Information Quality and Asset Pricing: The Implications of Heterogeneous Beliefs

Preliminary

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Abstract

We study the role of information quality in asset pricing models with heterogeneous beliefs. We find that the equity premium, equity volatility, and trading volume are bell-shaped functions of information quality. The bell-shape is driven by two opposite effects: while the speculation effect suggests that a higher information quality makes investors speculate more actively with a higher level of confidence, the learning effect indicates that a higher information quality leads to less aggressive speculation due to a lower level of heterogeneity in posterior beliefs. Consequently, signals with intermediate precision are associated with high trading volume, high return volatility, and high risk premium. Our model can help understand several challenges in finance such as the equity premium puzzle and the excess volatility puzzle. We calibrate the model to fundamental data and show that our model is consistent with observations in financial markets.

JEL Classifications: G12, G14, C1.

Keywords: Information quality, heterogeneous beliefs, the equity premium puzzle, excess volatility, trading volume.

1 Introduction

Information quality is related to how new information is incorporated into stock prices and trading. In modern financial markets, investors absorb a large amount of information, which includes both precise information and noise information. Because information of different quality has different implications for asset prices, investors have to learn about information quality. Indeed, information quality is an important factor in determining risk premia, as emphasized by Veronesi (2000), Easley and O'Hara (2004), and Epstein and Schneider (2008).

Though a few studies (e.g., Veronesi, 2000; Ai, 2010; Li, 2005) have provided theoretical foundation for understanding the effect of information quality on asset prices, they usually assume that investors hold homogeneous beliefs about the underlying state of the economy. While this representative agent paradigm with homogeneous beliefs sheds light on the effect of information quality on asset pricing, they fails to take into account the speculative behavior of different investors in the economy. Our insight is to recognize that heterogeneous beliefs have important implications for the role of information quality on asset pricing. Specifically, we focus on several questions arising regarding the role of information quality in asset pricing in the heterogenous beliefs framework: What are the general equilibrium implications of information quality for asset prices? How does information quality affect investors' speculating behavior? What is the implication of information quality for trading volume?

Toward this end, we develop a theoretical asset pricing model to provide insights on these questions. We build the model upon a continuous-time, general-equilibrium Lucas exchange economy where rational agents have identical risk preferences and endowments. In the model, stock dividends are stochastic and are generated by a diffusion process whose drift rate is unknown to investors. Economic agents hold heterogenous priors about dividend growth. Agents update their beliefs about future dividend growth by rationally using all available public information and using a Bayesian learning approach.[1] Due to heterogenous prior beliefs, agents obtain different posterior estimates and "agree to disagree" about the state of the underlying economy. In our setting, information quality naturally affects the learning process and the resulting posterior estimates.

In our model, information quality affects asset pricing properties in two ways. The speculation effect suggests that more precise signals make investors speculate more aggressively. The intuition is that precise signals make heterogenous investors more confident on their respective posterior estimates of dividend growth. Naturally, higher confidence encourages heterogenous investors to speculate with each other more aggressively. The learning effect implies that accurate information makes heterogenous investors to speculate less. Due to the improved efficiency in learning, higher information quality leads to a lower level of posterior heterogeneity. Accordingly, heterogenous investors become more likely to share risks with each other and less likely to take bets on their different beliefs.

The speculative behavior based on posterior beliefs changes as information quality improves. In equilibrium, asset prices are determined by market clearing conditions. We manage to solve the equilibrium in a closed form. Specifically, we derive the price of stocks, the volatility of stock returns, and the trading volume of stocks as a function of prior heterogenous beliefs and information quality.

The closed form solutions clearly indicate how information quality and heterogenous prior beliefs affect asset prices and trading. Given the level of prior heterogeneity, we find that both trading volume, return volatility, and the equity premium are bell-shaped functions of information quality. Initially, as signal precision increases, the speculation effect dominates the learning effect. Note signal precision can be interpreted as a proxy for the degree of investors' confidence. As such, a high level of signal precision makes investors speculate more aggressively and, equivalently, magnifies the effect of disagreement. The active speculation leads to a higher trading volume, a higher return

[1] Agents observe the same time series of public information. Because agents do not learn from each other and adhere to different beliefs on growth rate of dividend, agents can "agree to disagree".

volatility. Moreover, aggressive speculation also give rise to higher market prices of risks, since investors may require additional compensation for bearing the risk that the stock price moves more in line with the prediction of other agents than with their own. As a result, speculation effect leads to a higher equity premium.

Beyond some benchmark, the learning effect dominates the speculation effect. This is because that as information quality improves, the level of disagreement in posterior beliefs becomes lower. The speculative trading is at a low level. In addition, a low level of posterior heterogeneity induced by precise signals implies lower prices of risks. Overall, the trading volume, return volatility, and equity premium fall as information quality improves. Therefore, we find that trading volume, return volatility, and equity premium are bell-shaped functions of information quality. It is worth noting that the slope of the bell-shape changes as the level of heterogenous prior beliefs varies.

To date, information quality and heterogenous beliefs are largely separately studied in the asset pricing literature. On the theoretical side, we contribute to the literature by incorporating disagreement and information quality into a single model. The model provides new insights on asset pricing. First, we show that, due to the two opposite effects (the speculation effect and learning effect), the relationship between information quality and the risk premium is nonlinear. We thus offer extensions of the existing results (e.g., Veronesi, 2000, Ai, 2010) that suggest that a higher precision of signals tends to increase the risk premium monotonically. Second, our model contributes to resolves several well-known puzzles such as the equity premium puzzle and the excess volatility puzzle. Third, our model can generate large trading volume, this contrasts to asset pricing models with homogeneous beliefs.

We also make the empirical contribution to the literature. Since Mehra and Prescott (1985) posed the equity premium puzzle, academics have recognized that asset pricing anomalies are quantitative and an explanation must be consistent with observations in financial markets. In this spirit, we evaluate the ability of our model to account for asset prices and trading volume observed in the market. One of our main innovations is to use the inverse of analyst forecast error as a measure of signal precision[2] and run a threshold regression to recover the bell-shape function. Our empirical analysis confirms that signals with intermediate precision give rise to high trading volume, high return volatility, and high risk premium.

In the spirit of Buraschi and Jiltsov (2006) and David (2008), we calibrate the structural model using moment conditions on the dividend process and signal precision process. We use inverse of analyst forecast errors as a proxy for information quality (e.g., Anderson, Ghysels, and Juergens, 2005; Armstrong, Banerjee, and Corona, 2010; Loughran and Mcdonald, 2014). Our empirical analysis provides the supporting evidence on the important link between information quality and (a) the stock volatility, (b) the equity premium, and (c) the stock trading volume, as predicted by our analytical analysis. Furthermore, the statistical test based on overidentifying pricing restrictions cannot reject our model. Overall, our model is largely consistent with observations in financial markets.

Our paper is also related to the literature on the relation between information quality and asset prices. On the one hand, many studies suggest that increased public information quality reduces informational asymmetry, which, in turn, increases liquidity and reduces the expected return. Some examples include Diamond and Verrecchia (1991) and Easly and O'hara (2004). On the other hand, the seminal work of Veronesi (2000) builds a dynamic asset pricing model and investigates the relationship between information quality and asset returns.[3] He finds that increased information quality drives up the risk premium. Consequently, a number of studies, including Ai (2010), Li (2005), Gollier and Schlee (2009), and Croce, Lettau, and Ludvigson (2009) extend the Veronesi (2000) model and further investigate the link between information quality and asset pricing. While some of these studies suggest a positive relationship

[2] Barron, Kim, Lim, and Stevens (1998) use dispersion-adjusted forecast error as the measure of information quality.

[3] Besides providing evidence on a negative or positive relationship between information quality and asset prices, some studies (e.g., Botosan and Plumlee, 2002; Core, Guay, and Verdi, 2008; Duarte and Young, 2009) find no significant link between information quality and asset prices.

between information quality and asset prices, others show a negative one. Our paper contributes to this stream of literature by allowing information quality to affect asset prices through two opposite effects: the speculation effect and the learning effect.

Our paper is also closely related to the recent literature on asset pricing models with heterogenous beliefs.[4] The literature typically suggests that a positive risk premium should be associated with heterogenous beliefs.[5] In addition, a number of studies have looked at the link between heterogenous beliefs, price volatility, and trading volume. Some examples include Banerjee and Kremer (2010), Carlin, Longstaff, and Matoba (2014), Harris and Raviv (1993), Kandel and Pearson (1995), and Shalen (1993). These studies generally indicate a positive relation between heterogenous beliefs, trading volume, and price volatility. Our paper contributes to this strand of literature by taking information quality into consideration.

Furthermore, our paper relates to Christensen and Qin (2014), Ottaviani and Søensenz (2014), and Qin (2013), who propose some discrete-time asset pricing models that take into account heterogeneous beliefs and information quality simultaneously. We differ from these studies by proposing a continuous-time asset pricing model with CRRA agents. More importantly, while Christensen and Qin (2014) and Qin (2013) needs heterogenous beliefs on the second moment to generate multi-round trading, we show that heterogenous beliefs on only the first moment allow sequential speculations. In so doing, our model generates the speculation effect and the learning effect simultaneously. Our model thus allows us to obtain bell-shaped return volatility and risk premium as functions of signal precision. In contrast, return volatility are largely ne-

[4] Some important examples include Anderson, Ghysels, and Juergens (2005), Bhamra and Uppal (2014), Buraschi and Jiltsov (2006), Constantinides and Duffie (1996), David (2008), Dumas, Gallmeyer, and Hollifield (2008), Dumas, Kurshev, and Uppal (2009), Ferson and Lin (2014), Jiang and Sun (2014), Kogan, Wang, Ross, and Westerfield (2006), Kurshev and Uppal (2009), Scheinkman and Xiong (2003), Shefrin (2001), Xiong and Yan (2009). Basak (2005), Carlin, Longstaff and Matoba (2014), and Jouini and Napp (2007) provide excellent reviews of the literature on asset pricing with heterogenous beliefs.

[5] Another strand of literature suggests that short-sale constraints should lead to a negative relation between heterogenous beliefs and expected returns. Some examples include Miller (1977), Chen, Hong, and Stein (2002), and Yu (2011). However, some recent studies (e.g., Avramov, Chordia, Jostova, and Philipov, 2009) argue that the negative relation could simply explained by financial distress risk.

glected and the risk premium is independent of signal precision in Christensen and Qin (2014).

The remainder of the paper is organized as follows: Section 2 presents the primitives of the economy and establish the equilibrium. Section 3 discusses the effects of heterogenous beliefs and signal precision on asset pricing properties such as the equity premium, volatility, trading volume, and the risk-free rate. Section 4 conducts the empirical analysis based on the moment conditions implied by the general equilibrium model. This section also reports the empirical results. Section 5 concludes the paper and briefly discusses future research.

2 The Model

We investigate an economy in which two types of infinitely-lived agents are endowed with shares in a production technology that generates a dividend flow. Agents have identical preferences and endowments but differ in their beliefs about the dividend growth rate (see also Detemple and Murthy (1994)). This is a generalization of the standard Lucas model.

2.1 Information Structure, Investors' Perceptions

The model uncertainty in our economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ on which we define a two-dimensional Brownian motion $B(t) = [B_D, B_e]^{'}$. $\{\mathcal{F}_t^B\}$ denotes the augmented filtration generated by $B(t)$, and $\hbar$ is a σ-field independent of $\mathcal{F}_\infty^B$. The field $\hbar$ whose role is to allow for heterogeneity in investors' priors consists of all possible initial beliefs. The complete information filtration $\{\mathcal{F}_t\}$ is the augmentation of the filtration $\hbar \times \{\mathcal{F}_t^B\}$.

Assumption 1 (Dividend and Signal Processes) *Our economy is populated by two sets of investors ($i = 1, 2$) who commonly observe the aggregate endowment and the corresponding signal process. The exogenous aggregate endowment (or dividend)* *process follows*

$$\frac{dD_t}{D_t} = \left(\theta + \frac{1}{2}\sigma_D^2\right) dt + \sigma_D dB_D(t) \Longleftrightarrow d\ln D_t = \theta dt + \sigma_D dB_D(t), \tag{1}$$

and the noisy signal follows

$$de_t = \theta dt + \sigma_e dB_e(t). \tag{2}$$

The dividend growth rate θ is assumed to be a constant which is not observed by the investors. The investors form optimal estimations of the growth rate by filtering the data of dividend D_t and signal e_t through the incomplete information filtration $\mathcal{F}_t^B \subset \mathcal{F}_t, t \in [0, \infty]$. Particularly, the prior beliefs about θ at time $t = 0$ are heterogeneous for each investor and, thus, is $\hbar$-measurable. The beliefs of the investors about θ are updated in a Bayesian fashion, via $m_\theta^i = E^i [\theta | \mathcal{F}_t]$, where $E^i [\cdot]$ denotes the expectation relative to the subjective probability measure P^i, which is equivalent to the true measure P. Due to their heterogeneous priors, investors may draw different inferences about θ at all times[6]. In contrast, investors are aware of the volatility of dynamics of dividend and signal, these assumptions capture the feature that expected return are much more difficult to estimate than volatilities[7].

Although difficult to estimate, dividend growth rate is of vital importance for the calculation of fundamental value of stocks (Gordon, 1956). Financial analysts continuously actively produce forecasting reports about dividend growth rate or earning growth rate of firms, and the signal process in our model can be viewed as an analog of these forecasting reports. Recent empirical evidence suggests that analysts usually have some disagreements about expected dividend growth rate (e.g., Carlin, Longstaff, and Matoba, 2014; Jiang and Sun, 2014; Yu, 2011). Hence, it is reasonable to model

[6] Morris (1995) proposes a method to endogenize the difference in beliefs and formulations. He argues that it is fully consistent with rationality to have heterogeneous priors. We also note that since investors have common and not asymmetric information, they are aware of each others' different inferences, arising from their different priors. Under our heterogeneous beliefs formulation, the investors agree to disagree.

[7] As Merton (1980) points out that even if the expected return on the market were known to be a constant for all time, it would take a very long history of returns to obtain an accurate estimate, no mention when the expected return is believed to be changing through time.

heterogeneity in beliefs on the expected growth rate.

By (1) and (2), the innovation processes B_D^i and B_e^i induced by investor i's beliefs and filtration are given by

$$dB_D^i(t) = \frac{1}{\sigma_D}\left(d\ln D_t - m_\theta^i dt\right), \quad \text{and} \quad dB_e^i(t) = \frac{1}{\sigma_e}(de_t - m_\theta^i dt). \tag{3}$$

The innovation processes of each investor are such that given his perceived growth rate, m_θ^i, the observed aggregate endowment obeys

$$d\ln D_t = m_\theta^i dt + \sigma_D dB_D^i(t), \tag{4}$$

and the observed noisy signal follows

$$de_t = m_\theta^i dt + \sigma_e dB_e^i(t). \tag{5}$$

These individual perspective expressions of dividend and signal indicate that investors may hold different opinions about the composition of the dividend and signal processes, though agreeing with the processes they observe. By Girsanov's theorem, B^i is a two dimensional Brownian motion on the endowed probability space $(\Omega, \mathcal{F}^i, \{\mathcal{F}_t^i\}, P^i)$ for each investor.

