Long-Term Capital Budgeting Mechanism in Multidivision Firms*

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Abstract

We study the optimal dynamic mechanisms for capital budgeting and managerial compensation in a firm that consists of two divisions. Division managers privately observe the characteristics of their own project, which govern the evolution of the project types, and the project's subsequent profitability. At the same time, managers supply costly efforts to run the projects. The optimal mechanism incorporates the distortion driven by persistent private information and the production complementarity among the divisions. The headquarters should commit to investment policy that excludes or terminates projects, and controls the growth rate and volatility of project scales. With proportional distortions, the investment for good projects grows faster but is more volatile. In addition, we show that a linear contract implements the optimal investment and effort policy in an ex-post equilibrium, and the evolution of the power of incentives depends on the nature of initial private information. When a manager's private information generates decreasing distortion, her pay-performance sensitivity grows stronger over time, and the headquarters provides incentives through firm-level pay instead of divisional-level pay.

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1 Introduction

The success of corporations often requires investment in profitable long-term projects. Prominent examples include the development of iPhones and Windows. However, a firm's divisions, which are directly responsible for the operations of investment projects, typically possess better information about the profitability, quality, and characteristics of the projects at hand compared to their headquarters. For long-term projects, the managers' private information is likely to be persistent and has non-trivial impact on the firm's capital budgeting process. As shown in a recent work by Glaser, Lopez-de-Silanes, and Sautner (2013) who "open the black box" of a large conglomerate firm, capital budgeting process is not static. The firm carefully plans its capital allocations: from the identification of potential investment projects to their execution. The process repeats in each budgetary year, and managers who have superior knowledge about the firm's long-term projects have to report the status of the projects under their management.

In a dynamic economic environment, how should the headquarters allocate capital across divisions for investment in long-term projects? In what way the divisions' dynamic private information affects investment? How should the headquarters compensate and incentivize division managers to cooperate and invest efficiently in the long run? Certainly these questions are important. In this paper, we build a continuous-time dynamic mechanism design model to address these research questions. The answer we provide will guide the design of an efficient capital budgeting mechanism.

Our model consists of a firm with a single headquarters and two divisions. Each division is run by a division manager, who manages a long-term investment project. Both the headquarters (the principal) and the managers (the agents) are risk-neutral. Division managers have independent and private information regarding their own project type, which is Markovian in nature. In addition, the managers possess initial private information on a parameter that governs the future distributions of project types. Both divisional projects contribute to the firm's aggregate cash flows. However, project management requires both capital investments and managerial skills. The divisions have no access to external funding opportunities, therefore they rely on capital provided by the headquarters. On top of capital allocations, the managers can exert costly effort on their own project as well as the other's project. Thus, in our framework, the agency problem stems from both dynamic private information and moral hazard in teams.[1]

[1] Essentially, our dynamic model incorporates elements from Bernado, Cai, and Luo (2001, 2004) who analyze static capital budgeting problems with both asymmetric information and moral hazard using a mechanism design approach.

The objective of the headquarters is to find a mechanism that maximizes the firm's total expected discounted profits. Under the assumption of full commitment made by both the headquarters and the division managers, we study the optimal dynamic mechanism using tools developed recently by Pavan, Segal, and Toikka (2014) and Bergemann and Strack (2015). The mechanism specifies (1) performance-based compensations to both managers, (2) capital allocations to both divisions, and (3) recommended effort choices, including own effort and help effort.

We characterize the optimal allocation in closed-form. The optimal allocation incorporates distortions, which are summarized through a stochastic flow that captures the impact of initial private information on future project types. We also identify primitive conditions that guarantee the existence of an incentive contract that implements the optimal allocation. We then illustrate the dynamics of capital and incentives and the impact of the nature of initial private information through three examples of stochastic processes.

Specifically, we consider project types that follow geometric Brownian motions, arithmetic Brownian motions, or geometric Ornstein-Uhlenbeck process. These examples suggest that a number of features of our optimal capital budgeting mechanism depend on the nature of distortions.

First, the headquarters may commit to a project exclusion or termination policy when distortions are time-varying. In particular, when the impact of initial private information is increasing over time (increasing distortion), it is optimal for the headquarters to terminate the projects in a finite deterministic time or exclude the projects temporarily. On the opposite side, the headquarters commit to a slacker investment threshold over time with decreasing distortion. More importantly, bad projects (those with a lower initial profitability or lower growth rate) will be excluded for a longer period or get terminated earlier. Spillover effect implies the dependence of exclusion or termination policy on the quality of the other-division project: a bad project from another division can speed up termination of the own project. The rejection of positive NPV projects can be viewed as a way that headquarters economizes the information rents.

Second, the headquarters should control the dynamics of capital and incentives in respond to the persistent nature of initial private information. When time-t distortions are proportional to the current project types, the scale (capital) of bad projects grows slower and is less sensitive to shocks to project types.[2] Intuitively, the headquarters can provide incentives for the managers

[2]Distortions are proportional when the project types follow "geometric" processes: geometric Brownian motions or geometric Ornstein-Uhlenbeck processes. That is, when we model the relative increments of types. In contrast, when we model directly the increments of types, we obtain absolute distortions. In this case, initial private information may still affect the drift or volatility of the capital dynamics.

to truthfully reveal their initial private information by committing to such a dynamic investment policy. Consider a manager with a good project, given such a policy, she knows that once she deviates and reports a lower initial type, she will receive less capital, supply less effort, and obtain a lower wage over time with high chances. The dynamic investment policy provides a way to limit payoffs for a deviating high type.

Third, divisional investment increases with the past cash flows of own and the other division. As project cash flows are positively related to the project types, capital allocated to any division co-moves with the past cash flows of that division. Production complementarity also implies that investment in a division is positively related to the past quality of the project in the other division as well.

Forth, the performance-based compensation to any manager is a linear contract: it consists of a base salary and two incentive components: pay-for-own-division performance and pay-for-other-division performance. Hence the incentive contract takes the form of a joint performance evaluation. This is because both the managerial own effort and help effort are productive. To encourage cooperation and address the problem of moral hazard in teams, the headquarters incentivizes the managers by linking their compensation to the cash flows generated by the other division. Interestingly, in our time-separable and Markov environment, the linear contract implements the optimal allocation in a belief-free ex-post equilibrium. As a result, our characterization of dynamic mechanism is robust to belief specification.

Lastly, we also characterize the evolution of the power of incentives. In particular, when the impact of the initial private information of a manager vanishes over time, her incentive pays, both for her own division and the other division, increase over time. In our examples, this arises when the project types follow a geometric mean-reverting process with the initial value of the process being the manager's initial private information. To understand this result, note that decreasing distortion implies that the manager's ability to generate information rents from her initial knowledge about the project evolution diminishes in the distant future. As a result, the headquarters can relax spending limits and raise the investment to the efficient level over time. Complementarity of effort and capital implies more efforts to be supplied. To incentivize the managers, the performance-based compensations have to be raised as well.

The last two results carry significant implications on how the headquarters should provide effort incentives: as pay-performance sensitivity for the own division and the other division grows stronger over time, the firm relies more on firm-level pay instead of division-level pay. In contrast, in a firm in which cooperative effort is unimportant, division-level pay will dominate in the long-run with the same type of distortion. Overall, our results provide guidance on the optimal design of dynamic investment and compensation policy in environments where persistent private information is a non-trivial agency problem.

The paper makes three main contributions. First, we add to the capital budgeting literature by providing a tractable dynamic model with multiple divisions. Despite the importance of repeated interactions within a firm, the existing literature on capital budgeting mainly focuses on the static framework, with the exception of Malenko (2013), Fu (2015), and Roper and Ruckes (2012). In terms of the analytical scope, the first two papers study capital budgeting with i.i.d. projects and a single division, and the last paper works with a two-period model with financial constraints. Our model adds to this line of theoretical works by exploring a fully dynamic environment with multiple divisions managing long-term projects for which characteristics are related over time.

In addition, our model offers a number of novel implications that can be further tested against data. The implications are also useful in guiding the design of a profit-maximizing capital budgeting mechanism in practice. The implications on the dynamics of capital and incentives, and project exclusion and termination policy are new and have yet not been tested directly in the literature. Nevertheless, our predictions receive support from a number of empirical works that focus on capital allocation in internal capital market, including Mukherjee and Hingorani (1999), Graham, Harvey, and Puri (2015), and Alok and Gopalan (2014) etc. Our work provides guidance for further empirical investigations into conglomerate firms' capital budgeting process and their organization of internal capital market.

Finally our work makes contribution to the dynamic mechanism design literature. We introduce team production and variable project scales in Pavan, Segal, and Toikka (2014) and Bergemann and Strack (2015). We explore the applicability of their methodology and evaluate the role of stochastic flows in delivering the dynamic properties of optimal allocations in a multi-agent environment with both private information and moral hazard. Although the key focus of the paper is on capital budgeting, our analysis also shed lights on other areas in economics, for example, government regulation, procurement contract, or joint venture, where a dynamic mechanism design analysis is potentially applicable.

1.1 Related Literature

This paper is related to two lines of research. One is on internal capital market and capital budgeting in corporate finance and the other is on optimal dynamic mechanism design. Literature that studies how agency problems shape internal capital allocations was started by Harris, Kriebel and Raviv (1982) and Antle and Eppen (1985). They focus on the role of transfer prices in allocating capital across divisions. Harris and Raviv (1996, 1998) rationalize optimal budgeting procedures that involve initial spending limits which the division manager can relax subject to potential audit by the headquarters. Holmstrom and Ricard i Costa (1986) and Garcia (2013) study both capital allocation and performance-based incentive compensation. Using a model with managerial private information, Zhang (1997) explains why some firms voluntarily impose capital rationing.

In a recent paper, Almazan, Chen and Titman (2012) consider the “top-down” approach to capital budgeting. In their model, the headquarters possessing private information regarding the firm’s prospect over-invests to induce the division manager to take the most favorable action in the good state. In contrast, our model features the “bottom-up” procedures, where division managers request capital by reporting the prospects of their project.[3]

Most closely related to our paper are Bernado, Cai and Luo (2001, 2004, BCL hereafter). In a one-shot set up, BCL(2001) study the how interactions of asymmetric information and moral hazard in a firm with a single division affect internal capital allocation. They show that only sufficiently good projects will receive funding by the headquarters and characterize the optimal performance-based incentive scheme. BCL (2004) extend their earlier analysis to a firm with two divisions and find similar implications. We incorporate elements from BCL (2001, 2004) and study a dynamic environment where divisions invest in long-term projects of which qualities vary stochastically over time. Compared to their static analysis, the headquarters in our model can control and commit to future capital allocation and performance-based compensation in order to provide incentives for the managers to reveal their true project quality and to cooperate. Therefore, our model is able to make time-series predictions regarding the optimal capital budgeting policy in a changing world.

