Mutual Funds' Reputations and Star Ratings

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Abstract

We propose a theory of mutual fund's reputation in which a fund's outstanding performance depends on its information superiority. We show that an endogenous star rating system, by employing only a few stars, provides investors with sufficient information about the fund's reputation. The uninformed fund's incentives to acquire information vary with its status in the star rating system, making its reputation exhibit star rating properties: intra-group catch-up and inter-group jump. These equilibrium properties provide potential rational explanations for recent empirical findings of Morningstar ratings. We relate funds' activeness to their status in the star rating system, generating new empirical predictions.

Keywords: Mutual Fund, Reputation, Information Superiority, Information Acquisition, Obsolete Information, Morningstar

JEL Classification Codes: C73, D83, G23

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1 Introduction

Agency problems are inevitably present in the mutual fund industry, especially under the current prevailing fee structure by which funds charge fees based on the assets under management, rather than their performance.[1] Indeed, not only the financial service profession has been paying attentions to fund managers' incentives for a long time,[2] but also empirical researches have documented such moral hazard problems. For example, Bergstresser, Chalmers, and Tufano (2009) and Del Guercio and Reuter (2014) find that direct-sold funds who face stronger incentives have significantly higher alphas than broker-sold funds. Hence, as pointed out by Berk (2005), a mutual fund may work hard mainly because of future investor flows (and thus future revenues) that positively correlate to the fund's current investment performance. These kinds of substantial "indirect incentives" are recently documented by Lim, Sensoy, and Weisbach (2015).

A natural "indirect incentive" of mutual funds comes from their reputations concern. In practice, investors often use mutual funds' Morningstar ratings as proxies for funds' reputations, because Moringstar ratings are risk-adjusted performance measures that are updated monthly and freely available at popular investment Web sites such as Morningstar.com, Yahoo!Finance, and Fidelity.com. The "Morningstar effect" on funds' flows is documented by Del Guercio and Tkac (2008), who show, for example, when a fund is upgraded from the four-star group to the five-star group, it receives on average 53% to 61% positive abnormal flows.[3] Newer evidence about the Morningstar effect based on data from 2003 to 2014 is presented in Morningstar research, and the observed pattern is very similar. Importantly, these estimated impacts on flows are after controlling for funds' past performance and other characteristics, implying that investors place substantial importance on Morningstar ratings that is not captured by past performance alone. Hence, mutual funds will have strong incentives to work hard to boost their star ratings and thus their reputations.

However, a star rating system usually employs only a few stars to summarize a mutual funds' past risk-adjusted performance, which provides very coarse information to investors, especially to those with the fund's detailed performance records. In addition, because of the small number of possible stars assigned by a star rating system, funds with very similar past performance may

[1] We analyze the agency problem between funds and investors in this paper. There are also agency problems between funds and their managers, but such problems are not the focus of our paper. Correspondingly, we assume that there is a contract that perfectly aligns the preferences of funds and their managers.

[2] Morningstar, a highly impacting rating agency in the mutual fund industry, has been providing Stewardship Grade for funds since 2004, which is largely determined by its evaluations of fund managers' incentives. Note the Stewardship Grade for funds is not related to Morningstar Ratings.

[3] The abnormal flow can be interpreted as a fund's actual flow relative to that expected if it had maintained its pre-change star rating.

be assigned different star ratings, but funds with relatively different past performance could be categorized into one star group. Then, in a rational setting, how do investors rely on star ratings to pick funds?[4] More importantly, how do a fund's incentives vary with its status in the star ratings system?

We resolve these questions in an infinite-horizon reputation model. We show that, when investors focus on funds' information superiority, an endogenous star rating system provides them with sufficient information to pick funds, even if they possess much finer information about the fund's past performance. The fund's incentives to acquire information superiority vary with its status in the star rating system, neither monotonically nor continuously, leading to star rating properties of the fund's reputations: intra-group catch-up and inter-group jump.

We define a fund's reputation as investors' belief that the fund has information superiority. Information superiority has two important features. First, since information can be acquired, an uninformed mutual fund can become informed. Thus, a mutual fund's reputation in each period contains investors' belief about the fund's information acquisition behavior. Second, information has short-run persistence, but becomes obsolete in the long run. As a result, given a fund is informed in the current period, it is possible that the fund will remain informed in the next period, even without the next period's information acquisition; however, suppose that an uninformed fund never acquires information, then the fund's reputation will deteriorate over time. These two features determine that a mutual fund's reputation by our definition comprises investors' belief over a both exogenously and endogenously changing variable. As a result, as opposed to classical reputation models, in which an agent's reputation is defined as investors' belief over his exogenous types,[5] our model contains no exogenous type.

The critical role of information superiority in investment and the possibility of a star rating system in an equilibrium motivate our reputation definition. Information superiority is a prerequisite for a fund's good performance, which is a direct implication of the Efficient Market hypothesis: since stock prices incorporate all available information in the market, a mutual fund cannot persistently outperform its peers unless it persistently possesses information superiority.

[4]We are not arguing whether the existing Morningstar Ratings are equilibrium outcomes or not. Instead, we analyze whether a star rating system using a small number of stars can be part of an equilibrium.

[5]Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenberg and Levine (1989) define an agent's reputation as investors' belief whether the agent is of a commitment type. Mailath and Samuelson (2001) regard an agent's reputation as investors' belief that the agent is not an inept person. In these two kinds of reputation models, the agent's type is the agent's private information and unknown to investors. Mailath and Samuelson (2014) survey recent works on reputations in repeated games of incomplete information. Another strand of infinite-horizon reputation models are about career concerns, in which the agent's type is unknown to any player, but the agent still have incentives to manage investors' belief about his type (Holmström 1999).

Moreover, information superiority is well known as the reason why mutual funds are endogenously built (García and Vanden (2009) and He (2010)). Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014, 2015) further show that mutual funds need to acquire different types of information in different scenarios to maintain good performance records. Recent empirical studies further demonstrate information superiority's key role in determining mutual funds' performance, gained from the funds' efforts to build and maintain social networks (Cohen, Frazzini, and Malloy (2008), Gao and Huang (2015), and Tang (2013)) or from industry concentrations (Kacperczyk, Sialm, and Zheng (2005)).

On the other hand, mutual funds' reputations for information superiority creates possibilities for a star rating system with a small number of groups in an equilibrium. Because of information superiority's short-run persistence, a mutual fund will have temporary types. Such temporary types, while largely determined by the fund's past performance, also depend on the fund's current behavior, differing from exogenous types in classic reputation models. Hence, the fund's reputation also includes investors' belief about the fund's current behavior. Because a mutual fund's current action may be neither continuous nor monotonic in its past performance, its reputation may not be either. Consequently, it is possible to classify funds into a small number of groups in an equilibrium, with each group receiving the same star rating and reputation.

In our model, a mutual fund's reputation evolves as follows. At the beginning of any period t, investors hold prior beliefs about whether the fund is currently informed. This is the fund's *prior reputation* in period t. Given its prior reputation, the uninformed fund makes an information acquisition decision. If it acquires information, it becomes informed; otherwise, it remains uninformed. Investors do not observe the fund's choice but form a belief over its information acquisition behavior. Such a belief, together with the fund's prior reputation, constitutes the fund's *interim reputation*. Investors then make their purchase decisions based on the interim reputation, so the equilibrium properties analyzed in this paper are mainly about the fund's interim reputation. The fund's period payoff depends on the number of investors who purchase the fund's shares. After observing its investment outcome, investors reevaluate whether the fund is informed in Period t and form a *posterior reputation*. An investment outcome is drawn from the real line, and the informed fund's investment outcome dominates that of the uniformed fund in the sense of first-order stochastic dominance. Information may become obsolete, so the fund's posterior reputation in Period t, discounted by the information depreciation, turns into the prior reputation at the beginning of Period $t + 1$.

Motivated by mutual fund star ratings that are popular among individual investors, we search for a star rating equilibrium, a Markov perfect equilibrium in which investors' belief is consistent with a star rating system. We focus on a simple two-star rating system consisting of two components: a time-invariant increasing rating function, mapping from a fund's prior reputations to star ratings; and a time-invariant transition rule, such that if funds with the same rating generate the same investment outcome in the current period, they will be assigned the same rating in the next period.[6]

In a star rating equilibrium, there is an endogenously determined threshold in funds' prior reputation space, such that funds with prior reputations above the threshold receive a two-star rating, whereas funds with prior reputations below the threshold are assigned a one-star rating. In the equilibrium, the probability that an uninformed fund acquires information is a function of the fund's prior reputation. This probability is strictly between zero and one for almost all prior reputations. Given the fund's star rating, the information acquisition probability strictly decreases in the fund's prior reputation, such that the fund reaches a fixed interim reputation regardless of its prior reputation. By this information acquisition strategy, we say that funds with lower prior reputations "catch up" those with higher prior reputations in the same star rating group, leading to an "intra-group catch-up" property. However, when the fund is upgraded from a one-star rating to a two-star rating, the information acquisition probability discretely jumps, leading to a discrete increment of the fund's interim reputation. We name such an equilibrium property the "inter-group jump" property. Therefore, in an equilibrium, the star rating system assigns funds with the same interim reputation the same star rating, and assigns funds with the higher (lower) interim reputation a two-star (one-star) rating. What's more, to receive a two-star rating in the next period, funds in the one-star rating group need to perform much better than funds in the two-star rating group.

Both the "intra-group catch-up" property and the "inter-group jump" property result from the star rating system and its corresponding investors' belief system, which supports the star rating equilibrium. First, uninformed funds will never acquire information with probability one in an equilibrium. Otherwise, investors will believe the fund to be informed with probability one in the current period. Given such a belief, the fund's future investor flows will be independent of its current investment performance, and hence it will not acquire information to save the information acquisition cost. More importantly, the star rating system and the investors' belief system will reward funds with a two-star rating by assigning a higher interim reputation to them. Funds with a two-star rating will obtain more investor flows and thus higher period revenues. Consequently, funds will work hard in the current period to manage their star ratings and thus their reputations. However, within a star rating group, funds' probabilities of information acquisition must be strictly decreasing in their prior reputations, so that they can have the same interim

[6] A star rating equilibrium with more than two stars may exist but is rather intractable. However, provided such an equilibrium exists, its properties will be very similar to those of the equilibrium with only two star ratings.

reputation to accommodate investors' belief system.

A fund is said to have stronger incentives to invest in its reputation, if it acquires information with a higher probability in the equilibrium. Hence, in a star rating equilibrium, a fund's incentive is determined by its status in the star rating system, and it is neither monotonic nor continuous. Specifically, the "intra-group catch-up" property implies that, in a star rating group, as funds' prior reputations increase, their incentives decrease, such that they reach the same interim reputation; therefore, in a star rating system, two funds with different past performance, and thus different prior reputations, can have the same rating and the same interim reputation. By contrast, as implied by the "inter-group jump" property, a fund on the bottom of the two-star rating group has significantly stronger incentive than a fund on the top of the one-star rating group. Because the fund on the bottom of the two-star rating group and the fund on the top of the one-star rating group have very similar prior reputations, the "inter-group jump" property further implies that two funds with very similar past performance and very similar prior reputations may have different star ratings and thus different interim reputations.

