Time-Inconsistent Risk Preferences and the Term Structure of Dividend Strips

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Abstract

Recent empirical research finds that the term structures of risk premia, return volatilities and Sharpe ratios on dividend strips are all downward-sloping (van Binsbergen et al. (2012)), but these observations cannot be explained by most asset-pricing theories. In this paper, I resolve this discrepancy using time-inconsistent risk preferences: agents' risk aversion differs in the short-run from the long-run. I solve three variants of the model: i) I allow the agent to commit to her future plan; assuming committing is not possible, I consider ii) a naive agent unaware of her time inconsistency, and iii) a sophisticated agent aware of it. I show that the naive agent case generates a flat term structure when endowment growth is i.i.d., as with standard time-consistent preferences. In the commitment and sophisticated agent cases, the term structures are downward-sloping when the agent is less averse to immediate risks than to future risks. The reasoning is that time inconsistency makes the state prices depend on the current state of the economy. If the agent is less averse to immediate risk, one-period future consumption is valued less if the current state is the good state than if it is the bad state, which causes the payoff structure of long-maturity dividend strips to become less risky than short-maturity dividend strips, leading to a downward-sloping term structure.

Keywords: term structure, time inconsistency, risk preferences, general equilibrium

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The equity market literature closely examines the dynamics of the value of the aggregate stock market, which is computed as the sum of discounted future dividend payments. The literature has focused less on the value of individual terms in the sum, which can be viewed as separate assets known as dividend strips. The term structure of dividend strips displays their risk premia, return volatilities and Sharpe ratios across maturities. Are short-maturity dividend strips discounted at the same rate as long-maturity dividend strips? Is the high equity premium observed in many countries attributable to short-maturity dividends or long-maturity dividends? These are interesting questions that can help us better understand investors' risk preferences and stocks' dividend process.

The first empirical paper to study the pricing of dividend strips is van Binsbergen, Brandt and Koijen (2012), who recover the price of a short-term asset, which is an asset that pays dividends up to a terminal date T and nothing thereafter. These authors find that the expected return, return volatility and Sharpe ratio on the short-term asset are all higher than those on the stock index itself, which implies a downward-sloping term structure of dividend strips.[1] However, these authors also show that the empirical pattern cannot be explained by many leading asset-pricing theories, including Campbell and Cochrane (1999)'s habit formation model, Bansal and Yaron (2004)'s long-run risk model and Gabaix (2012)'s rare disaster model. My paper provides a general-equilibrium framework that explains the downward-sloping term structure of dividend strips using non-standard risk preferences.

Several researchers have worked to solve the puzzle raised by van Binsbergen et al. (2012), and most have focused on altering the dividend process. In this

[1] Given that the sample only spans the January 1996-October 2009 period, the authors cannot reject the null hypothesis that the average return on the dividend strategy is equal to the average return on the index. The difference in the Sharpe ratios between the dividend strategy and the index is also not statistically significant. The authors can reject the null hypothesis that the variance of the return on the dividend strategy is equal to the return variance on the index at the 1% level.

paper, I maintain a standard i.i.d. process for dividend growth, but instead assume non-standard risk preferences. I develop a general-equilibrium model that can explain the downward-sloping term structure of dividend strips by assuming that investors' risk attitudes vary over the horizon of cash flows. Thus, the paper can be interpreted as an attempt to determine whether a particular modification of preferences can, alone explain the observed patterns. Because the risk premium is mainly a function of risk aversion, it is natural to construct a model in which investors have different risk aversions toward short-run risk and long-run risk. Such preferences are called "time-inconsistent" because what is long-run risk for the agent today becomes short-run risk as time passes, such that the agent has different valuations of the same risky cash flows at different time periods. Experimental evidence shows that risk aversion tends to vary with the risk's temporal distance, but the conclusion on whether people are more or less averse to immediate risks is mixed (see Noussair and Wu (2006), Abdellaoui, Diecidue, and Onculer (2011), Ko and Huang (2012)). I consider both the case in which the agent has a higher risk aversion to immediate risk than to future risks, and the opposite case in which the agent is less averse to immediate risk.

When addressing time inconsistency, researchers often treat the investor at different time periods as different intertemporal "selves". There are generally three ways to solve the model with time-inconsistent preferences. The first is to assume a mechanism that allows the agent to commit to his plan of action. All future selves must maximize the preferences of the current self who made the commitment. In the models without commitment, the different selves at each point in time choose their own behavior to maximize their own preferences. Two distinct cases emerge: the "naive agent case" and the "sophisticated agent case", depending on whether the agent has rational expectations of her future behavior. The naive agent believes her future selves' preferences will be identical to those of her current self, not realizing that they will change as she moves closer to executing decisions. She constantly modifies her planned consumption and investment choices. The optimal decision rule is a time-inconsistent strategy, because future selves keep deviating from the initial plan. In contrast, the sophisticated agent knows exactly what her future selves' preferences will be. She rationally predicts her future selves' behavior and takes these predictions into account in her present-day optimization decision. Her objective is to "find the best plan among those that she will actually follow" (Stroz (1956)). Therefore, her optimal decision is a time-consistent strategy. Such an intertemporal decision problem can be transformed into an extensive game in which decisions are taken sequentially by different selves, and the solution should be constructed by looking for the subgame perfect equilibrium.

I solve for the general equilibrium in an endowment economy for all three cases and analyze their implications for the term structure of dividend strips. I first present a two-period model to build intuition, and I then extend the model to multiple periods using a recursive method. To isolate the effect of time-inconsistent preferences on the term structure, I assume that the endowment grows at an i.i.d. rate, which leads to a flat term structure under standard time-consistent preferences. I do not intend to explain the term structure of Sharpe ratios in my model. I assume that the endowment follows a binomial tree process, so that the Sharpe ratios of dividend strips of all maturities are equal by construction.[2] I show that the naive agent case is observationally equivalent to the standard case of time-consistent preferences and therefore generates a flat term structure of dividend strips. In the commitment and sophisticated agent cases, time-inconsistency induces a slope in the term structure. Somewhat surprisingly, short-maturity dividends have a higher risk premium and

[2]With a binomial tree process, the Sharpe ratio coincides with the price of risk, which is the same across all assets.

return volatility than long-maturity dividends when the investor is less averse to immediate risk, implying a downward-sloping term structure. By contrast, when the investor is more averse to immediate risk, the term structures for risk premia and volatilities are both upward-sloping.

The naive agent case generates a flat term structure because, although she plans ahead, she only implements her plan for the current period and replaces it with a new plan in the next period. Hence, only short-run risk aversion matters to the current period's decision. In essence, the naive agent behaves as if she was forever young: she acts like a standard time-consistent agent with constant short-run risk aversion. The equilibrium is also identical with the standard case of time-consistent preferences.

The results for the commitment case and the sophisticated agent case go against common intuition. When an investor is less risk-averse toward short-run risk, she would be expected to require a lower risk premium for holding short-maturity dividends. However, this is not the case in my model for the following reasons. First, I am comparing the one-period risk premium on the short-maturity dividend to the one-period risk premium on the long-maturity dividend. I am not comparing the average returns on assets held to maturity, which implies different holding periods. Therefore, this is not simply a matter of comparing short-run risk aversion with long-run risk aversion. Second, one important feature of my model with time-inconsistent preferences-which differs from the standard time-consistent preferences-is that the state prices become dependent on the current state even with the i.i.d. growth assumption and isoelastic utility. If short-run risk aversion is relatively lower, state prices decrease with the consumption of the current state. One-period future consumption is valued less if the current state is the good state than it is if the current state is the bad state. This state dependency causes the long-maturity dividend to have a less risky payoff than the short-maturity dividend, which leads to a downward-sloping term structure for dividend strips.

My research thus makes three contributions. First, it offers an explanation for the downward-sloping term structure of dividend strips based on investors' preferences. Second, although there have been quite a few works on time-inconsistent discounting, a model with time-inconsistent risk preferences has not yet been solved dynamically. My solution method can also be applied to other studies with dynamic inconsistency. Finally, the results from this study help explain the "value premium" of the stock returns as well. Value stocks can be considered as short-horizon equity because more cash flows are expected to be generated in the short run, whereas growth stocks are long-horizon equity because much of the cash flows are expected to be generated in the future. The downward-sloping term structure of risk premia implies that the value stocks will have a higher expected return than the growth stocks.

1 Related literature

Some theoretical work has already been performed in response to the puzzle posed by van Binsbergen, Brandt and Koijen (2012). Belo, Collin-Dufresne and Goldstein (2012) show that the discrepancy between the empirical evidence and the two leading asset-pricing models (the habit-formation model and the long-run risk model) can be reconciled if the dynamics of earnings before interest and taxes (EBIT) are combined with a dynamic capital structure strategy that generates stationary leverage ratios. This combination endogenously determines dividend dynamics that are cointegrated with EBIT, implying that long-horizon dividend strips are no riskier than long-horizon EBIT strips. Other studies, such as Ai et al. (2012) and Favilukis and Lin (2013), focus on the production side to explain the downward-sloping term structure of dividend strips. Muir

(2013) constructs a model of a financial crisis and shows that the term structure of risky assets is time-varying and downward-sloping in bad times when crisis probabilities are concentrated in the short term. Lettau and Wachter (2011) explain simultaneously the upward-sloping yield curve for bonds and a downward-sloping term structure of equity premia by exogenously specifying a parsimonious stochastic discount factor for the economy, and their model is thus not a general equilibrium model. An important feature of their model is that shocks to expected and unexpected dividend growth are negatively correlated. This relationship implies that long-maturity dividend strips are less risky than short-maturity dividend strips on a per-period basis. Whereas the previous literature focuses on changing the dividend process, I study the problem from the perspective of investor preferences.

There is also a stream of literature that studies time-inconsistent preferences. In general, time-inconsistent preferences mean people's long-run preferences are different from their short-run preferences. One type of time-inconsistent preferences which is well documented is hyperbolic discounting, with people having different time-discounting factors for the short-run consumption and long-run consumption. Thaler (1981) provides the experimental evidence and several papers show its asset pricing implications. Harris and Laibson (2001) derive the Euler equation for the hyperbolic discounting investor. Luttmer and Mariotti (2003) obtain a continuous-time approximate solution for the risk-free rate and the risk premium of stock in an exchange economy. They show that subjective rates of time preference affect risk-free rates but not instantaneous risk-return trade-offs. Both papers examine only the value of stock but not on the term structure of dividend strips. In fact, hyperbolic discounting cannot explain the puzzle of downward-sloping term structure of dividend strips.[3] Another

[3] I can show that the hyperbolic discounting preference generates a flat term structure with an i.i.d. endowment growth rate. This structure is generated because the state prices of the economy with hyperbolic discounting preferences do not depend on the current state, which

equally interesting dimension to study time-inconsistent preferences is through risk preferences. There have been experimental evidences showing that people's risk attitudes are horizon dependent, and the reason is that people's decision making with respect to immediate risk is easily affected by emotional impulses, such as excitement or anxiety. However, the experimental results and their psychological explanations vary in the literature. Noussair and Wu (2006) and Baucells and Heukamp (2010) use the choice between two binary lotteries to elicit people's risk aversions and they find that more subjects are more risk averse for the present than for the future. The potential reason for the increased aversion to immediate risk is that as the risk draws closer, people's emotional reactions to risk such as fear can anxiety increase. Ko and Huang (2012) argue that the previous studies use abstract choice between binary lotteries and do not simulate actual portfolio allocation decisions over time. When they ask the subjects to plan out all the contigent betting decisions in the initial session of a multi-period game and later play the game in a second session, they find that majority of subjects took more risk than they had planned, indicating a lower risk aversion to present than to future. And their reasoning for the excessive risk taking behavior is that the immediacy of payoffs drives people's emotional impulse of excitement and greed. In my model, I do not take standpoint on the psychological origin of the time-inconsistency. I take the revealed preferences from these experiments as given and consider both the cases when the agent is more or less averse to immediate risk than to future risks.