Note the signal is the real drift plus a noise. The inverse of the diffusion parameter, $h_e = 1/\sigma_e$, reflects the precision of the external signal. We say that investors have precise signals when h_e is relatively high. Similarly, the precision of the "dividend signal" is $h_D = 1/\sigma_D$. Again, the investors are aware of the values of precision h_D, h_e and that the growth rate is a constant. With the knowledge of the dividend growth rate model, they rationally estimate and update their beliefs using the information of the dividend and signal processes according to some optimal filtering equations which is introduced in the following subsection.

2.2 Investors' Learning Mechanism

In our economy, given the initial beliefs and the observed realizations of the dividend process and signal, the investors rationally update the posterior estimates of the drift of the dividend. In this section, we introduce the learning mechanism. We will show that the learning play important role in determining the asset pricing properties. Using standard results in filtering theory (see Theorems 12.6 and 12.7 in Liptser and Shiryaev (2001)), it is possible to prove the following results.

Lemma 1 (Learning): *Let $m(t) = E[\theta(t)|\mathcal{F}_t]$ and $\gamma(t) = E[((\theta(t) - m(t))^2 |\mathcal{F}_t]$. Under some technical regularities, $m(t)$ and $\gamma(t)$ are unique, continuous $\mathcal{F}_t$- measurable for any t solutions of the system of equations*

$$dm(t) = \gamma(t)\left(h_D^2 d\ln D_t + h_e^2 de_t\right) - \gamma(t)\left(h_D^2 + h_e^2\right) m(t)dt, \tag{6}$$

$$\dot{\gamma}(t) = -\left(h_D^2 + h_e^2\right)\gamma(t)^2, \tag{7}$$

with initial conditions $m(0) = E[\theta|\mathcal{F}_0]$ and $\gamma(0) = E[((\theta - m(0))^2 |\mathcal{F}_0]$.

In our model, the investors hold homogeneous prior variance at time 0, $\gamma(0)$. The solution to the Riccati equation $\dot{\gamma}(t) = -(h_D^2 + h_e^2)\gamma(t)^2$ is given as $\gamma(t) = 1/\left(\gamma_0^{-1} + (h_D^2 + h_e^2)t\right)$, and thus we obtain the following dynamics of the stochastic mean as

$$dm(t) = \frac{h_D^2 d\ln D_t + h_e^2 de_t}{\gamma_0^{-1} + (h_D^2 + h_e^2)t} - \frac{(h_D^2 + h_e^2)m(t)dt}{\gamma_0^{-1} + (h_D^2 + h_e^2)t},$$

so the difference in beliefs evolves according to a deterministic function[8]

$$m_\theta^1(t) - m_\theta^2(t) = \frac{(m_\theta^1(0) - m_\theta^2(0))}{\gamma_0}\frac{1}{\gamma_0^{-1} + (h_D^2 + h_e^2)t}.$$

Intuitively, as time evolves the difference in beliefs converges to zero, since the investors tend to agree as information accumulate to a huge amount. Moreover, the difference in

[8] Assumption of heterogeneous prior variances can lead to stochastic dynamic of difference in beliefs. Our assumption can simplify the analysis and highlight the impact of signal precision.

beliefs decreases in the signal precision. Since investors learn faster with higher signal precision and, thus, their beliefs converge at a higher speed.

2.3 Disagreement Process

In this section, we define the disagreement processes which are important quantities for characterizing the equilibrium. Define disagreement process scaled by dividend precision as

$$\psi_D(h_e) \equiv \sigma_D^{-1}\left(m_\theta^1(t) - m_\theta^2(t)\right) = h_D\left(m_\theta^1(t) - m_\theta^2(t)\right) = \frac{\left(m^1(0) - m^2(0)\right)h_D}{\gamma_0\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)t\right)}, \tag{8}$$

define disagreement process scaled by signal precision as

$$\psi_e(h_e) \equiv \sigma_e^{-1}\left(m_\theta^1(t) - m_\theta^2(t)\right) = h_e\left(m_\theta^1(t) - m_\theta^2(t)\right) = \frac{\left(m^1(0) - m^2(0)\right)h_e}{\gamma_0\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)t\right)}. \tag{9}$$

By (3), we have

$$dB_D^2(t) = dB_D^1(t) + \psi_D(h_e)\,dt, \qquad dB_e^2(t) = dB_e^1(t) + \psi_e(h_e)\,dt. \tag{10}$$

The disagreement process scaled by signal precision (dividend precision) capture the investors' disagreement on the mean of endowment growth rate, normalized by the signal precision (dividend precision). When investor 1 is more "optimistic", $\psi_e(h_e)$ and $\psi_D(h_e)$ is positive, and conversely. Note $\psi_e(h_e)$ has already reflected the two opposite effect of signal precision: The speculation effect suggests that a signal precision h_e leading to a higher value of $\psi_e(h_e)$, makes investors speculate more actively with a higher level of confidence, while the learning effect indicates that higher information quality h_e leading to a lower value of $\psi_e(h_e)$, makes investors speculate less aggressively due to a lower level of heterogeneity in posterior beliefs. On the contrast, $\psi_D(h_e)$ only reflects the learning effects. The impact of signal precision on these two quantities are characterized by the following Proposition 1.

Proposition 1 *(Disagreement Process) At any time t, the difference in investors' expected dividend growth rate,* $m^1(t) - m^2(t)$, *decreases with respect to the signal precision. Consequently, the disagreement process weighted by dividend precision,* $\psi_D(h_e)$, *decreases with respect to signal precision. While the disagreement process weighted by signal precision,* $\psi_e(h_e)$, *is bell-shaped with respect to the signal precision. The unique maximum for* $\psi_e(h_e)$ *is attained when the signal precision* $\bar{h}_e = \sqrt{1/(\gamma_0 t) + h_D^2}$ *and its minimum is attained for uninformative signal* $(h_e = 0)$ *and for perfect signal* $(h_e = +\infty)$. *Moreover, at any finite time for* $h_e = \bar{h}_e$, *the value of* $\psi_e(\bar{h}_e)$ *is always higher than* $\psi_D(\bar{h}_e)$, *i.e.,* $\psi_e(\bar{h}_e) > \psi_D(\bar{h}_e), 0 \leq t < +\infty$.

Note at any finite time, the value of $\psi_e(h_e)$ can be always higher than $\psi_D(h_e)$ for some intermediate signal precision, which indicates the investors can speculate more aggressively based on imperfect signal than based on dividend. Moreover, a lower prior variance γ_0, and shorter time of updating lead to stronger this kind of effects. This model property matches the fact that the dividend volatility is easy to estimate, however, there can be much more disagreement about the analysts' forecasts of growth rate based on which the speculation occurs. As a result, the investors may require considerate risk premium from bearing uncertainty from the public signals. As we will see in the following subsections, this behavior can contribute to explain not only the equity premium puzzle, but also the well-known excess volatility of stock, since speculative trading based imperfect signals can generate significant extra volatility of stock returns.

[Insert Figure 1 about Here]

Figure 1 illustrates the impact of signal precision on the disagreement processes $\psi_D(h_e)$ and $\psi_e(h_e)$. It is evident that the impact of information quality on $\psi_e(h_e)$ depends on the strength of the two opposite effects, i.e., the speculation effect and the learning effect. The learning effect gradually dominates the speculation effects as signal precision increases. Consequently, the counterbalancing result is that $\psi_e(h_e)$ first decreases and then increases with signal precision and, thus, $\psi_e(h_e)$ is bell-shaped with respect to the signal precision.

Even the impacts of uninformative signal and perfect signal on the disagreement process weighted by signal precision are the same, however, the uninformative signal and perfect signal affect disagreement process weighted by dividend precision, $\psi_D(h_e)$, differently. With uninformative signal, term $\psi_D(h_e)$, attains its highest value. This result indicates uninformative signal can generate more speculative activities than perfect signal, echoing with the findings (in following subsections) about the impacts of signal precision on asset pricing properties: imperfect signal can facilitate speculation at the highest level, uninformative signals have lower impacts, while perfect signals have the lowest ability to facilitate side-betting.

2.4 The General Equilibrium

To verify the ability of our model to address to equity premium puzzle and the excess volatility puzzle, we assume the investors have power utility functions.

Assumption 2 (Preferences): *Two sets of constant relative risk aversion (CRRA) agents act as infinite lifetime utility maximizers, i.e.,*

$$\max E^i\left[\int_t^\infty e^{-\rho t}\frac{c_i(t)^\gamma}{\gamma}dt\,\middle|\mathcal{F}_t^i\right].$$

The two sets of agents differ in terms of their beliefs, which affect their expectations. Using martingale techniques (Cox and Huang, 1989; Karatzas et al., 1987), we can express the intertemporal budget constraint in its martingale form for each investor's dynamic optimization problem given that investor's individual-specific state price density, ξ^i

$$E^i\left(\int_t^\infty \xi^i c_i(t)dt\,\middle|\mathcal{F}_t^i\right) = X_t^i.$$

The initial wealth is given by $X_t^i = e^i P_1(0)$, where e^i is the number of stock units with which agent i is initially endowed with and $P_1(t)$ is the (endogenous) stock price at time zero. To focus on the impact of signal precision, we assume $e^1 = e^2$, i.e., the investors have identical endowment.

To ensure a unique state price density for each investor, we assume that investor can trade continuously in three long-lived securities, i.e., a stock, a consol bond and a risk-free bond. Let $r(t)$ be the equilibrium riskless rate and the stock price and consol bond price evolve according to the general stochastic dynamics

$$\begin{aligned} dP(t) &= P(t)\left[\mu_P(t)dt + \sigma_{PD}(t)\, dB_D + \sigma_{Pe}(t)\, dB_e\right], \\ dO(t) &= O(t)\left[\mu_O(t)dt + \sigma_{Oe}(t)\, dB_D + \sigma_{Oe}(t)\, dB_e\right], \end{aligned}$$

where $r(t), \mu_P(t), \sigma_{PD}(t), \sigma_{Pe}(t), \mu_O(t), \sigma_{OD}(t)$, and $\sigma_{Oe}(t)$ are endogenized in the equilibrium. The consol bonds paying a continuous coupon of $\bar{c}$ per instant. Consol bonds is in zero net supply, while the stock is in positive net supply.

Note that the number of long-lived assets equals the number of stochastic shocks driving the economy, so there exists a unique state price density process for each investor ξ^i, consistent with no-arbitrage, given by

$$d\xi^i(t) = -\xi^i(t)\left(r(t)dt - \phi_D^i dB_D^i(t) - \phi_e^i dB_e^i(t)\right),$$

where ϕ^i is the perceived market price of risk (or the Sharpe ratio) process and is endognized in the equilibrium.

Let us define $\pi_i(t) = (\pi_{i1}(t), \pi_{i2}(t))$ as the number of consumption good units agent i invests in the stock and consol bond. We define the equilibrium concept as follows:

Definition 2 (Equilibrium) *: An equilibrium is a price system* $(r(t), P(t), O(t))$ *and consumption–portfolio processes* $(c_i(t), \pi_i(t))$ *such that: (i) investors choose their optimal consumption–portfolio strategies given their perceived price processes in* $(\Omega, \mathcal{F}^i, \{\mathcal{F}_t^i\}, P^i)$

and (ii) goods and security markets clear, i.e.,

$$
\begin{aligned}
c_1(t) + c_2(t) &= D(t), \\
\pi_{11}(t) + \pi_{21}(t) &= 1, \\
\pi_{12}(t) + \pi_{22}(t) &= 0.
\end{aligned}
$$

To solve for the equilibrium, it is convenient to use the following aggregation technique of Cuoco and He (1994) and Karatzas and Shreve (1998) in which the representative agent utility function is constructed by taking a (state-dependent) weighted average of individual utilities:

$$
U(c, \lambda) = e^{-\rho t} \max_{c_1(t)+c_2(t)=c} \frac{c_1(t)^\gamma}{\gamma} + \lambda_t \frac{c_2(t)^\gamma}{\gamma},
$$

with $\lambda > 0$. The structure of the utility function of the representative investor reflects that each investor wants to maximize his own utility; the larger the weight λ is, the more pricing implication investor 2 has since he dominates the market more relative to the investor 1.

The first-order conditions for the optimal consumption plan of agent i give $c_i(t) = \left(y_i \xi^i(t)\right)^{1/(\gamma-1)}$, where y_i is the (constant) Lagrange multiplier associated with the budget constraint. The consumption can be written in terms of the state price deflator. Thus, each agent's Lagrange multiplier y_i must satisfy

$$
E^i \left(\int_t^\infty \xi^i \left(y_i \xi^i(t)\right)^{1/(\gamma-1)} dt \,\middle|\, \mathcal{F}_t^i \right) = X_0^i.
$$

Imposing the market clearing condition $c_1(t) + c_2(t) = D(t)$, we obtain the equilibrium consumption allocation of the two agents and the two stochastic discount factors. The following proposition summarizes those results:

Proposition 3 ***(Equilibrium):*** *In equilibrium, the individual state price densities* *are equal to:*

$$\xi^1(t) = \frac{e^{-\rho t}}{y_1}\left(D(t)\right)^{\gamma-1}\lambda_t(1+\lambda_t^{\frac{1}{\gamma-1}})^{1-\gamma}, \quad \xi^2(t) = \frac{e^{-\rho t}}{y_2}\left(D(t)\right)^{\gamma-1}(1+\lambda_t^{\frac{1}{\gamma-1}})^{1-\gamma},$$

and the relative weight of the second agent λ_t *is state dependent and equal to* $\lambda_t = (y_1/y_2)\,\eta_t$, *with* $\eta_t = \xi^1(t)/\xi^2(t)$. *The ratio of the two agents' state price densities evolves according to*

$$\frac{d\lambda_t}{\lambda_t} = \frac{d\eta_t}{\eta_t} = -\psi_D\left(h_e\right)dB_D^1(t) - \psi_e\left(h_e\right)dB_e^1(t). \tag{11}$$

Finally, the individual optimal consumption allocations are given by

$$c_1(t) = \frac{D(t)\lambda_t^{\frac{1}{\gamma-1}}}{1+\lambda_t^{\frac{1}{\gamma-1}}}, \quad c_2(t) = \frac{D(t)}{1+\lambda_t^{\frac{1}{\gamma-1}}}. \tag{12}$$

Proof: See Appendix.

