mathbb{R}\mathbb{D}_t^\intercal)^{-1} \tag{27}$$

$$\mathbb{P}_{t|t} = (\mathbb{I} - \mathbb{K}_t\mathbb{C})\mathbb{P}_{t|t-1} \tag{28}$$

The projecting equations are responsible for projecting forward the current state and error covariance estimate. They produce an a priori estimate for the next time step. The updating equations are responsible for the feedback. They incorporate the new measurement into the a priori estimate and produce an improved a posteriori estimate. The matrix $\mathbb{K}_t$ is the Kalman gain, a result from the estimate error minimization problem in each step. It controls the relative weights on the a priori estimate and new measurement.

$$\mathbb{x}_{t+1|t} = \mathbb{A}\mathbb{x}_{t|t} \tag{29}$$

$$\mathbb{P}_{t+1|t} = \mathbb{A}\mathbb{P}_{t|t}\mathbb{A}^\intercal + \mathbb{B}\mathbb{Q}\mathbb{B}^\intercal \tag{30}$$

3.2 Time-Variant Bayesian Learning

With updating and projecting equations, I can express the Kalman filter problem in any recursive form. Particularly, in the form of a priori error covariance matrix, I derive the algebratic Riccati equation:

$$\mathbb{P}_{t+1|t} = \mathbb{A}(\mathbb{I} - \mathbb{K}_t\mathbb{C})\mathbb{P}_{t|t-1}\mathbb{A}^\intercal + \mathbb{B}\mathbb{Q}\mathbb{B}^\intercal \tag{31}$$

When system matrices are all time-invariant, Kalman gain, a priori and a posteriori error covariance matrices converge monotonically to a time-invariant solution. I can get these steady state values by recursively solving the Riccati equation. Using $\mathring{\mathbb{P}}_t$ and $\widehat{\mathbb{P}}_t$ to denote the steady a priori and a posteriori error covariance matrix, the recursive Riccati algorithm is expressed in the following three equations.

$$\mathring{\mathbb{P}} = \mathbb{A}(\mathbb{I} - \mathbb{K}\mathbb{C})\mathring{\mathbb{P}}\mathbb{A}^\intercal + \mathbb{B}\mathbb{Q}\mathbb{B}^\intercal \tag{32}$$

$$\mathbb{K} = \mathring{\mathbb{P}}\mathbb{C}^\intercal(\mathbb{C}\mathring{\mathbb{P}}\mathbb{C}^\intercal + \mathbb{D}\mathbb{R}\mathbb{D}^\intercal)^{-1} \tag{33}$$

$$\widehat{\mathbb{P}} = (\mathbb{I} - \mathbb{K}\mathbb{C})\mathring{\mathbb{P}} \tag{34}$$

However, in my model, the learning problem is time-variant. The matrix $\mathbb{D}_t$ is time-varying because of the σ_{st}. Thus, the Kalman learning variables $\mathbb{K}_t$, $\mathring{\mathbb{P}}_t$ and $\widehat{\mathbb{P}}_t$ are all time-varying, and depend on the state of σ_{st} at time t. I assume σ_{st} is public information to agent at time t. Unfortunately, there are no close form solutions for the $\mathbb{K}_t/\mathring{\mathbb{P}}_t/\widehat{\mathbb{P}}_t$-to-$\sigma_{st}$ mapping. I use a numerical method to approximate these mappings. First, I select a reasonably wide range of σ_s space $\left[\sigma_s^{LB}, \sigma_s^{UB}\right]$, and discretize it with n_s points. Then, for each σ_s^i, $i = 1, 2, ...n_s$, I recursively solve the Riccati equation, and get the corresponding Kalman gain $\mathbb{K}^i$, similarly for the a priori and a posteriori error covariance matrix $\mathring{\mathbb{P}}^i$ and $\widehat{\mathbb{P}}^i$. Finally, I use an n_p order of Chebyshev polynomials or power polynomials to approximate these n_s pair mappings with a reasonably low approximation error level. Generally, an n_p = 5 order approximation for n_s = 100 pair mapping is good enough to maintain the approximation error below 1×10^{-4} level.

3.3 Dynamics of Bayesian Beliefs

To formulate the representative agent's optimization problem in the recursive form and solve the model, I need to solve the law of motion of the agent's a posteriori beliefs $\mathbb{x}_{t|t}$ regarding fundamental productivity. In other words, $\mathbb{x}_{t|t}$ are state variables. The fundamental productivity variables, x_t, g_t or z_t are not state variables, since they are unobservable. To derive the dynamics of $\mathbb{x}_{t|t}$, I re-write the following two equations:

$$\mathbb{x}_{t|t} = \mathbb{A}\mathbb{x}_{t-1|t-1} + \mathbb{K}_t \left(\mathbb{s}_t - \mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1}\right) \tag{35}$$

$$\mathbb{s}_t = \mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1} + \left(\mathbb{s}_t - \mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1}\right) \tag{36}$$

The first equation comes from (26) and (29); the second equation is a mathematical identity. Let me define $\mathbb{u}_t = \mathbb{s}_t - \mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1}$. This term represents the measurement surprise, because $\mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1}$ is the best estimate of time t signal, $\mathbb{s}_{t|t-1}$, based on all available information at $t-1$. Let me also define $\mathbb{\Sigma}_t = Var_{t-1}[\mathbb{u}_t]$. I derive the relationship of $\mathbb{\Sigma}_t$ and system matrices in equation (37).

$$\mathbb{\Sigma}_t = \mathbb{C}\left(\mathbb{A}\mathbb{P}_{t-1|t-1}\mathbb{A}^{\intercal} + \mathbb{B}\mathbb{Q}\mathbb{B}^{\intercal}\right)\mathbb{C}^{\intercal} + \mathbb{D}_t\mathbb{R}\mathbb{D}_t^{\intercal} \tag{37}$$

Then I decompose the matrix $\mathbb{\Sigma}_t$ as

$$\mathbb{\Sigma}_t = \mathbb{H}_t\mathbb{H}_t^{\intercal} \tag{38}$$

and re-write the joint dynamics of $\mathbb{x}_{t|t}$ and $\mathbb{s}_t$ in the following form:

$$\mathbb{x}_{t|t} = \mathbb{A}\mathbb{x}_{t-1|t-1} + \mathbb{K}_t\mathbb{H}_t\hat{\mathbb{u}}_t \tag{39}$$

$$\mathbb{s}_t = \mathbb{C}\mathbb{A}\mathbb{x}_{t-1|t-1} + \mathbb{H}_t\hat{\mathbb{u}}_t \tag{40}$$

where $\hat{\mathbb{u}}_t$ is a vector of mutually independent i.i.d. standard normal shocks. Equations (39) and (40) fully characterize the evolution of the agent's beliefs. With these two equations, I convert the original model with unobservable information to an equivalent model with full information and correlated shocks.

4 Quantitative Analysis

The goal of this section is to evaluate the quantitative effects of stochastic information friction. To do so, I numerically solve the DSGE model with the 3rd-order perturbation method. A real business cycle model with information friction shocks can simultaneously match the key moments of macroeconomic variables and asset returns. The macroeconomic effects are captured by the impulse responses to the shocks. Though simulation, I also show that information friction shocks are important for capturing the dynamics of real business cycle quantities.

4.1 Numerical Method

The body of literature on DSGE computation methods is very large. Commonly used methods include value function iteration, perturbation, Chebychev polynomials, finite element methods, etc. Here I select the perturbation method that I find most promising in terms of accuracy and efficiency and implement a 3rd-order perturbation of the model. The 1st-order perturbation is useless here, particularly because of the recursive preference and second moment shocks ingredients in the model. Note that the most common solution methods, linearization and log-linearization, are particular cases of 1st-order perturbation. Essentially, the decision rules from 1st-order approximation are certainty equivalent. Therefore, they depend on ψ, but not on γ or σ_{st}. In order to allow recursive preference and second moment shocks to play a role, I need to go at least to 2nd-order perturbation to have terms that depend on γ or σ_{st}. Even in 2nd-order approximation, the effect of uncertainty shocks is limited. I say limited because the mean is affected but not the dynamics. For this reason, we are interested especially in the time-varying risk premia; a 3rd-order or even higher order perturbation is preferred. I solve the model using a 3rd-order perturbation method with `Dynare` and `MatLab`. For a 4th-order approximation, I use `Mathematica`, which works very well with symbolic algebra and symbolic equation manipulations. I perform the higher order perturbation to mitigate the concerns that lower order approximation may contain large errors if the model exhibits high non-linearity. The accuracy of 3rd-order perturbation in terms of Euler equation errors is excellent – even far way from the steady state. Throughout the paper, I report the numerical results from the 3rd-order perturbation solutions.