My work is most closely related to that of Eisenbach and Schmalz (2014), and Andries, Eisenbach and Schmalz (2014), who also use time-inconsistent preferences to explain the equity term structure. However, my results are opposite to theirs. Using a static two-period model, Eisenbach and Schmalz (2014) show that if the agent is an anxiety-prone investor who is more averse to im-

is an important difference from my model.

mediate risk, then the term structure of risk premia is downward-sloping in equilibrium. There are two differences between my model and that of Eisenbach and Schmalz (2014) that lead to different results. First, their model is static in which the decision is only made at time 0; thus, dynamic inconsistency does not play a role. However, in my model, the agent can rebalance her portfolio in every period. Second, the returns we compare are different. Eisenbach and Schmalz (2014) compare the returns of two dividend strips of different maturities by holding them to maturity. In my model, I compare the one-period expected returns for the short-maturity dividend strip with those of the long-maturity dividend strip, which is more comparable to the approach adopted by van Binsbergen et al. (2012). Andries, Eisenbach and Schmalz (2014) extend Eisenbach and Schmalz (2014) to a multi-period dynamic model that allows the agent to re-trade. However, their result depends critically on the assumption of the stochastic volatility of endowment growth. Their model cannot generate a slope on the term structure with the assumption of time-inconsistent preferences alone, without any additional structural assumption on the endowment.

2 Time-inconsistent risk preferences

The main difference between my model and the standard asset-pricing model is that I use time-inconsistent risk preferences. I assume that the investor has a different risk aversion to short-run risks than to long-run risks. To simplify the model, I assume that there are only two different levels of risk aversion. In the current period (with no risk) and the next one period, the agent has short-run risk aversion γ_s, whereas in all other future periods, she has long-run risk aversion γ_l. Mathematically, the agent maximizes her expected lifetime utility

$$U\left(c_t\right)+E_t\left[\beta U\left(c_{t+1}\right)+\beta^2 V\left(c_{t+2}\right)+\cdots+\beta^{T-t} V\left(c_T\right)\right] \tag{1}$$

where I assume power utility,

$$U(c_t) = \frac{c_t^{1-\gamma_s}}{1-\gamma_s},\ V(c_t) = \frac{c_t^{1-\gamma_l}}{1-\gamma_l},\ \gamma_s \neq \gamma_l. \tag{2}$$

This form of the utility function is similar to that considered in the hyperbolic discounting preferences discussed in Harris and Laibson (2001). While the authors use different time-discounting factors in the short run and in the long run, I focus on having different risk aversion parameters. This type of utility function is "time-inconsistent" because the agent making a decision in time t has preferences that differ from those at time t' for his future consumption stream. For example, at time t, the agent would use the long-run utility $V(.)$ to evaluate future cash flows c_{t+2}, whereas at time $t+1$, the same investor would change her preferences and use the short-run utility $U(.)$ to evaluate the same cash flow. Such an agent has a self-control problem. What is long-run risk now will become short-run risk as time passes, and the same risky cash flow would be evaluated differently by the same agent at different time periods. Note that this utility is not simply a time-changing utility because every period, the agent will shift her entire sequence of utility functions one period forward, and the agent who makes the decision always has the short-run utility $U(.)$ for the current period's consumption. It is like "resetting" time t to be the new time 0 in every period, with the agent having the same sequence of utility functions to evaluate future cash flows according to their proximity to the current period.

Three cases to be considered generate different consumption and portfolio choices for the investors' dynamic optimization problem. For a committed agent, her decision rule is a time-consistent strategy because her plan and the actual behavior coincide. However, her plan is optimal only from the perspective of the time-0 self and not for later selves. For a naive agent, the decision rule planned is optimal from the current self's perspective, but the plan is only implemented by the current self, and the future self will modify the calculated choices for the whole path in the next period. Therefore, the realized consumption path is not optimal from the perspective of any period's self, and the decision rule is generally time-inconsistent. For a sophisticated agent, the decision rule is both optimal and time-consistent. The sophisticated agent takes into account the preferences of all her future selves and forms a plan that is the best among those that will actually be followed. Thus, the consumption path is optimal from all selves' perspectives.

In the model, I assume a standard Lucas exchange economy. For analytical tractability, the aggregate endowment is assumed to follow a binomial-tree process. This assumption allows for better intuition by comparing state prices across different states. The growth rate of aggregate endowment is assumed to be i.i.d. The i.i.d. growth endowment process with the standard isoelastic utility generates a flat term structure, which is a good benchmark for comparison with the time-inconsistent case. The initial endowment is normalized to be 1, i.e., $\delta_0 = 1$. At every node, the endowment has a probability $\frac{1}{2}$ to go up with a gross return of u and a probability $\frac{1}{2}$ to go down with a gross return of d. The time-discounting factor is denoted as β. The market is dynamically complete, and three assets are traded: i) a riskless bond; ii) the stock, which is a claim of all future aggregate endowment; and iii) the first dividend strip, which is a claim of next period's aggregate endowment. Assume there is only one agent in the economy who has time-inconsistent preferences shown in (1). She chooses her consumption and portfolio in each period to maximize her expected utility subject to budget constraints. Because there is only one investor, no-trade equilibrium is obtained, in which equilibrium prices are such that the agent is content with not trading.

In this paper, I solve all three cases and examine their implications for the term structure of dividend strips. The most interesting case is the sophisticated agent case, but I start with the commitment case to provide an intuitive explanation of the model. The comparison of the naive agent case with the sophisticated agent case helps explain how investors' awareness of their own time inconsistency affects equilibrium prices. I first solve the simplest version of the model, in which $T = 2$, for the commitment case and the naive agent case, and I then solve the model in which $T = 3$ for the sophisticated agent case. I compare the risk premia and return volatilities of the first and the second dividend strips and examine the intuition gathered from each case. With the binomial-tree setting, the Sharpe ratios of the two strips coincide because their payoffs at $t = 1$ are always perfectly correlated. Then, I extend the model to the multi-period case with $T \geq 3$ by means of a recursive method. I obtain a general Euler condition for each case and show that the result that held for a two-period model remains true.

3 The commitment case

The commitment case is easy to solve because the agent makes the plan for the whole tree at $t = 0$, and all the future selves will follow this plan. Therefore, I solve the equilibrium with only one optimization problem at the initial node. I begin with a simple two-period model.

3.1 Two-period model

With a two-period model, there are three points in time: $t = 0, 1, 2$. The optimization problem of the agent is

$$Max\ U(c_0) + \frac{1}{2}\beta U(c_u) + \frac{1}{2}\beta U(c_d) + \frac{1}{4}\beta^2 V(c_{uu}) + \frac{1}{4}\beta^2 V(c_{ud}) + \frac{1}{4}\beta^2 V(c_{du}) + \frac{1}{4}\beta^2 V(c_{dd})$$

where

$$U(c_t) = \frac{c_t^{1-\gamma_s}}{1-\gamma_s},\ V(c_t) = \frac{c_t^{1-\gamma_l}}{1-\gamma_l}$$

subject to the dynamic budget constraint.

$$\begin{aligned} W_0 &= c_0 + \phi_u W_u + \phi_d W_d, \\ W_u &= c_u + \phi_{uu} W_{uu} + \phi_{ud} W_{ud}, \\ W_d &= c_d + \phi_{du} W_{du} + \phi_{dd} W_{dd} \end{aligned}$$

where $\{\phi_u, \phi_d, \phi_{uu}, \phi_{ud}, \phi_{du}, \phi_{dd}\}$ are first-period and second-period state prices. Furthermore, the Lagrangian is

$$\begin{aligned} L_0 = {} & U(c_0) + \frac{1}{2}\beta U(c_u) + \frac{1}{2}\beta U(c_d) + \frac{1}{4}\beta^2 V(c_{uu}) + \frac{1}{4}\beta^2 V(c_{ud}) + \frac{1}{4}\beta^2 V(c_{du}) \\ & + \frac{1}{4}\beta^2 V(c_{dd}) + \lambda_0 (W_0 - c_0 - \phi_u W_u - \phi_d W_d) + \lambda_u (W_u - c_u - \phi_{uu} W_{uu} \\ & - \phi_{ud} W_{ud}) + \lambda_d (W_d - c_d - \phi_{du} W_{du} - \phi_{dd} W_{dd}) \end{aligned}$$

where $\{c_0, W_u, W_d, c_u, W_{uu}, W_{ud}, c_d, W_{du}, W_{dd}\}$ are decision variables and $\{\lambda_0, \lambda_u, \lambda_d\}$ are Lagrangian multipliers. Taking first-order conditions, I obtain equilibrium state prices for the commitment case in the two-period model.

$$\begin{aligned}
\phi_u &= \frac{1}{2}\beta\frac{U'(c_u)}{U'(c_0)},\ \phi_d = \frac{1}{2}\beta\frac{U'(c_d)}{U'(c_0)}, \\
\phi_{uu} &= \frac{1}{2}\beta\frac{V'(c_{uu})}{U'(c_u)},\ \phi_{ud} = \frac{1}{2}\beta\frac{V'(c_{ud})}{U'(c_u)}, \\
\phi_{du} &= \frac{1}{2}\beta\frac{V'(c_{du})}{U'(c_d)},\ \phi_{dd} = \frac{1}{2}\beta\frac{V'(c_{dd})}{U'(c_d)}
\end{aligned}$$

The first-period state prices of the commitment case $\{\phi_u, \phi_d\}$ are the same as those in the standard time-consistent case and only depend on the aggregate growth rate of the endowment. Unlike in the time-consistent case in which the second-period state prices are independent of the state at $t = 1$ (i.e. $\phi_{uu} = \phi_{du},\ \phi_{ud} = \phi_{dd}$), they now differ from the time-1 states in the commitment case. How they differ depends upon the relationship between the short-run risk aversion and the long-run risk aversion. To see this more clearly, I rewrite the second period state prices as follows:

$$\begin{aligned}
\phi_{uu} = \frac{1}{2}\beta\frac{V'(c_{uu})}{U'(c_u)} = \frac{1}{2}\beta\frac{U'(c_{uu})}{U'(c_u)}\frac{V'(c_{uu})}{U'(c_{uu})} \\
\phi_{du} = \frac{1}{2}\beta\frac{V'(c_{du})}{U'(c_d)} = \frac{1}{2}\beta\frac{U'(c_{du})}{U'(c_d)}\frac{V'(c_{du})}{U'(c_{du})}
\end{aligned}$$

Thus, the second period state prices of the commitment case can be written as the state prices of the standard time-consistent case ($\frac{1}{2}\beta\frac{U'(c_2)}{U'(c_1)}$) multiplied by another term ($\frac{V'(c_2)}{U'(c_2)}$), which I call the "time-inconsistent component". This time-inconsistent component measures the valuation difference on c_2 between time-0 and time-1 selves, because c_2 is a long-run cash flow for the time-0 self but a short-run cash flow for the time-1 self. And this term depends not only on the aggregate consumption growth rate, but also on the consumption level at $t = 1$. With $U(.)$ and $V(.)$ being isoelastic utility,

$$\frac{U'(c_{uu})}{U'(c_u)} = \frac{U'(c_{du})}{U'(c_d)} = u^{-\gamma_s}$$
$$\frac{V'(c_{uu})}{U'(c_{uu})} = c_u^{\gamma_s-\gamma_l} u^{\gamma_s-\gamma_l}$$
$$\frac{V'(c_{du})}{U'(c_{du})} = c_d^{\gamma_s-\gamma_l} u^{\gamma_s-\gamma_l}$$

Suppose the agent is less averse to immediate risk, i.e.$\gamma_s < \gamma_l$, then

$$\gamma_s - \gamma_l < 0 \ and \ c_u > c_d$$

$$\Rightarrow \frac{V'(c_{uu})}{U'(c_{uu})} < \frac{V'(c_{du})}{U'(c_{du})}$$

$$\Rightarrow \phi_{uu} < \phi_{du}$$

By the same argument,

$$\phi_{ud} < \phi_{dd}$$

The second-period state prices are decreasing with consumption at $t = 1$ when $\gamma_s < \gamma_l$. Similar to the opposite case in which $\gamma_s > \gamma_l$.