Models that involve repeated interaction often generate new insights and allow us to answer a wider set of questions. Nevertheless, concerning capital budgeting, dynamic analysis is scarce. To the

[3] Our paper only focuses on the internal capital allocation and we do not address the question regarding the benefits and costs of using internal capital market. See Gertner, Scharfstein, and Stein (1994), Stein (1997) and Scharfstein and Stein (2000). See also Stein (2003) for a survey.

best of our knowledge, Malenko (2013), Fu (2015), and Roper and Ruckes (2012) are the only works that are devoted to dynamic capital budgeting.

Malenko (2013) incorporates elements from Harris and Raviv (1996, 1998) and models, in continuous-time, the capital budgeting procedure between a headquarters, which possesses an audit technology, and a manager with empire-building preferences. His main result is that the optimal contract can be implemented by a "threshold budgeting mechanism" which is an investment account held by the division. In a continuous-time model with i.i.d. projects, Fu (2015) explores how a simple budgeting account can alleviate the problem of capital diversion by the manager. Our key differences with Malenko (2013) and Fu (2015) are that, first, we characterize the budgeting mechanism in firms with multiple divisions. And second, in their models, projects arrive stochastically over time and all projects are ex-ante identical. In other words, his firm faces *a sequence of unrelated projects*. Our focus is on investment in long-term projects or short-term multi-stage projects that are correlated over time. Overall, our works are complementary and provide a better picture of dynamic capital budgeting.

This paper builds on recent developments in the literature on dynamic mechanism design with the agent's private information evolving persistently over time. Pavan, Segal and Toikka (2014) develop the most general results on mechanism design problems with dynamic adverse selection.[4] They provide a characterization of incentive compatibility and show how impulse responses, which capture the persistence of private information, enter the information rent equation. Bergemann and Strack (2015) provide a continuous-time mechanism design analysis. Our solution method levers on them by introducing moral hazard problems. In the environment with persistent private information, Garret and Pavan (2012) characterize the optimal design of managerial turnover policy and show that a firm's optimal retention decision will become more permissive over time. Garrett and Pavan (2014) study the interaction of the manager's degree of risk aversion and the power of incentives. In contrast with these works, we do not study retention decisions and managerial risk aversion, instead we focus on the joint dynamics of effort, cooperation, incentives, and internal capital allocation.

Our paper is also related to a growing literature on the theory of dynamic financial contract. The seminal work by Sannikov (2008) provides a tractable martingale approach to solve dynamic

[4] This literature dates back to Baron and Besanko (1984) who study optimal regulation of a monopolist in a multi-period adverse selection model. Subsequent works include Besanko (1985), Courty and Li (2000), Battaglini (2005), Eso and Szentes (2007), Krahmer and Strausz (2011), Toikka and Skrzypacz (2014), and many others.

principal-agent model in continuous time.[5] Nevertheless, it remains to be a challenge to apply the continuous-time recursive method in solving multi-agent contracting problems for the reason that the contract requires multiple state variables, i.e., the promised utilities, to keep track of the performance of each agent. This renders the principal's stochastic control problem difficult to solve.[6] In contrast, our work adopts the Myersonian approach. The solution to the optimal contract in our model can be obtained by point-wise maximizing the principal's dynamic virtual surplus, where each point represents the initial private information and the current project types in both divisions. As a consequence, we can characterize the explicit dependence of the allocation on the private information, and the closed-form solution permits application of stochastic calculus to deliver the dynamics of capital and incentives.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the efficient mechanism. Section 4 characterizes the optimal mechanism and identify conditions on the primitives that guarantee the implementability of the optimal allocation. Section 5 develops some examples and the empirical implications of the model. Section 6 concludes and all proofs are delegated to the appendix.

2 The Model

Consider a firm that consists of a single headquarters and two divisions. The headquarters (the principal) acts on behalf of the shareholders of the firm and each division is run by a division manager (the agent), who manages investment projects and is indexed by $i = A, B$. The headquarters hires and writes the employment contract with the division managers. The operation of the investment projects requires both the skills of the division managers and capital invested by the headquarters. Since the key focus of the paper is on internal capital market, we assume that the headquarters is the only source of capital and it has unlimited access to capital.

Time is continuous and indexed by t. The horizon is infinite. The headquarters and the division managers are risk-neutral and discount future cash flows at rate $r > 0$. At each moment of time,

[5] See, for example, DeMarzo and Sannikov (2006), Biais et al. (2010) , Williams (2011), DeMarzo et al. (2012) , and Wong (2016) for recent contributions. For a literature survey and applications of the tool in security design and corporate financing, see Sannikov (2013).

[6] Wong (2014) obtains the optimal contract in a continuous-time team production problem by restricting the preference of all the agents to be exponential.

the managers exert costly and unobservable efforts (e_{it}, e_{ijt}) and the headquarters allocates capital k_{it} for division i's investment. Division i produces cash flows at rate

$$\pi_{it} = (\alpha_o e_{it} + \alpha e_{jit} + \theta_{it} + v\theta_{jt})\, k_{it} \tag{1}$$

from its long-term project. In (1), e_{it} is the "own" effort of manager i exerted on her division's project, e_{jit} is the "help" effort exerted by manager j to help manager i, and θ_{it} represents the time-t project quality in division i. The cash flows specification (1) is an "AK" technology, with the technology level "A" depending on the division manager's effort choice and project quality. Moreover, effort cost of manager i is $h(e_{it}, e_{ijt}) = \frac{1}{2}(e_{it}^2 + e_{ijt}^2)$.[7] And when the headquarters provides capital k_{it} to division i, it pays an investment cost $c(k_{it}) = \frac{1}{2}k_{it}^2$. We assume quadratic costs mainly for tractability.[8] The parameters α_o, α, and v reflect marginal productivity of own effort, help effort, and project quality, respectively.

Our specification is rich enough to shed light on capital budgeting in a conglomerate firm with highly related business units. Suppose $v > 0$, a better project in one division will improve the project's cash flows in the other division. This dependence can arise due to production synergies, spillover effects, or asset complementarity in both divisions. Similarly, a higher α captures more productive value-enhancing help effort because related projects often require similar managerial skills. As a result, α and v also measure how close the divisions' lines of business are.

Before signing the employment contract with the headquarters, manager i privately observes an initial signal $\vartheta_i \in \Theta \equiv (\underline{\vartheta}, \bar{\vartheta})$, drawn from a common prior distribution G_i, at time 0 regarding the project. The initial signals are independently distributed across managers. The initial signal ϑ_i, together with the time-t value Z_{it} of a process of contemporaneous shocks $(Z_{it})_{t\geq 0}$, determine the project quality θ_{it} at time t which is privately observed by manager i. More specifically, the project quality is generated by

$$\theta_{it} = \phi^i(t, \vartheta_i, Z_{it}) \tag{2}$$

where $\phi^i : \mathbb{R}_+ \times \Theta \times \mathbb{R} \to \mathbb{R}$ is an aggregator. Since the quality of project i is its manager's private

[7] This setup provides a tractable way to model cooperation among divisions. Itoh (1991,1992) first develop this idea in static optimal contract with team production. Auriol, Friebel, and Pechlivanos (2002), and Bernado, Cai and Luo (2004) also use this modeling approach.

[8] Our formulation of investment cost implicitly assumes that putting all capital in one division is inefficient. This is because, the convexity in the cost functions implies that "spreading out" capital allocations incurs a lower total cost. An alternative is to assume $\frac{1}{2}\psi(k_{it} + k_{jt})^2$ so that capital in division i and j are perfect substitute. However, corner solutions arise and for tractability we ignore this case. The same comment applies to effort cost. See footnote 4 in Bernado, Cai and Luo (2004) as well.

information, we also refer $(\theta_{it})_{\geq 0}$ as the *type* process of manager i. Same as the initial signals, the types are independent across managers.[9] The type process $(\theta_{it})_{t\geq 0}$ is thus a Markov process: θ_{it} depends on $(\theta_{is})_{s<t}$ only through the cumulative shock Z_{it}. Note that the initial private information ϑ_i needs not be the initial type θ_{i0}, but could be other parameters that govern the evolution of the type $(\theta_{it})_{\geq 0}$. For example, if $(\theta_{it})_{t\geq 0}$ is the arithmetic or geometric Brownian motion, then θ_{i0} is the initial value of the process, but ϑ_i could be the initial value, the drift, or the volatility of the process.

Following Bergemann and Strack (2015), we make a few technical assumptions on the primitives. First, the distribution G_i has a density g_i and has a full support on Θ. The associated inverse hazard rate $\psi_i(\vartheta_i) \equiv \frac{1-G_i(\vartheta_i)}{g_i(\vartheta_i)}$ is assumed to be decreasing in ϑ_i. Second, the aggregator ϕ^i is twice differentiable in every direction and its partial derivative with respect to the initial signal ϑ_i and the value of contemporaneous shock z_i are denoted as

$$\phi^i_\vartheta(t,\vartheta_i,z_i) \equiv \frac{\partial\phi^i(t,\vartheta_i,z_i)}{\partial\vartheta};\ \phi^i_z(t,\vartheta_i,z_i) \equiv \frac{\partial\phi^i(t,\vartheta_i,z_i)}{\partial z}$$

respectively. The process $\left(\phi^i_\vartheta(t,\vartheta_i,Z_{it})\right)_{t\geq 0}$ is referred as a *generalized stochastic flow* process. It is the continuous-time analogue of the impulse response functions defined in the theory of discrete-time dynamic mechanism design.[10] Essentially, the generalized stochastic flow captures the infinitesimal variation of the initial signal ϑ_i to θ_{it}, holding constant the cumulative shocks Z_{it}. As a result, the process summarizes the dynamic effect of a small change in initial private information on future project types, and the object is critical in determining optimal dynamic allocations.

For regularity, we assume that for both projects and for all values (t,ϑ_i,z_i), a higher initial signal ϑ_i generates higher future types, $\phi^i_\vartheta(t,\vartheta_i,z_i) \geq 0$. This amounts to first-order stochastic dominance ranking of the distribution of θ_{it} in terms of the initial signal ϑ_i. Moreover, a larger value of shocks Z_{it} leads to a higher type, $\phi^i_z(t,\vartheta_i,z_i) > 0$. Lastly, for regularity, we assume the expected impact of the initial signals on the type grows at most exponentially: there exists a constant $C > 0$, $q \in (0,r)$ such that $\mathbb{E}\left[\phi^i_\vartheta(t,\vartheta_i,Z_{it})\right] \leq Ce^{qt}$ for all $t \geq 0$ and $\vartheta_i \in \Theta$.

At time 0, the headquarters offers a contract to the division managers. Both the headquarters and the division managers can fully commit to the contract. A contract specifies wage compensations, capital allocations and recommended effort choices to both agents. Therefore, each

[9]That is, Z_{At} and Z_{Bt} are independent. The independence of the managers' private information is assumed to rule out the logic of Cremer-McLean (1988).

