Falling into Traps?

Patent Thickets and Stock Returns[*]

Po-Hsuan Hsu[†], **Hsiao-Hui Lee**[‡], and **Tong Zhou**[§]

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ABSTRACT. When firms innovate on the basis of prior patents dispersedly owned by different patent assignees, this fragmented ownership results in patent thickets that adversely affect the value of firms' inventions. We develop a real option model that suggests a negative effect of patent thickets on expected stock returns due to licensing costs and delays in realizing options. In addition, such an effect is expected to be stronger for firms with larger patent portfolios and lower innovation efficiency. Our empirical analysis based on U.S. patent and financial data supports these model implications and highlights a new channel through which the dynamics of technology development are incorporated in the cross-section of stock returns.

KEYWORDS. Patent Thicket, Innovation, Technology Application, Stock Return

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[†] Correspondent author. Faculty of Business and Economics, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 28591049; E-mail address: `paulhsu@hku.hk`.

[‡] Faculty of Business and Economics, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 39171082; E-mail address: `hhlee@hku.hk`.

[§] Faculty of Business and Economics, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 96044959; E-mail address: `zhoutong@hku.hk`.

1 Introduction

Human civilization is built on accrued knowledge generated from past generations' innovative activities; as Sir Isaac Newton remarked, "*If I have seen further it is by standing on the shoulders of giants*". In particular, the economics and innovation literature has attributed the majority of technological achievements to the combination of pre-existing ideas with new knowledge and techniques.[1] In order to accelerate knowledge accumulation and technical progress, governments introduced patent systems to encourage inventors to share their ideas, discoveries, and inventions with the public in exchange for exclusive usage rights of inventions for a certain period. The idea is that, employing effective systematic disclosure and enforcing patent rights, these patent systems allow patentees and producers to negotiate licensing contracts efficiently, as well as generate more innovative products and higher incentives to innovate thanks to shared economic rents yielded by innovations.

Patentees collect economic rents from their patents through three channels: 1) they can collect royalty from users, including individuals and organizations that negotiate with them to use their patents; 2) they can file, or threaten to file, a patent-infringement lawsuit against individuals or organizations that potentially infringe their patents, in which case patentees collect settlement or royalty if they win the lawsuit or negotiate a deal with or without initiating a lawsuit; or 3) they can decide to enjoy market advantages as other individuals and organizations bear the monetary and time cost to circumvent existent patents.

However, for patent users, the coordination inherent in patent licensing and litigation could be costly if the ownership of these patents is dispersed, which is often referred to as a "patent thicket" (Shapiro, 2001; Ziedonis, 2004; Galasso and Schankerman, 2010).[2] Such obstacles usually make any participant in a fragmented technology field hesitate to convert

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[1] Perhaps the most famous quote about this recombinant viewpoint is Schumpeter's words regarding innovation as the "carrying out of new combinations" (Schumpeter, 1934, pp. 65-68). Such a viewpoint is also supported by Nelson and Sidney (1982), Weitzman (1998), and Singh and Fleming (2010).

[2] As far as we know, "patent thicket" is first used by Shapiro (2001) to refer to an overlapping set of patent rights that require that those trying to commercialize new technology to obtain permission from multiple patentees.

inventions into products and revenue due to high royalty and litigation risks. It appears that existent patents form a thicket which traps new inventors in and prevents them from fully exploiting their new patents. In the finance literature, firms' innovation activities, such as R&D and patenting, are commonly regarded as growth options and largely determine firms' market value.[3] Since firms confronted with patent thickets suffer from lower expected profits and delays in exercising options, the relationship between patent thickets and firms' value leads to asset pricing implications.

To illustrate how patent thickets affect both firms' innovation progress and asset prices, we first build two tractable partial equilibrium models based on the real option literature.[4] In the Basic Model with closed-form solutions, we illustrate that when a firm granted a patent plans to exercise this option and produce profits, that firm has to pay royalty to the owners of other patents surrounding its patent. Recognizing the expected royalty payments, the firm then decides when to exploit the option; thus, the option value depends on the number of surrounding patents. We show that a deeper patent thicket (i.e., more surrounding patent owners) hinders new technology application and hurts the focal firm's value. Due to higher royalty costs and delayed exercising time,[5] the focal firm is expected to provide a lower subsequent stock return. This argument yields our first theoretical prediction related to cross-sectional predictability: expected stock return decreases with patent thicket. Such a decrease in expected return is attributed to lower systematic risk, due to reduced option

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[3] The different properties, such as risk exposure and future cash flow, of asset in place and growth options are widely studied in the literature, including Berk, Green, and Naik (1999), Berk, Green, and Naik (2004), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Aguerrevere (2009), Garleanu, Panageas, and Yu (2012), Ai and Kiku (2013), and Kogan and Papanikolaou (2014), among many others.

[4] It is worthwhile to point out the limitation and advantage of our partial equilibrium setting. The major limitation is that by assumption the pricing kernel is given. That said, one may understandably express concern that innovation by nature influences the pricing kernel. However, we argue that this concern should be minor, since we focus on cross-sectional firm characteristics instead of aggregate shocks, to a given firm the cash flow effect should dominate the discount rate effect. Meanwhile, the biggest advantage of our model is its tractability. With closed-form solutions, simple comparative statics can produce a rich set of implications. We further show that our results still hold under a wide range of reasonably calibrated parameters.

[5] The total cost of royalty increases with the number of surrounding patents due to population and coordination effects. For the population effect, given the same royalty fee per patent, the total royalty cost increases with the number of surrounding patents. For the coordination effect, a larger number of surrounding patents leads to an increase in coordination difficulty and results in higher coordination cost.

value and delayed exercise timing. The intuition of our model is that a firm encountering a deeper patent thicket has a lower implicit leverage (due to reduced option value) as well as lower systematic risk, thereby providing lower expected returns. Following this argument, our model leads to a conditional CAPM in which market beta decreases with patent thicket.

The Full Model with numerical solutions enriches the Basic Model by allowing the focal firm to have more than one patent in the same field and to bear related exploitation costs. This Full Model delivers two conditional predictability implications. First, the explanatory power of patent thickets for expected stock returns is stronger for firms with more patents. The intuition is that firms with more patents carry larger exposure to systematic risk; thus, these firms' total systematic risk premia are lower given the same level of patent thickets. Second, we propose that the explanatory power of patent thickets for expected returns is stronger for firms with lower innovation efficiency. Firms that are less efficient in converting ideas into products tend to bear higher costs in innovation, leading to lower option value as well as lower expected stock returns.

The model robustness is discussed in Online Appendix. For simplicity, our main models assume that the patent endowment is independent from patent thicket, i.e., a firm faces an exogenous patent thicket when it receives patents. We adopt such a setting becasue 1) we focus on the cross-sectional variation of stock returns instead of time-series dynamics, and 2) a firm does not know patent thicket it faces until its and competitors' patents are granted. We appreciate that some may question the extent to which our results will change if we consider the endogeneity of patent endowment, i.e., the value of patents produced by a firm is subject to patent thicket the firm is facing. We prove under the condition that patent value decreases with patent thicket,[6] our model implications still hold. In another model extension, we also show that considering alternative bargaining processes and heterogeneous bargaining costs do not alter our implications.

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[6] Intuitively, firms confronted with deeper patent thicket may produce patents with lower value. A deeper patent thicket is ex post harmful to a firm's value because it delays patent exploitation and lowers patent value, and as a result it provides less ex ante incentive for a firm to innovate, especially break-through discoveries.

In our empirical analysis, we measure patent thicket using the definition of Ziedonis (2004) and the patent data collected by Kogan, Papanikolaou, Seru, and Stoffman (2012). Ziedonis' (2004) patent thicket measure is defined as one minus the Herfindahl index based on the shares of other firms' patents being cited by the focal firm's patents that are approved over the past five years. When the focal firm's patents cite prior patents owned by many different owners, this firm is regarded as being confronted with a deeper patent thicket, i.e., more fragmented ownership of related patents. It is noteworthy that patent applicants to U.S. patents are legally required to include a comprehensive list of prior patents and works that are relevant to their application ("duty of candor"). This list is subject to patent examiners' reviews. When patent applications are approved, the published patent documents will include a full list of prior patents and works based on self-reporting or being requested by patent examiners. These patent citations potentially lead to patent litigations as patentees tend to prosecute the infringement of a patent when the patent is cited by other firms that are active in related technology areas (Lanjouw and Schankerman, 2001).

We construct portfolios sorted by the patent thicket measure and other characteristics to test our model implications. First of all, we do one-way portfolio sorting to test our main theoretical prediction that expected stock returns decrease with patent thicket and obtain supportive evidence. Monthly value-weighted excess stock returns of firms in the top quintile of patent thicket underperform those of firms in the bottom quintile by 0.37% to 0.77%, given various systematic risk factors being controlled including the market factor, the size factor and the value factor by Fama and French (1993), the momentum factor by Carhart (1997), the profitability factor and the investment factor by Fama and French (2015), the investment-minus-consumption factor by Papanikolaou (2011), and the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2013).[7] Such a difference is significant at the 1% or 5% level, depending on model specification. In the robustness checks, we

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[7] Our findings that return spreads driven by patent thickets remain significant even after controlling for various risk factors can be attributed to two reasons: First, the portfolio approach and factor regressions cannot appropriately capture the conditional beta; Second, traditional risk factors cannot fully reflect systematic risk that innovation-intensive firms are facing.

independently sort two-way portfolios on patent thicket and one of the control variables of sample firms, including size, book-to-market ratio, momentum, and short-term reversal, counter-parties' bargaining power, product market competition, and patent quality. We obtain consistent results. Further tests based on industry-adjusted returns and within-industry sorted portfolios suggest that industry heterogeneity has little influence on our results.

We find empirical support for the mechanism of our model: conditional CAPM and growth options. First, the market beta of each quintile portfolio decreases with patent thicket, and the difference between the top quintile and the bottom quintile is -0.07 and significant at 5% level. Second, for all stocks in each month, we regress the cross-sectional market betas on patent thickets and then test the time-series average of the slope. We find that if a firm moves from the lowest patent thicket quintile to the highest, its market beta drops by 0.07 or 0.13 with significance at the 1% level, depending on whether industry fixed effect is included or not. The third test is about the predictive ability of patent thicket for the volatility of future firm fundamental, because firm fundamentals fluctuate with the likelihood of growth option exercising. We find that the volatility of ROA, investment, and sales in the next five years of a firm with deeper patent thicket is smaller, suggesting its greater difficulty in transforming growth options into asset in place.

We then test two conditional predictability implications by independently sort portfolios on patent thicket and one of the following two characteristics: patent portfolio size and innovation efficiency. We measure patent portfolio size using alternative proxies: the focal firm's total patent number divided by book equity or total assets, and total R&D investment divided by book equity or total assets. We measure innovation efficiency using the focal firm's patent number divided by accumulated R&D capital following Hirshleifer, Hsu, and Li (2013). Our results support these two predictions, as the predictive power of patent thicket is stronger for firms with bigger patent portfolios or lower innovation efficiency.

This paper is related to the finance and economics literature. This study adds to the finance literature by highlighting the role of patent competition in asset prices. Prior studies concerning about innovation and asset pricing, such as Lin (2012), Kogan, Papanikolaou, Seru, and Stoffman (2012), Cohen, Diether, and Malloy (2013), and Hirshleifer, Hsu, and Li (2013), among others, focus on the value implication of individual firms' technological development.[8] However, when firms race for filing new patents in order to slow down potential competitors and deter potential new entrants, the dynamics of such patent competition becomes important and value-relevant. Our model and empirical results suggest that patent thicket affects expected stock returns through its influence on option value of patents. This study thus provides a new perspective to the contemporary research on financial valuation of technological innovation.

Our paper relates to but differentiates from the studies on product market competition. Hou and Robinson (2006) link product quantity competition to stock returns and find that higher concentration of firm sales leads to lower returns. Hoberg and Phillips (2012) relate product quality competition to stock returns and document that product uniqueness does not have any explanatory power on future stock returns. We study the linkage between patent ownership and stock returns, and show that higher concentration of patent ownership predicts higher returns. All of these three papers examine how industry structure (patent ownership fragmentation in ours, sales concentration in Hou and Robinson (2006), and characteristics competition in Hoberg and Phillips (2012)) influences risk exposure of firms and therefore explains expected stock returns. Our evidence that the patent thicket effect is still significant even after industry effects being controlled implies that patent thicket documented in this paper is a distinct firm characteristic from product market competition. More importantly, our focus on patent portfolios should be more value-relevant than the other two studies' focus on product portfolios because patent portfolios reflect firms' prospects in future competition.

The economics literature has examined the phenomenon of patent thicket, associated

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[8] Moreover, Greenwood, Hercowitz, and Krusell (1997), Hobijn and Jovanovic (2001), Laitner and Stolyarov (2003), Pástor and Veronesi (2009), Hsu (2009), Papanikolaou (2011), and Garleanu, Panageas, and Yu (2012) investigate the value implication of technological innovation from an aggregate perspective.

problems, and possible solutions for long.[9] We contribute to this area by examining the impact of patent thicket from the perspective of equity market: we model and empirically verify a relation between patent thicket and firms' market value. Moreover, by implementing tests pooling all industries and controlling for industry effects, we show that the impact of patent thicket on economic value is a general pattern instead of an industry-specific phenomenon.[10] This study thus extends the scope of patent competition and has policy implications.

The rest of the paper is organized as follows: Section 2 first presents a simplified model with the goal of delivering the key intuition, and then elaborates a complete version of the model and develops theoretical predictions. Section 3 discusses data and presents the empirical results for three testable predictions and robustness checks. Section 4 concludes. All proofs as well as some ancillary explanations are provided in the appendixes.

2 Theoretical Discussion

In this session, we adopt a real option model to explain the effect of patent thicket on a firm's stock return. Specifically, we study a firm's behavior after a patent is granted, instead of its behavior in undertaking R&D projects to generate patents. A patent's value over time depends on the random evolutions of market fundamentals, and can be realized as a new product to the market. Our model captures the firm's decision on the timing of such a new product introduction. We first present a simplified model with closed-form solutions to discuss its basic intuition in Section 2.1, and then construct a full model with numerical

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[9] Shapiro (1985 and 2001) and Baron and Pohlmann (2015) investigate whether cross-licensing, patent pools, or cooperative standard setting can mitigate the problem of patent thicket. Ziedonis (2004) studies how firms expand their patent portfolios in response to patent thicket. Bessen (2004) considers patent thicket in a strategic patenting context, while Clark and Konrad (2008) relate patent thicket to incentive for R&D investment. Cockburn and MacGarvie (2009) focus on the initial acquisition of VC funding of start-up software companies facing patent thicket. And Cockburn, MacGarvie, and Mueller (2010) look at innovative performance and in-licensing of firms under the impact of patent thicket.

[10] Ziedonis (2004) and Cockburn, MacGarvie, and Mueller (2010) focus on semiconductor industry and software industry respectively, while Cockburn and MacGarvie (2009) focus on start-up firms only.

solutions and develop the full set of theoretical predictions in Section 2.2.

2.1 The Basic Model

2.1.1 Setup

We consider a single firm in the economy that holds a Lucas' tree (i.e., asset in place) producing an instantaneous dividend, θ_t, which follows a geometric Brownian motion, i.e.,

$$\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dz_t,$$

where $\mu > 0$ and $\sigma > 0$. At $t = 0$, the firm receives an opportunity to plant a new tree (i.e., a new discovery or invention, hereafter a "patent" for short), which produces an instantaneous dividend $\xi\theta_t$ ($\xi > 0$) once it is successfully exploited to production. We interpret a larger ξ as a larger patent valuableness.[11]

However, the realized patent is surrounded by n owners, and the patent will be successfully exploited to production only if all of the n owners are compensated with a royalty payment. We interpret a larger n as deeper patent thicket.[12] We assume that at $t = 0$, the n owners negotiate with the firm simultaneously on the royalty payments, i.e., owner i charges the firm a royalty payment, q_i, and pays a private cost, c_i,[13] when the patent is exploited. We define the patent exploitation cost to the firm as $Q \equiv \sum_{i=1}^{n} q_i$.[14]

The owners decide the royalty to be charged on the patent, given the royalty payments, the firm decides the timing to exploit the patent. As a result, we can use backward induc-

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[11] In Section C.1 in Online Appendix, we study the case with endogenous patent endowment, i.e., ξ is a function of n. We prove that the negative relation between patent thicket and firm's expected return still holds under the condition that ξ is decreasing in n.

[12] In this economy, the number patent owners, n, captures the spirit of patent thicket. A firm with a larger n is confronted with a more fragmented market of innovation ownership, thereby suffering from a deeper patent thicket.

[13] This private cost can be interpreted as the cost of authorizing the firm to start using the new technology, such as negotiation cost and filing cost. It can also be interpreted as the cost of "exclusive licensing" that prevents the owner from licensing its patent to the focal firm's competitors.

[14] In Section C.2 in Online Appendix, we look into two elements that may affect the bargaining between patent owners and the focal firm. One is that each of the patent owners bears different private cost c_i. The other is that each patent owner initiates bargaining with the focal firm at different timing.

tion to tackle the model. Given patent exploitation cost, Q, we first solve the real-option exercising problem to obtain the patent exploitation timing and the resulting price, P_t, for the firm. Next, with the firm's decision rule, the owner royalty payments are decided by solving a Cournot equilibrium among the owners.

We assume that the stochastic discount factor, M_t, also follows a geometric Brownian motion, i.e.,

$$\frac{dM_t}{M_t} = -rdt - \kappa dz_t,$$

where $r > 0$, $\kappa > 0$, and $\rho \equiv (r + \kappa\sigma - \mu)^{-1}$. To make the model meaningful, we also assume $r + \kappa\sigma - \mu > 0$. Note that, following Berk, Green, and Naik (1999) and Zhang (2005), we parameterize the stochastic discount factor without explicitly modeling the consumer's problem. We argue that this modelling strategy is acceptable, as our focus is on the production side and small-scaled innovations (i.e., ξ is sufficiently small).

2.1.2 The Firm's Decision

The focal firm's price, P_t, includes two part, the price of its asset in place, P_t^I, and that of its patent, P_t^O. The price of asset in place follows,

$$P_t^I = E_t\left[\int_t^\infty \frac{M_s}{M_t}\theta_s ds\right] = \rho\theta_t, \tag{1}$$

and the price of patent is determined by the optimization problem of the stopping (exploitation) time, τ, that maximizes,

$$P_t^O = \sup_\tau E_t\left[\int_\tau^\infty \frac{M_s}{M_t}\xi\theta_s ds - \frac{M_\tau}{M_t}Q\right],$$

in which the first term represents the present value of the (potential) real product, and the second term represents the present value of royalty paid. The following lemma characterizes the optimal stopping time and the patent value.

Lemma 1 *The optimal stopping time is reached when the market fundamentals reach* θ^**,* *i.e.,* $\theta_{\tau^*} = \theta^*$, *in which,*

$$\theta^* = \frac{\phi^+}{\phi^+ - 1}\frac{Q}{\rho\xi},\ \textit{and} \tag{2}$$

$$\phi^+ = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} > 1.$$

The value of patent is,

$$P_t^O = \left(\frac{\rho\xi}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+ - 1}{Q}\right)^{\phi^+ - 1}\theta_t^{\phi^+}. \tag{3}$$

See all proofs of lemmas and propositions in Appendix A.

From Equation (2), we have the following inequalities:

$$\frac{\partial\theta^*}{\partial Q} > 0, \text{ and} \tag{4}$$

$$\frac{\partial\theta^*}{\partial\xi} < 0. \tag{5}$$

From Equation (3), we find that,

$$\frac{\partial P_t^O}{\partial Q} < 0, \text{ and} \tag{6}$$

$$\frac{\partial P_t^O}{\partial\xi} > 0. \tag{7}$$

As a result, we obtain the following remarks.

Remark 1 *As the cost of patent exploitation increases, it is more difficult to transform a patent into asset in place, so 1) the threshold of patent exploitation increases (Equation 4), and 2) the value of patent decreases (Equation 6).*

Remark 2 *As the systematic risk exposure (or value) of the patent increases, it is more valuable to transform it into asset in place earlier, so 1) the threshold of patent exploitation decreases (Equation 5), and 2) the value of the patent increases (Equation 7).*

Because the price of the firm is the sum of prices of its asset in place and patent, we have,

$$P_t = P_t^I + P_t^O. \tag{8}$$

Combining Equations (8) with (1) and (A4)[15] and applying Ito's lemma gives,

$$\begin{aligned} dP_t &= \frac{\partial P_t^I}{\partial \theta_t} d\theta_t + \frac{1}{2}\frac{\partial^2 P_t^I}{\partial \theta_t^2}\left(d\theta_t\right)^2 + \frac{\partial P_t^O}{\partial \theta_t} d\theta_t + \frac{1}{2}\frac{\partial^2 P_t^O}{\partial \theta_t^2}\left(d\theta_t\right)^2 \\ &= \left[\mu P_t^I + \phi^+ \mu P_t^O + \frac{1}{2}\phi^+\left(\phi^+ - 1\right)\sigma^2 P_t^O\right] dt + \left[\sigma P_t^I + \phi^+ \sigma P_t^O\right] dz_t. \end{aligned}$$

Therefore, the expected future return of the firm is,

$$\begin{aligned} E_t\left[R_t\right] \equiv& E_t\left[\frac{dP_t + \theta_t dt}{P_t}\right] \\ =& \frac{\mu P_t^I + \mu\phi^+ P_t^O + \frac{1}{2}\phi^+\left(\phi^+ - 1\right)\sigma^2 P_t^O + \theta_t}{P_t^I + P_t^O} dt > 0. \end{aligned}$$

With simple algebra, we have,

$$E_t\left[R_t\right] = \left(\mu + \theta_t\right) dt + \Omega \frac{P_t^O}{P_t^I + P_t^O} dt, \tag{9}$$

where $\Omega = \left(\phi^+ - 1\right)\mu + \phi^+\left(\phi^+ - 1\right)\sigma^2/2 > 0$, because $\phi^+ > 1$.

Taking partial derivative of Equations (9) w.r.t. Q on both sides yields,

$$\frac{\partial E_t\left[R_t\right]}{\partial Q} = \left[\frac{\Omega P_t^I}{\left(P_t^I + P_t^O\right)^2}\right]\frac{\partial P_t^O}{\partial Q} dt.$$

As the partial derivative of P_t^O to Q is negative (Equation 6) and the value in the bracket is positive, we know that,

$$\frac{\partial E_t\left[R_t\right]}{\partial Q} < 0, \tag{10}$$

as well as the following remark.

Remark 3 *A firm which has a higher cost in patent exploitation suffers from a lower expected future return, ceteris puribus.*

2.1.3 The Patent Owners' Decision

With the firm's optimal exploitation time, we proceed to find the optimal royalty payments by solving a Cournot Equilibrium among the n owners. Owner i's maximization problem is

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[15] See Appendix A.

given by,

$$\max_{q_i} E_t\left[\frac{M_{\tau^*}}{M_t}(q_i - c_i)\right] = \max_{q_i} E_t\left[e^{-r(\tau^*-t)}(q_i - c_i)\right].$$

Solving this optimization problem simultaneous for n owners, we have Lemma 2.

Lemma 2 *The optimal royalty payment, q_i^*, can be expressed as,*

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n\right)c_i + \sum_{j=1}^{n} c_j\right]. \tag{11}$$

When $c_i = c_j = c$, the optimum can be simplified to $q_i^* = q_j^* = q^*$ for $i, j = 1 ... n$, and,

$$q^* = \frac{\phi^+}{\phi^+ - n}c > c. \tag{12}$$

For simplicity, we continue our discussion under the assumption that $c_i = c_j = c$.

2.1.4 Theoretical Implications

From Equation (12) we can show that,

$$\frac{dq^*}{dn} = \frac{\phi^+}{(\phi^+ - n)^2}c > 0, \text{ and} \tag{13}$$

$$\frac{dQ}{dn} = q^* + n\frac{dq^*}{dn} = \frac{(\phi^+)^2}{(\phi^+ - n)^2}c > 0. \tag{14}$$

Combining Equations (10) and (14) yields,

$$\frac{dE_t\left[R_t\right]}{dn} = \frac{\partial E_t\left[R_t\right]}{\partial Q}\frac{dQ}{dn} < 0, \tag{15}$$

which leads to the following Proposition 1.

Proposition 1 *A firm with a deeper patent thicket produces a lower expected future return.*

Proposition 1 implies that a firm that has a deeper patent thicket, n, has a higher cost in patent exploitation, Q, and hence delays the exploitation of the patent, i.e., lower θ^*, and suffers from a lower expected future return. Equation (14) shows the reasons why a deeper patent thicket leads to a higher exploitation cost—the population effect (the term q^*) as well as the coordination effect (the term $n \cdot dq^*/dn$). The population effect captures that given the individual royalty fees, the total royalty cost increases with the number of patent owners surrounding the patent. The coordination effect is in a similar spirit with Cournot and Fisher's (1897) "complements problem" and Heller and Eisenberg's (1998) "tragedy of anti-commons". Because of the complementary ownership relations, each owner will be better off if they can coordinate among themselves. However, each owner does not has any incentive to coordinate with others, as they share the public benefits of coordination but bear the private costs. Therefore, the positive externality of coordination induces them to over-use this "commons-like" patent, over-charge royalty payments, and hinder the exploitation of the patent.

We further interpret Proposition 1 from the perspective of asset pricing. The firm's total exposure to the systematic risk can be defined as,

$$\beta_t = -\frac{E_t\left[\frac{dM_t}{M_t} R_t\right]}{Var_t\left[\frac{dM_t}{M_t}\right]}.$$

Using the property of the stochastic discount factor, M_t, and the return, dP_t/P_t, we can rewrite β_t as,

$$\beta_t = \frac{\sigma}{\kappa}\left[\frac{P_t^I + \phi^+ P_t^O}{P_t^I + P_t^O}\right].$$

Note that the value in the bracket is larger than one, as $\phi^+ > 1$, and hence $\beta_t > 1$ is true under our calibrated parameters.

Taking the first derivative of β_t with respect to n, we have,

$$\frac{d\beta_t}{dn} = \frac{\sigma}{\kappa}\left[\frac{(\phi^+ - 1) P_t^I}{\left(P_t^I + P_t^O\right)^2}\right]\frac{dP_t^O}{dn},$$

where the value in the bracket is always positive. Therefore, using Equations (6) and (14) we know that,

$$\frac{d\beta_t}{dn} < 0,$$

and hence we have Proposition 2.

Proposition 2 *A firm with a deeper patent thicket has a smaller exposure to the systematic risk.*

The intuition rising from the two propositions are intertwined. A firm with a deeper patent thicket suffers from a higher cost in patent exploitation and delays the timing of exploiting the patent to become real product. Because it is more difficult for that firm to transform its patent into asset in place, it has a smaller exposure to the systematic risk (Proposition 2). Because such firm has a smaller exposure to the systematic risk, it has a lower future return, leading back to Proposition 1.

2.2 The Full Model

In this section, we extend the basic model to incorporate patent portfolio size (i.e., the number of patents a firm holds) and innovation efficiency (i.e., the firm's capability to turn patents into real products). These two new features enable us to further understand the impact of patent thicket on asset prices, and generalize the insights from the previous section. Specifically, we will deliver five theoretical predictions; not only are they provable in the extreme cases, but also shown to be true under a wide range of reasonable calibrated parameters.

2.2.1 Setup

This economy has a single firm and n owners surrounding each patent granted to the firm. The property of asset in place, stochastic discount factor, and patents are the same as described in the Basic Model.

We add patent portfolio size into the model by assuming that at $t = 0$ the firm receives an opportunity to plant A independent and homogeneous new trees.[16] Hereafter we interpret

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[16] We assume the A patents are independent because we would like to show the "mean" effect of patent thicket on asset prices. We could show that when the A patents are substitutes, the effect of patent thicket is relieved to some extent, and that when the A patents are complements, the effect of patent thicket is even more severe.

a large A as a bigger patent portfolio size. The role of innovation efficiency is modeled by the assumption that at any time t, each patent requires an instantaneous investment $i_t dt$ to maturate until it becomes a real product.[17] To make our model tractable, following Berk, Green, and Naik (2004), we consider a linear instantaneous investment, i.e. $i_t = a + b\theta_t$, which captures both the fixed and time-varying components. Hereafter we interpret a larger a or b as lower innovation efficiency.

The firm decides the timing of patent exploitation given the royalty payment, whereas the owners decide the royalty fees to collect from the firm when it exploits the patent. As a result, it leads to a similar backward induction procedure as in Section 2.1.

2.2.2 The Firm's Decision

The focal firm's price P_t includes the value of its asset in place P_t^I and the patent value AP_t^O. Similar to the Basic Model, its price of asset in place is given by $P_t^I = \rho\theta_t$. But the price of patent is modified to the optimization problem of the stopping time (τ) that maximizes,

$$P_t^O = \sup_{\tau} E_t \left[\int_{\tau}^{\infty} \frac{M_s}{M_t} \xi \theta_s ds - \frac{M_\tau}{M_t} Q - \int_t^{\tau} \frac{M_s}{M_t} i_s ds \right],$$

in which the first term represents the present value of the (potential) real product, the second term represents the present value of royalty paid, *and* the third term represents the present value of the cumulative investments since time t to exploiting time τ.

The following lemma characterizes the optimal stopping time and the patent value.

Lemma 3 *The optimal stopping time is reached when the market fundamentals reach θ^*, i.e., $\theta_{\tau^*} = \theta^*$, in which,*

$$\theta^* = \frac{\phi^+}{\phi^+ - 1} \frac{Q - a/r}{\rho\xi + b\rho}, \text{ and} \tag{16}$$

$$\phi^+ = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} > 1.$$

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[17] In practice, there is still a gap between a patent and a real product. In order to turn a patent into real product, a firm has to spend efforts and money in refining the technology, studying market demand, and building product lines. Investment i_t aims to capture these expenditures.

The value of patent is,

$$P_t^O = \left(\frac{\rho\xi + b\rho}{\phi^+}\right)^{\phi^+} \left(\frac{\phi^+ - 1}{Q - a/r}\right)^{\phi^+ - 1} \theta_t^{\phi^+} - b\rho\theta_t - \frac{a}{r} \equiv P_t^H - P_t^P, \tag{17}$$

in which P_t^H *represents the first term, the value growth effect on patent value, and* P_t^P *represents the latter two terms, the investment cost effect on patent value.*

Similar to the Basic Model, we use Equations (16) and (17) to obtain the following inequalities: $\partial\theta^*/\partial Q > 0$, $\partial\theta^*/\partial\xi < 0$, $\partial P_t^O/\partial Q < 0$, and $\partial P_t^O/\partial\xi > 0$. As a result, Remark 1 and Remark 2 are valid in the full model as well.

Because the price of the firm is the sum of its asset in place and patents, we have,

$$P_t = P_t^I + AP_t^O. \tag{18}$$

Combining Equations (18) with (1) and (A9), and applying Ito's lemma, we have,

$$\begin{aligned} dP_t &= \frac{\partial P_t^I}{\partial\theta_t} d\theta_t + \frac{1}{2}\frac{\partial^2 P_t^I}{\partial\theta_t^2}(d\theta_t)^2 + A\frac{\partial P_t^O}{\partial\theta_t} d\theta_t + \frac{1}{2}A\frac{\partial^2 P_t^O}{\partial\theta_t^2}(d\theta_t)^2 \\ &= \left[\mu P_t^I + \phi^+\mu A P_t^H - Ab\mu\rho\theta_t + \frac{1}{2}\phi^+\left(\phi^+ - 1\right)\sigma^2 A P_t^H\right] dt \\ &+ \left[\sigma P_t^I + \phi^+\sigma A P_t^H - Ab\sigma\rho\theta_t\right] dz_t. \end{aligned}$$

The expected future return of the firm becomes,

$$\begin{aligned} E_t\left[R_t\right] &\equiv E_t\left[\frac{dP_t + \theta_t dt}{P_t}\right] \\ &= \frac{\mu P_t^I + \mu A\left(\phi^+ P_t^H - b\rho\theta_t\right) + \frac{1}{2}\phi^+\left(\phi^+ - 1\right)\sigma^2 A P_t^H + \theta_t}{P_t^I + AP_t^O} dt \\ &= \left(\mu + \theta_t\right) dt + \Omega\frac{AP_t^H}{P_t^I + AP_t^O} dt + \frac{a}{r}\frac{\mu A}{P_t^I + AP_t^O} dt > 0, \end{aligned} \tag{19}$$

where $\Omega = \left(\phi^+ - 1\right)\mu + \phi^+\left(\phi^+ - 1\right)\sigma^2/2 > 0$. Finally, we have the following relationship,

$$\frac{\partial E_t\left[R_t\right]}{\partial Q} = A\left[\frac{(1 - Ab)\rho\Omega\theta_t - aA(\Omega + \mu)/r}{\left(P_t^I + AP_t^O\right)^2}\right]\frac{\partial P_t^H}{\partial Q} dt.$$

As the partial derivative of P_t^H to Q is equivalent to that of P_t^O to Q, which is negative, the sign of $\partial E_t[R_t]/\partial Q$ becomes the opposite sign of the value in the bracket. Note that when $a = 0$ and $b = 0$, the value in the bracket is always positive for any $\theta_t > 0$, leading to,

$$\frac{\partial E_t\left[R_t\right]}{\partial Q} < 0. \tag{20}$$

As a result, when a and b are sufficiently small, Equation (20) still holds. We will use this assumption as well as some minor conditions to derive our theoretical preditions in Section 2.2.4.

2.2.3 The Patent Owners' Decision

With the optimal stopping time, we proceed to find the optimal patent royalty by solving a Cournot Equilibrium. Owner i finds optimal its royalty to maximize its net present profits, i.e.,

$$\max_{q_i} E_t\left[\frac{M_{\tau^*}}{M_t}(q_i - c_i)\right] = \max_{q_i} E_t\left[e^{-r(\tau^*-t)}(q_i - c_i)\right].$$

Solving the simultaneous maximization problems for all owners, we have Lemma 4.

Lemma 4 *The optimal royalty of owner i is,*

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n\right)c_i + \sum_{j=1}^{n} c_j - \frac{a}{r}\right], \text{ and}$$

when $c_i = c_j = c$, the optimum can be simplified to $q_i^* = q_j^* = q^*$ for $j = 1...n$, in which,

$$q^* = \frac{\phi^+ c - a/r}{\phi^+ - n}. \tag{21}$$

Similar to the Basic Model, we continue our discussion under the assumption that $c_i = c_j = c$.[18]

2.2.4 Theoretical Predictions

In this section, we first derive insights based sufficiently small a, b, and ξ, and sufficiently large θ_t. This assumption helps us to find theoretical predictions without complicating the model.

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[18] The royalty, q^*, has to be positive, otherwise the patent owner will not participate in exploiting the patent. We apply the assumptions that $a < ncr$ and that $\phi^+ > n$ throughout this model.