The existence of a star rating equilibrium has an important implication about investors' use of information. When investors need to pick funds, it is costly for them to collect and analyze all funds' performance records. However, if a rating agency commits to a star rating system, as characterized in the star rating equilibrium in our model, and publicizes ratings assigned to funds according to the committed star rating system, investors can ignore their finer information and merely rely on the ratings to make their investment decisions. (They need not even collect information about funds' prior reputation and performance in the previous period.) Therefore, even if investors have huge costs to collect and process information, they can still rely on star ratings to make investment decisions. This argument also implies that, in a perfectly rational environment, a well-designed star rating system can significantly promote investors' welfare by reducing their large information costs.

In our model, the information acquisition action is highly correlated to a fund's activeness, because when a fund has new private information, it will change its portfolio holding based on the new information. In our model, funds that acquire information are definitely informed, while funds who do not acquire information may be informed or uninformed. Therefore, on average, more active funds are more likely to be informed and thus perform better. Such a prediction has been confirmed by empirical studies employing several different measures of funds' activeness, such as "industry concentration index" (Kacperczyk, Sialm, and Zheng 2005), "Reliance on Public Information" (Kacperczyk and Seru 2007), "return gap" (Kacperczyk, Sialm, and Zheng 2008), and "active share" (Cremers and Petajisto 2009).

While the positive correlation between funds' activeness and their current performance is a direct implication of our assumptions, our equilibrium analysis shows how funds' past performance affects their current activeness. The two properties of a star rating equilibrium imply that a fund's current activeness is neither monotonic nor continuous in its past performance. Within a star rating group, a higher ranked fund is less active; but funds at the bottom of the two-star group are much more active than funds on the top of the one-star group. Therefore, these measures of funds' activeness can be employed to test the effects of funds' past performance on their activeness, and thus to test the hypotheses of funds' incentives and reputations.[7]

A star rating equilibrium's properties also provide potential rational explanations for recent empirical observations of Morningstar Ratings' effects. Reuter and Zitzewitz (2015) document several interesting observations regarding Morningstar ratings. First, they find that mutual funds just above the threshold for a Morningstar rating receive incremental net flows, and are thus larger in size (than those funds just below the threshold). Second, funds just above the threshold for a Morningstar rating, though larger, on average perform as well as those just below the threshold, with some types of funds performing better. Reuter and Zitzewitz (2015) argue that Morningstar ratings' discreteness and investors' imperfect rationality explain the first observation; therefore, the incremental net flows are exogenous. As a result, they conclude by the second observation that the effects of diseconomies of scale assumed by Berk and Green (2004) are too small to justify the downward bias in performance persistence estimates.

We provide rational explanations for these empirical observations. The discrete jump in size is due to the inter-group jump property; therefore, it is endogenously determined in our model. Although our model is not suitable for discussing economic return to scales, the combination of the assumption of diseconomies of scale and the two properties of the star rating equilibrium can rationalize the second empirical observation in Reuter and Zitzewitz (2015). That is, when the effect of diseconomies of scale just offsets the inter-group jump effect, the size effects on returns are not significantly different from zero. And, when the inter-group jump effect dominates, we should observe that funds that are just above the threshold for a Morningstar rating perform significantly better.

Our paper complements existing literature that discusses mutual funds' reputations in terms of investors' belief over their permanent types. In a seminal paper by Berk and Green (2004), a fund's skill is inherent but unknown to all players, and a fund's expected performance is determined by its skill and its size. Therefore, there is no moral hazard problem. Dasgupta and Prat (2006, 2008),

[7]We should note that within a star-group, though lower ranked funds are more active, and more active funds perform better, we do not conclude that lower ranked funds will perform better than higher ranked funds. This is because higher ranked funds are more likely to be informed at the beginning of the period, so even though they are not that active, their performance may still be as good as lower ranked funds who are more active.

Dasgupta, Prat, and Verardo (2011), and Malliaris and Yan (2015) analyze fund managers' reputations as investors' belief about their abilities in finite-horizon models, where managers' abilities are only known to themselves. Guerrieri and Kondor (2012) define fund managers' reputations as investors' belief about whether managers are informed; yet, whether a manager is informed is exogenous and fixed forever. In all these reputation models, a discrete star rating system cannot guide investors' decisions, because it only provides investors with very coarse information. We define a mutual fund's reputation for its information superiority, which implies that the fund's reputation comprises investors' belief over an endogenous variable. We show that funds' incentives are determined by their status in a star-rating system, and the discrete star-rating system provide investors with sufficient information to make investment decisions.

Our paper also contributes to recent discussions about various ratings' discreteness. Nowadays, most rating systems have only finite rating notches, but fundamentals of entities under evaluation are usually measured by continuous variables. To explain such a phenomenon, Lizzeri (1999) models an information intermediary who commits to a disclosure rule, demonstrating that the optimal rating system is only reveals whether quality is above some minimal standard. Goel and Thakor (2015) study coarse credit ratings in a cheap talk model. As opposed to ratings assigned by certified information intermediaries, such as credit rating agencies, mutual fund star ratings in our theory provide no more information than what investors already know. We show that the star rating system's discreteness arises from the assumption that a mutual fund's reputation lies in its investors' belief about its information status, which is an endogenous variable.

This paper enriches the reputation literature by studying a discrete-time infinite-horizon model in which reputation is defined as investors' belief over an endogenous variable. In a recent paper, Board and Meyer-ter Vehn (2013) study a model in which a firm's reputation comprises the consumers' belief about its product quality. Their model differs from ours in the following aspects. First, in their setting, a shock periodically arrives and resets the product's quality; thereafter, the firm's current effort determines the new quality.[8] Hence, in their model, the firm stochastically controls the quality, and thus plays pure strategy in the equilibrium. In our model, the uninformed fund can directly choose whether to be informed, which naturally leads to mixing in the equilibrium. More importantly, in our equilibrium, both the fund's information acquisition effort and its expected performance exhibit the intra-group catch-up property, which is lacking in Board and Meyer-ter Vehn (2013).[9]

[8]Halac and Prat (2014) adopt a similar setting to study a dynamic inspection game between a worker and a manager.

[9]Dilme (2012) proposed an alternative model of firm reputation in which actions have lasting effects because of switching costs. Dilme and Garrett (2015) introduce switching cost into a repeated inspection game to rationalize the residual deterrence effect of the law enforcement decisions. Bohren (2014) also considers a reputation model in

The remainder of this paper is organized as follows. In Section 2, we present an infinite-horizon reputation model that captures the two special features of information superiority. Section 3 constructs a star rating equilibrium and analyzes the effects of reputation on a fund's incentive and the effects of a star rating system on investors' use of information. Section 4 is devoted to empirical implications. Section 5 concludes.

2 The Model

We consider a discrete-time infinite-horizon game, where time is indexed by $t = 1, 2, \ldots$. The economy consists of a mutual fund and a continuum of perfectly rational investors with measure one.

Mutual Fund. The mutual fund may or may not have superior information for investment. At the beginning of each period t, the uninformed fund decides whether to acquire information. Denote by σ_t the probability that the uninformed fund acquires information. If the uninformed fund does not acquire information, it remains uninformed; if the uninformed fund acquires information, it becomes informed in period t, but it needs to pay an effort cost $c > 0$. We assume that the fund knows whether it possesses superior information, but this is the fund's private information. Investors cannot observe the fund's information acquisition action.

Information may become *obsolete* in the sense that an informed fund in period t will become uninformed in period $t+1$ with probability $\lambda \in (0, 1)$. The assumption of $\lambda \in (0, 1)$ captures important features of information superiority. On one hand, $\lambda < 1$ implies that information superiority has short-run persistence, so the informed fund can still have information superiority in the next period. On the other hand, $\lambda > 0$ suggests that information superiority may quickly disappear in the financial market.

Investment Outcomes. In every period t, the informed fund's investment outcome dominates that of the uninformed fund, in the sense of first-order stochastic dominance. In particular, we denote the fund's investment outcome in period t by x_t. We assume

$$x_t \sim \begin{cases} \mathcal{N}(1, \phi^2), & \text{if the fund is informed in period } t; \\ \mathcal{N}(0, \phi^2), & \text{if the fund is uninformed in period } t. \end{cases} \tag{1}$$

which the reputation of a firm is the persistent quality of its product. However, in her model, there is no asymmetric information. A firm's reputation is an observable quality stock, which can be enhanced through the firm's costly investment.

Denote the distribution functions to be $F^I(x)$ and $F^U(x)$, respectively; and the density functions to be $f^I(x)$ and $f^U(x)$, respectively. We assume that before making period t decisions, the fund and the investors know the whole history of investment outcomes up to period t, $h^t = \{x_1, x_2, \ldots, x_{t-1}\}$.

Reputation. We interpret the investors' belief that the fund is informed as the fund's *reputation*. Denote by $\rho_1 \in [0, 1]$ investors' belief that the fund is informed at the beginning of period 0, which is common knowledge among all players. Because of the fund's potential hidden information acquisition action and its investment outcome, the fund's reputation evolves within any period $t \geq 1$. Formally, at the beginning of each period t, the fund has a *prior reputation* defined as

$$\rho_t = \Pr(\text{the fund is informed at the beginning of period } t | h^{t-1}).$$

Whether the fund has superior information in period t depends not only on its initial information status in period t, but also on its information acquisition decision in period t if it is uninformed at the beginning of period t. Therefore, given the fund's prior reputation in period t and investors' belief that the fund, if uninformed, acquires information, investors form a belief about whether the fund is informed in period t, which is named as the fund's *interim reputation* in period t.

$$\mu_t = \rho_t + (1 - \rho_t)\hat{\sigma}_t, \tag{2}$$

where $\hat{\sigma}_t$ is investors' belief that the uninformed fund acquires information in period t.

Once the investment outcome x_t is realized, investors will update their belief about whether the fund is informed in period t, which is the fund's *posterior reputation* in period t,

$$\zeta_t = \frac{f^I(x_t)\mu_t}{f^I(x_t)\mu_t + f^U(x_t)(1 - \mu_t)}. \tag{3}$$

Because the information will be obsolete with probability λ, given the fund's period t posterior reputation, the fund's prior reputation in period $t + 1$ is:

$$\rho_{t+1} = (1 - \lambda)\zeta_t. \tag{4}$$

Because the fund's interim reputation $\zeta_t \in [0, 1]$ for any $t \geq 1$, Equation (4) implies that the fund's prior reputation in any period $t \geq 2$ belongs to the interval $[0, 1 - \lambda]$. Therefore, without losing any generality, we restrict the fund's prior reputation space to $[0, 1 - \lambda]$.

Payoff. In period t, investors make investment decisions solely based on the fund's interim reputation, because the fund's interim reputation represents investors' belief about the fund's information status in period t (right before investment), and thus the fund's investment outcome in period t. Hence, for simplicity, we assume that there are μ_t measure investors who buy the fund's shares, if and only if the fund's interim reputation is μ_t. That is, in period t, the fund's size is the fund's interim reputation.[10]

We further assume the fund's period revenue equals the fund's size. This is consistent with the fixed management fees as percentages of the assets under management. We do not consider the fund's possible management fee changes. Because the fund has private knowledge, the fund offering management fees every period will have signaling effects, which make the model rather intractable.