[10]For the general definition of impulse response functions, see Pavan, Segal, and Toikka (2014).

contract can be interpreted as a long-term capital budgeting mechanism, together with the managerial compensations to the division managers. By the revelation principle, we can without loss of generality restrict attention to direction revelation mechanisms. Formally, denote a mechanism as $\Gamma = \langle (w_{it}, e_{it}, e_{ijt}, k_{it})_{t \geq 0} \rangle_{i=A,B}$, where, to manager i, w_{it} is the wage compensation, (e_{it}, e_{ijt}) is the recommended effort choices, and k_{it} is capital invested. All of them are functions of past reports of initial signals ϑ and project types $(\theta_s)_{s<t}$ in both divisions. In addition, w_{it} is also a function of the past cash flows $(\pi_s)_{s<t}$.[11] In the rest of the paper, the term "allocation" refers to $\langle (e_{it}, e_{ijt}, k_{it})_{t \geq 0} \rangle_{i=A,B}$ part of the mechanism. We restrict the allocations to be non-negative.

At each moment of time, given the past reports $(\hat{\vartheta}, (\hat{\theta}_s)_{s<t})$ and past observed cash flows $(\pi_s)_{s<t}$, the sequence of events that occur during the small time interval $[t, t+dt)$ are:

1. The project types $(\theta_{At}, \theta_{Bt})$ realize and manager i privately observes θ_{it};
2. The managers report their own project type $\hat{\theta}_{it}$ to the headquarters simultaneously;
3. The headquarters allocates capitals (k_{At}, k_{Bt}) and recommends (e_{it}, e_{ijt}) to the managers;
4. The managers choose their efforts after observing the allocations simultaneously;
5. Cash flows (π_{At}, π_{Bt}) realize and the headquarters pays manager i her promised wage w_{it}.

Given a mechanism Γ, we can now define the payoffs of the principal and agents. The headquarters's expected discounted profits is given by

$$\mathbb{E}\left[\int_0^\infty e^{-rt} \sum_{i=A,B} (\pi_{it} - c(k_{it}) - w_{it}) dt\right] \tag{3}$$

and the division manager i's expected discounted payoff, conditional on her initial signal ϑ_i, is given by

$$\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}(w_{it} - h(e_{it}, e_{ijt}))dt\right] \tag{4}$$

As we restrict attention to direct revelation mechanisms, the constraints facing the headquarters in the optimal contracting problem are the incentive-compatibility and participation constraints.

[11] We assume both efforts and capitals are independent of past cash flows. This is because for any type of manager i, fixing the type and strategy of manager j, she is able to generate the same distribution of cash flows in both divisions, regardless of the other type i's effort. Hence cash flows are not informative enough for future effort decisions. We adopt this logic from GP.

In particular, since two managers are involved in the contracting problem, incentive compatibility requires that each manager employs a truthful and obedient strategy, given the other manager is also truthful and obedient. That is, each manager reports truthfully their private information and follows the headquarters' recommended effort choices in every moment of time. We say that a mechanism Γ is *perfect Bayesian incentive compatible (PBIC)* if a *truthful* and *obedient* strategy profile forms an equilibrium strategy profile in a *perfect Bayesian equilibrium* (PBE).[12] In addition, we normalize the managers' reservation utility to 0 so that Γ is *individually rational* if it delivers non-negative payoffs to both managers at time 0. As a result, the headquarters's decision problem is to search for a *PBIC* and *individually rational* mechanism that maximizes (3).

3 Symmetric Information: The Efficient Mechanism

We first examine an economic environment with symmetric information. In this environment, both the project types and the effort choices are observable and verifiable. In particular, the headquarters chooses a contract to maximize the expected discounted profits (3) subject to the individual rationality constraints. The individual rationality constraints are obviously binding at the optimum and the resulting first-best contract is the one that maximizes the following ex-ante social surplus

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\sum_{i=A,B}(\pi_{it}-c(k_{it})-h(e_{it},e_{ijt}))dt\right] \tag{5}$$

Thus, the ex-ante social surplus is the expected discounted sum of the firm's total cash flows and the managers' disutility of efforts minus all the capital expenditures. The efficient mechanism is described as follows.

Proposition 1. *Suppose $H \equiv 1-\alpha^2-\alpha_o^2>0$. The efficient mechanism specifies that for all t, i and j*

$$k_{it}^{FB}(\theta_t) = \frac{1}{H}\left[\theta_{it}+v\theta_{jt}\right];\ e_{it}^{FB}(\theta_t) = \frac{\alpha_o}{H}\left[\theta_{it}+v\theta_{jt}\right];\ e_{ijt}^{FB}(\theta_t) = \frac{\alpha}{H}\left[\theta_{jt}+v\theta_{it}\right]$$

The wages are set such that individually rational constraints are binding.

[12] The conditional expectation in (4) is taken with respect to manager i's beliefs about the other manager's initial signal and continuation type process. We omit the description of the beliefs for ease of notation.

In the efficient mechanism, both the capital allocations and effort choices are increasing in the project types. This is because both the cash flows and the marginal value of capital investment are increasing in the project types. Moreover, as efforts and capital are complement, when more capital is allocated to the division, the marginal value of effort provision increases. This chain effect implies both the first-best own effort and help effort are increasing in the project types. Extending this line of reasoning, as more efforts are provided, complementarity implies higher marginal value of capital and thus more capital investment. This feedback effect in turn encourages more effort provisions. Therefore, complementarity has to be small enough, that is $1 > \alpha_o^2 + \alpha^2$, to ensure the solution satisfies the second-order condition for maximization. We maintain this parametric assumption in the rest of the paper.

Observe that the efficient mechanism is dynamic in nature. By the aggregator (2), the current type θ_{it} depend on past types through the shocks Z_{it}. Interestingly, the efficient allocation does not depend on the initial signals ϑ_i, except when it is the initial value of the type process θ_{i0}. In case where the initial signal is the drift or volatility of the type processes, the headquarters does not contract on them in the first-best, as these characteristics of the types are not directly payoff relevant. Finally, note that the headquarters will exclude projects when $\theta_{it} + v\theta_{jt} \leq 0$.

4 The Optimal Dynamic Mechanism

In this section, we characterize the optimal dynamic mechanism under the assumption that both the project types and the effort choices are managers' private information. To solve the model, we adopt the Myersonian approach in continuous time.[13] First, we obtain a *relaxed program* by replacing the global incentive constraints with the local incentive constraints using a dynamic envelope condition. The envelope condition is a necessary condition for incentive compatibility and it summarizes the marginal impact of the initial signal on the manager's equilibrium payoff. Then we express the objective of the headquarters as dynamic virtual surplus. Maximization of the dynamic virtual surplus leads to allocations. Lastly, we identify conditions on the primitives that guarantee the solution to the relaxed program also satisfies the global incentive constraints.

[13] The approach is a dynamic extension of Myerson's (1981) classic approach to static mechanism design problems.

4.1 Dynamic Information Rent

Given any PBIC mechanism Γ, define division manager i's equilibrium payoff associated with each initial signal when she reports truthfully and follows an obedient effort choice as

$$V_i^{\Gamma}(\vartheta_i) \equiv \mathbb{E}_{\vartheta_i}\left[\int_0^{\infty} e^{-rt}\left(w_{it} - h(e_{it}, e_{ijt})\right) dt\right]$$

Proposition 2 characterizes the derivative of the value function.

Proposition 2. *In any PBIC mechanism Γ, the value function of manager i is Lipschitz continuous and its derivative with respect to the initial signal ϑ_i is given by*

$$\frac{\partial V_i^{\Gamma}(\vartheta_i)}{\partial \vartheta_i} = \mathbb{E}_{\vartheta_i}\left[\int_0^{\infty} e^{-rt}\phi_{\vartheta}^{i}(t, \vartheta_i, Z_{it})\left(\frac{e_{it}(\vartheta, (\theta_s)_{s\leq t})}{\alpha_o} + \frac{ve_{ijt}(\vartheta, (\theta_s)_{s\leq t})}{\alpha}\right) dt\right] \tag{6}$$

when the division manager i reports truthfully and employs an obedient effort choice.

The information rent equation in the dynamic environment is then obtained by integrating the dynamic envelope condition (6)

$$V_i^{\Gamma}(\vartheta_i) = V_i^{\Gamma}(\underline{\vartheta}) + \int_{\underline{\vartheta}}^{\vartheta_i} \mathbb{E}_{\tilde{\vartheta}_i}\left[\int_0^{\infty} e^{-rt}\phi_{\vartheta}^{i}(t, \tilde{\vartheta}_i, Z_{it})\left(\frac{e_{it}(\tilde{\vartheta}, (\theta_s)_{s\leq t})}{\alpha_o} + \frac{ve_{ijt}(\tilde{\vartheta}, (\theta_s)_{s\leq t})}{\alpha}\right) dt\right] d\tilde{\vartheta}_i \tag{7}$$

The information rent equation (7) admits an intuitive interpretation. The left-hand side of (7) is the expected discounted payoff of a manager with an initial signal ϑ_i in any PBIC mechanism. It is the expected payoff that the headquarters must leave to the manager in order to induce truth-telling and obedient effort choice. The right-hand side of (7) equals the expected payoff of the lowest initial signal $\underline{\vartheta}$ plus the information rent of the type-ϑ_i manager.

Manager i earns information rent because she is privately informed about the initial signal ϑ_i and subsequent project types θ_{it}. Consider a manager who privately observes θ_{it} at time t. This manager will be able to mimic any lower types $\tilde{\theta}_{it} < \theta_{it}$ by producing the same cash flows in the own (other) division as in a type-$\tilde{\theta}_{it}$ division. She can achieve this by shirking, that is, by providing less own (help) effort, in which case she saves certain effort costs. The disutility saved is captured by the terms $\frac{e_{it}}{\alpha_o}$ and $\frac{ve_{ijt}}{\alpha}$, which contribute to the flow information rent, on the right-hand side of (7).

In the dynamic environment, the critical feature of (7) is that the total information rent incorporates the effect of a manager's initial signal on all the future project types. Note that the time-t allocations of the optimal mechanism could depend on the history of past reports in general. This implies that for a manager, not only can she derive rent from current allocations, but any reports she sends today affect future allocations too. More importantly, the initial signal reported at time 0 affects the all future allocations. This generates substantial rents for the manager at each point in time. In addition, a manager, who privately observes the initial signal ϑ_i, understands the evolution of the future types and her potential ability to derive information rents in the future. As a result, the total information rent is the expected discounted and weighted sum of rent at each point in time, with the weights given by the stochastic flow.