For example, given sufficiently small a and b, $\partial E_t[R_t]/\partial Q$ is negative, and hence delivers the same outcome as Equation (20). But we later relax this assumption by numerically reinforcing the insights with a wide range of reasonable calibrated parameters.

The Extreme Cases: Sufficiently Small a, b and ξ, and Sufficiently Large θ_t. From Equation (21), we can show that,

$$\frac{dq^*}{dn} = \frac{\phi^+ c - a/r}{(\phi^+ - n)^2} > 0, \text{ and} \tag{22}$$

$$\frac{dQ}{dn} = q^* + n\frac{dq^*}{dn} = \phi^+ \frac{\phi^+ c - a/r}{(\phi^+ - n)^2} > 0. \tag{23}$$

Combining Equations (20) and (23) yields,

$$\frac{dE_t\left[R_t\right]}{dn} = \frac{\partial E_t\left[R_t\right]}{\partial Q}\frac{dQ}{dn} < 0, \tag{24}$$

which leads to Proposition 3, the counterpart of Proposition 1 in the Basic Model.

Proposition 3 *For sufficiently small a and b, a firm with a deeper patent thicket produce a lower expected future return, ceteris puribus.*

After simple algebra, the firm's exposure to the systematic risk can be rearranged as,

$$\beta_t = \frac{\sigma}{\kappa}\frac{P_t^I + \phi^+ A P_t^H - bA\rho\theta_t}{P_t^I + AP_t^O}, \tag{25}$$

which is greater than 1 when $a \to 0$ and $b \to 0$. We show that this market beta decreases in n for small a and b by taking the first derivative of β_t with n, i.e.,

$$\frac{d\beta_t}{dn} = \frac{\sigma}{\kappa}A\left[\frac{\left(\phi^+ - 1 + Ab\right)P_t^I - \phi^+ AP_t^P}{\left(P_t^I + AP_t^O\right)^2}\right]\frac{dP_t^H}{dn}. \tag{26}$$

Notice that the value of the bracket is always positive for any θ_t when $a \to 0$ and $b \to 0$. Moreover, dP_t^H/dn can be decomposed to $(\partial P_t^H/\partial Q)(dQ/dn)$. The combination of $\partial P_t^O/\partial Q < 0$ and (23) and the definition of P_t^H implies that $dP_t^H/dn < 0$, and hence,

$$\frac{d\beta_t}{dn} < 0.$$

As a result, we have Proposition 4, which is consistent with Proposition 2 in the simplified model.

Proposition 4 *For sufficiently small a and b, a firm with a deeper patent thicket have a smaller exposure to the systematic risk, ceteris puribus.*

Next, we discuss the impact of patent portfolio size on the predictive power of the patent thicket to the future return, i.e., the sign of,

$$\frac{d\left|dE_t\left[R_t\right]/dn\right|}{dA}.$$

As shown in Equation (24), a deeper patent thicket reduces the future return, and hence we can remove the absolute value operator and simplify it to,

$$\begin{aligned}\frac{d\left|dE_t\left[R_t\right]/dn\right|}{dA} &= -\frac{d\left(dE_t\left[R_t\right]/dn\right)}{dA} \\ &= -\frac{d}{dA}\left(A\left(\frac{\Omega\left(P_t^I - AP_t^P\right) - a\mu A/r}{\left(P_t^I + AP_t^O\right)^2}\right)\frac{\partial P_t^H}{\partial Q}\frac{dQ}{dn}dt\right).\end{aligned}$$

As $dQ/dn > 0$ (Equation 23) and $\partial P_t^H/\partial Q < 0$ (as $\partial P_t^O/\partial Q < 0$) are both independent of A, its sign depends on the the derivative of the rest of the terms with A, which is

$$\begin{aligned}&\frac{d}{dA}\left(A\frac{\Omega\left(P_t^I - AP_t^P\right) - a\mu A/r}{\left(P_t^I + AP_t^O\right)^2}\right) \\ &= \frac{\Omega\left(P_t^I - AP_t^P\right) - a\mu A/r}{\left(P_t^I + AP_t^O\right)^2} + A\frac{-\Omega P_t^P P_t^I - a\mu P_t^I/r + \left(-2\Omega P_t^I + \Omega AP_t^P + a\mu A/r\right)P_t^O}{\left(P_t^I + AP_t^O\right)^3} \\ &= \frac{P_t^I\left[\Omega P_t^I - A\Omega P_t^O - 2A\Omega P_t^P - a\mu A/r\right]}{\left(P_t^I + AP_t^O\right)^3}.\end{aligned}$$

Notice that when $a \to 0$, $b \to 0$, and $\xi \to 0$, the magnitude of ΩP_t^I dominates the rest three negative terms, and hence, the sign of the previous fraction is positive. Therefore, we have,

$$\frac{d\left|dE_t\left[R_t\right]/dn\right|}{dA} > 0, \tag{27}$$

which leads to Proposition 5. Proposition 5 results from the argument that a firm with more patents has a larger exposure to the challenges of patent thicket, and thereby, such firm is more vulnerable to a given level of patent thicket.

Proposition 5 *For sufficiently small a, b, and ξ, as the patent portfolio size increases, the predictive power of the patent thicket to the future return increases, ceteris puribus.*

Next, we discuss the effect of patent value on the predictive power of the patent thicket to the future return, i.e., the sign of

$$\frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{d\xi}.$$

Similar to Proposition 5, the expression above can be rewritten as,

$$\frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{d\xi}=-\frac{d}{d\xi}\left(A\left(\frac{\Omega\left(P_t^I-AP_t^P\right)-a\mu A/r}{\left(P_t^I+AP_t^O\right)^2}\right)\frac{\partial P_t^H}{\partial Q}dt\frac{dQ}{dn}\right).$$

We know that $dQ/dn>0$ (Equation 23) does not depend on ξ, and hence the sign of the above expression depends on,

$$-A\left(\frac{d}{d\xi}\left(\frac{\Omega\left(P_t^I-AP_t^P\right)-a\mu A/r}{\left(P_t^I+AP_t^O\right)^2}\right)\right)\frac{\partial P_t^H}{\partial Q}dt-A\left[\frac{\Omega\left(P_t^I-AP_t^P\right)-a\mu A/r}{\left(P_t^I+AP_t^O\right)^2}\right]\frac{d}{d\xi}\left(\frac{\partial P_t^H}{\partial Q}\right)dt.$$

First, when ξ increases, P_t^I and P_t^P remains the same whereas P_t^O increases (Equation 17). Combining this property with $\partial P_t^O/\partial Q<0$, the first term is positive. Second, using the definition of P_t^H, we have,

$$\begin{aligned}\frac{d}{d\xi}\left(\frac{\partial P_t^H}{\partial Q}\right)&=\frac{d}{d\xi}\left[-\left(\frac{\rho\xi+b\rho}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+-1}{Q-a/r}\right)^{\phi^+}\theta_t^{\phi^+}\right]\\&=-\rho\left(\frac{\rho\xi+b\rho}{\phi^+}\right)^{\phi^+-1}\left(\frac{\phi^+-1}{Q-a/r}\right)^{\phi^+}\theta_t^{\phi^+}<0,\end{aligned}$$

and hence, the second term is also positive. As a result, we obtain,

$$\frac{d\left|dE_t\left[dP_t/\,P_t\right]/\,dn\right|}{d\xi}>0, \tag{28}$$

thus leading to Proposition 6. Intuitively, a higher ξ implies that the patents granted are potentially more valuable. A firm with more valuable patents suffers more from patent thicket, and therefore, such firm is more vulnerable to a given level of patent thicket.

Proposition 6 *As the patent value increases, the predictive power of the patent thicket to future return increases, ceteris puribus.*

Finally, we discuss the impact of innovation efficiency on the predictive power of the patent thicket to the future return, i.e., the sign of,

$$\frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{da} \text{ and } \frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{db}.$$

In Section A.5 of Appendix A we prove that when $a \to 0$, $b \to 0$, $\xi \to 0$, and $\theta_t \to \theta^*$, we have,

$$\frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{db} > 0. \tag{29}$$

We further prove that when ϕ^+ is sufficiently large and when $a \to 0$, $b \to 0$, $\xi \to 0$, and $\theta_t \to \theta^*$, we have,

$$\frac{d\left|dE_t\left[R_t\right]/\,dn\right|}{da} > 0. \tag{30}$$

Inequalities (29) and (30) lead to the following proposition.

Proposition 7 *For sufficiently small a, b, and ξ, and sufficiently large θ_t, as the innovation efficiency (the inverse of a or b) decreases, the predictive power of the patent thicket to the future return increases, ceteris puribus.*

Two conflicting effects–the optimal timing and investment cost effects–affect the conditional predictability of the patent thicket to the future returns on innovation efficiency. We explain the two forces by studying the role of a played on θ^* and P_t^O. We know that,

$$\frac{d\theta^*}{da} < 0, \text{ and} \tag{31}$$

$$\frac{dP_t^O}{da} = \frac{1}{r}\left(\frac{\rho\xi + b\rho}{Q - a/r}\frac{\phi^+ - 1}{\phi^+}\theta_t\right)^{\phi^+} - \frac{1}{r}, \tag{32}$$

in which we can further prove that $dP_t^O/da \leq 0$, and the equality holds only when $\theta_t = \theta^*$. Equation (31) shows the optimal timing effect: a higher a leads to a lower patent exploitation threshold, thus enhancing the patent value. On the other hand, Equation (32) is affected by a positive royalty bargaining effect (see the first term and Equation 21) and a negative cummulative cost effect (see the second term). Although the former implies that a higher a entitles the firm with stronger bargaining power and reduce the royalty, thereby enhancing the patent value, it is always dominated by the latter, the negative cummulative investment cost effect, which reduces the patent value. Therefore, a firm with a lower innovation efficiency suffers from both a lower market value and a lower future return. We skip the discussion on b as the discussion is similar to the one on a.

Numerical Illustration with Calibrated Parameters. Propositions 3-7 are proved around the extreme case of small a, b, and ξ, and big θ_t. To show the validity of the propositions, we relax these conditions and test the propositions numerically under a wide range of reasonably calibrated parameters. Borrowed from Campbell and Cochrane (1999) and Garleanu, Panageas, and Yu (2012), μ, σ, r, and κ are assumed to be 1.2%, 3%, 6.8%, and 3%, respectively. n and A are set as 3 and 30, respectively, to fit the data sample mean. Other parameters are set as follows: $a = 1\%$, $b = 0.1\%$, $\xi = 0.1$, and $c = 10$. This choice of parameters satisfies the implicit restrictions on the parameters in the model. See Appendix B for more details about our choice of parameters.

To show the numerical support of Proposition 3, we plot the value of $E_t[R_t]$ for $n \in [1, 5]$ and feasible θ_t in Panel A of Figure 1, and the relationship between $E_t[R_t]$ and n, for the 25^{th}, 50^{th}, and 75^{th} percentiles of θ_t in Panel B of Figure 1. Both panels show that $E_t[R_t]$ is decreasing in n for any θ_t, which means that a deeper patent thicket predicts a lower future return. The combination of Proposition 3 and Figure 1 leads us to Testable Hypothesis I.

Testable Hypothesis I (Unconditional Predictive Ability of Patent Thicket). *Firms with deeper patent thicket have lower expected future returns, ceteris paribus.*

[Figure 1 here.]

With a proxy of patent thicket, we run direct asset pricing tests on Testable Hypothesis I in the coming Section 3.4.1.

Figure 2 shows the relationship between β_t and n. We report the relationship for all feasible θ_t in Panel A and its cross-sections for the 25^{th}, 50^{th}, and 75^{th} percentiles of θ_t in Panel B. As we can see, β_t is decreasing in n for any θ_t. The combination of Proposition 4 and Figure 2 gives Testable Hypothesis II.

Testable Hypothesis II (Systematic Risk Exposure and Patent Thicket). *Firms with deeper patent thicket have lower exposure to the systematic risk, ceteris paribus.*

[Figure 2 here.]

It is worthwhile to mention that Theoretical Prediction II implies the predictability of the patent thicket to the future return is due to factor loading rather than abnormal return or mispricing if we can construct a perfect proxy of the systematic risk. Even though admittedly the market factor is not a perfect proxy of systematic risk, the Fama-MacBeth regressions conducted in Section 3.4.4 to examine the relationship between market beta and patent thicket shed light to Testable Hypothesis II.

Panel A of Figure 3 is a surface chart illustrating the relationship between $\left| dE_t \left[R_t\right] / dn \right|$ and A for each feasible θ_t, and Panel B is the cross-sections for the 25^{th}, 50^{th}, and 75^{th} percentiles of θ_t corresponding to the surface chart in Panel A. Consistent with Proposition 5, Figure 3 displays the pattern that for any θ_t, $\left| dE_t \left[R_t\right] / dn \right|$ is increasing in A. Therefore, we develop Testable Hypothesis III on the basis on Proposition 5 and Figure 3.

Testable Hypothesis III (Predictive Ability of Patent Thicket Conditional on Patent Portfolio Size). *As patent portfolio size increases, the predictive power of the patent thicket to the future return is stronger, ceteris paribus.*

[Figure 3 here.]

We run empirical tests on Testable Hypothesis III in Section 3.4.6, with a proxy of patent thicket and various proxies of patent portfolio size.

The relationship between $\left| dE_t \left[R_t\right] / dn \right|$ and ξ for each feasible θ_t is illustrated in Panel A of Figure 4 and the relationship between $\left| dE_t \left[R_t\right] / dn \right|$ and ξ for the 25^{th}, 50^{th}, and 75^{th}

percentiles of θ_t is drawn in Panel B of Figure 4. Both show that for any θ_t, $\left| dE_t\left[R_t\right]/\,dn \right|$ is increasing in ξ, which is consistent with Proposition 6. We develop Untestable Hypothesis I.

Untestable Hypothesis I (Predictive Ability of Patent Thicket Conditional on Patent value). *As patent value increases, the predictive power of the patent thicket to the future return is stronger, ceteris paribus.*

[Figure 4 here.]

From the prospective of empirical studies, the patent value, ξ, is not measurable so far, thus we leave the test of Untestable Hypothesis I to future research.

Panel A's of Figures 5 and 6, respectively, displays the relationship between $\left| dE_t\left[R_t\right]/\,dn \right|$ and a and between $\left| dE_t\left[R_t\right]/\,dn \right|$ and b for all feasible θ_t. Panel B's represent the relationships for the 25^{th}, 50^{th}, and 75^{th} percentiles of θ_t. Consistent with Proposition 7, Figures 5 and 6 deliver the message that $\left| dE_t\left[R_t\right]/\,dn \right|$ is increasing in a or b for any θ_t. As innovation efficiency is the inverse of a or b, we conclude Testable Hypothesis IV based on Proposition 7 and the combination of Figure 5 and Figure 6.

Testable Hypothesis IV (Predictive Ability of Patent Thicket Conditional on Innovation Efficiency). *As innovation efficiency decreases, the predictive power of the patent thicket to the future return is stronger, ceteris paribus.*

[Figure 5 and Figure 6 here.]

In Section 3.4.7, we will empirically test Testable Hypothesis IV with proxies of patent thicket and innovation efficiency.

3 Empirical Analysis

In this section, we describe our data collection, empirical strategies, and test results for the following testable hypotheses developed in Section 2. First, in a cross-sectional setting, firms facing deeper patent thickets provide lower expected stock returns (Testable Hypothesis I). Second, such a negative relation is stronger for firms with larger patent portfolios and firms with lower innovation efficiency (Testable Hypotheses II, III and IV, respectively).

3.1 Data

Our sample includes firm-year observations in the intersection of stock transaction data from Center for Research in Security Prices (CRSP), accounting data from Compustat, and the patent database constructed by Kogan, Papanikolaou, Seru, and Stoffman (2012). We then limit our sample to only firm-years with domestic common shares traded on NYSE, AMEX, and NASDAQ from 1963 July to 2012 June and exclude financial and other firms whose Fama-French 48 industry classification codes (Fama and French, 1997) are between FF44 and FF48 (banking, insurance, real estate, trading, and others).[19] Patent data constructed by Kogan, Papanikolaou, Seru, and Stoffman (2012) includes detailed information (patent number, patent assignee's CRSP identifier, and patent grant date) on all U.S. patents granted by the U.S. Patent and Trademark Office (USPTO) between 1926 and 2010. However, the references (citations) made by every granted patent that is necessary for our patent thicket proxy is available only since 1962. We restrict our sample from 1977 to 2010, since the accounting treatment of R&D expense reporting was standardized in 1976, according to Financial Accounting Standards Board Statement No. 2, and take one more year to ensure data comprehensiveness. Note that all of the results in this paper will not change much if we use the sample from 1962 to 2010 (not reported).

3.2 Empirical Measures

We discuss empirical proxies for patent thicket, patent portfolio size, R&D investment, innovation efficiency, and other control variables as follows.

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[19] We also exclude closed-end funds, trusts, American Depository Receipts, Real Estate Investment Trusts, units of beneficial interest, and firms with negative book equity following Fama and French (1993).

Patent thicket. We use the definition of Ziedonis (2004) and define a firm's patent thicket index as adjusted value of one minus concentration index of the ownership of cited patents as follows,[20]

$$PT_{i,t} = Frag_{i,t} \cdot \frac{Numpats_{i,t}}{Numpats_{i,t} - 1}, \text{ when } Numpats_{i,t} > 1;$$

$$PT_{i,t} = Frag_{i,t}, \text{ when } Numpats_{i,t} = 1,$$

where

$$Frag_{i,t} = 1 - \sum_{j=1}^{J} \left(\frac{Numcites^{j}_{i,t}}{Numcites_{i,t}} \right)^2, \ i \neq j.$$

$PT_{i,t}$ refers to an adjusted patent thicket index of firm i in year t which is adjusted for downward bias pointed out by Hall (2005), and $Frag_{i,t}$ refers to an "ownership fragmentation index" of patents cited by firm i in year t based on patents granted to firm i in years $t-4$ to year t.[21] $Numcites^{j}_{i,t}$ denotes the total number of citation made by firm i's patents granted in the most recent five years (from year $t-4$ to year t) to firm j's patents granted earlier ($j \neq i$),[22] $Numcites_{i,t}$ stands for the total number of citations made by firm i's patents granted in the most recent five years (from year $t-4$ to year t) to all other firms' patents. By definition, $Numcites_{i,t} = \sum_{j=1, j\neq i}^{J} Numcites^{j}_{i,t}$. $Numpats_{i,t}$ represents the total number of firm i's patents granted in the most recent five years (from year $t-4$ to year t). When $Numpats_{i,t} > 1$, we use the adjustment factor $Numpats_{i,t}/(Numpats_{i,t} - 1)$ to help mitigate the inflation driven by firm i's patent number as firms owning more patents naturally make more citations.[23] Thus, our patent thicket proxy is patent portfolio size-neutral, consistent

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[20] This patent thicket measure is applied in other studies such as Cockburn and MacGarvie (2009), Cockburn, MacGarvie, and Mueller (2010), Galasso and Schankerman (2010), Cockburn and MacGarvie (2011), Graevenitz, Wagner, and Harhoff (2013), Noel and Schankerman (2013), and others.

[21] It is common to use a five year window to construct proxies related to innovation in the literature, see Chan, Lakonishok, and Sougiannis (2001) and Hirshleifer, Hsu, and Li (2013).

[22] Citations to a firm's own patents are excluded from the proxy since they are unrelated to any patent royalty and infringement issues.

[23] Let us consider the following examples: (i) if firm i has three patents and each of them cites only one prior patent (owned by different firms), its fragmentation index is $1 - 3 \times (0.3)^2 = 0.67$ before adjustment and its PT is 1 after the adjustment; (ii) if firm j has two patents and each of them cites only one prior patent (owned by different firms), its fragmentation index is $1 - 2 \times (0.2)^2 = 0.50$, and its PT is also 1 after the adjustment; and (iii) if firm k has only one patent that cites only one prior patent, its fragmentation index and PT are both 1. Since it is natural to cite more prior patents when a firm owns more patents, the

with the setting of our simple model that each firm is endowed with one patent in the beginning. Also, such size-neutrality facilitates our interpretation of test results as they will be less subject to various size issues. Nevertheless, the role of patent portfolio and its interaction with patent thicket are considered in our full model and later analyses.

Based on the patent data of Kogan, Papanikolaou, Seru, and Stoffman (2012), we calculate public firms' annual PT variable from 1981 to 2010 as it takes five years (1977-1981) to calculate the first PT value. A high PT index suggests that a firm invents in deeper patent thicket because it is confronted with less concentrated (more fragmented) patent ownership. Based on our model, this firm will have to negotiate royalties or cross-license with more patent owners in order to exploit its growth option in patents, such as converting a patent into a new product. A major assumption of our model setting is that such a negotiation process is costly and increases with the number of owners of related prior patents. Thus, alternative proxies of patent thicket would be the number of firms that are cited by a firm's patent portfolio (i.e., the number J). Although such a proxy is intuitive, it likely increases with the firm's patent portfolio size. A firm with more patents naturally cites prior patents from more different firms and thus has a higher value of J.[24] Moreover, such a proxy neglects the reality that a firm may directly exploit a patent without negotiating with some minor patent owners.[25] Such an action is intuitive and reasonable because, even if the firm is sued for patent infringement later, the associated litigation risk and settlements associated these minor shares will be lower.

Four important issues related to our patent thicket proxy merit more discussions. First, using the patent data sample of Kogan, Papanikolaou, Seru, and Stoffman (2012), we identify

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adjustment helps us mitigate the inflation due to patent size of a firm.

[24] In unreported tests, we replicate all of our analysis using the number J as the proxy of patent thicket and obtain qualitatively consistent results.

[25] Let us consider the following two scenarios: (i) if 50% of the citations made by firm i's only patent belong to firm j and the other 50% of the citations made belong to firm k, firm i's PT is $1-2\times(0.5)^2=0.5$; (ii) if 90% of the citations made by firm h's only patent belong to firm j and the other 10% of the citations made belong to firm k, firm i's PT is $1-(0.9)^2-(0.1)^2=0.18$. In comparison with firm i, it is easier for firm h to exploit the patent without reaching an agreement with firm k because the expected penalty (if any) is relatively lower. The value J is 2 in both scenarios, thus our PT measure is better than J.

patent ownership by their CRSP identifiers (permno), which require both firms i and j to be publicly listed. As robustness checks, we construct an alternative patent thicket index using all patent assignees defined by PDPASS (including all entities such as universities, hospitals, governments, etc.) in the NBER Patent database that was first constructed by Hall, Jaffe, and Trajtenberg (2001) and obtain consistent results (unreported).

Second, we use the citations made by a firm's patents to measure its patent thicket based on two reasons. U.S. patent laws require patent applicants to provide a full list of references including prior patents and documents known to be relevant ('duty of candor'). Such a reference list will later be reviewed and supplemented by patent examiners. Such a legal requirement of completeness enables us to assume that the reference list reflects the distribution of prior knowledge a patent is based on. More importantly, the reference list allows us to track the owners of prior patents a firm may have to negotiate in order to exploit its patent.[26]

Third, we use the grant date of patents that is public information to construct our PT measure. Since granted patents and their references are fully disclosed by the United States Patent and Trademark Office (USPTO) in the weekly Official Gazette, the PT measure based on granted patents until year t is publicly observable at the end of year t and can thus be used for portfolio sorting at the end of June of year $t + 1$.

Fourth, our PT measure is silent on the bargaining power of other patent owners because it only considers their shares of being cited rather than the strength of their patent portfolios. However, in reality, some patent owners are more powerful than others because they have stronger patent portfolios or are more aggressive in initiating patent lawsuits (Lanjouw and Schankerman, 2004). In later section, we will propose empirical proxies and control for the bargaining power of counterparties (i.e., patent owners cited by the focal firm).

Patent portfolio size. One implication of our model is that the negative relation

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[26] Even if the list is incomplete or some citations it contains are irrelevant, it still appropriately approximates the fragmentation of patent ownership a firm faces unless the missing and irrelevant citations affect the distribution of patent ownership in any systematic way.

between patent thicket and expected stock returns is stronger for firms with a bigger patent portfolio (or more growth options), due to higher systematic exposure. We cannot directly use the total number of firm i's patents granted in the most recent five years (i.e., $Numpats_{i,t}$ mentioned above) as a proxy for patent portfolio size because this number often increases with the firm's assets in place. In the transition from our simple model to our full model, we increase the number of patents owned by a firm from 1 to A with fixing assets in place (P_t^I). Thus, we measure the firm's patent portfolio size using $Numpats_{i,t}$ divided by the firm's book equity (total asset) at the end of year t, denoted by CTBE (CTA). When a firm's CTBE or CTA is higher in a period, it is regarded as having a larger patent portfolio and thus carrying more growth options.

Moreover, we also use R&D investment as alternative proxies of the size of growth options (A in our model). Our R&D investment measures include RDBE and RDA that are defined as accumulated R&D expenses in years $t-4$ to t (with 20% obsolescence rate) scaled by book equity and total asset at the end of year t, respectively.

Innovation efficiency. Another implication of our model is that the negative relation between patent thicket and expected stock return is stronger for firms that is less efficient in innovation. To be precise, when a firm's parameters a and b are larger, it is less efficient in innovation as it incurs higher costs to exercise the option (i.e., convert its patent into new products as well as profits). We measure firm i's innovation efficiency ("IE" hereafter) in year t using firm i's number of granted patents in year t divided by its one plus total R&D expenses accumulated in years $t-6$ to $t-2$ (with 20% obsolescence rate). We adopt a two-year lag in the denominator because it usually takes two years for an invention (and a related patent application) to be granted (Hall et al., 2001). This proxy is a modified version based on Cohen, Diether, and Malloy (2013) and Hirshleifer, Hsu, and Li (2013) and has been shown by these studies that firms being more innovatively efficient perform better in future profitability. Such an empirical design is based on a premise that a firm's efficiency in converting R&D into patents approximates its efficiency in converting patents into new products and profits.[27]

Other control variables. We also consider the following firm characteristics: Characteristics conventional in asset pricing literature, such as market size, book-to-market ratio, momentum, and reversal, and characteristics considered in industrial organization literature, such as counter-parties' bargaining power, product market competition, and patent quality of the focal firm. We use a firm's stock market capitalization at the end of June of year $t+1$ as the proxy of market size (SIZE). A firm's book-to-market ratio (B/M) is defined as its book value of equity scaled by its market size at the end of year t. Accumulated stock returns over the previous five and eleven months (with a one-month gap between the holding period and the current month) are defined as two proxies of momentum (MOM11 and MOM6), and stock returns in the previous one month as a the proxy of reversal (REV). Following the idea of Bessen (2004) and Noel and Schankerman (2013), we define citation-based counter-parties' bargaining power (BPC) as the total accumulative forward citations received by the other firms which are ever cited by the focal firm until year t. Similarly, patent-stock-based counter-parties' bargaining power (BPP) is defined as the total number of patent granted to the other firms which are ever cited by the focal firm until year $t-1$. We proxy product market competition (PMC) as one minus the concentration index of sales in industry in year t, following Hou and Robinson (2006). Finally, According to Harhoff, Narin, Scherer, and Vopel (1999), Hall, Jaffe, and Trajtenberg (2005), and Moser, Ohmstedt, and Rhode (2011), patent quality (PQ) is proxied with the total accumulative forward citations of the patents in the previous five years granted to the focal firm.

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[27] Our alternative choices are to collect and estimate a firm's new products and innovation-related profits by its patent number, but these approaches are subject to the following issues: first, even if we collect the number of new products, the economic value and quality of each new product cannot be measured as there is no standard for new products. Second, the estimation of innovation-driven profits could be difficult and subject to estimation errors. We view our use of patents scaled by R&D as a proxy of IE as a more straightforward and objective choice.

3.3 Summary Statistics

Table I Panel A presents the proportion of patents granted to listed firms among all patents in each year from 1981 to 2010. It increases smoothly from about 35% to 40% during our sample period. We do not observe significant spikes and drops in the time series of the ratio, suggesting a reasonably steady propensity in patenting among public firms since 1981.

[Table I here.]

In Table I Panel B, we report the summary statistics of the PT measures for firms in each industry accordingly to the FF48 industry classification (Fama and French, 1997) since 1981. We first find that the industries with the most valid PT observations are those often regarded as high-tech ones, including medical equipment, pharmaceutical products chemicals, construction materials, machinery, business services, computers, and electronic equipment. Aircraft industry has the highest average PT (0.94), while agriculture industry and restaraunts, hotels, and motels industry have the lowest average PT (0.72). In addition, the standard deviations of PT range from 0.08 to 0.36. These statistics suggest heterogeneity in industry-level patent thicket: Some industries are highly fragmented in patent ownership, while some industries are less fragmented. To further relieve the concern of bias potentially caused by industry heterogeneity, we also consider two ways of industry adjustment in our empirical tests, one is substracting variable of interst from the weighted average in industry, and the other is sorting characteristics within industry.

In Table II Panel A, we present the time-series averages of cross-sectional averages of PT and other firm characteristics that are either related to our model implications or known to predict stock returns. We categorize all firm-year observations into six groups: for each year, firms with PT are assigned to quintile portfolios labelled "Low", "2", "3", "4", and "High" groups in order based on the quintiles of PT. Firms without PT are grouped together as the "No" group. On average, there are about 190 firms in each quintile and $3,366$ firms in the "No" group. Although there are much more firms without PT, the firms in the "No" group are much smaller. The average sizes of firms in the "No", "Low", "2", "3", "4", and "High" groups are \$1,066, \$2,212, \$3,448, \$4,329, \$6,972, and \$6,329 respectively in millions. In other words, firms with PT cover around 55% of the total stock market capitalization, which form a substantial set that merits in-depth investigation from both the perspectives of stock market and innovation.

We also report average patent portfolio sizes (CTBE/CTA and RDBE/RDA), innovation efficiency (IE), and other firm characteristics of quintile portfolios in Table II. We do not find any clear pattern of these variables across quintile PT portfolios, suggesting that PT is a unique dimension of technology competition dynamics as it is not associated with patent portfolio size and efficiency. Firms with PT are more R&D-intensive (measured by RDBE and RDA) than firms without PT. We also find that R&D intensity slightly decreases with PT. According to Cohen and Levinthal (1989), firms invest in R&D not only to create patents but also to absorb their competitors' innovation and knowledge. Thus, such a decreasing pattern can be interpreted as that R&D-intensive firms absorb more knowledge from different firms and are thus less dependent on only a few prior patents owners.[28] The book-to-market ratios (B/M) of firms with PT measure are much lower than those of firms without. This is intuitive because firms with patents, a necessary condition for our PT calculation, often carry higher growth opportunities due to their innovativeness and patenting activities. We do not observe any clear pattern in momentums (measured by MOM11 and MOM6), reversals (proxied by REV), counter-parties' bargaining power (measured by BPC and BPP), and product market competition (proxied by PMC) across all portfolios. Firms with a deeper patent thicket tend to grant patents with higher quality (i.e., larger PQ).

[Table II here.]

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[28] We note that the predictive power of R&D intensity for stock returns is sensitive to the scaling factor, as shown in Deng, Lev, and Narin (1999), Chan, Lakonishok, and Sougiannis (2001), and Lev, Sarath, and Sougiannis (2005). These studies find that only R&D scaled by market equity is able to predict stock returns, which makes us doubt if the R&D effect is mainly attributed to the size effect that small firms in general provide higher subsequent stock returns. Thus, we have to take the size effect into account when we attempt to differentiate the PT effect from the R&D effect.

Table II Panel B presents the time-series averages of the cross-sectional correlation coefficients between PT and all other variables. We find that PT only weakly correlates with other firm characteristics. The Pearson correlation coefficients between PT and other patent-based variables (CTBE, CTA, and IE) are only 0.01. Such a weak correlation confirms our observation in Panel A that PT is a unique dimension of technology competition dynamics different from patent portfolio size and efficiency. On the other hand, although Panel A shows a decreasing pattern of R&D intensity along PT, the Pearson correlation coefficients between PT and R&D-based variables (RDBE and RDA) range from -0.06 to 0.00, which is low in absolute magnitude. In addition, PT positively correlates with size and negatively correlates with the book-to-market ratio, consistent with Panel A. PT is almost uncorrelated with momentums and reversals. PT is only weakly correlated with firm's characteristics in counter-parties' bargaining power, product market competition, and patent quality (the correlation coefficient is 0.07 for BPC and BPP, -0.01 for PMC, and 0.06 for PQ). Lastly, firm fundamentals correlates with PT to a small extent (the correlation coefficient is 0.06 for ROA, -0.02 for IA, and 0.00 for SA). Overall, these weak correlations indicate that patent thicket is a firm characteristic which is distinct from these commonly known return predictors and proxies of firm fundamentals.

3.4 Results

3.4.1 Patent Thicket and Expected Stock Returns

We use portfolio sorting analysis to test Testable Hypothesis I of our model, i.e., firms with deeper patent thicket provide lower expected stock returns. At the end of June of year t from 1982 to 2011, we sort firms with non-missing PT measure into five PT groups (Low, 2, 3, 4 and High) based on the quintiles of PT in year $t-1$. In addition, we label firms with missing PT measure as members in the "No" group. We also construct a zero-cost portfolio by longing a unit of "High" PT portfolio and shorting a unit of "Low" PT portfolio and label this portfolio as "High-Low (or H-L)". Since our PT measure is based on granted patents and references made by these patents that are disclosed to the public in the weekly Official Gazette of the USPTO, all information related to PT in year $t-1$ is publicly observable at the end of year $t-1$ (or the first week of year t). Nevertheless, to make our results comparable to prior studies, we use a six-month lag in forming the PT portfolios at the end of June of year t and hold these portfolios for the next twelve months until June of year $t+1$. All portfolios are value-weighted as we use each firm's lagged market capitalization to determine its weight in a portfolio.

Table III reports the monthly stock returns (the time-series averages), alphas, and betas of all PT portfolios. We use the monthly stock returns in excess of one-month Treasury bill rate ("Excess Returns") as our main results. To further relieve the concern of bias potentially caused by industry heterogeneity, in Panel A of Table F.I in Online Appendix we report "Industry-adjusted Excess Returns", which is defined as the difference between Excess Returns and the weighted average Excess Return in the industry (Fama-French 48 industry classifications), and in Panel B of Table F.I we sort patent thicket within industry to ensure that all firms in each industry are evenly distributed across quintiles.

[Table III here.]

In the first column of Table III, we first find that the average excess returns of the quintile PT portfolios (from Low to High) are higher than that of the no group (-0.39%). This is intuitive that patenting firms' returns are in general higher than non-patenting firms' (Hirshleifer et al., 2014). More importantly, we find a decreasing pattern in excess stock returns from the low group to the high group, as the average excess returns of the "Low", "2", "3", "4", and "High" groups are 1.46%, 1.44%, 1.25%, 1.28%, and 1.04% per month, respectively. The excess returns of the High-Low portfolio average -0.42% per month, which is statistically significant at the 5% level and provides preliminary support to Testable Hypothesis I.