4.2 Parameterization

I select a benchmark calibration for the numerical computations. The parameter values are chosen to match basic observations of the US economy, and they align closely with common choices in the literature. Table 1 summarizes the full set of parameters in my benchmark calibration. I take one period in the model to represent one quarter.

On the preference side, the quarterly discount factor β is set to 0.994 to match the risk-free interest rate. The value for labor share θ is set to 0.36, implying that the share of time devoted to work is one-third in the steady state. Aligned with the long-run risk literature, the relative risk aversion and EIS are set as $\gamma = 8$ and $\psi = 2$.

On the production side, the long run deterministic growth rate μ is set to 0.0049 to match the GDP growth in the US. The capital exponent in production function α is calibrated to match the capital income share. The capital depreciation rate is set as 2.5%, which is standard for a quarterly frequency model. The steady state value of capital adjustment cost $\bar{\xi}$ is set to 20.

On the information friction side, the average information friction level is set as $\bar{\sigma}_s = 0.0147$, which is consistent with Blanchard et al. (2013). The persistence of information friction shock κ_s is set to 0.9, and the standard deviation of information friction shock ω_s is set to 0.02.

There is no debt in the model. In data, equity returns are levered, and some portion of dividend growth volatility is due to idiosyncratic payout shocks. To better compare the data with the model, I multiply risk premia and standard deviations of stock returns by a leverage parameter of 2.

4.3 Impulse Responses

Figure 1 shows the impulse responses for four observed macro variables to one standard deviation of permanent technology shock, transitory technology shock, and noise shock respectively. All four variables show similar patterns in response to the three shocks. They respond primarily to the transitory productivity shock and noise shock in the short run. In response to a permanent shock, they build up slowly over time, because it takes a longer period for agents to recognize the permanent shocks. Noise shock generates some short-term fluctuations, but the response also dies down very fast. In response to a noise shock, agents consume more and invest less, because they assign a positive probability to the case that the shock might be a permanent productivity shock. Agents quickly learn that the shock is just noise; those responses quickly return to zero.

By nature, the information friction shocks are second moment shocks. They affect the shape of impulse responses by affecting the initial state when other shocks hit the economy. In Figures 2, 3 and 4, I plot the impulse responses to three shocks in two different scenarios, in which the initial state is high ($\sigma_{st} = H$) or low ($\sigma_{st} = L$) information friction. I also plot the impulse responses for the perfect information case. This perfect information scenario is a special case where agents fully observe the permanent and transitory components; it is captured as $\sigma_{st} = 0$.

With perfect information, in response to a permanent productivity shock, agents consume more, cut investments, and work less, because they know productivity will be high tomorrow. However, when there is information friction, agents assign a positive probability to the case that the shock might just be noise or transitory. The more severe information friction is, the less probability agents assign to a permanent shock. So agents consume less, invest more, and work harder when information friction is severe, as shown in Figure 2. Agents display consumption smoothing behavior.

In response to a transitory productivity shock, the magnitude of initial response is very similar under the three scenarios shown in Figure 3. However, the degree of information friction does affect persistence. When σ_{st} is high, responses to transitory shocks die out very fast, because productivity shocks are less informative when information friction is high. Thus, the transitory productivity shocks are more likely to be treated as noises.

With perfect information, agents do not respond to any noise shocks. With an imperfect information environment, noise shocks generate short-term fluctuations. With Bayesian learning, agents quickly recognize the disturbance is just noise, so responses die out very fast. When σ_{st} is high, responses to noise shock are more persistent, and die out slowly.

4.4 Business Cycle Moments

I report the model-implied moments of macroeconomic quantities and asset prices from the baseline model in Table 2. The moments in actual data are estimated using quarterly US time series data of GDP, consumption, investment and employment from the period 1948:I to 2014:IV. Further details of variables' definition and construction may be found in the Appendix. The model matches the key moments of the US data closely. For the simulated data, the table shows the mean values across simulations, along with the 5th and 95th percentile values in brackets.

For most of the moments of interest, the range of empirical estimates falls inside the 90 percent confidence intervals generated by the model. The volatility of output growth is 1.53%, compared with 1.38% in the data. But the volatility of investment growth and consumption growth in the model are a little bit off their empirical counterpart. The ratio of consumption growth volatility to output growth volatility is 0.61, and the ratio of investment growth volatility to output growth volatility is 3.40. The implied levered equity premium of the model is 4.23%, matching the historical mean of the CRSP stock market log excess returns.

5 Asset Pricing Implication

The goal of this section is to explore asset pricing implications from the model. To do so, I first derive the price of information friction shock from the model solutions. IFS carries a negative price of risk. Then I investigate the mechanism of generating a negative price of risk by showing the impulse responses to the information friction shocks.

5.1 Price of Risk

The innovations to the stochastic discount factor are important as they characterize the risks that affect investors' marginal utility and determine risk compensation. To pin down the price of information friction shock risk, I express the innovations to the log of stochastic discount factor $m_t \equiv \log(M_t)$ from the model solutions.

$$m_t - E_{t-1}[m_t] = -\lambda_t^{\hat{g}} \epsilon_t^{\hat{g}} - \lambda_t^{\hat{z}} \epsilon_t^{\hat{z}} - \lambda_t^f \epsilon_t^f - \lambda_t^s \eta_t^s \tag{41}$$

Since I am interested in the price of risk associated with information friction, I focus on the λ_t^s part. I solve the model with 3rd-order perturbation; therefore, the price of risk is time-variant, and λ_t^s includes the 2nd-order cross terms among all the state variables and exogenous shocks in the model. Here I consider the unconditional mean of the λ_t^s. In such a way, all the interaction terms are eliminated, leaving only a constant term.

$$\bar{\lambda}^s \equiv E[\lambda_t^s] = -0.0032 \tag{42}$$

Note that this approach is essentially the same as solving the model with 1st-order perturbation, or the log-linearization technique. The price of information friction risk is negative. A positive shock to the information friction induces the marginal utility of consumption to increase, i.e. in a bad state of world. On average, the price of risk for one standard deviation[2] shock to the information friction is 0.32%.

5.2 Mechanism Inspection

Figure 5 shows the impulse responses for six observed macro variables, as well as the kernel and dividend, to one standard deviation of information friction shock. In response to a positive shock to information friction, agents consume less, invest

[2] All the shocks in the model are standardized normal shocks with unit variance.

more, and work more hours. As a result, capital and output increase and dividend drops significantly. However, agents' marginal utility increases in response to a positive information friction shock. Dividend and SDF move in opposite directions. This means dividend payments are low when agents are in a bad state of world. Thus, dividend claim carries a positive risk premium associated with information friction shock.

Financial friction also amplifies this co-movement effect. Information friction shocks and financial friction shocks are negatively correlated in the model. A positive information friction shock also decreases the ξ_t, as shown in Figure 5. Note, the lower ξ_t represents higher capital adjustment costs. The friction in capital markets becomes more severe when information friction is high. On average, dividend drops more when capital market friction is higher. In Figure 6, I plot the impulse responses of SDF and dividend to the information friction shock, under two scenarios. Dividend is more volatile when the correlation between information friction and financial market friction is higher. There is vast evidence in the literature that investigating the relationship between capital markets and imperfect information showing external financing is more expensive with the existence of asymmetrical information. To sum up, the interaction between information friction and financial market friction amplifies the effect of information uncertainty on asset valuations.