To derive (6) as a necessary condition for incentive compatibility, we follow Bergemann and Strack (2015) by focusing on a small class of deviations called *consistent deviations.* Intuitively, a manager who misreports as if she has an initial signal $\vartheta_i^{'}$ will continue to misreport as if all the future project types are generated by $\vartheta_i^{'}$. Formally:

Definition (Consistent deviation). *A manager i with initial signal ϑ_i and type $\theta_{i0} = \phi^i(0, \vartheta_i, Z_{i0})$* consistently deviates *if she misreports $\hat{\theta}_{i0} = \phi^i(0, \vartheta_i^{'}, Z_{i0})$ at time 0 and continues to misreport $\hat{\theta}_{it} = \phi^i(t, \vartheta_i^{'}, z(t, \vartheta_i, \theta_{it}))$ instead for the true type θ_{it} for all $t > 0$. The function z is implicitly defined by $\theta_{it} = \phi^i(t, \vartheta_i, z(t, \vartheta_i, \theta_{it}))$ for all t and ϑ_i.*[14]

Consistent deviation requires a manager with a true initial signal ϑ_i to report the truthfully shocks $z(t, \vartheta_i, \theta_{it})$ at each moment time, after she misreports $\vartheta_i^{'}$ initially. Because of the independence Z_{it} and ϑ_i, the smaller class of deviation implies that changes in the initial signal causes no variation in the continuation reporting strategy. This helps establishing the differentiability of the managers' payoffs.[15] Together with the effort concealment argument as in Laffont and Tirole (1986), we can turn the managers' reporting problem into a one-dimensional problem. Then Envelope theorem allows us to derive the condition (6) as a necessary condition for incentive compatibility.

[14] z is well-defined because ϕ^i is strictly increasing in the value of contemporaneous shock z_i.

[15] Eso and Szentes (2007) first use this class of deviations. Pavan, Segal, and Toikka (2014) apply it to identify the sufficient conditions for the infinite-horizon version of the envelope condition in discrete time. For more discussion on consistent deviation, see Bergemann and Strack (2015).

4.2 The Optimal Allocation

We now use the dynamic information rent equation (7) to eliminate the wage compensation to both division managers in the principal's expected discounted profits (3). The resulting expression (8) below is the dynamic virtual surplus of the headquarters. As only the local incentive constraints are embedded in the dynamic virtual surplus, it is the objective function of the *relaxed program.* In the problem, the headquarters chooses capital allocations and effort recommendations $\langle (k_{it}, e_{it}, e_{ijt})_{t\geq 0}\rangle_{i=A,B}$ to maximize (8) subject to the participation constraints. As usual, $V_i^{\Gamma}(\underline{\vartheta}) = 0$ in equilibrium for both divisions. The following proposition provides the dynamic virtual surplus and the solution to the relaxed program.

Proposition 3. *In any PBIC and individually rational mechanism* Γ*, the headquarters's expected discounted profits is given by the dynamic virtual surplus*

$$\int_{\Theta^2} \mathbb{E}_{\vartheta}\left[\int_0^{\infty} e^{-rt}\left(\begin{array}{c}\pi_{it} - c(k_{it}) - h(e_{it}, e_{ijt}) \\ -\left(\frac{e_{it}}{\alpha_o} + \frac{v e_{ijt}}{\alpha}\right)\phi_{\vartheta}^i(t, \vartheta_i, Z_{it})\frac{1-G_i(\vartheta_i)}{g_i(\vartheta_i)}\end{array}\right) dt\right] dG(\vartheta) - \sum_i V_i^{\Gamma}(\underline{\vartheta}) \tag{8}$$

In the optimal mechanism, the allocations for division i *are given by*

$$k_{it}(\vartheta, \theta_t) = \frac{1}{H}\left(\left(\theta_{it} - \phi_{\vartheta}^i(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)\right) + v\left(\theta_{jt} - \phi_{\vartheta}^j(t, \vartheta_j, Z_{jt})\psi_j(\vartheta_j)\right)\right) \tag{9}$$

$$e_{it}(\vartheta, \theta_t) = \frac{\alpha_o}{H}\left(\left(\theta_{it} - \frac{1-\alpha^2}{\alpha_o^2}\phi_{\vartheta}^i(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)\right) + v\left(\theta_{jt} - \phi_{\vartheta}^j(t, \vartheta_j, Z_{jt})\psi_j(\vartheta_j)\right)\right) \tag{10}$$

$$e_{ijt}(\vartheta, \theta_t) = \frac{\alpha}{H}\left(v\left(\theta_{it} - \frac{1-\alpha_o^2}{\alpha^2}\phi_{\vartheta}^i(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)\right) + \left(\theta_{jt} - \phi_{\vartheta}^j(t, \vartheta_j, Z_{jt})\psi_j(\vartheta_j)\right)\right) \tag{11}$$

whenever positive. The optimal mechanism delivers $V_i^{\Gamma}(\underline{\vartheta}) = 0$ *for* $i = A, B$*.*

Expression (8) is the headquarters's expected payoffs (dynamic virtual surplus) under any PBIC and individually rational mechanism. It is the expected discounted sum of the headquarters's flow virtual surplus, which contains two components: the first one is the social value of capital investment and productive efforts, given by the first line in the bracket of (8). The second line in the bracket represents the time-t information rent that the headquarters must deliver to a division manager with type θ_{it} to truthfully reveal the her time-t private information. The last term $V_i^{\Gamma}(\underline{\theta})$ is the total rent given to the division manager with the lowest initial signal in order to induce her to participate in the mechanism.

The dynamic capital allocations and effort choices that solve the relaxed program are stated in the second part of the proposition. Two features of the optimal allocations are noteworthy. First, there are under-investment and under-provision of efforts. This is because $\phi^i_\vartheta \geq 0$. The result is standard: the headquarters distorts the allocations downward to limit the payoffs that a misreporting manager can get, and hence economizing the information rents. The result immediately implies that the headquarters may exclude projects with large enough distortions, even though the projects are acceptable (those with $\theta_{it} + v\theta_{jt} > 0$) under the efficient mechanism.

Second, the time-t optimal allocations depend only on the reports about the initial signal ϑ and the time-t project type θ_t. This is because of the time separability of the investment problem: the cash flows generated by the project at time t only depend on the current project quality, and indirectly on the initial signal through its effect on the current type.

The optimal allocations need not be fully incentive compatible. We now identify conditions on the primitives that ensure the optimal allocations are monotonic in the initial signals and the project types. This parallel static mechanism design analysis in which the envelope condition and the monotonicity of the allocation are sufficient conditions for incentive compatibility. Following Bergemann and Strack (2015), we use the following conditions.

Definition (Decreasing influence of initial signal). *The relative impact of the initial signal on the type:* $\frac{\phi^i_\vartheta(t,\vartheta_i,z_i)}{\phi^i(t,\vartheta_i,z_i)}$ *is decreasing in* z_i *for all* (t, ϑ_i, z_i).

Definition (Decreasing influence of initial signal vs. contemporaneous shock). *The ratio of the marginal impact of initial signal and contemporaneous shocks:* $\frac{\phi^i_\vartheta(t,\vartheta_i,z_i)}{\phi^i_z(t,\vartheta_i,z_i)}$ *is decreasing in* ϑ_i *for all* (t, ϑ_i, z_i).

Then for a constant $C > 0$, we define the virtual quality of project i as

$$\theta_{it} - C\phi^i_\vartheta(t, \vartheta_i, z(t, \vartheta_i, \theta_{it}))\psi_i(\vartheta_i) \tag{12}$$

where C can take value 1, $\frac{1-\alpha^2}{\alpha_o^2}$, or $\frac{1-\alpha_o^2}{\alpha^2}$ in the optimal allocations. A straightforward modification of proposition 2 in Bergemann and Strack (2015) verifies that (12) is increasing in the initial signal ϑ_i and the time-t type θ_{it}.[16] This implies that the optimal allocation with respect to the initial signals ϑ and the time-t project types. We summarize the discussion below.

[16]Specifically, decreasing influence of initial signal implies that (12) is increasing in θ_{it}; and decreasing influence of initial signal vs. contemporaneous shock implies that (12) is increasing in ϑ_i.

Proposition 4. *Suppose $\phi^i(t, \vartheta_i, z_i)$ exhibits decreasing influence of initial signal and initial signal vs. contemporaneous shocks for both i, then the time-t optimal allocations $k_{it}(\vartheta, \theta_t)$, $e_{it}(\vartheta, \theta_t)$, and $e_{ijt}(\vartheta, \theta_t)$ are increasing in ϑ and θ_t for all i, j and $t \geq 0$, as long as they are positive.*

4.3 Implementation and Managerial Compensations.

With the optimal allocation being monotonic, we can now construct wage payments that guarantee the incentives for the managers to truthfully to reveal their project types and follow the recommended effort choice obediently. We first state a condition.

Definition (Ex-post monotonicity). *An allocation $\left\langle (k_{it}(\vartheta, \theta_t), e_{it}(\vartheta, \theta_t), e_{ijt}(\vartheta, \theta_t))_{t\geq 0} \right\rangle_{i=A,B}$ satisfies* ex-post monotonicity *if for all i, t, and ϑ_i,*

$$\int_0^\infty e^{-rt} \phi^i_\vartheta(t, \vartheta_i, Z_{it}) \left(\frac{e_{it}(\hat{\vartheta}, \hat{\theta}_t)}{\alpha_o} + \frac{v e_{ijt}(\hat{\vartheta}, \hat{\theta}_t)}{\alpha} \right) dt \tag{13}$$

is increasing in $\hat{\vartheta}_i$ and $\hat{\theta}_{it}$, for all reports $(\hat{\vartheta}_j, (\hat{\theta}_{jt})_{t\geq 0})$ by manager j.

Ex-post monotonicity is the continuous-time analogue of ex-post monotonicity in discrete-time dynamic mechanism design.[17] The condition requires that the discounted weighted sum of manager i's flow information rent (13), with the weights given by the stochastic flow process, is increasing in her reports, regardless of the other manager's reports. More specifically, note that the stochastic flow is independent of the reports, hence the monotonicity of (13) with respect to $\hat{\vartheta}_i$ and $\hat{\theta}_{it}$ requires that the recommended efforts $e_{it}(\hat{\vartheta}, \hat{\theta})$ and $e_{ijt}(\hat{\vartheta}, \hat{\theta}_t)$ being increasing in $\hat{\vartheta}_i$ and $\hat{\theta}_{it}$. Given the primitive conditions, proposition 4 implies that the optimal allocation satisfies ex-post monotonicity. We can now state a result.

Proposition 5. *Suppose the optimal allocation $\left\langle (k_{it}(\vartheta, \theta_t), e_{it}(\vartheta, \theta_t), e_{ijt}(\vartheta, \theta_t))_{t\geq 0} \right\rangle_{i=A,B}$ is ex-post monotonic. There exists a linear compensation scheme $(w_t(\vartheta, (\theta_s)_{s\leq t}; (\pi_s)_{s\leq t}))_{t\geq 0}$ with*

$$w_{it}(\vartheta, \theta_t, \pi_t) = B_{it}(\vartheta, \theta_t) + S_{it}(\vartheta, \theta_t)\pi_{it} + S_{ijt}(\vartheta, \theta_t)\pi_{jt}$$

for $i = A, B$, where to manager i, $B_{it}(\vartheta, \theta_t)$ is the base salary, $S_{it}(\vartheta, \theta_t)$ is her share of own-division's cash flows, and $S_{ijt}(\vartheta, \theta_t)$ is her share of other-division's cash flows, such that in the mechanism Γ,

[17] See corollary 1 and expression (10) in Pavan, Segal, and Toikka (2014).

both managers report their initial signal ϑ_i and project type θ_{it} truthfully for all $t \geq 0$, regardless of the past reports $\hat{\vartheta}$ and $(\hat{\theta}_s)_{s<t}$.