As we have argued in the modeling part, expected stock returns decrease with patent thicket due to lower systematic risk exposure ($d\beta_t/dn < 0$). Thus, it is necessary for us to examine the risk loadings of the PT portfolios on various systematic risk factors. We take systematic risk into account by conducting time-series regressions based on the CAPM model (including a single market factor), the Fama-French three-factor model (including the market factor, the size factor, and the value factor, following Fama and French (1993)), the Fama-French five-factor model (including Fama-French three factors plus the profitability factor and investment factor, following Fama and French (2015)), the Fama-French six-factor model (including Fama-French five factors and the momentum factor, following Carhart (1997)) and the Fama-French six-factor model augmented by the investment-minus-consumption (IMC) factor proposed by Papanikolaou (2011), or the innovation originality factor (IO) proposed by Hirshleifer, , Hsu, and Li (2014), or the innovative efficient-minus-inefficient (EMI) factor proposed by Hirshleifer, Hsu, and Li (2013), or all of the three.[29] Controlling for these additional factors in time-series regressions helps adjust for risk premia associated with common systematic risk factors as well as other technology-related factors.

We find that the negative relation between PT and subsequent stock returns remain after we have controlled for conventional systematic risk factors. First, the alphas estimated as the regression intercept term decreases from the Low group to the High Group. Second, we observe a decreasing pattern in market betas, confirming our risk-based argument that firms with deeper patent thicket have lower expected future returns because of their smaller exposure to systematic risk. This finding will be further examined in Section 3.4.4. Third, we find that low-PT firms have positive loadings on the size factor and negative loadings on the value factor, and the opposite pattern is found in high-PT firms. Fourth, the low-PT firms have insignificant loadings on the profitability factor and the investment factor, while high-PT firms have positive and significant loadings on these two factors. Fifth, low-PT and high-PT firms load positively and negatively on the momentum factor. Lastly, the alphas of the high-low portfolio are −0.37%, −0.53%, −0.71%, and −0.62% for the CAPM model, the Fama-French three-factor model, the Fama-French five-factor model, and the Fama-French

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[29] Our sample period for regressions including IO or EMI are limited to 1982 July to 2008 June due to availability of these two factors.

six-factor model, respectively.

We then examine the alphas when technology-related factors are included in the Fama-French six-factor model and find even stronger results. We do not find an important role of IMC or IO in explaining the PT effect, and find that low-PT and high-PT firms respectively load negatively and positively on the EMI factor. These results suggest that even though firms in high-PT group suffer from deeper patent thicket they are more efficient in innovation. However, all these technology-related factors cannot explain the PT effect; instead, their existence strengthens such pattern: the alphas of the high-low portfolio are -0.67%, -0.70%, -0.75%, and -0.77% when IMC, IO, EMI, and all three exist in the Fama-French six-factor model, respectively.

Similar patterns show up in Table F.I in Online Appendix. The average monthly industry-adjusted returns of the "No", "Low", "2", "3", "4", and "High" groups in Panel A are -0.40%, 0.13%, -0.02%, -0.08%, -0.05%, and -0.23%, respectively. Those in Panel B are -0.39%, 1.49%, 1.41%, 1.27%, 1.24%, and 1.07%, respectively. The adjusted returns of the High-Low portfolio average -0.36% per month in Panel A and -0.42% in Panel B, both of which are statistically significant at the 1% level and further supports Testable Hypothesis I. For brevity, we focus on the High-Low portfolio in our discussions. The alphas of the high-low portfolio range from -0.30% to -0.38% in Panel A and from -0.35% to -0.54% in different model specifications.

Our portfolio results presented in Table III and Table F.I in Online Appendix not only support our Testable Hypothesis I but also indicate a unique role of patent thicket in expected stock returns. The negative relation between patent thicket and subsequent stock returns cannot be explained by common risk factors, technology-related factors, or industry heterogeneity. In our model, we argue that such a relation can be attributed to lower systematic exposure of firms being confronted with greater patent thicket. However, empirically, we cannot find a risk factor to fully explain the spreads in returns and alphas of portfolios sorted on patent thicket. Our interpretation is that existing factors, even those based on patent performance, do not fully capture the risk premia associated with technology competition dynamics.[30] It is reasonable in the sense that a firm's risk in technology competition includes not only its performance in R&D and patents, but also its competitors' performance. On the other hand, when we use CAPM model we do see high-PT firms have low loadings on the market factor. To further understand the return predictability, we will consider the conditional CAPM tests in Section 3.4.4.

Figure 7 visualizes the performance of our long-short strategies from July 1982 to June 2012. Over the past 30 years, $1 investment in our zero-cost portfolio in 1982 becomes $3.78 in 2010. The performance of our strategy is quite stable over time except that it soars during the ".com bubble". Industry adjustments does not change much the measurement of strategy performance. For example, if we compare the performance with the industry weighted averages, the accumulative return until 2012 is $3.36, and if we construct the zero-cost portfolio by longing the bottom quintile and shorting the top quintile within each industry, the accumulative return is $3.97.

[Figure 7 here.]

3.4.2 Controlling for Common Return Predictors in Asset Pricing

The negative relation between patent thicket and expected stock returns reported in Section 3.4.1 demands necessary robustness checks. Specifically, we examine if the return predictability associated with patent thicket is distinct from the effects of well-know return predictors including size, book-to-market ratio, momentum, and reversal. We construct two-way sorted portfolios on our PT measure and one of these predictors. At the end of June of year t from 1982 to 2011, we independently sort firms with non-missing PT measure on both PT and one of the control variables (SIZE, B/M, MOM11, MOM6, and REV) into 15 groups based on the quintiles of PT in year $t-1$ and the 30^{th} and 70^{th} percentiles of SIZE, MOM11,

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[30] Other explanations for significant alphas include misspecified regression models, non-linearity issues, and market frictions.

MOM6, or REV in June of year t, or B/M at the end of year $t-1$.[31] We then calculate the value-weighted excess returns of these 15 portfolios from July of year t to June of year $t+1$.

The average returns of these 15 portfolios are reported in Table IV (Panels A, B, C, D, and E are for SIZE, B/M, MOM11, MOM6, and REV respectively). We first focus on Panel A controlling for firm size. In the upper part of Panel A, we report the average excess returns of 15 portfolios and the average excess returns of one particular PT quintile portfolio across all three size groups (in the column labelled "Average"). We find that, within each size group and the average column, top-quintile PT portfolio always underperform bottom-quintile portfolio in average returns.

[Table IV here.]

In addition, we construct the following four long-short portfolios to examine if the PT effect remains after other effects being controlled. Within the each group (Small, 2, and Big) of the control variable, we form a high-minus-low portfolios by long a unit of the top-quintile PT portfolio and short a unit of the bottom-quintile PT portfolio. The returns of three portfolios are used to measure the return predictability associated with PT within each control group. Moreover, we construct the fourth portfolio (the average high-minus-low portfolio) by longing one third in the top-quintile PT portfolio of each group of the control variable and shorting one third in the bottom-quintile PT portfolio of each group. The monthly return of this portfolio is equivalent to the average of three long-short portfolios' monthly returns. More importantly, the average monthly returns of the average portfolio can be regarded as the PT effect net of the influence of the control variable (e.g., Fama and French, 1993).

In the lower part of Panel A, we first report the average returns of the high-minus-low portfolio and standard errors (in parentheses) across three size groups (Small, 2, and Big). Although these returns are not always significantly negative, the returns on the average high-minus-low portfolio are -0.37% with statistical significance at the 5% level. We further

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[31] The breakpoints for SIZE and B/M are based on NYSE-listed stocks, following Fama and French (1993).

control for systematic risk by regressing the returns of each high-minus-low portfolio on common systematic risk factors and technology-related factors, and report the alphas of nine factor models employed in Table III. We find that all alphas are negative; more importantly, the alphas of the average high-minus-low portfolio are significantly negative in all factor models. Industry-adjusted results presented in Panels A1 and B1 of Table F.II in Online Appendix are similar. Consequently Panels A of Table IV, as well as Panels A1 and B1 of Table F.II, collectively support that the negative relation between PT and subsequent stock returns cannot be attributed to size. More importantly, the return predictability of big firms, especially in Panels A1 and B1 of Table F.II, suggests that the PT effect is not a small-size effect as the high-minus-low portfolio provides significantly negative alphas among big firms.

In Panels B, we find that the PT effect remains when the book-to-market ratio is controlled. The average returns and alphas of all high-minus-low portfolios are negative. One intriguing observation is that the PT effect is not simply a growth-firm phenomenon as we also find strong PT effect among firms with high book-to-market ratios. This suggests that the patent thicket is a prevailing phenomenon and its influence on expected stock returns is distinct from that of the growth or value effect. Similar results show up in Panel A2 and Panel B2 of Table F.II.

The PT effect prevails in all momentum and reversal groups in Panels C, D, and E of Table IV as we find all negative average returns and alphas in different high-minus-low portfolios. The average returns and alphas of high-minus-low portfolios are negative (and largely significant) in both high- and low- momentum and reversal groups. Moreover, the PT effect remains when we control for either momentum or reversal as shown in the Average column. Overall, Table IV not only confirms the one-way sorted portfolio result but also highlight the unique role of patent thicket in explaining the cross-section of stock returns. Similar results can be observed in Table F.II.

3.4.3 Controlling for Variables in Industrial Organization

To ensure the robustness of our main result, we not only check the return predictors commonly known in asset pricing literature, but also consider firm characteristics of interest in industrial organization literature, such as counterparties' bargaining power, product market competition, and patent quality. Following the same empirical practice, we construct two-way sorted portfolios on our PT measure and one of these characteristics. At the end of June of year t from 1982 to 2011, we independently sort firms with non-missing PT measure on both PT and one of the control variables (BPC, BPP, PMC, and PQ, as defined in Section 3.2) into 15 groups based on the quintiles of PT in year $t-1$ and the 30^{th} and 70^{th} percentiles of BPC, BPP, PMC, or PQ at the end of year $t-1$. We then calculate the value-weighted excess returns of these 15 portfolios from July of year t to June of year $t+1$.

Panels F and G imply that the PT effect remains even when we control for counterparties' bargaining power. The alphas of average H-L spreads are negative and significant at 10% level. PT is in effect not only for firms facing giant counter-parties (i.e., big BPC or BPP) but also for firms held up by small counter-parties (i.e., small BPC or BPP). These small counter-parties may be "patent trolls". One important observation is that giants pose a larger hindering impact on the focal firm through patent thicket channel. For example, in Panel G the H-L spread for big-BPP group is -0.52% and that for small-BPP group is -0.28%. Panels A6, A7, B6, and B7 of Table F.II. show that our results are not driven by industry heterogeneity.

The PT effect remains even when we control for product market competition, according to Panel H of Table IV, where we observe all negative average returns and alphas in different high-minus-low portfolios. There is evidence supporting that firms in more competitive industries suffer more from patent thicket. For example, when we control for all of the factors mentioned in the paper (FF6+3 specification), the H-L spread for big-PMC group is about twice bigger than that for small-PMC group in absolute value.

Panel I provides evidence that firms with patent of better quality suffer more from patent thicket problem. For instance, the H-L spread for big-PQ group is larger in absolute value and more signficant than that for small-PQ group. Again the average H-L alphas are negative and significant at 1% level, implying that the PT effect is not mainly attributable to patent quality. Table F.II further confirms the robustness.

All of the analysis mentioned above suggest that the patent thicket effect is more pronounced when counter-parties' bargaining power is greater, when product market competition is more fierce, and when the focal firm's patent quality is better. Nevertheless, even after controlling for all these conditions, we still find a significantly negative relation between patent thicket and subsequent stock returns, which confirms the important and distinct role of patent competition in asset pricing.

3.4.4 Tests of Conditional CAPM

Our main argument is that (a) firms with deeper patent thicket produce lower expected future returns because (b) these firms are less exposed to systematic risk due to lower option value. In Section 3.4.1 we have provided supportive evidence to part (a), and in this and the next sections, we implement direct tests for part (b) of our main argument. In this section, we employ the two-step Fama-MacBeth regression to test Testable Hypothesis II. In the first step, at each month from July 1982 to June 2012, for each firm we regress its monthly excess return on the market factor using the past 60-month window and get its time-series factor loading (beta) on the market factor. In the second step, at each month between 1982 and 2012, we do a cross-sectional regression of beta on patent thicket, and get the intercept and slope of patent thicket. We report the time-series averages of them, and test their significance based on the Newey-West standard errors. We also try to incorporate industry fixed effects. In Specificantion 1 of Table V, the slopes of PT are negative, and are significant when industry fixed effect is considered. We are concerned that Specification 1 may be biased because PT is heavily negatively skewed (Figure 8). To relieve this concern, we consider two ways of monotonic transformation of PT, the Ranked PT and the Transformed PT. For

Ranked PT, we sort PT into quintiles each year so that Ranked PT takes the integers from 1 to 5. Transformed PT is defined as $1/(1 - Frag)$, where $Frag$ is defined in Section 3.2.[32] As Figure 8 displays, the distribution of Transformed PT approximates to normal.

Specifications 2 and 3 of Table V show that the negative relationship between market beta and patent thicket remains for ranked PT and Transformed PT respectively. The slopes of PT in different model specifications are all negative and significant at 1% level. We can interpret the economic magnitude of the slopes: According to Specification 2, controlling for industry fixed effects, if a firm jumps from the lowest PT quintile to the highest, its market beta drops by around 0.13 (0.0315×4).

[Table V here.]

3.4.5 Tests of Future Fundamentals Volatility

In this section, we look into the future performance of the firms as a response to patent thicket. In our model, firms with deeper patent thicket are facing more difficulties in transforming real options into asset in place, therefore the volatility of their future performance should be smaller. We consider three dimensions of firm performance, profitability, investment, and sales. When a firm exercises its real options in the future, it should generate more profits, make more tangible investments, and sell more products. As a result, firms with deeper patent thicket are expected to reveal less volatile profitability, investment, and sales in the future. We measure profitability with ROA, which is defined as income before extraordinary items plus interest expenses divided by lagged total assets, investment with IA, which is defined as capital expenditure divided by lagged total assets, and sales with SA, which is defined as total sales divided by lagged total assets. Future performance volatility is proxied by the standard deviations of one of these three variables in the next five years.

To give a brief overview of the relationship between patent thicket and volatility of future fundamentals, we redo the one-way portfolio sorting analysis conducted in Section 3.4.1 and

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[32] Perhaps the most intuitive reason why we define Transformed PT as $1/(1 - Frag)$ is that in the ideal economy assumed by our model, we have $Frag = 1 - 1/n$ and Transformed PT is equal to n.

replace excess return with future performance volatility. That is, at the end of year t from 1981 to 2005, we sort firms with non-missing PT measure into five PT groups (Low, 2, 3, 4 and High) based on the quintiles of PT in year t, and track the value-weighted performance volatility for each PT group in year $t+1$ to $t+5$.

Panel A of Table VI reports the time-series averages of performance volatility in the five PT groups. It is shown that the future volatility of ROA, IA, and SA decreases with patent thicket. The t-test for the difference between high-PT group and low-PT group (labelled with H-L) suggests the statistical significance of such a decreasing pattern. For example, compared with their peers in low-PT group, firms in high-PT group are 2.13% lower (1% significance) in ROA, 0.79% lower (1% significance) in IA, and 1.94% lower (not significant) in SA. These differences are substantial in comparison with the volatility in each group.

Furthermore, we control for firms' patent portfolio size and check the relationship between patent thicket and volatility of future fundamentals free of the influence of the number of real options. At the end of year t from 1981 to 2005, we independently sorts firms with non-missing PT measure on both PT and one of patent portfolio size proxies (CTBE and CTA) into 25 groups based on their quintiles in year t. Then we track the value-weighted performance volatility in the next five years for each portfolio.

In Panel B of Table VI, we report the time-series averages of performance volatility in each portfolio. We observe that in general given the level of patent portfolio size, the future volatility of performance decreases with patent thicket, and given the level of patent thicket, the future volatility of performance increases with patent portfolio size. For example, in Panel B1, for the median level of CTBE (CTBE rank=3) ROA volatility decreases with PT from 5.08% to 2.54%, and for the median level of PT (PT rank=3) ROA volatility increases with CTBE from 3.41% to 6.76%. To estimate the effect of patent thicket on future performance volatility net of the number of real options (i.e., patent portfolio size), we follow the method mentioned in Section 3.4.2 and Section 3.4.3 and calculate the average future performance volatility of one particular PT quintile portfolio across all five patent portfolio size groups, and run t-test for the difference between the average of high-PT quintiles and that of low-PT quintiles (labelled as High-Low). Our results indicate that in general deeper patent thicket predicts lower future performance volatility, even after we control for patent portfolio size. For example, when we use CTBE to proxy for patent portfolio size, compared with their peers in low-PT group, firms in high-PT group are 4.12% lower (1% significance) in ROA, 1.20% lower (1% significance) in IA, and 4.06% lower (1% significance) in SA. Robustness checks for industry heterogeneity reported in Table F.III in Online Appendix provide similar results as Table V.

[Table VI here.]

3.4.6 Patent Thicket and Expected Stock Returns Conditional on Patent Portfolio Size

We construct two-way sorted portfolios to test Testable Hypothesis III of our model, i.e., the predictive ability of patent thicket for stock return increases with patent portfolio size. At the end of June of year t from 1982 to 2011, we independently sort firms with non-missing PT measure on both PT and one of patent portfolio size proxies (CTBE and CTA) into 25 groups based on their quintiles in year $t-1$. We sort patent portfolio size by quintiles instead of terciles in order to generate greater dispersion to highlight changes in the PT effect, if any. We then calculate the value-weighted returns of these 25 portfolios. In addition, within each patent portfolio size quintile, we construct five high-minus-low PT portfolios by longing a unit of the top-quintile PT portfolio and shorting a unit of the bottom-quintile PT portfolio. All these portfolios are held over the next twelve months (July of year t to June of year $t+1$).

Table VII presents the average returns and alphas of the 25 portfolios and the five long-short portfolios. In Table VII, we find that the average returns of the high-minus-low portfolios for the Small, 2, 3, 4, and Big size portfolios are -0.13%, -1.03%, -0.85%, -0.73%, and -1.76% respectively in Panel A for CTBE, and -0.29%, -0.68%, -0.56%, -1.23%, and

−1.24% in Panel B for CTA. All of them are significantly negative except the one for the Small portfolio. Robustness checks for industry heterogeneity are reported in Table F.III. For example, in Panel A of Table E.IV in Online Appendix, the industry-adjusted returns of the high-minus-low portfolios present a decreasing pattern (−0.17%, −0.58%, −0.54%, −1.22%, and −1.28% for the Small, 2, 3, 4, and Big CTA portfolios, respectively).

[Table VII here.]

Furthermore, the difference between high-minus-low spread in the biggest CTBE/CTA portfolio and that in the smallest CTBE/CTA portfolio, called "spread of spreads", is reported in the column labelled "B-S". Table VII supports Testable Hypothesis III, in the sense that with various proxies of patent portfolio size, the spread of spreads is always negative and significant. For example, the spread of spreads is −1.62% (1% significance) for CTBE, and −0.95% (5% significance) for CTA. We also find that the PT effect still survive when we include all factors in regressions. For example, the Fama-French six-factor model produces a smaller alpha of −1.56% (1% significance) and the nine-factor model considered in this paper yields a even smaller alpha of −0.95% (10% significance).

Another way to test Testable Hypothesis III is to use proxies of growth options associated with innovation activities. In Table VIII, we use two proxies of R&D intensity based on the premise that firms' R&D activities result in growth options, or some intellectual property that cannot be fully patented. Similar to Table VII, we independently sorts firms with non-missing PT measure on both PT and one of R&D investment proxies (RDBE and RDA) into 25 groups and track the value-weighted returns on these portfolios. We also construct five high-minus-low portfolios by longing a unit of the top-quintile PT portfolio and shorting a unit of the bottom-quintile PT portfolio within each R&D intensity quintile.

[Table VIII here.]

Evidence based on R&D intensity reported in Table VIII also supports Testable Hypothesis II. In Panel A for RDBE, the returns of the high-minus-low portfolios are 0.03%, −0.39%,

-0.83%, -0.45%, and -1.58% for the Small, 2, 3, 4, and Big RDA portfolios, respectively. A similar pattern is observed in Panels B for RDA. The difference between the high-minus-low spread in the biggest RDBE/RDA portfolio and that in the smallest RDBE/RDA portfolio ("spread of spreads") is presented in the column labelled "B-S". The "spread of spreads" and corresponding alphas are all negative and significant for RDBE but less significant for RDA. Overall, Table VIII supports Testable Hypothesis II by using different proxies of growth options.

3.4.7 Patent Thicket and Expected Stock Returns Conditional on Innovation Efficiency

We test Testable Hypothesis IV that the predictive ability of patent thicket for stock return decreases with innovation efficiency using two-way sorted portfolios. At the end of June of year t from 1982 to 2011, we independently sorts firms with non-missing PT measure on both PT and innovation efficiency (IE) into 25 groups based on their quintiles in year $t-1$. We then calculate the value-weighted returns of these 25 portfolios. Within each IE quintile, we construct five high-minus-low portfolios by longing a unit of the top-quintile PT portfolio and shorting a unit of the bottom-quintile PT portfolio. All these portfolios are held over the next twelve months (July of year t to June of year $t+1$), and their value-weighted returns are calculated.

Table IX presents the average returns and alphas of the 25 portfolios and the five high-minus-low portfolios. We find that the average returns of the high-minus-low portfolios are -0.92%, -0.58%, -0.83%, -0.04%, and 0.27% for the Small, 2, 3, 4, and Big IE portfolios, respectively. Such an increasing pattern remains when we control for other conventional risk factors and innovation-related factors. In fact, the spread becomes even greater when more factors are controlled for. For example, the alphas of the high-minus-low portfolios using the Fama-French six factors plus three innovation-related factors model are -1.49%, -1.18%, -0.70%, -0.30%, and -0.12% for the Small, 2, 3, 4, and Big IE portfolios, respectively. All these findings support Testable Hypothesis IV.

[Table IX here.]

To directly examine different PT effects under small and big innovation efficiency, we report the difference between the high-minus-low spread in the biggest IE portfolio and that in the smallest IE portfolios in the column labelled "B-S". The observation that the "spread of spreads" and corresponding alphas are negative and signficant at 1% level again supports Testable Hypothesis III that the negative effect of PT on expected stock returns is more pronounced among firms less efficient in innovation activities. Robustness checks with industry adjustments are reported in Table F.VI in Online Appendix.

4 Concluding Remarks

In this paper, we examine the effect of fragmented patent ownership (patent thicket) on asset pricing. We develop a model which proposes that the fragmented ownership of patents reduces the option value of patents and thus leads to low expected stock returns. In addition, the negative effect of patent thicket on expected stock returns is stronger for firms with bigger patent portfolios or lower innovation efficiency. These model implications are supported by our empirical tests using patent data. Our one-way sorted portfolio analysis indicates that firms confronted with deeper patent thicket are associated with significantly lower stock returns in the next twelve months. In addition, these firms have smaller market betas and smaller volatilities of firm performance in the next five years. Such a pattern cannot be explained by the exposure to common systematic risk factors, the predictive ability of firm characteristics, and industry heterogeneity. Our two-way sorted portfolio analysis supports the two conditional predictability implications, as we find stronger negative effects of patent thicket on subsequent stock returns among firms with more patents or R&D investment scaled by firm size and firms with fewer patents per R&D input. Our theoretical model and empirical collectively point to an important role of technology competition in asset pricing.

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A Appendix A: Proofs

A.1 Proof of Lemma 1

Stochastic integration gives $E_t\left[\int_\tau^\infty M_s\theta_s/M_t ds\right] = E_t\left[\rho\theta_t e^{-r(\tau-t)}\right]$ and $E_t\left[\int_\tau^\infty M_\tau/M_t\right] = E_t\left[e^{-r(\tau-t)}\right]$, which simplifies the expression of the firm's expected profit from exploiting the patent to,

$$E_t\left[\int_\tau^\infty \frac{M_s}{M_t}\xi\theta_s ds - \frac{M_\tau}{M_t}Q\right] = E_t\left[e^{-r(\tau-t)}\left(\rho\xi\theta_t - Q\right)\right].$$

We construct a risk-free portfolio, H_t, by longing one unit of the patent, P_t^O, and shorting $\partial P_t^O/\partial\theta_t$ units of fundamental asset θ_t, and the process of this risk-free portfolio follows,

$$\begin{aligned} dH_t &= dP_t^O - \frac{\partial P_t^O}{\partial\theta_t}\left[d\theta_t + (r+\kappa\sigma-\mu)\,\theta_t dt\right] \\ &= \left(\frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 P_t^O}{\partial\theta_t^2} - (r+\kappa\sigma-\mu)\,\theta_t\frac{\partial P_t^O}{\partial\theta_t}\right)dt. \end{aligned}$$

Imposing $dH_t = rdt$ gives the following ODE of P_t^o in the continuation region,

$$\frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 P_t^O}{\partial\theta_t^2} + (\mu-\kappa\sigma)\theta_t\frac{\partial P_t^O}{\partial\theta_t} - rP_t^O = 0, \tag{A1}$$

which should satisfy three boundary conditions,

$$\begin{aligned} \text{Absorbing-Barrier Condition: } & P_t^O(\theta_t\to 0) < \infty; \\ \text{Value-Matching Condition: } & P_t^O(\theta_t = \theta^*) = \rho\xi\theta^* - Q; \\ \text{Smooth-Pasting Condition: } & \left.\frac{\partial P_t^O}{\partial\theta_t}\right|_{\theta_t=\theta^*} = \rho\xi. \end{aligned}$$

Equation (A1) yields the general solution,

$$P_t^O = D\theta_t^\phi. \tag{A2}$$

Plugging equation (A2) into (A1) leads to,

$$\phi^\pm = \frac{-\left(\mu-\kappa\sigma-\frac{1}{2}\sigma^2\right) \pm \sqrt{\left(\mu-\kappa\sigma-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2}.$$

Because $r + \kappa\sigma - \mu > 0$, we have $\phi^+ > 1$ and $\phi^- < 0$. Absorbing-Barrier Condition implies that ϕ^- should be dropped. Plugging $P_t^O = D\theta_t^{\phi^+}$ into the Value-Matching Condition and Smooth-Pasting Condition gives,

$$\theta^* = \frac{\phi^+}{\phi^+ - 1}\frac{Q}{\rho\xi}, \text{ and} \tag{A3}$$

$$P_t^O = \left(\frac{\rho\xi}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+ - 1}{Q}\right)^{\phi^+ - 1}\theta_t^{\phi^+}. \tag{A4}$$

A.2 Proof of Lemma 2

Because we have shown in the proof of Lemma 1 that,

$$P_t^O = \sup_{\tau} E_t\left[e^{-r(\tau - t)}\left(\rho\xi\theta_\tau - Q\right)\right],$$

we can get

$$E_t\left[e^{-r(\tau^* - t)}\right] = \left[\frac{\phi^+ - 1}{\phi^+}\frac{\rho\xi}{Q}\right]^{\phi^+}\theta_t^{\phi^+}.$$

Therefore, the maximization problem of owner i is equivalent to,

$$\max_{q_i} \frac{q_i - c_i}{\left(\sum_{j=1}^{n} q_j\right)^{\phi^+}},$$

with the first-order condition (FOC) as,

$$\sum_{j=1}^{n} q_j - \phi^+(q_i - c) = 0. \tag{A5}$$

Summing the FOC across all i's, we have,

$$\sum_{j=1}^{n} q_j = \frac{\phi^+ \sum_{j=1}^{n} c_j}{\phi^+ - n},$$

and hence, the optimum can be obtained, i.e.,

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n\right)c_i + \sum_{j=1}^{n} c_j\right].$$

If $c_i = c_j = c$, then $q_i = q_j = q^*$ for $j = 1...n$, and we have,

$$q^* = \frac{\phi^+}{\phi^+ - n} c.$$

Note that, the expression leads to an implicit assumption that $\phi^+ > n$.

A.3 Proof of Lemma 3

Similar to the proof of Lemma 1, we can derive that,

$$E_t \left[\int_\tau^\infty \frac{M_s}{M_t} \xi \theta_s ds - \frac{M_\tau}{M_t} Q \right] = E_t \left[e^{-r(\tau - t)} \left(\rho \xi \theta_t - Q \right) \right], \text{ and}$$

$$\begin{aligned} E_t \left[\int_t^\tau \frac{M_s}{M_t} i_s ds \right] &= E_t \left[\int_t^\tau \frac{M_s}{M_t} (a + b\theta_t) ds \right] \\ &= \left\{ \frac{a}{r} - E_t \left[e^{-r(\tau - t)} \frac{a}{r} \right] \right\} + \left\{ b\rho\theta_t - E_t \left[e^{-r(\tau - t)} b\rho\theta_\tau \right] \right\}, \end{aligned}$$

in which we used the following property in the second equality:

$$\begin{aligned} E_t \int_t^\tau \frac{M_s}{M_t} \frac{\theta_s}{\theta_t} ds &= E_t \int_t^\infty \frac{M_s}{M_t} \frac{\theta_s}{\theta_t} ds - E_t \int_\tau^\infty \frac{M_s}{M_t} \frac{\theta_s}{\theta_t} ds \\ &= \rho - \frac{M_\tau}{M_t} \frac{\theta_\tau}{\theta_t} E_t \int_\tau^\infty \frac{M_s}{M_\tau} \frac{\theta_s}{\theta_\tau} ds = \rho - \frac{M_\tau}{M_t} \frac{\theta_\tau}{\theta_t} \rho. \end{aligned}$$

Hence, the price of the focal patent can be written as,

$$P_t^O = \sup_\tau E_t \left[e^{-r(\tau - t)} \left(\left(\rho\xi + b\rho \right) \theta_\tau - \left(Q - \frac{a}{r} \right) \right) \right] - \left(b\rho\theta_t + \frac{a}{r} \right).$$

We construct a risk-free portfolio H_t by longing one unit of the patent P_t^O and shorting $\partial P_t^O / \partial \theta_t$ units of fundamental asset θ_t, and the process of this risk-free portfolio follows,

$$\begin{aligned} dH_t &= dP_t^O - i_t dt - \frac{\partial P_t^O}{\partial \theta_t} \left[d\theta_t + \left(r + \kappa\sigma - \mu \right) \theta_t dt \right] \\ &= \left(\frac{1}{2} \sigma^2 \theta_t^2 \frac{\partial^2 P_t^O}{\partial \theta_t^2} - i_t - \left(r + \kappa\sigma - \mu \right) \theta_t \frac{\partial P_t^O}{\partial \theta_t} \right) dt. \end{aligned}$$

Imposing $dH_t = rdt$ gives the following ODE of P_t^o in the continuation region,

$$\frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 P_t^O}{\partial\theta_t^2} + (\mu - \kappa\sigma)\theta_t\frac{\partial P_t^O}{\partial\theta_t} - rP_t^O = a + b\theta_t, \tag{A6}$$

which should satisfy three boundary conditions,

$$\begin{aligned} \text{Absorbing-barrier Condition: } & P_t^O(\theta_t \to 0) < \infty; \\ \text{Value-matching Condition: } & P_t^O(\theta_t = \theta^*) = \rho\xi\theta^* - Q; \\ \text{Smooth-pasting Condition: } & \left.\frac{\partial P_t^O}{\partial\theta_t}\right|_{\theta_t=\theta^*} = \rho\xi. \end{aligned}$$

Equation (A1) yields the general solution,

$$P_t^O = D\theta_t^\phi - b\rho\theta_t - \frac{a}{r}. \tag{A7}$$

Plugging Equation (A7) into (A6) leads to,

$$\phi^\pm = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) \pm \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2}.$$

Because $r + \kappa\sigma - \mu > 0$, we have $\phi^+ > 1$ and $\phi^- < 0$. Absorbing-barrier Condition implies that ϕ^- should be dropped. Plugging $P_t^O = D\theta_t^{\phi^+}$ into the Value-matching Condition and Smooth-pasting Condition gives,

$$\theta^* = \frac{\phi^+}{\phi^+ - 1}\frac{Q - a/r}{\rho\xi + b\rho}, \text{ and} \tag{A8}$$

$$P_t^O = \left(\frac{\rho\xi + b\rho}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+ - 1}{Q - a/r}\right)^{\phi^+ - 1}\theta_t^{\phi^+} - b\rho\theta_t - \frac{a}{r}, \tag{A9}$$

in which we have an implicit assumption that $a < rnc$. Under such an assumption, $Q - a/r > 0$ so that the patent is worth of exploiting.

A.4 Proof of Lemma 4

Because we have shown in the proof of Lemma 3 that,

$$P_t^O = \sup_\tau E_t\left[e^{-r(\tau-t)}\left((\rho\xi + b\rho)\,\theta_\tau - \left(Q - \frac{a}{r}\right)\right)\right] - \left(b\rho\theta_t + \frac{a}{r}\right),$$

we can get,

$$E_t\left[e^{-r(\tau^*-t)}\right] = \left[\frac{(\phi^+ - 1)(\rho\xi + b\rho)}{\phi^+(Q - a/r)}\right]^{\phi^+} \theta_t^{\phi^+}.$$

Therefore, the maximization problem of owner i is equivalent to,

$$\max_{q_i} \frac{q_i - c_i}{\left(\sum_{j=1}^n q_j - a/r\right)^{\phi^+}},$$

with the first-order condition (FOC):

$$\sum_{j=1}^n q_j - \frac{a}{r} - \phi^+(q_i - c) = 0.$$

Summing the FOC across all i's, we have,

$$\sum_{j=1}^n q_j = \frac{\phi^+ \sum_{j=1}^n c_j - na/r}{\phi^+ - n},$$

and hence, we can have the optimum,

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n\right) c_i + \sum_{j=1}^n c_j - \frac{a}{r}\right].$$

If $c_i = c_j = c$, then $q_i = q_j = q^*$ for $j = 1...n$, and we have,

$$q^* = \frac{\phi^+ c - a/r}{\phi^+ - n},$$

which leads to an implicit assumption that $\phi^+ > n$ so that the royalty is positive.