By (11) we have

$$\lambda(s) = \lambda(t)e^{-\int_t^s \psi_D(h_e,u)dB_D^1(u) - \int_t^s \psi_e(h_e,u)dB_e^1(u) - \int_t^s \left(\frac{1}{2}(\psi_D(h_e,u))^2 + \frac{1}{2}(\psi_e(h_e,u))^2\right)du},$$

which shows that the heterogeneity in prior beliefs and the signal precision have direct impact on the weight λ_t via the disagreement processes. Since the expected value of $\eta(s)$ decreases with the value of the disagreement processes, a relatively high λ_t arises when investor 1's prediction of mean growth has tended to be relatively poor in the past or relatively unlucky in his prediction. Furthermore, investor 1's consumption level decreases as λ_t increases, in the limit $lim_{\lambda(t)\to\infty}c_1(t) = 0$; The intuition is that a pessimistic investor save more for future consumption, since they think their consumption level tends to be low in the future.

3 Asset Pricing Properties

We are interested in how the informativeness of the public signal, i.e., the signal precision, affects the asset pricing properties, equilibrium stock return volatility, equilibrium equity premium, and the trading volume when the investors have heterogeneous prior means. In this section, we provide analyze plots of comparative statics to address the impact of information quality.

3.1 Equilibrium Asset Prices and Stock Return Volatility

Under the assumption of common prior variances, the difference in beliefs follows an ODE, and we can proof that the stochastic discount factor $\xi^1(t)$ is driven by two log-normally distributed state variables, $\eta(s)$ and $D(s)$, for $s > t$. It is easy to simulate the random paths the state variables and, thus, the distribution of the stochastic discount factor $\xi^1(t)$. Therefore, we can directly price assets by the following proposition.

The equilibrium stock price is given by

$$P(t) = \frac{1}{\xi^1(t)} E_t^1 \left(\int_t^{\infty} \xi^1(s) D(s) ds \right).$$

Note although $\psi_D(h_e)$ and $\psi_e(h_e)$ are two important quantities that carry the impacts of signal precision, these two disagreement processes cannot summarize all the asset pricing implications of signal precision. Because $\psi_D(h_e)$ and $\psi_e(h_e)$ can only describe the stochastic discount factor, while the distribution of future dividend is directly affected by signal precision and priors, being independent of $\psi_D(h_e)$ and $\psi_e(h_e)$. Therefore, to study the overall impacts of signal precision on asset pricing, we must endogenize the asset prices with primitive assumptions of information system.

Let the stock price from the first agent's perspective be

$$dP(t)/P(t) = \mu_p^1(t)\, dt + \sigma_{pD}(t)\, dB_D^1(t) + \sigma_{pe}(t)\, dB_e^1(t),$$

where

$$\sigma_{pD}(t) = (\partial P/\partial D)\, D(t)\, \sigma_D - (\partial P/\partial \eta)\, \psi_D(t)\eta(t), \tag{13}$$

$$\sigma_{pe}(t) = (\partial P/\partial \eta)\, \psi_e(t)\eta(t). \tag{14}$$

Thus, we have the following Proposition.

Proposition 4 *In equilibrium, the total stock volatility is equal to*

$$\sigma^2\left(\frac{dP(t)}{P(t)}\right) = \left(\frac{\partial P}{\partial D}D(t)\sigma_D - \frac{\partial P}{\partial \lambda}\psi_D(t)\lambda(t)\right)^2 + \left(\frac{\partial P}{\partial \lambda}\psi_e(t)\lambda(t)\right)^2.$$

Note that the terms $\frac{\partial P}{\partial D}D(t)\sigma_D$ arise from the volatility of the dividend processes, the fundamental volatility, while the term $\frac{\partial P}{\partial \lambda}\psi_D(t)\lambda(t)$ and $\frac{\partial P}{\partial \lambda}\psi_e(t)\lambda(t)$ are from the volatility from the process of state variable $\lambda(t)$. Asset prices and related derivatives are deterministic integrals that can be computed at any desired level of accuracy using Monte Carlo methods. See Appendix for a description of Monte Carlo methods to calculate the prices and related derivatives. By endogenizing the stock price we are able to show how the endogenous volatility of stock returns depends on the signal precision.

[Insert Figure 2 about Here]

Figure 2 plots stock return volatility as a function of information quality. We can see that the signal precision impacts the stock return volatility with two different effects: speculation effect and the learning effect. And these two countervailing effects give rise to that the stock return volatility is bell-shaped with respect to the signal volatility, since with imperfect signal precision the investors speculate the stock market most aggressively. Empirically, the high volatility of the stock price series combined with the low volatility of the dividend series has suggested that prices may be "too volatile" to be consistent with a rational model. Our model help to explain the excess volatility, since in our setting, model uncertainty of the expected dividend growth rate and difference in priors introduces an element of learning which generates the heterogeneity in posterior beliefs and speculative trading. Speculations based on public signal and heterogeneous beliefs can generate endogenous volatility, which is much higher than the volatility of dividend.

Our results also help to understand the equity premium puzzle. Since, the combination of the low volatility of dividend and the high volatility associated with stock returns is a fundamental element of the equity premium puzzle. Only can we explain why equities are risky, we can hope to understand why they command a significant risk premium. As we can see in the following subsection, investor requires significant compensation of risk premium for bearing signal risk, arising from speculation.

3.2 Market Prices of Risk and Equity Premium

Since there are two different random sources in the information system, which are captured by the volatility of dividend and the volatility of signal, the investors can speculate with both kind of uncertainty and price the risks differently. We obtain analytical expressions market prices of risk.

Proposition 5 ***(Market Prices of Risk):*** *In equilibrium, the agents are averse to both dividend and signal shocks. The two agent-specific prices of risk are equal to*

$$\begin{aligned} \phi_D^1(t) &= (1-\gamma)\,\sigma_D(t) + \frac{\lambda_t^{\frac{1}{1-\gamma}}\psi_D\left(h_e\right)}{1+\lambda_t^{\frac{1}{1-\gamma}}}, & \phi_D^2(t) &= (1-\gamma)\,\sigma_D(t) - \frac{\psi_D\left(h_e\right)}{1+\lambda_t^{\frac{1}{1-\gamma}}}, \\ \phi_e^1(t) &= \frac{\lambda_t^{\frac{1}{1-\gamma}}}{1+\lambda_t^{\frac{1}{1-\gamma}}}\psi_e\left(h_e\right), & \phi_e^2(t) &= -\frac{1}{1+\lambda_t^{\frac{1}{1-\gamma}}}\psi_e\left(h_e\right). \end{aligned}$$

Proof: See Appendix.

The disagreement of the investors on the expected growth rate of dividend leads to their difference in the perceived market prices of risk. Under homogeneous beliefs, investors would price risk equally. With heterogeneous beliefs, however, the more optimistic investor is willing to bear more risk than the pessimistic investor. Hence the optimistic investor holds risky asset and price the signal risk positively. Furthermore, from the expressions of the market prices of risk, we can see that the signal precision affects the market prices of risk through the disagreement process. With imperfect signal, investors speculate actively and the optimistic investor perceive higher market price of signal risk. The intuition is that when an agent is more optimistic and speculates with long position in stock, he faces potentially a larger correction of prices moving in line with the pessimistic's beliefs. Hence, his market price of signal risk is the highest with a imperfect public signal.

[**Insert Figure 3 about Here**]

With the expression of prices of risk, we can compute the required equity premium. In equilibrium, the individually specific required excess return is an inner product of the market prices of risk of that agent and endogenized equity volatility. For agent 1, i.e.,

$$\bar{\mu}^1 = \sigma_{pD}(t)\,\phi_D^1(t) + \sigma_{pe}(t)\,\phi_e^1(t),$$

the expression of $\sigma_{pD}(t)$ and $\sigma_{pe}(t)$ are given by (13) and (14) in the subsection of stock return volatility. We plot the agent 1's equity premium as a function of signal precision in Figure 3. We find that the risk premium inherits many of the features of the prices of risk, stock return volatility: Intermediate signal precision give rise to the highest stock volatility and the highest prices of risk for investor 1, which naturally also yields a maximum point for the required premium for investor 1.

3.3 Trading Volume

Trading volume is defined to measure the absolute value of the change in investors' position in the risky asset. It is straightforward to measure trading volume in discrete time model (e.g., Christensen and Qin, 2014). However, in continuous time model, usually at each point of time, the change of position is infinitesimal. Therefore, we follow Xiong and Yan (2010) and turn to another proxy of trading volume: The volatility of investors' position.

[Insert Figure 4 about Here]

The stock holding of investor 1 is given as (see Appendix for a proof)

$$\pi_{11}(t) = \left(\frac{\partial X^1}{\partial D}\right) / \left(\frac{\partial P(t)}{\partial D}\right),$$

and, thus, the volatility of $\pi_{11}(t)$ is given as

$$\begin{aligned}
\sigma^2_{\pi_{11}} &= \left(\frac{\partial \pi_{11}}{\partial D}\sigma_D D(t) - \frac{\partial \pi_{11}}{\partial \lambda}\psi_D(h_e)\lambda(t)\right)^2 + \left(\frac{\partial \pi_{11}}{\partial \lambda}\psi_e(h_e)\lambda(t)\right)^2 \\
&= \left(\psi_D(h_e)^2 + \psi_e(h_e)^2\right)\left(\frac{\partial \pi_{11}}{\partial \lambda}\lambda(t)\right)^2.
\end{aligned}$$

See Appendix for the derivation of the expression of $\frac{\partial \pi_{11}}{\partial D}$ and $\frac{\partial \pi_{11}}{\partial \lambda}$. We plot the trading volume as a function of signal precision in Figure 4. The figure clearly shows that trading volume is a bell-shaped function of information quality. This is consistent with the combined effect of the speculation mechanism and the learning mechanism.

4 Empirical Analysis and Calibration

In this section, we empirically test our model. Firstly, we describe the data and discuss how to construct the signal precision index. Secondly, we run threshold regressions to recover the bell-shape. Thirdly, we present the GMM method for estimating parameters with the moment conditions implied by the theoretical model. Empirically, we do find that the equity premium and return volatility are bell-shaped functions of signal precision. Moreover, with the calibrated parameters, we show that imperfect signal can generate high level of return volatility and risk premium.

4.1 Data

The fundamental data set includes quarterly data series spanning from 1968:04 to 2013:04. The quarterly closing price for S&P 500 index, the cumulative dividend payment within the last 12 months on that index are from Goyal's website.[9] We obtain daily returns on S&P 500 index from CRSP and follow Chen and Petkova (2012) by using within-quarter daily returns to compute quarterly realized variance:

$$V_i = \sum_{d=1}^{D_i} R_d^2 + 2\sum_{d=2}^{D_i} R_d R_{d-1},$$

where D_i is the number of days in quarter i and R_d is the index's return on day d. The second term on the right-hand side adjusts for the autocorrelation in daily returns. The daily trading volume on S&P 500 index is also obtained from CRSP. The quarterly trading volume is computed as the sum of daily trading volume within that quarter.

[**Figure 5 about Here**]

For empirically testing our model, we need to measure signal precision process. For this purpose, we use Survey of Professional Forecasters available at the Federal Reserve Bank of Philadelphia. Reliable data start from 1968:04. For each quarter, private sector economists are asked to forecast approximately 27 economic variables over the subsequent five quarters. Let $FD_i(t, \tau)$ be the forecast of a variable τ quarters ahead

[9] This is an extended data set used by Welch and Goyal (2008). We thank Amit Goyal for kindly providing the data.

of time t by forecaster i, and let $\overline{FD}(t,\tau)$ be the mean of these forecasts. We focus on corporate profits after tax. We take the inverse of the absolute difference between the mean of forecasts $\overline{FD}(t,\tau)$ for this process and the realization of it at time $t+\tau$ as the measure of signal precision. This measure is intuitive: High information quality should lead to low average forecast errors. Figure 5 presents the signal process. It is evident that the signal process is volatile. Interestingly, it becomes more volatile during crisis periods, i.e. the early 1970s (the oil crisis), the early 2000s (dotcom bubble), and the period between 2007-2009 (the subprime crisis). In addition, we use the inverse of the historical volatility calculated from the dividend growth rate as the proxy for dividend signal precision.

4.2 Threshold Regressions

The time variation in information quality generates endogenous equity premium, return volatility, and trading volume. In particular, our model implies that these quantities are bell-shaped functions of information quality. To investigate the economic significance of the link between these quantities and information quality, we run the following threshold regression to uncover the bell-shape:

$$\begin{aligned} y_t &= (1-I(x_t))(\alpha_1 + \sum\nolimits_{j=1}^{3} \beta_{1j} X_{j,t-1} + \gamma_{11} x_{t-1} + \gamma_{21} x_{t-1}^2) + \\ &\quad I(x_t)(\alpha_2 + \sum\nolimits_{j=1}^{3} \beta_{2j} X_{j,t-1} + \gamma_{12} x_{t-1} + \gamma_{22} x_{t-1}^2), \end{aligned} \tag{15}$$

where y_t is the equity premium, stock return volatility, or trading volume, X_t is a vector of control variables, and x_t is information quality. The control variables for the equity premium include the dividend yield (DY), the Treasury bill rate (TBL), CAY (see also Lettau and Ludvigson, 2001), which are widely believed to predict asset prices.[10] The control variables for trading volume (e.g., Kaniel, Ozoguz, Starks, 2012) have lagged

[10] We also tried to include the default premium, the yield spread, and inflation in the threshold regression. However, these variables are generally insignificant when the dividend yield, the short-term interest rate, and CAY are included in the regression.

trading volume (TV), trading spread (TS), and lagged stock returns (SR). The control variable for stock volatility includes lagged volatility (Vol) and stock returns. $I(x_t)$ is an indicator variable with

$$I(x_t) = \begin{cases} 0 \; if \; x_t \leq c \\ 1 \; if \; x_t > c \end{cases},$$

where c is the threshold value. In our setting, it is very natural to take information quality (x_t) as the threshold variable.

[**Insert Table 1, Figure 6 about Here**]

The intuition of threshold regression (15) is straightforward: if the effect of x_{t-1} on y_t increases below c and decreases above c, the regression implies a bell-shape function. Empirically, we use the maximum likelihood method to estimate the threshold regression. Table 2 reports the empirical results. When y_t denotes the equity premium, it is evident that y_t is an increasing function of information quality initially, beyond the benchmark c, y_t is decreasing as information quality improves. If y_t represents the stock return volatility,[11] the results also indicate a bell-shape. The threshold regression therefore provides supporting evidence for the theoretical model. However, when y_t is trading volume, the evidence is not supportive. To recover the bell-shape from the regression results, Panel A and B of Figure 6 respectively plot the equity premium and stock return volatility against signal precision. The plots clearly show that the equity premium and volatility are bell-shaped functions of information quality.