The proof of the proposition consists of two steps. In the first step, we construct flow wage processes that ensure the managers' incentives to tell the truth about the time-t projects. The construction goes as follows: we choose the shares so that they satisfy $\alpha_o k_{it}(\vartheta, \theta_t) S_{it}(\vartheta, \theta_t) = e_{it}(\vartheta, \theta_t)$ and $\alpha k_{jt}(\vartheta, \theta_t) S_{ijt}(\vartheta, \theta_t) = e_{ijt}(\vartheta, \theta_t)$. Intuitively, the shares equalize the managers' marginal benefits, which is the marginal productivity of effort times the pay-sensitivity with respect to project cash flows, and marginal cost of exerting own (help) effort. The shares thus provide incentives for the managers to exert efforts in an obedient way. Then, given the shares S_{it} and S_{ijt}, we construct a base salary b_{it} to control for time-t adverse selection. In fact, b_{it} is set such that manager i's time-t payoff equals her time-t information rent. Since the optimal allocation only depends on the initial signals and time-t project types due to time separability, the shares and base payment share this feature as well. This dependence is sufficient to ensure the managers' incentives to reveal truthfully their time-t private information.

In the second step, we construct a compensation $P_i(\vartheta)$, which depends only on the initial signal, to induce the managers to truthfully reveal their initial signals. The idea behind the payment is that it accounts for the expected discounted information rent generated by the manager's initial private information. Naturally, the payment depends on the stochastic flow $\phi^i_\vartheta(t, \vartheta_i, Z_{it})$. A larger impact of initial signal on the future types implies that, on expectation, the manager will have a better ability to mimic other types in the future. Hence she will to be compensated more, with a larger $P_i(\vartheta)$, to reveal her initial signal. Since $P_i(\vartheta)$ is independent of reported types $\hat{\theta}_t$, the payment can be spread over time so that $B_{it}(\vartheta, \theta_t) \equiv b_{it}(\vartheta, \theta_t) + \beta_i(\vartheta)$, where $\beta_i(\vartheta) = rP_i(\vartheta)$. (See (36) in the appendix.)

It is also worth to note that the ex-post monotonicity condition requires (13) be increasing in $\hat{\vartheta}_i$ and $\hat{\theta}_{it}$, *independent* of manager i's beliefs about the other manager's types and initial signal. Proposition 5 implies that the linear contracts implement the optimal allocations in an ex-post equilibrium. That is, at time t, truthful revelation of θ_{it}, and at time 0, truthful revelation of ϑ_i, are dominant strategy for manager i.[18] As a result, the managers' reporting strategy satisfies PBIC and the optimal allocations constitute the solution to the full program.

[18] This result is reminiscent of Mookherjee and Reichelstein (1992). They show that in a static quasilinear environment with independent types, a Bayesian incentive compatible allocation satisfying a one-period monotonicity condition can be equivalently implemented in dominant strategies. Hence, dominant strategy implementation can be obtained for free in static optimal Bayesian mechanism design problems.

Before we turn to the next section, we note that the linear contracts of Proposition 5 can violate limited liability. For some initial signals and project types, the payment made to a manager is negative. When we impose limited liability constraints on the headquarters's problem, we can introduce a fixed cost of efforts that ensure the payment is non-negative to the managers at each point in time. The additional fixed costs obviously leaves the optimal allocation unchanged and shifts the wage payment upward uniformly only. We conclude that our key results and model implications continue to hold with limited liability constraints.

Corollary. *In a modified economic environment with a fixed cost of effort c not too low, linear compensation contracts as in Proposition 5 can be chosen so as to entail non-negative payments to the managers. The allocation stated in Proposition 3 remains optimal and implementable when managers are protected by limited liability.*

5 Model Implications

In this section, we explore the model implications on capital investment and managerial compensations through some examples. We focus on the case with symmetric projects. That is, both projects follow the same form of stochastic process. We focus on three examples of type processes: geometric Brownian motions, arithmetic Brownian motions, and geometric Ornstein-Uhlenbeck process. Then we draw some general implications and relate them with empirical evidence.

5.1 Geometric Brownian Motion

Suppose $(\theta_{it})_{t\geq 0}$ follows a geometric Brownian motion

$$\frac{d\theta_{it}}{\theta_{it}} = \mu_i dt + \sigma_i dZ_{it},\ i = A, B \tag{14}$$

where $\mu_i > 0$ is the drift rate, $\sigma_i > 0$ is the volatility, and Z_{it} is a standard Brownian motion. The solution to the stochastic differential equation (14) is well-known:

$$\theta_{it} = \theta_{i0} \exp\left((\mu_i - \frac{1}{2}\sigma_i)t + \sigma_i Z_{it} \right)$$

for an initial value θ_{i0}. Hence, the aggregator for the type process can be defined as $\phi^i(t, \vartheta_i, Z_{it}) = \theta_{i0} \exp\left((\mu_i - \frac{1}{2}\sigma_i)t + \sigma_i Z_{it}\right)$, where the initial signal ϑ_i can be the initial value, the drift, or the volatility.

5.1.1 Privately known initial value

First, consider the case when the initial value of the process is privately known by the manager, $\vartheta_i = \theta_{i0}$. The generalized stochastic flow is the derivative of $\phi^i(t, \theta_{i0}, Z_{it})$ with respect to θ_{i0},

$$\phi^i_{\theta_0}(t, \theta_{i0}, Z_{it}) = \exp\left((\mu_i - \frac{1}{2}\sigma_i)t + \sigma_i Z_{it}\right) = \frac{\theta_{it}}{\theta_{i0}} \tag{15}$$

Since the inverse hazard rate depends only on θ_{i0}, the above equation implies that the distortion with a geometric Brownian motion is proportional to the project type. With the stochastic flow (15), it follows from (9) that the capital allocation takes a closed-form:

$$k_{it}(\theta_0, \theta_t) = \frac{1}{H}\left[\theta_{it}\left(1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}\right)\right] \tag{16}$$

There are a few implications follow from (16). First, when the type processes satisfy geometric Brownian motions, $\theta_{it} > 0$. Thus, in the efficient mechanism, the headquarters do not exclude any projects. However, under the optimal mechanism, the headquarters will exclude projects due to screening of the initial value of the project types. For example, for initial values $(\theta_{A0}, \theta_{B0})$ such that $\max\left\{1 - \frac{\psi_A(\theta_{A0})}{\theta_{A0}}, 1 - \frac{\psi_B(\theta_{B0})}{\theta_{B0}},\right\} \leq 0$, the projects in both divisions are not promising at the initial capital budgeting phase and the headquarters excludes these projects forever.

Second, application of Ito's lemma to (16) allows us to characterize the capital dynamics:

$$dk_{it}(\theta_0, \theta_t) = (\mathcal{K}_i\theta_{it} + v\mathcal{K}_j\theta_{jt})\, dt + \sigma\mathcal{K}_i\theta_{it}dZ_{it} + v\mathcal{K}_j\theta_{jt}dZ_{jt} \tag{17}$$

where $\mathcal{K}_i \equiv \frac{1}{H}\left(1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}\right)$. While (16) shows that initial private information affects the level of capital investment, (17) reveals that initial private information affects the drift and volatility of the capital dynamics. For project with a low initial value θ_{i0}, the inverse hazard rate $\psi_i(\theta_{i0})$ is high. Thus, $\mathcal{K}_i$ is decreasing in θ_{i0}. As a result, projects with low initial value $(\theta_{A0}, \theta_{B0})$ will exhibit a low drift and volatility. This results from the headquarters' allocation of information rent over time: as the headquarters understands the persistent nature of initial value, it designs a low-growth and steady capital path for bad projects. This prevents managers with good project to pretend to be the low types, and thus economizes information rent.

Third, in a multidivision firm, the spillover effect v is also critical to the capital dynamics. Suppose projects are not excluded and $\mathcal{K}_j > 0$, a larger production complementarity implies that the size of the projects grow faster, and is more volatile. As long as $v \neq 0$, initial private information of the other manager will also affect the capital dynamic of the own project. Although we did not compute explicitly the dynamics of own and help efforts, the above discussions apply to these objects as well. This is because the virtual project quality enters into (9), (10), and (11) in a similar way. In fact, it is easy to see that $k_{it}(\theta_0, \theta_t)$, $e_{it}(\theta_0, \theta_t)$, and $e_{ijt}(\theta_0, \theta_t)$ are positively correlated, hence they share a similar dynamics.

Next, we turn to the pay-performance sensitivity (PPS). With the stochastic flow given by (15), the PPS for the own division satisfies

$$S_{it}(\theta_0, \theta_t) = \frac{\theta_{it}\left(1 - \frac{1-\alpha^2}{\alpha_0^2}\frac{\psi_i(\theta_{i0})}{\theta_{i0}}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}\right)}{\theta_{it}\left(1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}\right)} \tag{18}$$

It is interesting to note that when divisions are independent, $v = 0$, $S_{it}(\theta_0, \theta_t) = \frac{1 - \frac{1-\alpha^2}{\alpha_0^2}\frac{\psi_i(\theta_{i0})}{\theta_{i0}}}{1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}}$ and is independent of the current project type θ_{it}. This stems from proportional distortions driven by the geometric Brownian motion. An immediately implication is that our model predicts constant effort incentives in a single division firm, or firms with unrelated business. When the spillover effect v is large, (18) implies a higher PPS, as long as $\theta_{j0} > \psi_j(\theta_{j0})$. This result is intuitive: with a larger spillover, the headquarters injects more capital in both divisions. This leads to a higher marginal productivity of effort and calls for a higher pay sensitivity to induce the optimal effort provisions.