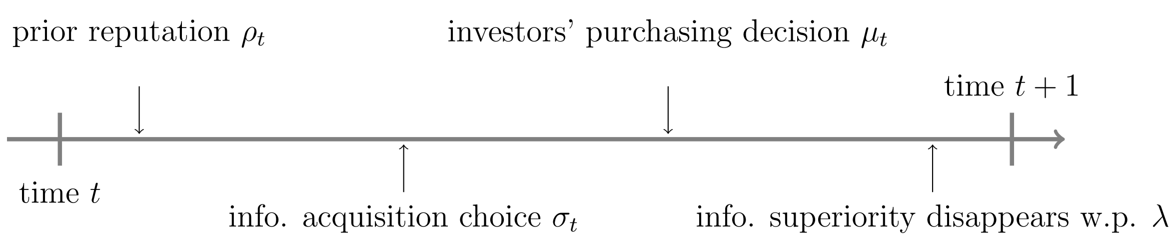
Therefore, given that the fund will discount its future revenues by a discount factor $\delta \in (0, 1)$, the fund's continuation value in any period t is

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[\mu_\tau - \mathbb{1}\{\text{the fund acquires information at time } \tau\}c\right], \tag{5}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function.

Timing. We summarize the timing in each period t in Figure 1 below.

Figure 1: Timing of the game.



Star Raings. Motivated by the highly influential star ratings of mutual funds in the investment profession, in this paper we search for an equilibrium with star rating properties. For simplicity, we focus on a simple "two stars" rating system consisting of an increasing rating function $\iota : [0, 1-\lambda] \to \{1, 2\}$ and transition thresholds $(x_1^*, x_2^*) \in \mathbb{R}^2$. The fund, given its prior

[10]This reduced form assumption is just for simplicity. It is sufficient to capture the fact that the number of investors is increasing in the fund's interim reputation. In fact, our result is robust if we assume that in each period, the size of the fund is any increasing function of the fund's interim reputation.

reputation ρ_t in period t, will be assigned a rating $\iota_t = \iota(\rho_t) \in \{1, 2\}$ by the rating function. Furthermore, the simple rating system will have a simple transition rule: if $\iota_t = j$, then

$$\iota_{t+1} = \begin{cases} 1, & \text{if } x_t \leq x_j^*; \\ 2, & \text{if } x_t > x_j^*. \end{cases} \tag{6}$$

Note that both the rating function and the transition rule are time-invariant, because the rating mechanism is rather stable in the real world.

While the assumption of the simple "two stars" rating system is mainly for the model's tractability, it sufficiently captures the discreteness of mutual fund ratings in the investment profession. Discrete ratings provide investors with very coarse information, and the simple "two stars" rating system uses the least amount of information to assign ratings. Since one of our aims in this paper is to study whether rational investors can ignore finer information and merely rely on the rating to make investment decisions, it is sufficient for us to check whether such a simple form of rating system can support an equilibrium.[11]

The transition rule specified in the simple "two stars" rating system uses only the fund's current rating and current investment performance as inputs. However, without using the transition rule, investors can calculate the fund's period $t+1$ rating, given the rating function ι, the fund's period t prior reputation ρ_t, the fund's period t investment outcome x_t, and their belief system $\hat{\sigma}$. Specifically, investors can use Equations (2), (3), and (4) to calculate ρ_{t+1} and thus get the period $t+1$ rating $\iota(\rho_{t+1})$.

We say that investors' belief system is *consistent with* a simple star rating system, if and only if there is a simple star rating system, such that ratings generated by the system are identical to those calculated by investors using their belief system and the rating function. That is, given $\hat{\sigma}$, $\iota_{t+1} = \iota(\rho_{t+1})$, for any ρ_t and x_t. If in an equilibrium investors' belief system is consistent with some simple star rating system, we say the star rating system *supports* the equilibrium.

Equilibrium. We are interested in a *star rating Equilibrium*, which is a Markov perfect equilibrium supported by a star rating system.

Definition 1. *The fund's information acquisition strategy, a system of investors' belief, and a simple star rating system constitute a star rating equilibrium, if*

1. *the fund's strategy depends on the history through its prior reputation ρ only. Namely, there exists a function $\sigma : [0, 1-\lambda] \to [0, 1]$ such that $\sigma_t = \sigma(\rho_t)$;*

[11] Though we analyze the simple "two stars" rating system in this paper, we acknowledge that other star rating systems, especially discrete rating systems with more than two stars, may also support some star rating equilibria. However, such equilibria, if exist, should have similar equilibrium properties as those characterized in this paper.

2. the equilibrium strategy $\sigma(\rho_t)$ maximizes the fund's expected continuation payoff (5);

3. investors' belief is consistent with the fund's strategy: $\hat{\sigma}_t = \sigma(\rho_t)$;

4. over time, investors' belief evolves according to Bayes' rule: Equations (2), (3), and (4); and

5. the system of investors' belief is consistent with the simple star rating system.

Because we focus on (stationary) Markov strategies, we can ignore the subscript t in the rest of this paper. Hence, we denote ρ (and μ) as the current prior (and interim) reputation and denote ρ' (and μ') as the next period prior (and interim) reputation.

3 Star Rating Equilibrium and Reputation Effects

In this section, we show that a star rating equilibrium exists. We then analyze how reputation concerns affect a mutual fund's incentives, and why investors can ignore finer information and merely rely on star ratings to make investment decisions.

3.1 Preliminary Analysis

Denote by $V_I(\rho)$ the fund's continuation value, if it is currently informed and has a prior reputation ρ. Then given investors' belief $\hat{\sigma}$, we have

$$V_I(\rho) = \mu + \delta \int_{-\infty}^{\infty} [(1-\lambda)V_I(\rho') + \lambda \max\{V_U(\rho'), V_I(\rho') - c\}] \, \mathrm{d}F^I(x) \tag{7}$$

where $\mu = \rho + (1-\rho)\hat{\sigma}(\rho)$, and ρ' is a function of x derived from Equations (2), (3), and (4).

The economic interpretations of Equation (7) are as follows. First, given $\hat{\sigma}$, investors believe the fund will acquire information with probability $\hat{\sigma}(\rho)$; hence, the fund's current period payoff, which is also the fund's interim reputation, is $\mu = \rho + (1-\rho)\hat{\sigma}(\rho)$. Note that the fund's current period payoff is independent of its true information status. Second, because the fund is informed, the fund's investment outcome will be drawn from the distribution with the cdf $F^I(x)$. Given each realized investment outcome x, investors will calculate the fund's next period prior reputation ρ', which is a function of the fund's interim reputation μ and current investment outcome x. Third, with probability $1-\lambda$, the fund will stay informed, so its continuation value in the next period will be $V^I(\rho')$. But with probability λ, the fund loses information superiority. In such a case, the fund needs to decide whether to acquire information in the next period, and thus its next period continuation value is $\max\{V_U(\rho'), V_I(\rho') - c\}$.

Similarly, denote by $V_U(\rho)$ the fund's continuation value, if it is currently uninformed, has a prior reputation ρ, and chooses to remain uninformed. Then we have

$$V_U(\rho) = \mu + \delta \int_{-\infty}^{\infty} \max\{V_U(\rho'), V_I(\rho') - c\} \mathrm{d}F^U(x). \tag{8}$$

Given the prior reputation ρ, we define $V_I(\rho) - V_U(\rho)$ as the *reputation premium* at ρ. Then, the fund's optimal information acquisition rule, given investors' belief $\hat{\sigma}$, is

$$\sigma(\rho) \begin{cases} = 0, & \text{if} \quad V_I(\rho) - V_U(\rho) < c; \\ \in [0,1], & \text{if} \quad V_I(\rho) - V_U(\rho) = c; \\ = 1, & \text{if} \quad V_I(\rho) - V_U(\rho) > c. \end{cases}$$

That is, an uninformed fund acquires information if and only if the reputation premium is greater than the information acquisition cost. Because the reputation premium is bounded, $\sigma(\rho) = 0$ for any $\rho \in [0, 1-\lambda]$ when c is large. To avoid such a trivial case, in the rest of the paper we assume that c is sufficiently small, and we will derive the range of c in which a star rating equilibrium exists.

With the fund's continuation value functions, we now analyze whether there is an equilibrium in which the fund with some prior reputation ρ acquires information with probability one. Generally, the fund can obtain two benefits from acquiring information. First, the fund is more likely to have a better investment outcome, so that its next period prior reputation will be higher. Second, there is a positive probability that the fund will remain informed in the next period.

However, when investors believe the fund will acquire information with probability one, the first benefit from information acquisition disappears. Suppose there is an equilibrium in which the uninformed fund with a particular prior reputation ρ acquires information with probability one, investors' belief is $\hat{\sigma}(\rho) = 1$. Then the fund's interim reputation is

$$\mu = \rho + (1-\rho)\hat{\sigma}(\rho) = 1,$$

and for any investment outcome x, the fund's posterior reputation is one. That is, if investors believe that the fund is surely informed, they will not use the investment outcome to update their belief. Consequently, the fund's next period prior reputation is independent of the fund's current investment outcome, implying that the first benefit from information acquisition disappears.

Although the fund can still obtain the second benefit from information acquisition, such a benefit is dominated by the information acquisition cost. This can be seen by considering a deviation in which the fund delays acquiring information in the next period. By such a deviation, the fund will also be informed in the next period, but the fund will pay the information acquisition cost later. Given the fund's discount factor $\delta < 1$, such a deviation is profitable.

Lemma 1 below summarizes the arguments above.

Lemma 1. *In any equilibrium (if exists),* $\sigma(\rho) < 1$ *for each* $\rho \in [0, 1-\lambda]$.

Lemma 1 largely simplifies our analysis. Generally, $V_U(\cdot)$ in Equation (8) is not the value function of the uninformed fund but the continuation value by assuming that the fund chooses to remain uninformed. The value function of an uninformed fund is $\max\{V_U(\rho), V_I(\rho) - c\}$. However, because of Lemma 1, in any equilibrium we have

$$\max\{V_U(\rho), V_I(\rho) - c\} = V_U(\rho)$$

for any $\rho \in [0, 1-\lambda]$. Consequently, in any equilibrium, Equations (7) and (8) can be simplified as

$$V_I(\rho) = \mu + \delta \int_{-\infty}^{\infty} [(1-\lambda)V_I(\rho') + \lambda V_U(\rho')] \,\mathrm{d}F^I(x) \tag{9}$$

and

$$V_U(\rho) = \mu + \delta \int_{-\infty}^{\infty} V_U(\rho')\mathrm{d}F^U(x). \tag{10}$$

Lemma 1 is a property of any Markov perfect equilibrium of our model. Since we are interested in a star rating equilibrium that features star rating properties, we now analyze how a star rating system sets restrictions on investors' equilibrium beliefs. Suppose there is a star rating equilibrium. Because the rating function ι is increasing, it can be rewritten as a step function with the threshold $\hat{\rho} \in (0, 1-\lambda)$, such that

$$\iota(\rho) = \begin{cases} 1, & \text{if } \rho \leq \hat{\rho}; \\ 2, & \text{if } \rho > \hat{\rho}. \end{cases} \tag{11}$$

With any prior reputation $\rho \leq \hat{\rho}$, the fund's rating $\iota = 1$. Then, according to the transition rule of the rating system, if the fund's investment outcome is $x \leq x_1^*$, the fund's next period rating will be $\iota' = 1$; otherwise, $\iota' = 2$.