6 Empirical Evidence

In this section, I test the model's asset pricing implication empirically. To do so, I construct a proxy for information friction shock using Survey of Professional Forecasters (SPF) data and measure of macroeconomic uncertainty from Jurado et al. (2015). Specifically, I first construct the measure of belief dispersion from the

SPF data; then, I orthogonalize the time series of belief dispersion using estimates from Jurado et al. (2015) to control any effects from the fundamental uncertainty channel. The remaining part is driven by the information uncertainty channel; I attribute it as the proxy for information friction shock. With this empirical proxy for information friction shock, I use Fama-French 25 size-value portfolios to test whether IFS is a priced risk factor. The results show a significant negative price of risk for these test assets. I also explore the cross-sectional return predictability using portfolio sorts methodology. My main finding is that firms with high exposure to information friction shock tend to have lower returns, on average, than firms with low information friction shock exposure. This finding is consistent with the model prediction of negative price of information friction risk.

6.1 Measure of Information Friction Shocks

The main interest of this paper is to study the uncertainty effect from the information friction channel. In the set up of my model, time-varying information friction directly affects agents' a posteriori estimate variance. When the information friction becomes more severe, agents are more unsure about true fundamental productivity and the variance of their beliefs become wider. Thus, I first construct the measure of belief dispersion using the Survey of Professional Forecasters data.

The Federal Reserve Bank of Philadelphia provides extensive panel data on economic variable forecasts by professional economists. Each economist is asked to forecast a large set of macroeconomic and financial variables over the current quarter and the subsequent four quarters. To ensure panelists have the same information set, the Philadelphia Fed synchronizes the survey timing with the release of Bureau of Economic Analysis' advance report, which contains the first estimate of GDP from the previous quarter. After this report is released to the public, the Philadelphia Fed sends out survey questionnaires with all recent data from the advance report. Usually, the deadline for survey responses is set one or two weeks before the second month of each quarter, and the Philadelphia Fed releases the survey results at the end of second month of each quarter. Figure 11 shows the timing of the Survey of Professional Forecasters.

Although SPF provides forecasts over different horizons, I focus on the forecasts over the current quarter, not any future quarters. The reason is imperfect information prevents the agents from observing the true current state, not future states. The forecasts of future states are more likely to be driven by the fundamental uncertainty, not imperfect information. As shown in Figure 11, when economists are asked to forecast the current quarter GDP, they are almost at the end of the second month of the current quarter. Therefore, a significant portion of the individual-level forecast dispersion is due to information friction. To some extent, this helps me control some fundamental uncertainty effects. SPF also provides forecasts of many macroeconomic and financial variables. However, I focus on the nominal GDP because this directly relates to the productivity in my model set up. As a result, the belief dispersion series is constructed using the individual-level forecast of nominal GDP of the current quarter from the SPF data. I define the belief dispersion (BD) as the difference between the 75 and 25 percentiles of logarithm of nominal GDP across all individual forecasts for each quarter. For robustness check, I also consider the 80-20 and 90-10 dispersion measures. The results do not change much.

$$BD_t = Pctile(\log(NGDP_{it}), 75) - Pctile(\log(NGDP_{it}), 25) \tag{43}$$

The cross-sectional forecasts dispersion is used in other studies as a proxy for macroeconomic uncertainty, e.g. Bloom (2009), Bachmann et al. (2013) and Bali et al. (2014). In the model, belief dispersion is driven by two channels: the information channel, and the fundamental uncertainty channel. To get a clean proxy for information friction shocks, I orthogonalize the measure of belief dispersion with a macro uncertainty proxy from Jurado et al. (2015). Jurado et al. (2015) exploit a very rich data environment and provide direct estimates of macroeconomic uncertainty (MU). I use two methods in this orthogonalization practice. The first method is simply regressing belief dispersion (BD) over macro uncertainty (MU). The information friction shocks (IFS) are defined as the residuals of this OLS regression.

$$BD_t = \beta MU_t + IFS_t \tag{44}$$

The second method involves two steps. In the first step, I get the unexpected change of belief dispersion (BDS), and unexpected change of macro uncertainty (MUS), by fitting a AR(1) regression of the BD and MU series. Then, in the second step, I regress MUS over BDS; the residuals are defined as information friction shocks (IFS).

$$BD_t = \rho BD_{t-1} + BDS_t \tag{45}$$

$$MU_t = \rho MU_{t-1} + MUS_t \tag{46}$$

$$MUS_t = \rho BDS_t + IFS_t \tag{47}$$

The empirical results are not sensitive to the choice of these two methods. I report the result with the second orthogonalization method. Note that all the residuals from OLS regressions are standardized, so that BDS, MUS and IFS all have unit variance.

6.2 Risk Pricing

The model implies a negative price of information friction risk. I evaluate the performance of the baseline model using 25 Fama-French size and book-to-market portfolios as test assets. I follow the two-pass regression procedure in Boguth and Kuehn (2013). First, for each test asset, I obtain unconditional risk loadings from a time-series regression of excess returns on information friction shock (IFS) and other factors, depending on the model specification. In the second pass, I estimate the prices of risk by cross-sectionally regressing average excess returns on the first-pass loadings. The results from the second-pass regression are reported in Table 5. I consider several specifications. The results confirm that IFS is a negatively priced factor. The Fama-French 3 factors specification explains the test asset returns with an R^2 of 69%. Adding the IFS factor into the Fama-French 3 factors specification improves the explanatory power with an R^2 of 78%. The last full model specification, including the IFS factor, Fama-French 3 factors and momentum factor, achieves an R^2 of 82%.

6.3 Portfolio Sort

I obtain risk loadings as slope coefficients from time-series regressions of individual stock returns on information friction shocks, controlling the Fama-French 3 factors and the momentum factor. In particular, for each security, I estimate factor loadings in each quarter using the previous 5 years of quarterly observations.

$$r_t^i - r_t^f = \alpha_t^i + \beta_t^i IFS_t + \gamma_t^i Control_t + \varepsilon_t^i \tag{48}$$

With the estimates of each stock's risk loadings, I am able to test whether future returns are predicted by the exposure to innovations in information friction, using the portfolio sort methodology. To make an accurate estimate of the price of dispersion shock, I need assets with substantial dispersion in their exposure to the dispersion shock. Thus, I create portfolios of firms sorted on their past sensitivity to the dispersion shock, and focus on the spread between highest and lowest decile portfolios. At the end of each quarter, I sort all stocks into portfolios based on their estimated risk loadings from the time-series regression (48). Portfolios are held for three months and re-balanced every quarter.

Table 6 shows the average excess returns and risk characteristics for the 10 portfolios of stocks sorted on their past sensitivity to the information friction shock. First, these portfolios display a declining pattern of average excess returns, ranging from 85 bps to 46 bps per month. Second, the volatility of these portfolios displays a U-shape from low decile to high decile. The volatility in the middle is around 4.6 percent per month, where the volatility of low and high portfolios is above 7.7 percent per month.

In the last column, I also report the excess returns of a long-short strategy that invests in the high exposure portfolio and sells the low exposure portfolio. The average monthly returns of this zero investment portfolio is -0.39%, yielding an annual return of around -5%. Cross-sectional differences in returns might not be surprising if the IFS betas covary with other variables known to predict returns. To mitigate this concern, I regress the portfolio returns on the most commonly used factors, such as Fama-French 3 factors and the momentum factor. The α and its t-statistic are also reported in Table 6. The long-short portfolio earns a significant -55 bps excess return per month, or about -6.8% per annum.

7 Information Uncertainty and Fundamental Uncertainty

So far, I have shown the importance of stochastic information friction for capturing the dynamics of real business cycle quantities and asset prices. The main mechanism is time-varying information friction affecting agents' a posteriori estimate variance. When information friction becomes more severe, the agent is more unsure about the true fundamental productivity, belief dispersion becomes wider, uncertainty becomes larger. Importantly, this time-variant uncertainty feature is solely generated from the information channel; it is not related to fundamental productivity uncertainty. In the baseline model, productivity uncertainty is constant. In this section, I allow the fundamental productivity uncertainty to be time-varying. As a result, time-variant uncertainty is generated from two channels.