Figure 1 illustrates the dynamics of capital allocation and pay-performance sensitivity with different initial value of the process. In the figure, the blue (red) path corresponds to the good (bad) project. The top-left panel of figure 1 confirms the analysis: the path of capital investment for a good project has a higher drift and a higher volatility. However, the incentives have the reversed pattern: the manager holding a good project faces less volatile incentives. Following BCL(2004), we can rewrite the wage contract as

$$w_{it}(\vartheta, \theta_t) = b_{it}(\vartheta, \theta_t) + S_{ijt}(\vartheta, \theta_t)\left(\pi_{it} + \pi_{jt}\right) + \left(S_{it}(\vartheta, \theta_t) - S_{ijt}(\vartheta, \theta_t)\right)\pi_{it}$$

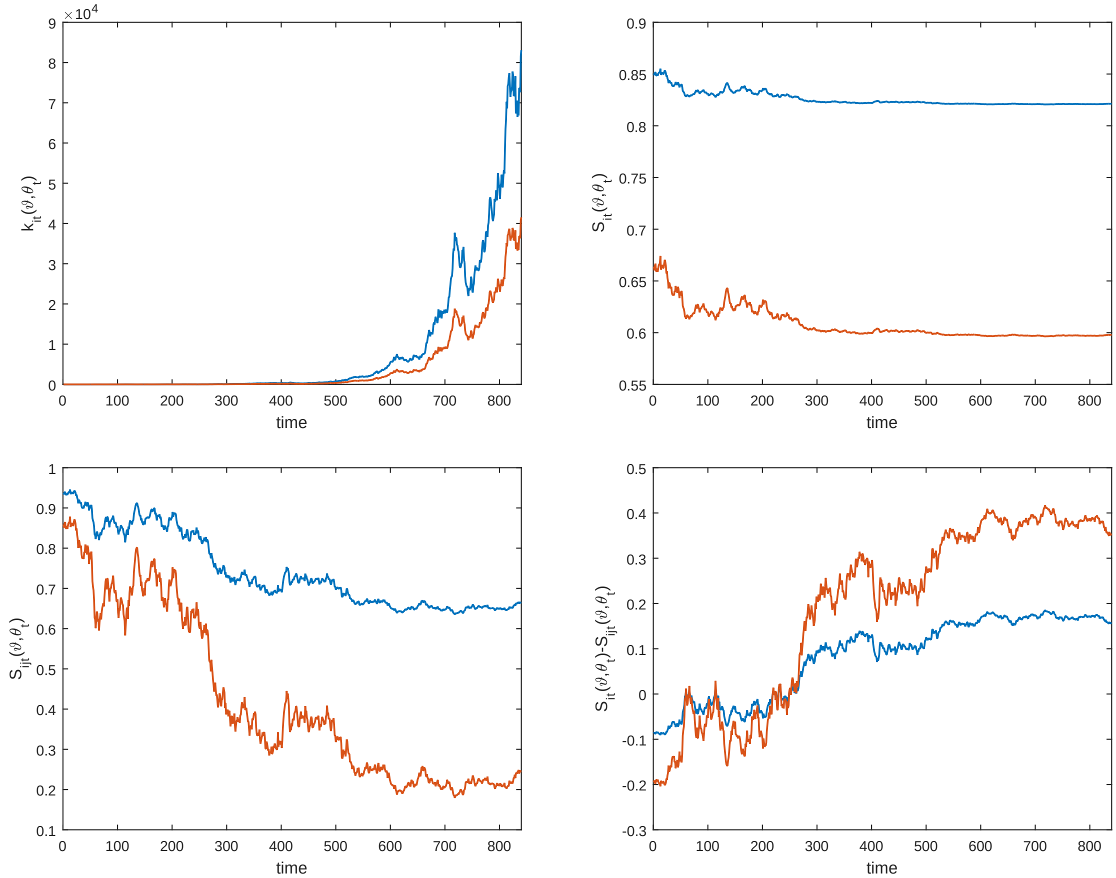

Figure 1: Dynamics of capital and incentives with geometric Brownian motions. The initial value of the type processes are $\theta_{i0} = 0.55$ (red) and $\theta_{i0} = 0.6$ (blue) for both $i = A, B$. We assume a uniform distribution for the initial value on $(0, 1)$. The paths are drawn by holding constant the shocks Z_{it}. Other parameters: $\alpha_o = 0.8$, $\alpha = 0.55$, $v = 0.2$, $\mu_A = 18\%$, $\mu_B = 14\%$, and $\sigma = 20\%$.

where S_{ijt} can be interpreted as the manager's firm-level performance pay and $(S_{it} - S_{ijt})$ can be interpreted as the manager's division-level pay-performance sensitivity. The division-level PPS is illustrated in the bottom-right panel of figure 1. In the early stage of the firm, the headquarters relies more on firm-level pay. However, since $\mu_A > \mu_B$, project A grows larger relative to project B eventually and the headquarters compensates the managers more with division-level pay.

Figure 2 illustrates the dynamics of capital and incentives with different level of complementarity. The top panel verifies our analysis: with a higher v, projects become more related, and the capital invested in a division increases. Moreover, both the drift and volatility of the capital increase as well. In the bottom panel, the same result is found when cooperative effort becomes more

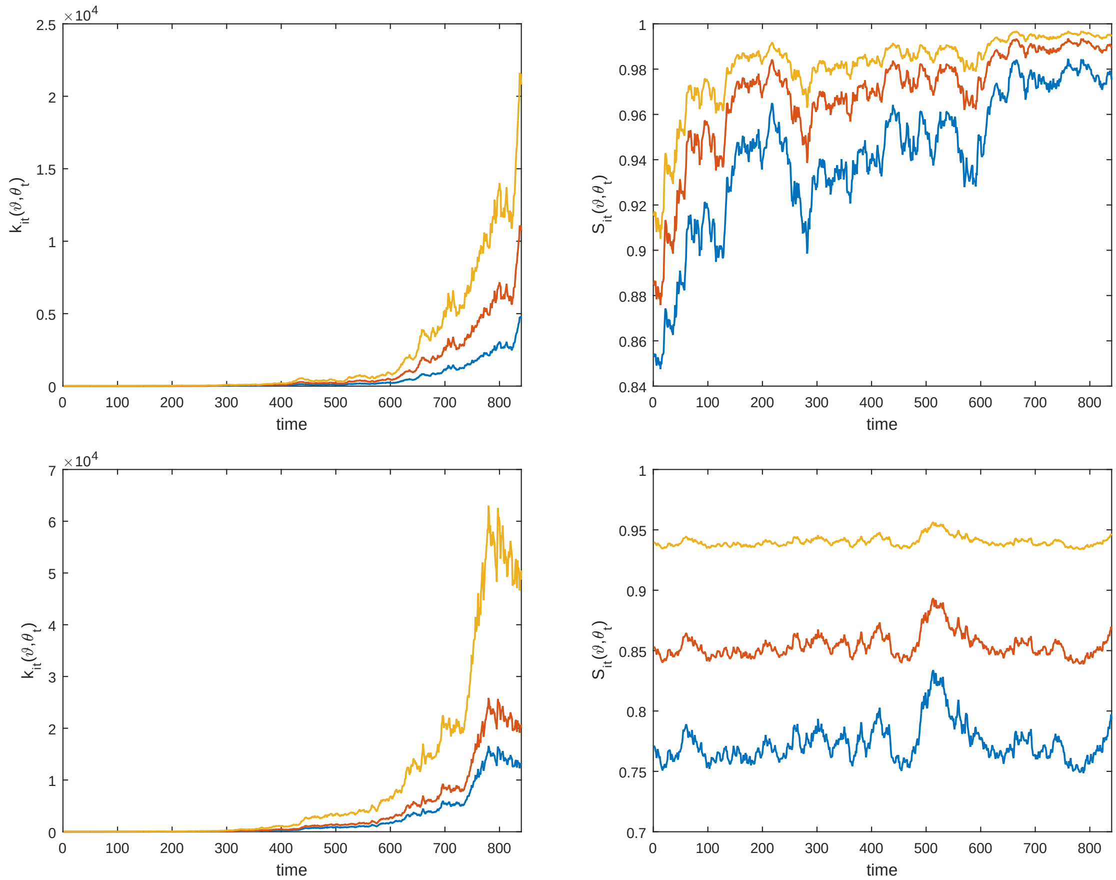

Figure 2: Comparative dynamics of capital and incentives with respect to υ and α. Top panel: $\upsilon = 0.2$ (blue), $\upsilon = 0.5$ (red), and $\upsilon = 1$ (yellow). Bottom panel: $\alpha = 0.5$ (blue), $\alpha = 0.55$ (red), and $\alpha = 0.58$ (yellow). Other parameters: $\alpha_o = 0.8$, $\mu_A = \mu_B = 14\%$, and $\sigma = 20\%$.

productive, i.e., when α increases.[19]

[19]In both figure 1 and 2, the incentives are varying over time and display no clear pattern with respect to time. This is because the distortion (15) in this example is time-independent. In section 5.3.1, we provide an example with decreasing distortion and discuss the long-run pattern of incentives.

5.1.2 Privately known drift

We now turn to the case when the drift of the geometric Brownian motion is initial private information. In this case, $\vartheta_i = \mu_i$ and the generalized stochastic flow is

$$\phi_\mu^i(t, \mu_i, Z_{it}) = \theta_{it} \cdot t$$

The distortion is still proportional to the current type θ_{it}, but at the same time increasing in the time variable. Increasing distortion implies that projects may be excluded over time. Formally, the capital allocation (9) becomes

$$k_{it}(\mu, \theta_t) = \frac{1}{H} \left[\theta_{it} \left(1 - \psi_i(\mu_i)t\right) + v\theta_{jt} \left(1 - \psi_j(\mu_j)t\right)\right] \tag{19}$$

From (19), the headquarters will accept both projects at time 0 (as $\theta_{it}, \theta_{jt} > 0$). However, the projects in both divisions will be terminated no later than a deterministic finite time $t_* = \max\left\{\frac{g_A(\mu_A)}{1-G_A(\mu_A)}, \frac{g_B(\mu_B)}{1-G_B(\mu_B)}\right\}$. A project with a higher drift rate μ_i has a higher hazard rate $\frac{g_i(\mu_i)}{1-G_i(\mu_i)}$, and thus will last longer in general. Moreover, project which is not so profitable (with a low μ_i) along may not be excluded, due to positive spillover effect.

By Ito's lemma, the capital dynamics satisfies

$$dk_{it}(\mu, \theta_t) = \left((\mathcal{K}_i\mu_i - \psi_i(\mu_i))\theta_{it} + v(\mathcal{K}_j\mu_j - \psi_j(\mu_j))\theta_{jt}\right) dt + \sigma_i\mathcal{K}_i\theta_{it}dZ_{it} + \sigma_j v\mathcal{K}_j\theta_{jt}dZ_{jt} \tag{20}$$

where we have abused notation by defining $\mathcal{K}_i = \frac{1}{H}\left(1 - \psi_i(\mu_i)t\right)$. How the initial private information affects the drift and volatility of (20) is the same as (17). Thus, capital investment for better projects (with higher drift rate μ_i's) displays a higher drift rate and is more volatile. With increasing proportional distortions, the drift and volatility of (20) will decrease over time. This has significant implication on the capital dynamics when the projects are close to their termination time. For simplicity, consider $v = 0$. Then the termination time for project i is thus $t_* = \frac{g_i(\mu_i)}{1-G(\mu_i)} = \frac{1}{\psi_i(\mu_i)}$. At time t_*, $\mathcal{K}_i = 0$, so the drift of $k_{it}(\mu_i, \theta_{it})$ is $-\psi_i(\mu_i)\theta_{it}$, and its volatility vanishes.