LEMMA 1: For the commitment case in a two-period binomial-tree setting, if $\gamma_s < \gamma_l$, then $\phi_{uu} < \phi_{du}$, $\phi_{ud} < \phi_{dd}$; if $\gamma_s > \gamma_l$, then $\phi_{uu} > \phi_{du}$, $\phi_{ud} > \phi_{dd}$; if $\gamma_s = \gamma_l$, then $\phi_{uu} = \phi_{du}$, $\phi_{ud} = \phi_{dd}$.

How does this state dependency of the second period's state prices affect the term structure of dividend strips? I have the following Lemma 2. This conclusion is not limited to the commitment case and holds for the naive agent case and the sophisticated agent case as well. The proof is presented in the

Appendix.

LEMMA 2: For an equilibrium two-period binomial-tree model with an i.i.d. growth rate, if $\phi_{uu} < \phi_{du,} \phi_{ud} < \phi_{dd}$, then the first dividend strip has a higher risk premium and higher return volatility than the second dividend strip. If $\phi_{uu} > \phi_{du,} \phi_{ud} > \phi_{dd}$, then the first dividend strip has a lower risk premium and lower return volatility than the second dividend strip. If $\phi_{uu} = \phi_{du}$, $\phi_{ud} = \phi_{dd}$, then the first and second dividend strips have the same risk premium and return volatility.

With Lemma 1 and Lemma 2, I immediately obtain the following proposition regarding the term structure of dividend strips for the commitment case. $R^{(n)}$ denotes the return for the n-th dividend strip, meaning the dividend to be paid n periods later.

PROPOSITION 1: For a commitment agent with short-run risk aversion γ_s and long-run risk aversion γ_l, there are three different scenarios for the term structures of risk premia and return volatilities, in a two-period binomial-tree model with an i.i.d. growth rate:

(a) if $\gamma_s < \gamma_l$, then $E\left[R^{(1)}\right] - R_f > E\left[R^{(2)}\right] - R_f$, $\sigma\left(R^{(1)}\right) > \sigma\left(R^{(2)}\right)$. The term structures of risk premia and return volatilities are downward-sloping.

(b) if $\gamma_s > \gamma_l$, then $E\left[R^{(1)}\right] - R_f < E\left[R^{(2)}\right] - R_f$, $\sigma\left(R^{(1)}\right) < \sigma\left(R^{(2)}\right)$. The term structures of risk premia and return volatilities are upward-sloping.

(c) if $\gamma_s = \gamma_l$, then $E\left[R^{(1)}\right] - R_f = E\left[R^{(2)}\right] - R_f$, $\sigma\left(R^{(1)}\right) = \sigma\left(R^{(2)}\right)$. The term structures of risk premia and return volatilities are flat.

The main reasoning for the commitment case is as follows. The risk premia of dividend strips depend on the state prices and future payoffs. Because the first period's state prices are the same for both dividend strips, the term structure only depends on how the payoff structure of the second dividend strip differs from that of the first dividend strip at $t = 1$. The payoff of the first dividend strip is exogenously given to be the aggregate endowment. The payoff of the second dividend strip is its price at $t = 1$ and depends on the second period's state prices. In the standard time-consistent case in which $\gamma_l = \gamma_s$, the state prices depend only on the growth rate of aggregate endowment and are universal in all one-period sub-trees, i.e., $\phi_u = \phi_{uu} = \phi_{du,}$ $\phi_d = \phi_{ud} = \phi_{dd}$. The second dividend strip is priced in the same manner as the first dividend strip; therefore, the term structure is flat. When preferences are time-inconsistent, the second-period state price becomes state-dependent. Specifically, when the agent is less averse to immediate risk, the second-period state price is lower when time-1 state is the good state and higher when time-1 state is the bad state. This relationship causes the return volatility of the second dividend strip to be lower than that of the first dividend strip at $t = 1$. Because the price of risk is the same for all assets with the binomial-tree assumption, the first dividend strip has a higher risk premium than the second dividend strip. The same argument applies to the opposite case when the agent is more averse to immediate risk: the state dependency of the second-period state price increases the volatility of the second dividend strip and makes it riskier than the first dividend strip, leading to an upward-sloping term structure.

3.2 Multi-period model

Now, I extend the commitment case to a T-period model with $T > 2$. I obtain the Euler condition with the recursive method and then present the results for the term structure. I assume that the agent makes the commitment at time t such that the Euler condition is comparable to the naive agent case and the sophisticated agent case. The optimization of a committed agent at time t is

$$Z_t(W_t) \triangleq Max_{\{c_t,\{\theta_t\}\}} U\left(c_t\right) + \beta E_t\left[F_{t+1}\left(W_{t+1}\right)\right]$$

subject to

$$
\begin{aligned}
W_t &= c_t + \sum\nolimits_{i=1}^{N} \theta_{t,i} S_{t,i}, \\
W_{t+1} &= \sum_{i=1}^{N} \theta_{t,i} \left(S_{t+1,i} + \delta_{t+1,i}\right)
\end{aligned}
$$

where $F_{t+1}(W_{t+1})$ is the future continuation value function. It is the expectation, conditional on W_{t+1}, of the present discounted value of the utility stream that begins in period $t+1$.

$$F_{t+1}(W_{t+1}) = U(c_{t+1}) + \beta E_{t+1}\left[M_{t+2}(W_{t+2})\right]$$

And

$$M_{t+n}(W_{t+n}) = V(c_{t+n}) + \beta E_{t+2}\left[M_{t+n+1}(W_{t+n+1})\right], \; for \; n = 2, 3, 4 \ldots$$

where $M_{t+n}(W_{t+n})$ is the expectation of the present discounted value of the utility stream that begins in period $t+n$. Taking first-order conditions, I obtain the following proposition about the Euler condition of the commitment case. The proposition is the Euler condition with the general utility form $U(.)$ and $V(.)$ and the general endowment process.

PROPOSITION 2: If a committed agent has time-inconsistent preferences with current and the next one period's utility $U(.)$ and all future periods' utility $V(.)$, then the Euler conditions are

$$
\begin{aligned}
\beta E_t \left[\frac{U'(c_{t+1})}{U'(c_t)} R_{t,t+1}\right] &= 1 \\
\beta E_{t+1} \left[\frac{V'(c_{t+2})}{U'(c_{t+1})} R_{t+1,t+2}\right] &= 1 \\
\beta E_{t+n} \left[\frac{V'(c_{t+n+1})}{V'(c_{t+n})} R_{t+n,t+n+1}\right] &= 1 \; for \; n = 2, 3, 4 \ldots \qquad (3)
\end{aligned}
$$

where $R_{t,t+1}$ denotes the return on asset from t to $t+1$.

Again, the second-period pricing kernel of the commitment case can be rewritten as the product of two parts: the pricing kernel of the standard time-consistent case and the time-inconsistent component:

$$m^C_{t+1,t+2} = \beta \frac{V'(c_{t+2})}{U'(c_{t+1})} = \beta \frac{U'(c_{t+2})}{U'(c_{t+1})} \frac{V'(c_{t+2})}{U'(c_{t+2})} = m^B_{t+1,t+2} \frac{V'(c_{t+2})}{U'(c_{t+2})}$$

where $m^C_{t+1,t+2}$ and $m^B_{t+1,t+2}$ denote the second period pricing kernel of the commitment case and the benchmark time-consistent case, respectively. The time-inconsistent component measures the valuation difference on c_{t+2}between time-t and time-$t+1$ selves.

$$\frac{V'(c_{t+2})}{U'(c_{t+2})} = c_{t+2}^{\gamma_s - \gamma_l} = c_{t+1}^{\gamma_s - \gamma_l} g^{\gamma_s - \gamma_l}$$

When short-run risk aversion is lower than long-run risk aversion, this term is smaller than 1 and is decreasing with the consumption level at time t, which indicates that the larger the consumption level at time t is, the greater the valuation difference between time-t and time-$t+1$ selves becomes. This reduces the payoff volatility of the long-maturity dividend strips at time $t+1$ while the payoff of the first dividend strip is exogenously given and therefore not affected by the time-inconsistent preferences. Thus, when the agent is less averse to immediate risk, it produces a downward-sloping term structure of dividend strips.

From time $t+n$ onwards, the pricing kernels of the commitment case are the same as those for the time-consistent case with the long-run utility $V(.)$. Therefore, all the long-maturity dividend strips with $t > 2$ will be priced similarly to the second dividend strip, and the term structure of risk premia and return volatilities will be flat from $t = 2$ onwards. The pricing kernels in the first two periods are the same as those in the two-period model. From Proposition 1, we know that the first dividend strip will have a higher risk premium and return volatility than the second dividend strip if $\gamma_l > \gamma_s$, and vice versa. The result is stated in the following proposition.

PROPOSITION 3: For a committed agent with short-run risk aversion γ_s and long-run risk aversion γ_l, there are three different scenarios for the term structure of risk premia and return volatilities in a T-period binomial-tree model with an i.i.d. growth rate:

(a) if $\gamma_s < \gamma_l$, then $E\left[R^{(1)}\right] - R_f > E\left[R^{(2)}\right] - R_f = E\left[R^{(3)}\right] - R_f = \ldots = E\left[R^{(T)}\right] - R_f$, $\sigma\left(R^{(1)}\right) > \sigma\left(R^{(2)}\right) = \sigma\left(R^{(3)}\right) = \ldots = \sigma\left(R^{(T)}\right)$. The term structures of risk premia and return volatilities are first downward-sloping and then flat.

(b) if $\gamma_s > \gamma_l$, then $E\left[R^{(1)}\right] - R_f < E\left[R^{(2)}\right] - R_f = E\left[R^{(3)}\right] - R_f = \ldots = E\left[R^{(T)}\right] - R_f$, $\sigma\left(R^{(1)}\right) < \sigma\left(R^{(2)}\right) = \sigma\left(R^{(3)}\right) = \ldots = \sigma\left(R^{(T)}\right)$. The term structures of risk premia and return volatilities are first upward-sloping and then flat.

(c) if $\gamma_s = \gamma_l$, then $E\left[R^{(1)}\right] - R_f = E\left[R^{(2)}\right] - R_f = \ldots = E\left[R^{(T)}\right] - R_f$, $\sigma\left(R^{(1)}\right) = \sigma\left(R^{(2)}\right) = \ldots = \sigma\left(R^{(T)}\right)$. The term structures of risk premia and return volatilities are flat.