4.3 GMM Estimation and Endogenized Return Volatility and Risk Premium

To verify the model' ability to generate high return volatility and high risk premium with imperfect signal, we now calibrate the model to obtain the value of fundamental

[11]Because trading volume shows a upward-sloping trend, we add a time trend in the threshold regression when the dependent variable is trading volume.

parameters. With these parameters, we can endogenize the return volatility and risk premium. Note the value of the signal volatility and the dividend volatility can be easily estimated from the times series by definition, we only need to estimate three parameters, i.e., prior beliefs ($m_\theta^1(0)$, $m_\theta^2(0)$, and γ_0).

We estimate the prior beliefs by matching the generated dividend growth and signal process with observed ones. More specifically, let $\Theta = (m_\theta^1(0), m_\theta^2(0), \gamma_0)'$ be the vector of parameters to be estimated. We estimate Θ by minimizing a GMM quadratic criterion defined in terms of the estimation errors for the dividend process and signal process. The GMM objective function to be minimized is as follows:

$$\min \sum_{t=1}^{T} g_t(\Theta) W_t(\Theta) g_t(\Theta),$$

where

$$g_t(\Theta) = \begin{bmatrix} \frac{\ln D_t^1(\Theta)}{\ln D_t} - 1 \\ \frac{\ln D_t^2(\Theta)}{\ln D_t} - 1 \\ \frac{e_t^1(\Theta)}{e_t} - 1 \\ \frac{e_t^2(\Theta)}{e_t} - 1 \end{bmatrix} \otimes Z_t,$$

and Z_t is a vector of instrumental variables. $D_t^i(\Theta), i = 1$ or 2, refers to the perceived dividend process by investor 1 or 2, respectively[12]. Similarly, $e_t^i(\Theta)$, $i = 1$ or 2, refers to the perceived signal process by investor 1 or 2, respectively. The weighting matrix is the Newey-West (1987) covariance matrix of the estimation errors. The details of the GMM estimation procedure are presented in Appendix B.

Panel A of Table 2 reports the descriptive statistics for the quarterly data of dividend growth rate, stock returns, signal process, and trading volume (in log form). It is obvious that dividend growth rate is compatible with stock returns, yet the later is much more volatile than the former. The standard deviation for the dividend growth rate is 2.1%, yet the standard deviation for stock returns is 8.4%. It seems that stock

[12] Investors perceive identical information, i.e., $D_t^1 = D_t^2$. The superscripts indicate the investors disagree about the component of the perceive dividend, i.e., they perceive different drift and different shocks.

returns are much more volatile than dividend growth is. This finding is consistent with previous studies (e.g., Shiller, 1981; Zhu, 2013).

[Insert Table 2 about here]

Panel B reports the GMM estimates for parameters.[13] As we expect, the prior belief of the dividend growth rate for investor 1, $m_\theta^1(0)$, and the prior belief for investor 2, $m_\theta^2(0)$ sandwich the mean of dividend growth rate from the data. $m_\theta^1(0)$ is larger than 0.015, while $m_\theta^2(0)$ is smaller than 0.015. Since investor 1 has a higher prior belief about the dividend growth rate, we call she as an optimistic investor. On the contrary, we call investor 2 as the pessimistic investor for the similar reason. Also note that γ_0, the prior variance between these two types of investors, is much larger than the realized volatility of the dividend growth rate. Both investors will update their belief with the coming of new data.

With the calibrated values of parameters, our model can generate an annual risk premium of 6.5% and an volatility of the stock return of 0.23. Our model shows that speculations based on the imperfect signals can generate large amount of the endogenized risk premium and the volatility of the stock return. The generated risk premium and excess volatility are largely consistent with the observations in the stock market.

5 Conclusions

In this paper, we investigate both theoretically and empirically the link between information quality and asset pricing with heterogeneous beliefs in a continuous-time general equilibrium model. In our setting, differences in beliefs have important implications for asset pricing and trading behavior. Information quality affects asset prices

[13]To avoid the over-identification problem, we also did the Hansen test for over-identification restrictions. The result shows that we cannot reject the null.

and trading volume by affecting posterior heterogenous beliefs. In particular, information quality has two opposite effect: (a) the speculation effect suggests that higher information quality makes investors speculate more aggressively with a higher level of confidence; (b) the learning effect indicates that higher information quality makes investors speculate less actively due to a lower level of heterogeneity in posterior beliefs. Driven by the two effects, we show that the equity premium, stochastic volatility, and trading volume are bell-shaped functions of information quality. Our simulation analysis replicates the bell shape and providing the supporting evidence for the model.

We then estimate and test the structural model. We construct an information quality index using survey data. We test the structural overidentifying pricing restrictions of the model by using the GMM test. We cannot reject the model. More importantly, our calibrated model can generate reasonable return volatility and equity premium. Hence, our model help resolve several puzzles in finance such as the equity premium puzzle and the excess volatility. We also run a threshold regression and document an empirical bell shape between information quality and the equity premium/trading volume. This further provides the supporting evidence for our model.

The calibration and empirical analysis gives strong support to the role played by information quality and heterogenous beliefs in the dynamics of asset prices and volume. We view our results as complementary to the line of research on information quality (e.g., Veronesi, 2000; Ai, 2010) because our model sharpens our understanding on the effect of information quality on asset prices. In particular, we show that information quality has two opposite effect on asset prices, this contrasts to the previous literature, which usually suggests a monotonic relation between information quality and asset prices. We also view our results as complementary to the literature on heterogenous beliefs by taking information quality into consideration. For future research, the role of information quality on the pricing of financial derivatives is an interesting topic.

Appendix A: Proofs

A.1. Derivation of Dynamics of Difference in Beliefs

We have a matrix-form representation of the dividend and the signal as

$$\begin{pmatrix} d\ln D_t \\ de_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_t \\ \theta_t \end{pmatrix} dt + \begin{pmatrix} \sigma_D & 0 \\ 0 & \sigma_e \end{pmatrix} \begin{pmatrix} dB_D(t) \\ dB_e(t) \end{pmatrix} \Longleftrightarrow$$

$$\begin{pmatrix} d\ln D_t \\ de_t \end{pmatrix} = \begin{pmatrix} \theta_t \\ \theta_t \end{pmatrix} dt \begin{pmatrix} \sigma_D dB_D(t) \\ \sigma_e dB_e(t) \end{pmatrix} \Longleftrightarrow \begin{pmatrix} d\ln D_t \\ de_t \end{pmatrix} = \begin{pmatrix} \theta_t dt + \sigma_D dB_D(t) \\ \theta_t dt + \sigma_e dB_e(t) \end{pmatrix},$$

hence, we can express the signal processes as

$$\begin{pmatrix} d\ln D_t \\ de_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_t \\ \theta_t \end{pmatrix} dt + \begin{pmatrix} \sigma_D & 0 \\ 0 & \sigma_e \end{pmatrix} \begin{pmatrix} dB_D(t) \\ dB_e(t) \end{pmatrix}.$$

According to the standard filtering theory (see Theorems 12.6 and 12.7 in Liptser and Shiryaev (2001)), the dynamics of the posterior mean follow

$$\begin{aligned}
& d\begin{pmatrix} m(t) \\ m(t) \end{pmatrix} \\
= & \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1} \end{pmatrix} \left(\begin{pmatrix} d\ln D_t \\ de_t \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m(t) \\ m(t) \end{pmatrix} dt \right) \\
= & \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1} \end{pmatrix} \begin{pmatrix} d\ln D_t - m(t)dt \\ de_t - m(t)dt \end{pmatrix} \\
= & \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} (d\ln D_t - m(t)dt) \\ (\sigma_e^2)^{-1} (de_t - m(t)dt) \end{pmatrix},
\end{aligned}$$

and dynamics of the posterior variance evolve according to

$$
\begin{aligned}
\begin{pmatrix} \dot{\gamma}(t) & \dot{\gamma}(t) \\ \dot{\gamma}(t) & \dot{\gamma}(t) \end{pmatrix} &= -\begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \\
&= -\begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1} \end{pmatrix} \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix}.
\end{aligned}
$$

Simplifying yields

$$
\begin{aligned}
d\begin{pmatrix} m(t) \\ m(t) \end{pmatrix} &= \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1}(d\ln D_t - m(t)dt) \\ (\sigma_e^2)^{-1}(de_t - m(t)dt) \end{pmatrix} \\
&= \begin{pmatrix} \gamma(t)\left((\sigma_D^2)^{-1}(d\ln D_t - m(t)dt) + (\sigma_e^2)^{-1}(de_t - m(t)dt)\right) \\ \gamma(t)\left((\sigma_D^2)^{-1}(d\ln D_t - m(t)dt) + (\sigma_e^2)^{-1}(de_t - m(t)dt)\right) \end{pmatrix},
\end{aligned}
$$

and

$$
\begin{aligned}
\dot{\gamma}(t) &= -\begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \begin{pmatrix} (\sigma_D^2)^{-1} & 0 \\ 0 & (\sigma_e^2)^{-1} \end{pmatrix} \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \\
&= -\begin{pmatrix} \gamma(t)(\sigma_D^2)^{-1} & \gamma(t)(\sigma_e^2)^{-1} \\ \gamma(t)(\sigma_D^2)^{-1} & \gamma(t)(\sigma_e^2)^{-1} \end{pmatrix} \begin{pmatrix} \gamma(t) & \gamma(t) \\ \gamma(t) & \gamma(t) \end{pmatrix} \\
&= -\gamma^2(t)\left(\left(\sigma_D^2\right)^{-1} + \left(\sigma_e^2\right)^{-1}\right) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.
\end{aligned}
$$

Hence, we achieve the dynamics of the posterior beliefs as

$$
\begin{aligned}
dm(t) &= \gamma(t)\left(\left(\sigma_D^2\right)^{-1}(d\ln D_t - m(t)dt) + \left(\sigma_e^2\right)^{-1}(de_t - m(t)dt)\right), \\
\dot{\gamma}(t) &= -\gamma^2(t)\left(\left(\sigma_D^2\right)^{-1} + \left(\sigma_e^2\right)^{-1}\right),
\end{aligned}
$$

which can be rewritten as

$$\begin{aligned}
dm(t) &= \gamma(t)\left(h_D^2\left(d\ln D_t - m(t)dt\right) + h_e^2\left(de_t - m(t)dt\right)\right) \\
&= \gamma(t)\left(h_D^2 d\ln D_t - h_D^2 m(t)dt + h_e^2 de_t - h_e^2 m(t)dt\right) \\
&= \gamma(t)\left(h_D^2 d\ln D_t + h_e^2 de_t - \left(h_D^2 + h_e^2\right) m(t)dt\right) \\
&= \gamma(t)\left(h_D^2 d\ln D_t + h_e^2 de_t\right) - \gamma(t)\left(h_D^2 + h_e^2\right) m(t)dt, \\
\dot{\gamma}(t) &= -\left(h_D^2 + h_e^2\right)\gamma^2(t).
\end{aligned}$$

Therefore, the disagreement process follows

$$\begin{aligned}
dm^1(t) - dm^2(t) &= -\gamma(t)\left(h_D^2 + h_e^2\right)\left(m^1(t) - m^1(t)\right)dt \\
\dot{\gamma}(t) &= -\left(h_D^2 + h_e^2\right)\gamma^2(t).
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{d\gamma}{dt} &= -\left(h_D^2 + h_e^2\right)\gamma^2(t) \Longrightarrow \frac{d\gamma}{\gamma^2(t)} = -\left(h_D^2 + h_e^2\right)dt \Longrightarrow \int \frac{1}{\gamma^2(t)}d\gamma = \int -\left(h_D^2 + h_e^2\right)dt \\
\Longrightarrow & -\frac{1}{\gamma} + \frac{1}{\gamma_0} = -\left(h_D^2 + h_e^2\right)t \Longrightarrow \gamma(t) = \frac{1}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)t},
\end{aligned}$$

hence, the disagreement process follows

$$\begin{aligned}
d\left(m^1(t) - m^2(t)\right) &= -\gamma(t)\left(h_D^2 + h_e^2\right)\left(m^1(t) - m^2(t)\right)dt \Longrightarrow \\
d\ln\left(m^1(t) - m^2(t)\right) &= -\gamma(t)\left(h_D^2 + h_e^2\right)dt \Longrightarrow \\
d\ln\left(m^1(t) - m^2(t)\right) &= -\frac{h_D^2 + h_e^2}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)t}dt = -\frac{1}{\frac{\gamma_0^{-1}}{h_D^2 + h_e^2} + t}dt \Longrightarrow \\
d\ln\left(m^1(t) - m^2(t)\right) &= -\frac{1}{\frac{\gamma_0^{-1}}{h_D^2 + h_e^2} + t}d\left(\frac{\gamma_0^{-1}}{h_D^2 + h_e^2} + t\right) \Longrightarrow \\
d\ln\left(m^1(t) - m^2(t)\right) &= -d\ln\left(\frac{\gamma_0^{-1}}{h_D^2 + h_e^2} + t\right) \Longrightarrow
\end{aligned}$$

$$\ln\left(m^1(t)-m^2(t)\right)-\ln\left(m^1(0)-m^2(0)\right)=\ln\left(\frac{\gamma_0^{-1}}{h_D^2+h_e^2}\right)-\ln\left(\frac{\gamma_0^{-1}}{h_D^2+h_e^2}+t\right)\Longrightarrow$$

$$\begin{aligned}
\ln\left(m^1(t)-m^2(t)\right) &= \ln\left(m^1(0)-m^2(0)\right)+\ln\left(\frac{\gamma_0^{-1}}{h_D^2+h_e^2}\frac{1}{\left(\frac{\gamma_0^{-1}}{h_D^2+h_e^2}+t\right)}\right)\Longrightarrow \\
\ln\left(m^1(t)-m^2(t)\right) &= \ln\left(\frac{\left(m^1(0)-m^2(0)\right)\gamma_0^{-1}}{h_D^2+h_e^2}\frac{1}{\left(\frac{\gamma_0^{-1}}{h_D^2+h_e^2}+t\right)}\right)\Longrightarrow \\
m^1(t)-m^2(t) &= \frac{\left(m^1(0)-m^2(0)\right)}{\gamma_0}\frac{1}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}.
\end{aligned}$$

A.2. Derivation of Stationary Point of Disagreement Process

Taking the first-order condition of the disagreement process weighted by signal precision $\psi_e\left(h_e\right)$ gives

$$\frac{1}{\gamma_0^{-1}+h_D^2t+h_e^2t}-\frac{2h_e^2t}{\left(\gamma_0^{-1}+h_D^2t+h_e^2t\right)^2}=0\Longleftrightarrow$$

$$\begin{aligned}
\frac{1}{\gamma_0^{-1}+h_D^2t+h_e^2t} &= \frac{2h_e^2t}{\left(\gamma_0^{-1}+h_D^2t+h_e^2t\right)^2}\Longleftrightarrow \\
\frac{2h_e^2t}{\gamma_0^{-1}+h_D^2t+h_e^2t} &= 1\Longleftrightarrow\gamma_0^{-1}+h_D^2t+h_e^2t=2h_e^2t\Longleftrightarrow \\
\gamma_0^{-1}+h_D^2t &= h_e^2t\Longleftrightarrow\frac{\gamma_0^{-1}+h_D^2t}{t}=h_e^2\Longleftrightarrow \\
h_e^2 &= \frac{\gamma_0^{-1}}{t}+h_D^2\Longleftrightarrow h_e=\sqrt{\frac{\gamma_0^{-1}}{t}+h_D^2}.
\end{aligned}$$

Hence, maximum point is achieved at $h_e=\sqrt{\frac{\gamma_0^{-1}}{t}+h_D^2}$, which is higher than the dividend precision h_D.