A.5 Proof of Proposition 7

We first discuss the derivative with respect to b, which is,

$$\begin{aligned}\frac{d\left|dE_t\left[R_t\right]/dn\right|}{db} &= -\frac{d\left(dE_t\left[R_t\right]/dn\right)}{db}\\ &= \chi_{b,1}\frac{A}{\left(P_t^I+AP_t^O\right)^2}\frac{dP_t^O}{db} - \chi_{b,2}\frac{1}{\left(P_t^I+AP_t^O\right)^2}\frac{d^2P_t^O}{dbdn}\\ &\quad + 2\chi_{b,2}\frac{A}{\left(P_t^I+AP_t^O\right)^3}\frac{dP_t^O}{db}\frac{dP_t^O}{dn},\end{aligned}$$

where,

$$\begin{aligned}\chi_{b,1} &= (\phi^+-1)\mu A\rho\theta_t + \frac{1}{2}\phi^+(\phi^+-1)\sigma^2 A\rho\theta_t > 0, \text{ and}\\ \chi_{b,2} &= (\phi^+-1)\mu A\left(P_t^I - AP_t^P\right) + \frac{1}{2}\phi^+(\phi^+-1)\sigma^2 A\left(P_t^I - AP_t^P\right) - \mu A^2\frac{a}{r}.\end{aligned}$$

Next, we can derive,

$$\begin{aligned}\frac{dP_t^O}{db} &= \rho\theta_t\left[\left(\frac{\rho\xi+b\rho}{Q-a/r}\frac{\phi^+-1}{\phi^+}\theta_t\right)^{\phi^+-1} - 1\right] \to 0,\\ \frac{dP_t^O}{dn} &= -\phi^+\frac{\phi^+c-a/r}{\left(\phi^+-n\right)^2}\left(\frac{\rho\xi+b\rho}{Q-a/r}\frac{\phi^+-1}{\phi^+}\theta_t\right)^{\phi^+} \to -\frac{\left(\phi^+\right)^2c}{\left(\phi^+-n\right)^2}, \text{ and}\\ \frac{d^2P_t^O}{dbdn} &= -\frac{\rho\left(\phi^+c-a/r\right)\left(\phi^+-1\right)}{\left(\phi^+-n\right)\left(nc-a/r\right)}\theta_t\left(\frac{\rho\xi+b\rho}{Q-a/r}\frac{\phi^+-1}{\phi^+}\theta_t\right)^{\phi^+-1} \to -\frac{\rho\phi^+\left(\phi^+-1\right)}{n\left(\phi^+-n\right)},\end{aligned}$$

where the limits are taken when $a \to 0$, $b \to 0$, $\xi \to 0$, and $\theta_t \to \theta^*$. Therefore, we have,

$$\frac{d\left|dE_t\left[R_t\right]/dn\right|}{db} \to \chi_{b,2}\frac{\theta^*}{\left(P_t^I+AP_t^O\right)^2}\frac{\left(\phi^+\right)^2c}{\left(\phi^+-n\right)^2} > 0.$$

Finally, we discuss the derivative with respect to a, which is,

$$\begin{aligned}\frac{d\left|dE_t\left[R_t\right]/dn\right|}{da} &= -\frac{d\left(dE_t\left[R_t\right]/dn\right)}{da}\\ &= \chi_{a,1}\frac{A}{\left(P_t^I+AP_t^O\right)^2}\frac{dP_t^O}{da} - \chi_{a,2}\frac{1}{\left(P_t^I+AP_t^O\right)^2}\frac{d^2P_t^O}{dadn}\\ &\quad + 2\chi_{a,2}\frac{A}{\left(P_t^I+AP_t^O\right)^3}\frac{dP_t^O}{da}\frac{dP_t^O}{dn},\end{aligned}$$

in which,

$$\begin{aligned}
\chi_{a,1} &= \frac{1}{r}\phi^{+}\mu A + \frac{1}{2r}\phi^{+}(\phi^{+}-1)\sigma^{2}A > 0, \text{ and} \\
\chi_{a,2} &= (\phi^{+}-1)\mu A\left(P_t^I - AP_t^P\right) + \frac{1}{2}\phi^{+}(\phi^{+}-1)\sigma^{2}A\left(P_t^I - AP_t^P\right) - \mu A^{2}\frac{a}{r}.
\end{aligned}$$

Similarly, we can derive,

$$\begin{aligned}
\frac{dP_t^O}{da} &= \frac{1}{r}\left[\frac{\phi^{+}}{\phi^{+}-n}\left(\frac{\rho\xi + b\rho}{Q-a/r}\frac{\phi^{+}-1}{\phi^{+}}\theta_t\right)^{\phi^{+}} - 1\right] \to \frac{1}{r}\frac{n}{\phi^{+}-n}, \text{ and} \\
\frac{d^{2}P_t^O}{dadn} &= \frac{\phi^{+}}{r\left(\phi^{+}-n\right)^{2}}\left(1-\phi^{+}\frac{\phi^{+}c-a/r}{nc-a/r}\right)\left(\frac{\rho\xi + b\rho}{Q-a/r}\frac{\phi^{+}-1}{\phi^{+}}\theta_t\right)^{\phi^{+}} \to -\frac{\phi^{+}\left(\left(\phi^{+}\right)^{2}-n\right)}{rn\left(\phi^{+}-n\right)^{2}},
\end{aligned}$$

where the limits are taken when $a \to 0$, $b \to 0$, $\xi \to 0$, and $\theta_t \to \theta^*$. Thus, we have,

$$\begin{aligned}
\frac{d\left|dE_t\left[R_t\right]/dn\right|}{da} &\to -\frac{A}{r\left(P_t^I + AP_t^O\right)^{2}}\left(\frac{\phi^{+}}{\phi^{+}-n}\right)^{2} \\
&\cdot\left[\begin{array}{c}
-\left(\frac{1}{2n}\sigma^{2}P_t^I\right)\left(\phi^{+}\right)^{3} \\
+\left(\frac{1}{2}\sigma^{2}Ac - \frac{1}{n}\mu\frac{\phi^{+}-1}{\phi^{+}}P_t^I + \frac{1}{2n}\sigma^{2}P_t^I\right)\left(\phi^{+}\right)^{2} \\
+\left(-\frac{1}{2}\sigma^{2}Ac + \mu Ac + \frac{1}{2}\sigma^{2}P_t^I\right)\phi^{+} \\
+\left(\mu\frac{\phi^{+}-1}{\phi^{+}}P_t^I - \frac{1}{2}\sigma^{2}P_t^I + \frac{Anc(\phi^{+}-1)\sigma^{2}}{(\phi^{+}-n)}\frac{P_t^I}{P_t^I + AP_t^O}\right) \\
+\left(\frac{2A\mu nc(\phi^{+}-1)}{(\phi^{+}-n)}\frac{P_t^I}{P_t^I + AP_t^O}\right)\frac{1}{\phi^{+}}
\end{array}\right] \\
&\equiv \Xi.
\end{aligned}$$

Notice that,

$$\theta^* = \frac{\left(\phi^{+}\right)^{2}}{\left(\phi^{+}-1\right)\left(\phi^{+}-n\right)}\frac{nc}{\rho\xi},$$

and $P_t^I = \rho\theta^*$ and $P_t^O = \rho\xi\theta^*$. Ξ is a polynomial of ϕ^{+}, and its coefficient of the term with the highest (3) moment of ϕ^{+} is negative. Thus for large enough ϕ^{+}, we have $\Xi > 0$, i.e.,

$$\frac{d\left|dE_t\left[R_t\right]/dn\right|}{da} > 0.$$

B Appendix B: Choice of Parameters

Restrictions on the parameters:

$$\phi^{+} - n > 0; \tag{B1}$$

$$nc - a/r > 0; \tag{B2}$$

$$\theta^{*} > 0; \text{ and} \tag{B3}$$

$$P_t^{O}(\theta^{*}) > 0. \tag{B4}$$

Choice of patent thicket and patent portfolio size. n can be calculated from our patent thicket proxy, i.e., $n = (1 - PT)^{-1}$, where PT is our empirical proxy of patent thicket. Note that n is strictly increasing in PT. The sample mean of n is about 3, and it is assumed to vary from 1 to 5. The annual sample average of the patent portfolio size is about 30, and therefore we set $A = 30$, and it is assumed to vary from 20 to 40.

Choice of economic growth and stochastic discount factor. Borrowed from Garleanu, Panageas, and Yu (2012), the economic growth trend parameter μ is chosen as 1.2% to match the annual aggregate growth of (neutral) total factor productivity, and both of the volatility parameters, r and κ, are chosen to be 3% to match the volatility of annual time-integrated consumption. Following Campbell and Cochrane (1999), we set r as 6.8% to match the mean of equity annual return. To summarize, $\mu = 1.2\%$, $\sigma = 3\%$, $r = 6.8\%$, and $\kappa = 3\%$.

Choice of patents' exposure to systematic risk. We set $\xi = 0.1$. Notice that the risk exposure of asset in place is one, so $\xi = 0.1$ implies that the total risk exposure of new patents granted each year for a firm is 3 (i.e., $\xi A = 3$) times the risk exposure of asset in place, which seems sufficiently huge to sever as an upper bound. Therefore, we set $\xi = 0.1$ and check Proposition 5 within the range of $\xi \in [0, 0.1]$.

Choice of innovation efficiency. If we borrow from Berk, Green, and Naik (2004) that $a = 1$ and $b = 0.1$, for any c, Inequalities (B3) and (B4) will be violated. We argue that the estimation of a and b should be much lower in our model than in Berk, Green, and Naik (2004) for two reasons. First, Berk, Green, and Naik (2004) assume away the royalty fee charged on patents. And second, Berk, Green, and Naik (2004) are interested in the suspension of R&D whereas we focus on the continuation. If we set $a = 1\%$ and $b = 0.1\%$, then the annual R&D cost of each firm to maintain its growth options is 80% $\left(b\left(\mu\xi\right)^{-1} \approx 0.8\right)$ of the annual total factor productivity (TFP) growth rate, which seems sufficiently high as upper bounds of a and b. Therefore, we set $a = 1\%$ and $b = 0.1\%$ and check Propositions 6 and 7 within the range of $a \in [0, 1\%]$ and $b \in [0, 0.1\%]$.

Choice of owners' personal cost. For simplicity, we set $c = 10$. This choice satisfies all of the restrictions mentioned above. Mathematically, c just plays a role as a scale factor, thereby insignificantly altering the properties. For robustness checks, we also try $c = 20$, $c = 50$, and $c = 100$, and the figures are approximately similar.

C Model Extensions in Online Appendix

C.1 Endogeneity of Patent Endowment

In our model, we assume patent value (ξ) and patent thicket (n) are independent. However, a deeper patent thicket is ex post harmful to a firm's value because it delays patent exploitation and lowers patent value, and as a result it provides less ex ante incentive for a firm to innovate, especially break-through discoveries. Taking into account the endogeneity of patent endowment yields the following proposition.

Proposition 8 *Under the condition that patent value is negatively correlated with patent thicket, a firm with a deeper patent thicket produces a lower expected future return.*

Proof. *Under the assumption that ξ is a continuous differentiable function of n, such that $d\xi/dn < 0$, Equation (15) can be written as,*

$$\begin{aligned}\frac{dE_t\left[R_t\right]}{dn} &= \frac{\partial E_t\left[R_t\right]}{\partial Q}\frac{dQ}{dn} + \frac{\partial E_t\left[R_t\right]}{\partial \xi}\frac{d\xi}{dn} \\ &= \left(\frac{\phi^+}{\xi}\frac{d\xi}{dn} - \frac{\phi^+ - 1}{Q}\frac{dQ}{dn}\right)\left[\frac{\Omega P_t^I P_t^O}{\left(P_t^I + P_t^O\right)^2}\right] dt.\end{aligned}$$

So $dE_t\left[dP_t/P_t\right]/dn < 0$ if and only if,

$$\frac{d\xi}{dn} < \frac{\phi^+ - 1}{\phi^+}\frac{\xi}{Q}\frac{dQ}{dn}. \tag{C1}$$

Because the term on the right-hand side of inequality (C1) is positive, $dE_t\left[dP_t/P_t\right]/dn < 0$ for $d\xi/dn < 0$. ■

C.2 Effect of Patent Owners' Bargaining Power

The effect of patent owners' individual bargaining power on innovation competition and technology exploitation could be different. For example, they show that small patent owners suffer from higher cost in protecting their patents and are more active in launching lawsuits (Lanjouw and Schankerman, 2004). In the following two subsections we take these two effects into consideration.

C.2.1 Effect of Heterogeneous Private Costs

In this subsection, we assume patent owners have different private cost c_i. We can prove the following proposition.

Proposition 9 *The patent owners suffering from higher private cost c_i are more influential to the future expected return of the focal firm.*

Proof. *Equation (11) can by written as,*

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n + 1\right) c_i + \sum_{j=1, j\neq i}^{n} c_j\right].$$

So we have,

$$\frac{\partial q_i^*}{\partial c_i} > \frac{\partial q_i^*}{\partial c_j} > 0. \tag{C2}$$

Combining Inequalities (C2) and (10) yields,

$$\frac{\partial E_t\left[R_t\right]}{\partial c_i} < 0. \tag{C3}$$

■

Remark 4 *Inequalities (C2) and (C3) imply that the patent owners suffering from higher private cost c_i are more harmful to the focal firm, because these patent owners tend to charge a higher royalty fee and as a result they bring more difficulty to the focal firm during the patent exploitation process.*

C.2.2 Effect of Different Bargaining Timing

A sequential (Stackelberg) bargaining game is considered in this subsection, where patent owner i moves first to charge a royalty fee q_i, and the rest $(n-1)$ of them move second to charge q_{-i}. We assume that all of them have the same level of private costs c, in order to rule out the effect of private cost. We have the following proposition.

Proposition 10 *The patent owners, who move first to initiate patent negotiation, charge a higher royalty fee from the focal firm, and are more influential to the future expected return of the focal firm.*

Proof. *For the second stage of the Stackelberg game, Equation (A5) can be reorganized as,*

$$\sum_{-i} q_{-i} + q_i + \phi^+ \left(q_{-i} - c\right) = 0.$$

Summing it across the $n-1$ patent owners yields,

$$(n-1) \sum_{-i} q_{-i} + (n-1)\, q_i + \phi^+ \left(\sum_{-i} q_{-i} - (n-1)\, c \right) = 0,$$

or equivalently,

$$q_{-i} = \frac{\phi^+ c - q_i}{\phi^+ + n - 1}. \tag{C4}$$

For the first stage of the Stackelberg game, the profit maximization problem is,

$$\max_{q_i} \frac{q_i - c}{\left(\sum_{-i} q_{-i}(q_i) + q_i\right)^{\phi^+}},$$

with the FOC:

$$\left(\sum_{-i} q_{-i}(q_i) + q_i \right) - \phi^+ \left((n-1) \frac{dq_{-i}}{dq_i} + 1 \right) (q_i - c) = 0.$$

Given Equation (C4), the FOC above can be solved as,

$$q_i^* = \frac{\phi^+ + n - 1}{\phi^+ - 1} c < \frac{\phi^+}{\phi^+ - n} c, \tag{C5}$$

and plugging Equantion (C5) into Equation (C4) gives,

$$q_{-i}^* = \frac{\left(\phi^+ - 1\right)^2 - n}{\left(\phi^+ + n - 1\right)\left(\phi^+ - 1\right)} c < \frac{\phi^+}{\phi^+ - n} c. \tag{C6}$$

Comparing Equation (C5) with Equation (C6) yields,

$$q_i^* > q_{-i}^*. \tag{C7}$$

Combining Inequalities (C7) and (10) yields,

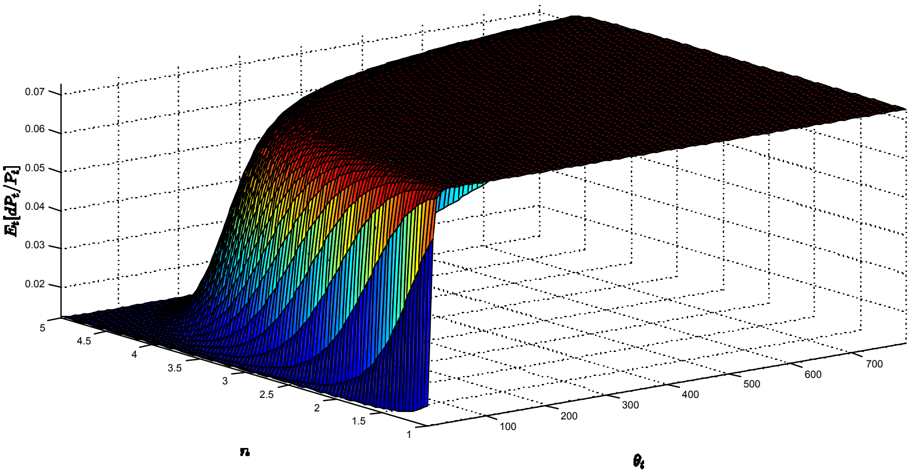
$$\frac{\partial E_t\left[R_t\right]}{\partial q_i^*} < \frac{\partial E_t\left[R_t\right]}{\partial q_{-i}^*}. \tag{C8}$$

■

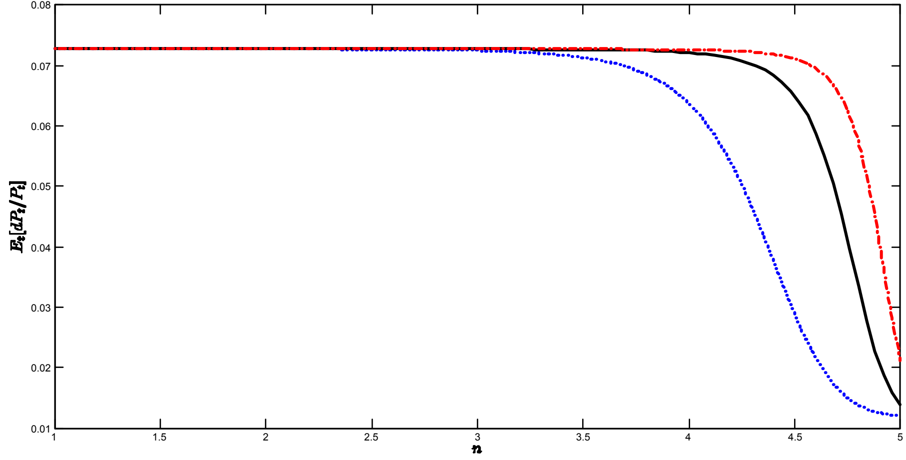
Remark 5 *The reason why the patent owners who are more active to initiate patent negotiation charge a higher royalty fee is that they enjoy the first-mover advantage. As a result, they pose more hindering effect to technology exploitation and lower the focal firm's expected futurn return.*

D Figures

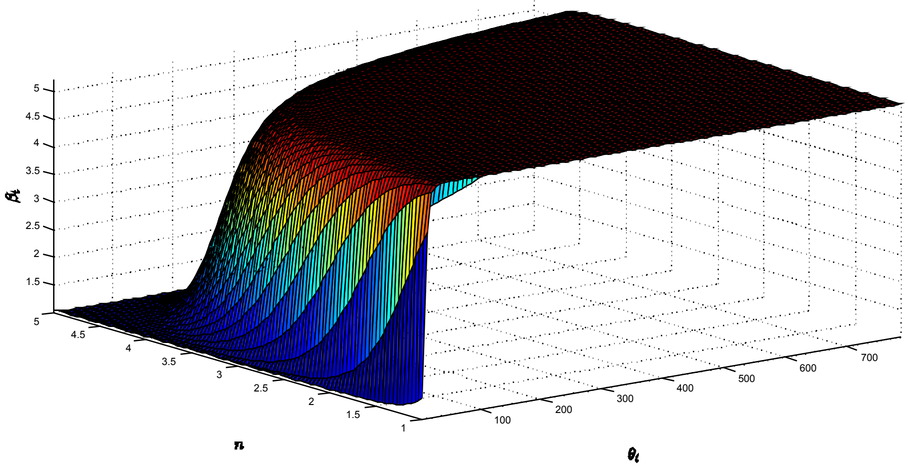
Panel A. The relationship among $E_t[dP_t/P_t]$, n, and θ_t.



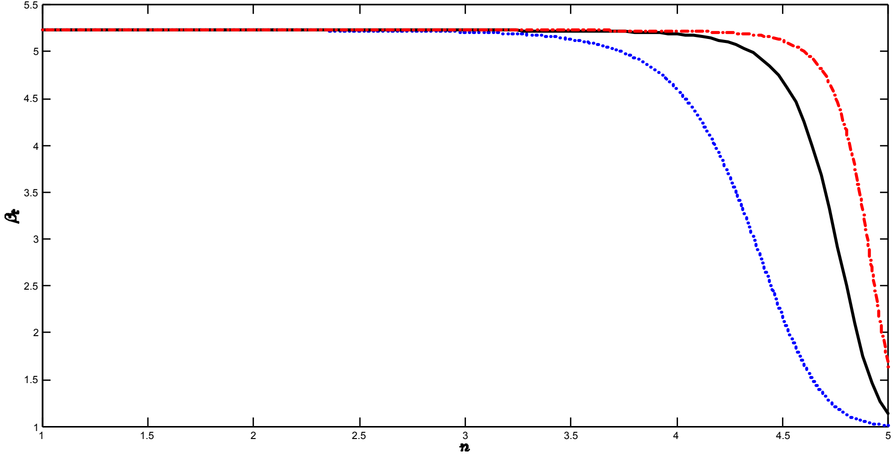
Panel B. The relationship between $E_t[dP_t/P_t]$ and n, given θ_t.



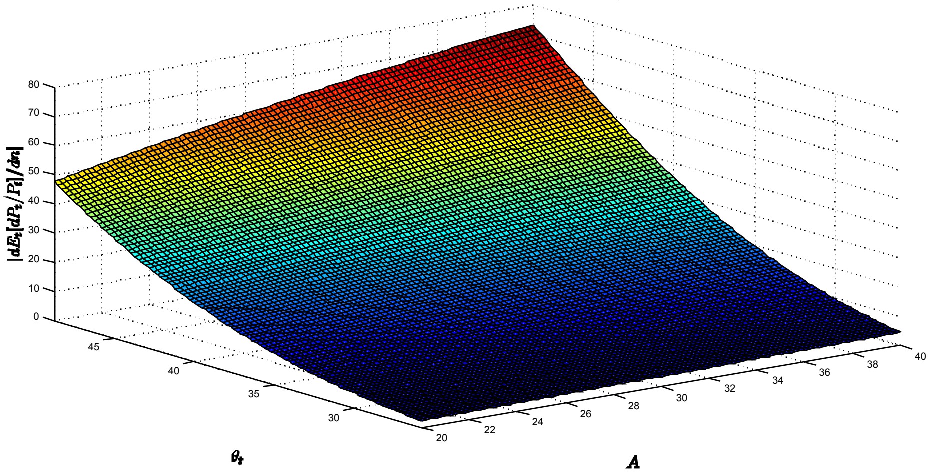
Panel A. The relationship among β_t, n, and θ_t.



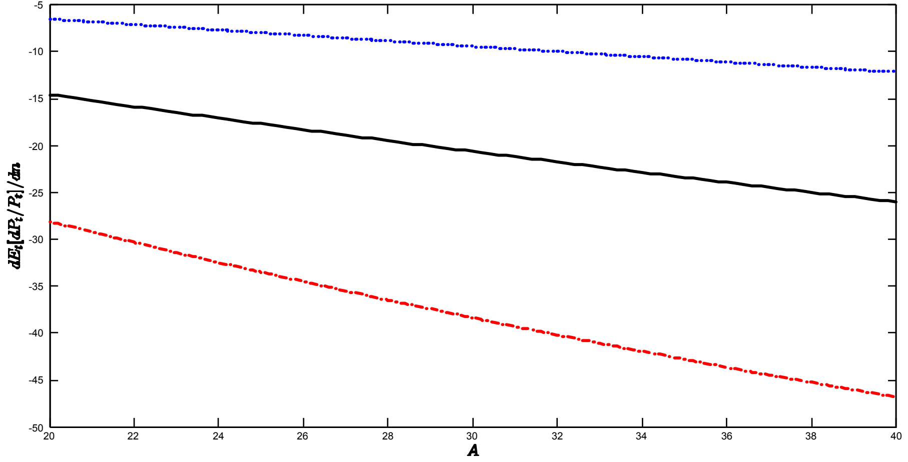
Panel B. The relationship between β_t and n, given θ_t.



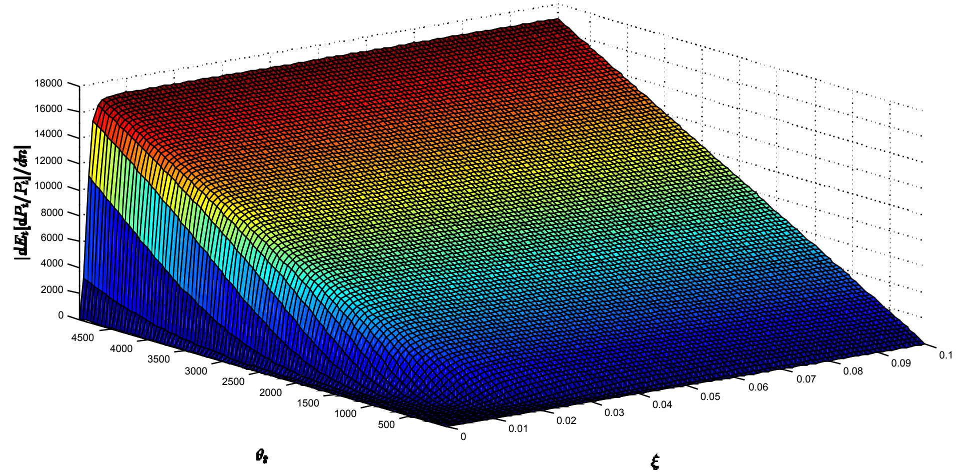
Panel A. The relationship among $|dE_t[dP_t/P_t]/dn|$, A, and θ_t.



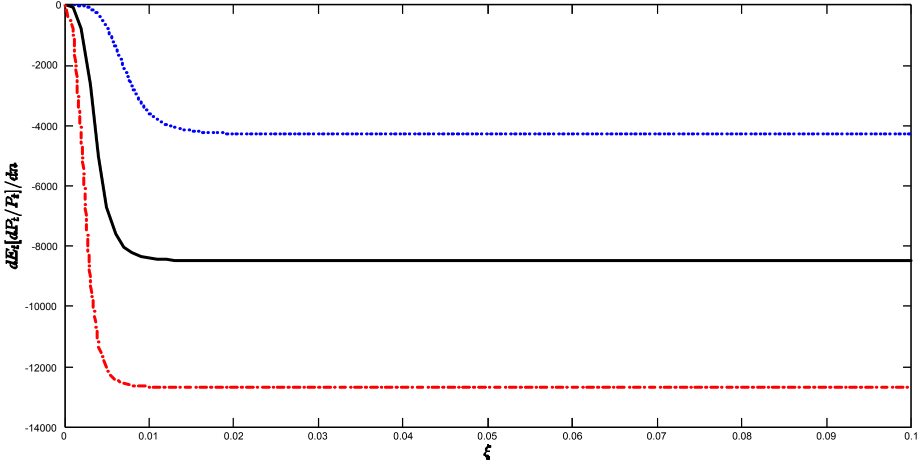
Panel B. The relationship between $dE_t[dP_t/P_t]/dn$ and A, given θ_t.



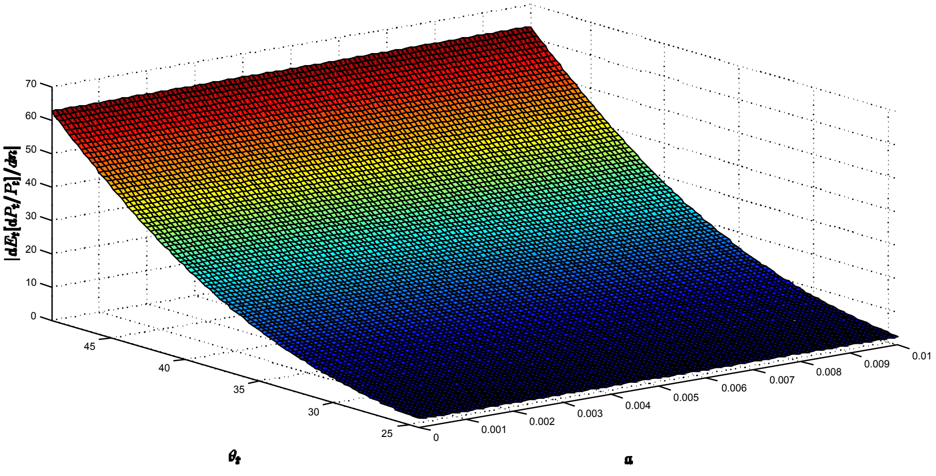
Panel A. The relationship among $|dE_t[dP_t/P_t]/dn|$, ξ, and θ_t.



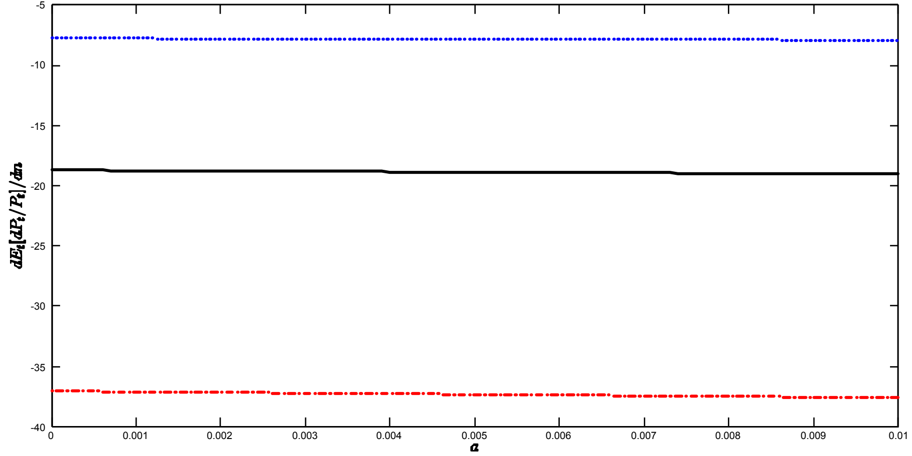
Panel B. The relationship between $dE_t[dP_t/P_t]/dn$ and ξ, given θ_t.



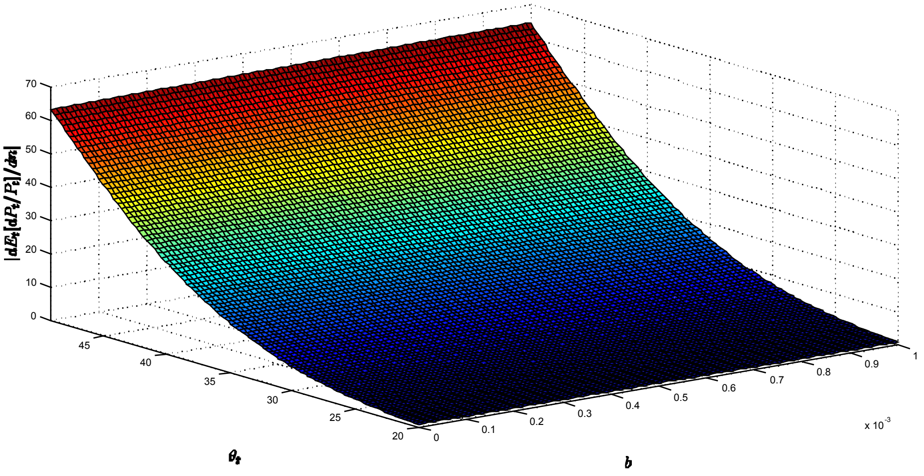
Panel A. The relationship among $|dE_t[dP_t/P_t]/dn|$, α, and θ_t.



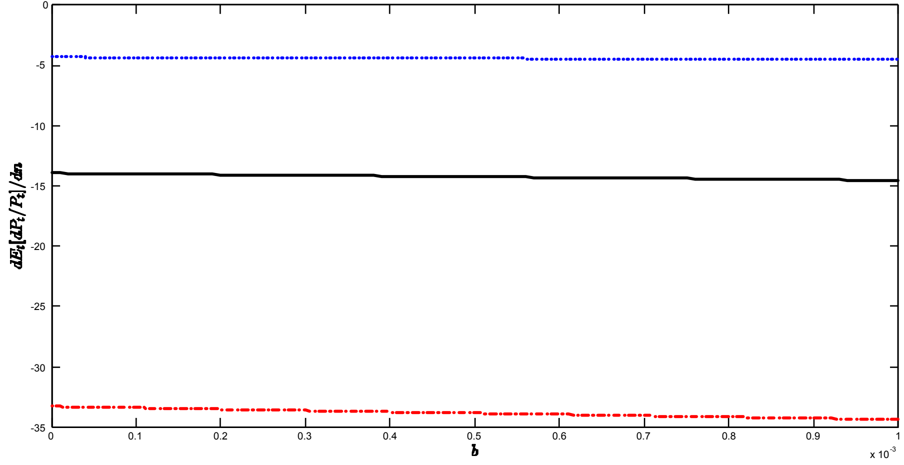
Panel B. The relationship between $dE_t[dP_t/P_t]/dn$ and α, given θ_t.

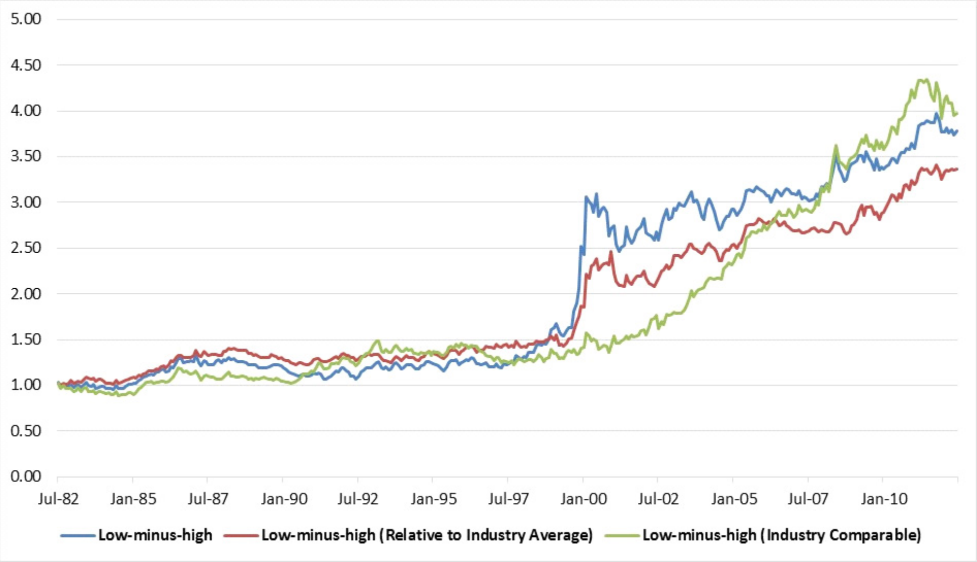


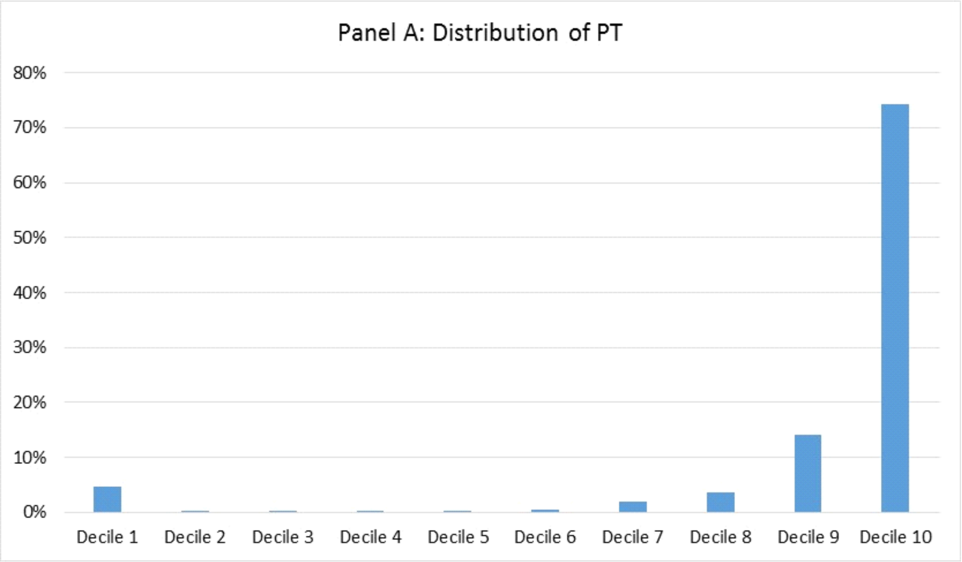
Panel A. The relationship among $|dE_t[dP_t/P_t]/dn|$, b, and θ_t.