Now, given investors' equilibrium belief system $\hat{\sigma}$, investors can calculate the fund's next period prior reputation ρ', for any given investment outcome x. Specifically, the fund's interim reputation is

$$\mu(\rho) = \rho + (1-\rho)\hat{\sigma}(\rho),$$

and the next period prior reputation is

$$\rho' = (1-\lambda)\frac{f^I(x)\mu(\rho)}{f^I(x)\mu(\rho) + f^U(x)(1-\mu(\rho))}.$$

Because the likelihood ratio

$$\ell(x) = \frac{f^I(x)}{f^U(x)} \tag{12}$$

is continuous and strictly increasing in x, for any given current prior reputation ρ, the next period ρ' is continuous and strictly increasing in the current investment outcome x.

Since investors' belief must be consistent with the star rating system in a star rating equilibrium, we have

$$\rho' \begin{cases} \leq \hat{\rho}, & \text{if } \; x \leq x_1^*; \\ > \hat{\rho}, & \text{if } \; x \leq x_1^*, \end{cases}$$

for all $\rho \leq \hat{\rho}$. This implies that for any two prior reputations $\rho_1 < \rho_2 \leq \hat{\rho}$, if the realized investment outcome is x_1^*, the fund will have the same prior reputation $\hat{\rho}$ in the next period. Because the fund's next period prior reputation depends on the current interim reputation and the current investment outcome only, the fund should have the same interim reputation, if its prior reputation ρ is between 0 and $\hat{\rho}$.

Similarly, we argue that at any prior reputation $\rho > \hat{\rho}$, the fund has the same interim reputation. These arguments lead to Lemma 2 below.

Lemma 2. *In a star rating equilibrium (if it exists), funds with the same star rating will have the same interim reputation.*

We then denote the fund's interim reputation by μ_1^* and μ_2^*, if its current rating is $\iota = 1$ and $\iota = 2$, respectively.

Lemma 2 describes an important property, but we will defer the discussion about it to Section 3.3. We need first to characterize a star rating equilibrium, because otherwise we can neither conclude that it is an equilibrium property, nor figure out its intuition.

3.2 Star Rating Equilibrium

In this subsection, we first describe the algorithm for constructing a star rating equilibrium. Because such an algorithm is always feasible when the information acquisition cost is sufficiently small, we will conclude in Proposition 1 that a star rating equilibrium exists. Detailed proofs are presented in the Appendix.

By Lemma 2, funds can be divided into two groups according to their star ratings, with funds in the same group having the same interim reputation. Hence, any star rating equilibrium outcome can be represented by a star rating system $\{\hat{\rho}, x_1^*, x_2^*\}$ and the fund's interim reputations μ_1^* and μ_2^*. Given $\{\hat{\rho}, x_1^*, x_2^*\}$ and $\{\mu_1^*, \mu_2^*\}$, the value functions in Equations (9) and (10) can be simplified to

$$V_I^1 = \mu_1^* + \delta\left[(1-F^I(x_1^*))(1-\lambda)V_I^2 + (1-F^I(x_1^*))\lambda V_U^2 + F^I(x_1^*)(1-\lambda)V_I^1 + F^I(x_1^*)\lambda V_U^1\right] \quad (13)$$

$$V_U^1 = \mu_1^* + \delta\left[(1-F^U(x_1^*))V_U^2 + F^U(x_1^*)V_U^1\right] \quad (14)$$

$$V_I^2 = \mu_2^* + \delta\left[(1-F^I(x_2^*))(1-\lambda)V_I^2 + (1-F^I(x_2^*))\lambda V_U^2 + F^I(x_2^*)(1-\lambda)V_I^1 + F^I(x_2^*)\lambda V_U^1\right] \quad (15)$$

$$V_U^2 = \mu_2^* + \delta\left[(1-F^U(x_2^*))V_U^2 + F^U(x_2^*)V_U^1\right] \quad (16)$$

Here, V_I^j (V_U^j) is the fund's continuation value when it is informed (uninformed) and has a rating $\iota = j$.

According to the star rating system, if the fund is assigned the rating $\iota = 2$, its interim reputation is μ_2^*. Because the fund's highest possible prior reputation is $1-\lambda$, we have $\mu_2^* \geq 1-\lambda$. Similarly, the interim reputation μ_1^* must be greater than or equal to $\hat{\rho}$, the highest possible prior reputation with which the fund is assigned the rating $\iota = 1$.

According to the star rating system, if the fund is assigned the rating $\iota = 1$ and generates an investment outcome x_1^*, its next period rating will be $\iota' = 1$. In a star rating equilibrium, investors' belief is consistent with the star rating system; thus, the fund with an interim reputation μ_1^* and an investment outcome x_1^* must have the prior reputation in the next period $\rho' = \hat{\rho}$. That is,

$$\hat{\rho} = (1-\lambda)\frac{\ell_1\mu_1^*}{\ell_1\mu_1^* + (1-\mu_1^*)}, \quad (17)$$

where ℓ_1 is the likelihood ratio evaluated at the investment outcome x_1^*.

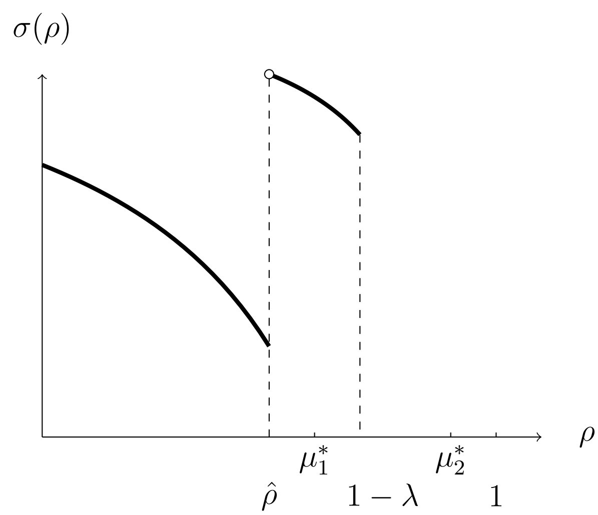
Similarly, if the fund is assigned the rating $\iota = 2$ and generates an investment outcome x_2^*, its next period rating will also be $\iota' = 1$. Then, by calculating the fund's next period prior reputation using its current interim reputation μ_2^* and its current investment outcome x_2^*, we have

$$\hat{\rho} = (1-\lambda)\frac{\ell_2\mu_2^*}{\ell_2\mu_2^* + (1-\mu_2^*)}, \quad (18)$$

where ℓ_2 is the likelihood ratio evaluated at the investment outcome x_2^*.

Finally, to achieve the same interim reputation, when the fund's prior reputation is $\rho \leq \hat{\rho}$, the fund will acquire information with probability $\sigma(\rho) = \frac{\mu_1^* - \rho}{1-\rho} \in (0, 1)$, leading to the interim

Figure 2: Fund's Equilibrium Strategy. The figure shows the probability that the fund acquire information, σ, as a function of its prior belief, ρ.



reputation $\mu_1^* = \rho + (1-\rho)\sigma(\rho)$. Similarly, when the fund's prior reputation is $\rho > \hat{\rho}$, the fund will acquire information with probability $\sigma(\rho) = \frac{\mu_2^*-\rho}{1-\rho} \in (0,1)$, leading to the interim reputation $\mu_2^* = \rho + (1-\rho)\sigma(\rho)$. In sum, in a star rating equilibrium with the star rating system $\{\hat{\rho}, x_1^*, x_2^*\}$ and the fund's interim reputations μ_1^* and μ_2^*, the fund's equilibrium strategy is

$$\sigma(\rho) = \begin{cases} \frac{\mu_1^*-\rho}{1-\rho}, & \text{if } \rho \leq \hat{\rho}; \\ \frac{\mu_2^*-\rho}{1-\rho}, & \text{if } \rho > \hat{\rho}. \end{cases} \tag{19}$$

The following Proposition 1 implies that a star rating equilibrium always exists when the cost of acquiring information, c, is sufficiently small.

Proposition 1. *There exists $\bar{c} > 0$ such that for any $c < \bar{c}$, the model has a star rating equilibrium. In any star rating equilibrium:*

1. *the fund's strategy follows Equation (19); and*

2. *investors belief system and a star rating system are characterized in Equation (17) and Equation (18).*

Figure 2 below illustrates the fund's information acquisition strategy in any star rating equilibrium.

3.3 Star Rating Properties

A mutual fund's reputation for information superiority significantly impacts its incentives to acquire information. We use a fund's equilibrium information probability to measure its incentive; the reputation effects can be seen in Figure 2.

In a star rating equilibrium, if the fund has a prior reputation $\rho \leq \hat{\rho}$, it is assigned a rating $\iota = 1$ and has an interim reputation μ_1^*; if the fund's prior reputation ρ is greater than $\hat{\rho}$, the fund's star rating is $\iota = 2$ and has an interim reputation μ_2^*. This is the "intra-group catch-up" property in a star rating equilibrium.

Corollary 1. *In a star rating equilibrium, funds' interim reputations depend only on their star ratings, with funds in the same star rating group having the same interim reputation. Specifically, as a function of the prior reputation, the fund's interim reputation in the equilibrium is*

$$\mu(\rho) = \begin{cases} \mu_1^*, & \text{if } \rho \leq \hat{\rho}; \\ \mu_2^*, & \text{if } \rho > \hat{\rho}. \end{cases} \tag{20}$$

While funds within a star group catch up to one another, funds in different star groups significantly differ in their interim reputations, and thus also in their sizes and expected investment outcomes. Specifically, let's consider two funds, Fund A and Fund B, whose prior reputations are ρ_A and ρ_B, respectively. Suppose their prior reputations are both very close to $\hat{\rho}$, the equilibrium threshold of ratings, but $\rho_A > \hat{\rho} > \rho_B$. According to the equilibrium star rating system, Fund A is assigned a star rating $\iota = 2$, while Fund B is assigned a star rating $\iota = 1$. Therefore, according to investors' belief system, Fund A's interim reputation is μ_2^*, and Fund B's interim reputation is μ_1^*. In the equilibrium, investors' belief is consistent with the fund's strategy, so we have Corollary 2 below, which states the "inter-group jump" property in a star rating equilibrium.

Corollary 2. *In a star rating equilibrium, $\mu_2^* > \mu_1^*$.*

Both the intra-group catch-up property and the inter-group jump property are direct implications of the fund's equilibrium information acquisition strategy, which strictly decreases when the fund's prior reputation is below $\hat{\rho}$, has a discrete jump at $\hat{\rho}$, and then strictly decreases when the fund's prior reputation is above $\hat{\rho}$. Because these two properties are special star rating properties in our model, and they have not been identified in reputation models with exogenous types, it is interesting to investigate the intuition why such an information acquisition strategy could be part of a star rating equilibrium.