The main result obtained from the two-period model still holds with $T > 2$. When $\gamma_l = \gamma_s$, it corresponds to the standard time-consistent case, which generates a flat term structure. When $\gamma_l \neq \gamma_s$, the term structure is non-flat. Because there are only two different risk-aversion levels in my model, all of the long-term dividends have the same risk premia and return volatilities as the second dividend strip. When the agent is less averse to immediate risk, the term structures of risk premia and return volatilities on dividend strips are first downward-sloping and then flat. When the agent is more averse to immediate risk, the term structures are first upward-sloping and then flat. To generate a continuously downward-sloping term structure in the commitment case, the further away the risk is, the more the agent must be risk-averse.

4 The naive agent case

In the naive agent case, the current self constructs the sequence of actions that maximizes her own preferences. The current self then implements the first action in that sequence, expecting future selves to implement the remaining ones. However, those future selves conduct their own optimization and therefore implement actions that do not maximize earlier selves' preferences. In other words, the naive agent constantly wants to commit her future selves but then constantly fails. In solving the two-period model, I must construct a new optimization problem at every node.

4.1 Two-period model

The optimization problem of the naive agent at $t = 0$ is the same as that in the commitment case; thus, the Lagrangian is

$$\begin{aligned} L_0 \quad = \quad & U\left(c_0\right) + \frac{1}{2}\beta U\left(c_u\right) + \frac{1}{2}\beta U\left(c_d\right) + \frac{1}{4}\beta^2 V\left(c_{uu}\right) + \frac{1}{4}\beta^2 V\left(c_{ud}\right) + \frac{1}{4}\beta^2 V\left(c_{du}\right) \\ & + \frac{1}{4}\beta^2 V\left(c_{dd}\right) + \lambda_0\left(W_0 - c_0 - \phi_u W_u - \phi_d W_d\right) + \lambda_u(W_u - c_u - \phi_{uu} W_{uu} \\ & - \phi_{ud} W_{ud}) + \lambda_d\left(W_d - c_d - \phi_{du} W_{du} + \phi_{dd} W_{dd}\right) \end{aligned}$$

where $\{c_0, W_u, W_d, c_u, W_{uu}, W_{ud}, c_d, W_{du}, W_{dd}\}$ are decision variables and $\{\lambda_0, \lambda_u, \lambda_d\}$ are Lagrangian multipliers. The solution is the same as that in the commitment case. However, the naive agent only implements her plan for time-0's decision

$\{c_0, W_u, W_d\}$. Therefore, only the first period's state price will be realized.

$$\phi_u = \frac{1}{2}\beta\frac{U'(c_u)}{U'(c_0)},\ \phi_d = \frac{1}{2}\beta\frac{U'(c_d)}{U'(c_0)}$$

At $t = 1$, the naive agent constructs a new optimization according to her new preferences. At the node-u, the Lagrangian is

$$L_u = U(c_u) + \frac{1}{2}\beta U(c_{uu}) + \frac{1}{2}\beta U(c_{ud}) + \mu_u (W_u - c_u - \phi_{uu}W_{uu} - \phi_{ud}W_{ud})$$

where $\{c_u, W_{uu}, W_{ud}\}$ are decision variables and μ_u is Lagrangian multiplier. Taking first-order conditions and combining with the market clearing condition, I obtain the second period's state prices.

$$\phi_{uu} = \frac{1}{2}\beta\frac{U'(c_{uu})}{U'(c_u)},\ \phi_{ud} = \frac{1}{2}\beta\frac{U'(c_{ud})}{U'(c_u)}$$

Similarly, for node-d, the second period's state prices are

$$\phi_{du} = \frac{1}{2}\beta\frac{U'(c_{du})}{U'(c_d)},\ \phi_{dd} = \frac{1}{2}\beta\frac{U'(c_{dd})}{U'(c_d)}$$

The state prices in the naive agent case are observationally equivalent to those in the time-consistent case with constant short-run risk aversion γ_s. The naive agent plans as a committed agent, but only the consumption and portfolio choice of the current period are implemented. Therefore, the state prices in the naive agent case should be equal to the first period's state prices of the commitment case repeated for every period. The first period's state prices in the commitment case are identical to those in the time-consistent case because only the short-run utility is involved. Therefore, the naive agent acts like a time-consistent agent, and the equilibrium prices are also the same. It follows that the term structure of the equity premia is flat, as in the standard time-consistent case with the i.i.d. growth assumption.

PROPOSITION 4: For a naive agent with short-run risk aversion γ_s and long-run risk aversion γ_l, the term structure of expected returns and return volatilities are both flat in a two-period binomial-tree model with an i.i.d. growth rate. In other words, the first dividend strip and the second dividend strip have identical risk premia and return volatilities.

$$\begin{aligned} E\left[R^{(1)}\right] - R_f &= E\left[R^{(2)}\right] - R_f \\ \sigma\left(R^{(1)}\right) &= \sigma\left(R^{(2)}\right) \end{aligned}$$

4.2 Multi-period model

I first derive the Euler condition of the naive agent case using a recursive method and then examine the term structure of dividend strips. In a T-period model, the optimization of a naive agent at time t is

$$Z_t\left(W_t\right) \triangleq Max_{\{c_t,\{\theta_t\}\}}\, U\left(c_t\right) + \beta E_t\left[\hat{F}_{t+1}\left(W_{t+1}\right)\right]$$

subject to

$$\begin{aligned} W_t &= c_t + \sum\nolimits_{i=1}^{N} \theta_{t,i} S_{t,i} \\ W_{t+1} &= \sum_{i=1}^{N} \theta_{t,i}\left(S_{t+1,i} + \delta_{t+1,i}\right) \end{aligned}$$

where $\hat{F}_{t+1}\left(W_{t+1}\right)$ is self t's perception of the future continuation value function. From time-t self's perspective, this continuation value function should be identical to that in the commitment case. Time-t self believes that all future selves would choose consumption and portfolios such that the continuation value function $\hat{F}_{t+1}(W_{t+1})$ is maximized.

$$\hat{F}_{t+1}(W_{t+1}) = Max_{\{\hat{c}_{t+1},\{\hat{\theta}_{t+1}\}\}}\, u(c_t) + \beta E_t\left[\hat{F}_{t+2}(W_{t+1})\right]$$

The hat symbol that is used in $\hat{F}_{t+1}, \hat{c}_{t+1}$ and $\left\{\hat{\theta}_{t+1}\right\}$ indicates that this is the value that the naive agent at time t expects to be at time $t+1$ but will in fact not be realized by time-$t+1$ self. By the Envelope Theorem,

$$\hat{F}'_{t+1}(W_{t+1}) = U'(c_{t+1})$$

The following proposition provides the Euler condition for the naive agent case, which is derived using the general utility form and the general endowment process.

PROPOSITION 5: If a naive agent has time-inconsistent preferences with the current and next period's utility $U(.)$ and all future periods' utility $V(.)$, then the Euler condition between t and $t+1$ is

$$\beta E_t\left[\frac{U'(c_{t+1})}{U'(c_t)}R_{t,t+1}\right] = 1\ for\ t = 0, 1, 2\ldots \tag{4}$$

where $R_{t,t+1}$ denotes the return on asset from t to $t+1$.

The equilibrium state prices in the naive agent case remain the same as those in the time-consistent case with multi-period models. Therefore, the term structures of dividend strips are flat with the T-period model, as stated in the following proposition.

PROPOSITION 6: For a naive agent with short-run risk aversion γ_s and long-run risk aversion γ_l, the term structures of the expected returns and return volatilities are both flat in a T-period binomial-tree model with an i.i.d. growth rate.

It can be observed that just in the two-period model, the one-period state prices in the naive agent case remain the same as in the time-consistent case with a constant short-run utility function, and the equilibrium of the naive agent case is identical to that of the time-consistent case. The main reasoning is as follows. In every period, the naive agent would behave just like a committed agent, expecting future selves to commit. However, in the naive agent case, future selves fail to commit. The naive agent only implements her plan for one period, and she will move to the next period and make a new plan on the tree. Essentially, the manner in which the state prices are set in the naive agent case is similar to taking the first period's state prices of the commitment case and repeating them for every period. Therefore, only the short-run utility function matters, and the long-run utility function $V(.)$ does not enter into the one-period state price. The naive agent is myopic in the sense that she does not know that her plan of the future will never be implemented and she does not learn from her mistakes. In every period, the agent has irrational expectations for her future decisions, and she makes the same mistake again and again.

5 The sophisticated agent case

The sophisticated agent will take into consideration the way future selves react to today's choice. From a game-theoretic point of view, we can consider this intertemporal decision problem as a sequential game in which the decisions at different nodes are taken by different selves of the decision maker sequentially. Time t-self will take into account the reaction of all time t'-selves, for $t' > t$. Time t-self is the leader, and she looks for the best plan that will be coordinated by all future selves as followers. Therefore, the solution should be constructed by looking for the subgame perfect equilibrium of the corresponding sequential game. Note that even though we consider the agent to be strategic in making her own intertemporal consumption and portfolio choices, she is one of many in the financial market and remains to be competitive.

5.1 Three-period model

For the sophisticated agent case, I begin with $T = 3$ because in a two-period model, the agent only has the short-run utility $U(.)$ in her optimization at $t = 1$ and will act like a time-consistent agent in the second period. The second period's state price will be the same as that in the time-consistent case, and the term structure will be flat according to Lemma 2.

To determine the subgame perfect equilibrium, I solve by backward induction. At $t = 2$, the optimization problem of the agent at the node-uu is

$$Max \quad U(c_{uu}) + \frac{1}{2}\beta U(c_{uuu}) + \frac{1}{2}\beta U(c_{uud})$$

subject to the budget constraint

$$W_{uu} = c_{uu} + \phi_{uuu} W_{uuu} + \phi_{uud} W_{uud}$$

Thus, the Lagrangian at the node-uu is

$$L_{uu} = U(c_{uu}) + \frac{1}{2}\beta U(c_{uuu}) + \frac{1}{2}\beta U(c_{uud}) + \lambda_{uu}(W_{uu} - c_{uu} - \phi_{uuu}c_{uuu} - \phi_{uud}c_{uud})$$

where $\{c_{uu}, c_{uuu}, c_{uud}\}$ are decision variables and λ_{uu} is a Lagrangian multiplier. Taking first-order conditions, I have the third-period state prices.

$$\phi_{uuu} = \frac{1}{2}\beta\frac{U'(c_{uuu})}{U'(c_{uu})}, \ \phi_{uud} = \frac{1}{2}\beta\frac{U'(c_{uud})}{U'(c_{uu})}$$

Similarly, I can obtain the state prices at node-ud, node-du, and node-dd. The last period's state prices are the same as those in the time-consistent case because there is only short-run utility involved.