A.3. Proof of Proposition 2 (Equilibrium)

We now derive the difference of the market prices of risk as functions of disagreement processes. By simple substitution, the agent-specific Brownian motions are related to the true innovations as follows:

$$\begin{aligned} dB_D(t) &= \frac{1}{\sigma_D}(m_\theta^i(t) - \theta(t))dt + dB_D^i(t), \\ dB_e(t) &= \frac{1}{\sigma_e}(m_\theta^i(t) - \theta(t))dt + dB_e^i(t). \end{aligned}$$

Let asset prices follow the diffusion processes

$$dP_z(t) = P_z(t)\left[\mu_z(t)\,dt + \sigma_{zD}(t)\,dB_D(t) + \sigma_{ze}(t)\,dB_e(t)\right],$$

with $P_1(t) = P(t)$ and $P_2(t) = O(t)$. Substituting the agent-perceived Brownian motions, we obtain

$$\begin{aligned} dP_z(t) &= P_z(t)\left[\mu_z^i(t)\,dt + \sigma_{zD}(t)\,dB_D^i(t) + \sigma_{ze}(t)\,dB_e^i(t)\right], \\ \mu_z^i(t) &= \mu_z(t) + (\sigma_{zD}(t) + \sigma_{ze}(t))\,\frac{m_\theta^i(t) - \theta(t)}{\sigma_D}, \end{aligned}$$

where $\mu_z^i(t)$ is the expected instantaneous return for asset z from the perspective of agent i. Thus, the difference in expected returns from the perspective of two different agents is given by

$$\begin{aligned} \mu_z^1(t) - \mu_z^2(t) &= \sigma_{zD}(t)\,\frac{m_\theta^1(t) - m_\theta^2(t)}{\sigma_D} + \sigma_{ze}(t)\,\frac{m_\theta^1(t) - m_\theta^2(t)}{\sigma_e} \\ &= \sigma_{zD}(t)\,\psi_D(h_e) + \sigma_{ze}(t)\,\psi_D(h_e). \end{aligned}$$

Let us define $\phi_D^i(t)$ to be agent i's price of dividend risk and $\phi_e^i(t)$ to be his price of signal risk. By no-arbitrage, excess returns need to satisfy

$$\mu_z^i(t) - r(t) = \sigma_{zD}(t)\,\phi_D^i(t) + \sigma_{ze}(t)\,\phi_e^i(t).$$

From the previous two equations we have

$$\sigma_{zD}(t)\left(\phi_D^1(t) - \phi_D^2(t)\right) + \sigma_{ze}(t)\left(\phi_e^1(t) - \phi_e^2(t)\right) = \sigma_{zD}(t)\,\psi_D(h_e) + \sigma_{ze}(t)\,\psi_D(h_e).$$

Because the last equation has to hold for any $\sigma_{zD}(t)$ and $\sigma_{ze}(t)$, it follows that

$$\phi_D^1(t) - \phi_D^2(t) = \psi_D(h_e) \quad \text{and} \quad \phi_e^1(t) - \phi_e^2(t) = \psi_e(h_e). \tag{16}$$

The optimal program is

$$e^{-\rho t} \max_{c_1, c_2,\ c_1 + c_2 = D} \frac{c_1^\gamma}{\gamma} + \lambda \frac{c_2^\gamma}{\gamma}.$$

In equilibrium, $u_1'(c_1) = \lambda u_2'(c_2)$, i.e., $c_1^{\gamma-1} = \lambda c_2^{\gamma-1}$. Solving for the optimal allocation of the aggregate consumption between the two agents, we obtain $c_1 = D\lambda_t^{\frac{1}{\gamma-1}}/(1+\lambda_t^{\frac{1}{\gamma-1}})$ and $c_2 = D/(1+\lambda_t^{\frac{1}{\gamma-1}})$. Substituting, we obtain the indirect utility function

$$\begin{aligned} V(D;\lambda) &= e^{-\rho t}\left(\frac{D^\gamma \lambda_t^{\frac{\gamma}{\gamma-1}}}{\left(1+\lambda_t^{\frac{1}{\gamma-1}}\right)^\gamma \gamma} + \frac{\lambda D^\gamma}{\left(1+\lambda_t^{\frac{1}{\gamma-1}}\right)^\gamma \gamma}\right) = e^{-\rho t}\frac{D^\gamma}{\gamma}\left(1+\lambda_t^{\frac{1}{\gamma-1}}\right)^{-\gamma}\left(\lambda + \lambda_t^{\frac{\gamma}{\gamma-1}}\right) \\ &= e^{-\rho t}\frac{D^\gamma}{\gamma}\lambda\left(1+\lambda_t^{\frac{1}{\gamma-1}}\right)^{-\gamma}\left(1+\lambda_t^{\frac{\gamma}{\gamma-1}-1}\right) = e^{-\rho t}\frac{D^\gamma}{\gamma}\lambda\left(1+\lambda_t^{\frac{1}{\gamma-1}}\right)^{1-\gamma}. \end{aligned}$$

The first-order conditions of each agent require $u_i'(c_i) = y_i \xi^i$ and $V_D'(D;\lambda) = u'(c_1) = \lambda u'(c_2)$, thus

$$\xi^1(t) = e^{-\rho t}\frac{1}{y_1}(D(t))^{\gamma-1}\lambda_t(1+\lambda_t^{\frac{1}{\gamma-1}})^{1-\gamma} \text{ and } \xi^2(t) = e^{-\rho t}\frac{1}{y_2}(D(t))^{\gamma-1}(1+\lambda_t^{\frac{1}{\gamma-1}})^{1-\gamma}, \tag{17}$$

with $\lambda_t = y_1\xi^1(t)/\left(y_2\xi^2(t)\right) = y_1\eta_t/y_2$, so that

$$c_1(t) = D(t)\frac{\lambda_t^{\frac{1}{\gamma-1}}}{1+\lambda_t^{\frac{1}{\gamma-1}}}; \qquad c_2(t) = D(t)\frac{1}{1+\lambda_t^{\frac{1}{\gamma-1}}}.$$

The constants $\lambda(0)$, y_1, and y_2 solve the static individual first-order conditions and budget constraints $E^i\left(\int V_D' \cdot \left(y_i\xi^i(t)\right)^{1/\gamma-1} dt \,|\mathcal{F}_t^i\right) = X_0^i$, which imply

$$\begin{aligned}
E^1\left(\int D(t)\cdot\left(\frac{\lambda_t^{\frac{1}{\gamma-1}}}{1+\lambda_t^{\frac{1}{\gamma-1}}}\right)^{\gamma} dt \,\middle|\mathcal{F}_t^i\right) &= X_0^1,\\
E^2\left(\int D(t)\lambda_t\left(\frac{1}{1+\lambda_t^{\frac{1}{\gamma-1}}}\right)^{\gamma} dt \,\middle|\mathcal{F}_t^i\right) &= X_0^2.
\end{aligned}$$

Using Ito's rule, λ_t satisfies the differential equation

$$\frac{d\lambda_t}{\lambda_t} = \frac{d\eta_t}{\eta_t} = -(\phi_D^1 - \phi_D^2)dB_D^1(t) - (\phi_e^1 - \phi_e^2)dB_e^1(t).$$

thus, by Eq. (16) we retrieve

$$\frac{d\lambda_t}{\lambda_t} = -\psi_D\left(h_e\right)dB_D^1(t) - \psi_e dB_e^1(t).$$

A.4. Proof of Proposition 3 (Asset Prices and Return Volatility)

(a) Stock price: From the Euler equation, the stock price

$$P(t) = \frac{1}{\xi^1(t)}E_t^1\left(\int_t^{\infty}\xi^1(s)D(s)ds\right).$$

Thus

$$P(t) = \frac{E_t^1\left(\int_t^{\infty}\left(D(s)\right)^{\gamma-1}\lambda(s)(1+\lambda(s)^{\frac{1}{\gamma-1}})^{1-\gamma}D(s)ds\right)}{\left(D(t)\right)^{\gamma-1}\lambda(t)(1+\lambda(t)^{\frac{1}{\gamma-1}})^{1-\gamma}}.$$

Note the dividend $D(s)$ and the state variable $\lambda(s)$ are both lognormal distributed, and the stock price can be calculated by Monte Carlo simulation. The distribution of λ_s and $D(t)$ are derived as follows.

Marginal Distribution of λ_s

The dynamics of the difference in beliefs process λ_t is given by

$$\begin{aligned}
\frac{d\lambda_t}{\lambda_t} &= -h_D\left(m_\theta^1(t) - m_\theta^2(t)\right)dB_D^1(t) - h_e\left(m_\theta^1(t) - m_\theta^2(t)\right)dB_e^1(t) \\
&= -\psi_D(t)dB_D^1(t) - \psi_e(t)dB_e^1(t),
\end{aligned}$$

so that

$$\lambda(s) = \lambda(t)\exp\left(-\int_t^s \psi_D(u)dB_D^1(u) - \int_t^s \psi_e(u)dB_e^1(u) - \int_t^s \left(\frac{1}{2}\left(\psi_D(u)\right)^2 + \frac{1}{2}\left(\psi_e(u)\right)^2\right)du\right). \tag{18}$$

Note that the Ito integrals are normal random variables because the subintegral function is deterministic. Therefore, the values of the difference in beliefs process at time s conditional of time t can be presented as

$$\lambda(s) = \lambda(t)\exp\left(M_\lambda(t,s) - \sqrt{V_\lambda(t,s)}Z_\lambda\right), Z_\lambda \sim N(0,1),$$

where the mean and variance are given by

$$\begin{aligned}
M_\lambda(t,s) &= -\frac{1}{2}M_{\lambda,e}(t,s) - \frac{1}{2}M_{\lambda,D}(t,s), \\
V_\lambda(t,s) &= V_{\lambda,e}(t,s) + V_{\lambda,D}(t,s),
\end{aligned}$$

and

$$\begin{aligned}
&M_{\lambda,D}(t,s) = V_{\lambda,D}(t,s) \\
&= \int_t^s \left(\psi_D(u)\right)^2 du = \left(\frac{h_D\left(m^1(0) - m^2(0)\right)}{\gamma_0}\right)^2 \int_t^s \left(\frac{1}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)u}\right)^2 du \\
&= \left(\frac{m^1(0) - m^2(0)}{\gamma_0}\right)^2 \frac{h_D^2}{\left(h_D^2 + h_e^2\right)} \int_t^s \left(\frac{1}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)u}\right)^2 d\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)u\right) \\
&= \left(\frac{m^1(0) - m^2(0)}{\gamma_0}\right)^2 \frac{h_D^2}{\left(h_D^2 + h_e^2\right)} \int_{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}^{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s} \frac{1}{v^2}dv
\end{aligned}$$

$$
\begin{aligned}
&= \left(\frac{m^1(0)-m^2(0)}{\gamma_0}\right)^2 \frac{h_D^2}{(h_D^2+h_e^2)} \left(-\frac{1}{v}\Big|_{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}^{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}\right) \\
&= \left(\frac{m^1(0)-m^2(0)}{\gamma_0}\right)^2 \frac{h_D^2}{(h_D^2+h_e^2)} \left(\frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,t} - \frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,s}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{h_D^2\left(m^1(0)-m^2(0)\right)^2}{\gamma_0^2\,(h_D^2+h_e^2)} \left(\frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,t} - \frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,s}\right) \\
&= \frac{h_D^2\left(m^1(0)-m^2(0)\right)^2(s-t)}{\gamma_0^2\left(\gamma_0^{-1}+h_D^2+h_e^2\right)^2 ts}
\end{aligned}
$$

$$
\begin{aligned}
& M_{\lambda,e}\,(t,s) = V_{\lambda,e}\,(t,s) \\
&= \int_t^s (\psi_e(u))^2\,du = \int_t^s \left(\frac{(m^1(0)-m^2(0))}{\gamma_0}\frac{h_e}{\gamma_0^{-1}+(h_D^2+h_e^2)\,u}\right)^2 du \\
&= \left(\frac{(m^1(0)-m^2(0))}{\gamma_0}\right)^2 \int_t^s \left(\frac{h_e}{\gamma_0^{-1}+(h_D^2+h_e^2)\,u}\right)^2 du \\
&= \left(\frac{(m^1(0)-m^2(0))}{\gamma_0}\right)^2 h_e^2 \int_t^s \left(\frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,u}\right)^2 du
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{(m^1(0)-m^2(0))}{\gamma_0}\right)^2 \frac{h_e^2}{(h_D^2+h_e^2)} \left(\frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,t} - \frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,s}\right) \\
&= \frac{h_e^2\left(m^1(0)-m^2(0)\right)^2}{\gamma_0^2\,(h_D^2+h_e^2)} \left(\frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,t} - \frac{1}{\gamma_0^{-1}+(h_D^2+h_e^2)\,s}\right) \\
&= \frac{h_e^2\left(m^1(0)-m^2(0)\right)^2}{\gamma_0^2\,(h_D^2+h_e^2)} \left(\frac{\gamma_0^{-1}+(h_D^2+h_e^2)\,s-\gamma_0^{-1}-(h_D^2+h_e^2)\,t}{\left(\gamma_0^{-1}+(h_D^2+h_e^2)\right)\left(\gamma_0^{-1}+(h_D^2+h_e^2)\right)ts}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{h_e^2 \left(m^1(0) - m^2(0)\right)^2}{\gamma_0^2 \left(h_D^2 + h_e^2\right)} \left(\frac{\left(h_D^2 + h_e^2\right)(s-t)}{\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)\right)\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)\right) ts} \right) \\
&= \frac{h_e^2 \left(m^1(0) - m^2(0)\right)^2 (s-t)}{\gamma_0^2 \left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)\right)\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)\right) ts} = \frac{h_e^2 \left(m^1(0) - m^2(0)\right)^2 (s-t)}{\gamma_0^2 \left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right)\right)^2 ts} \\
&= \frac{h_e^2 \left(m^1(0) - m^2(0)\right)^2 (s-t)}{\left(1 + \gamma_0 \left(h_D^2 + h_e^2\right)\right)^2 ts} = \frac{h_e^2 \left(m^1(0) - m^2(0)\right)^2 (s-t)}{\gamma_0^2 \left(\gamma_0^{-1} + h_D^2 + h_e^2\right)^2 ts}.
\end{aligned}
$$