Since the seminal work of Bloom (2009), a large and growing body of literature has studied the effect of uncertainty shock in explaining macroeconomic dynamics. In this literature, the information environment is usually perfect; agents know the fundamentals today, but are not sure about the volatility of productivity tomorrow due to uncertainty shock. This clearly differentiates from the information uncertainty channel. Information uncertainty from the baseline model refers to agents' belief uncertainty regarding the fundamentals today. To better understand the time-varying uncertainty from these two channels, I build a DSGE model with both information friction shocks and uncertainty shocks in next section.

7.1 Extended Model with Uncertainty Shock

As in my baseline model, the productivity a_t follows the process (3) with component x_t and transitory component z_t. Permanent growth $g_t \equiv \Delta x_t$ follows the same

AR(1) process in (4).

$$a_t = x_t + z_t \tag{49}$$

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_t^g \tag{50}$$

The transitory productivity z_t follows a similar AR(1) process in (5), but with time-varying σ_{zt} to incorporate the uncertainty shocks. Fundamental uncertainty level is perturbed by i.i.d. standard normal shocks η_t^z, with variance ω_z^2.

$$z_t = \rho_z z_{t-1} + \sigma_{zt} \epsilon_t^z \tag{51}$$

$$\log(\sigma_{zt}) = (1 - \kappa_a) \log(\bar{\sigma}_z) + \kappa_a \log(\sigma_{zt-1}) + \omega_a \eta_t^a \tag{52}$$

The information environment is still imperfect, and features time-varying frictions. As in my baseline model, the agent observes a_t and an additional signal s_t, but is imperfectly informed about the true decomposition of x_t and z_t.

$$s_t = x_t + \sigma_{st} \epsilon_t^s \tag{53}$$

$$\log(\sigma_{st}) = (1 - \kappa_s) \log(\bar{\sigma}_s) + \kappa_s \log(\sigma_{st-1}) + \omega_s \eta_t^s \tag{54}$$

All the other setups are the same as the baseline model.

7.2 Quantitative Analysis

In this extended model, with both information friction shock and uncertainty shock, the agent's beliefs dispersion is time-varying and is affected by both shocks simultaneously. To better understand this, I take a look at modified equations (37), (38) and (39) in agents' Bayesian learning problem.

$$\begin{aligned}\mathbb{\Sigma}_t &= \mathbb{C}\left(\mathbb{A}\mathbb{P}_{t-1|t-1}\mathbb{A}^\intercal + \mathbb{B}_t\mathbb{Q}\mathbb{B}_t^\intercal\right)\mathbb{C}^\intercal + \mathbb{D}_t\mathbb{R}\mathbb{D}_t^\intercal \\ \mathbb{\Sigma}_t &= \mathbb{H}_t\mathbb{H}_t^\intercal \\ \mathbb{x}_{t|t} &= \mathbb{A}\mathbb{x}_{t-1|t-1} + \mathbb{K}_t\mathbb{H}_t\hat{\mathbb{u}}_t\end{aligned}$$

Note that the matrix $\mathbb{B}_t$ is time-varying now. It contains σ_{zt}, which controls fundamental uncertainty. The matrix $\mathbb{D}_t$ contains σ_{st}, which controls time-varying information friction. Beliefs dispersion is affected by both $\mathbb{B}_t$ and $\mathbb{D}_t$ matrices. Figure 7, 8 and 9 show the beliefs dispersion against σ_{st} and σ_{zt}. Beliefs dispersion is a monotonically increasing function of both information uncertainty and fundamental uncertainty.

The model is solved with a 3rd-order perturbation method. Table 3 summarizes the full set of parameters used in this model. Most of the parameters remain the same as the baseline model. Table 4 shows the ability of the extended model, with both information friction shock and uncertainty shock, to match business cycle moments. The model matches key moments of the US data closely as reported in the second column. Simulated moments from the baseline mode and extended model are reported in the third and fourth column. For the simulated data, the table shows the mean values across simulations, along with the 5th and 95th percentile values in brackets. For most of the moments of interest, the range of empirical estimates falls inside the 90 percent confidence intervals generated by the model.

To study the asset pricing implication of this extended model, I focus on the innovations to the log of stochastic discount factor, which characterize the risk compensation in this economy.

$$m_t - E_{t-1}[m_t] = -\lambda_t^{\hat{g}}\epsilon_t^{\hat{g}} - \lambda_t^{\hat{z}}\epsilon_t^{\hat{z}} - \lambda_t^{f}\epsilon_t^{f} - \lambda_t^{s}\eta_t^{s} - \lambda_t^{a}\eta_t^{a} \tag{55}$$

I focus on the λ_t^s and λ_t^a terms, which determine the price of risk associated with information uncertainty risk and fundamental uncertainty risk. Similarly, I examine the unconditional mean of λ_t^s and λ_t^a.

$$\bar{\lambda}^s \equiv E[\lambda_t^s] = -0.0025$$
$$\bar{\lambda}^a \equiv E[\lambda_t^a] = -0.0025$$

Both information uncertainty and fundamental uncertainty carry a negative price of risk. A positive shock to the information friction or fundamental uncertainty induces the marginal utility of consumption to increase, i.e. in a bad state of world. On average, the price of risk for one standard deviation shock to the information friction or fundamental uncertainty is 0.25%. Under the assumptions of this extended model, both information uncertainty and fundamental productivity uncertainty contribute 50% to the total uncertainty risk in this economy.

7.3 Social Welfare

To examine any policy implication, I plot the social welfare over $[\sigma_{st}\ \sigma_{zt}]$ space under average economic conditions in Figure 10. High information friction decreases social welfare. A clear policy implication is to reduce the information uncertainty and enhance social welfare. Increasing the quality of public news, increasing the accuracy of public reports, increasing transparency, and reducing policy uncertainty are all effective ways to reduce information uncertainty or costs associated with information acquisition in the economy. Reducing information friction is always better because agents make more efficient allocation decisions.

8 Conclusions

In an economy with a time-varying imperfect information environment, rational agents' beliefs, with Bayesian learning, feature time-variant second moments. This creates belief uncertainties from the information channel. I investigate the effect of stochastic information friction by analyzing a DSGE model with Epstein-Zin preferences. In particular, the interaction between imperfect information and financial market friction provides an important channel to amplify the effect of information uncertainty. Information friction shocks carry a negative price of risk. This source of risk also harms social welfare by preventing efficient allocation. A policy implication from the model is to reduce information uncertainty or costs associated with information acquisition, thus enhancing market allocation efficiency.

Empirical evidence supports the asset pricing prediction from the DSGE model. I construct an empirical measure to proxy for information friction shocks. Innovation to information uncertainty is a negatively priced source of risk for a wide variety of test portfolios. Cross-sectionally, exposure to information uncertainty risk strongly predicts future asset returns. Firms with high exposure to information friction shock generate significantly lower returns than firms with low information friction shock exposure. The estimated risk premium associated with information friction shock is negative and statistically significant. A mimicking portfolio, IFS factor, generates 6% excess return per annum.