Now, moving back one period to $t = 1$, the optimization problem at the node-u is

$$\begin{aligned} Max \quad & U(c_u) + \frac{1}{2}\beta U(c_{uu}(W_{uu})) + \frac{1}{2}\beta U(c_{ud}(W_{ud})) + \frac{1}{4}\beta^2 V(c_{uuu}(W_{uu})) + \\ & \frac{1}{4}\beta^2 V(c_{uud}(W_{uu})) + \frac{1}{4}\beta^2 V(c_{udu}(W_{ud})) + \frac{1}{4}\beta^2 V(c_{udd}(W_{ud})) \end{aligned}$$

subject to the budget constraint

$$W_u = c_u + \phi_{uu} W_{uu} + \phi_{ud} W_{ud}$$

The Lagrangian at the node-u is

$$\begin{aligned} L_u = \; & U(c_u) + \frac{1}{2}\beta U(c_{uu}(W_{uu})) + \frac{1}{2}\beta U(c_{ud}(W_{ud})) + \frac{1}{4}\beta^2 V(c_{uuu}(W_{uu})) + \\ & \frac{1}{4}\beta^2 V(c_{uud}(W_{uu})) + \frac{1}{4}\beta^2 V(c_{udu}(W_{ud})) + \frac{1}{4}\beta^2 V(c_{udd}(W_{ud})) + \\ & \lambda_u (W_u - c_u - \phi_{uu} W_{uu} - \phi_{ud} W_{ud}) \end{aligned}$$

where $\{c_u, W_{uu}, W_{ud}\}$ are decision variables and λ_u is a Lagrangian multiplier. $c_{uu}(W_{uu}), c_{ud}(W_{ud}), c_{uuu}(W_{uu}), c_{uud}(W_{uu}), c_{udu}(W_{ud}), c_{udd}(W_{ud})$ are the decision rules of the time-2 self solved at $t = 2$ optimization. As the leader, the time-1 self takes into consideration how her choice in the next period's wealth will affect time-2 self's optimal choices, and substitutes the reaction functions of the followers into her own objective function. Similarly, I can write the Lagrangian at the node-d. Taking the first order conditions,

$$\phi_{uu} = \frac{1}{2}\beta\frac{U'(c_{uu})}{U'(c_u)}c'_{uu}(W_{uu})+\frac{1}{4}\beta^2\frac{V'(c_{uuu})}{U'(c_u)}c'_{uuu}(W_{uu})+\frac{1}{4}\beta^2\frac{V'(c_{uud})}{U'(c_u)}c'_{uud}(W_{uu}) \tag{5}$$

$$\phi_{ud} = \frac{1}{2}\beta\frac{U'(c_{ud})}{U'(c_u)}c'_{ud}(W_{ud})+\frac{1}{4}\beta^2\frac{V'(c_{udu})}{U'(c_u)}c'_{udu}(W_{ud})+\frac{1}{4}\beta^2\frac{V'(c_{udd})}{U'(c_u)}c'_{udd}(W_{ud}) \tag{6}$$

$$\phi_{du} = \frac{1}{2}\beta\frac{U'(c_{du})}{U'(c_d)}c'_{du}(W_{du})+\frac{1}{4}\beta^2\frac{V'(c_{duu})}{U'(c_d)}c'_{duu}(W_{du})+\frac{1}{4}\beta^2\frac{V'(c_{dud})}{U'(c_d)}c'_{dud}(W_{du}) \tag{7}$$

$$\phi_{dd} = \frac{1}{2}\beta\frac{U'(c_{dd})}{U'(c_d)}c'_{dd}(W_{dd})+\frac{1}{4}\beta^2\frac{V'(c_{ddu})}{U'(c_d)}c'_{ddu}(W_{dd})+\frac{1}{4}\beta^2\frac{V'(c_{ddd})}{U'(c_d)}c'_{ddd}(W_{dd}) \tag{8}$$

At $t = 1$, time inconsistency and the agent's sophistication begin to play a role. The state prices between $t = 1$ and $t = 2$ in the sophisticated agent case contain not only the one-period marginal rate of substitution (MRS) between $t = 1$ and $t = 2$ but also the two-period MRS between $t = 1$ and $t = 3$. Additionally, there are derivative terms, which is a measure of how future consumption choices will be affected by today's choice on the next period's wealth. Compared with two other cases, only in the sophisticated agent case do all of the future consumptions appear in the one-period Euler condition, which represents the strategic component that comes into play.

Now, I proceed to the equilibrium analysis of the term structure of dividend strips. Because the state price in the last period is the same as that in the time-consistent case and only depends on the growth rate of the endowment, the third dividend strip has a payoff that is proportional to the payoff of the second dividend strip. Therefore, they have the same risk premium and return volatility. The term structure between the first dividend strip and the second dividend strip depends on the second-period state prices. I show that from the first-order conditions of the $t = 1$ and $t = 2$ optimization, these derivative terms of consumption with respect to wealth are independent of the wealth level and equal among all states, i.e.,

$$\begin{aligned} c'_{uu}(W_{uu}) &= c'_{ud}(W_{ud}) = c'_{du}(W_{du}) = c'_{dd}(W_{dd}) \\ c'_{uuu}(W_{uu}) &= c'_{udu}(W_{ud}) = c'_{duu}(W_{du}) = c'_{ddu}(W_{dd}) \\ c'_{uud}(W_{uu}) &= c'_{udd}(W_{ud}) = c'_{dud}(W_{du}) = c'_{ddd}(W_{dd}) \end{aligned}$$

The future selves will react the same way to time-1 self's decision on the wealth level irrespective of what wealth level time-1 self chooses. Thus, the one-period MRS between $t = 1$ and $t = 2$, $\frac{U'(c_2)}{U'(c_1)}$, is not state-dependent and the same across all states with power utility and an i.i.d. growth rate. The only term left to compare is the two-period MRS between $t = 1$ and $t = 3$, which is $\frac{V'(c_3)}{U'(c_1)}$. This term will be either increasing or decreasing in c_1 depending on the relationship between γ_s and γ_l. When $\gamma_s < \gamma_l$, $\frac{V'(c_3)}{U'(c_1)}$ is decreasing in c_1, and $\phi_{uu} < \phi_{du}$, $\phi_{ud} < \phi_{dd}$. By Lemma 2, the first dividend strip has a higher risk premium and higher return volatility than the second dividend strip, and the same argument applies to the opposite case. A detailed proof is presented in the Appendix.

PROPOSITION 7: For a sophisticated agent with short-run risk aversion γ_s and long-run risk aversion γ_l, there are three different scenarios for the term structure of risk premia and return volatilities in a three-period binomial-tree model with an i.i.d. growth rate:

(a) if $\gamma_s < \gamma_l$, then $E\left[R^{(1)}\right] - R_f > E\left[R^{(2)}\right] - R_f = E\left[R^{(3)}\right] - R_f$, $\sigma\left(R^{(1)}\right) > \sigma\left(R^{(2)}\right) = \sigma\left(R^{(3)}\right)$.

(b) if $\gamma_s > \gamma_l$, then $E\left[R^{(1)}\right] - R_f < E\left[R^{(2)}\right] - R_f = E\left[R^{(3)}\right] - R_f$, $\sigma\left(R^{(1)}\right) < \sigma\left(R^{(2)}\right) = \sigma\left(R^{(3)}\right)$.

(c) if $\gamma_s = \gamma_l$, then $E\left[R^{(1)}\right] - R_f = E\left[R^{(2)}\right] - R_f = E\left[R^{(3)}\right] - R_f$, $\sigma\left(R^{(1)}\right) = \sigma\left(R^{(2)}\right) = \sigma\left(R^{(3)}\right)$.

Because there is no dynamic inconsistency in the last period, we focus on the relationship between the first two dividend strips. The sophisticated agent case shows a pattern similar to that of the commitment case: when the agent is less averse to immediate risk, the short-maturity dividend strip has both a higher risk premium and volatility than the long-maturity dividend strip. The mechanism is also similar to that in the commitment case. Because the state prices for calculating one-period returns are the same, the risk premium depends on the payoff structure of the two assets at $t = 1$. The one-period state prices become state-dependent in the sophisticated agent case, such that the long-maturity dividend strip has a different payoff structure at $t = 1$ from the short-maturity dividend. When the agent is less risk averse toward immediate risk, the next one-period consumption is valued lower if the current state is the good state than if the current state is the bad state. The long-maturity dividend becomes less risky than the short-maturity dividend, which creates a downward-sloping term structure. In the opposite case when the agent is more averse to immediate risk, the term structures of risk premia and return volatilities are both upward-sloping.

5.2 Euler condition derived from the multi-period model

Because it is a special case for $T = 3$ that the derivative terms in the second-period state price are equal and independent of wealth, I aim to determine whether the result for the term structure still holds when $T > 3$. In fact, the state prices in the three-period model can be generalized to the multi-period model with $T > 3$. I first derive the Euler condition of the T-period model with a recursive method. This part of the model is solved with the general utility function form and the general endowment process. The optimization problem of a sophisticated agent is

$$Max\, U(c_t) + E_t\left[\beta U(c_{t+1}) + \beta^2 V(c_{t+2}) + \cdots + \beta^{T-t} V(c_T)\right]$$

subject to

$$c_t + \sum\nolimits_{i=1}^{N} \theta_{t,i} S_{t,i} = \sum_{i=1}^{N} \theta_{t-1,i}\left(S_{t,i} + \delta_{t,i}\right)$$

Define the value function $Z_t(W_t)$ as the maximum value of the agent's life time utility. The optimization problem becomes

$$Z_t(W_t) \triangleq Max_{c_t,\{\theta_t\}}\, U(c_t) + \beta E_t\left[F_{t+1}(W_{t+1})\right]$$

subject to

$$\begin{aligned} W_t &= c_t + \sum\nolimits_{i=1}^{N} \theta_{t,i} S_{t,i} \\ W_{t+1} &= \sum_{i=1}^{N} \theta_{t,i}\left(S_{t+1,i} + \delta_{t+1,i}\right) \end{aligned}$$

where $F_{t+1}(W_{t+1})$ is the continuation value function at time $t+1$.

$$F_{t+1}(W_{t+1}) = U(c_{t+1}) + E_{t+1}\left[\beta V(c_{t+2}) + \beta^2 V(c_{t+3}) + \cdots + \beta^{T-t-1} V(c_T)\right] \tag{9}$$

Take first-order conditions,

$$E_t\left[\beta \frac{F'_{t+1}(W_{t+1})}{U'(c_t)} \frac{S_{t+1} + \delta_{t+1}}{S_t}\right] = 1 \tag{10}$$

So the pricing kernel between t and $t+1$ is

$$m^S_{t,t+1} = \beta \frac{F'_{t+1}(W_{t+1})}{U'(c_t)} = \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{F'_{t+1}(W_{t+1})}{Z'_{t+1}(W_{t+1})} = m^B_{t,t+1} \frac{F'_{t+1}(W_{t+1})}{Z'_{t+1}(W_{t+1})} \tag{11}$$

where $m^S_{t,t+1}$ and $m^B_{t,t+1}$ denote the pricing kernel of the sophisticated agent case and the time-consistent case respectively. Equation (11) holds, because by Envelope Theorem,

$$Z'_t(W_t) = U'(c_t)$$

Just like the commitment case, the pricing kernel of the sophisticated agent case is equal to the pricing kernel of the standard time-consistent case ($m^B_{t,t+1}$) multiplied by a time-inconsistent component ($\frac{F'_{t+1}(W_{t+1})}{Z'_{t+1}(W_{t+1})}$). The time-inconsistent component measures the valuation difference on the consumption stream from c_{t+1} onwards between time-t self and time-$t+1$ self. Compare F_{t+1} in (9) with Z_{t+1}, the two selves disagree only on the valuation of c_{t+2}.

$$Z_{t+1}(W_{t+1}) = U(c_{t+1}) + E_{t+1}\left[\beta U(c_{t+2}) + \beta^2 V(c_{t+3}) + \cdots + \beta^{T-t-1} V(c_T)\right]$$

If the agent has a higher risk aversion for the long run risk than the short run risk, then $F'_{t+1} < Z'_{t+1}$ so, $m^S_{t,t+1} < m^B_{t,t+1}$. And the pricing kernel of the sophisticated agent case will again be decreasing with the consumption level at time t , as in the commitment case. This will decrease the price volatility of the long-maturity dividend strips, making it less risky than the first dividend strip and thus creating a downward-sloping term structure of dividend strips.