Marginal Distribution of $D(s)$

The dynamics of the dividend process $D(s)$ under the perceptions of the first agent is given by

$$
d \ln D_t = m_\theta^1 dt + \sigma_D dB_D^1(t),
$$

Since

$$
\begin{aligned}
dm^i(t) &= \gamma(t) \left(h_D^2 d \ln D_t + h_e^2 de_t\right) - \gamma(t) \left(h_D^2 + h_e^2\right) m^i(t) dt \\
&= \frac{h_D^2 d \ln D_t + h_e^2 de_t}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} - \frac{\left(h_D^2 + h_e^2\right) m^i(t) dt}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} \\
&= \frac{h_D^2 \left(m_\theta^i dt + \sigma_D dB_D^i(t)\right) + h_e^2 \left(m_\theta^i dt + \sigma_e dB_e^i(t)\right)}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} - \frac{\left(h_D^2 + h_e^2\right) m^i(t) dt}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} \\
&= \frac{\left(h_D^2 + h_e^2\right) m_\theta^i dt + h_D dB_D^i(t) + h_e dB_e^i(t)}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} - \frac{\left(h_D^2 + h_e^2\right) m^i(t) dt}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} \\
&= \frac{\left(h_D^2 + h_e^2\right) m_\theta^i dt + h_D dB_D^i(t) + h_e dB_e^i(t) - \left(h_D^2 + h_e^2\right) m^i(t) dt}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t} \\
&= \frac{h_D dB_D^i(t) + h_e dB_e^i(t)}{\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) t}
\end{aligned}
$$

hence

$$
dm_\theta^1 = \gamma(t) h_D dB_D^1 + \gamma(t) h_e dB_e^1.
$$

so

$$
m_\theta^1(t) = m_\theta^1(0) + \gamma(t) h_D B_D^1(t) + \gamma(t) h_e B_e^1(t).
$$

Thus,

$$d\ln D_t = m_\theta^1 dt + \sigma_D dB_D^1(t) = \left(m_\theta^1(0) + \gamma(t) h_D B_D^1(t) + \gamma(t) h_e B_e^1(t)\right) dt + \sigma_D dB_D^1(t),$$

therefore, the dynamics of the log-dividend process becomes

$$\log D(s) = \log D(t) + m_\theta^1(0)(s-t) + \int_t^s \gamma(u) h_D B_D^1(u) du + \int_t^s \gamma(u) h_e B_e^1(u) du + \sigma_D B_D^1(s-t). \tag{19}$$

Since

$$E\left(\int_t^s \gamma(u) h_D B_D^1(u) du + \int_t^s \gamma(u) h_e B_e^1(u) du\right) = 0,$$

and we have

$$E\left(\int_t^s \gamma(u) h_D B_D^1(u) du\right)^2 = \int_t^s \gamma^2(u) h_D^2 E\left[B_D^1(u)\right]^2 du = h_D^2 \int_t^s \gamma^2(u) E\left[B_D^1(u)\right]^2 du$$

since $B_D^1(u) \sim N(0,u) = \sqrt{u} N(0,1)$, we have

$$\begin{aligned} E\left(\int_t^s \gamma(u) h_D B_D^1(u) du\right)^2 &= h_D^2 \int_t^s \gamma^2(u) u du = h_D^2 \int_t^s \left(\frac{1}{\gamma_0^{-1} + (h_D^2 + h_e^2) u}\right)^2 u du \\ &= \frac{h_D^2}{2} \int_t^s \frac{1}{\left(\gamma_0^{-1} + (h_D^2 + h_e^2) u\right)^2} du^2, \end{aligned}$$

note

$$\begin{aligned} d\left(\gamma_0^{-1} + \left(h_D^2 + h_e^2\right) u\right)^2 &= d\left(\gamma_0^{-2} + \left(h_D^2 + h_e^2\right)^2 u^2 + 2\gamma_0^{-1}\left(h_D^2 + h_e^2\right) u\right) \\ &= \left(h_D^2 + h_e^2\right)^2 du^2 + 2\gamma_0^{-1}\left(h_D^2 + h_e^2\right) du, \end{aligned}$$

and

$$du^2 = \frac{d\left(\gamma_0^{-1} + (h_D^2 + h_e^2) u\right)^2 - 2\gamma_0^{-1}(h_D^2 + h_e^2) du}{\left(h_D^2 + h_e^2\right)^2}.$$

Hence, we have

$$
\begin{aligned}
& \frac{h_D^2}{2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}du^2 \\
= \;& \frac{h_D^2}{2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}\frac{d\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2-2\gamma_0^{-1}\left(h_D^2+h_e^2\right)du}{\left(h_D^2+h_e^2\right)^2} \\
= \;& \frac{h_D^2}{2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}\frac{d\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}{\left(h_D^2+h_e^2\right)^2} \\
& -\frac{h_D^2}{2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}\frac{2\gamma_0^{-1}\left(h_D^2+h_e^2\right)du}{\left(h_D^2+h_e^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
= \;& \frac{h_D^2}{2\left(h_D^2+h_e^2\right)^2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}d\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2 \\
& -\frac{\gamma_0^{-1}h_D^2}{\left(h_D^2+h_e^2\right)^2}\int_t^s \frac{1}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right)^2}d\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)u\right) \\
= \;& \frac{h_D^2}{2\left(h_D^2+h_e^2\right)^2}\int_{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t\right)^2}^{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s\right)^2}\frac{1}{v}dv-\frac{\gamma_0^{-1}h_D^2}{\left(h_D^2+h_e^2\right)^2}\int_{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}^{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}\frac{1}{v^2}dv
\end{aligned}
$$

$$
\begin{aligned}
= \;& \frac{h_D^2}{2\left(h_D^2+h_e^2\right)^2}\ln v\,\Bigg|_{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t\right)^2}^{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s\right)^2}+\frac{\gamma_0^{-1}h_D^2}{\left(h_D^2+h_e^2\right)^2}\frac{1}{v}\,\Bigg|_{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}^{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s} \\
= \;& \frac{h_D^2}{2\left(h_D^2+h_e^2\right)^2}\ln\frac{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s\right)^2}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t\right)^2} \\
& +\frac{\gamma_0^{-1}h_D^2}{\left(h_D^2+h_e^2\right)^2}\left(\frac{1}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}-\frac{1}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \;& \frac{h_D^2}{\left(h_D^2+h_e^2\right)^2}\ln\frac{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}+\frac{\gamma_0^{-1}h_D^2}{\left(h_D^2+h_e^2\right)^2}\frac{\left(h_D^2+h_e^2\right)(t-s)}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)\right)^2 ts} \\
= \;& \frac{h_D^2}{\left(h_D^2+h_e^2\right)^2}\ln\frac{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}+\frac{\gamma_0^{-1}h_D^2}{h_D^2+h_e^2}\frac{t-s}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)\right)^2 ts}.
\end{aligned}
$$

Similarly, we have

$$E\left(\int_t^s \gamma(u)\,h_e B_e^1(u)\,du\right)^2 = \frac{h_e^2}{\left(h_D^2+h_e^2\right)^2}\ln\frac{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t}+\frac{\gamma_0^{-1}h_e^2}{h_D^2+h_e^2}\frac{t-s}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)\right)^2 ts}.$$

Hence the distribution of $\log(D(s))$ is normal with

$$\begin{aligned}
M_D(t,s) &= \log D(t) + m_\theta^1(0)(s-t),\\
V_D(t,s) &= \sigma_D^2(s-t) + \frac{1}{h_D^2+h_e^2}\ln\frac{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)s}{\gamma_0^{-1}+\left(h_D^2+h_e^2\right)t} + \frac{\gamma_0^{-1}(t-s)}{\left(\gamma_0^{-1}+\left(h_D^2+h_e^2\right)\right)^2 ts}.
\end{aligned}$$

And we have

$$D(s) = D(t)\exp\left(M_D(t,s) - \sqrt{V_D(t,s)}Z_D\right), Z_D \sim N(0,1).$$

A.5. Stock Return Volatility and Asset Prices Sensitivity to $D(t)$ and $\lambda(t)$

Since the stock price is a function of the state variables $D(t)$ and $\lambda(t)$, applying Ito's Lemma yields

$$dP(t) - E\left[dP\,\middle|\mathcal{F}_t^i\right] = \left(\frac{\partial P}{\partial D}\sigma_D D(t) - \frac{\partial P}{\partial \lambda}\psi_D(h_e)\lambda(t)\right)dB_D^1(t) - \frac{\partial P}{\partial \lambda}\psi_e(h_e)\lambda(t)\,dB_e^1(t). \tag{20}$$

stock return volatility is equal to

$$\sigma^2\left(\frac{dP(t)}{P(t)}\right) = \left(\frac{\partial P}{\partial D}D(t)\sigma_D - \frac{\partial P}{\partial \lambda}\psi_D(h_e)\lambda(t)\right)^2 + \left(\frac{\partial P}{\partial \lambda}\psi_e(h_e)\lambda(t)\right)^2.$$

The stock price sensitivities with respect to $D(t)$ and $\lambda(t)$ are computed from the results of Proposition 3. Let $\zeta(t,s,\lambda(t)) = \frac{\xi^1(s)}{\xi^1(t)}$, hence $P(t) = E_t^1\int_t^\infty \left(\zeta(t,s,\lambda(t))\,D(s)ds\right)$, from Fubini's Theorem, the derivative of the stock price $P(t)$ with respect to $\eta(t)$ is

$$\frac{\partial P\left(t\right)}{\partial \lambda\left(t\right)}=E_{t}^{1}\int_{t}^{\infty}\frac{\partial \zeta\left(t,s,\lambda(t)\right)}{\partial \lambda(t)}D(s)ds$$

Similarly, the derivative of the stock price with respect to $D(t)$ is

$$\begin{aligned}\frac{\partial P\left(t\right)}{\partial D\left(t\right)} &= E_{t}^{1}\int_{t}^{\infty}\zeta\left(t,s,\lambda(t)\right)e^{M_{D}(t,s)-\sqrt{V_{D}(t,s)}Z_{D}}ds \\ &= E_{t}^{1}\int_{t}^{\infty}\zeta\left(t,s,\lambda(t)\right)\frac{D(t)}{D(t)}e^{M_{D}(t,s)-\sqrt{V_{D}(t,s)}Z_{D}}ds=E_{t}^{1}\int_{t}^{\infty}\zeta\left(t,s,\lambda(t)\right)\frac{D(s)}{D(t)}ds,\end{aligned}$$

Monte Carlo Simulation can be employed to calculate the above expectation: First, we simulate two paths of two independent Brownian motions. Second, calculate the value of $D(s)$, and $\lambda(s)$ for $s>t$ according to (19), (18) and the definition of stochastic integration. Third, calculate the value of the integrand in $\partial P\left(t\right)/\partial \lambda\left(t\right)$ and, thus the value the integration. Fourth, simulate n times of the paths of two independent Brownian motions and obtain n difference realizations of the value the integrations, and the expectation equals to the sum of n integrations divided by n. Note all the following related expectation can be calculated according to this algorithm.

The expression of $\frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda(t)}$ can be derived as follows. Note

$$\zeta\left(t,s,\lambda(t)\right)=\frac{\xi^{1}\left(s\right)}{\xi^{1}\left(t\right)}=\frac{\frac{1}{y_{1}}\left(D(s)\right)^{\gamma-1}\lambda_{s}(1+\lambda_{s}^{\frac{1}{\gamma-1}})^{1-\gamma}}{\frac{1}{y_{1}}\left(D(t)\right)^{\gamma-1}\lambda_{t}(1+\lambda_{t}^{\frac{1}{\gamma-1}})^{1-\gamma}}=\frac{\left(D(s)\right)^{\gamma-1}\lambda_{s}(1+\lambda_{s}^{\frac{1}{\gamma-1}})^{1-\gamma}}{\left(D(t)\right)^{\gamma-1}\lambda_{t}(1+\lambda_{t}^{\frac{1}{\gamma-1}})^{1-\gamma}},$$

and note

$$\frac{\partial\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\right)^{1-\gamma}}{\partial\lambda} = (1-\gamma)\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\right)^{-\gamma}\times$$

$$\left(\frac{1}{\gamma-1}\frac{\lambda_t^{\frac{1}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}-\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2}\frac{1}{\gamma-1}\lambda(t)^{\frac{1}{\gamma-1}-1}\right)$$

$$\begin{aligned}
= &\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\right)^{-\gamma}\times \\
&\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2}\lambda(t)^{\frac{1}{\gamma-1}-1}-\frac{\lambda_t^{\frac{1}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\right) \\
= &\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2\right)^{-\gamma}\times \\
&\left(\left(1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)\lambda(t)^{\frac{1}{\gamma-1}-1}-\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)\lambda_t^{\frac{1}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)
\end{aligned}$$

$$\begin{aligned}
= &\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2\right)^{-\gamma}\times \\
&\left(\begin{array}{c}\lambda_t^{\frac{1}{\gamma-1}-1}+\lambda_t^{\frac{2}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}-\lambda_t^{\frac{1}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)} \\ -\lambda_t^{\frac{2}{\gamma-1}-1}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\end{array}\right) \\
= &\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2\right)^{-\gamma}\lambda_t^{\frac{1}{\gamma-1}-1}\left(1-e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)
\end{aligned}$$

so that

$$\begin{aligned}\frac{\partial}{\partial\lambda(t)}\zeta\left(t,s,\lambda(t)\right) &= e^{M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda}\left(e^{M_D(t,s)-\sqrt{V_D(t,s)}Z_D}\right)^{\gamma-1} \\ &\times\lambda_t^{\frac{1}{\gamma-1}-1}\left(1-e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right) \\ &\times\left(\frac{1+\lambda_t^{\frac{1}{\gamma-1}}e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1+\lambda(t)^{\frac{1}{\gamma-1}}}\left(1+\lambda(t)^{\frac{1}{\gamma-1}}\right)^2\right)^{-\gamma}.\end{aligned}$$

A.6. Derivation of Market Prices of Risk

The diffusion process for the sum of individual consumptions should be identical to the diffusion process for the dividend. Hence, the following restrictions follow. First,

$$-\frac{c_1\left(t\right)}{\gamma-1}\phi_D^1\left(t\right)-\frac{c_2\left(t\right)}{\gamma-1}\phi_D^2\left(t\right)=\sigma_D D\left(t\right).$$

From $\phi_D^1(t)-\phi_D^2(t)=\psi_D\left(h_e\right)$, the price of dividend risk is equal to

$$\phi_D^1(t)=\left[\frac{c_1\left(t\right)}{\gamma-1}+\frac{c_2\left(t\right)}{\gamma-1}\right]^{-1}\left[\sigma_D D\left(t\right)+\frac{\psi_D\left(h_e\right)}{1-\gamma}\right].$$