For the PPS for the own division,

$$S_{it}(\mu, \theta_t) = \frac{\theta_{it}\left(1 - \frac{1-\alpha^2}{\alpha_o^2}\psi_i(\mu_i)t\right) + v\theta_{jt}\left(1 - \psi_j(\mu_j)t\right)}{\theta_{it}\left(1 - \psi_i(\mu_i)t\right) + v\theta_{jt}\left(1 - \psi_j(\mu_j)t\right)}$$

When projects are independent, $v = 0$, $S_{it}(\mu_i, \theta_{it}) = \frac{1 - \frac{1-\alpha^2}{\alpha_o^2}\psi_i(\mu_i)t}{1-\psi_i(\mu_i)t}$ and the PPS is independent of θ_{it}. However, PPS is a deterministic function of time. As time moves on, the drift term in (20) decreases and the project size shrinks over time. Marginal productivity of effort will start to decline eventually and the required incentives to motivate the managers decrease as well. With positive spillover effect, the above logic still holds even though $S_{it}(\mu, \theta_t)$ will be stochastic.

5.2 Arithmetic Brownian Motion

We now turn to type processes that satisfy arithmetic Brownian motions. Suppose $(\theta_{it})_{t\geq 0}$ follows

$$d\theta_{it} = \mu_i dt + \sigma_i dZ_{it},\ i = A, B \tag{21}$$

where $\mu_i > 0$ is the drift rate and $\sigma_i > 0$ is the volatility. For θ_{i0} being the initial value of the process, the solution to (21) is

$$\theta_{it} = \theta_{i0} + \mu_t t + \sigma_i Z_{it}$$

Hence the aggregator of the type process can be written as $\phi^i(t, \vartheta_i, Z_{it}) = \theta_{i0} + \mu_t t + \sigma_i Z_{it}$. We again focus on two cases as in section 5.1: with initial value being the initial signal $\vartheta_i = \theta_{i0}$ and the drift rate being the initial signal $\vartheta_i = \mu_i$. In the former case, the generalized stochastic flow is $\phi^i_\theta(t, \theta_{i0}, Z_{it}) = 1$; and in the latter case, $\phi^i_\mu(t, \mu_i, Z_{it}) = t$. Compared to geometric Brownian motion, the generalized stochastic flow feature *absolute* distortion, rather than *proportional* distortion. That is, the magnitude of distortion at time t is independent of current type θ_{it}. Yet, there are similarity: private information regarding the initial value of the type process generates a time-independent stochastic flow; and that regarding the drift of the type process introduces positive dependence of the stochastic flow on the time variable, hence the distortion, be it absolute or proportional, is increasing in time. We briefly discuss the implications on capital allocation.

5.2.1 Privately known initial value

With $\phi^i_\theta(t, \theta_{i0}, Z_{it}) = 1$, the optimal capital allocation (9) becomes

$$k_{it}(\theta_0, \theta_t) = \frac{1}{H}\left[\theta_{it} - \psi_i(\theta_{i0}) + v\left(\theta_{jt} - \psi_j(\theta_{j0})\right)\right]$$

and its dynamics satisfies

$$dk_{it}(\theta_0, \theta_t) = \frac{1}{H}(\mu_i + v\mu_j)dt + \frac{1}{H}\left(\sigma_i dZ_{it} + v\sigma_j dZ_{jt}\right) \tag{22}$$

whenever $k_{it} > 0$. Constant absolute distortion drives a time-independent project exclusion threshold: at every point in time, the headquarters rejects projects if their time-t types are in the set $\{(\theta_A, \theta_B)|\theta_A + v\theta_B \leq \psi_A(\theta_{A0}) + v\psi_B(\theta_{B0})\}$. It is clear that this set of "acceptable projects" expands when projects have a better initial value $(\theta_{A0}, \theta_{B0})$. Another implication of constant absolute distortion is that the initial private information produces no influence on the dynamics of capital allocation: $k_{it}(\theta_0, \theta_t)$ evolves in exactly the same way as the $k_{it}^{FB}(\theta_t)$ in the efficient mechanism. This manifests that instead of manipulating the dynamics of capital paths, the headquarters only uses project exclusions to resolve adverse selection problem.

5.2.2 Privately known drift

With $\phi_\mu^i(t, \mu_i, Z_{it}) = t$, the optimal capital allocation (9) becomes

$$k_{it}(\theta_0, \theta_t) = \frac{1}{H}\left[\theta_{it} - \psi_i(\theta_{i0})t + v\left(\theta_{jt} - \psi_j(\theta_{j0})t\right)\right]$$

and by Ito's lemma, the capital has dynamics

$$dk_{it}(\mu, \theta_t) = \frac{1}{H}\left(d\theta_{it} + vd\theta_{jt}\right) - \frac{1}{H}\left[\psi_i(\mu_i) + v\psi_j(\mu_j)\right]dt \tag{23}$$

whenever $k_{it} > 0$. The first term on the RHS of (23) is exactly (22). Thus, increasing absolute distortion reduces the drift, but not the volatility, of capital allocation. The drift reduction is decreasing in the type growth rate μ_A and μ_B. As a result, better projects (with higher growth rate) have faster-growing project size. The intuition behind this result is the same as in the geometric Brownian motion case.

Increasing absolute distortion implies a project-exclusion threshold which is increasing in time. As in section 5.2.1, the set $\{(\theta_A, \theta_B)|\theta_A + v\theta_B \leq \left(\psi_A(\mu_{A0}) + v\psi_B(\mu_{B0})\right)t\}$ describes the projects to be excluded at time t. This rejection set expands in time and shrinks in the drift rate of the project types.[20] Thus, capital rationing becomes more and more stringent over time and the headquarters adopts this dynamic policy to screen out project growth rates.

[20]The set of projects to be excluded form a triangle on the (θ_A, θ_B)-plane: given θ_A, the minimum θ_B required

5.3 Geometric Ornstein-Uhlenbeck process

As a last example, we study the model implications if the project type is given by the exponential Ornstein-Uhlenbeck process, which is a continuous-time analogue of the discrete-time geometric AR(1) process. Consider the process

$$\theta_{it} = e^{x_{it}} \text{ where } dx_{it} = \eta_i(\bar{x}_i - x_t)dt + \sigma dZ_{it}$$

where x_{it} is an Ornstein-Uhlenbeck process, with mean reversion speed $\eta_i > 0$, long-run average $\bar{x}_i$, and volatility $\sigma > 0$. As usual, Z_{it} is a standard Brownian motion. Using Ito's formula, θ_{it} satisfies the following stochastic differential equation

$$d\theta_{it} = \theta_{it}\left(\eta_i(\bar{x}_i - \log\theta_{it}) + \frac{1}{2}\sigma^2\right)dt + \sigma\theta_{it}dZ_{it} \tag{24}$$

The solution to (24) is given by

$$\theta_{it} = \exp\left(e^{-\eta_i t}\log\theta_{i0} + \bar{x}\left(1 - e^{-\eta_i t}\right) + \sigma e^{-\eta_i t}B_{it}^{\eta_i}\right) \tag{25}$$

where $B_{it}^{\eta_i} = \tilde{Z}_{\frac{e^{2\eta_i t}-1}{2\eta_i}}$ is a time-changed Brownian motion.[21] Then we can define the aggregator for the exponential Ornstein-Uhlenbeck process via (25). Note that the project type θ_{it} is log-normally distributed.[22]

5.3.1 Privately known initial value

In the case where the initial value of the type process is the initial private information of the managers, the stochastic flow is given by

$$\phi_\theta^i(t, \theta_{i0}, B_{it}^{\eta_i}) = \frac{\theta_{it}}{\theta_{i0}}e^{-\eta_i t} \tag{26}$$

for project acceptance is defined by the line $\left[\frac{\psi_A(\mu_{A0})}{v} + \psi_B(\mu_{B0})\right]t - \frac{\theta_A}{v}$. An increase in t raises the intercept. An increase in v reduces both the intercept and the slope of this line. As a result, a larger spillover effect does not necessarily leads to more project acceptance.

[21] $B_{it}^{\eta_i}$ is a weak solution to the stochastic differential equation $dM_t = e^{\eta_i t}dZ_{it}$. The quadratic variation of M is $\langle M\rangle_t = \frac{1}{2\eta_i}(e^{2\eta_i t} - 1)$ and so by Dambis, Dubins-Schwarz theorem, $\tilde{Z}_{\frac{e^{2\eta_i t}-1}{2\eta_i}}$ is a Brownian motion equals M_t. See, theorem 4.6 in Karatzas and Shreve (1991).

[22] The process corresponds to the exponential Vasicek model in the analysis of the term structure of interest rate.

The stochastic flow (26) is the same as the stochastic flow under a geometric Brownian motion (15), except for an extra term $e^{-\eta_i t}$. As a result, the implications on the drift and volatility of the paths of capital investment are the same as in section 5.1.1, as those effects are captured by the term $\frac{\theta_{it}}{\theta_{i0}}$. The extra term $e^{-\eta_i t}$ reflects the mean-reversion speed of the relative increment of the type θ_{it}. Holding $\frac{\theta_{it}}{\theta_{i0}}$ constant, the proportional distortion is decreasing over time deterministically. Moreover, when η_i is small, the effect of the initial value θ_{i0} on the future types will be more persistent. The managers will be able to derive more information rent and hence the required distortion is large in the optimal mechanism.

Another implication of (26) is that project exclusion could be persistent, but declining over time. Consider the optimal capital allocation under (26),

$$k_{it}(\theta_0, \theta_t) = \frac{1}{H}\left[\theta_{it}\left(1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}e^{-\eta_i t}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}e^{-\eta_j t}\right)\right] \tag{27}$$

As θ_{it} and θ_{jt} must be positive, $k_{it}(\theta_0, \theta_t) = 0$ when $\max\left\{1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}e^{-\eta_i t}, 1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}e^{-\eta_j t}\right\} \leq 0$. Consider the case without spillover effect $v = 0$, then given θ_{i0}, there is a finite time $t_* = \frac{1}{\eta_i}\log\frac{\psi_i(\theta_{i0})}{\theta_{i0}}$ for which project is excluded before time t_*. A better project (with a higher θ_{i0}) will have a shorter exclusion time.

Application of Ito's lemma to (27) allows us to derive the dynamics of capital as well. One can see from (27) that the higher the η_i, the more volatile the path of capital is. This is illustrated in figure 3 where we provide a numerical example for the dynamics of capital and incentives with different, but common, persistence parameters η. In the figure, the incentives $S_{it}(\vartheta, \theta_t)$ and $S_{ijt}(\vartheta, \theta_t)$ both converge to 1 as time grows. This can be easily seen from the PPS equation:

$$S_{it}(\theta_0, \theta_t) = \frac{\theta_{it}\left(1 - \frac{1-\alpha^2}{\alpha_0^2}\frac{\psi_i(\theta_{i0})}{\theta_{i0}}e^{-\eta_i t}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}e^{-\eta_j t}\right)}{\theta_{it}\left(1 - \frac{\psi_i(\theta_{i0})}{\theta_{i0}}e^{-\eta_i t}\right) + v\theta_{jt}\left(1 - \frac{\psi_j(\theta_{j0})}{\theta_{j0}}e^{-\eta_j t}\right)} \tag{28}$$

As distortion vanishes in the long rune, we have $S_{it}(\theta_0, \theta_t) \to \frac{\theta_{it}+v\theta_{jt}}{\theta_{it}+v\theta_{jt}} = 1$. Similarly for $S_{ijt}(\theta_0, \theta_t)$. This implies that $S_{it} - S_{ijt} \to 0$, as shown in the bottom-right panel of figure 3. As a result, for projects with mean-reverting returns, the headquarters will rely on firm-level performance pay in the long run as opposed to division-level pay.[23]

[23] Bergemann and Strack (2015) analyze an optimal sequential auction with Ornstein-Uhlenbeck process. The distortion is absolute in that case.