By substituting (9) into (10), I obtain the following proposition for the Euler condition of the sophisticated agent case.

PROPOSITION 8: If a sophisticated agent has time-inconsistent preferences with current utility and next period's utility $U(.)$ and all future periods' utility $V(.)$, then the Euler condition between t and $t+1$ is

$$E_t[(\beta \frac{U'(c_{t+1})}{U'(c_t)} c'_{t+1}(W_{t+1}) + \beta^2 \frac{V'(c_{t+2})}{U'(c_t)} c'_{t+2}(W_{t+1}) + \cdots$$
$$+ \beta^{T-t} \frac{V'(c_T)}{U'(c_t)} c'_T(W_{t+1})) R_{t,t+1}] = 1 \qquad (12)$$

where where $R_{t,t+1}$ denotes the return on asset from t to $t+1$.

Similar to the three-period model, the one-period Euler condition of the sophisticated agent case between t and $t+1$ contains not only the MRS between c_t and c_{t+1}, but also the MRS between c_t and all future consumptions c_{t+n}. There are also derivative terms of future consumptions on tomorrow's wealth $c'_{t+n}(W_{t+1})$, which is a measure of how future selves will react to today's choice regarding W_{t+1}. The sophisticated agent is aware that by giving up consumption today, she can increase tomorrow's wealth, which will affect all future selves' consumption decisions. The equilibrium state prices are such that the utility of consuming one unit less today is equal to the aggregate utility of all future periods' increase in consumption. As in the three-period model, the state prices will be state-dependent, which will lead to a non-flat term structure of dividend strips.

The derivative terms $c'_{t+n}(W_{t+1})$ in the Euler condition are difficult to obtain analytically; therefore, I resort to the numerical method. To simplify the numerical process, I first simplify the Euler condition using the Envelope Theorem.

$$Z'_t(W_t) = U'(c_t)$$

The relation between F_t and Z_t is

$$F_t = Z_t - \beta E_t[U(c_{t+1}) - V(c_{t+1})]$$

$$
\begin{aligned}
F'_{t+1}\left(W_{t+1}\right) &= Z'_{t+1}\left(W_{t+1}\right) - \beta E_{t+1}\left[\left(U'\left(c_{t+2}\right) - V'\left(c_{t+2}\right)\right) c'_{t+2}\left(W_{t+1}\right)\right] \\
&= U'\left(c_{t+1}\right) - \beta E_{t+1}\left[\left(U'\left(c_{t+2}\right) - V'\left(c_{t+2}\right)\right) c'_{t+2}\left(W_{t+1}\right)\right]
\end{aligned}
$$

By substituting the relation into the first-order condition, I obtain the simplified form of the Euler condition with the recursive method.

$$
\beta E_t \left[\left(\frac{U'\left(c_{t+1}\right)}{U'\left(c_t\right)} - \frac{\beta E_{t+1}\left[\left(U'\left(c_{t+2}\right) - V'\left(c_{t+2}\right)\right) c'_{t+2}\left(W_{t+1}\right)\right]}{U'\left(c_t\right)} \right) R_{t,t+1} \right] = 1 \quad (13)
$$

Equation (13) is equivalent to the Euler condition (12) in Proposition 8. In this equation, there are two parts in the pricing kernel. The first part is the time-consistent part. The second part is the time-inconsistency part, and the numerator only contains c_{t+2}. This result is obtained because the time-t agent rationally expects that time-$t+1$ self will have a different valuation relative to that of time-t self with respect to c_{t+2} only. When the long-run utility $V(.)$ is identical to the short-run utility $U(.)$, the Euler condition of the sophisticated agent case collapses to the standard time-consistent case with only $U(.)$. I will use equation (13) for the following numerical analysis.

5.3 Numerical results

In this section, instead of comparing the dividend strips with different maturities, I compare the risk premia and return volatilities between the stock and the first dividend strip, following van Binsbergen et al. (2012). I assume the aggregate endowment follows a binomial-tree process in which the growth rate of

Table 1: Choice of Parameter Values for the Model

parameter	value
e_0	1
π_u	0.5
π_d	0.5
u	1.06
d	0.97
β	0.98
T	5

the endowment is i.i.d.. The tree is assumed to be recombining. The parameter values I use are listed below:

I assume the growth rate of endowment is i.i.d., with a probability $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ to go to the up-node and the down-node. The magnitude of the gross rate of growth is $u = 1.06$ for the up-node and $d = 0.97$ for the down-node. The mean of the growth rate of endowment is therefore 1.5%, and the standard deviation is 4.5%, which is a reasonable assumption for the aggregate economy.[4] I normalize the initial endowment $\delta_0 = 1$. The time-discounting factor is $\beta = 0.98$. There are five periods in total. The model still holds with more periods. Here, I use the $T = 5$ case for a simple illustration. For the preference parameters, it can be observed that when the long-run utility is the same as the short-run utility, the second part of the pricing kernel in equation (10) becomes 0, and the Euler condition is the same as that in the time-consistent case with utility $U(.)$. Therefore, it is natural to set the time-consistent case as the benchmark and vary the long-run risk aversion to observe how it affects the equity term

[4]The historical record of the consumption growth rate from 1889 to 1978 in the U.S is 1.83% and the standard deviation is 3.57% (Mehra and Prescott (1985)).

structure. I choose short-run risk aversion to be fixed at $\gamma_s = 5$ and vary the long-run risk aversion γ_l from 1 to 10. This process provides an overview of how the risk premia and return volatilities of the dividend strip and stock vary as a function of time inconsistency. The numerical result serves only to demonstrate how the term structure changes with the dynamic inconsistency and is not a detailed calibration. The purpose is to demonstrate the direction of the slope of the term structure, but not to explain the equity premium puzzle. The results are as follows:

Table 2: Term Structure of Dividend Strips for the Sophisticated Agent Case.

$T = 5, \gamma_s = 5$		$\gamma_s > \gamma_l$				$\gamma_s = \gamma_l$	$\gamma_s < \gamma_l$				
		$\gamma_l = 1$	$\gamma_l = 2$	$\gamma_l = 3$	$\gamma_l = 4$	$\gamma_l = 5$	$\gamma_l = 6$	$\gamma_l = 7$	$\gamma_l = 8$	$\gamma_l = 9$	$\gamma_l = 10$
risk premium	stock	3.292%	2.670%	2.115%	1.582%	1.043%	0.483%	-0.107%	-0.733%	-1.399%	-0.211%
	strip	0.973%	0.958%	0.971%	1.002%	1.043%	1.091%	1.143%	1.196%	1.249%	1.301%
signed return volatility	stock	16.13%	13.25%	10.36%	7.53%	4.78%	2.12%	-0.45%	-2.95%	-5.40%	-7.81%
	strip	4.77%	4.76%	4.76%	4.77%	4.78%	4.79%	4.80%	4.81%	4.82%	4.82%

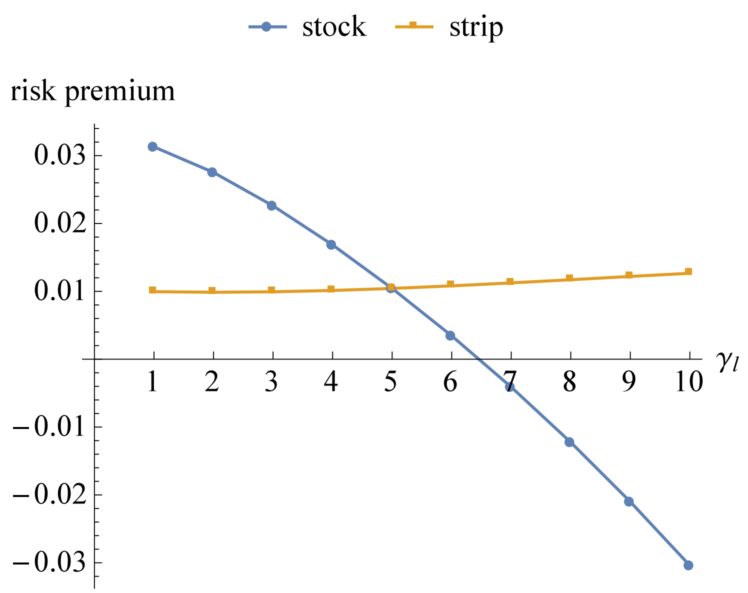

Figure 1: Risk Premia of the Stock and the First Dividend Strip with Different Values of γ_l.

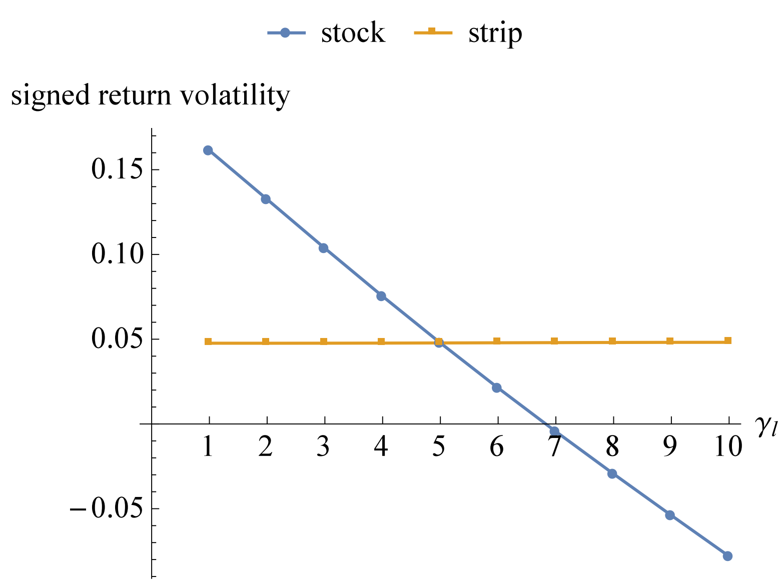

Figure 2: Return Volatilities of the Stock and the First Dividend Strip with Different Values of γ_l.

From the graphs, it can be observed that the general pattern of the three-period model still holds. First, the curves of stock and the first dividend strip intersect at the point at which $\gamma_l = \gamma_s = 5$, which means that in the time-consistent case, the dividend strip has the same risk premium and return volatility as the stock. Second, when $\gamma_l > \gamma_s = 5$, i.e. the agent is less averse to immediate risk, the dividend strip has a higher risk premium and return volatility than the stock, which implies a downward-sloping term structure. Another interesting finding is that the stock can even have a negative risk premium when the time inconsistency is sufficiently large. This negative value is obtained when the stock price at $t = 1$ moves in the opposite direction from the endowment to the extent that it offsets the first dividend payment. The stock pays off more in the bad state than in the good state, and it has a negative risk premium. Third, when $\gamma_l < \gamma_s = 5$, i.e. the agent is more averse to immediate risk, the dividend strip has a lower risk premium and return volatility than the stock, which implies an upward-sloping term structure. Fourth, the slope of the term structure becomes steeper as the degree of the time inconsistency increases (i.e., there is a greater difference between γ_s and γ_l).