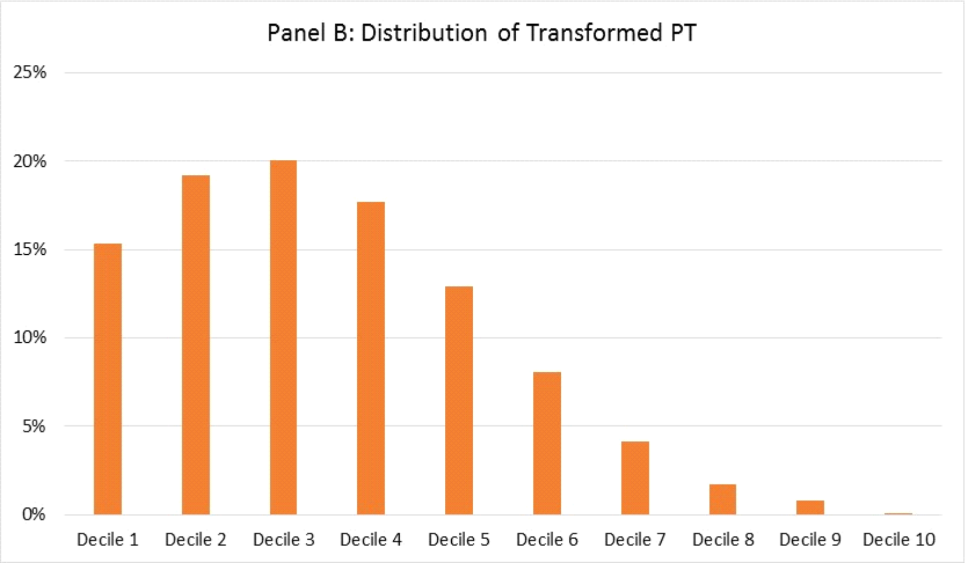


Panel B. The relationship between $dE_t[dP_t/P_t]/dn$ and b, given θ_t.




5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Jul-82
Jan-85
Jul-87
Jan-90
Jul-92
Jan-95
Jul-97
Jan-00
Jul-02
Jan-05
Jul-07
Jan-10
Low-minus-high
Low-minus-high (Relative to Industry Average)
Low-minus-high (Industry Comparable)


Panel A: Distribution of PT
80%
70%
60%
50%
40%
30%
20%
10%
0%
Decile 1
Decile 2
Decile 3
Decile 4
Decile 5
Decile 6
Decile 7
Decile 8
Decile 9
Decile 10


Panel B: Distribution of Transformed PT
25%
20%
15%
10%
5%
0%
Decile 1
Decile 2
Decile 3
Decile 4
Decile 5
Decile 6
Decile 7
Decile 8
Decile 9
Decile 10

E Tables

Table I

Annual Proportions of Patents Granted to Listed Firms and Patent Thicket across Industries

Panel A reports the proportion of patents granted to listed firms among all patents in each year. Year denotes the year when patents were granted. The sample period from 1981 to 2010. Panel B reports the pooled mean (Mean), standard deviation (Std), minimum (Min), 25th percentile (Perc25), median (Perc50), 75th percentile (Perc75), and maximum (Max) of the patent thicket (IO) measure for firms in industries based on Fama-French 48 industry classifications (FF48), financial and other industries (FF44-48) excluded. The sample ranges from 1985 to 2010. Obs is the number of firm-year observations with PT measure in each industry.

Panel A

Year	Ratio	Year	Ratio	Year	Ratio
1981	34.69%	1991	29.11%	2001	36.26%
1982	34.71%	1992	30.12%	2002	36.97%
1983	35.19%	1993	31.01%	2003	37.42%
1984	34.50%	1994	31.54%	2004	37.96%
1985	33.92%	1995	31.82%	2005	45.09%
1986	32.95%	1996	32.91%	2006	45.02%
1987	32.43%	1997	32.56%	2007	43.81%
1988	31.62%	1998	33.23%	2008	43.23%
1989	30.26%	1999	33.42%	2009	43.00%
1990	29.33%	2000	34.34%	2010	40.06%

Table I (continued)

Panel B

FF48	Industry	Obs	Mean	Std	Min	Perc25	Perc50	Perc75	Max
1	Agriculture	36	0.72	0.34	0.00	0.73	0.83	0.93	1.00
2	Food Products	419	0.90	0.17	0.00	0.90	0.95	0.97	1.00
3	Candy & Soda	122	0.89	0.16	0.00	0.86	0.96	0.98	1.00
4	Beer & Liquor	75	0.93	0.07	0.57	0.91	0.96	0.98	1.00
5	Tobacco Products	68	0.91	0.08	0.67	0.87	0.96	0.97	1.00
6	Recreation	357	0.86	0.22	0.00	0.86	0.94	0.96	1.00
7	Entertainment	113	0.85	0.20	0.00	0.84	0.90	0.95	1.00
8	Printing and Publishing	102	0.77	0.34	0.00	0.80	0.93	0.96	1.00
9	Consumer Goods	922	0.86	0.21	0.00	0.86	0.93	0.96	1.00
10	Apparel	173	0.80	0.32	0.00	0.83	0.93	0.98	1.00
11	Healthcare	125	0.79	0.26	0.00	0.75	0.89	0.95	1.00
12	Medical Equipment	1,902	0.87	0.19	0.00	0.85	0.93	0.96	1.00
13	Pharmaceutical Products	2,390	0.82	0.25	0.00	0.80	0.91	0.95	1.00
14	Chemicals	1,162	0.90	0.16	0.00	0.91	0.95	0.97	1.00
15	Rubber and Plastic Products	341	0.86	0.21	0.00	0.85	0.93	0.97	1.00
16	Textiles	187	0.90	0.18	0.00	0.89	0.96	0.98	1.00
17	Construction Materials	1,018	0.89	0.21	0.00	0.90	0.96	0.98	1.00
18	Construction	84	0.81	0.31	0.00	0.83	0.94	0.97	1.00
19	Steel Works Etc	555	0.89	0.18	0.00	0.90	0.94	0.98	1.00
20	Fabricated Products	127	0.85	0.23	0.00	0.85	0.93	0.96	1.00
21	Machinery	2,312	0.91	0.16	0.00	0.92	0.95	0.97	1.00
22	Electrical Equipment	1,328	0.88	0.18	0.00	0.88	0.94	0.97	1.00
23	Automobiles and Trucks	846	0.91	0.16	0.00	0.91	0.95	0.97	1.00
24	Aircraft	331	0.94	0.11	0.00	0.95	0.97	0.98	1.00
25	Shipbuilding, Railroad Equipment	57	0.91	0.19	0.00	0.95	0.96	0.98	1.00
26	Defense	128	0.88	0.21	0.00	0.90	0.94	0.98	1.00
27	Precious Metals	13	0.85	0.27	0.00	0.81	0.94	1.00	1.00
28	Non-Metallic and Industrial Metal Mining	98	0.89	0.22	0.00	0.90	0.95	0.98	1.00
29	Coal	18	0.89	0.24	0.00	0.90	0.95	1.00	1.00
30	Petroleum and Natural Gas	679	0.90	0.18	0.00	0.92	0.95	0.97	1.00
31	Utilities	308	0.85	0.26	0.00	0.86	0.94	0.98	1.00
32	Communication	268	0.89	0.17	0.00	0.89	0.94	0.96	1.00
33	Personal Services	69	0.84	0.29	0.00	0.89	0.94	0.96	1.00
34	Business Services	2,346	0.86	0.21	0.00	0.87	0.92	0.95	1.00
35	Computers	1,758	0.90	0.13	0.00	0.90	0.94	0.96	1.00
36	Electronic Equipment	3,441	0.91	0.15	0.00	0.92	0.95	0.97	1.00
37	Measuring and Control Equipment	1,333	0.90	0.19	0.00	0.91	0.96	0.97	1.00
38	Business Supplies	611	0.91	0.15	0.00	0.92	0.95	0.97	1.00
39	Shipping Containers	286	0.92	0.12	0.00	0.90	0.95	0.97	1.00
40	Transportation	99	0.81	0.29	0.00	0.80	0.93	0.97	1.00
41	Wholesale	489	0.81	0.29	0.00	0.85	0.93	0.96	1.00
42	Retail	199	0.81	0.29	0.00	0.83	0.95	0.97	1.00
43	Restaraunts, Hotels, Motels	63	0.72	0.36	0.00	0.62	0.91	0.96	1.00

Table II

Summary Statistics by PT Groups and Correlation Matrix

At the end of June of year *t* from 1982 to 2011, we sort firms with non-missing patent thicket (PT) into five PT groups (Lowest, Low, Middle, High and Highest) based on the quintiles of PT in year *t* – 1. In addition, we assign firms with missing PT (i.e., no patent) into the “No” group. The time-series average of cross-sectional mean characteristics of firms in each PT group is reported in Panel A, and the time-series average of cross-sectional correlation is reported in Panel B. Obs denotes the average number of firms in each group across years. CTBE is the number of patents issued to a firm in the previous five years divided by the firm’s book equity at the end of year *t* – 1. CTA is the number of patents issued to a firm in the previous five years divided by the firm’s total assets at the end of year *t* – 1. RDBE is R&D capital in fiscal year ending in year *t* – 1 divided by book equity at the end of year *t* – 1. RDA is R&D capital in fiscal year ending in year *t* – 1 divided by total assets at the end of year *t* – 1. IE is the patent-based innovative efficiency measure, proxied by the number of patent granted in year *t* divided by one plus accumulative R&D capital from year *t* – 3 to *t* – 7 (with 20% obsolescence rate). SIZE is market capitalization (in millions) at the end of June of year *t*. B/M is book-to-market ratio of fiscal year ending in year *t* – 1. MOM11 is the accumulation of previous eleven-month returns (in percentages, with a one-month gap between the holding period and the current month). MOM6 is the accumulation of previous 6-month returns (in percentages, with a one-month gap between the holding period and the current month). REV is the previous one-month return, in percentages. BPC (in thousands) is the total accumulative forward citations received by the other firms which are cited by the focal firm until year *t* – 1. BPP (in thousands) is the total number of patent granted by the other firms which are cited by the focal firm until year *t* – 1. PMC is one minus the concentration index of sale in industry in year *t* – 1. PQ (in thousands) is the total accumulative forward citations of the patents in the previous five years of the focal firm. ROA is defined as income before extraordinary items plus interest expenses in year *t* – 1 divided by total assets in year *t* – 2. IA is defined as capital expenditure in year *t* – 1 divided by total assets in year *t* – 2. SA is defined as total sales in year *t* – 1 divided by total assets in year *t* – 2.

Panel A

PT group	Obs	PT	CTBE	CTA	RDBE	RDA	IE	SIZE	B/M	MOM11
No	3,366	----	----	----	0.09	0.03	0.00	1,066	0.85	9.70
Low	190	0.61	0.44	0.18	0.24	0.10	1.08	2,212	0.65	10.87
2	188	0.90	0.50	0.18	0.24	0.09	1.35	3,448	0.68	11.14
3	189	0.94	0.47	0.19	0.23	0.08	1.30	4,329	0.65	11.13
4	188	0.96	0.53	0.17	0.19	0.07	1.14	6,972	0.63	11.45
High	188	0.99	0.37	0.16	0.20	0.06	1.22	6,329	0.66	11.03

PT group	MOM6	REV	BPC	BPP	PMC	PQ	ROA	IA	SA
No	11.28	-0.05	----	----	0.77	----	0.02	0.08	1.24
Low	11.01	-0.10	3.45	1.50	0.77	0.14	-0.03	0.07	1.12
2	11.73	-0.45	4.27	1.71	0.77	0.41	0.00	0.07	1.11
3	11.82	-0.67	4.47	1.77	0.77	0.84	0.01	0.07	1.14
4	10.29	-0.26	4.39	1.75	0.76	1.29	0.04	0.07	1.16
High	11.00	-0.41	3.63	1.52	0.75	0.87	0.03	0.06	1.20

Table II (continued)

Panel B

	PT	CTBE	CTA	RDBE	RDA	IE	SIZE	B/M	MOM11	MOM6	REV	BPC	BPP	PMC	PQ	ROA	IA	SA
PT	1																	
CTBE	0.01	1																
CTA	0.01	0.33	1															
RDBE	0.00	0.50	0.05	1														
RDA	-0.06	0.11	0.28	0.16	1													
IE	0.01	0.00	0.00	0.00	-0.04	1												
SIZE	0.05	-0.02	-0.07	-0.01	-0.06	0.05	1											
B/M	-0.01	-0.04	-0.05	-0.04	-0.17	0.01	-0.10	1										
MOM11	0.01	0.01	0.03	-0.01	0.01	-0.01	0.02	-0.11	1									
MOM6	0.01	0.02	0.07	0.01	0.03	-0.01	-0.01	0.08	0.65	1								
REV	-0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.10	0.09	1							
BPC	0.07	0.00	-0.02	0.02	0.06	0.00	0.10	-0.07	0.02	-0.01	-0.04	1						
BPP	0.07	0.01	0.00	0.02	0.02	0.00	0.07	-0.04	0.02	-0.01	-0.02	0.83	1					
PMC	-0.01	0.02	0.05	0.02	0.22	-0.02	-0.03	-0.08	0.01	-0.02	0.00	0.13	0.07	1				
PQ	0.06	0.00	0.00	0.00	-0.01	0.01	0.37	-0.06	0.00	-0.02	-0.01	0.11	0.11	-0.01	1			
ROA	0.06	-0.06	-0.15	-0.07	-0.36	0.01	0.09	0.02	0.02	-0.01	0.00	-0.02	0.00	-0.03	0.02	1		
IA	-0.02	-0.02	-0.05	-0.01	-0.07	0.01	-0.01	-0.07	-0.01	-0.02	-0.01	-0.12	-0.05	-0.02	0.01	-0.15	1	
SA	0.00	-0.03	-0.07	-0.04	-0.26	0.01	-0.04	0.01	0.04	0.01	0.02	-0.13	-0.07	-0.06	-0.04	0.22	0.38	1

Table III

One-way Portfolio Sorting on Patent Thicket

At the end of June of year *t* from 1982 to 2011, we sort firms with non-missing patent thicket (PT) into five PT groups (Low, 2, 3, 4 and High) based on the quintiles of PT in year *t* – 1. In addition, we assign firms with missing PT (i.e., no patent) into the "No" group. We also construct a high-minus-low (High–Low) portfolio by holding a long (short) position in the high (low) PT portfolio and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We report the monthly returns in excess of one-month Treasury bill rate (Excess Returns) and their corresponding alphas and betas in different model specifications. Following Fama and French (1993), MKT denotes the market factor. SMB denotes the size factor, and HML represents the value factor. Similarly to Fama and French (2015), we use RMW and CMA to represent profitability and investment factors respectively. UMD stands for the momentum factor (Carhart, 1997). IMC represents the investment-minus-consumption factor by Papanikolaou (2011). IO stands for the innovation originality factor by Hirshleifer, Hsu, and Li (2014). EMI denotes the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. Sample period: 1982Q3-2012Q2 for MKT, SMB, HML, RMW, CMA, and UMD, 1982Q3-2008Q2 for IMC, IO, and EMI.

PT	Raw	CAPM		FF3				FF5					
	Time-series Mean	Alpha	MKT	Alpha	MKT	SMB	HML	Alpha	MKT	SMB	HML	RMW	CMA
No	-0.39**	-0.48***	0.99***	-0.63***	0.98***	0.14***	0.05**	-0.49***	0.98***	0.17***	0.06*	0.10***	-0.06
	(0.18)	(0.18)	(0.01)	(0.11)	(0.01)	(0.02)	(0.02)	(0.12)	(0.02)	(0.02)	(0.03)	(0.03)	(0.05)
Low	1.46***	0.82***	0.99***	0.98***	0.89***	0.08**	-0.39***	1.00***	0.89***	0.06	-0.40***	-0.07	0.06
	(0.27)	(0.13)	(0.03)	(0.11)	(0.03)	(0.04)	(0.04)	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)
2	1.44***	0.72***	1.12***	0.94***	1.01***	-0.03	-0.53***	1.11***	0.97***	-0.08	-0.40***	-0.21***	-0.24**
	(0.32)	(0.17)	(0.04)	(0.15)	(0.03)	(0.05)	(0.05)	(0.15)	(0.04)	(0.05)	(0.07)	(0.08)	(0.10)
3	1.25***	0.59***	1.02***	0.66***	0.99***	-0.05	-0.17***	0.64***	1.00***	-0.08**	-0.24***	-0.06	0.19**
	(0.27)	(0.11)	(0.0231)	(0.11)	(0.02)	(0.04)	(0.04)	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.08)
4	1.28***	0.66***	0.96***	0.71***	0.94***	-0.06**	-0.12***	0.63***	0.97***	-0.07**	-0.23***	0.02	0.26***
	(0.25)	(0.08)	(0.02)	(0.08)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
High	1.04***	0.45***	0.91***	0.45***	0.95***	-0.19***	0.00	0.29***	0.99***	-0.16***	-0.14***	0.15***	0.28***
	(0.24)	(0.09)	(0.02)	(0.08)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
High-Low	-0.42**	-0.37**	-0.07**	-0.53***	0.05	-0.28***	0.39***	-0.71***	0.10***	-0.22***	0.27***	0.22***	0.22**
	(0.16)	(0.17)	(0.04)	(0.14)	(0.03)	(0.05)	(0.05)	(0.15)	(0.04)	(0.05)	(0.07)	(0.07)	(0.10)

Table III continued

PT	FF6							FF6+IMC							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC
No	-0.38***	0.98***	0.17***	0.07**	0.09***	-0.07	0.01	-0.40***	0.98***	0.16***	0.11***	0.07*	-0.11**	0.03*	0.00
	(0.11)	(0.02)	(0.02)	(0.03)	(0.03)	(0.05)	(0.01)	(0.12)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	-0.02	(0.00)
Low	0.95***	0.91***	0.04	-0.35***	-0.11*	0.01	0.10***	0.99***	0.90***	0.03	-0.39***	-0.11*	0.01	0.11***	-0.01
	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)	(0.02)	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.01)
2	1.11***	0.97***	-0.08	-0.40***	-0.21***	-0.24**	0.01	1.25***	1.00***	-0.06	-0.32***	-0.25***	-0.34***	-0.03	0.00
	(0.16)	(0.04)	(0.05)	(0.07)	(0.08)	(0.11)	(0.03)	(0.17)	(0.04)	(0.06)	(0.09)	(0.08)	(0.11)	(0.04)	(0.01)
3	0.67***	0.99***	-0.08*	-0.27***	-0.04	0.22***	-0.05**	0.66***	1.00***	-0.09*	-0.25***	-0.07	0.21**	-0.05*	-0.00
	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.08)	(0.02)	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.01)
4	0.64***	0.97***	-0.07**	-0.24***	0.03	0.27***	-0.02	0.65***	0.94***	-0.12***	-0.32***	-0.01	0.27***	-0.02	-0.01**
	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.09)	(0.02)	(0.03)	(0.05)	(0.05)	(0.06)	(0.02)	(0.01)
High	0.34***	0.98***	-0.15***	-0.18***	0.18***	0.33***	-0.08***	0.32***	1.01***	-0.15***	-0.15***	0.18***	0.31***	-0.10***	-0.01
	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.01)
High-Low	-0.62***	0.07**	-0.19***	0.16**	0.29***	0.32***	-0.18***	-0.67***	0.12***	-0.17***	0.24***	0.29***	0.30***	-0.21***	0.00
	(0.14)	(0.03)	(0.05)	(0.07)	(0.07)	(0.10)	(0.03)	(0.16)	(0.04)	(0.06)	(0.08)	(0.08)	(0.11)	(0.04)	(0.01)

Table III continued

PT	FF6+IO								FF6+EMI							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IO	Alpha	MKT	SMB	HML	RMW	CMA	UMD	EMI
No	-0.37***	0.98***	0.16***	0.11***	0.07*	-0.11	0.03**	0.01	-0.34***	0.98***	0.15***	0.09**	0.06*	-0.1	0.03*	-0.14
	(0.12)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.02)	(0.03)	(0.12)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.02)	(0.04)
Low	1.02***	0.90***	0.02	-0.40***	-0.09	0.03	0.10***	-0.09	1.06***	0.90***	0.02	-0.41***	-0.10*	0.02	0.10***	-0.21***
	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.06)	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.07)
2	1.23***	1.00***	-0.05	-0.31***	-0.27***	-0.36***	-0.03	0.07	1.16***	1.00***	-0.04	-0.28***	-0.25***	-0.35***	-0.03	0.25***
	(0.17)	(0.04)	(0.06)	(0.09)	(0.08)	(0.11)	(0.04)	(0.08)	(0.17)	(0.04)	(0.06)	(0.09)	(0.08)	(0.11)	(0.04)	(0.09)
3	0.62***	1.00***	-0.06	-0.23***	-0.08	0.19**	-0.05*	0.15***	0.57***	1.00***	-0.06	-0.21***	-0.05	0.21**	-0.05*	0.26***
	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.09)	(0.03)	(0.06)	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.06)
4	0.62***	0.95***	-0.09***	-0.29***	-0.00	0.27***	-0.02	0.12***	0.53***	0.95***	-0.08***	-0.26***	0.02	0.28***	-0.01	0.34***
	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.04)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.04)
High	0.32***	1.02***	-0.14***	-0.15***	0.18***	0.32***	-0.10***	0.02	0.31***	1.02***	-0.14***	-0.15***	0.19***	0.32***	-0.10***	0.03
	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.04)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.05)
High-Low	-0.70***	0.12***	-0.16***	0.25***	0.27***	0.29***	-0.21***	0.12	-0.75***	0.12***	-0.16***	0.27***	0.29***	0.30***	-0.20***	0.24***
	(0.16)	(0.04)	(0.06)	(0.08)	(0.08)	(0.11)	(0.04)	(0.07)	(0.16)	(0.04)	(0.06)	(0.08)	(0.08)	(0.11)	(0.04)	(0.08)

Table III continued

PT	FF6+IMC+IO+EMI									
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC	IO	EMI
No	-0.32***	0.98***	0.15***	0.09**	0.05	-0.11**	0.03*	0.00	0.03	-0.15
	(0.12)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.02)	(0.00)	(0.03)	(0.04)
Low	1.08***	0.89***	0.00	-0.43***	-0.11*	0.02	0.10***	-0.01	-0.06	-0.20***
	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.01)	(0.06)	(0.07)
2	1.15***	1.00***	-0.04	-0.28***	-0.25***	-0.35***	-0.03	0.00	0.03	0.24***
	(0.17)	(0.04)	(0.06)	(0.09)	(0.08)	(0.11)	(0.04)	(0.01)	(0.08)	(0.09)
3	0.55***	1.01***	-0.05	-0.21***	-0.08	0.19**	-0.04	-0.00	0.11*	0.23***
	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.01)	(0.06)	(0.06)
4	0.52***	0.95***	-0.09***	-0.26***	-0.08	0.26***	-0.01	-0.01*	0.06	0.32***
	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.01)	(0.04)	(0.04)
High	0.31***	1.01***	-0.14***	-0.15***	0.17***	0.31***	-0.10***	-0.01	0.02	0.02
	(0.09)	(0.02)	(0.03)	(0.05)	(0.05)	(0.06)	(0.02)	(0.01)	(0.04)	(0.05)
High-Low	-0.77***	0.12***	-0.14**	0.28***	0.29***	0.29***	-0.20***	0.00	0.08	0.22***
	(0.16)	(0.04)	(0.06)	(0.08)	(0.08)	(0.11)	(0.04)	(0.01)	(0.07)	(0.08)

Table IV

Two-way Portfolio Sorting on Patent Thicket and Control Variables

At the end of June of year *t* from 1982 to 2011, we conduct independent double sorts on patent thicket (PT) and common return predictors into 15 groups based on the quintiles of PT at the end of year *t* – 1 and the 30th and 70th percentiles of SIZE, MOM11, MOM6, or REV in June of year *t*, or B/M, BPC, BPP, PMC, or PQ at the end of year *t* – 1. All these variables are defined in Table II. Then we construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each control variable group and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We report the monthly returns in excess of one-month Treasury bill rate (Excess Returns), and their corresponding alphas in different model specifications. Following Fama and French (1993), MKT denotes the market factor. SMB denotes the size factor, and HML represents the value factor. Similarly to Fama and French (2015), we use RMW and CMA to represent profitability and investment factors respectively. UMD stands for the momentum factor (Carhart, 1997). IMC represents the investment-minus-consumption factor by Papanikolaou (2011). IO stands for the innovation originality factor by Hirshleifer, Hsu, and Li (2014). EMI denotes the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. Sample period: 1982Q3-2012Q2 for MKT, SMB, HML, RMW, CMA, and UMD, 1982Q3-2008Q2 for IMC, IO, and EMI.

Panel A. Conditional Predictive Power on SIZE

PT \ SIZE	Small	2	Big	Average
Low	3.67	2.24	1.26	2.39
2	3.54	2.09	1.31	2.31
3	3.67	2.19	1.16	2.34
4	3.31	2.23	1.24	2.26
High	3.18	1.90	0.98	2.02
		High-Low		
Raw	-0.48	-0.35	-0.28	-0.37**
	(0.32)	(0.24)	(0.17)	(0.16)
CAPM Alpha	-0.31	-0.38	-0.25	-0.32*
	(0.32)	(0.25)	(0.18)	(0.16)
FF3 Alpha	-0.36	-0.57**	-0.42***	-0.45***
	(0.32)	(0.22)	(0.16)	(0.15)
FF5 Alpha	-0.51	-0.77***	-0.62***	-0.63***
	(0.34)	(0.23)	(0.17)	(0.16)
FF6 Alpha	-0.78**	-0.68***	-0.51***	-0.65***
	(0.32)	(0.23)	(0.16)	(0.16)
FF6+IMC Alpha	-0.53*	-0.70***	-0.56***	-0.59***
	(0.28)	(0.25)	(0.18)	(0.15)
FF6+IO Alpha	-0.53*	-0.70***	-0.60***	-0.61***
	(0.28)	(0.26)	(0.18)	(0.16)
FF6+EMI Alpha	-0.48*	-0.65**	-0.65***	-0.59***
	(0.28)	(0.26)	(0.18)	(0.16)
FF6+3 Alpha	-0.49*	-0.66***	-0.67***	-0.61***
	(0.29)	(0.25)	(0.18)	(0.16)

Panel B. Conditional Predictive Power on B/M

PT \ B/M	Small	2	Big	Average
Low	1.50	1.35	1.83	1.56
2	1.34	1.57	1.70	1.54
3	1.26	1.24	1.67	1.39
4	1.25	1.20	2.08	1.51
High	0.99	1.35	1.16	1.17
		High-Low		
Raw	-0.51***	0.00	-0.67**	-0.39**
	-0.19	-0.22	-0.33	-0.16
CAPM Alpha	-0.40**	-0.1	-0.79**	-0.43***
	-0.19	-0.22	-0.33	-0.16
FF3 Alpha	-0.57***	-0.15	-0.97***	-0.56***
	(0.16)	(0.23)	(0.33)	(0.15)
FF5 Alpha	-0.74***	-0.22	-0.85**	-0.61***
	(0.17)	(0.24)	(0.35)	(0.16)
FF6 Alpha	-0.67***	-0.16	-0.67*	-0.50***
	(0.17)	(0.24)	(0.34)	(0.15)
FF6+IMC Alpha	-0.67***	-0.22	-0.84**	-0.57***
	(0.19)	(0.26)	(0.34)	(0.16)
FF6+IO Alpha	-0.67***	-0.27	-0.91***	-0.62***
	(0.19)	(0.26)	(0.34)	(0.16)
FF6+EMI Alpha	-0.73***	-0.39	-0.83**	-0.65***
	(0.19)	(0.26)	(0.35)	(0.16)
FF6+3 Alpha	-0.73***	-0.42	-0.88**	-0.68***
	(0.19)	(0.26)	(0.35)	(0.16)

Panel C. Conditional Predictive Power on MOM11

PT \ MOM11	Small	2	Big	Average
Low	1.99	1.31	1.54	1.61
2	1.82	1.04	1.69	1.52
3	1.69	0.99	1.50	1.40
4	1.61	1.14	1.28	1.34
High	1.72	0.85	1.01	1.19
		High-Low		
Raw	-0.26	-0.46**	-0.53**	-0.42**
	(0.36)	(0.19)	(0.23)	(0.17)
CAPM Alpha	-0.19	-0.47**	-0.52**	-0.39**
	(0.37)	(0.19)	(0.24)	(0.17)
FF3 Alpha	-0.47	-0.57***	-0.61***	-0.55***
	(0.34)	(0.19)	(0.23)	(0.15)
FF5 Alpha	-0.72**	-0.60***	-0.76***	-0.69***
	(0.36)	(0.20)	(0.24)	(0.15)
FF6 Alpha	-0.68*	-0.53***	-0.68***	-0.63***
	(0.36)	(0.20)	(0.24)	(0.15)
FF6+IMC Alpha	-0.84**	-0.48**	-0.67**	-0.66***
	(0.41)	(0.22)	(0.27)	(0.17)
FF6+IO Alpha	-0.81*	-0.50**	-0.71***	-0.67***
	(0.41)	(0.22)	(0.27)	(0.17)
FF6+EMI Alpha	-1.00**	-0.52**	-0.80***	-0.77***
	(0.41)	(0.22)	(0.27)	(0.17)
FF6+3 Alpha	-0.96**	-0.53**	-0.81***	-0.77***
	(0.41)	(0.22)	(0.27)	(0.17)

Panel D. Conditional Predictive Power on MOM6

PT \ MOM6	Small	2	Big	Average
Low	1.54	1.44	1.67	1.55
2	1.21	1.23	1.70	1.38
3	1.63	0.96	1.71	1.43
4	1.31	1.19	1.47	1.32
High	1.18	1.03	0.94	1.05
		High-Low		
Raw	-0.37	-0.41*	-0.73***	-0.50***
	(0.35)	(0.21)	(0.25)	(0.17)
CAPM Alpha	-0.28	-0.40*	-0.75***	-0.48***
	(0.35)	(0.22)	(0.25)	(0.18)
FF3 Alpha	-0.56*	-0.47**	-0.89***	-0.64***
	(0.33)	(0.20)	(0.25)	(0.16)
FF5 Alpha	-0.98***	-0.47**	-0.98***	-0.81***
	(0.35)	(0.22)	(0.27)	(0.16)
FF6 Alpha	-0.94***	-0.34	-0.91***	-0.73***
	(0.35)	(0.21)	(0.27)	(0.16)
FF6+IMC Alpha	-0.95**	-0.27	-0.87***	-0.70***
	(0.39)	(0.23)	(0.29)	(0.18)
FF6+IO Alpha	-1.03***	-0.27	-0.85***	-0.71***
	(0.39)	(0.23)	(0.29)	(0.18)
FF6+EMI Alpha	-0.97**	-0.3	-1.05***	-0.78***
	(0.40)	(0.23)	(0.29)	(0.18)
FF6+3 Alpha	-1.03**	-0.3	-1.02***	-0.78***
	(0.40)	(0.23)	(0.29)	(0.18)

Panel E. Conditional Predictive Power on REV

PT \ REV	Small	2	Big	Average
Low	1.56	1.36	1.76	1.56
2	1.68	1.26	1.35	1.43
3	1.45	1.23	1.26	1.31
4	1.53	1.22	1.15	1.30
High	1.37	0.91	1.08	1.12
		High-Low		
Raw	-0.19	-0.45**	-0.68**	-0.44**
	(0.29)	(0.21)	(0.32)	(0.17)
CAPM Alpha	-0.08	-0.42*	-0.62*	-0.37**
	(0.29)	(0.21)	(0.32)	(0.17)
FF3 Alpha	-0.12	-0.53***	-0.88***	-0.51***
	(0.29)	(0.20)	(0.30)	(0.16)
FF5 Alpha	-0.18	-0.71***	-0.93***	-0.60***
	(0.31)	(0.22)	(0.32)	(0.17)
FF6 Alpha	-0.23	-0.63***	-0.82**	-0.56***
	(0.31)	(0.21)	(0.32)	(0.17)
FF6+IMC Alpha	0.08	-0.61**	-1.07***	-0.54***
	(0.30)	(0.24)	(0.36)	(0.18)
FF6+IO Alpha	0.01	-0.59**	-1.03***	-0.54***
	(0.30)	(0.24)	(0.36)	(0.18)
FF6+EMI Alpha	-0.01	-0.66***	-1.18***	-0.62***
	(0.30)	(0.24)	(0.36)	(0.18)
FF6+3 Alpha	-0.05	-0.64***	-1.14***	-0.61***
	(0.31)	(0.25)	(0.36)	(0.18)