To make the fund's strategy part of a star rating equilibrium, the star rating system and the investors' belief system must be able to reward the fund's effort to acquire information; however, these two systems cannot provide the fund with too strong incentives, because the belief that the fund will surely acquire information cannot support an equilibrium, as shown in Lemma 1. The star rating system and the investors' belief system constructed in Section 3.2 satisfy these two requirements.

First, the star rating system and investors' belief reward the fund's effort to acquire information by accordingly assigning a high rating and a higher interim reputation when the fund exhibits good investment performance. Specifically, the threshold of star ratings $\hat{\rho}$ is strictly between 0 and $1-\lambda$, so when the fund's investment performance is sufficiently good, the fund will receive a rating $\iota = 2$. In addition, μ_2^* is strictly greater than μ_1^*, so a fund assigned the rating $\iota = 2$ due to good investment performance will have a higher interim reputation and thus more investor flows. Therefore, given the equilibrium star rating system and investors' belief system, the fund has incentives to acquire information.

Second, the star rating system and the investors' belief system make the fund's reputation premium equal to its information acquisition cost, so that the fund is willing to employ a mixed strategy to accommodate these two systems. Indeed, by adjusting the threshold of star ratings ($\hat{\rho}$) and the difference between interim reputations ($\mu_2^* - \mu_1^*$), the star rating system and the investors' belief system can make an uninformed fund indifferent between acquiring information and staying uninformed. Therefore, the intra-group similary property and the inter-group jump property coexist in a star rating equilibrium.

It is interesting to point out another feature of the star rating system that can also help to understand the inter-group jump property: the star rating system "discriminates" against funds with a rating $\iota = 1$ in the sense that given the information status (uninformed), it is harder for a fund with $\iota = 1$ (than a fund with $\iota = 2$) to receive a rating $\iota = 2$ in the next period.

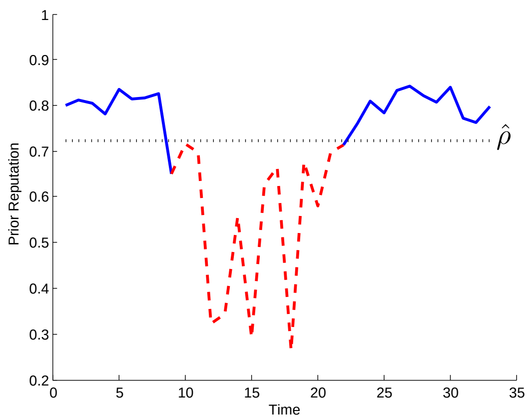
Corollary 3. *In a star rating equilibrium, $x_1^* > x_2^*$.*

Let's consider the uninformed fund with a prior reputation less than but very close to $\hat{\rho}$. While the fund is more likely to receive a star rating $\iota = 2$ if it acquires information, the benefit from the upgrading is largely offset by the harsh upgrading requirement – it has to generate an investment outcome greater than x_1^*. Then, the fund may want to stay uninformed and wait for a good luck (to generate an investment outcome better than x_2^*, and then to exert efforts). That is, such a fund has even lower incentive to invest in its reputation.

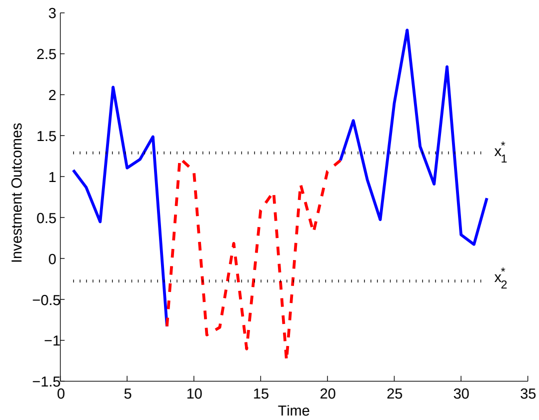
Corollary 3 suggests the importance of the fund's initial prior reputation, which provides a rationality for mutual fund incubation (Evans 2010). Suppose Fund A includes incubation, and Fund B does not, so when open to the public, Fund A is more likely to receive a rating $\iota = 2$. Further assume that Fund A and Fund B are initially assigned ratings $\iota = 2$ and $\iota = 1$, respectively, and generate investment outcomes $\{x_\tau^A\}_{\tau=1}^t$ and $\{x_\tau^B\}_{\tau=1}^t$, respectively. Then, even if Fund B's performance is better than that of Fund A such that $x_1^* > x_\tau^B > x_\tau^A > x_2^*$, these two funds will retain their initial ratings, and Fund A will attract more investors flows than Fund B. Because such an event can occur with a rather large probability in the short-run but is less likely in the long-run, the incubation effect is much more significant in the short-run than in the long-run.

We simulate the dynamics of a fund's star ratings and its investment outcomes when the fund's initial rating is $\iota = 2$ in Figure 3.

Figure 3: Dynamics of a fund's prior reputation and investment outcome. The figures depict a simulated path of a fund's prior reputation and investment outcome. The solid line corresponds to the periods when the fund has a two-star rating, while the dashed line corresponds to the periods when the fund has a one-star rating. The parameters are $\delta = 0.8$, $\lambda = 0.15$, $\phi = 1$, $c = 0.2475$, such that $x_1^* = 1.2777$, $x_2^* = -0.2777$ and $\hat{\rho} = 0.7225$.



(a) Prior reputation



(b) Investment outcome

Figure 3 demonstrates interesting dynamics of a mutual fund's reputation in a star rating equilibrium: whereas a disastrous investment outcome can immediately ruin a fund's good reputation, it takes the fund a very long time to rebuild its reputation, because a one-star fund needs to get very good investment outcome to receive a two-star rating. As shown in Figure 3b, the fund gets a very bad investment outcome in $t = 8$; as a result, the fund is downgraded from a two-star rating to a one-star rating in $t = 9$. Then, from $t = 9$ to $t = 23$, the fund is assigned only a one-star rating, which represents a bad reputation. During these periods, the fund generates investment outcomes greater than x_2^* in several periods, with which the fund can receive a two-star rating if it were in the two-star rating group. The fund finally obtains a very good investment outcome (greater than x_1^*) in $t = 24$, when it earns a good reputation back.

3.4 Use of Star Ratings

Both the intra-group catch-up effect and the inter-group jump effect imply how investors can use star ratings to guide their investment decisions. In particular, the star rating system can largely reduce investors' reliance on past information. Given the huge costs of collecting and processing information about a fund's past performance, our argument implies that a well-designed star rating system can greatly improve investors' welfare by reducing their information costs.

Let's consider a rating agency, who commits to publishing star ratings of mutual funds according to the star rating system in a star rating equilibrium constructed in Proposition 1. Investors' only information is the current star ratings of mutual funds, and they assign interim reputations to funds according to their star ratings: funds with the star rating $\iota = j$ have the interim reputation μ_j^*, which is also characterized in Proposition 1. In such a scenario, the fund will employ the information acquisition strategy specified by Equation (19). This implies Proposition 2 below.

Proposition 2. *In the scenario in which a rating agency commits to employing a star rating system and investors make investment decisions based on the fund's current star rating, there is an equilibrium that is exactly the one characterized in Proposition 1.*

Proposition 2 shows that the discrete rating system could be an equilibrium phenomenon. It is endogenously determined after taking into account investors' belief and the mutual fund's best response. Therefore, the equilibrium star ratings provide investors with sufficient statistics about the fund's information status, and they affect the mutual fund's information acquisition behavior. As a result, although the discrete rating system only provides investors with redundant and rather coarse information, investors can ignore their finer information and merely rely on star ratings to make investment decisions.

3.5 Long-run Reputation

In this section, we analyze how the star rating system and the fund's long-run expected performance change, as the model's parameters change. We are particularly interested in the comparative statics of the fund's information acquisition cost (c), the volatility of the fund's investment outcome (ϕ), and the information obsolete probability (λ).

We first derive the stationary probability over the fund's star ratings. In a star rating equilibrium with the star rating system $\{\hat{\rho}, x_1^*, x_2^*\}$ and the fund's interim reputations μ_1^* and μ_2^*, an investor, without the exact knowledge of the fund's information status, can calculate the probability that the fund with the star rating $\iota = 2$ will be assigned the rating $\iota = 1$ in the next period as

$$p_{21} = \mu_2^* F^I(x_2^*) + (1 - \mu_2^*) F^U(x_2^*). \tag{21}$$

In the equilibrium, if the fund has a star rating $\iota = 2$, its interim reputation is μ_2^*, independent of the prior reputation. So, investors believe that the fund is informed with probability μ_2^*, and uninformed with the complementary probability. Then, according to the star rating system, the fund will receive the rating $\iota = 1$ in the next period, if and only if the investment outcome is lower than x_2^*. From investors' points of view, this occurs with probability $\mu_2^* F^I(x_2^*) + (1 - \mu_2^*) F^U(x_1^*)$.

Similarly, investors will calculate the probability that a one-star fund is assigned the rating $\iota = 2$ as

$$p_{12} = \mu_1^*(1 - F^I(x_1^*)) + (1 - \mu_1^*)(1 - F^U(x_1^*)). \tag{22}$$

As a result, the evolution of the rating constitutes a Markov chain. Its transition probability matrix can be written as

$$\mathbf{P} = (p_{ij})_{i,j \in \{1,2\}} = \begin{array}{c c} & \begin{array}{cc} 1 & \quad 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 1 - p_{12} & p_{12} \\ p_{21} & 1 - p_{21} \end{pmatrix} \end{array}. \tag{23}$$

Let $\mathbf{Q} = (q_1, q_2)'$ be the stationary probability distribution, given $\mathbf{P}$. Note that $q_i \in [0, 1]$, and $q_1 + q_2 = 1$. Denote by $\mathbf{P}'$ the transpose of $\mathbf{P}$, the stationary probability distribution Q is given by

$$\mathbf{Q} = \mathbf{P}'\mathbf{Q}. \tag{24}$$

Therefore, in a star rating equilibrium, the fund will in the long-run receive the star rating $\iota = 1$ with probability q_1 and the star rating $\iota = 2$ with probability q_2.

Denote the stationary ratio of the one-star rating probability to the two-star rating probability by

$$q = \frac{q_1}{q_2} \tag{25}$$

and the fund's expected investment performance in the long-run by

$$\mathrm{E}(\mu) = q_1 \mu_1^* + q_2 \mu_2^*. \tag{26}$$

Solving the stationary rating distribution, we can now analyze how q and $\mathrm{E}(\mu)$ change when parameters change.

In Appendix B, we perform detailed comparative static analysis numerically in a star rating equilibrium, where the uninformed fund with the highest prior reputation among one-star funds chooses not to acquire information. We first show that as the information acquisition cost c increases, the cutoff prior reputation ($\hat{\rho}$) below which the fund is assigned the one-star rating decreases. This is due to the particular equilibrium condition that the fund with the prior reputation $\hat{\rho}$ does not acquire information and thereby has the interim reputation $\mu_1^* = \hat{\rho}$: an increase in the information acquisition cost leads to lower incentives of the uninformed fund to acquire information regardless of the fund's star rating, implying lower μ_1^*. On the other hand, since the fund's incentives to acquire information decrease as the information acquisition cost increases, the fund is more likely to have a bad performance and thus receives a one-star rating. These two facts together suggest that the fund's long-run expected performance is decreasing in the information acquisition cost.