Simplifying the terms and using the market clearing condition, we obtain the price of the dividend risk for each agent as

$$\phi_D^1(t)=\left(1-\gamma\right)\sigma_D+\psi_D\left(h_e\right)\frac{c_2\left(t\right)}{D\left(t\right)},\qquad\phi_D^2(t)=\left(1-\gamma\right)\sigma_D+\psi_D\left(h_e\right)\frac{c_1\left(t\right)}{D\left(t\right)}.$$

Substituting the solution for the individual consumptions, we obtain the solution for the dividend price of risk

$$\phi_D^1(t)=\left(1-\gamma\right)\sigma_D+\frac{\psi_D\left(h_e\right)}{1+\lambda_t^{\frac{1}{\gamma-1}}},\quad\phi_D^2(t)=\left(1-\gamma\right)\sigma_D-\frac{\psi_D\left(h_e\right)\lambda_t^{\frac{1}{\gamma-1}}}{1+\lambda_t^{\frac{1}{\gamma-1}}}.$$

Second, for the individual price of signal risk the restriction is

$$\frac{c_1(t)}{1-\gamma}\phi_e^1(t) + \frac{c_2(t)}{1-\gamma}\phi_e^2(t) = 0.$$

Because the difference of the individual prices of signal risk is equal to $\phi_e^1(t) - \phi_e^2(t) = \psi_e(h_e)$, the prices of signal risk are equal to

$$\phi_e^1(t) = \psi_D(h_e)\frac{c_2(t)}{D(t)}, \qquad \phi_e^2(t) = -\psi_D(h_e)\frac{c_1(t)}{D(t)}.$$

Substituting the solution for individual consumptions into the equation above, we obtain the solution for the signal price of risk

$$\phi_e^1(t) = \frac{\psi_e(h_e)}{1+\lambda_t^{\frac{1}{\gamma-1}}}, \quad \text{and} \quad \phi_e^2(t) = -\frac{\psi_e(h_e)\lambda_t^{\frac{1}{\gamma-1}}}{1+\lambda_t^{\frac{1}{\gamma-1}}}.$$

A.7. Derivation of Trading Volume

The dynamic budget constraint satisfies the stochastic differential equation

$$dX^1(t) = -c_1(t)\,dt + X^1(t)\,r(t)\,dt + \pi_1(t)\left(\mu^1(t) - r(t)\right)dt + \pi_1(t)\,\sigma(t)\begin{bmatrix} dB_D^1(t) \\ dB_e^1(t) \end{bmatrix}, \tag{21}$$

where $\pi_1(t)$ is the amount of wealth invested in stock. Given the equilibrium stochastic discount factor, at any time t, the wealth level must also satisfy the static budget constraint

$$X^1(t) = \frac{1}{\xi^1(t)}E^1\left[\int_t^\infty c_1(s)\,\xi^1(s)\,ds \,\middle|\, \mathcal{F}_t^i\right].$$

Since $c_1(t)$ and $\xi^1(t)$ are functions of the state variables $D(t)$ and $\lambda(t)$, Ito's Lemma applied to the static budget constraint yields

$$dX^1(t) - E\left[dX^1(t)\,\middle|\,\mathcal{F}_t^i\right] = \left(\frac{\partial X^1}{\partial D}\sigma_D D(t) - \frac{\partial X^1}{\partial \lambda}\psi_D(h_e)\,\lambda(t)\right)dB_D^1(t) - \frac{\partial X^1}{\partial \lambda}\psi_e(h_e)\,\lambda(t)\,dB_e^1(t). \tag{22}$$

The stochastic differential equations (21) and (22) are Ito representations of the same wealth process with respect to the same vector of Brownian motions. Thus, the factor loading on the Brownian motions must be identical in any state of the world. This yields the following system of two equations with two unknowns:

$$\pi_1(t)^\intercal \sigma(t) = \left[\frac{\partial X^1}{\partial D}\sigma_D D(t) - \frac{\partial X^1}{\partial \lambda}\psi_D(h_e)\lambda(t), -\frac{\partial X^1}{\partial \lambda}\psi_e(h_e)\lambda(t)\right],$$

where

$$\sigma(t) = \left[\begin{array}{c} \frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial \lambda}\psi_D(h_e)\lambda(t), -\frac{\partial P(t)}{\partial \lambda}\psi_e(h_e)\lambda(t) \\ \frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial O(t)}{\partial \lambda}\psi_D(h_e)\lambda(t), -\frac{\partial O(t)}{\partial \lambda}\psi_e(h_e)\lambda(t) \end{array}\right],$$

where $O(t)$ is the price of the consel bond. However, as we can see later, it will not have an impact on the trading volume.

Solving for the portfolio holding, we obtain the first agent's optimal position as

$$\pi_1(t) = \left(\sigma(t)^\intercal\right)^{-1}\left[\begin{array}{c} \frac{\partial X^1}{\partial D}\sigma_D D(t) - \frac{\partial X^1}{\partial \lambda}\psi_D(h_e)\lambda(t) \\ -\frac{\partial X^1}{\partial \lambda}\psi_e(h_e)\lambda(t) \end{array}\right],$$

where

$$\begin{aligned}
\sigma(t)^{-1} &= \Pi^{-1}\left[\begin{array}{c} -\frac{\partial O(t)}{\partial \lambda}\psi_e(h_e)\lambda(t), \frac{\partial P(t)}{\partial \lambda}\psi_e(h_e)\lambda(t) \\ \frac{\partial O(t)}{\partial \lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t), \frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial \lambda}\psi_D(h_e)\lambda(t) \end{array}\right], \\
\left(\sigma(t)^\intercal\right)^{-1} &= \left(\sigma(t)^{-1}\right)^\intercal = \Pi^{-1}\left[\begin{array}{c} -\frac{\partial O(t)}{\partial \lambda}\psi_e(h_e)\lambda(t), \frac{\partial O(t)}{\partial \lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t) \\ \frac{\partial P(t)}{\partial \lambda}\psi_e(h_e)\lambda(t), \frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial \lambda}\psi_D(h_e)\lambda(t) \end{array}\right],
\end{aligned}$$

and

$$\begin{aligned}
\Pi &= \frac{\partial P(t)}{\partial \lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial O(t)}{\partial \lambda}\psi_D(h_e)\lambda(t)\right) \\
&\quad -\frac{\partial O(t)}{\partial \lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial \lambda}\psi_D(h_e)\lambda(t)\right).
\end{aligned}$$

Hence, the dynamic of stock holding is given as

$$\pi_{11}(t) = \frac{\left(\begin{array}{c} -\frac{\partial O(t)}{\partial\lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial X^1}{\partial D}\sigma_D D(t) - \frac{\partial X^1}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) \\ -\frac{\partial X^1}{\partial\lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t)\right) \end{array}\right)}{\left(\begin{array}{c} \frac{\partial P(t)}{\partial\lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) \\ -\frac{\partial O(t)}{\partial\lambda}\psi_e(h_e)\lambda(t)\left(\frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) \end{array}\right)}$$

$$\begin{aligned}
&= \frac{-\frac{\partial O(t)}{\partial\lambda}\left(\frac{\partial X^1}{\partial D}\sigma_D D(t) - \frac{\partial X^1}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) - \frac{\partial X^1}{\partial\lambda}\left(\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t)\right)}{\frac{\partial P(t)}{\partial\lambda}\left(\frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) - \frac{\partial O(t)}{\partial\lambda}\left(\frac{\partial P(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial\lambda}\psi_D(h_e)\lambda(t)\right)} \\
&= \frac{\left(-\frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial D}\sigma_D D(t) + \frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial\lambda}\psi_D(h_e)\lambda(t)\right) - \frac{\partial X^1}{\partial\lambda}\left(\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t)\right)}{\frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial D}\sigma_D D(t) + \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial\lambda}\psi_D(h_e)\lambda(t)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{-\frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial D}\sigma_D D(t) + \frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial X^1}{\partial\lambda}\left(\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial O(t)}{\partial D}\sigma_D D(t)\right)}{\frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial D}\sigma_D D(t) - \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial D}\sigma_D D(t)} \\
&= \frac{-\frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial D}\sigma_D D(t) + \frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial\lambda}\psi_D(h_e)\lambda(t) - \frac{\partial X^1}{\partial\lambda}\frac{\partial O(t)}{\partial\lambda}\psi_D(h_e)\lambda(t) + \frac{\partial X^1}{\partial\lambda}\frac{\partial O(t)}{\partial D}\sigma_D D(t)}{\left(\frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial D} - \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial D}\right)\sigma_D D(t)} \\
&= \frac{-\frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial D}\sigma_D D(t) + \frac{\partial X^1}{\partial\lambda}\frac{\partial O(t)}{\partial D}\sigma_D D(t)}{\left(\frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial D} - \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial D}\right)\sigma_D D(t)} = \frac{\frac{\partial X^1}{\partial\lambda}\frac{\partial O(t)}{\partial D} - \frac{\partial O(t)}{\partial\lambda}\frac{\partial X^1}{\partial D}}{\frac{\partial P(t)}{\partial\lambda}\frac{\partial O(t)}{\partial D} - \frac{\partial O(t)}{\partial\lambda}\frac{\partial P(t)}{\partial D}} = \frac{\frac{\partial X^1}{\partial D}}{\frac{\partial P(t)}{\partial D}}
\end{aligned}$$

Hence, the volatility of the portfolio in stock, π_{11}, is given as

$$\begin{aligned}
\sigma^2_{\pi_{11}} &= \left(\frac{\partial\pi_{11}}{\partial D}\sigma_D D(t) - \frac{\partial\pi_{11}}{\partial\lambda}\psi_D(h_e)\lambda(t)\right)^2 + \left(\frac{\partial\pi_{11}}{\partial\lambda}\psi_e(h_e)\lambda(t)\right)^2, \\
&= \left(\frac{\partial\pi_{11}}{\partial\lambda}\psi_D(h_e)\lambda(t)\right)^2 + \left(\frac{\partial\pi_{11}}{\partial\lambda}\psi_e(h_e)\lambda(t)\right)^2 \\
&= \psi_D(h_e)^2\left(\frac{\partial\pi_{11}}{\partial\lambda}\lambda(t)\right)^2 + \psi_e(h_e)^2\left(\frac{\partial\pi_{11}}{\partial\lambda}\lambda(t)\right)^2 \\
&= \left(\psi_D(h_e)^2 + \psi_e(h_e)^2\right)\left(\frac{\partial\pi_{11}}{\partial\lambda}\lambda(t)\right)^2
\end{aligned}$$

where the expression of $\frac{\partial \pi_{11}}{\partial D}$ and $\frac{\partial \pi_{11}}{\partial \lambda}$ will be derived as follows.

Wealth dynamics sensitivity to $D(t)$ and $\lambda(t)$

Because $X(t) = E_t^1 \int_t^\infty (\zeta(t,s,\lambda(t))\, c(s) ds)$ and by Fubini's Theorem, the derivative of the wealth process $X(t)$ with respect to $\lambda(t)$ is

$$
\begin{aligned}
\frac{\partial X(t)}{\partial \lambda(t)} &= E_t^1 \int_t^\infty \frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda(t)} \frac{\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)} D(s)}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} ds \\
&\quad + E_t^1 \int_t^\infty \left(\zeta(t,s,\lambda(t)) \left(\begin{array}{c} \frac{e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} \\ -\frac{\lambda(t) e^{\frac{2}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{\left(1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)^2} \end{array} \right) \right) D(s)\, ds
\end{aligned}
$$

$$
\begin{aligned}
&= E_t^1 \int_t^\infty \frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda(t)} \frac{\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)} D(s)}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} ds \\
&\quad + E_t^1 \int_t^\infty \frac{\zeta(t,s,\lambda(t)) \left(\begin{array}{c} e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)} \left(1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right) \\ -\lambda(t) e^{\frac{2}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)} \end{array} \right)}{\left(1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)^2} D(s)\, ds
\end{aligned}
$$

$$
\begin{aligned}
&= E_t^1 \int_t^\infty \left(\frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda(t)} \frac{\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} \right) D(s)\, ds \\
&\quad + E_t^1 \int_t^\infty \left(\frac{\zeta(t,s,\lambda(t)) (1-2\lambda(t))\, e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{\left(1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)^2} \right) D(s)\, ds
\end{aligned}
$$

Similarly, the derivative of the wealth process with respect to $D(t)$ is

$$\frac{\partial X(t)}{\partial D(t)} = E_t^1 \int_t^\infty \zeta(t,s,\lambda(t)) \frac{\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} e^{M_D(t,s)-\sqrt{V_D(t,s)}Z_D} ds$$

Moreover, note $\partial \frac{\partial X^1}{\partial D}/\partial D = \partial \frac{\partial P}{\partial D}/\partial D = 0$, hence

$$\frac{\partial \pi_{11}}{\partial D} = \frac{\partial \frac{\partial X^1}{\partial D}/\partial D}{\frac{\partial P(t)}{\partial D}} - \frac{\partial X^1}{\partial D}\left(\frac{1}{\frac{\partial P(t)}{\partial D}}\right)^2 \left(\partial \frac{\partial P}{\partial D}/\partial D\right) = 0,$$

Now we turn to $\frac{\partial \pi_{11}}{\partial \lambda}$. Note

$$\frac{\partial \pi_{11}}{\partial \lambda} = \frac{\partial \frac{\partial X^1}{\partial D}/\partial \lambda}{\frac{\partial P(t)}{\partial D}} - \frac{\partial X^1}{\partial D} \frac{1}{\left(\frac{\partial P(t)}{\partial D}\right)^2} \left(\partial \frac{\partial P}{\partial D}/\partial \lambda\right),$$

and

$$\begin{aligned} \partial \frac{\partial X^1}{\partial D}/\partial \lambda &= E_t^1 \int_t^\infty \frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda} \frac{\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}} e^{M_D(t,s)-\sqrt{V_D(t,s)}Z_D} ds \\ &\quad + E_t^1 \int_t^\infty \frac{\zeta(t,s,\lambda(t))(1-2\lambda(t)) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}}{\left(1-\lambda(t) e^{\frac{1}{\gamma-1}\left(M_\lambda(t,s)-\sqrt{V_\lambda(t,s)}Z_\lambda\right)}\right)^2} e^{M_D(t,s)-\sqrt{V_D(t,s)}Z_D} ds \end{aligned}$$

and

$$\begin{aligned} \partial \frac{\partial P}{\partial D}/\partial \lambda &= E_t^1 \int_t^\infty \frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda} e^{M_D(t,s)-\sqrt{V_D(t,s)}Z_D} ds \\ &= E_t^1 \int_t^\infty \frac{\partial \zeta(t,s,\lambda(t))}{\partial \lambda} \frac{D(s)}{D(t)} ds. \end{aligned}$$

Appendix B: Moment Conditions

In this appendix, we provide moment conditions for the GMM estimation that uses information in fundamentals like dividend growth rate, signal process, and a series of forecasters' dispersion about future macro economics from the surveys to estimate the parameter values.