Panel F. Conditional Predictive Power on BPC

PT \ BPC	Small	2	Big	Average
Low	1.56	1.18	1.53	1.42
2	1.36	1.27	1.58	1.40
3	1.37	0.99	1.59	1.31
4	1.17	1.24	1.36	1.26
High	1.16	1.01	0.96	1.05
		High-Low		
Raw	-0.40*	-0.16	-0.56*	-0.38**
	(0.24)	(0.18)	(0.29)	(0.15)
CAPM Alpha	-0.24	-0.18	-0.67**	-0.36**
	(0.24)	(0.18)	(0.30)	(0.16)
FF3 Alpha	-0.37*	-0.23	-0.88***	-0.50***
	(0.20)	(0.18)	(0.29)	(0.14)
FF5 Alpha	-0.64***	-0.19	-0.90***	-0.58***
	(0.21)	(0.19)	(0.31)	(0.15)
FF6 Alpha	-0.59***	-0.11	-0.78**	-0.49***
	(0.21)	(0.19)	(0.30)	(0.14)
FF6+IMC Alpha	-0.57**	-0.16	-0.85**	-0.53***
	(0.23)	(0.21)	(0.33)	(0.15)
FF6+IO Alpha	-0.59**	-0.11	-0.86**	-0.52***
	(0.23)	(0.21)	(0.34)	(0.16)
FF6+EMI Alpha	-0.51**	-0.22	-1.05***	-0.59***
	(0.23)	(0.21)	(0.33)	(0.16)
FF6+3 Alpha	-0.53**	-0.18	-1.03***	-0.58***
	(0.23)	(0.21)	(0.34)	(0.16)

Panel G. Conditional Predictive Power on BPP

PT \ BPP	Small	2	Big	Average
Low	1.52	1.12	1.51	1.38
2	1.39	1.33	1.57	1.43
3	1.29	1.08	1.47	1.28
4	1.12	1.33	1.28	1.24
High	1.23	1.02	0.99	1.08
		High-Low		
Raw	-0.28	-0.1	-0.52*	-0.30**
	(0.21)	(0.21)	(0.28)	(0.15)
CAPM Alpha	-0.15	-0.09	-0.51*	-0.25*
	(0.21)	(0.21)	(0.29)	(0.15)
FF3 Alpha	-0.26	-0.13	-0.82***	-0.40***
	(0.19)	(0.21)	(0.26)	(0.13)
FF5 Alpha	-0.44**	-0.13	-1.05***	-0.54***
	(0.20)	(0.22)	(0.28)	(0.14)
FF6 Alpha	-0.40**	-0.04	-0.97***	-0.47***
	(0.20)	(0.22)	(0.28)	(0.13)
FF6+IMC Alpha	-0.37	-0.12	-1.01***	-0.50***
	(0.23)	(0.23)	(0.31)	(0.14)
FF6+IO Alpha	-0.35	-0.11	-1.06***	-0.50***
	(0.23)	(0.23)	(0.31)	(0.15)
FF6+EMI Alpha	-0.32	-0.2	-1.13***	-0.55***
	(0.23)	(0.23)	(0.31)	(0.15)
FF6+3 Alpha	-0.31	-0.19	-1.16***	-0.55***
	(0.23)	(0.23)	(0.31)	(0.15)

Panel H. Conditional Predictive Power on PMC

PT \ PMC	Small	2	Big	Average
Low	1.27	1.61	1.43	1.44
2	1.32	1.40	1.50	1.41
3	1.18	1.19	1.49	1.29
4	1.33	1.16	1.35	1.28
High	0.01	1.17	1.15	1.06
		High-Low		
Raw	-0.39*	-0.44**	-0.29	-0.37***
	(0.20)	(0.20)	(0.31)	(0.14)
CAPM Alpha	-0.40**	-0.38*	-0.23	-0.33**
	(0.20)	(0.20)	(0.32)	(0.14)
FF3 Alpha	-0.53***	-0.40**	-0.41	-0.44***
	(0.20)	(0.20)	(0.30)	(0.12)
FF5 Alpha	-0.35*	-0.47**	-0.71**	-0.51***
	(0.21)	(0.21)	(0.31)	(0.13)
FF6 Alpha	-0.27	-0.48**	-0.59*	-0.45***
	(0.20)	(0.21)	(0.31)	(0.13)
FF6+IMC Alpha	-0.29	-0.53**	-0.67*	-0.49***
	(0.23)	(0.24)	(0.35)	(0.14)
FF6+IO Alpha	-0.27	-0.53**	-0.74**	-0.51***
	(0.23)	(0.24)	(0.35)	(0.14)
FF6+EMI Alpha	-0.35	-0.58**	-0.79**	-0.57***
	(0.23)	(0.24)	(0.35)	(0.14)
FF6+3 Alpha	-0.34	-0.57**	-0.83**	-0.58***
	(0.23)	(0.25)	(0.35)	(0.14)

Panel I. Conditional Predictive Power on PQ

PT \ PQ	Small	2	Big	Average
Low	1.63	1.56	1.55	1.58
2	1.32	1.47	1.45	1.41
3	1.21	1.54	1.16	1.31
4	1.62	1.19	1.26	1.36
High	1.31	1.07	0.98	1.12
		High-Low		
Raw	-0.33	-0.49*	-0.57**	-0.46***
	(0.21)	(0.26)	(0.24)	(0.16)
CAPM Alpha	-0.29	-0.36	-0.51**	-0.39**
	(0.21)	(0.26)	(0.24)	(0.16)
FF3 Alpha	-0.40*	-0.54**	-0.71***	-0.55***
	(0.21)	(0.25)	(0.23)	(0.14)
FF5 Alpha	-0.33	-1.09***	-0.87***	-0.76***
	(0.22)	(0.25)	(0.24)	(0.14)
FF6 Alpha	-0.25	-1.10***	-0.76***	-0.70***
	(0.22)	(0.25)	(0.24)	(0.14)
FF6+IMC Alpha	-0.25	-1.31***	-0.78***	-0.78***
	(0.23)	(0.28)	(0.25)	(0.15)
FF6+IO Alpha	-0.23	-1.36***	-0.75***	-0.78***
	(0.23)	(0.28)	(0.25)	(0.15)
FF6+EMI Alpha	-0.33	-1.33***	-0.84***	-0.83***
	(0.23)	(0.28)	(0.25)	(0.15)
FF6+3 Alpha	-0.31	-1.36***	-0.81***	-0.83***
	(0.23)	(0.28)	(0.26)	(0.15)

Table V
Market Beta and Patent Thicket

We employ the two-step Fama-MacBeth regression. In the first step, at each month between 1981 and 2010, for each firm we regress its monthly excess return on MKT using the past 60-month window and get its time-series factor loading on MKT (beta). In the second step, at each month between 1981 and 2010, we do a cross-sectional regression of beta on patent thicket, with or without industry fixed effect, and get the intercept and slope of PT. We report the time-series averages of them, and test their significance calculating the Newey-West standard errors (in parentheses). Industry fixed effect (Industry FE) is constructed under Fama-French 48 industry classifications. Ranked PT takes the value of 1, 2, 3, 4, or 5, for firms with PT in the low, 2, 3, 4, and high quintiles respectively in each year. Transformed PT is a monotonic transformation of PT such that it is equal to 1/(1-Frag).

	Specification 1: PT		**Specification 2: Ranked PT**		**Specification 3: Transformed PT**	
Intercept	1.3187***	1.8162***	1.3217***	1.6312***	1.2923***	1.5764***
	(0.0601)	(0.1284)	(0.0211)	(0.0953)	(0.0206)	(0.0908)
PT	-0.0647	-0.3059***	-0.0199***	-0.0315***	-0.0016***	-0.0016***
	(0.0770)	(0.0663)	(0.0023)	(0.0036)	(0.0003)	(0.0003)
Industry FE	No	Yes	No	Yes	No	Yes

Table VI

Volatility of Future Fundamentals and Patent Thicket

At the end of year *t* from 1981 to 2005, for Panel A we sort firms on patent thicket (PT) based on the quintiles of PT, and then for each quintile of PT we compute the weighted average (by firm size) of standard deviation of ROA, IA, or SA in year *t* + 1 to *t* + 5, and for Panel B we conduct independent double sorts on patent thicket (PT) and patent portfolio size into 25 groups based on the quintiles of PT and patent portfolio size, and then for each of the 25 portfolios we compute the weighted average (by firm size) of standard deviation of ROA, IA, or SA in year *t* + 1 to *t* + 5. We also compare the highest quintile with the lowest quintile (High–Low) and test the significance of the difference. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. ROA in year *t* is defined as income before extraordinary items plus interest expenses in year *t* divided by total assets in year *t* – 1. IA in year *t* is defined as capital expenditure in year *t* divided by total assets in year *t* – 1. SA in year *t* is defined as total sales in year *t* divided by total assets in year *t* – 1.

Panel A. One-way Sorting Analysis

PT	ROA Volatility	PT	IA Volatility	PT	SA Volatility
Low	0.0495	Low	0.0263	Low	0.1634
2	0.0481	2	0.0224	2	0.1729
3	0.0420	3	0.0196	3	0.1703
4	0.0360	4	0.0169	4	0.1359
High	0.0282	High	0.0185	High	0.1441
High-Low	-0.0213***	High-Low	-0.0079***	High-Low	-0.0194
	(0.0057)		(0.0014)		(0.0141)

Table VI (continued)

Panel B1. PT Predicts ROA Volatility

Panel B1(a). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0443	0.0665	0.0508	0.0498	0.1462	0.0716
2	0.0436	0.0501	0.0591	0.0478	0.0804	0.0562
3	0.0341	0.0472	0.0432	0.0476	0.0676	0.0479
4	0.0285	0.0484	0.0303	0.0380	0.0625	0.0416
High	0.0319	0.0295	0.0254	0.0339	0.0311	0.0303
High-Low	-0.0124**	-0.0370***	-0.0255***	-0.0159***	-0.1151***	-0.0412***
	(0.0059)	(0.0117)	(0.0061)	(0.0046)	(0.0198)	(0.0052)

Panel B1 continued

Panel B1(b). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0421	0.0618	0.0433	0.0722	0.0981	0.0635
2	0.0451	0.0446	0.0719	0.0606	0.0993	0.0643
3	0.0344	0.0416	0.0462	0.0548	0.0947	0.0543
4	0.0271	0.0378	0.0377	0.0418	0.0594	0.0408
High	0.0269	0.0286	0.0292	0.0307	0.0456	0.0322
High-Low	-0.0152**	-0.0332***	-0.0141***	-0.0415***	-0.0525***	-0.0313***
	(0.0062)	(0.0096)	(0.0032)	(0.0076)	(0.0079)	(0.0039)

Panel B2. PT Predicts IA Volatility

Panel B2(a). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0271	0.0256	0.0274	0.0254	0.0419	0.0295
2	0.0247	0.0237	0.0219	0.0239	0.0250	0.0238
3	0.0195	0.0212	0.0186	0.0217	0.0242	0.0211
4	0.0162	0.0170	0.0148	0.0180	0.0270	0.0186
High	0.0180	0.0203	0.0164	0.0182	0.0147	0.0175
High-Low	-0.0091***	-0.0053*	-0.0110***	-0.0072***	-0.0272***	-0.0120***
	(0.0024)	(0.0029)	(0.0023)	(0.0025)	(0.0050)	(0.0011)

Panel B2 continued

Panel B2(b). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0278	0.0235	0.0198	0.0352	0.0326	0.0278
2	0.0246	0.0202	0.0257	0.0252	0.0343	0.0260
3	0.0197	0.0189	0.0209	0.0222	0.0342	0.0232
4	0.0152	0.0159	0.0167	0.0192	0.0307	0.0195
High	0.0164	0.0202	0.0176	0.0156	0.0193	0.0178
High-Low	-0.0114***	-0.0033	-0.0023	-0.0196***	-0.0132***	-0.0100***
	(0.0023)	(0.0021)	(0.0018)	(0.0034)	(0.0031)	(0.0010)

Panel B3. PT Predicts SA Volatility

Panel B3(a). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.1543	0.1828	0.1960	0.1456	0.2300	0.1818
2	0.1774	0.1723	0.1887	0.1836	0.2087	0.1861
3	0.1856	0.1747	0.1304	0.2025	0.1842	0.1755
4	0.1426	0.1350	0.1236	0.1418	0.1678	0.1421
High	0.1454	0.1676	0.1448	0.1355	0.1123	0.1411
High-Low	-0.0089	-0.0152	-0.0513**	-0.0101	-0.1177***	-0.0406***
	(0.0182)	(0.0171)	(0.0205)	(0.0122)	(0.0234)	(0.0075)

Panel B3 continued

Panel B3(b). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.1562	0.1704	0.1502	0.2202	0.1801	0.1754
2	0.1829	0.1650	0.2132	0.1718	0.2377	0.1941
3	0.1975	0.1675	0.1515	0.2007	0.2056	0.1845
4	0.1349	0.1205	0.1447	0.1448	0.1693	0.1428
High	0.1265	0.1721	0.1577	0.1153	0.1398	0.1423
High-Low	-0.0298	0.0017	0.0076	-0.1049***	-0.0403**	-0.0332***
	(0.0188)	(0.0165)	(0.0147)	(0.0163)	(0.0155)	(0.0071)

Table VII

Two-way Portfolio Sorting on Patent Thicket and Patent Portfolio Size

At the end of June of year *t* from 1982 to 2011, we conduct independent double sorts on patent thicket (PT) and patent portfolio size into 25 groups based on the quintiles of PT and patent portfolio size in year *t* – 1. Then we construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each patent portfolio size category and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We proxy patent portfolio size with 5-year total number of patent scaled by book equity (CTBE) in Panel A, and 5-year total number of patent scaled by total asset (CTA) in Panel B. We report the monthly returns in excess of one-month Treasury bill rate (Excess Returns). Following Fama and French (1993), MKT denotes the market factor. SMB denotes the size factor, and HML represents the value factor. Similarly to Fama and French (2015), we use RMW and CMA to represent profitability and investment factors respectively. UMD stands for the momentum factor (Carhart, 1997). IMC represents the investment-minus-consumption factor by Papanikolaou (2011). IO stands for the innovation originality factor by Hirshleifer, Hsu, and Li (2014). EMI denotes the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. Sample period: 1982Q3-2012Q2 for MKT, SMB, HML, RMW, CMA, and UMD, 1982Q3-2008Q2 for IMC, IO, and EMI.

Panel A. Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	B-S
Low	1.21	1.80	1.93	1.84	3.05	1.84
2	1.37	1.33	1.82	1.14	2.13	0.76
3	1.01	1.25	1.27	1.75	1.67	0.66
4	1.09	1.28	1.17	1.30	1.70	0.62
High	1.08	0.77	1.08	1.11	1.30	0.22
			High-Low			
Raw	-0.13	-1.03***	-0.85***	-0.73***	-1.76***	-1.62***
	(0.21)	(0.32)	(0.32)	(0.27)	(0.57)	(0.55)
CAPM Alpha	-0.04	-0.96***	-0.77**	-0.69**	-1.48***	-1.45***
	(0.21)	(0.32)	(0.32)	(0.27)	(0.57)	(0.55)
FF3 Alpha	-0.19	-1.22***	-0.90***	-0.74***	-1.55***	-1.36***
	(0.20)	(0.30)	(0.28)	(0.27)	(0.51)	(0.51)
FF5 Alpha	-0.49**	-1.44***	-0.88***	-0.51*	-2.00***	-1.51***
	(0.21)	(0.31)	(0.29)	(0.29)	(0.54)	(0.55)
FF6 Alpha	-0.41**	-1.33***	-0.80***	-0.49*	-1.97***	-1.56***
	(0.21)	(0.31)	(0.29)	(0.29)	(0.54)	(0.55)
FF6+IMC Alpha	-0.52**	-1.37***	-0.76**	-0.34	-1.67***	-1.15**
	(0.23)	(0.34)	(0.33)	(0.32)	(0.46)	(0.49)
FF6+IO Alpha	-0.58**	-1.44***	-0.71**	-0.24	-1.54***	-0.96*
	(0.23)	(0.34)	(0.33)	(0.32)	(0.46)	(0.49)
FF6+EMI Alpha	-0.54**	-1.39***	-0.69**	-0.39	-1.63***	-1.09**
	(0.23)	(0.35)	(0.33)	(0.32)	(0.47)	(0.50)
FF6+3 Alpha	-0.59**	-1.45***	-0.66**	-0.31	-1.54***	-0.95*
	(0.23)	(0.34)	(0.33)	(0.32)	(0.46)	(0.49)

Panel B. Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	B-S
Low	1.25	1.66	1.61	2.37	2.83	1.58
2	1.32	1.42	1.49	1.61	2.27	0.95
3	1.04	1.23	1.61	1.41	2.45	1.42
4	1.14	1.22	1.21	1.40	1.76	0.61
High	0.96	0.98	1.05	1.14	1.59	0.63
			High-Low			
Raw	-0.29	-0.68*	-0.56**	-1.23***	-1.24***	-0.95**
	(0.20)	(0.36)	(0.24)	(0.40)	(0.41)	(0.43)
CAPM Alpha	-0.23	-0.54	-0.55**	-0.99**	-1.18***	-0.95**
	(0.20)	(0.36)	(0.24)	(0.39)	(0.42)	(0.44)
FF3 Alpha	-0.39**	-0.88***	-0.61**	-1.02***	-1.24***	-0.85*
	(0.19)	(0.32)	(0.24)	(0.35)	(0.41)	(0.44)
FF5 Alpha	-0.55***	-1.14***	-0.61**	-1.09***	-1.50***	-0.95**
	(0.20)	(0.34)	(0.25)	(0.37)	(0.43)	(0.47)
FF6 Alpha	-0.47**	-1.05***	-0.55**	-1.16***	-1.38***	-0.91*
	(0.20)	(0.34)	(0.25)	(0.37)	(0.43)	(0.47)
FF6+IMC Alpha	-0.56**	-1.14***	-0.67**	-0.86***	-1.38***	-0.82
	(0.22)	(0.38)	(0.27)	(0.30)	(0.48)	(0.52)
FF6+IO Alpha	-0.61***	-1.22***	-0.61**	-0.76**	-1.40***	-0.78
	(0.22)	(0.39)	(0.27)	(0.31)	(0.49)	(0.53)
FF6+EMI Alpha	-0.63***	-1.10***	-0.74***	-0.83***	-1.55***	-0.92*
	(0.23)	(0.39)	(0.27)	(0.31)	(0.49)	(0.53)
FF6+3 Alpha	-0.67***	-1.18***	-0.69**	-0.76**	-1.55***	-0.88*
	(0.23)	(0.39)	(0.28)	(0.31)	(0.48)	(0.53)

Table VIII

Two-way Portfolio Sorting on Patent Thicket and R&D Investment

At the end of June of year *t* from 1967 to 2011, we conduct independent double sorts on patent thicket (PT) and R&D investment into 25 groups based on the quintiles of PT and R&D investment in year *t* – 1. Then we construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each R&D investment category and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We proxy R&D investment with R&D expenditure scaled by book equity (RDBE) in Panel A, and R&D expenditure scaled by total asset (RDA) in Panel B. We report the monthly returns in excess of one-month Treasury bill rate (Excess Returns). Following Fama and French (1993), MKT denotes the market factor. SMB denotes the size factor, and HML represents the value factor. Similarly to Fama and French (2015), we use RMW and CMA to represent profitability and investment factors respectively. UMD stands for the momentum factor (Carhart, 1997). IMC represents the investment-minus-consumption factor by Papanikolaou (2011). IO stands for the innovation originality factor by Hirshleifer, Hsu, and Li (2014). EMI denotes the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. Sample period: 1982Q3-2012Q2 for MKT, SMB, HML, RMW, CMA, and UMD, 1982Q3-2008Q2 for IMC, IO, and EMI.

Panel A. Conditional Predictive Power on RDBE

PT \ CTBE	Small	2	3	4	Big	B-S
Low	0.94	1.29	1.86	1.52	2.89	1.95
2	1.04	0.73	1.50	1.83	2.21	1.17
3	1.10	1.12	1.49	1.33	2.00	0.89
4	1.08	1.38	1.45	1.24	1.72	0.64
High	0.97	0.90	1.03	1.07	1.31	0.34
			High-Low			
Raw	0.03	-0.39	-0.83***	-0.45	-1.58***	-1.61***
	(0.19)	(0.24)	(0.27)	(0.37)	(0.41)	(0.45)
CAPM Alpha	0.01	-0.35	-0.77***	-0.3	-1.33***	-1.34***
	(0.19)	(0.25)	(0.28)	(0.37)	(0.40)	(0.45)
FF3 Alpha	0.01	-0.36	-1.00***	-0.66**	-1.64***	-1.65***
	(0.19)	(0.25)	(0.25)	(0.33)	(0.36)	(0.42)
FF5 Alpha	-0.08	-0.36	-1.17***	-1.04***	-1.80***	-1.72***
	(0.20)	(0.26)	(0.27)	(0.35)	(0.39)	(0.44)
FF6 Alpha	-0.02	-0.39	-1.08***	-0.92***	-1.86***	-1.84***
	(0.20)	(0.26)	(0.26)	(0.35)	(0.39)	(0.44)
FF6+IMC Alpha	-0.05	-0.42	-1.20***	-1.04***	-1.66***	-1.61***
	(0.22)	(0.29)	(0.30)	(0.39)	(0.42)	(0.48)
FF6+IO Alpha	-0.13	-0.29	-1.23***	-1.08***	-1.67***	-1.54***
	(0.22)	(0.29)	(0.30)	(0.40)	(0.43)	(0.48)
FF6+EMI Alpha	-0.11	-0.32	-1.26***	-1.21***	-1.83***	-1.72***
	(0.23)	(0.30)	(0.30)	(0.39)	(0.43)	(0.48)
FF6+3 Alpha	-0.16	-0.24	-1.27***	-1.22***	-1.82***	-1.66***
	(0.23)	(0.30)	(0.30)	(0.40)	(0.43)	(0.48)

Panel B. Conditional Predictive Power on RDA

PT \ CTA	Small	2	3	4	Big	B-S
Low	0.93	1.22	1.45	1.98	2.94	2.02
2	1.04	0.83	1.20	2.10	2.33	1.30
3	1.07	1.21	1.11	1.48	2.27	1.20
4	1.21	1.32	1.46	1.20	1.89	0.68
High	0.92	1.07	0.92	1.23	2.27	1.34
			High-Low			
Raw	0	-0.14	-0.53*	-0.74*	-0.68	-0.67
	(0.19)	(0.22)	(0.31)	(0.41)	(0.51)	(0.55)
CAPM Alpha	-0.05	-0.21	-0.38	-0.55	-0.85*	-0.81
	(0.19)	(0.22)	(0.31)	(0.41)	(0.51)	(0.56)
FF3 Alpha	-0.03	-0.32	-0.55*	-0.95**	-0.62	-0.59
	(0.19)	(0.22)	(0.30)	(0.37)	(0.51)	(0.56)
FF5 Alpha	-0.05	-0.28	-0.81**	-1.25***	-0.26	-0.22
	(0.21)	(0.24)	(0.32)	(0.39)	(0.52)	(0.58)
FF6 Alpha	0	-0.23	-0.78**	-1.22***	-0.39	-0.38
	(0.21)	(0.24)	(0.32)	(0.39)	(0.52)	(0.58)
FF6+IMC Alpha	-0.05	-0.15	-0.92**	-1.43***	0.29	0.34
	(0.23)	(0.27)	(0.36)	(0.45)	(0.54)	(0.60)
FF6+IO Alpha	-0.12	-0.06	-0.96***	-1.45***	0.33	0.44
	(0.23)	(0.27)	(0.37)	(0.45)	(0.55)	(0.60)
FF6+EMI Alpha	-0.1	-0.08	-1.04***	-1.64***	-0.03	0.07
	(0.23)	(0.27)	(0.37)	(0.45)	(0.54)	(0.60)
FF6+3 Alpha	-0.15	-0.02	-1.06***	-1.63***	0.03	0.18
	(0.23)	(0.27)	(0.37)	(0.45)	(0.54)	(0.60)

Table IX
Two-way Portfolio Sorting on Patent Thicket and Innovation Efficiency

At the end of June of year *t* from 1967 to 2011, we conduct independent double sorts on patent thicket (PT) and innovation efficiency into 25 groups based on the quintiles of PT and innovation efficiency in year *t* – 1. Then we construct a high-minus-low (H-L) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each innovation efficiency category and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We proxy innovation efficiency with the patent-based innovation efficiency (IE) by the number of patents granted in year *t* – 1 divided by one plus total R&D capital accumulated in year *t* – 7 to *t* – 3 (with 20% obsolescence rate). We report the monthly returns in excess of one-month Treasury bill rate (Excess Returns). Following Fama and French (1993), MKT denotes the market factor. SMB denotes the size factor, and HML represents the value factor. Similarly to Fama and French (2015), we use RMW and CMA to represent profitability and investment factors respectively. UMD stands for the momentum factor (Carhart, 1997). IMC represents the investment-minus-consumption factor by Papanikolaou (2011). IO stands for the innovation originality factor by Hirshleifer, Hsu, and Li (2014). EMI denotes the innovative efficient-minus-inefficient factor by Hirshleifer, Hsu, and Li (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate the significance level of 1%, 5%, and 10%, respectively. Sample period: 1982Q3-2012Q2 for MKT, SMB, HML, RMW, CMA, and UMD, 1982Q3-2008Q2 for IMC, IO, and EMI.

PT \ IE	Small	2	3	4	Big	B-S
Low	1.78	1.82	1.84	1.19	0.95	-0.83
2	1.67	1.34	1.46	0.97	1.45	-0.22
3	1.10	1.26	1.18	1.48	1.74	0.64
4	1.07	1.27	1.27	1.56	1.28	0.21
High	0.86	1.24	1.01	1.16	1.22	0.36
			High-Low			
Raw	-0.92***	-0.58*	-0.83***	-0.04	0.27	1.18***
	(0.32)	(0.30)	(0.28)	(0.22)	(0.25)	(0.38)
CAPM Alpha	-0.72**	-0.52*	-0.91***	-0.02	0.37	1.08***
	(0.32)	(0.30)	(0.29)	(0.23)	(0.25)	(0.38)
FF3 Alpha	-1.04***	-0.78***	-1.03***	-0.02	0.39	1.43***
	(0.29)	(0.28)	(0.28)	(0.23)	(0.25)	(0.35)
FF5 Alpha	-1.30***	-1.09***	-0.90***	-0.14	0.07	1.36***
	(0.30)	(0.29)	(0.30)	(0.24)	(0.26)	(0.38)
FF6 Alpha	-1.17***	-1.00***	-0.83***	-0.11	0.05	1.23***
	(0.30)	(0.29)	(0.30)	(0.24)	(0.26)	(0.37)
FF6+IMC Alpha	-1.35***	-1.06***	-0.70**	-0.24	-0.08	1.27***
	(0.33)	(0.32)	(0.33)	(0.26)	(0.29)	(0.41)
FF6+IO Alpha	-1.41***	-1.05***	-0.58*	-0.33	-0.13	1.28***
	(0.34)	(0.32)	(0.33)	(0.26)	(0.29)	(0.41)
FF6+EMI Alpha	-1.45***	-1.19***	-0.81**	-0.24	-0.08	1.37***
	(0.34)	(0.32)	(0.34)	(0.26)	(0.29)	(0.41)
FF6+3 Alpha	-1.49***	-1.18***	-0.70**	-0.30	-0.12	1.36***
	(0.34)	(0.32)	(0.33)	(0.26)	(0.29)	(0.42)

F Tables in Online Appendix

Table F.I

One-way Portfolio Sorting on Patent Thicket with Industry Adjustments

This table works as a robustness check of Table III. In Panel A we report the monthly industry-adjusted returns (Industry-adjusted Excess Returns), which are based on the difference between individual firms' excess returns and the weighted average excess returns of firms in the same industry (accordingly to Fama-French 48 industry classifications), and their corresponding alphas and betas in different model specifications. In Panel B we sort patent thicket within industries, then calculate the value-weighted excess return of each quintile portfolio, and report their corresponding alphas and betas in different model specifications.

Panel A. Predictive Power of PT on Industry-adjusted Excess Returns

PT	Raw	CAPM		FF3				FF5					
	Time-series Mean	Alpha	MKT	Alpha	MKT	SMB	HML	Alpha	MKT	SMB	HML	RMW	CMA
No	-0.40**	-0.45***	0.06***	-0.57***	0.05***	0.14***	0.06***	-0.38***	0.04***	0.13***	0.09***	-0.04**	-0.05*
	(0.16)	(0.16)	(0.01)	(0.11)	(0.01)	(0.01)	(0.01)	(0.12)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)
Low	0.13	0.14	-0.01	0.17*	-0.03	0.07**	-0.08**	0.09	-0.01	0.06*	-0.18***	0.04	0.22***
	(0.10)	(0.10)	(0.02)	(0.10)	(0.02)	(0.03)	(0.03)	(0.10)	(0.02)	(0.04)	(0.05)	(0.05)	(0.07)
2	-0.02	-0.04	0.03	0.03	0.00	-0.05	-0.17***	0.06	-0.01	-0.04	-0.13***	-0.01	-0.08
	(0.09)	(0.09)	(0.02)	(0.09)	(0.02)	(0.03)	(0.03)	(0.10)	(0.02)	(0.03)	(0.04)	(0.05)	(0.06)
3	-0.08	-0.08	0.00	-0.07	0.01	-0.09***	-0.02	-0.1	0.02	-0.09***	-0.04	0.03	0.04
	(0.07)	(0.07)	(0.02)	(0.07)	(0.02)	(0.02)	(0.02)	(0.07)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)
4	-0.05	-0.03	-0.03***	-0.03	-0.02**	-0.04**	-0.00	-0.03	-0.03**	-0.03**	0.01	0.00	-0.02
	(0.05)	(0.05)	(0.01)	(0.05)	(0.01)	(0.02)	(0.02)	(0.05)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)
High	-0.23***	-0.19***	-0.07***	-0.21***	-0.03***	-0.13***	0.07***	-0.25***	-0.02*	-0.12***	0.04	0.04	0.06
	(0.06)	(0.06)	(0.01)	(0.05)	(0.01)	(0.02)	(0.02)	(0.05)	(0.01)	(0.02)	(0.02)	(0.03)	(0.04)
High-Low	-0.36***	-0.32***	-0.06**	-0.38***	0.00	-0.20***	0.15***	-0.34***	-0.01	-0.19***	0.21***	0.00	-0.16**
	(0.12)	(0.12)	(0.03)	(0.11)	(0.03)	(0.04)	(0.04)	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)

Panel A continued

PT	FF6							FF6+IMC							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC
No	-0.29***	0.04***	0.13***	0.09***	-0.04**	-0.05**	0.00	-0.31***	0.03***	0.11***	0.07***	-0.04**	-0.03	0.03***	0.00
	(0.11)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)	(0.12)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.00)
Low	0.07	-0.00	0.06	-0.15***	0.03	0.20***	0.04**	0.07	-0.02	0.03	-0.20***	0.02	0.21***	0.06**	-0.01
	(0.10)	(0.02)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.11)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.01)
2	0.06	-0.01	-0.04	-0.14***	-0.01	-0.08	-0.00	0.11	0.03	-0.03	-0.09	-0.02	-0.12*	-0.04	0.00
	(0.10)	(0.02)	(0.03)	(0.05)	(0.05)	(0.07)	(0.02)	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.01)
3	-0.08	0.02	-0.08***	-0.06*	0.04	0.05	-0.03*	-0.1	0.04*	-0.04	-0.02	0.07	0.07	-0.04**	0.01**
	(0.07)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.02)	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.01)
4	-0.03	-0.02**	-0.04**	0.01	-0.00	-0.03	0.02	-0.03	-0.04***	-0.05***	-0.02	-0.02	-0.04	0.01	-0.00
	(0.05)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)	(0.05)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.00)
High	-0.23***	-0.03**	-0.12***	0.01	0.06**	0.08**	-0.04***	-0.23***	-0.00	-0.09***	0.05	0.06**	0.07*	-0.06***	0.00
	(0.05)	(0.01)	(0.02)	(0.02)	(0.03)	(0.04)	(0.01)	(0.06)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.00)
High-Low	-0.30**	-0.03	-0.18***	0.17***	0.03	-0.12	-0.08***	-0.30**	0.02	-0.13***	0.25***	0.04	-0.14	-0.12***	0.01
	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)	(0.02)	(0.13)	(0.03)	(0.05)	(0.07)	(0.07)	(0.09)	(0.03)	(0.01)

Panel A continued

PT	FF6+IO								FF6+EMI							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IO	Alpha	MKT	SMB	HML	RMW	CMA	UMD	EMI
No	-0.28**	0.03***	0.11***	0.07***	-0.04**	-0.04	0.03***	0.02	-0.28**	0.03***	0.11***	0.06***	-0.04**	-0.03	0.03***	-0.06***
	(0.12)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.12)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)
Low	0.11	-0.02	0.03	-0.21***	0.06	0.24***	0.05**	-0.13**	0.16	-0.02	0.03	-0.23***	0.03	0.22***	0.05**	-0.25***
	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.08)	(0.03)	(0.05)	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.06)
2	0.12	0.03	-0.03	-0.09*	-0.01	-0.12	-0.04	-0.04	0.07	0.03	-0.02	-0.07	-0.02	-0.12*	-0.03	0.11**
	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.05)	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.05)
3	-0.13	0.03	-0.05	-0.02	0.03	0.04	-0.03*	0.08**	-0.12	0.03	-0.05*	-0.02	0.04	0.05	-0.04*	0.04
	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.04)	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.04)
4	-0.04	-0.04***	-0.04**	-0.01	-0.01	-0.04	0.01	0.03	-0.07	-0.04***	-0.04**	0.00	-0.00	-0.03	0.01	0.12***
	(0.05)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.02)	(0.05)	(0.01)	(0.02)	(0.03)	(0.02)	(0.03)	(0.01)	(0.03)
High	-0.21***	-0.01	-0.10***	0.04	0.07***	0.08**	-0.06***	-0.08***	-0.19***	-0.001	-0.10***	0.03	0.06**	0.07*	-0.06***	-0.10***
	(0.06)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.03)	(0.06)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.03)
High-Low	-0.32**	0.02	-0.13***	0.25***	0.01	-0.16*	-0.11***	0.05	-0.36***	0.02	-0.13***	0.26***	0.02	-0.15*	-0.11***	0.14**
	(0.13)	(0.03)	(0.04)	(0.07)	(0.06)	(0.09)	(0.03)	(0.06)	(0.13)	(0.03)	(0.04)	(0.07)	(0.06)	(0.08)	(0.03)	(0.07)

Panel A continued

PT	FF6+IMC+IO+EMI									
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC	IO	EMI
No	-0.25**	0.03***	0.11***	0.06***	-0.04**	-0.04	0.03***	0.00	0.03*	-0.06***
	(0.12)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.00)	(0.02)	(0.02)
Low	0.18	-0.03	0.00	-0.24***	0.03	0.22***	0.05**	-0.01*	-0.09*	-0.23***
	(0.11)	(0.03)	(0.04)	(0.06)	(0.06)	(0.07)	(0.02)	(0.01)	(0.05)	(0.06)
2	0.08	0.03	-0.02	-0.08	-0.00	-0.11	-0.04	0.00	-0.06	0.12**
	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.01)	(0.05)	(0.06)
3	-0.13	0.04*	-0.03	-0.00	0.05	0.06	-0.04**	0.01**	0.08**	0.03
	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.01)	(0.04)	(0.04)
4	-0.07	-0.04***	-0.04**	0.00	-0.01	-0.04	0.01	-0.00	0.01	0.12***
	(0.05)	(0.01)	(0.02)	(0.03)	(0.03)	(0.03)	(0.01)	(0.00)	(0.02)	(0.03)
High	-0.18***	-0.01	-0.11***	0.03	0.07**	0.08**	-0.07***	-0.00	-0.06**	-0.09***
	(0.06)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.00)	(0.03)	(0.03)
High-Low	-0.36***	0.02	-0.11**	0.27***	0.04	-0.14*	-0.11***	0.01	0.03	0.14**
	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.01)	(0.06)	(0.07)