The reasoning is still the same. The one-period Euler condition of the sophisticated agent case depends on all future consumptions, and the state prices become dependent on the current state. When the agent is less averse to immediate risk, the state prices decrease with the consumption of the current state. This state dependency changes the payoff structure of the long-maturity dividend strips, making them less risky than the short-maturity ones and thereby creating a downward-sloping term structure of dividend strips.

6 Conclusion

In this paper, I construct a general equilibrium model in which the agent has time-inconsistent risk preferences to study the term structure of dividend strips. I solve the model in three different cases. The naive agent case has the same equilibrium prices as the standard time-consistent case and therefore leads to a flat term structure. In the commitment case and the sophisticated agent case, when the agent is less averse to immediate risk than to future risks, the dividend strip has a higher risk premium and return volatility than the stock, implying a downward-sloping term structure of equity returns. This result is consistent with the empirical findings of van Binsbergen, Brandt and Koijen (2012). I explain the mechanism of my model and show that the main reason why the time inconsistency generates a downward-sloping term structure is that the state prices become state-dependent and, in the case in which the agent is less risk-averse to immediate risk, future cash flows are valued higher when the current state is the bad state than when the current state is the good state. This effect reduces the return volatilities of the long-maturity dividend strips and causes them to be less risky than the short-maturity dividend strips. One limitation of the study is that the elasticity of intertemporal substitution also plays a role with time-additive utility. In future work, I will extend the model to the recursive utility case, in which I can separate risk aversion from the elasticity of intertemporal substitution so that I can obtain a clear view of how risk aversion alone affects the term structure of dividend strips.

Appendix

Proof for Lemma 1:

Combining the F.O.C.s and the market-clearing conditions, I have

$$\begin{aligned}\phi_{uu} &= \frac{\beta}{2}u^{\gamma_s-2\gamma_l}, \phi_{ud} = \frac{\beta}{2}u^{\gamma_s-\gamma_l}d^{-\gamma_l} \\ \phi_{du} &= \frac{\beta}{2}d^{\gamma_s-\gamma_l}u^{-\gamma_l}, \phi_{dd} = \frac{\beta}{2}d^{\gamma_s-2\gamma_l}\end{aligned}$$

So

$$\frac{\phi_{uu}}{\phi_{du}} = \left(\frac{u}{d}\right)^{\gamma_s-\gamma_l} \ \ and \ \frac{\phi_{ud}}{\phi_{dd}} = \left(\frac{u}{d}\right)^{\gamma_s-\gamma_l}$$

$u > 1 > d > 0$, so $\frac{u}{d} > 1$.

If $\gamma_s < \gamma_l$, then $\gamma_s - \gamma_l < 0$, $0 < \frac{\phi_{uu}}{\phi_{du}} < 1$ and $0 < \frac{\phi_{ud}}{\phi_{dd}} < 1$. Therefore, $\phi_{uu} < \phi_{du}$, $\phi_{ud} < \phi_{dd}$.

If $\gamma_s > \gamma_l$, then $\gamma_s - \gamma_l > 0$, $\frac{\phi_{uu}}{\phi_{du}} > 1$ and $\frac{\phi_{ud}}{\phi_{dd}} > 1$. Therefore, $\phi_{uu} > \phi_{du}$, $\phi_{ud} > \phi_{dd}$.

If $\gamma_s = \gamma_l$, then $\gamma_s - \gamma_l = 0$, $\frac{\phi_{uu}}{\phi_{du}} = 1$ and $\frac{\phi_{ud}}{\phi_{dd}} = 1$. Therefore, $\phi_{uu} = \phi_{du}$, $\phi_{ud} = \phi_{dd}$.

Proof for Lemma 2:

The first-period expected return and the return volatility of the first dividend strip is

$$\begin{aligned}E\left[R^{(1)}\right] &= \frac{\frac{1}{2}u + \frac{1}{2}d}{\phi_u u + \phi_d d} \\ \sigma\left(R^{(1)}\right) &= \frac{\frac{1}{2}u - \frac{1}{2}d}{\phi_u u + \phi_d d}\end{aligned}$$

The first-period expected return and the return volatility of the second dividend strip are

$$
\begin{aligned}
E\left[R^{(2)}\right] &= \frac{\frac{1}{2}P_u^{(2)} + \frac{1}{2}P_d^{(2)}}{\phi_u P_u^{(2)} + \phi_d P_d^{(2)}} \\
\sigma\left(R^{(2)}\right) &= \frac{\frac{1}{2}P_u^{(2)} - \frac{1}{2}P_d^{(2)}}{\phi_u P_u^{(2)} + \phi_d P_d^{(2)}}
\end{aligned}
$$

where

$$
\begin{aligned}
P_u^{(2)} &= \phi_{uu}u^2 + \phi_{ud}ud = u\left(\phi_{uu}u + \phi_{ud}d\right) \\
P_d^{(2)} &= \phi_{du}ud + \phi_{dd}d^2 = d\left(\phi_{du}u + \phi_{dd}d\right)
\end{aligned}
$$

If $\phi_{uu} = \phi_{du}$, $\phi_{ud} = \phi_{dd}$, then $\phi_{uu}u + \phi_{ud}d = \phi_{du}u + \phi_{dd}d$, and $\frac{P_u^{(2)}}{P_d^{(2)}} = \frac{u}{d}$. The payoff of the second dividend strip at $t = 1$ is proportional to that of the first dividend strip in both two states. Let $P_u^{(2)} = ku$, and $P_d^{(2)} = kd$ for some constant k. Therefore,

$$
\begin{aligned}
E\left[R^{(2)}\right] &= \frac{\frac{1}{2}ku + \frac{1}{2}kd}{\phi_u ku + \phi_d kd} = \frac{\frac{1}{2}u + \frac{1}{2}d}{\phi_u u + \phi_d d} = E\left[R^{(1)}\right] \\
\sigma\left(R^{(2)}\right) &= \frac{\frac{1}{2}ku - \frac{1}{2}kd}{\phi_u ku + \phi_d kd} = \frac{\frac{1}{2}u - \frac{1}{2}d}{\phi_u u + \phi_d d} = \sigma\left(R^{(1)}\right)
\end{aligned}
$$

If $\phi_{uu} < \phi_{du}$, $\phi_{ud} < \phi_{dd}$, then $\phi_{uu}u + \phi_{ud}d < \phi_{du}u + \phi_{dd}d$, and $\frac{P_u^{(2)}}{P_d^{(2)}} < \frac{u}{d}$. The second dividend strip has a smaller payoff volatility at $t = 1$ than the first dividend strip.

$$
\begin{aligned}
E\left[R^{(2)}\right] &= \frac{\frac{1}{2}P_u^{(2)}/P_d^{(2)}+\frac{1}{2}}{\phi_u P_u^{(2)}/P_d^{(2)}+\phi_d} \\
&= \frac{\frac{1/2}{\phi_u}\left(\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d\right)+\frac{1}{2}-\frac{1}{2}\frac{\phi_d}{\phi_u}}{\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d} \\
&= \frac{1/2}{\phi_u}-\frac{1}{2}\frac{\frac{\phi_d}{\phi_u}-1}{\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d} \\
&< \frac{1/2}{\phi_u}-\frac{1}{2}\frac{\frac{\phi_d}{\phi_u}-1}{\phi_u(\frac{u}{d})+\phi_d} = E\left[R^{(1)}\right]
\end{aligned}
$$

Thus, the risk premium $E\left[R^{(2)}\right]-R_f < E\left[R^{(1)}\right]-R_f$.

$$
\begin{aligned}
\sigma\left(R^{(2)}\right) &= \frac{\frac{1}{2}P_u^{(2)}/P_d^{(2)}-\frac{1}{2}}{\phi_u P_u^{(2)}/P_d^{(2)}+\phi_d} < \frac{1/2}{\phi_u}-\frac{1}{2}\frac{\frac{\phi_d}{\phi_u}-1}{\phi_u(\frac{u}{d})+\phi_d} = E\left[R^{(1)}\right] \\
&= \frac{\frac{1/2}{\phi_u}\left(\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d\right)-\frac{1}{2}-\frac{1}{2}\frac{\phi_d}{\phi_u}}{\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d} \\
&= \frac{1/2}{\phi_u}-\frac{1}{2}\frac{\frac{\phi_d}{\phi_u}+1}{\phi_u \frac{P_u^{(2)}}{P_d^{(2)}}+\phi_d} \\
&< \frac{1/2}{\phi_u}-\frac{1}{2}\frac{\frac{\phi_d}{\phi_u}+1}{\phi_u(\frac{u}{d})+\phi_d} = \sigma\left(R^{(1)}\right)
\end{aligned}
$$

If $\phi_{uu} > \phi_{du}$, $\phi_{ud} > \phi_{dd}$, then $\phi_{uu}u+\phi_{ud}d > \phi_{du}u+\phi_{dd}d$, and $\frac{P_u^{(2)}}{P_d^{(2)}} > \frac{u}{d}$. The second dividend strip has a larger payoff volatility at $t=1$ than the first dividend strip. By the same argument, $E\left[R^{(2)}\right]-R_f > E\left[R^{(1)}\right]-R_f$ and $\sigma\left(R^{(2)}\right) > \sigma\left(R^{(1)}\right)$ in this case.

Proof for Proposition 1:

The conclusion for the risk premium and return volatility directly follows from Lemma 1 and Lemma 2. For the Sharpe ratio,

$$\begin{aligned}
E\left[mR\right] &= 1 \\
E\left[m\right]E\left[R\right] + Cov\left(m,R\right) &= 1 \\
E\left[m\right]E\left[R\right] - 1 &= -\rho_{m,R}\sigma(m)\sigma(R) \\
E\left[R\right] - \frac{1}{E\left[m\right]} &= -\frac{\rho_{m,R}\sigma(m)\sigma(R)}{E\left[m\right]} \\
\frac{E\left[R\right] - R_f}{\sigma(R)} &= -\rho_{m,R}\frac{\sigma(m)}{E\left[m\right]}
\end{aligned}$$

On a binomial-tree with only two states, $\rho_{m,R} = 1$ for all assets. Therefore the Sharpe ratio $SR = \frac{E[R]-R_f}{\sigma(R)} = -\frac{\sigma(m)}{E[m]}$ and is equal across all assets. $SR\left(R^{(1)}\right) = SR\left(R^{(2)}\right)$.