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Figures and Tables

Figure 1: Impulse Responses

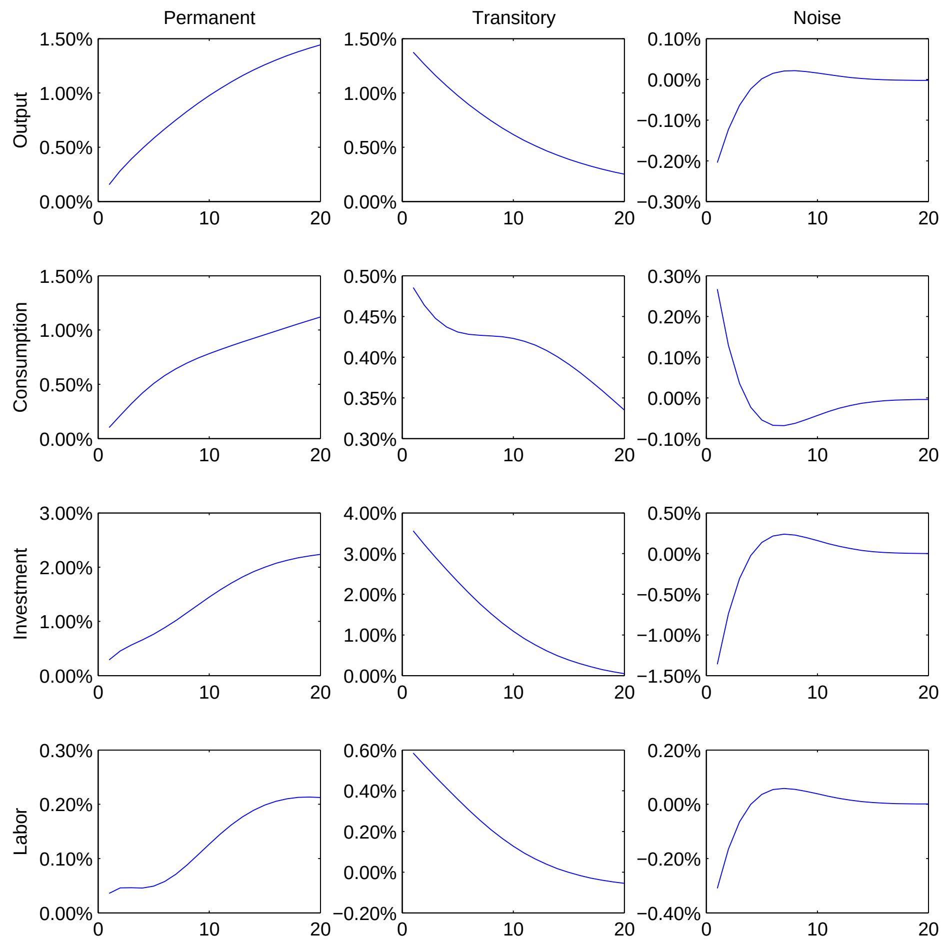

Figure 2: Impulse Responses to Permanent Productivity Shock

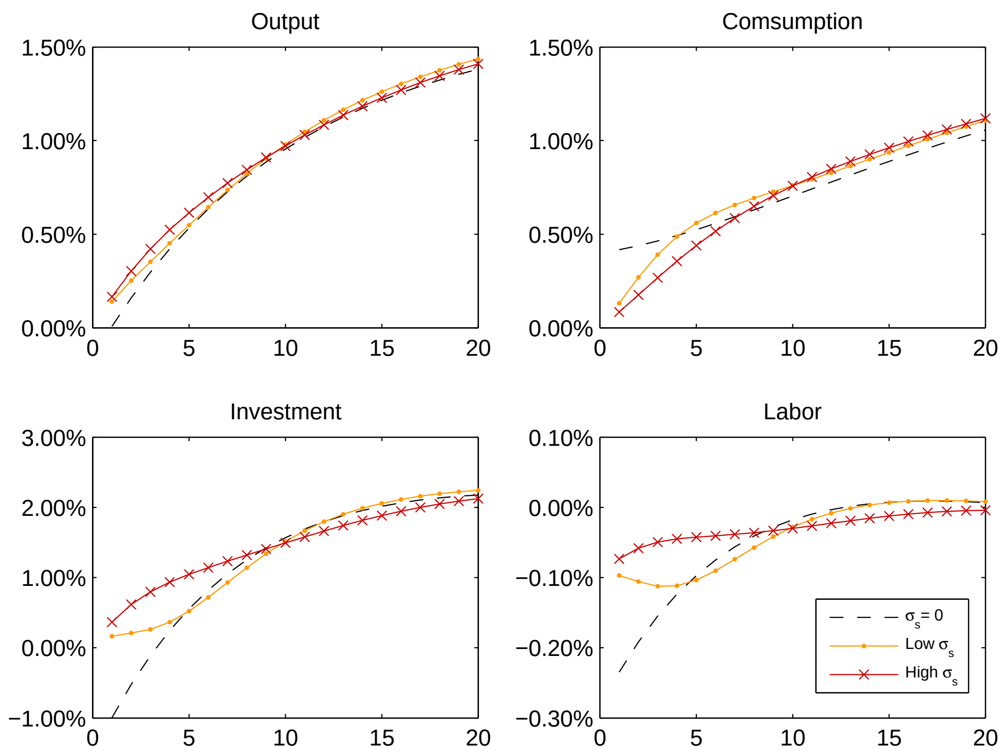

Figure 3: Impulse Responses to Transitory Productivity Shock

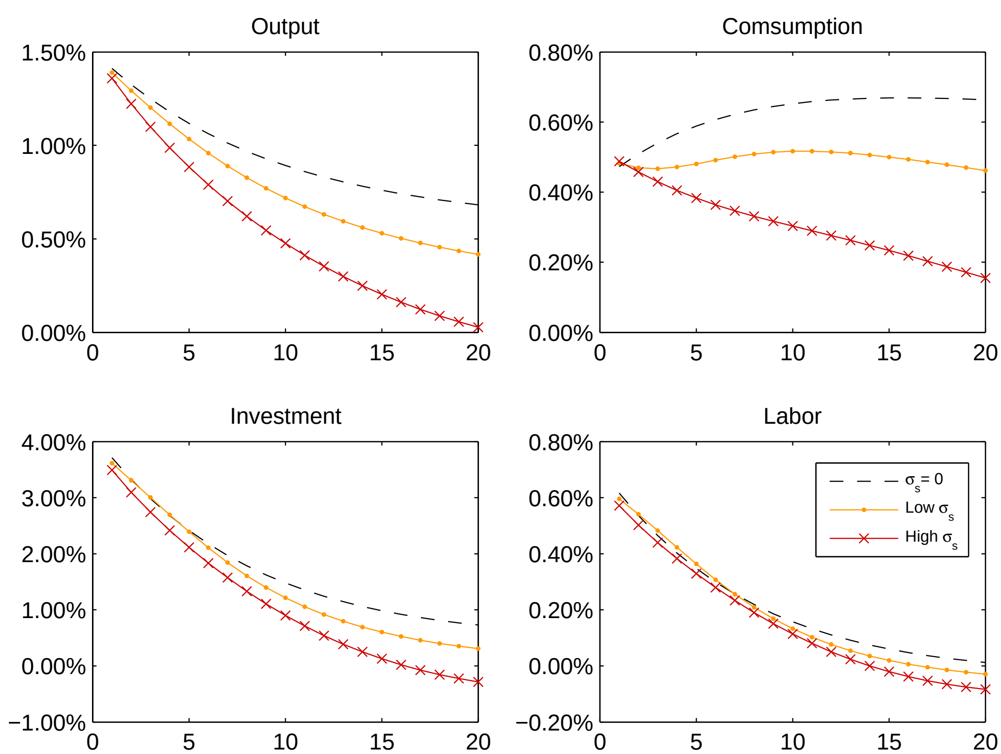

Figure 4: Impulse Responses to Noise Shock

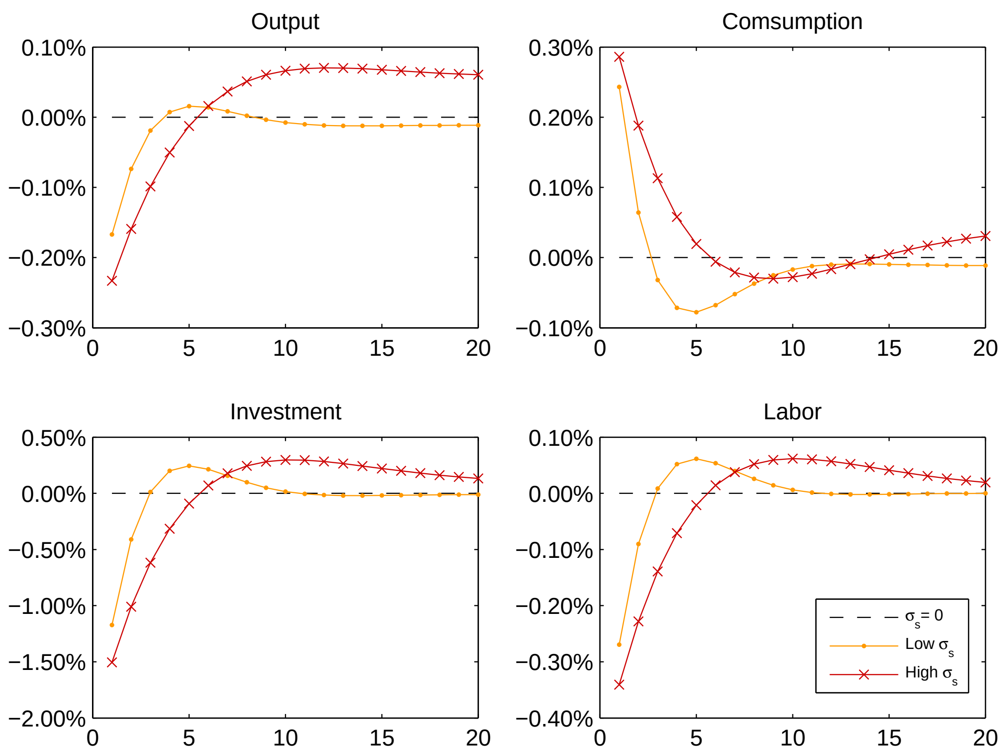

Figure 5: Impulse Responses to Information Friction Shock

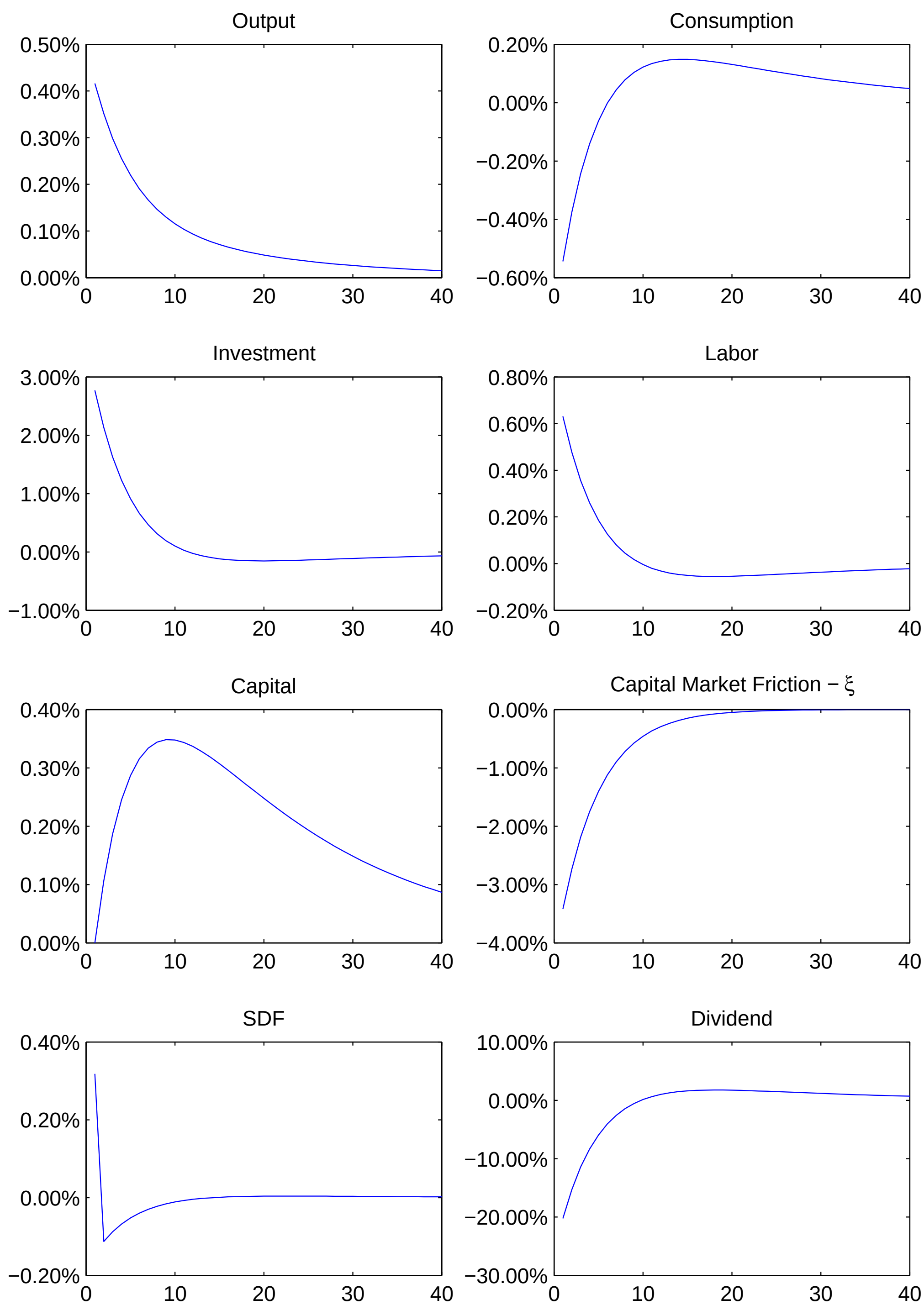

Figure 6: Impulse Response of SDF and Dividend to Information Friction Shock

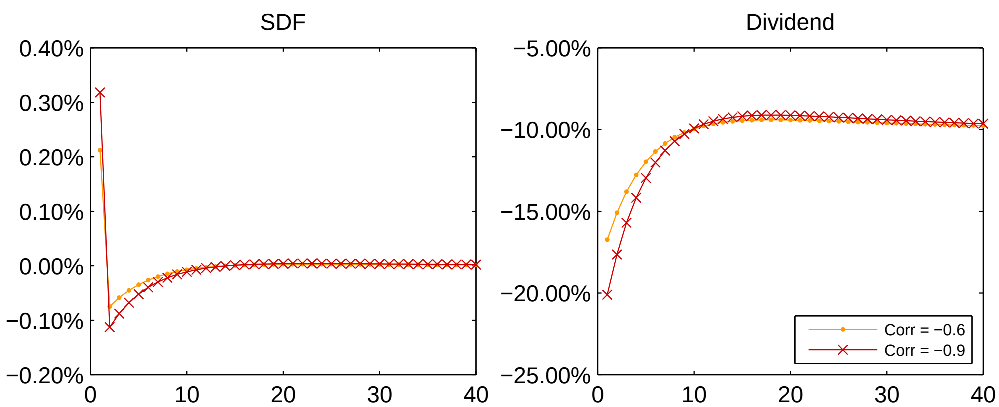