The moment conditions that we fit jointly in the GMM procedure are as follows:

1. Difference in belief evolves according to

$$m^1(t) - m^2(t) = \frac{m_\theta^1(0) - m_\theta^2(0)}{\gamma_0}\gamma(t),$$

where $\gamma(t) = \frac{1}{\gamma_0^{-1} + (h_D^2 + h_e^2)t}$.

2. From the perspective of investor 1, the log dividend evolves as a function of the prior beliefs and signal precision

$$\begin{aligned} d\ln D_t &= m_\theta^1(t)dt + \sigma_D dB_D^1(t) \\ &= (m_\theta^1(0) + \gamma(t)B_D^1(t) + \gamma(t)h_e B_e^1(t))dt + \sigma_D dB_D^1(t). \end{aligned}$$

3. The observed signals evolves as a function of the prior beliefs and signal precision

$$\begin{aligned} de_t &= m_\theta^1(t)dt + \sigma_e dB_e^1(t) \\ &= (m_\theta^1(0) + \gamma(t)B_D^1(t) + \gamma(t)h_e B_e^1(t))dt + \sigma_e dB_e^1(t). \end{aligned}$$

4. From the perspective of investor 2, the log dividend evolves as a function of the prior beliefs and signal precision

$$\begin{aligned} d\ln D_t &= m_\theta^2(t)dt + \sigma_D dB_D^2(t) \\ &= (m_\theta^2(0) + \gamma(t)B_D^2(t) + \gamma(t)h_e B_e^2(t))dt + \sigma_D dB_D^2(t). \end{aligned}$$

5. The observed signals evolves as a function of the prior beliefs and signal precision

$$\begin{aligned} de_t &= m_\theta^2(t)dt + \sigma_e dB_e^2(t) \\ &= (m_\theta^2(0) + \gamma(t)B_D^2(t) + \gamma(t)h_e B_e^2(t))dt + \sigma_e dB_e^2(t). \end{aligned}$$

/ ADD MOMNET CONDITION FOR TRADING VOLUME HERE /

Use these conditions, we can form the GMM objective function and the apply the standard GMM estimation procedure.

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Table 1: The Estimates of Threshold Regressions

	equity premium		volatility		trading volume	
	$i=0$	$i=1$	$i=0$	$i=1$	$i=0$	$i=1$
$\beta_{i,DY}$	0.076 (1.78)	0.039 (2.05)	–	–	–	–
$\beta_{i,TBL}$	−0.058 (−1.39)	−0.493 (−2.23)	–	–	–	–
$\beta_{i,CAY}$	−0.113 (−0.89)	0.871 (3.74)	–	–	–	–
$\beta_{i,TV}$	–	–	–	–	1.004 (195.4)	0.973 (70.2)
$\beta_{i,TS}$	–	–	–	–	0.002 (0.79)	−0.003 (0.65)
$\beta_{i,SR}$	–	–	−0.008 (−0.20)	−0.006 (−1.21)	0.156 (1.17)	−0.001 (−0.02)
$\beta_{i,Vol}$	–	–	0.447 (1.29)	0.062 (2.87)	–	–
$\gamma_{i,1}$	3.72 (1.88)	−0.247 (−0.47)	0.231 (1.35)	−0.080 (−1.72)	−0.844 (−0.78)	−2.619 (−1.13)
$\gamma_{i,2}$	−2.241 (−1.97)	−3.430 (−2.43)	−0.656 (−0.70)	0.097 (0.29)	−4.05 (−0.42)	8.11 (0.93)

The table reports the estimation results of the threshold regressions using quarterly data. The threshold variable is signal precision. The depedent variables are respectively the equity premium, stock return volatility, and trading volume. The data sample is 1968:01-2013:04. t-values are reported in parentheses.

Table 2: Descriptive Statistics and Parameter Estimates

Panel A: Descriptive Statistics				
	Mean	Maximum	Minimum	Std. Err.
Signal Process	0.066	0.277	0.01	0.048
Stock Returns	0.0278	0.228	-0.252	0.084
Stock Return Volatility	0.185	0.586	0.079	0.073
Equity Premium	0.0148	0.214	-0.272	0.084
Trading Volume (in logs)	14.1	17.5	11.1	2.0
Dividend Growth Rate	0.015	0.067	-0.076	0.021
Panel B: GMM Estimates				
Parameter	$m^1(0)$	$m^2(0)$	γ_0	
Value	0.031^{**}	0.0041^{***}	0.14^{**}	
Std. Err.	0.012	0.0009	0.06	

This table reports the descriptive statistics in Panel A for the quarterly dividend growth rate, stock returns, signal process, riskfree rate, and trading volume for the period from 1968 : 04 to 2013 : 04. Panel B presents the GMM estimates for the parameters. *, **, *** represent the signifiance level at 10%; 5%; and 1%; respectively.

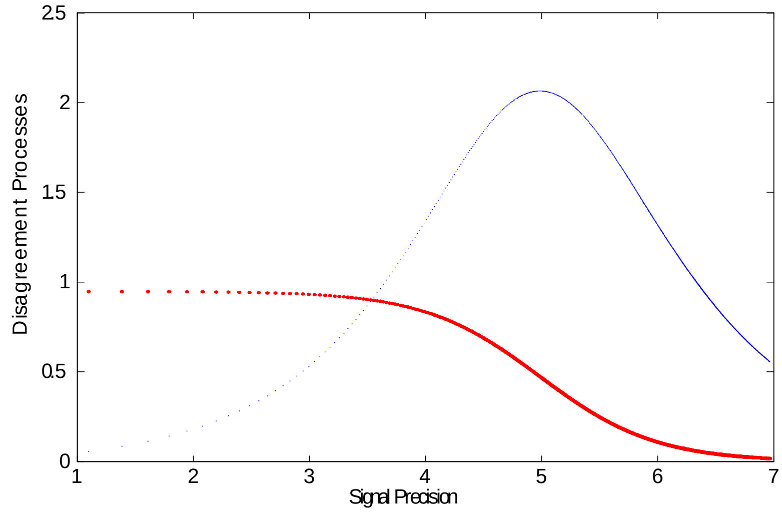

Figure 1: **Disagreement Processes.** The figure plots the two disagreement processes, ψ_D and ψ_e, as the functions of signal precision with σ_D= $0.2, m^1(0) = 0.5, m^2(0) = 0.1, t = 0.01$. The scale on the axis of the public signal precision is $x = \ln(1 + h_\varepsilon)$.

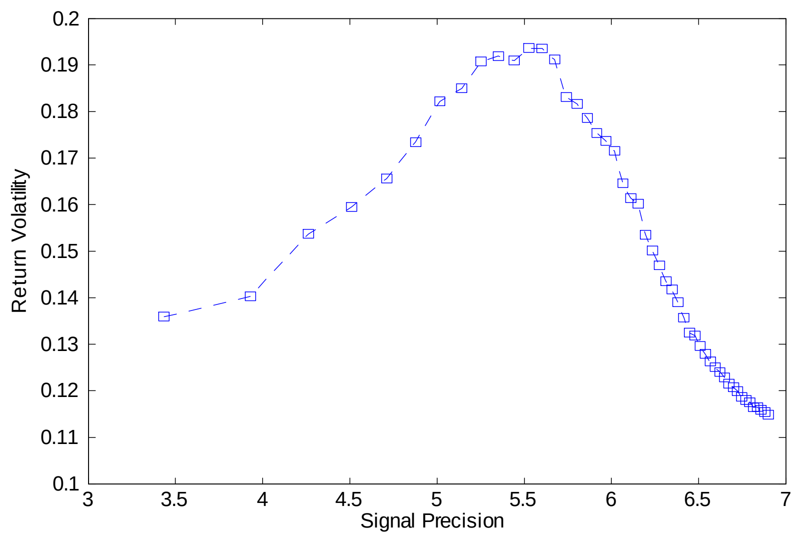

Figure 2: **The stock return volatility.** The figure plots the stock return volatility as a function of signal precision. The scale on the axis of the public signal precision is $x = \ln(1 + h_\varepsilon)$. The parameters used when plotting are as follow: $m^1(0) = 0.04; m^2(0) = 0.01; \gamma = 0.5; \sigma_D = 0.03; \gamma_0 = 0.1$. We use identical value of the parameters for plotting other asset pricing properties throughout this section, and do not repeat the parameter values again.

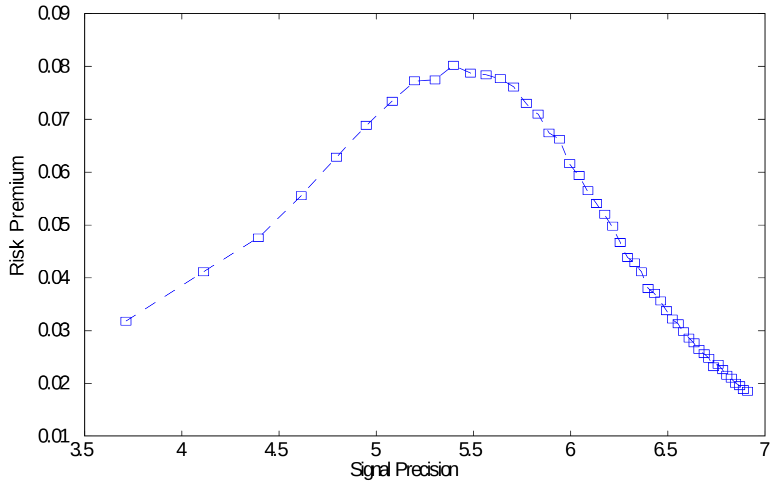

Figure 3: **The investor 1's conditional equity premium.** The figure plots agent 1's equity premium as a function of signal precision. The scale on the axis of the public signal precision is $x = \ln(1 + h_\varepsilon)$.

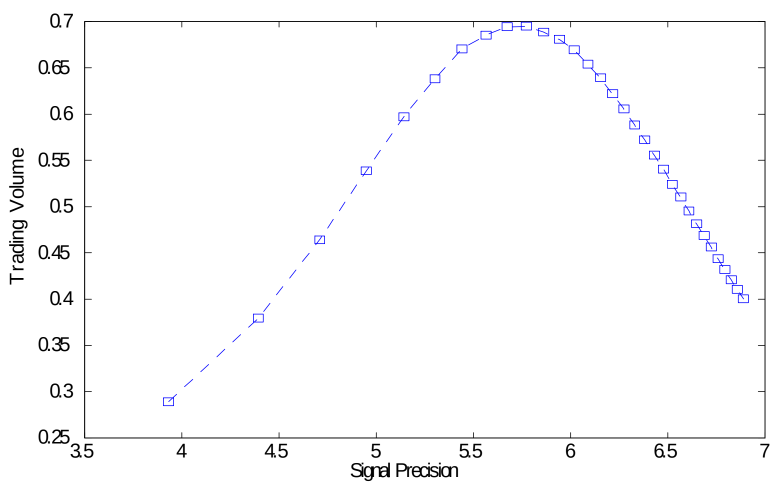

Figure 4: **The trading volume.** The figure plots trading volume as a function of signal precision. The scale on the axis of the public signal precision is $x = \ln(1 + h_\varepsilon)$.

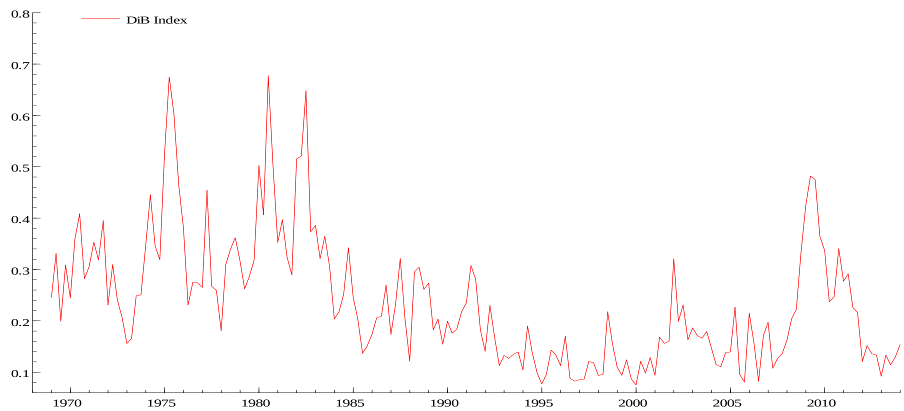

Figure 5: **The Difference-in-Belief Index.** This figure plots the time series of the Difference-in-Belief (DiB) index from 1968:04 to 2013:04. The DiB index is obtained by calculating the forecasters' dispersion on macroeconomic fundamental variables from Survey of Professional Forecasters available at the Federal Reserve Bank of Philadelphia.

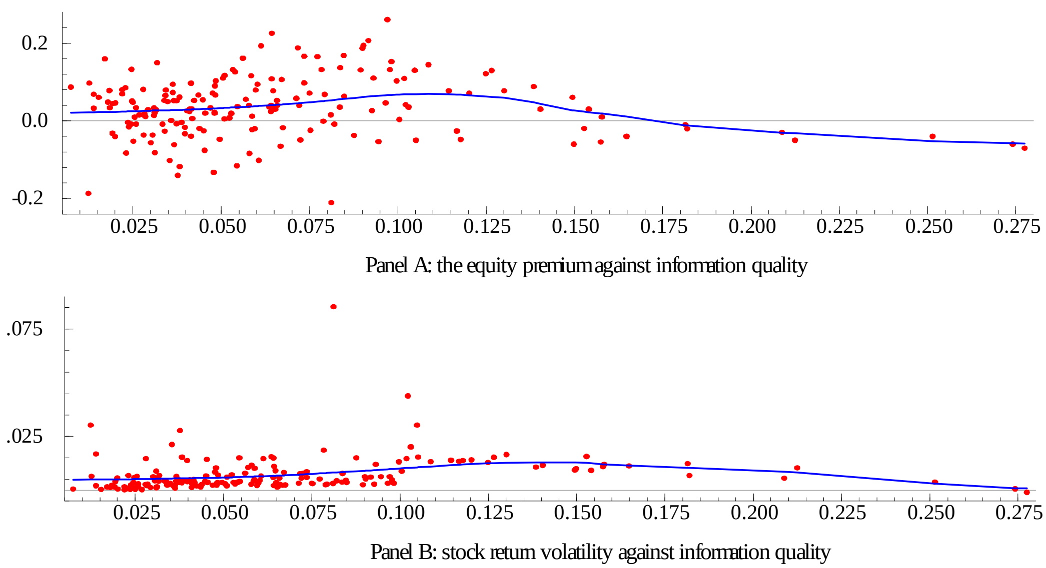

Figure 6: **The Bell-Shaped Functions**. The figure plots the threshold-regression-implied equity premium (Panel A) and trading volume (Panel B) against information quality (x-axis). The solid curves are a cubic spline implied by the data.