Panel B. Predictive Power of PT sorted within industries on Excess Returns

PT	Raw	CAPM		FF3				FF5					
	Time-series Mean	Alpha	MKT	Alpha	MKT	SMB	HML	Alpha	MKT	SMB	HML	RMW	CMA
Low	1.49***	0.84***	1.00***	0.91***	0.95***	0.09***	-0.15***	0.81***	0.98***	0.09**	-0.26***	0.05	0.24***
	(0.26)	(0.10)	(0.02)	(0.10)	(0.02)	(0.03)	(0.03)	(0.10)	(0.02)	(0.04)	(0.05)	(0.05)	(0.07)
2	1.41***	0.71***	1.09***	0.84***	1.02***	0.07*	-0.32***	0.90***	1.01***	-0.01	-0.36***	-0.20***	0.15**
	(0.29)	(0.12)	(0.03)	(0.11)	(0.03)	(0.04)	(0.04)	(0.11)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)
3	1.27***	0.58***	1.07***	0.75***	1.00***	-0.03	-0.40***	0.88***	0.97***	-0.09**	-0.33***	-0.20***	-0.10
	(0.29)	(0.13)	(0.03)	(0.12)	(0.03)	(0.04)	(0.04)	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)
4	1.24***	0.63***	0.94***	0.65***	0.95***	-0.12***	-0.03	0.52***	0.99***	-0.10***	-0.17***	0.11**	0.28***
	(0.24)	(0.08)	(0.02)	(0.08)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
High	1.07***	0.49***	0.89***	0.53***	0.90***	-0.17***	-0.08**	0.42***	0.94***	-0.17***	-0.21***	0.06	0.28***
	(0.24)	(0.09)	(0.02)	(0.09)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.05)	(0.06)
High-Low	-0.42***	-0.35***	-0.10***	-0.38***	-0.05	-0.26***	0.07	-0.39***	-0.04	-0.26***	0.05	0.01	0.04
	(0.14)	(0.13)	(0.03)	(0.13)	(0.03)	(0.04)	(0.04)	(0.14)	(0.03)	(0.05)	(0.06)	(0.07)	(0.09)

Panel B continued

PT	FF6							FF6+IMC							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC
Low	0.80***	0.98***	0.09**	-0.25***	0.05	0.23***	0.02	0.83***	0.99***	0.06*	-0.26***	0.00	0.22***	0.02	-0.02***
	(0.10)	(0.02)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.01)
2	0.92***	1.01***	-0.01	-0.38***	-0.19***	0.17**	-0.03	1.01***	1.03***	0.01	-0.35***	-0.20***	0.18**	-0.05	0.00
	(0.12)	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)	(0.02)	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.01)
3	0.88***	0.96***	-0.09**	-0.34***	-0.19***	-0.09	-0.02	0.94***	0.98***	-0.07	-0.27***	-0.22***	-0.17*	-0.05	0.01
	(0.12)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.13)	(0.03)	(0.05)	(0.07)	(0.07)	(0.09)	(0.03)	(0.01)
4	0.54***	0.98***	-0.10***	-0.20***	0.12***	0.31***	-0.05**	0.53***	0.95***	-0.15***	-0.27***	0.09*	0.32***	-0.03	-0.01**
	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.01)
High	0.44***	0.93***	-0.17***	-0.23***	0.07	0.30***	-0.03*	0.44***	0.95***	-0.19***	-0.26***	0.06	0.32***	-0.04*	-0.01
	(0.10)	(0.02)	(0.03)	(0.04)	(0.05)	(0.06)	(0.02)	(0.10)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.01)
High-Low	-0.36***	-0.05	-0.25***	0.02	0.03	0.07	-0.06**	-0.39***	-0.04	-0.25***	-0.01	0.06	0.10	-0.06*	0.01
	(0.14)	(0.03)	(0.05)	(0.06)	(0.07)	(0.09)	(0.03)	(0.15)	(0.04)	(0.05)	(0.08)	(0.07)	(0.10)	(0.03)	(0.01)

Panel B continued

PT	FF6+IO								FF6+EMI							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IO	Alpha	MKT	SMB	HML	RMW	CMA	UMD	EMI
Low	0.88***	0.99***	0.07*	-0.26***	0.07	0.26***	0.02	-0.16***	0.86***	0.99***	0.08**	-0.26***	0.04	0.24***	0.02	-0.09
	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.05)	(0.11)	(0.03)	(0.03)	(0.06)	(0.05)	(0.07)	(0.02)	(0.06)
2	0.98***	1.03***	0.02	-0.35***	-0.21***	0.16*	-0.04	0.08	0.93***	1.03***	0.03	-0.33***	-0.19***	0.17**	-0.04	0.22***
	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.09)	(0.03)	(0.06)	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.06)
3	0.90***	0.98***	-0.06	-0.25***	-0.26***	-0.21**	-0.04	0.16***	0.78***	0.98***	-0.05	-0.21***	-0.22***	-0.19**	-0.04	0.47***
	(0.13)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.06)	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.06)
4	0.51***	0.95***	-0.13***	-0.25***	0.10**	0.33***	-0.03	0.07	0.49***	0.95***	-0.13***	-0.24***	0.12***	0.34***	-0.03	0.10**
	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.04)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.05)
High	0.41***	0.95***	-0.17***	-0.25***	0.06	0.32***	-0.04*	0.08*	0.37***	0.96***	-0.16***	-0.23***	0.08*	0.32***	-0.04*	0.19***
	(0.10)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.05)	(0.10)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.05)
High-Low	-0.46***	-0.03	-0.23***	0.01	-0.00	0.05	-0.06*	0.24***	-0.49***	-0.04	-0.24***	0.03	0.04	0.08	-0.06*	0.28***
	(0.15)	(0.04)	(0.05)	(0.07)	(0.07)	(0.10)	(0.03)	(0.07)	(0.15)	(0.04)	(0.05)	(0.07)	(0.07)	(0.10)	(0.03)	(0.08)

Panel B continued

PT	FF6+IMC+IO+EMI									
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC	IO	EMI
Low	0.90***	0.98***	0.04	-0.29***	0.03	0.24***	0.02	-0.02***	-0.15***	-0.07
	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.01)	(0.05)	(0.06)
2	0.92***	1.03***	0.03	-0.32***	-0.20***	0.17**	-0.04	0.00	0.05	0.21***
	(0.13)	(0.03)	(0.05)	(0.06)	(0.06)	(0.08)	(0.03)	(0.01)	(0.06)	(0.07)
3	0.76***	0.99***	-0.03	-0.19***	-0.21***	-0.19**	-0.04	0.01	0.08	0.46***
	(0.13)	(0.03)	(0.04)	(0.06)	(0.06)	(0.08)	(0.03)	(0.01)	(0.06)	(0.07)
4	0.48***	0.95***	-0.14***	-0.25***	0.08*	0.31***	-0.02	-0.01**	0.05	0.09*
	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.01)	(0.04)	(0.05)
High	0.36***	0.95***	-0.17***	-0.23***	0.06	0.31***	-0.04	-0.01	0.05	0.18***
	(0.10)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.01)	(0.05)	(0.05)
High-Low	-0.54***	-0.03	-0.21***	0.05	0.03	0.07	-0.05*	0.01	0.20***	0.25***
	(0.15)	(0.04)	(0.05)	(0.07)	(0.07)	(0.10)	(0.03)	(0.01)	(0.07)	(0.08)

Table F.II

Two-way Portfolio Sorting on Patent Thicket and Control Variables with Industry Adjustments

This table works as a robustness check of Table IV. In Panel A we report the monthly industry-adjusted returns (Industry-adjusted Excess Returns), which are based on the difference between individual firms' excess returns and the weighted average excess returns of firms in the same industry (accordingly to Fama-French 48 industry classifications), and their corresponding alphas in different model specifications. In Panel B we sort patent thicket within industries, then calculate the value-weighted excess return of each quintile portfolio, and report their corresponding alphas and betas in different model specifications.

Panel A. Industry-adjusted Excess Returns

Panel A1. Conditional Predictive Power on SIZE

PT \ SIZE	Small	2	Big	Average
Low	2.24	0.81	-0.02	1.01
2	2.14	0.67	-0.14	0.89
3	2.11	0.67	-0.15	0.87
4	1.84	0.75	-0.09	0.84
High	1.72	0.46	-0.28	0.63
High-Low				
Raw	-0.52	-0.35	-0.26**	-0.38**
	(0.35)	(0.23)	(0.13)	(0.16)
CAPM Alpha	-0.26	-0.33	-0.23*	-0.27*
	(0.34)	(0.23)	(0.13)	(0.15)
FF3 Alpha	-0.27	-0.48**	-0.29**	-0.35**
	(0.34)	(0.20)	(0.12)	(0.14)
FF5 Alpha	-0.45	-0.64***	-0.24*	-0.44***
	(0.37)	(0.21)	(0.13)	(0.15)
FF6 Alpha	-0.76**	-0.59***	-0.19	-0.51***
	(0.33)	(0.21)	(0.13)	(0.14)
FF6+IMC Alpha	-0.55**	-0.65***	-0.19	-0.46***
	(0.28)	(0.23)	(0.14)	(0.14)
FF6+IO Alpha	-0.56**	-0.65***	-0.21	-0.47***
	(0.28)	(0.24)	(0.14)	(0.14)
FF6+EMI Alpha	-0.47*	-0.54**	-0.25*	-0.42***
	(0.28)	(0.24)	(0.14)	(0.14)
FF6+3 Alpha	-0.48*	-0.56**	-0.26*	-0.43***
	(0.28)	(0.23)	(0.15)	(0.14)

Panel A2. Conditional Predictive Power on B/M

PT \ B/M	Small	2	Big	Average
Low	0.19	-0.02	0.54	0.24
2	-0.11	0.17	0.27	0.11
3	-0.15	0.05	0.16	0.02
4	-0.09	0.01	0.81	0.24
High	-0.27	-0.09	0.05	-0.11
High-Low				
Raw	-0.46***	-0.07	-0.50*	-0.34***
	(0.14)	(0.19)	(0.27)	(0.12)
CAPM Alpha	-0.41***	-0.07	-0.55**	-0.34***
	(0.15)	(0.19)	(0.27)	(0.12)
FF3 Alpha	-0.48***	-0.07	-0.68**	-0.41***
	(0.14)	(0.19)	(0.27)	(0.12)
FF5 Alpha	-0.45***	-0.02	-0.52*	-0.33***
	(0.15)	(0.20)	(0.28)	(0.13)
FF6 Alpha	-0.41***	0.01	-0.44	-0.28**
	(0.15)	(0.20)	(0.28)	(0.12)
FF6+IMC Alpha	-0.43***	0.05	-0.63**	-0.33**
	(0.16)	(0.23)	(0.26)	(0.13)
FF6+IO Alpha	-0.43***	0.04	-0.67**	-0.36***
	(0.16)	(0.23)	(0.26)	(0.13)
FF6+EMI Alpha	-0.47***	-0.04	-0.60**	-0.37***
	(0.16)	(0.23)	(0.27)	(0.13)
FF6+3 Alpha	-0.48***	-0.04	-0.64**	-0.38***
	(0.16)	(0.23)	(0.27)	(0.13)

Panel A3. Conditional Predictive Power on MOM11

PT \ MOM11	Small	2	Big	Average
Low	0.48	0.09	0.14	0.24
2	0.40	-0.15	0.07	0.11
3	0.23	-0.21	0.10	0.04
4	0.30	-0.17	-0.01	0.04
High	0.46	-0.36	-0.23	-0.04
High-Low				
Raw	-0.03	-0.45***	-0.37*	-0.28**
	(0.30)	(0.15)	(0.19)	(0.13)
CAPM Alpha	0.09	-0.42***	-0.35*	-0.23*
	(0.30)	(0.15)	(0.19)	(0.13)
FF3 Alpha	-0.01	-0.46***	-0.39**	-0.29**
	(0.29)	(0.14)	(0.19)	(0.12)
FF5 Alpha	-0.01	-0.36**	-0.38*	-0.25**
	(0.31)	(0.15)	(0.20)	(0.13)
FF6 Alpha	-0.06	-0.31**	-0.35*	-0.24*
	(0.31)	(0.15)	(0.20)	(0.13)
FF6+IMC Alpha	-0.16	-0.28*	-0.28	-0.24*
	(0.34)	(0.16)	(0.23)	(0.14)
FF6+IO Alpha	-0.17	-0.28*	-0.3	-0.25*
	(0.34)	(0.16)	(0.23)	(0.14)
FF6+EMI Alpha	-0.26	-0.28*	-0.34	-0.29**
	(0.34)	(0.16)	(0.23)	(0.14)
FF6+3 Alpha	-0.25	-0.28*	-0.35	-0.29**
	(0.34)	(0.16)	(0.23)	(0.14)

Panel A4. Conditional Predictive Power on MOM6

PT \ MOM6	Small	2	Big	Average
Low	0.19	0.18	0.26	0.21
2	0.02	0.01	0.08	0.04
3	0.20	-0.28	0.33	0.08
4	-0.05	-0.09	0.06	-0.02
High	-0.15	-0.24	-0.28	-0.22
High-Low				
Raw	-0.34	-0.42***	-0.54***	-0.43***
	(0.29)	(0.15)	(0.21)	(0.13)
CAPM Alpha	-0.25	-0.39**	-0.52**	-0.39***
	(0.29)	(0.16)	(0.21)	(0.14)
FF3 Alpha	-0.34	-0.42***	-0.56***	-0.44***
	(0.28)	(0.15)	(0.21)	(0.13)
FF5 Alpha	-0.35	-0.33**	-0.50**	-0.39***
	(0.30)	(0.16)	(0.22)	(0.13)
FF6 Alpha	-0.35	-0.28*	-0.50**	-0.38***
	(0.30)	(0.16)	(0.22)	(0.13)
FF6+IMC Alpha	-0.37	-0.26	-0.42*	-0.35**
	(0.33)	(0.17)	(0.25)	(0.15)
FF6+IO Alpha	-0.45	-0.23	-0.41	-0.36**
	(0.33)	(0.17)	(0.25)	(0.15)
FF6+EMI Alpha	-0.36	-0.27	-0.51**	-0.38**
	(0.34)	(0.17)	(0.25)	(0.15)
FF6+3 Alpha	-0.42	-0.25	-0.49*	-0.39**
	(0.34)	(0.17)	(0.25)	(0.15)

Panel A5. Conditional Predictive Power on REV

PT \ REV	Small	2	Big	Average
Low	0.22	0.02	0.45	0.23
2	0.30	-0.17	0.02	0.05
3	0.08	-0.10	-0.04	-0.02
4	0.18	-0.07	-0.16	-0.01
High	0.13	-0.36	-0.18	-0.14
High-Low				
Raw	-0.09	-0.38**	-0.63**	-0.37***
	(0.28)	(0.16)	(0.26)	(0.14)
CAPM Alpha	0.07	-0.33**	-0.61**	-0.29**
	(0.28)	(0.17)	(0.26)	(0.14)
FF3 Alpha	0.2	-0.39**	-0.74***	-0.31**
	(0.28)	(0.16)	(0.25)	(0.13)
FF5 Alpha	0.31	-0.37**	-0.67**	-0.24*
	(0.29)	(0.17)	(0.26)	(0.14)
FF6 Alpha	0.14	-0.31*	-0.59**	-0.26*
	(0.28)	(0.17)	(0.26)	(0.14)
FF6+IMC Alpha	0.32	-0.29	-0.70**	-0.22
	(0.25)	(0.19)	(0.30)	(0.14)
FF6+IO Alpha	0.25	-0.26	-0.66**	-0.22
	(0.25)	(0.19)	(0.30)	(0.15)
FF6+EMI Alpha	0.22	-0.33*	-0.71**	-0.27*
	(0.26)	(0.19)	(0.30)	(0.15)
FF6+3 Alpha	0.18	-0.3	-0.68**	-0.27*
	(0.26)	(0.19)	(0.31)	(0.15)

Panel A6. Conditional Predictive Power on BPC

PT \ BPC	Small	2	Big	Average
Low	0.57	0.19	0.50	0.42
2	0.19	0.23	0.63	0.35
3	0.06	0.12	0.53	0.24
4	-0.23	0.37	0.28	0.14
High	0.05	-0.09	-0.16	-0.07
High-Low				
Raw	-0.52***	-0.28	-0.66**	-0.49***
	(0.20)	(0.20)	(0.28)	(0.14)
CAPM Alpha	-0.40**	-0.28	-0.66**	-0.45***
	(0.20)	(0.21)	(0.28)	(0.14)
FF3 Alpha	-0.49***	-0.40**	-0.82***	-0.57***
	(0.18)	(0.20)	(0.27)	(0.13)
FF5 Alpha	-0.64***	-0.40*	-0.71**	-0.58***
	(0.19)	(0.21)	(0.29)	(0.14)
FF6 Alpha	-0.62***	-0.34	-0.62**	-0.53***
	(0.19)	(0.21)	(0.29)	(0.14)
FF6+IMC Alpha	-0.61***	-0.31	-0.74**	-0.55***
	(0.21)	(0.23)	(0.32)	(0.15)
FF6+IO Alpha	-0.60***	-0.29	-0.73**	-0.54***
	(0.21)	(0.23)	(0.32)	(0.15)
FF6+EMI Alpha	-0.56**	-0.3	-0.81**	-0.56***
	(0.22)	(0.23)	(0.33)	(0.15)
FF6+3 Alpha	-0.56***	-0.29	-0.80**	-0.55***
	-0.22	-0.23	-0.33	-0.15

Panel A7. Conditional Predictive Power on BPP

PT \ BPP	Small	2	Big	Average
Low	0.56	0.21	0.48	0.41
2	0.34	0.24	0.60	0.39
3	-0.01	0.23	0.39	0.20
4	-0.13	0.21	0.38	0.15
High	0.09	-0.11	-0.17	-0.06
High-Low				
Raw	-0.47**	-0.32	-0.64**	-0.48***
	(0.19)	(0.22)	(0.26)	(0.14)
CAPM Alpha	-0.35*	-0.33	-0.63**	-0.44***
	(0.19)	(0.23)	(0.26)	(0.14)
FF3 Alpha	-0.44**	-0.43*	-0.80***	-0.56***
	(0.18)	(0.22)	(0.25)	(0.13)
FF5 Alpha	-0.54***	-0.46*	-0.69**	-0.57***
	(0.19)	(0.24)	(0.27)	(0.14)
FF6 Alpha	-0.52***	-0.41*	-0.61**	-0.51***
	(0.19)	(0.24)	(0.27)	(0.14)
FF6+IMC Alpha	-0.52**	-0.35	-0.65**	-0.51***
	(0.21)	(0.25)	(0.29)	(0.14)
FF6+IO Alpha	-0.47**	-0.35	-0.69**	-0.50***
	(0.21)	(0.25)	(0.29)	(0.15)
FF6+EMI Alpha	-0.47**	-0.35	-0.70**	-0.51***
	(0.21)	(0.25)	(0.30)	(0.15)
FF6+3 Alpha	-0.44**	-0.36	-0.73**	-0.51***
	(0.22)	(0.25)	(0.30)	(0.15)

Panel A8. Conditional Predictive Power on PMC

PT \ PMC	Small	2	Big	Average
Low	0.27	0.65	0.36	0.43
2	0.26	0.41	0.53	0.40
3	0.29	0.06	0.43	0.26
4	0.36	0.06	0.31	0.24
High	-0.10	0.05	-0.12	-0.06
High-Low				
Raw	-0.37*	-0.60***	-0.48	-0.48***
	(0.21)	(0.19)	(0.34)	(0.15)
CAPM Alpha	-0.32	-0.60***	-0.39	-0.44***
	(0.21)	(0.20)	(0.34)	(0.15)
FF3 Alpha	-0.45**	-0.59***	-0.61**	-0.55***
	(0.21)	(0.20)	(0.30)	(0.13)
FF4 Alpha	-0.33	-0.55***	-0.80**	-0.56***
	(0.22)	(0.21)	(0.31)	(0.14)
FF6 Alpha	-0.30	-0.55**	-0.65**	-0.50***
	(0.22)	(0.21)	(0.31)	(0.14)
FF4+IMC Alpha	-0.27	-0.58**	-0.72**	-0.52***
	(0.24)	(0.24)	(0.34)	(0.15)
FF4+IO Alpha	-0.26	-0.60**	-0.76**	-0.54***
	(0.25)	(0.24)	(0.34)	(0.15)
FF4+EMI Alpha	-0.31	-0.59**	-0.75**	-0.55***
	(0.25)	(0.24)	(0.34)	(0.16)
FF4+3 Alpha	-0.30	-0.60**	-0.78**	-0.56***
	(0.25)	(0.24)	(0.35)	(0.15)

Panel A9. Conditional Predictive Power on PQ

PT \ PQ	Small	2	Big	Average
Low	0.23	0.69	0.49	0.47
2	0.25	0.43	0.39	0.36
3	0.22	0.50	0.16	0.29
4	0.44	0.08	0.26	0.26
High	0.18	0.06	-0.18	0.02
High-Low				
Raw	-0.05	-0.63***	-0.66***	-0.45***
	(0.21)	(0.22)	(0.25)	(0.13)
CAPM Alpha	-0.02	-0.65***	-0.59**	-0.42***
	(0.21)	(0.22)	(0.25)	(0.13)
FF3 Alpha	-0.04	-0.68***	-0.76***	-0.49***
	(0.21)	(0.22)	(0.24)	(0.13)
FF4 Alpha	0.15	-0.83***	-0.82***	-0.50***
	(0.22)	(0.23)	(0.26)	(0.14)
FF6 Alpha	0.18	-0.81***	-0.72***	-0.45***
	(0.22)	(0.23)	(0.25)	(0.14)
FF4+IMC Alpha	0.11	-0.76***	-0.73***	-0.46***
	(0.24)	(0.25)	(0.26)	(0.14)
FF4+IO Alpha	0.04	-0.77***	-0.69***	-0.47***
	(0.24)	(0.25)	(0.26)	(0.14)
FF4+EMI Alpha	-0.02	-0.70***	-0.73***	-0.48***
	(0.24)	(0.25)	(0.27)	(0.14)
FF4+3 Alpha	-0.05	-0.72***	-0.70***	-0.49***
	(0.24)	(0.25)	(0.27)	(0.14)

Panel B. Characteristics Sorted within Industries

Panel B1. Conditional Predictive Power on SIZE

PT \ SIZE	Small	2	Big	Average
Low	3.77	2.10	1.29	2.39
2	3.38	2.32	1.24	2.32
3	3.74	2.02	1.21	2.32
4	3.50	2.17	1.19	2.29
High	3.21	2.18	1.02	2.14
High-Low				
Raw	-0.56*	0.08	-0.27*	-0.25*
	(0.33)	(0.15)	(0.14)	(0.13)
CAPM Alpha	-0.38	0.04	-0.23	-0.19
	(0.33)	(0.15)	(0.14)	(0.13)
FF3 Alpha	-0.4	0.06	-0.25*	-0.2
	(0.33)	(0.15)	(0.14)	(0.13)
FF5 Alpha	-0.49	0.12	-0.26*	-0.21
	(0.35)	(0.16)	(0.15)	(0.14)
FF6 Alpha	-0.70**	0.09	-0.22	-0.28**
	(0.34)	(0.16)	(0.15)	(0.14)
FF6+IMC Alpha	-0.42	0.07	-0.25	-0.2
	(0.28)	(0.18)	(0.17)	(0.12)
FF6+IO Alpha	-0.42	0.05	-0.34**	-0.24*
	(0.28)	(0.18)	(0.16)	(0.13)
FF6+EMI Alpha	-0.33	0.09	-0.38**	-0.21
	(0.29)	(0.18)	(0.17)	(0.13)
FF6+3 Alpha	-0.33	0.07	-0.43***	-0.23*
	(0.28)	(0.18)	(0.16)	(0.13)

Panel B2. Conditional Predictive Power on B/M

PT \ B/M	Small	2	Big	Average
Low	1.54	1.25	1.78	1.52
2	1.30	1.56	1.96	1.61
3	1.27	1.37	1.73	1.46
4	1.22	1.29	1.85	1.45
High	0.98	1.17	1.12	1.09
High-Low				
Raw	-0.56***	-0.08	-0.67*	-0.43***
	(0.16)	(0.21)	(0.35)	(0.15)
CAPM Alpha	-0.47***	-0.12	-0.80**	-0.47***
	(0.16)	(0.21)	(0.35)	(0.15)
FF3 Alpha	-0.50***	-0.14	-0.88**	-0.51***
	(0.15)	(0.21)	(0.35)	(0.15)
FF4 Alpha	-0.57***	-0.07	-0.73*	-0.46***
	(0.16)	(0.22)	(0.37)	(0.16)
FF6 Alpha	-0.58***	0.05	-0.55	-0.36**
	(0.16)	(0.22)	(0.36)	(0.15)
FF4+IMC Alpha	-0.62***	0.01	-0.78**	-0.47***
	(0.18)	(0.23)	(0.36)	(0.16)
FF4+IO Alpha	-0.69***	-0.06	-0.85**	-0.54***
	(0.18)	(0.23)	(0.37)	(0.16)
FF4+EMI Alpha	-0.72***	-0.17	-0.80**	-0.57***
	(0.18)	(0.23)	(0.37)	(0.16)
FF4+3 Alpha	-0.76***	-0.21	-0.85**	-0.61***
	(0.18)	(0.23)	(0.37)	(0.16)

Panel B3. Conditional Predictive Power on MOM11

PT \ MOM11	Small	2	Big	Average
Low	1.39	1.27	1.79	1.49
2	1.49	1.14	1.54	1.39
3	1.13	1.26	1.33	1.24
4	0.99	1.13	1.48	1.20
High	1.30	0.97	1.04	1.11
High-Low				
Raw	-0.09	-0.3	-0.75***	-0.38***
	(0.30)	(0.19)	(0.21)	(0.14)
CAPM Alpha	-0.17	-0.24	-0.64***	-0.35**
	(0.30)	(0.19)	(0.21)	(0.14)
FF3 Alpha	-0.36	-0.26	-0.69***	-0.44***
	(0.29)	(0.19)	(0.21)	(0.14)
FF4 Alpha	-0.43	-0.17	-0.84***	-0.48***
	(0.31)	(0.20)	(0.23)	(0.15)
FF6 Alpha	-0.33	-0.09	-0.86***	-0.43***
	(0.31)	(0.19)	(0.23)	(0.14)
FF4+IMC Alpha	-0.12	-0.17	-0.84***	-0.38**
	(0.34)	(0.21)	(0.25)	(0.16)
FF4+IO Alpha	-0.27	-0.2	-0.89***	-0.45***
	(0.34)	(0.21)	(0.25)	(0.16)
FF4+EMI Alpha	-0.07	-0.25	-0.98***	-0.44***
	(0.35)	(0.21)	(0.25)	(0.16)
FF4+3 Alpha	-0.2	-0.27	-1.01***	-0.49***
	(0.34)	(0.21)	(0.25)	(0.16)

Panel B4. Conditional Predictive Power on MOM6

PT \ MOM6	Small	2	Big	Average
Low	1.32	1.28	1.94	1.51
2	1.20	1.37	1.55	1.37
3	1.39	1.20	1.48	1.35
4	1.17	1.10	1.48	1.25
High	0.93	1.19	1.17	1.10
High-Low				
Raw	-0.39	-0.09	-0.77***	-0.41***
	(0.25)	(0.18)	(0.26)	(0.13)
CAPM Alpha	-0.43*	-0.05	-0.63**	-0.37***
	(0.26)	(0.18)	(0.26)	(0.13)
FF3 Alpha	-0.44*	-0.05	-0.71***	-0.40***
	(0.26)	(0.18)	(0.26)	(0.13)
FF5 Alpha	-0.50*	0.08	-0.89***	-0.43***
	(0.27)	(0.19)	(0.28)	(0.14)
FF6 Alpha	-0.51*	0.14	-0.92***	-0.43***
	(0.27)	(0.19)	(0.28)	(0.14)
FF6+IMC Alpha	-0.57*	0.12	-0.86***	-0.44***
	(0.30)	(0.21)	(0.31)	(0.16)
FF6+IO Alpha	-0.64**	0.06	-0.90***	-0.49***
	(0.30)	(0.21)	(0.31)	(0.16)
FF6+EMI Alpha	-0.63**	0	-1.04***	-0.56***
	(0.30)	(0.22)	(0.31)	(0.15)
FF6+3 Alpha	-0.68**	-0.03	-1.05***	-0.59***
	(0.30)	(0.22)	(0.31)	(0.15)

Panel B5. Conditional Predictive Power on REV

PT \ REV	Small	2	Big	Average
Low	1.29	1.58	1.65	1.51
2	1.33	1.32	1.45	1.37
3	1.57	1.11	1.45	1.38
4	1.42	1.18	1.25	1.29
High	1.08	1.06	1.04	1.06
High-Low				
Raw	-0.21	-0.52***	-0.61**	-0.45***
	(0.23)	(0.18)	(0.27)	(0.14)
CAPM Alpha	-0.19	-0.48***	-0.50*	-0.39***
	(0.24)	(0.18)	(0.27)	(0.14)
FF3 Alpha	-0.15	-0.48***	-0.57**	-0.40***
	(0.23)	(0.18)	(0.27)	(0.13)
FF4 Alpha	-0.26	-0.43**	-0.51*	-0.40***
	(0.25)	(0.19)	(0.28)	(0.14)
FF6 Alpha	-0.27	-0.35*	-0.50*	-0.37***
	(0.25)	(0.19)	(0.28)	(0.14)
FF4+IMC Alpha	-0.2	-0.37*	-0.60*	-0.39**
	(0.28)	(0.20)	(0.32)	(0.15)
FF4+IO Alpha	-0.33	-0.38*	-0.62*	-0.44***
	(0.27)	(0.21)	(0.32)	(0.15)
FF4+EMI Alpha	-0.28	-0.46**	-0.78**	-0.51***
	(0.28)	(0.21)	(0.32)	(0.15)
FF4+3 Alpha	-0.37	-0.46**	-0.77**	-0.53***
	(0.28)	(0.21)	(0.32)	(0.15)

Panel B6. Conditional Predictive Power on BPC

PT \ BPC	Small	2	Big	Average
Low	1.36	1.52	1.56	1.48
2	1.32	1.48	1.38	1.40
3	1.14	1.19	1.37	1.24
4	1.02	1.33	1.18	1.18
High	1.10	1.11	0.97	1.06
High-Low				
Raw	-0.27	-0.41**	-0.59*	-0.42***
	(0.21)	(0.20)	(0.30)	(0.13)
CAPM Alpha	-0.31	-0.40**	-0.34	-0.35***
	(0.21)	(0.20)	(0.29)	(0.13)
FF3 Alpha	-0.19	-0.35*	-0.50*	-0.34***
	(0.21)	(0.20)	(0.26)	(0.12)
FF4 Alpha	0.01	-0.13	-0.96***	-0.36***
	(0.22)	(0.21)	(0.26)	(0.13)
FF6 Alpha	0.06	-0.16	-0.93***	-0.34***
	(0.22)	(0.21)	(0.26)	(0.13)
FF4+IMC Alpha	0.11	-0.11	-1.12***	-0.37**
	(0.25)	(0.23)	(0.29)	(0.14)
FF4+IO Alpha	0.08	-0.12	-1.15***	-0.40***
	(0.25)	(0.24)	(0.29)	(0.14)
FF4+EMI Alpha	-0.04	-0.17	-1.20***	-0.47***
	(0.25)	(0.24)	(0.29)	(0.14)
FF4+3 Alpha	-0.05	-0.17	-1.21***	-0.48***
	(0.25)	(0.24)	(0.29)	(0.14)

Panel B7. Conditional Predictive Power on BPP

PT \ BPP	Small	2	Big	Average
Low	1.35	1.55	1.56	1.49
2	1.30	1.42	1.44	1.39
3	1.18	1.26	1.31	1.25
4	1.09	1.33	1.24	1.22
High	1.41	1.05	1.02	1.16
High-Low				
Raw	0.05	-0.50**	-0.55*	-0.33***
	(0.25)	(0.20)	(0.29)	(0.12)
CAPM Alpha	0	-0.47**	-0.33	-0.27**
	(0.25)	(0.21)	(0.28)	(0.12)
FF3 Alpha	0.19	-0.42**	-0.49**	-0.24**
	(0.24)	(0.21)	(0.25)	(0.12)
FF5 Alpha	0.54**	-0.25	-0.92***	-0.21
	(0.24)	(0.22)	(0.25)	(0.13)
FF6 Alpha	0.49**	-0.27	-0.89***	-0.22*
	(0.24)	(0.22)	(0.25)	(0.13)
FF6+IMC Alpha	0.56**	-0.22	-1.09***	-0.25*
	(0.27)	(0.25)	(0.28)	(0.14)
FF6+IO Alpha	0.47*	-0.24	-1.09***	-0.29**
	(0.27)	(0.25)	(0.28)	(0.14)
FF6+EMI Alpha	0.35	-0.28	-1.12***	-0.35**
	(0.26)	(0.25)	(0.28)	(0.14)
FF6+3 Alpha	0.3	-0.3	-1.12***	-0.37**
	(0.26)	(0.25)	(0.28)	(0.14)

Panel B8. Conditional Predictive Power on PMC

PT \ PMC	Small	2	Big	Average
Low	----	----	----	----
2	----	----	----	----
3	----	----	----	----
4	----	----	----	----
High	----	----	----	----
High-Low				
Raw	----	----	----	----
	----	----	----	----
CAPM Alpha	----	----	----	----
	----	----	----	----
FF3 Alpha	----	----	----	----
	----	----	----	----
FF4 Alpha	----	----	----	----
	----	----	----	----
FF6 Alpha	----	----	----	----
	----	----	----	----
FF4+IMC Alpha	----	----	----	----
	----	----	----	----
FF4+IO Alpha	----	----	----	----
	----	----	----	----
FF4+EMI Alpha	----	----	----	----
	----	----	----	----
FF4+3 Alpha	----	----	----	----
	----	----	----	----

Panel B9. Conditional Predictive Power on PQ

PT \ PQ	Small	2	Big	Average
Low	1.61	1.60	1.43	1.55
2	1.10	1.52	1.55	1.39
3	1.18	1.24	1.27	1.23
4	1.56	1.25	1.20	1.33
High	1.57	1.37	1.01	1.32
High-Low				
Raw	-0.04	-0.22	-0.43*	-0.23*
	(0.23)	(0.22)	(0.23)	(0.12)
CAPM Alpha	-0.2	-0.23	-0.24	-0.22*
	(0.23)	(0.22)	(0.22)	(0.12)
FF3 Alpha	-0.02	-0.22	-0.36*	-0.2
	(0.22)	(0.22)	(0.21)	(0.12)
FF4 Alpha	0.43**	-0.28	-0.50**	-0.11
	(0.21)	(0.24)	(0.23)	(0.13)
FF6 Alpha	0.48**	-0.27	-0.44*	-0.08
	(0.21)	(0.24)	(0.23)	(0.13)
FF4+IMC Alpha	0.74***	-0.39	-0.51**	-0.05
	(0.23)	(0.25)	(0.24)	(0.14)
FF4+IO Alpha	0.69***	-0.49**	-0.51**	-0.1
	(0.24)	(0.25)	(0.24)	(0.14)
FF4+EMI Alpha	0.66***	-0.4	-0.64***	-0.13
	(0.24)	(0.25)	(0.24)	(0.14)
FF4+3 Alpha	0.63***	-0.48*	-0.63***	-0.16
	(0.24)	(0.25)	(0.24)	(0.14)

Table F.III

Volatility of Future Fundamentals and Patent Thicket with Industry Adjustments

Panel A works as a robustness check of Panel A of Table VI, and Panel B acts as a robustness check of Panel B of Table VI. In Panel A1 and B1 we report and industry-adjusted volatility of ROA, IA, and SA (i.e., we first calculate the volatility for each firm and then subtract it from the weighted (by firm size) average of the FF48 industry). In Panel A2 and B2 we sort patent thicket within industries. The definition of ROA, IA, and SA are provided in Table III.