Figure 7: Belief Dispersion in Permanent Component x_t

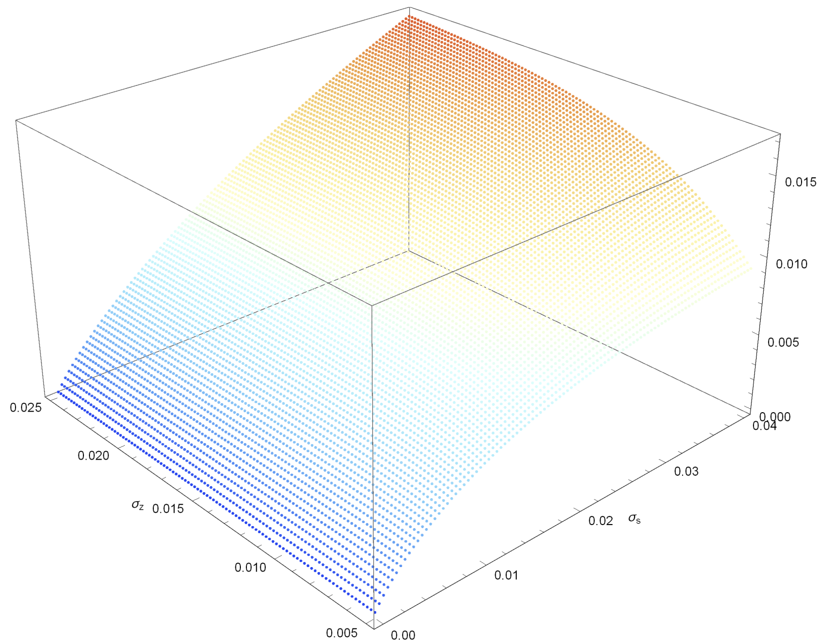

Figure 8: Belief Dispersion in Growth Component g_t

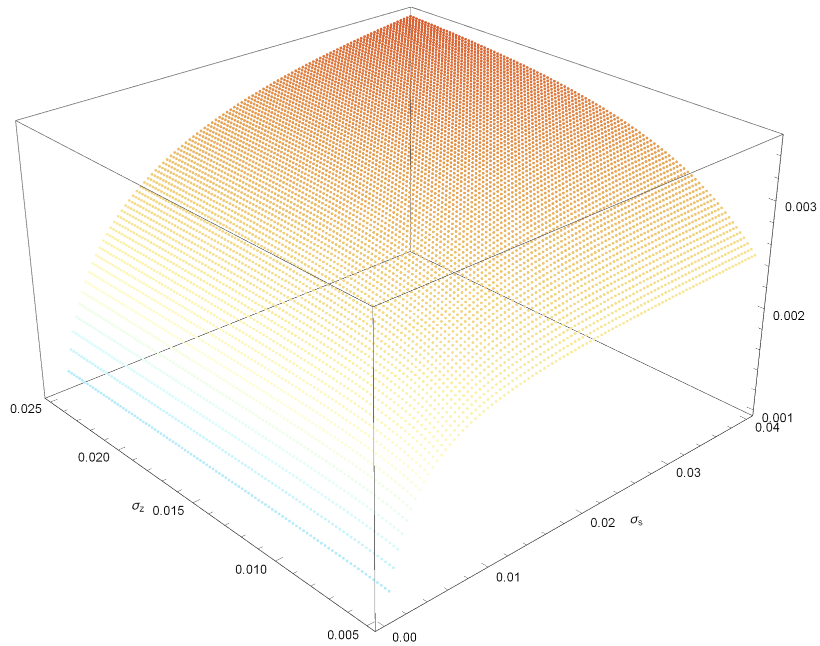

Figure 9: Belief Dispersion in Transitory Component z_t

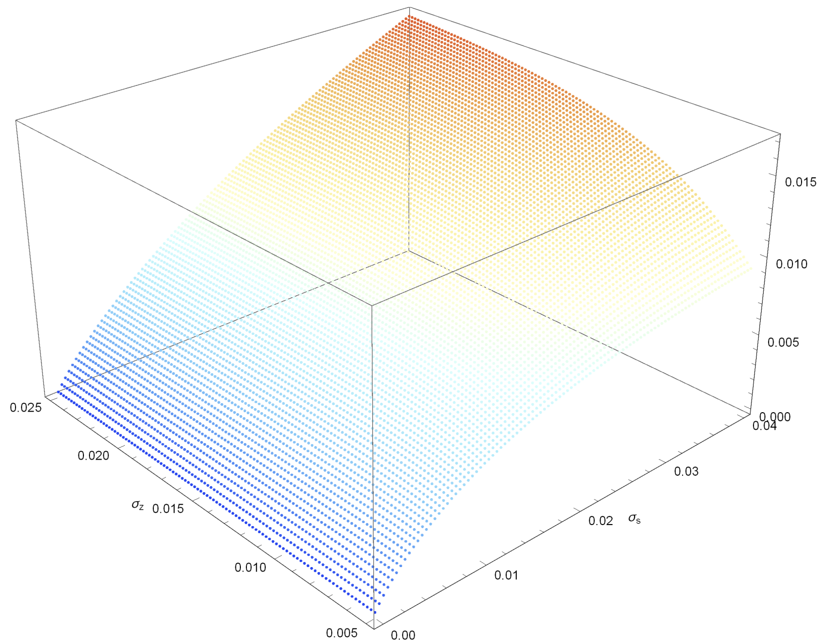

Figure 10: Social Welfare

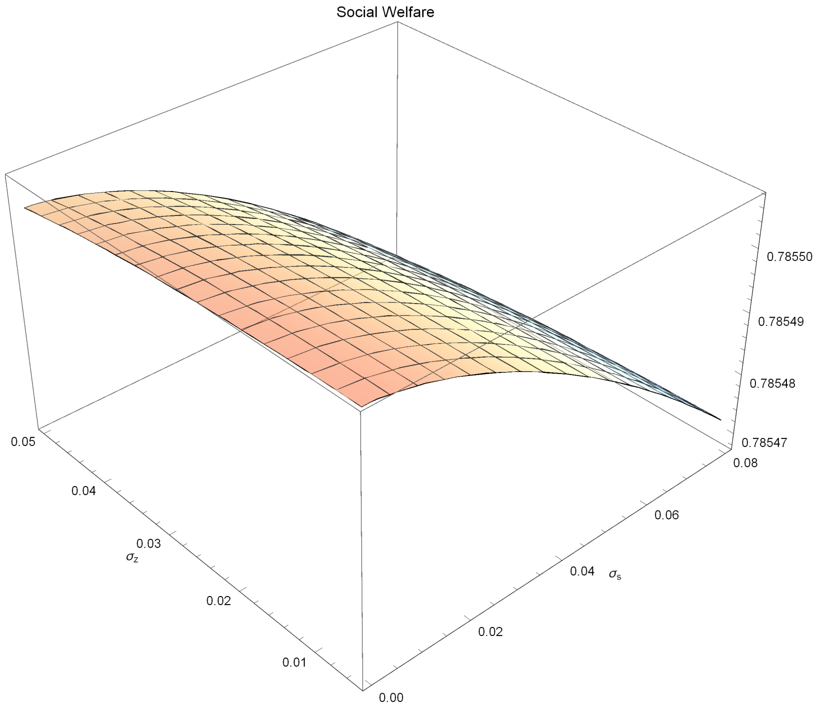

Figure 11: Timing of the Survey

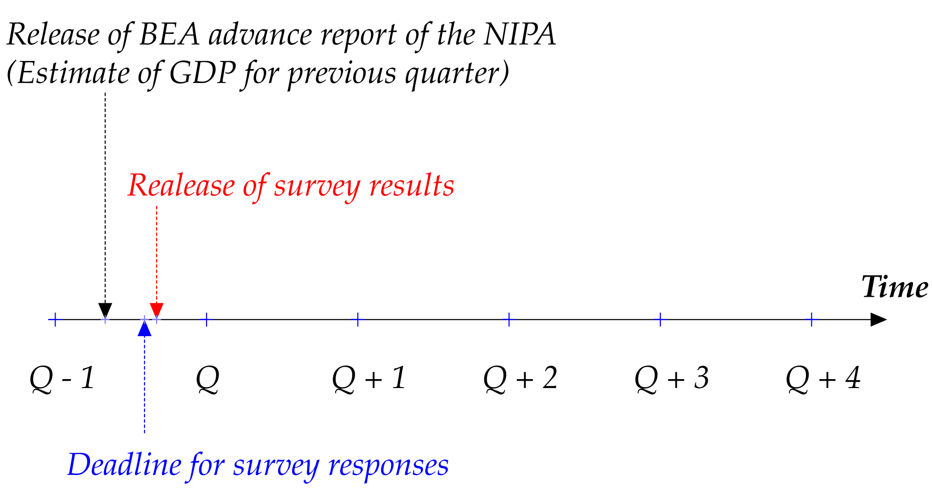

Table 1: Parameterization - Baseline Model

Parameters	Symbol	Value
Preference		
Discount factor	β	0.994
Risk aversion	γ	8.0
Elasticity of intertemporal substitution	ψ	2.0
Leisure exponent in utility	θ	0.36
Technology		
Capital exponent in production	α	0.34
Depreciation rate	δ	0.025
Deterministic growth rate	μ	0.0049
Persistence of permanent productivity shock	ρ_g	0.89
Volatility of permanent productivity shock	σ_g	0.002
Persistence of transitory productivity shock	ρ_z	0.89
Volatility of transitory productivity shock	σ_z	0.015
Capital adjustment cost	$\bar{\xi}$	20.0
Persistence of financial friction shock	ρ_f	0.80
Volatility of financial friction shock	σ_f	0.04
Operating cost	ι	0.0018
Wage adjustment cost	ξ_w	15.0
Information Friction		
Average level of information friction	$\bar{\sigma}_s$	0.0147
Persistence of information friction shock	κ_s	0.90
Volatility of information friction shock	ω_s	0.02
Correlation		
Corr(η_t^s, ϵ_t^f)	ϱ^{sf}	-0.90