An increase in the volatility of the fund's investment outcome has very similar effects as that of an increase in the information acquisition cost. Because when the fund's investment outcome increases, the investment outcome becomes a noisier signal of the fund's information superiority. Then, it is hard for an informed fund to distinct itself from an uninformed fund, hurting the uninformed fund's incentives to acquire information.

The effects the information obsolete probability, λ, is somehow surprising. First, as λ increases, the linkage between the fund's current information acquisition action and its next period information superiority becomes weaker. This is reflected by the fact that the fund's star rating is more persistent, because the cutoff performance for a one-star fund to be upgraded to the two-star rating increases, while the cutoff performance for a two-star fund to maintain the two-star rating decreases. This hurts one-star rating uninformed funds' incentives to acquire information. leading to a lower interim reputation of one-star rating funds and thus lower cutoff prior reputation below which the fund is assigned a one-star rating decreases in the equilibrum. On the other hand, when λ increases, the fund is more likely to receive a two-star rating, because otherwise it will have no incentive to acquire information. These two opposite effects then lead to a non-monotonic effect on the fund's long-run expected performance.

4 Empirical Predictions

The properties of the star rating equilibrium have several interesting empirical implications. First, because information acquisition closely relates to the mutual fund's activeness, our theoretical analysis relates the fund's activeness to its future performance, which has been tested using several measures. However, we also show how the fund's past performance, or the fund's current status in the star rating system, affect the fund's activeness. This is important because investors can observe the fund's current prior reputation and star rating, and thus choose funds based on such information. Second, the star rating system greatly affects on the fund's incentives. Our analysis thus provides potential rational explanations for empirical observations of Morningstar Ratings' effects.

4.1 Mutual Funds' Activeness

Identifying mutual funds that will generate superior investment outcomes is an important topic in financial economics. One hypothesis is that more active mutual funds should perform better. In our setting, if an uninformed fund acquires private information, it optimally changes its portfolio, and is thus regarded as more active. Also, in our setting, an informed fund is more likely to perform better than an uninformed fund, and so we have the empirical prediction 1 below.

Prediction 1. *Cross-sectionally, funds' performance increases in their activeness.*

Prediction 1 has been confirmed by empirical studies. Cremers and Petajisto (2009) measure a fund's activeness by the fund's "active shares," which is defined as the difference between the fund's portfolio and the market index. So, if a fund holds more active shares, it is more active. Then, they document that on average, funds' investment outcomes positively correlate to their active shares, or their activeness. Funds that acquire information are more likely to have private information. As a result, their portfolios are more likely to differ from the market index. So funds that acquire information are more likely to have active shares. Hence, our theoretical model directly provides a foundation for the "active share" measure. Besides "active shares," several other measures of funds' activeness have also been analyzed, such as "industry concentration index" (Kacperczyk, Sialm, and Zheng 2005), "Reliance on Public Information" (Kacperczyk and Seru 2007), and "return gap" (Kacperczyk, Sialm, and Zheng 2008). In all these empirical studies, funds' investment outcomes (relative to the market performance) increases in their activeness.

While the documented positive relation between funds' activeness and their future performance in empirical studies is directly implied by our model setting (without any equilibrium analysis), the properties of a star rating equilibrium show the relationship between a fund's past performance and its current activeness. This is important for investors because it could provide them with clues for identifying funds' activeness in the current period. In particular, as stated in Corollary 1 and Corollary 2, in a star rating equilibrium, a fund's activeness strictly decreases when its rank within a star group increases. But, when the fund just gets upgraded, its activeness exhibits a discrete jump up.

Prediction 2. *When a fund is higher ranked within a star group, its activeness decreases; however, the fund is much more active when it is at the bottom of a higher star rating group than when it is on the top of a lower star rating group.*

4.2 Morningstar Ratings' Effects

The star rating properties of a star rating equilibrium imply that a star rating system greatly affects a fund's incentives and thus its future performance. These effects, together with other features of mutual funds, provide rational explanations for many recent empirical observations about Morningstar Ratings, which is a highly influential rating system in the mutual fund industry.

In a recent empirical study, Reuter and Zitzewitz (2015) document several interesting facts. In particular, they find that funds in the bottom of a higher-star rating group on average receive much larger cash inflows than those on the top of a lower-star rating group. For example, consider two funds, Fund A and Fund B. Fund A is ranked in the 90th percentile and thus makes a five-star rating according to Morningstar rating system; Fund B is ranked in the 89th percentile and thus only receives a four-star rating only. Then, Fund A receives significantly more cash inflows than Fund B. Reuter and Zitzewitz (2015) attribute such an observation to investors' imperfect rationality: investors pay too many attentions to star ratings of mutual funds, and ignore the fact that the information that Fund A and Fund B have very similar past performance.

Our setting provides a rational explanation for this observation: this is actually the inter-group jump effect given the star rating system. Indeed, investors ignore some information and may only pay attentions to mutual funds' star ratings, but they do so for rational reasons. That is, they believe Fund A will have much higher incentives to acquire information in the next period, if it is uninformed. Therefore, investors will expect that Fund A's performance is much better than Fund B.

A second interesting observation in Reuter and Zitzewitz (2015) is that, in some classes, funds in the bottom of a higher-star rating group have significantly better investment outcomes than those on the top of a lower-star rating group. Yet, in some other classes, the difference between the average investment outcomes of such two groups of funds are almost zero. Reuter and Zitzewitz (2015) then conclude that the individual fund level diseconomy of scale ((Berk and Green 2004)) may not hold in their data.

However, the inter-group jump effect, together with the decreasing return to scale assumption, can also explain the second observation in Reuter and Zitzewitz (2015). When the inter-group jump effect dominates in some classes, funds in the bottom of a higher-star rating group should experience better investment outcomes; when the inter-group jump effect is offset by the decreasing return to scale effect, we may not see that funds in the bottom of a higher-star rating group perform better than those on the top of a lower-star rating group.

5 Conclusion

As many empirical studies document, the agency problems in the mutual fund industry are with us, especially under the current fee structure. Therefore, mutual funds are working hard for their future investor flows and thus future revenues. Such an "indirect incentive" is usually regarded as the mutual fund's reputations concern.

In practice, investors treat funds' star ratings as important measures of funds' reputations, and thereby rely on those discrete rating systems to pick funds. However, this kind of rating systems use only a few stars to describe funds' reputations on a continuum. So it seems that investors are losing some information, and the rating system cannot remedy funds' incentive problems.

In this paper, surprisingly, we show that in an equilibrium, an endogenously determined star rating system provides investors with sufficient information to pick funds. A fund's incentives to work hard in the current period are strictly decreasing in its rank within a star rating group. However, the fund's incentives jump up discretely, when the fund is upgraded to a higher star rating. The fund's behavior feeds back to investors' belief system, making their belief system consistent with the star rating system.

The star rating equilibrium properties provide potential rational explanations for recent empirical findings about Morningstar ratings. In particular, the discrete increment of investor flows received by funds at the bottom of the 5-star group relative to that received by funds on the top of the 4-star group may reflect investors' belief that the funds at the bottom of the 5-star group have higher incentives to acquire information and thus are more likely to be informed.

Our paper has both theoretical and applied contributions. From the theory perspective, we provide an algorithm to characterize a star rating equilibrium of a discrete-time infinite-horizon continuous-signal reputation model, which is free of exogenous types. From the applied perspective, we find that how mutual funds' reputations, which are represented by their star ratings, affect their incentives. We also add another possible reason why ratings of institutions can be discrete. Our theoretical framework can be potentially applied to areas beyond mutual funds, especially those in which reputation and information superiority play key roles.

Appendix A Omitted Proofs

Proof of Lemma 1. Suppose $\sigma(\rho) = 1$ for some prior reputation ρ. Then the manager with the prior reputation ρ will have the interim reputation $\mu = \rho + (1-\rho)\sigma(\rho) = 1$, and the manager's reputation in the next period will be $1-\lambda$, independent of the investment outcome. To support this equilibrium, we must have $V_I(\rho) - V_U(\rho) \geq c$, which implies that (from Equations (7) and (8))

$$V_I(1-\lambda) - \max\{V_I(1-\lambda) - c, V_U(1-\lambda)\} \geq \frac{c}{\delta(1-\lambda)}. \tag{27}$$

If $V_I(1-\lambda) - c > V_U(1-\lambda)$, Equation (27) above becomes $V_I(1-\lambda) - [V_I(1-\lambda) - c] = c \geq \frac{c}{\delta(1-\lambda)}$. Since $\delta < 1$ and $1-\lambda < 1$, this equation obviously does not hold. In the case that $V_I(1-\lambda) - c \leq V_U(1-\lambda)$, Equation (27) becomes $V_I(1-\lambda) - V_U(1-\lambda) \geq \frac{c}{\delta(1-\lambda)} > c$. As a result, $\sigma(1-\lambda) = 1$ in the equilibrium under consideration. Then $V_I(1-\lambda) - V_U(1-\lambda) = \delta(1-\lambda)[V_I(1-\lambda) - V_U(1-\lambda)]$, which is a contradiction!

Therefore, we conclude that Equation (27) can never hold, and hence $\sigma(\rho) < 1$ for all ρ. □

Proof of Proposition 1. The proof proceeds in two steps. In the fist step, we derive the explicit expression of $\bar{c} > 0$, and in the second step, we construct a star rating equilibrium for $c < \bar{c}$.

Step 1: Expression of $\bar{c}$. In a star rating equilibrium, the fund's equilibrium strategy (19) implies that $\sigma(\rho) \in (0,1)$, and hence, at all prior reputations, the uninformed fund must be indifferent between acquiring information and remaining uninformed. As a result, in a star rating equilibrium,

$$c = V_I^1 - V_U^1 = V_I^2 - V_U^2,$$

which implies

$$\left[\frac{1}{\delta} - (1-\lambda)\right] c = \left[F^U(x_1^*) - F^I(x_1^*)\right](V_U^2 - V_U^1) = \left[F^U(x_2^*) - F^I(x_2^*)\right](V_U^2 - V_U^1). \tag{28}$$

Equation (28) implies that

$$F^U(x_1^*) - F^I(x_1^*) = F^U(x_2^*) - F^I(x_2^*). \tag{29}$$

Since F^U and F^I are normal distribution functions with mean 0 and 1 respectively, Equation (29) implies that $x_2^* = 1 - x_1^*$. Moreover, the likelihood function satisfies

$$\ell(x) = \frac{f^I(x)}{f^U(x)} = e^{\frac{2x-1}{2\phi^2}}. \tag{30}$$

Therefore, $x_2^* = 1 - x_1^*$ implies that $\ell_2 = 1/\ell_1$. Plug $\ell_2 = 1/\ell_1$ into Equation (18), and we get:

$$\hat{\rho} = (1-\lambda)\frac{\mu_2^*}{\mu_2^* + \ell_1(1-\mu_2^*)}.$$

This equation together with Equation (17) yield

$$\ell_1 = \sqrt{\frac{(1-\mu_1^*)\mu_2^*}{\mu_1^*(1-\mu_2^*)}}, \tag{31}$$

which implies that both x_1^* and x_2^* can be expressed as a function of μ_1^* and μ_2^*.