Proof for Proposition 2:

$$\begin{aligned}
Z_t(W_t) &= Max_{\{\theta_t\}}\ U(W_t - \sum_{i=1}^{N}\theta_{t,i}S_{t,i}) + \beta E_t\left[F_{t+1}(\theta_{t,i}(S_{t+1,i} + \delta_{t+1,i}))\right] \\
F_{t+1}(W_{t+1}) &= Max_{\{\theta_{t+1}\}}\ U(W_{t+1} - \sum_{i=1}^{N}\theta_{t+1,i}S_{t+1,i}) + \\
&\quad \beta E_{t+1}\left[M_{t+2}(\theta_{t+1,i}(S_{t+2,i} + \delta_{t+2,i}))\right] \\
M_{t+n}(W_{t+n}) &= Max_{\{\theta_{t+n}\}}\ V(W_{t+n} - \sum_{i=1}^{N}\theta_{t+n,i}S_{t+n,i}) + \\
&\quad \beta E_{t+n}\left[M_{t+n+1}(\theta_{t+n,i}(S_{t+n+1,i} + \delta_{t+n+1,i}))\right]\ \ for\ n = 2,3,4\ldots
\end{aligned}$$

F.O.Cs:

$$\begin{aligned} U'(c_t)(-S_{t,i}) &= \beta E_t\left[F'_{t+1}(W_{t+1})\cdot(S_{t+1,i}+\delta_{t+1,i})\right] \\ U'(c_{t+1})(-S_{t+1,i}) &= \beta E_{t+1}\left[M'_{t+2}(W_{t+2})\cdot(S_{t+2,i}+\delta_{t+2,i})\right] \\ V'(c_{t+n})(-S_{t+n,i}) &= \beta E_{t+n}\left[M'_{t+n+1}(W_{t+n+1})\cdot(S_{t+n+1,i}+\delta_{t+n+1,i})\right] \end{aligned}$$

By the Envelope Theorem,

$$\begin{aligned} F'_{t+1}(W_{t+1}) &= U'(c_{t+1}) \\ M'_{t+n}(W_{t+n}) &= V'(c_{t+n})\ \ for\ n=2,3,4\dots \end{aligned}$$

By substituting this relation into the F.O.Cs, I obtain the Euler condition:

$$\begin{aligned} \beta E_t\left[\frac{U'(c_{t+1})}{U'(c_t)}R_{t,t+1}\right] &= 1 \\ \beta E_{t+1}\left[\frac{V'(c_{t+2})}{U'(c_{t+1})}R_{t+1,t+2}\right] &= 1 \\ \beta E_{t+n}\left[\frac{V'(c_{t+n+1})}{V'(c_{t+n})}R_{t+n,t+n+1}\right] &= 1\ for\ n=2,3,4\dots \end{aligned}$$

Proof for Proposition 3:

I first prove that when $\gamma_l = \gamma_s$, i.e in the time-consistent case, all the dividend strips have the same payoff structure at $t = 1$. In other words,

$$\frac{P_u^{(1)}}{P_d^{(1)}} = \frac{P_u^{(2)}}{P_d^{(2)}} = \dots = \frac{P_u^{(T)}}{P_d^{(T)}} = \frac{u}{d}\ at\ t=1$$

I use proof by induction to show that $\frac{P_u^{(t)}}{P_d^{(t)}} = \frac{u}{d}$ for all t . In the time-consistent case, the state prices are state-independent and universal at all nodes. At every node, I have

$$\phi_u = \frac{1}{2}\beta u^{-\gamma},\ \phi_d = \frac{1}{2}\beta d^{-\gamma}$$

When $t = 1$, $\frac{P_u^{(1)}}{P_d^{(1)}} = \frac{u}{d}$, the conclusion holds.
When $t = 2$, $P_u^{(2)} = \phi_u u^2 + \phi_d ud = u\left(\phi_u u + \phi_d d\right)$, $P_d^{(2)} = \phi_u ud + \phi_d d^2 = d\left(\phi_u u + \phi_d d\right)$, $\frac{P_u^{(2)}}{P_d^{(2)}} = \frac{u}{d}$ also holds.
Suppose that when $t = n$, $\frac{P_u^{(n)}}{P_d^{(n)}} = \frac{u}{d}$ holds. The payoff structure of the n-th dividend strip at $t = n$ in $n+1$ states is $\left\{u^n, u^{n-1}d, u^{n-2}d^2, \ldots, u^2d^{n-2}, ud^{n-1}, d^n\right\}$. The payoff structure of the $(n+1)$-th dividend strip at $t = n$ in $n+1$ states is

$$\left\{\phi_u u^{n+1} + \phi_d u^n d, \phi_u u^n d + \phi_d u^{n-1}d^2, \ldots, \phi_u u^2 d^{n-1} + \phi_d ud^n, \phi_u ud^n + \phi_d d^{n+1}\right\}$$
$$= \left(\phi_u u + \phi_d d\right)\left\{u^n, u^{n-1}d, u^{n-2}d^2, \ldots, u^2d^{n-2}, ud^{n-1}, d^n\right\}$$

Thus, the payoff of the $(n+1)$-th dividend strip at $t = n$ is proportional to that of the n-th dividend strip in all $n+1$ states. One unit of the $(n+1)$-th dividend strip pays $(\phi_u u + \phi_d d)$ times the payoff of the n-th dividend strip. Therefore, the price of the $(n+1)$-th dividend strip at $t = 1$ should also be $(\phi_u u + \phi_d d)$ times the price of the n-th dividend strip.

$$\begin{aligned} P_u^{(n+1)} &= \left(\phi_u u + \phi_d d\right) P_u^{(n)} \\ P_d^{(n+1)} &= \left(\phi_u u + \phi_d d\right) P_d^{(n)} \end{aligned}$$

As a result, $\frac{P_u^{(n+1)}}{P_d^{(n+1)}} = \frac{P_u^{(n)}}{P_d^{(n)}} = \frac{u}{d}$ holds true as well. By induction, $\frac{P_u^{(t)}}{P_d^{(t)}} = \frac{u}{d}$ holds true for all t. Because all dividend strips have the same payoff structure at $t = 1$, they should have the same risk premium and return volatility. Part

(c) of the Proposition 3 is proved.

When $\gamma_l \neq \gamma_s$, the committed agent still acts like a time-consistent one with constant risk aversion γ_l from $t = 2$ onwards. By the same argument, the payoff structure of all the future dividend strips with $t > 2$ should be proportional to that of the second dividend strip. The term structure is flat from $t = 2$ onwards. The comparison of the second dividend strip with the first dividend strip is the same as that in the two-period model. Part (a) and (b) of the proposition prove to be true.

Proof for Proposition 5:

$$Z_t(W_t) = Max_{\{\theta_t\}}\ U(W_t - \sum_{i=1}^{N} \theta_{t,i} S_{t,i}) + \beta E_t \left[\hat{F}_{t+1}(\theta_{t,i}(S_{t+1,i} + \delta_{t+1,i})) \right]$$

F.O.C.

$$U'(c_t)(-S_{t,i}) = \beta E_t \left[\hat{F}'_{t+1}(W_{t+1})(S_{t+1,i} + \delta_{t+1,i}) \right]$$

By the Envelope Theorem,

$$\hat{F}'_t(W_t) = U'(c_t)$$

By substituting the relation into F.O.C., I obtain the Euler condition.

$$\beta E_t \left[\frac{U'(c_{t+1})}{U'(c_t)} R_{t,t+1} \right] = 1$$

Proof for Proposition 7:

The last period's state price of the sophisticated agent case is the same as that in the time-consistent case; therefore, the third dividend strip should have its payoff structure proportional to that of the second dividend strip at $t = 1$. Therefore, the strips have the same risk premium and return volatility.

$$\begin{aligned} \frac{P_u^{(3)}}{P_d^{(3)}} &= \frac{P_u^{(2)}}{P_d^{(2)}} \\ E\left[R^{(2)}\right] &= E\left[R^{(3)}\right] \\ \sigma(R^{(2)}) &= \sigma(R^{(3)}) \end{aligned}$$

To compare the first and the second dividend strips, I must determine how the second period's state prices depend on the state at $t = 1$ by Lemma 2. The second period's state prices are listed in (5)-(8). With isoelastic utility and i.i.d. growth endowment,

$$\begin{aligned} \frac{U'(c_{uu})}{U'(c_u)} &= \frac{U'(c_{du})}{U'(c_d)} = u^{-\gamma_s}, \\ \frac{U'(c_{ud})}{U'(c_u)} &= \frac{U'(c_{dd})}{U'(c_d)} = d^{-\gamma_s} \end{aligned}$$

Because $c_{uu}, c_{ud}, c_{du}, c_{dd}$ are determined by the time-2 self and the optimization contains only short-run utility $U(.)$, the consumption rule should be identical to that in the time-consistent case, with c_t being proportional to W_t, and $\frac{c_t}{W_t}$ does not depend on W_t. Therefore, $c'_{uu}(W_{uu}) = c'_{ud}(W_{ud}) = c'_{du}(W_{du}) = c'_{dd}(W_{dd})$. From the Euler condition between $t = 2$ and $t = 3$,

$$\begin{aligned}
\phi_{uuu} &= \frac{1}{2}\beta\left(\frac{c_{uuu}}{c_{uu}}\right)^{-\gamma_s} \\
c_{uuu} &= \left(\frac{2\phi_{uuu}}{\beta}\right)^{-\gamma_s} c_{uu}, \\
c'_{uuu}(W_{uu}) &= \left(\frac{2\phi_{uuu}}{\beta}\right)^{-\gamma_s} c'_{uu}(W_{uu}),
\end{aligned}$$

similarly to other derivative terms. In equilibrium, $\phi_{uuu} = \phi_{udu} = \phi_{duu} = \phi_{ddu}$, $\phi_{uud} = \phi_{udd} = \phi_{dud} = \phi_{ddd}$. Therefore,

$$\begin{aligned}
c'_{uuu}\left(W_{uu}\right) &= c'_{udu}\left(W_{ud}\right) = c'_{duu}\left(W_{du}\right) = c'_{ddu}\left(W_{dd}\right), \\
c'_{uud}\left(W_{uu}\right) &= c'_{udd}\left(W_{ud}\right) = c'_{dud}\left(W_{du}\right) = c'_{ddd}\left(W_{dd}\right)
\end{aligned}$$

$$\frac{V'(c_3)}{U'(c_1)} = \frac{V'(c_3)}{V'(c_1)}\frac{V'(c_1)}{U'(c_1)} = g_{1,2}^{-\gamma_l} g_{2,3}^{-\gamma_l}\frac{V'(c_1)}{U'(c_1)}$$

where $g_{1,2}$ and $g_{2,3}$ are the growth rates in the first and second period respectively.

$$\begin{aligned}
& \gamma_l > \gamma_s \\
\Rightarrow \quad & -\tfrac{V'(c_1)}{V'(c_1)} > -\tfrac{U'(c_1)}{U'(c_1)} \\
\Rightarrow \quad & U'(c_1)V''(c_1) < V'(c_1)U''(c_1) \\
\Rightarrow \quad & \tfrac{U'(c_1)V''(c_1) - V'(c_1)U''(c_1)}{U'(c_1)^2} < 0 \\
\Rightarrow \quad & \left(\tfrac{V'(c_1)}{U'(c_1)}\right)' < 0
\end{aligned}$$

Thus, if the long-run risk aversion is higher than the short-run risk aversion, $\frac{V'(c_1)}{U'(c_1)}$ is decreasing in c_1. Therefore,

$$\frac{V'(c_{uuu})}{U'(c_u)} = u^{-2\gamma_l}\frac{V'(c_u)}{U'(c_u)} < u^{-2\gamma_l}\frac{V'(c_d)}{U'(c_d)} = \frac{V'(c_{duu})}{U'(c_d)}$$
$$\frac{V'(c_{uud})}{U'(c_u)} = u^{-\gamma_l}d^{-\gamma_l}\frac{V'(c_u)}{U'(c_u)} < u^{-\gamma_l}d^{-\gamma_l}\frac{V'(c_d)}{U'(c_d)} = \frac{V'(c_{dud})}{U'(c_d)}$$

I have $\phi_{uu} < \phi_{du}$. Similarly, $\phi_{ud} < \phi_{dd}$. By Lemma 2, $E[R^{(1)}] - R_f > E[R^{(2)}] - R_f$, $\sigma(R^{(1)}) > \sigma(R^{(2)})$. Part (a) of Proposition 7 is proved. By the same argument, (b) and (c) also hold true.

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