Table 2: Business Cycle Moments - Baseline Model

	Data	Model
Macroeconomic Quantities		
$\sigma[\Delta y]$	1.38%	1.53%
		[1.41% 1.67%]
$\sigma[\Delta c]/\sigma[\Delta y]$	0.37	0.61
		[0.54 0.68]
$\sigma[\Delta i]/\sigma[\Delta y]$	3.45	3.40
		[3.20 3.62]
$\sigma[\Delta l]/\sigma[\Delta y]$	0.37	0.67
		[0.64 0.72]
Asset Prices		
$E[r_t - r_t^f]$	4.09%	4.23%
		[3.65% 4.83%]
$\sigma[r_t - r_t^f]$	15.80%	18.50%
		[16.74% 20.10%]

Note – The table compares moments of the data to simulated moments from the baseline model. I consider log output growth variability, relative variance of consumption to output, relative variance of investment to output, relative variance of labor to output, equity premium, and equity excess return volatility. Moments of actual data are estimated using quarterly US time series data of GDP, consumption, investment and employment from 1948:I to 2014:IV. Moments of simulated data from the model are at quarterly frequency. I report mean moments, along with the 5th and 95th percentiles across 1,000 simulations, each with a length of 50 years.

Table 3: Parameterization - Extended Model

Parameters	Symbol	Value
Preference		
Discount factor	β	0.994
Risk aversion	γ	8.0
Elasticity of intertemporal substitution	ψ	2.0
Leisure exponent in utility	θ	0.36
Technology		
Capital exponent in production	α	0.34
Depreciation rate	δ	0.025
Deterministic growth rate	μ	0.0049
Persistence of permanent productivity shock	ρ_g	0.89
Volatility of permanent productivity shock	σ_g	0.002
Persistence of transitory productivity shock	ρ_z	0.89
Volatility of transitory productivity shock	σ_z	0.015
Capital adjustment cost	$\bar{\xi}$	20.0
Persistence of financial friction shock	ρ_f	0.80
Volatility of financial friction shock	σ_f	0.04
Operating cost	ι	0.0018
Wage adjustment cost	ξ_w	15.0
Persistence of uncertainty shock	κ_a	0.90
Volatility of uncertainty shock	ω_a	0.02
Information Friction		
Average level of information friction	$\bar{\sigma}_s$	0.0147
Persistence of information friction shock	κ_s	0.90
Volatility of information friction shock	ω_s	0.02
Correlation		
Corr(η_t^s, ϵ_t^f)	ϱ^{sf}	-0.70
Corr(η_t^a, ϵ_t^f)	ϱ^{af}	-0.70

Table 4: Business Cycle Moments - Extended Model

		Model	
	Data	Baseline	Extended
Macroeconomic Quantities			
$\sigma[\Delta y]$	1.38%	1.53%	1.54%
		[1.41% 1.67%]	[1.41% 1.69%]
$\sigma[\Delta c]/\sigma[\Delta y]$	0.37	0.61	0.62
		[0.54 0.68]	[0.54 0.68]
$\sigma[\Delta i]/\sigma[\Delta y]$	3.45	3.40	3.39
		[3.20 3.62]	[3.18 3.64]
$\sigma[\Delta l]/\sigma[\Delta y]$	0.37	0.67	0.67
		[0.64 0.72]	[0.62 0.73]
Asset Prices			
$E[r_t - r_t^f]$	4.09%	4.23%	4.27%
		[3.65% 4.83%]	[3.41% 5.01%]
$\sigma[r_t - r_t^f]$	15.80%	18.50%	18.52%
		[16.74% 20.10%]	[16.58% 20.68%]

Note – The table compares moments of the data to simulated moments from the baseline and extended models. I consider log output growth variability, relative variance of consumption to output, relative variance of investment to output, relative variance of labor to output, equity premium, and equity excess return volatility. Moments of actual data are estimated using quarterly US time series data of GDP, consumption, investment and employment from 1948:I to 2014:IV. Moments of simulated data from the model are at quarterly frequency. I report mean moments, along with the 5th and 95th percentiles across 1,000 simulations, each with a length of 50 years.

Table 5: IFS Pricing with Fama-French 25 Size-Value Test Portfolios

	IFS	MKT	SMB	HML	UMD	R^2
Model I	-0.344					9.27
	(-1.53)					
Model II		-0.019	0.004	0.012		69.13
		(-1.38)	(2.65)	(6.25)		
Model III	-1.490	-0.005	0.006	0.011		81.25
	(-3.56)	(-0.47)	(4.56)	(7.11)		
Model IV		-0.006	0.004	0.012	0.026	71.38
		(-0.34)	(2.60)	(6.40)	(1.17)	
Model V	-1.447	-0.001	0.006	0.011	0.020	81.62
	(-3.36)	(-0.04)	(4.31)	(7.02)	(1.07)	

Note – The table reports market prices of risk from quarterly cross-sectional regressions of average excess returns on estimated factor loadings, using Fama-French 25 size-value portfolios. The factors are information friction shock (IFS), Fama-French 3 factors (MKT, SMB, HML), and momentum factor (UMD). For each model, I report the estimated prices of risk, and the R^2. The t-statistics are reported in parentheses.

Table 6: Cross-Setional Return Predictability

	Value Weighted Portfolio Sorted on IFS Beta										
	Low	2	3	4	5	6	7	8	9	High	High - Low
Excess Return $E[r_t - r_t^f]$	0.85	0.63	0.53	0.66	0.65	0.63	0.50	0.52	0.61	0.46	-0.39
Volatility $\sigma[r_t - r_t^f]$	7.64	6.15	4.93	4.84	4.63	4.54	4.97	5.48	6.14	7.86	4.74
Sharpe Ratio	0.11	0.10	0.11	0.14	0.14	0.14	0.10	0.10	0.10	0.06	-0.08
Average β^{IFS}	-0.182	-0.083	-0.050	-0.026	-0.007	0.011	0.031	0.055	0.092	0.218	0.401
α - CAPM	0.08	-0.02	-0.01	0.14	0.14	0.13	-0.06	-0.08	-0.04	-0.34	-0.42
	(0.43)	(-0.16)	(-0.09)	(1.47)	(1.89)	(1.70)	(-0.77)	(-0.89)	(-0.34)	(-1.82)	(-1.98)
α - FF3	0.23	0.06	0.07	0.19	0.15	0.16	-0.03	-0.07	-0.07	-0.32	-0.55
	(1.40)	(0.51)	(0.78)	(1.99)	(1.89)	(2.11)	(-0.37)	(-0.70)	(-0.60)	(-2.18)	(-2.46)
α - FF3+UMD	0.30	0.14	0.03	0.29	0.15	0.13	-0.02	-0.06	-0.03	-0.28	-0.58
	(1.61)	(1.03)	(0.34)	(3.12)	(1.90)	(1.68)	(-0.32)	(-0.57)	(-0.27)	(-1.87)	(-2.29)

Note – The table reports summary statistics for 10 value-weighted portfolios sorted on IFS exposure. I report mean excess returns over the risk-free rate, excess return volatility, Sharpe ratio, and average IFS beta in each portfolio. I also report the abnormal returns for each portfolio. Monthly portfolio abnormal returns are computed by running time series regressions of portfolio excess returns on risk factors. The t-statistics are reported in parentheses. The sample includes monthly data from October 1973 to December 2013.