Plug the requirement $\mu_1 \geq \hat{\rho}$ into Equation (17), and we obtain

$$(1-\lambda)\ell_1 \geq \ell_1\mu_1^* + 1 - \mu_1^*.$$

From Corollary 3, $\ell_1 > 1$ and hence,

$$\mu_1^* \geq 1 - \frac{\lambda\ell_1}{\ell_1 - 1}.$$

Finally, substituting V_U^1 in Equation (14) and V_U^2 in Equation (16) into Equation (28), we have

$$c = \Omega(\mu_1^*, \mu_2^*) \triangleq \frac{\delta}{1-\delta(1-\lambda)} \frac{(\mu_2^* - \mu_1^*)\left[F^U(x_1^*) - F^I(x_1^*)\right]}{1 - \delta\left[F^U(x_1^*) - F^U(x_2^*)\right]}. \tag{32}$$

Define $\bar{c}$ such that:

$$\bar{c} = \max_{\mu_1,\mu_2} \Omega(\mu_1, \mu_2) \quad \text{s.t. } (1-\lambda) \leq \mu_2 \leq 1,\ 1 - \frac{\lambda\ell_1(\mu_1,\mu_2)}{\ell_1(\mu_1,\mu_2) - 1} \leq \mu_1 \leq \mu_2. \tag{33}$$

$\bar{c}$ is well-defined, because $\Omega(\mu_1, \mu_2)$ is a continuous function in $\{\mu_1, \mu_2\}$, and the constraints constitute a compact set of $\{\mu_1, \mu_2\}$.

Step 2: Construction of a star rating equilibrium for $c < \bar{c}$. First notice that $\Omega(\mu_1, \mu_2) = 0$ when $\mu_1 = \mu_2$. The continuity of function Ω then implies that for any $c < \bar{c}$, there exists a pair $\{\mu_1^*, \mu_2^*\}$ which satisfies $(1-\lambda) \leq \mu_2^* \leq 1$ and $1 - \frac{\lambda\ell_1(\mu_1^*,\mu_2^*)}{\ell_1(\mu_1^*,\mu_2^*)-1} \leq \mu_1^* \leq \mu_2^*$, and solves Equation (32). Given $\{\mu_1^*, \mu_2^*\}$, define $\{x_1^*, x_2^*, \hat{\rho}\}$ satisfying

$$\ell(x_1^*) = \ell_1 = \sqrt{\frac{(1-\mu_1^*)\mu_2^*}{\mu_1^*(1-\mu_2^*)}},$$

$$x_2^* = 1 - x_1^* \quad \text{and} \quad \hat{\rho} = (1-\lambda)\frac{\ell_1\mu_1^*}{\ell_1\mu_1^* + (1-\mu_1^*)}.$$

We want to show that the star rating system $\{\hat{\rho}, x_1^*, x_2^*\}$ and the fund's interim reputations $\{\mu_1^*, \mu_2^*\}$ constitute a star rating equilibrium.

From Equation (14) and (16), we obtain:

$$V_U^2 - V_U^1 = \frac{\mu_2^* - \mu_1^*}{1 - \delta\left[F^U(x_1^*) - F^U(x_2^*)\right]}.$$

Then Equation (28) is naturally satisfied since $\Omega(\mu_1^*, \mu_2^*) = c$. This implies that

$$c = V_I^1 - V_U^1 = V_I^2 - V_U^2.$$

Therefore, an uninformed fund will be indifferent between acquiring information and remaining uninformed, implying that the equilibrium strategy (19) maximizes the fund's expected continuation value. Hence, the proposed is indeed a star rating equilibrium. $\square$

Proof of Corollary 3. From Equation (31),

$$\ell_1 = \sqrt{\frac{(1-\mu_1^*)\mu_2^*}{\mu_1^*(1-\mu_2^*)}}.$$

Notice that μ_1^* cannot be the same as μ_1^* in equilibrium, because otherwise $\Omega(\mu_1^*, \mu_2^*) = 0 < c$. Therefore, we must get $\mu_2^* > \mu_1^*$, and hence $\ell_1 > 1$. From Equation (30), $\ell_1 > 1$ implies $x_1^* > \frac{1}{2}$ and hence, $x_2^* = 1 - x_1^* < \frac{1}{2}$. $\square$

Appendix B Comparative Static Analysis

To simplify the analysis, we will focus on a particular star rating equilibrium satisfying:

$$\mu_1^* = \hat{\rho} \tag{34}$$

With $\mu_1^* = \hat{\rho}$, we express x_1^*, x_2^* and μ_1^* as a function of μ_2^*:

$$\begin{aligned} \ell_2(x_2^*) &= \frac{-(1-\mu_2^*) + \sqrt{(1-\mu_2^*)^2 + 4\lambda(1-\lambda)\mu_2^*(1-\mu_2^*)}}{2\lambda\mu_2^*}, && (35) \\ x_1^* &= 1 - x_2^*, && (36) \\ \mu_1^* &= 1 - \frac{\lambda}{1-\ell_2(x_2^*)}. && (37) \end{aligned}$$

For a given μ_2^*, the above system of equations yields a unique solution $\{x_1^*, x_2^*, \mu_1^*\}$ since the likelihood ℓ_2 is a strictly increasing function. Finally, μ_2^* is determined by Equation (38) below:[12]

$$c = \frac{\delta}{1-\delta(1-\lambda)} \frac{(\mu_2^* - \mu_1^*)\left[F^U(x_1^*) - F^I(x_1^*)\right]}{1-\delta\left[F^U(x_1^*) - F^U(x_2^*)\right]}. \tag{38}$$

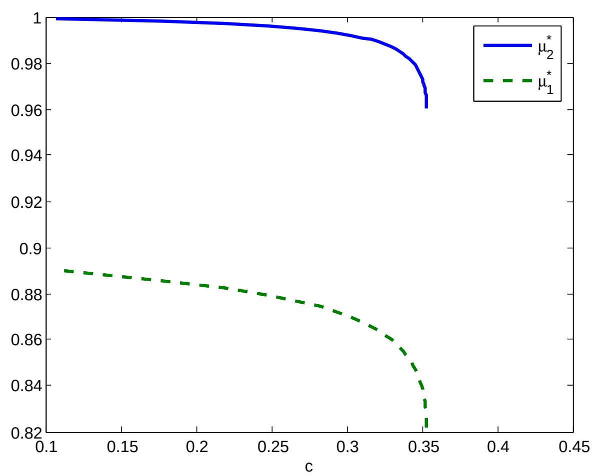
Corollary 4. *When $c \leq \bar{c}$, Equations (34), (35), (36), (37), and (38) characterize a star rating equilibrium. In this equilibrium, the fund's information acquisition strategy is*

$$\sigma(\rho) = \begin{cases} \frac{\mu_1^* - \rho}{1-\rho}, & \textit{if} \ \ \rho \leq \mu_1^*; \\ \frac{\mu_2^* - \rho}{1-\rho}, & \textit{if} \ \ \rho > \mu_1^*. \end{cases} \tag{39}$$

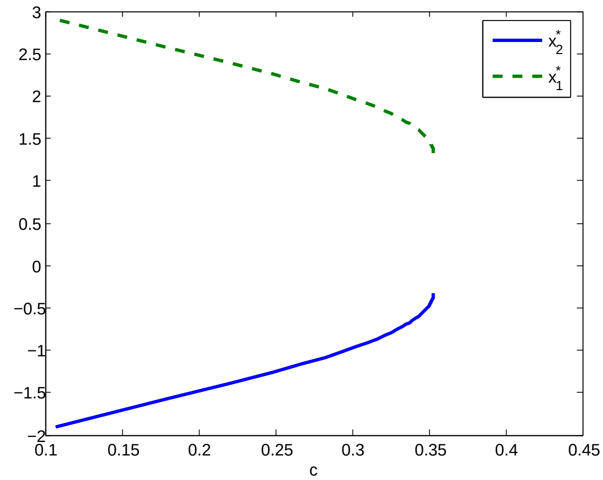
We now analyze the fund's reputation and expected investment outcome in the long-run in the equilibrium characterized in Corollary 4. Let's first consider the change of the information acquisition cost. When the information acquisition cost increases, the fund will have less incentive to acquire information. This leads to the decrease in the fund's interim reputation, regardless of whether it has a one- or two-star rating, as shown in Figure 4a. This directly causes the fund to be more likely to generate a bad investment outcome, and so the probability that it receives a one-star rating increases. In addition, Figure 4b shows that when the information acquisition cost increases, it is harder for a two-star fund to remain in the two-star group (x_2^* increases), but it is easier for a one-star fund to be promoted to the two-star group. However, the marginal increase in x_2^* and the marginal decrease in x_1^* are the same. Thus, when we consider the effect of an increase in the information acquisition cost c, the changes of the interim reputations dominate. Therefore,

[12] In the subsequent analysis, if there are multiple μ_2^* satisfying Equation (38), we pick the solution with the highest μ_2^*.

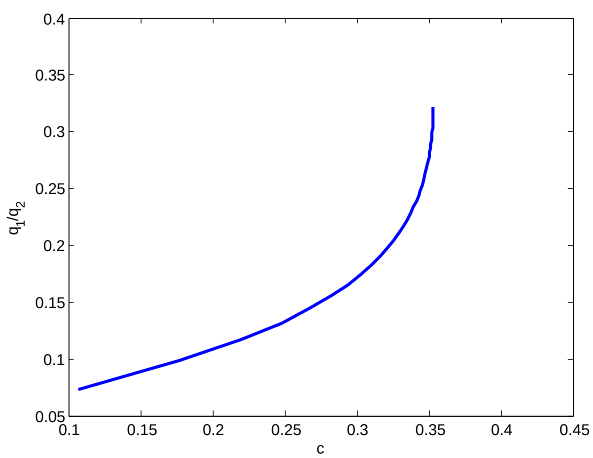
Figure 4: Change of c. The figures plot how interim reputations, thresholds of star ratings, the probability ratio of two star managers over one star managers, and the overall expected performance change with the information acquisition cost, c. The other parameters are chosen such that $\delta = 0.9$, $\lambda = 0.1$, and $\phi = 1$.