Panel A1. Industry-adjusted Excess Returns

PT	ROA Volatility	PT	IA Volatility	PT	SA Volatility
Low	0.0079	Low	0.0017	Low	0.0003
2	-0.0019	2	-0.0012	2	-0.0113
3	0.0011	3	-0.0015	3	0.0013
4	-0.0038	4	-0.0033	4	-0.0163
High	-0.0078	High	-0.0018	High	-0.0125
High-Low	-0.0157*** (0.0033)	High-Low	-0.0035*** (0.0010)	High-Low	-0.0128 (0.0110)

Panel A2. Patent Thicket Sorted within Industries

PT	ROA Volatility	PT	IA Volatility	PT	SA Volatility
Low	0.0427	Low	0.0235	Low	0.1550
2	0.0450	2	0.0212	2	0.1748
3	0.0429	3	0.0198	3	0.1604
4	0.0377	4	0.0184	4	0.1530
High	0.0295	High	0.0168	High	0.1330
High-Low	-0.0132*** (0.0030)	High-Low	-0.0067*** (0.0011)	High-Low	-0.0220** (0.0092)

Table F.III (continued)

Panel B1. Industry-adjusted Excess Returns

Panel B1(a). PT Predicts ROA Volatility

Panel B1(a1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0047	0.0192	0.0144	0.0106	0.1002	0.0298
2	-0.0017	-0.0045	0.0130	0.0029	0.0465	0.0113
3	-0.0008	0.0026	-0.0006	0.0082	0.0258	0.0071
4	-0.0046	0.0025	-0.0049	-0.0062	0.0121	-0.0002
High	0.0006	-0.0064	-0.0150	-0.0054	-0.0085	-0.0069
High-Low	-0.0041	-0.0255**	-0.0294***	-0.0160***	-0.1087***	-0.0367***
	(0.0032)	(0.0120)	(0.0076)	(0.0040)	(0.0175)	(0.0050)

Panel B1(a2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0029	0.0151	0.0086	0.0285	0.0504	0.0211
2	0.0001	-0.0092	0.0240	0.0144	0.0581	0.0175
3	-0.0007	0.0005	0.0015	0.0123	0.0399	0.0107
4	-0.0038	-0.0038	-0.0056	-0.0030	0.0047	-0.0023
High	-0.0019	-0.0082	-0.0132	-0.0117	0.0004	-0.0069
High-Low	-0.0048*	-0.0233**	-0.0218***	-0.0402***	-0.0500***	-0.0280***
	(0.0028)	(0.0109)	(0.0032)	(0.0068)	(0.0076)	(0.0029)

Panel B1(b). PT Predicts IA Volatility

Panel B1(b1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0002	0.0038	0.0090	0.0066	0.0269	0.0093
2	0.0001	0.0003	-0.0011	0.0027	0.0062	0.0017
3	-0.0021	-0.0004	-0.0013	0.0018	0.0031	0.0002
4	-0.0055	-0.0041	-0.0029	-0.0020	0.0038	-0.0021
High	-0.0030	0.0001	-0.0031	-0.0012	-0.0042	-0.0023
High-Low	-0.0032*	-0.0038*	-0.0121***	-0.0078***	-0.0310***	-0.0116***
	(0.0016)	(0.0021)	(0.0022)	(0.0020)	(0.0071)	(0.0015)

Panel B1(b2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0013	0.0022	0.0024	0.0158	0.0114	0.0066
2	-0.0007	-0.0024	0.0031	0.0036	0.0139	0.0035
3	-0.0018	-0.0017	-0.0004	0.0025	0.0097	0.0017
4	-0.0061	-0.0034	-0.0026	-0.0009	0.0057	-0.0015
High	-0.0027	-0.0008	-0.0023	-0.0047	-0.0008	-0.0023
High-Low	-0.0041**	-0.0030	-0.0047***	-0.0205***	-0.0122***	-0.0089***
	(0.0017)	(0.0018)	(0.0014)	(0.0035)	(0.0027)	(0.0010)

Panel B1(c). PT Predicts SA Volatility

Panel B1(c1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	-0.0058	-0.0027	0.0394	-0.0160	0.0804	0.0191
2	-0.0059	-0.0138	0.0230	-0.0047	0.0618	0.0120
3	0.0053	0.0113	-0.0295	0.0226	0.0121	0.0044
4	-0.0175	-0.0137	-0.0154	-0.0207	-0.0023	-0.0139
High	-0.0065	-0.0091	-0.0075	-0.0097	-0.0472	-0.0160
High-Low	-0.0007	-0.0064	-0.0469**	0.0063	-0.1277***	-0.0351***
	(0.0140)	(0.0163)	(0.0168)	(0.0146)	(0.0317)	(0.0083)

Panel B1(c2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	-0.0008	-0.0106	-0.0040	0.0548	0.0076	0.0094
2	-0.0014	-0.0215	0.0379	-0.0041	0.0724	0.0167
3	0.0139	0.0039	-0.0144	0.0281	0.0173	0.0098
4	-0.0194	-0.0178	-0.0159	-0.0145	0.0013	-0.0133
High	-0.0130	-0.0077	0.0005	-0.0543	-0.0026	-0.0154
High-Low	-0.0122	0.0029	0.0045	-0.1090***	-0.0101	-0.0248***
	(0.0136)	(0.0161)	(0.0158)	(0.0148)	(0.0146)	(0.0065)

Panel B2. Characteristics Sorted within Industries

Panel B2(a). PT Predicts ROA Volatility

Panel B2(a1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0439	0.0529	0.0369	0.0398	0.0953	0.0538
2	0.0451	0.0470	0.0446	0.0470	0.0603	0.0488
3	0.0433	0.0406	0.0384	0.0546	0.0525	0.0459
4	0.0413	0.0356	0.0334	0.0361	0.0553	0.0403
High	0.0446	0.0248	0.0326	0.0379	0.0286	0.0337
H-L	0.0006	-0.0282**	-0.0043	-0.0019	-0.0667***	-0.0201***
	(0.0053)	(0.0119)	(0.0040)	(0.0056)	(0.0113)	(0.0032)

Panel B2(a2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0368	0.0616	0.0447	0.0386	0.1114	0.0586
2	0.0451	0.0431	0.0481	0.0470	0.0699	0.0506
3	0.0404	0.0411	0.0408	0.0522	0.0562	0.0461
4	0.0354	0.0375	0.0338	0.0411	0.0504	0.0397
High	0.0282	0.0280	0.0334	0.0341	0.0349	0.0317
H-L	-0.0086**	-0.0336**	-0.0113**	-0.0046	-0.0766***	-0.0269***
	(0.0038)	(0.0145)	(0.0050)	(0.0029)	(0.0093)	(0.0038)

Panel B2(b). PT Predicts IA Volatility

Panel B2(b1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.0260	0.0263	0.0233	0.0168	0.0311	0.0247
2	0.0228	0.0209	0.0224	0.0214	0.0227	0.0220
3	0.0224	0.0175	0.0161	0.0231	0.0259	0.0210
4	0.0197	0.0180	0.0169	0.0187	0.0223	0.0191
High	0.0178	0.0148	0.0154	0.0195	0.0160	0.0167
H-L	-0.0083***	-0.0115***	-0.0079***	0.0027	-0.0152***	-0.0080***
	(0.0021)	(0.0026)	(0.0024)	(0.0026)	(0.0038)	(0.0007)

Panel B2(b2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.0247	0.0257	0.0249	0.0190	0.0378	0.0264
2	0.0230	0.0198	0.0218	0.0212	0.0277	0.0227
3	0.0204	0.0176	0.0171	0.0223	0.0276	0.0210
4	0.0169	0.0183	0.0181	0.0207	0.0213	0.0191
High	0.0143	0.0165	0.0179	0.0186	0.0188	0.0172
H-L	-0.0104***	-0.0091***	-0.0070*	-0.0004	-0.0190***	-0.0092***
	(0.0015)	(0.0025)	(0.0039)	(0.0028)	(0.0044)	(0.0012)

Panel B2(c). PT Predicts SA Volatility

Panel B2(c1). Conditional Predictive Power on CTBE

PT \ CTBE	Small	2	3	4	Big	Average
Low	0.1488	0.2119	0.1565	0.1477	0.2058	0.1742
2	0.1804	0.1737	0.1563	0.1847	0.1893	0.1769
3	0.1755	0.1441	0.1339	0.2084	0.1594	0.1642
4	0.1693	0.1400	0.1311	0.1567	0.2025	0.1599
High	0.1757	0.1309	0.1450	0.1518	0.1195	0.1446
H-L	0.0269*	-0.0810***	-0.0116	0.0040	-0.0864***	-0.0296***
	(0.0142)	(0.0277)	(0.0158)	(0.0181)	(0.0222)	(0.0085)

Panel B2(c2). Conditional Predictive Power on CTA

PT \ CTA	Small	2	3	4	Big	Average
Low	0.1496	0.1871	0.1616	0.1490	0.2260	0.1747
2	0.1943	0.1537	0.1636	0.1788	0.2121	0.1805
3	0.1649	0.1583	0.1396	0.1862	0.1702	0.1639
4	0.1573	0.1418	0.1463	0.1580	0.1734	0.1554
High	0.1460	0.1242	0.1585	0.1398	0.1563	0.1450
H-L	-0.0036	-0.0629***	-0.0032	-0.0091	-0.0698***	-0.0297***
	(0.0149)	(0.0211)	(0.0186)	(0.0173)	(0.0165)	(0.0095)

Table F.IV

Two-way Portfolio Sorting on Patent Thicket and Patent Portfolio Size with Industry Adjustments

This table works as a robustness check of Table VII. In Panel A we report the monthly industry-adjusted returns (Industry-adjusted Excess Returns), which are based on the difference between individual firms' excess returns and the weighted average excess returns of firms in the same industry (accordingly to Fama-French 48 industry classifications), and their corresponding alphas and betas in different model specifications. In Panel B we sort patent thicket within industries, then calculate the value-weighted excess return of each quintile portfolio, and report their corresponding alphas and betas in different model specifications.

Panel A. Industry-adjusted Excess Returns													
Panel A1. Conditional Predictive Power on CTBE							Panel A2. Conditional Predictive Power on CTA						
PT \ CTBE	Small	2	3	4	Big	B-S	PT \ CTA	Small	2	3	4	Big	B-S
Low	-0.10	0.44	0.67	0.43	1.78	1.87	Low	-0.04	0.31	0.28	1.05	1.45	1.49
2	-0.06	-0.06	0.33	-0.05	0.73	0.79	2	-0.05	-0.04	0.09	0.24	0.89	0.94
3	-0.18	-0.09	-0.10	0.23	0.36	0.54	3	-0.12	-0.14	0.10	0.09	0.82	0.94
4	-0.13	0.01	-0.04	-0.16	0.23	0.36	4	-0.11	0.01	-0.19	-0.06	0.22	0.33
High	-0.19	-0.39	-0.27	0.00	-0.07	0.12	High	-0.21	-0.27	-0.26	-0.18	0.17	0.38
			High-Low							High-Low			
Raw	-0.09	-0.84***	-0.94***	-0.43*	-1.85***	-1.75***	\	-0.17	-0.58*	-0.54***	-1.22***	-1.28***	-1.11**
	(0.15)	(0.26)	(0.28)	(0.23)	(0.58)	(0.58)		(0.13)	(0.30)	(0.21)	(0.39)	(0.43)	(0.44)
CAPM Alpha	-0.09	-0.73***	-0.83***	-0.44*	-1.46**	-1.37**	CAPM Alpha	-0.16	-0.49	-0.50**	-0.99**	-1.02**	-0.86**
	(0.15)	(0.26)	(0.28)	(0.23)	(0.57)	(0.57)		(0.13)	(0.30)	(0.21)	(0.39)	(0.42)	(0.43)
FF3 Alpha	-0.13	-0.86***	-0.95***	-0.43*	-1.51***	-1.38***	FF3 Alpha	-0.19	-0.67**	-0.51**	-0.96***	-1.21***	-1.02**
	(0.15)	(0.25)	(0.23)	(0.24)	(0.50)	(0.51)		(0.14)	(0.29)	(0.21)	(0.35)	(0.39)	(0.41)
FF5 Alpha	-0.03	-0.88***	-1.10***	-0.31	-2.20***	-2.17***	FF5 Alpha	-0.04	-0.62**	-0.57**	-1.10***	-1.83***	-1.79***
	(0.16)	(0.26)	(0.24)	(0.25)	(0.52)	(0.52)		(0.14)	(0.31)	(0.22)	(0.37)	(0.40)	(0.42)
FF6 Alpha	0.00	-0.79***	-1.04***	-0.31	-2.29***	-2.29***	FF6 Alpha	-0.03	-0.50*	-0.53**	-1.27***	-1.77***	-1.74***
	(0.16)	(0.26)	(0.24)	(0.25)	(0.52)	(0.52)		(0.14)	(0.30)	(0.22)	(0.37)	(0.40)	(0.42)
FF6+IMC Alpha	0.02	-0.91***	-1.02***	-0.19	-2.06***	-2.08***	FF6+IMC Alpha	-0.05	-0.54	-0.59**	-0.95***	-1.92***	-1.87***
	(0.17)	(0.28)	(0.27)	(0.27)	(0.43)	(0.45)		(0.16)	(0.34)	(0.23)	(0.28)	(0.44)	(0.46)
FF6+IO Alpha	0.00	-0.97***	-1.02***	-0.11	-1.98***	-1.98***	FF6+IO Alpha	-0.07	-0.59*	-0.55**	-0.93***	-1.96***	-1.89***
	(0.18)	(0.29)	(0.27)	(0.27)	(0.43)	(0.45)		(0.16)	(0.34)	(0.24)	(0.29)	(0.45)	(0.47)
FF6+EMI Alpha	-0.04	-0.93***	-0.92***	-0.19	-1.88***	-1.84***	FF6+EMI Alpha	-0.12	-0.52	-0.63***	-0.85***	-1.76***	-1.65***
	(0.18)	(0.29)	(0.27)	(0.28)	(0.43)	(0.45)		(0.16)	(0.35)	(0.24)	(0.29)	(0.45)	(0.47)
FF6+3 Alpha	-0.05	-0.98***	-0.93***	-0.13	-1.85***	-1.80***	FF6+3 Alpha	-0.12	-0.57	-0.60**	-0.85***	-1.81***	-1.69***
	(0.18)	(0.29)	(0.27)	(0.28)	(0.43)	(0.45)		(0.16)	(0.35)	(0.24)	(0.29)	(0.45)	(0.47)

Panel B. Characteristics Sorted within Industries													
Panel B1. Conditional Predictive Power on CTBE							Panel B2. Conditional Predictive Power on CTA						
PT \ CTBE	Small	2	3	4	Big	B-S	PT \ CTA	Small	2	3	4	Big	B-S
Low	1.30	1.91	1.76	1.59	2.80	1.50	Low	1.30	1.65	1.60	1.61	2.38	1.08
2	1.24	1.60	1.51	1.53	1.72	0.48	2	1.29	1.54	1.78	1.11	1.57	0.28
3	1.09	1.20	1.29	1.44	1.69	0.60	3	0.97	1.37	1.22	1.41	1.64	0.67
4	1.21	1.17	1.25	1.18	1.19	-0.02	4	1.25	1.31	1.16	1.23	1.29	0.04
High	1.13	0.85	1.15	1.21	1.24	0.11	High	0.97	1.01	1.01	1.29	1.08	0.10
			High-Low							High-Low			
Raw	-0.18	-1.06***	-0.61**	-0.38	-1.56***	-1.39***	Raw	-0.32	-0.63**	-0.59**	-0.32	-1.30***	-0.98**
	(0.20)	(0.31)	(0.28)	(0.26)	(0.46)	(0.51)		(0.20)	(0.27)	(0.23)	(0.25)	(0.39)	(0.41)
CAPM Alpha	-0.10	-0.93***	-0.63**	-0.29	-1.34***	-1.24**	CAPM Alpha	-0.27	-0.44*	-0.60***	-0.25	-1.13***	-0.85**
	(0.20)	(0.30)	(0.29)	(0.26)	(0.46)	(0.51)		(0.20)	(0.26)	(0.23)	(0.25)	(0.39)	(0.42)
FF3 Alpha	-0.07	-1.06***	-0.64**	-0.22	-1.57***	-1.50***	FF3 Alpha	-0.32	-0.49**	-0.58**	-0.23	-1.36***	-1.04***
	(0.21)	(0.28)	(0.28)	(0.27)	(0.38)	(0.45)		(0.20)	(0.24)	(0.24)	(0.24)	(0.34)	(0.40)
FF5 Alpha	-0.08	-1.31***	-0.47	-0.16	-1.79***	-1.72***	FF5 Alpha	-0.32	-0.67***	-0.34	-0.25	-1.81***	-1.50***
	(0.22)	(0.29)	(0.29)	(0.28)	(0.39)	(0.47)		(0.21)	(0.25)	(0.25)	(0.25)	(0.36)	(0.42)
FF6 Alpha	-0.08	-1.25***	-0.37	-0.24	-1.59***	-1.51***	FF6 Alpha	-0.33	-0.63**	-0.30	-0.17	-1.71***	-1.38***
	(0.22)	(0.29)	(0.29)	(0.28)	(0.38)	(0.46)		(0.21)	(0.25)	(0.25)	(0.25)	(0.36)	(0.42)
FF6+IMC Alpha	-0.16	-1.29***	-0.14	-0.09	-1.69***	-1.53***	FF6+IMC Alpha	-0.43*	-0.63**	-0.04	-0.39	-1.61***	-1.17***
	(0.24)	(0.32)	(0.32)	(0.27)	(0.43)	(0.51)		(0.23)	(0.28)	(0.27)	(0.27)	(0.39)	(0.45)
FF6+IO Alpha	-0.28	-1.24***	-0.11	-0.11	-1.62***	-1.34***	FF6+IO Alpha	-0.52**	-0.64**	0.01	-0.39	-1.72***	-1.20***
	(0.24)	(0.33)	(0.33)	(0.27)	(0.43)	(0.51)		(0.23)	(0.28)	(0.27)	(0.27)	(0.39)	(0.45)
FF6+EMI Alpha	-0.23	-1.35***	-0.15	-0.17	-1.77***	-1.54***	FF6+EMI Alpha	-0.52**	-0.68**	-0.16	-0.42	-1.64***	-1.12**
	(0.24)	(0.33)	(0.33)	(0.27)	(0.43)	(0.52)		(0.24)	(0.28)	(0.27)	(0.27)	(0.40)	(0.46)
FF6+3 Alpha	-0.32	-1.31***	-0.13	-0.18	-1.71***	-1.39***	FF6+3 Alpha	-0.57**	-0.69**	-0.11	-0.43	-1.73***	-1.15**
	(0.24)	(0.33)	(0.33)	(0.27)	(0.43)	(0.52)		(0.23)	(0.28)	(0.27)	(0.28)	(0.40)	(0.46)

Table F.V

Two-way Portfolio Sorting on Patent Thicket and R&D Investment with Industry Adjustments

This table works as a robustness check of Table VIII. In Panel A we report the monthly industry-adjusted returns (Industry-adjusted Excess Returns), which are based on the difference between individual firms' excess returns and the weighted average excess returns of firms in the same industry (accordingly to Fama-French 48 industry classifications), and their corresponding alphas and betas in different model specifications. In Panel B we sort patent thicket within industries, then calculate the value-weighted excess return of each quintile portfolio, and report their corresponding alphas and betas in different model specifications.

Panel A. Industry-adjusted Excess Returns													
Panel A1. Conditional Predictive Power on RDBE							**Panel A2. Conditional Predictive Power on RDA**						
PT \ CTBE	Small	2	3	4	Big	B-S	PT \ CTA	Small	2	3	4	Big	B-S
Low	-0.22	0.20	0.51	-0.04	1.41	1.63	Low	-0.23	0.08	0.04	0.46	1.52	1.75
2	-0.16	-0.43	0.05	0.15	0.80	0.95	2	-0.14	-0.28	-0.21	0.36	1.01	1.15
3	-0.14	0.07	0.05	-0.08	0.45	0.59	3	-0.17	0.07	-0.25	0.02	0.75	0.93
4	-0.19	0.07	-0.01	-0.16	0.32	0.51	4	-0.11	0.03	0.02	-0.11	0.42	0.53
High	-0.19	-0.29	-0.34	-0.19	0.07	0.26	High	-0.22	-0.16	-0.44	-0.01	0.75	0.96
			High-Low							High-Low			
Raw	0.03	-0.49**	-0.85***	-0.16	-1.34***	-1.36***	Raw	0.01	-0.24	-0.47*	-0.47	-0.78*	-0.79*
	(0.14)	(0.21)	(0.24)	(0.30)	(0.37)	(0.40)		(0.13)	(0.18)	(0.27)	(0.33)	(0.45)	(0.47)
CAPM Alpha	-0.02	-0.40*	-0.78***	-0.09	-1.05***	-1.04***	CAPM Alpha	-0.03	-0.22	-0.4	-0.36	-0.73	-0.7
	(0.14)	(0.21)	(0.24)	(0.30)	(0.36)	(0.39)		(0.14)	(0.19)	(0.28)	(0.33)	(0.46)	(0.47)
FF3 Alpha	0.04	-0.41**	-0.89***	-0.23	-1.11***	-1.14***	FF3 Alpha	0.02	-0.26	-0.39	-0.56*	-0.55	-0.58
	(0.14)	(0.20)	(0.23)	(0.29)	(0.34)	(0.36)		(0.14)	(0.19)	(0.28)	(0.32)	(0.46)	(0.47)
FF5 Alpha	0.25*	-0.40*	-1.01***	-0.26	-1.03***	-1.28***	FF5 Alpha	0.23*	-0.21	-0.44	-0.51	-0.4	-0.63
	(0.15)	(0.22)	(0.24)	(0.31)	(0.36)	(0.39)		(0.14)	(0.20)	(0.30)	(0.34)	(0.48)	(0.50)
FF6 Alpha	0.28*	-0.44**	-0.95***	-0.2	-1.16***	-1.44***	FF6 Alpha	0.24*	-0.2	-0.45	-0.52	-0.58	-0.82*
	(0.15)	(0.22)	(0.24)	(0.31)	(0.36)	(0.38)		(0.14)	(0.20)	(0.30)	(0.34)	(0.48)	(0.50)
FF6+IMC Alpha	0.32**	-0.51**	-1.02***	-0.21	-0.95**	-1.27***	FF6+IMC Alpha	0.25	-0.18	-0.48	-0.73*	-0.09	-0.35
	(0.16)	(0.24)	(0.27)	(0.35)	(0.39)	(0.42)		(0.15)	(0.23)	(0.34)	(0.39)	(0.47)	(0.50)
FF6+IO Alpha	0.31*	-0.43*	-1.03***	-0.26	-0.97**	-1.28***	FF6+IO Alpha	0.23	-0.09	-0.51	-0.75*	-0.06	-0.3
	(0.16)	(0.24)	(0.27)	(0.36)	(0.39)	(0.42)		(0.16)	(0.23)	(0.34)	(0.39)	(0.48)	(0.50)
FF6+EMI Alpha	0.25	-0.41*	-1.03***	-0.35	-1.06***	-1.30***	FF6+EMI Alpha	0.2	-0.1	-0.54	-0.89**	-0.16	-0.36
	(0.16)	(0.24)	(0.27)	(0.36)	(0.39)	(0.42)		(0.16)	(0.23)	(0.34)	(0.39)	(0.48)	(0.51)
FF6+3 Alpha	0.24	-0.36	-1.04***	-0.37	-1.06***	-1.31***	FF6+3 Alpha	0.19	-0.04	-0.56	-0.90**	-0.14	-0.32
	(0.17)	(0.24)	(0.27)	(0.36)	(0.39)	(0.43)		(0.16)	(0.23)	(0.34)	(0.39)	(0.48)	(0.51)

Panel B. Characteristics Sorted within Industries

Panel B1. Conditional Predictive Power on RDBE							Panel B2. Conditional Predictive Power on RDA						
PT \ CTBE	Small	2	3	4	Big	B-S	PT \ CTA	Small	2	3	4	Big	B-S
Low	1.12	1.58	1.58	1.69	2.44	1.32	Low	1.09	1.57	1.71	1.61	2.24	1.16
2	1.10	1.33	1.65	1.23	1.92	0.82	2	1.07	1.34	1.82	1.23	2.03	0.96
3	0.95	1.31	1.13	1.35	1.73	0.78	3	0.86	1.08	1.56	1.28	1.60	0.73
4	1.33	1.32	0.97	1.34	1.22	-0.11	4	1.40	1.07	1.10	1.24	1.43	0.03
High	0.89	1.01	1.09	1.44	1.04	0.16	High	0.91	1.12	1.25	1.25	1.08	0.17
			High-Low							High-Low			
Raw	-0.24	-0.56*	-0.49**	-0.25	-1.40***	-1.16***	Raw	-0.18	-0.45	-0.46*	-0.36	-1.16***	-0.98**
	(0.22)	(0.31)	(0.25)	(0.32)	(0.37)	(0.44)		(0.21)	(0.29)	(0.27)	(0.28)	(0.39)	(0.44)
CAPM Alpha	-0.21	-0.39	-0.4	-0.16	-1.23***	-1.01**	CAPM Alpha	-0.17	-0.24	-0.50*	-0.27	-1.06***	-0.88**
	(0.23)	(0.30)	(0.25)	(0.32)	(0.37)	(0.44)		(0.22)	(0.29)	(0.28)	(0.29)	(0.39)	(0.44)
FF3 Alpha	-0.11	-0.51*	-0.37	-0.29	-1.53***	-1.42***	FF3 Alpha	-0.07	-0.29	-0.56**	-0.47*	-1.31***	-1.24***
	(0.23)	(0.28)	(0.25)	(0.32)	(0.33)	(0.41)		(0.21)	(0.28)	(0.28)	(0.28)	(0.37)	(0.42)
FF5 Alpha	-0.16	-0.66**	-0.49*	-0.18	-1.53***	-1.37***	FF5 Alpha	-0.05	-0.50*	-0.38	-0.56*	-1.45***	-1.40***
	(0.24)	(0.30)	(0.27)	(0.34)	(0.36)	(0.44)		(0.23)	(0.29)	(0.29)	(0.29)	(0.39)	(0.45)
FF6 Alpha	-0.18	-0.63**	-0.48*	-0.02	-1.55***	-1.37***	FF6 Alpha	-0.08	-0.44	-0.29	-0.58**	-1.40***	-1.31***
	(0.24)	(0.30)	(0.27)	(0.33)	(0.36)	(0.44)		(0.23)	(0.29)	(0.29)	(0.30)	(0.39)	(0.45)
FF6+IMC Alpha	-0.2	-0.63*	-0.43	-0.14	-1.72***	-1.52***	FF6+IMC Alpha	-0.1	-0.47	-0.25	-0.64**	-1.45***	-1.35***
	(0.27)	(0.33)	(0.30)	(0.36)	(0.39)	(0.48)		(0.25)	(0.34)	(0.33)	(0.31)	(0.41)	(0.47)
FF6+IO Alpha	-0.32	-0.66*	-0.37	-0.21	-1.71***	-1.40***	FF6+IO Alpha	-0.21	-0.5	-0.23	-0.77**	-1.47***	-1.27***
	(0.27)	(0.34)	(0.30)	(0.36)	(0.40)	(0.49)		(0.25)	(0.34)	(0.33)	(0.31)	(0.42)	(0.48)
FF6+EMI Alpha	-0.26	-0.69**	-0.52*	-0.32	-1.69***	-1.43***	FF6+EMI Alpha	-0.17	-0.63*	-0.37	-0.75**	-1.39***	-1.22**
	(0.27)	(0.34)	(0.30)	(0.36)	(0.40)	(0.50)		(0.25)	(0.34)	(0.33)	(0.32)	(0.42)	(0.48)
FF6+3 Alpha	-0.34	-0.71**	-0.47	-0.35	-1.69***	-1.36***	FF6+3 Alpha	-0.24	-0.63*	-0.34	-0.84***	-1.41***	-1.17**
	(0.27)	(0.34)	(0.30)	(0.36)	(0.40)	(0.49)		(0.25)	(0.34)	(0.33)	(0.31)	(0.42)	(0.48)

Table F.VI

Two-way Portfolio Sorting on Patent Thicket and Innovation Efficiency with Industry Adjustments

This table works as a robustness check of Table IX. In Panel A we report the monthly industry-adjusted returns (Industry-adjusted Excess Returns), which are based on the difference between individual firms' excess returns and the weighted average excess returns of firms in the same industry (accordingly to Fama-French 48 industry classifications), and their corresponding alphas and betas in different model specifications. In Panel B we sort patent thicket within industries, then calculate the value-weighted excess return of each quintile portfolio, and report their corresponding alphas and betas in different model specifications.

Panel A. Industry-adjusted Excess Returns

PT \ IE	Small	2	3	4	Big	B-S
Low	0.33	0.41	0.52	-0.04	-0.21	-0.55
2	0.13	-0.16	0.00	-0.09	0.15	0.02
3	-0.19	-0.13	-0.08	0.24	0.32	0.51
4	-0.21	0.05	-0.05	0.07	-0.06	0.15
High	-0.30	-0.14	-0.20	-0.15	-0.03	0.28
			High-Low			
Raw	-0.63***	-0.55**	-0.72***	-0.11	0.19	0.82***
	(0.24)	(0.24)	(0.26)	(0.21)	(0.16)	(0.29)
CAPM Alpha	-0.53**	-0.49**	-0.72***	-0.14	0.22	0.75**
	(0.24)	(0.25)	(0.26)	(0.21)	(0.16)	(0.29)
FF3 Alpha	-0.66***	-0.62***	-0.81***	-0.08	0.28*	0.94***
	(0.24)	(0.24)	(0.25)	(0.22)	(0.17)	(0.28)
FF5 Alpha	-0.53**	-0.78***	-0.78***	0.03	0.35*	0.88***
	(0.25)	(0.25)	(0.26)	(0.23)	(0.18)	(0.30)
FF6 Alpha	-0.46*	-0.76***	-0.75***	0.08	0.34*	0.80***
	(0.25)	(0.25)	(0.26)	(0.23)	(0.18)	(0.30)
FF6+IMC Alpha	-0.45	-0.87***	-0.71**	0.06	0.3	0.76**
	(0.28)	(0.27)	(0.30)	(0.24)	(0.18)	(0.33)
FF6+IO Alpha	-0.52*	-0.85***	-0.61**	0.02	0.3	0.81**
	(0.29)	(0.28)	(0.30)	(0.25)	(0.19)	(0.33)
FF6+EMI Alpha	-0.56*	-0.98***	-0.66**	0.02	0.28	0.84**
	(0.29)	(0.28)	(0.30)	(0.25)	(0.19)	(0.34)
FF6+3 Alpha	-0.59**	-0.96***	-0.59*	0	0.28	0.87**
	(0.29)	(0.28)	(0.30)	(0.25)	(0.19)	(0.34)

Panel B. Characteristics Sorted within Industries

PT \ IE	Small	2	3	4	Big	B-S
Low	1.38	1.48	2.02	1.16	1.50	0.12
2	1.29	1.53	1.29	1.46	1.66	0.37
3	1.00	1.64	1.02	1.23	1.43	0.43
4	1.10	1.17	1.14	1.74	1.68	0.58
High	0.97	1.05	0.91	1.36	1.42	0.45
			High-Low			
Raw	-0.42*	-0.43	-1.11***	0.2	-0.09	0.33
	(0.22)	(0.27)	(0.30)	(0.30)	(0.33)	(0.38)
CAPM Alpha	-0.32	-0.3	-1.15***	0.27	0.04	0.35
	(0.22)	(0.26)	(0.31)	(0.30)	(0.33)	(0.38)
FF3 Alpha	-0.39*	-0.29	-1.15***	0.41	0.04	0.43
	(0.22)	(0.25)	(0.30)	(0.30)	(0.33)	(0.39)
FF5 Alpha	-0.3	-0.36	-1.18***	0.45	-0.2	0.1
	(0.23)	(0.26)	(0.32)	(0.31)	(0.34)	(0.40)
FF6 Alpha	-0.26	-0.4	-1.16***	0.39	-0.17	0.08
	(0.24)	(0.26)	(0.32)	(0.32)	(0.34)	(0.40)
FF6+IMC Alpha	-0.36	-0.51*	-0.86***	0.22	-0.14	0.22
	(0.26)	(0.29)	(0.33)	(0.35)	(0.38)	(0.45)
FF6+IO Alpha	-0.46*	-0.53*	-0.79**	0.08	-0.28	0.18
	(0.26)	(0.30)	(0.33)	(0.35)	(0.38)	(0.46)
FF6+EMI Alpha	-0.46*	-0.50*	-0.95***	-0.01	-0.11	0.35
	(0.26)	(0.30)	(0.33)	(0.35)	(0.39)	(0.46)
FF6+3 Alpha	-0.53**	-0.52*	-0.88***	-0.09	-0.21	0.32
	(0.26)	(0.30)	(0.33)	(0.35)	(0.38)	(0.46)