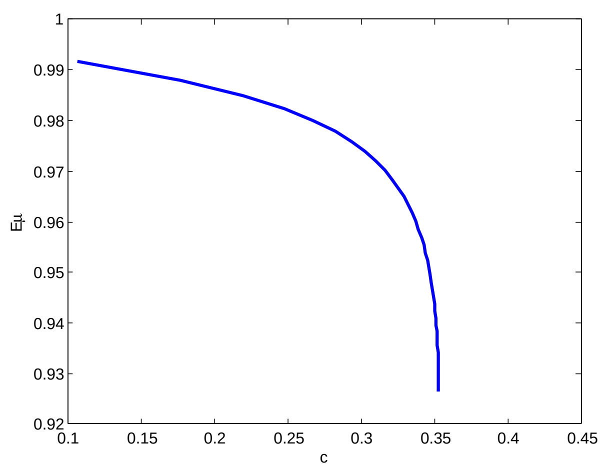
(a) Interim reputations



(b) Thresholds of star ratings



(c) The probability ratio



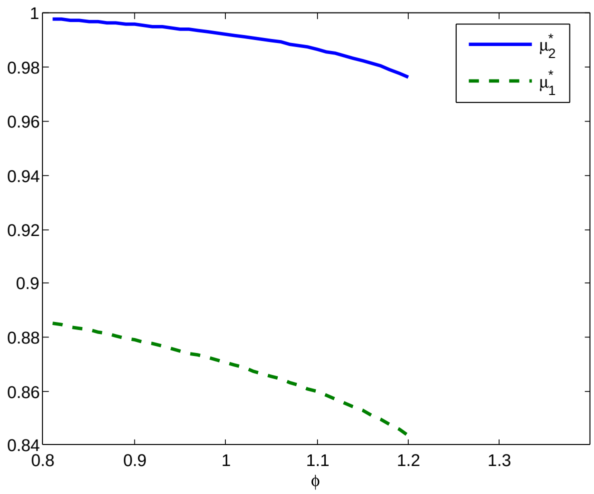
(d) The expected performance

the ratio q increases and the expected performance $\mathrm{E}(\mu)$ decreases, as shown in Figure 4c and Figure 4d, respectively.

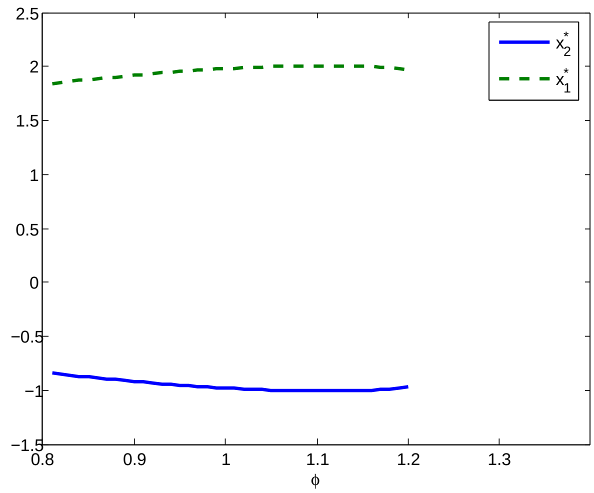
Now, let's consider that the fund's investment outcomes are more volatile (that is, ϕ increases). In such a case, investment outcomes are less informative about the fund's information status. This will also hurt the fund's incentives to acquire information, because a good investment outcome cannot help investors distinguish whether the fund is informed or not. Therefore, we expect that both μ_1^* and μ_2^* will decrease. Also, the star rating system will have a higher x_1^* and a lower x_2^* to help investors screen. Consequently, the ratio q will increase, and the fund's expected investment outcome $\mathrm{E}(\mu)$ will decrease. The numerical results in Figure 5 confirm such a hypothesis.

Finally, we study the effect of an increase in the decay rate of information superiority (that is, λ increases). Surprisingly, while the interim reputation of the one-star fund strictly decreases in λ, the interim reputation of the two-star fund is not monotonic in λ. In addition, the ratio q strictly decreases in λ, but the expected investment outcome $\mathrm{E}(\mu)$ is not monotonic in λ. The numerical results are presented in Figure 6 below.

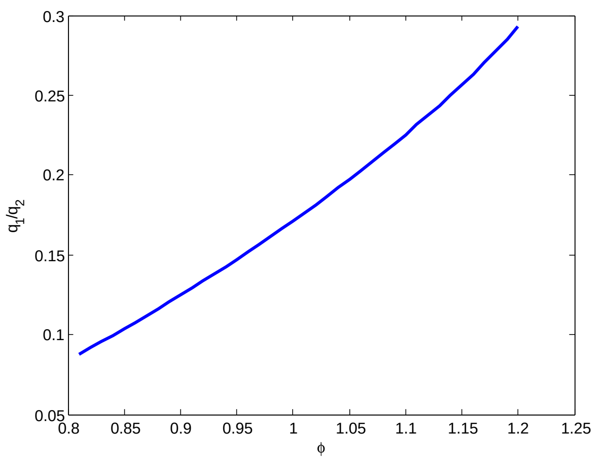
Figure 5: Change of ϕ. The figures plot how interim reputations, thresholds of star ratings, the probability ratio of two star managers over one star managers, and the overall expected performance change with the standard deviation of investment outcome, ϕ. The other parameters are chosen such that $\delta = 0.9$, $\lambda = 0.1$, and $c = 0.3$.



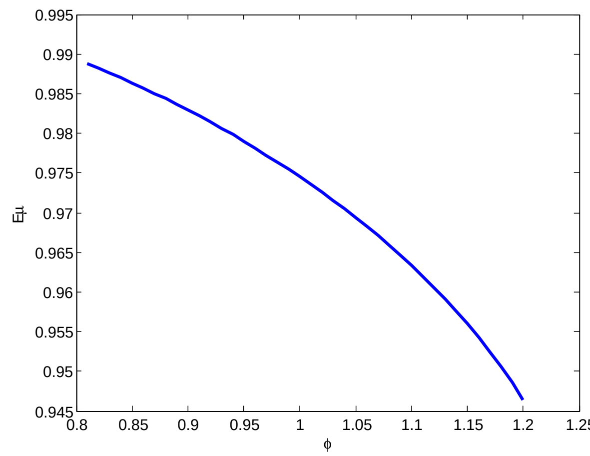
(a) Interim reputations



(b) Thresholds of star ratings

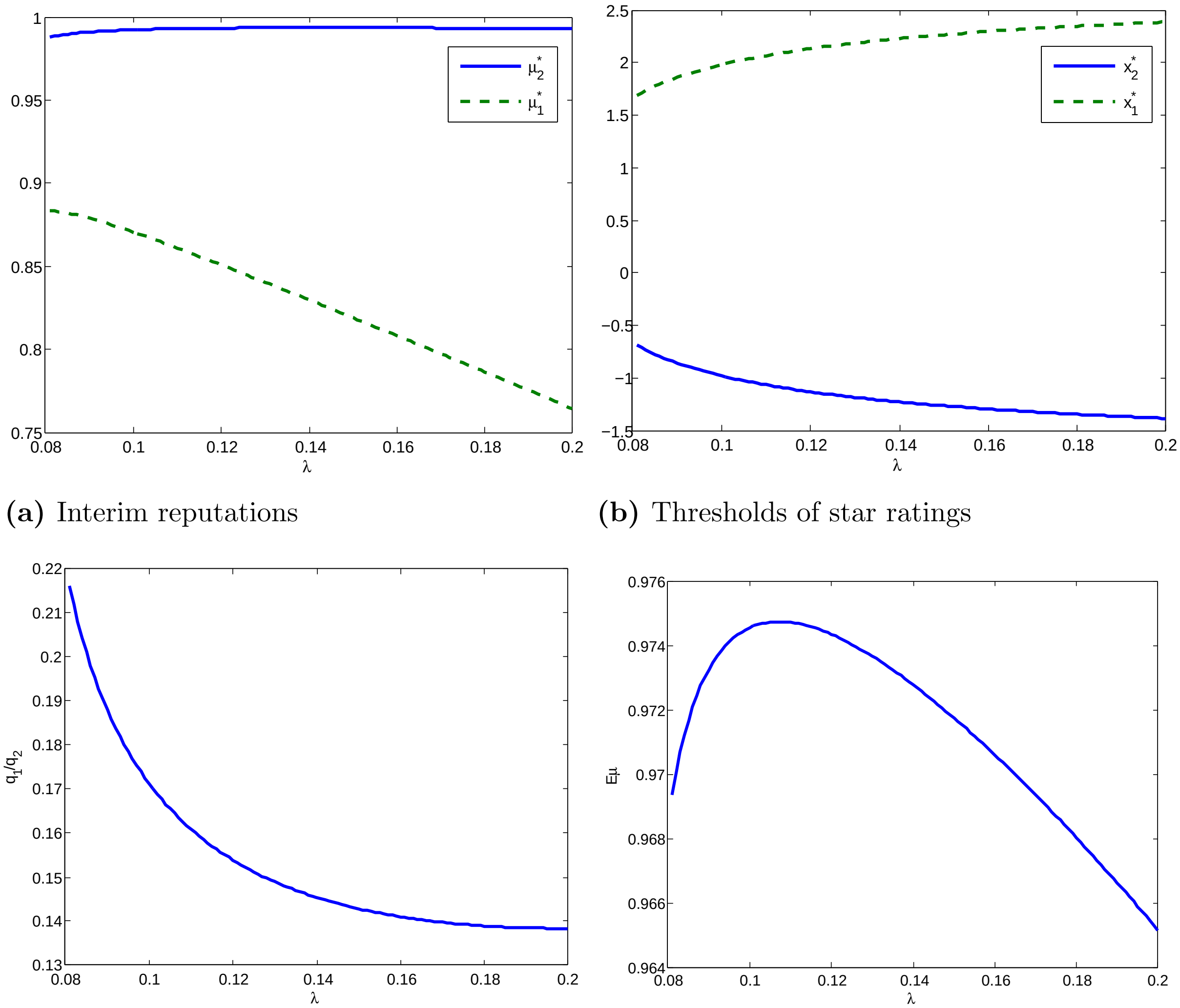


(c) The probability ratio



(d) The expected performance

Figure 6: Change of λ. The figures plot how interim reputations, thresholds of star ratings, the probability ratio of two star managers over one star managers, and the overall expected performance change with λ. The other parameters are chosen such that $\delta = 0.9$, $\phi = 1$, and $c = 0.3$.



(a) Interim reputations

(b) Thresholds of star ratings

(c) The probability ratio

(d) The expected performance

The intuition behind such surprising comparative statics is as follows. When λ increases, the highest possible prior reputation decreases. To reward good investment outcomes, the star rating system must adjust down $\hat{\rho}$, the cutoff of star ratings. Otherwise, it is harder for the fund to receive a two-star rating. In the equilibrium characterized in Corollary 4, $\mu_1^* = \hat{\rho}$, so lower $\hat{\rho}$ means lower μ_1^*. So, this argument, combined with the fund's lower incentives to acquire information, implies that μ_1^* strictly decreases in λ. But in the equilibrium characterized in Corollary 4, μ_1^* is also the prior belief when the two-star fund generates the investment outcome x_2^*, so a smaller μ_1^* implies a smaller μ_2^*, mechanically. On the other hand, when λ increases, an informed fund will more quickly lose its information superiority. As a result, the value of an informed fund (one-star or two-star) decreases. Then, investors' belief system will adjust to keep the fund's reputation premium equal to the information acquisition cost. That is, investors need to assign a higher μ_2^* to compensate the faster decay rate of information superiority, so that the fund can still have incentives to acquire information. Finally, the competition between these two forces leads to the non-monotonicity of μ_2^* and the non-monotonicity of $\mathrm{E}(\mu)$.

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