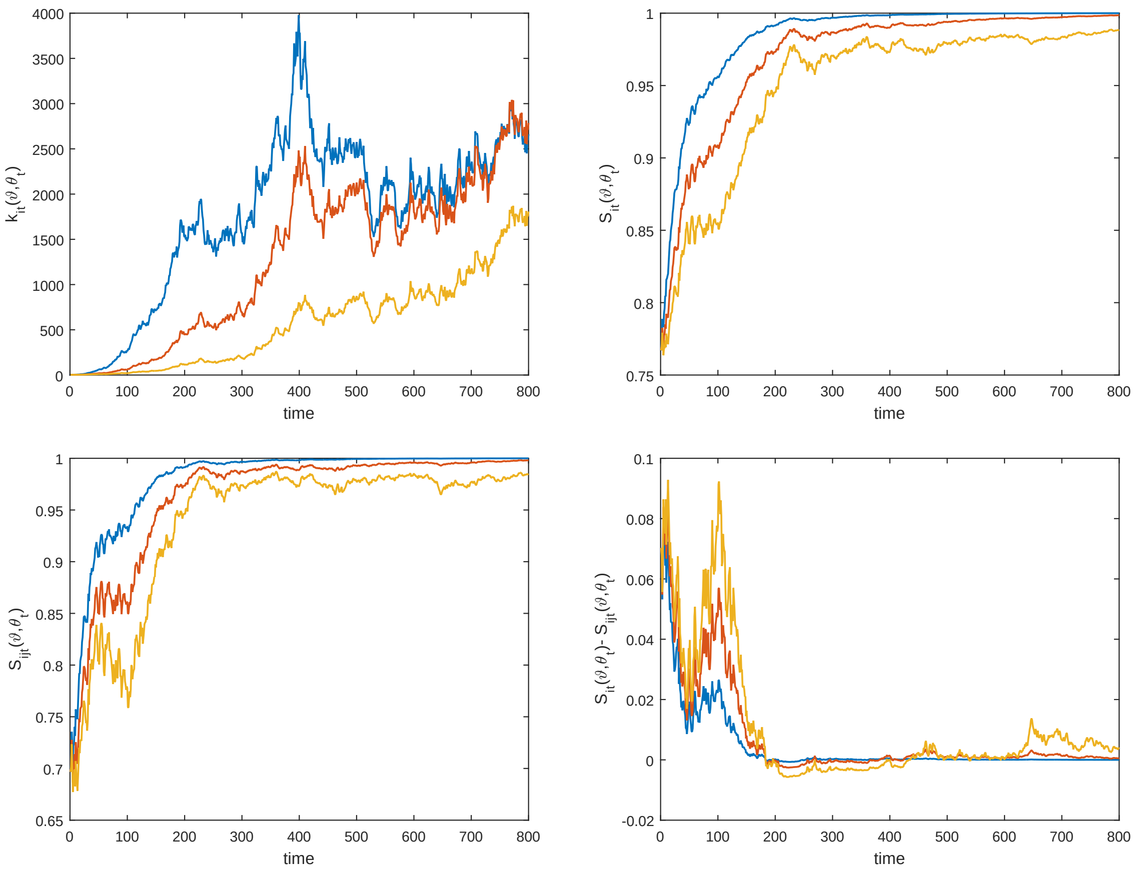

Figure 3: Dynamics of capital and incentives under geometric Ornstein-Uhlenbeck processes with different persistence. $\eta = 0.01$ (blue), $\eta = 0.005$ (red), and $\eta = 0.0025$ (yellow). The initial signals are the initial value of the processes, which are assumed to be uniformly distributed on $(0, 1)$. All the paths start with $\theta_{i0} = 0.6$ for both $i = A, B$ and are drawn holding the shocks Z_{it} constant. Other parameters: $\bar{x} = 5$, $\alpha_o = 0.8$, $\alpha = 0.5$, $v = 0.5$, and volatility $\sigma = 20\%$.

5.4 Relation with the Empirical Literature

Our model generates a number of new implications regarding the dynamics of capital and incentives in a multidivision firm. Although our analysis shows that the exact implications depend on the types of the stochastic processes that model the project evolution and the nature of initial private information, we can still draw a few general observations from the examples and compare them with the empirical literature.

Implication 1: A multidivision firm adopts a dynamic project exclusion policy: the headquarters rejects less projects over time with decreasing distortions, and it terminates or stops funding temporarily more projects over time with increasing distortions.

Decreasing distortions happen when the project types follow the geometric Ornstein-Uhlenbeck process and the initial private information is about the initial value of the processes. In our model, we can interpret underinvestment and project exclusion as capital rationing. Then the implication is consistent with early studies by Gittman and Foster (1977) and Ross (1986), who point out that firms do adopt capital rationing: they restrict capital expenditures and forgo profitable projects.

The novel implication of our model is that restrictions on capital allocation and exclusion of projects are persistent. The most direct evidence is given by Mukherjee and Hingorani (1999). In their survey of 102 Fortune 500 firms, 26.5% of the firms ration capital for one to three years, and 37.2% of the respondents adopts the same policy for more than four years. This clearly indicates that some firms operate under capital rationing for an extensive period of time. In terms the spending limits, 34% of the firms would lower the limits when it currently has low-NPV projects, for which the firms reject. And 43% of the firms would raise the investment ceiling to accommodate high-NPV projects.

Implication 2: With proportional distortion, capital investment for good (bad) projects exhibits a higher (lower) drift and a higher (lower) volatility. The drift and volatility of own-division and the other-division PPS for a manager with good (bad) project is lower (higher).

The implication holds true in the case of geometric Brownian motions and geometric Ornstein-Uhlenbeck process. With absolute distortion, we also found that persistent private information may distort the drift of the dynamics of capital investment, as in the case of arithmetic Brownian motions with the drift being the initial signal. Overall, the implication suggests that the headquarters can allocate information rents by controlling the paths of capital and incentives.[24]

Implication 3: A larger spillover effect and a higher productivity of cooperative effort implies a higher drift and volatility for capital investment in a division, as long as the type of the other division is sufficiently high.

The effect of spillover on the drift and volatility of capital dynamics can be observed from (17) and (27). As a higher v implies a greater dependence of capital allocated to division i on the other division, it magnifies the effect of the initial private information on the drift and volatility of the capital dynamics as long as both projects are sufficiently profitable. Similarly for the productivity of the help effort α.

Implication 4: Divisional investment is positively related to past cash flows of both the own and other division on expectation.

To see the time-series correlation, observe that cash flows of a division are positively related to the project types and hence capital investment. As an example, take expectation of (17) and observes that its drift is the time-t capital investment: $\mathbb{E}\left[dk_{it}(\vartheta, \theta_t)\right] = k_{it}(\vartheta, \theta_t)dt$. Thus, on expectation, the increment of divisional investment is increasing in the past cash flows of both divisions. As a result, we expect a higher divisional investment in the next moment when the current investment is large.

The implication is consistent with a recent work by Graham, Harvey, and Puri (2015). They survey more than 1,000 Chief Executive Officers (CEOs) and Chief Financial Officers (CFOs). The data shows that more than 71% of U.S. CEOs rely on manager's reputation as a criteria for capital allocation. Obviously, division manager's reputation and past cash flows are highly correlated. A more direct, although a weaker, piece of evidence is that 51% of U.S. CEOs and 64% of U.S. CFOs indicate historical return is an important decision criteria for capital allocation. Meanwhile, more than 75% of non-U.S. CFOs agree divisional return is very important.

Overall, the survey indicates divisional investment is positively correlated with the past performance of divisional manager. However, our implication suggests that if divisions collaborate on investment projects, own-divisional capital allocation should be positively correlated with other division's past

[24]Implication 2 immediately implies that the dynamics of optimal capital has lower drift and volatility compared to the first-best solution.

performance as well. In the static analysis, the positive relation between a division's investment and the cash flows in the other division appears in BCL(2004). Empirically, the works by Lamont (1997), Shin and Stulz (1998), and more recently by Fee, Hadlock, and Pierce (2008) support this result. While the empirical works focus on firms that are financially constrained, BCL(2004) and our model analyze unconstrained firm. Hence, when divisions are independent, our model predicts no time-series correlation between divisional investment and other division's performance.

The last result predicts the power of incentives will grow over time when distortion is decreasing. To state the result, we start with the following definition.

Definition (Vanishing distortion). *The type process i satisfies the property of* vanishing distortion *if for any $\epsilon > 0$, there exists t_ϵ such that, for all $t > t_\epsilon$, $\phi^i_\vartheta(t, \vartheta_i, z(t, \vartheta_i, \theta_i))\psi_i(\vartheta_i) < \epsilon$ for all $(\vartheta_i, \theta_i) \in \Theta \times \mathbb{R}$.*

The condition says that the effect of the initial signal on the future project types eventually vanishes after a sufficiently long time, and this happens over all possible values of initial signal and type uniformly.[25] The condition holds true in the example in section 5.3.1, because for all possible values of $(\theta_{i0}, \theta_{it})$ the quantity (26) decreases to 0 as $t \to \infty$.

Implication 5: If project i's type process satisfies the property of vanishing distortion, then manager i's faces a steeper incentive contract over time: $S_{it}(\vartheta, \vartheta_t)$ and $S_{ijt}(\vartheta, \theta_t)$ converge to 1 uniformly as $t \to \infty$.

Unlike implication 2, this result does not rely on the distortion being proportional or absolute. In particular, it holds when $(\theta_{it})_{t\geq 0}$ follows a plain Ornstein-Uhlenbeck process. At a practical level, the result implies that the headquarters should not use the divisional-pay in the long run because $S_{it}(\vartheta, \theta_t) - S_{ijt}(\vartheta, \theta_t)$ converges to 0 uniformly. Instead, the firm should use more firm-level pay when the impact of initial private information diminishes over time.[26]

We make two more remarks. First, note that this result on the power of incentives for a manager only requires the distortion generated by her private information being vanishing for her own

[25] Our definition is a continuous-time analogue of "vanishing impulse response" in definition 2 of Garrett and Pavan (2012). In studying long-run distortions, Bergemann and Strack (2015) use a weaker condition: $\lim_{t\to\infty} \mathbb{E}\left[\phi^i_\vartheta(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)\right] = 0$ because they are interested in the long-run expected social welfare.

[26] This also implies joint performance evaluation is important in the long rune. For empirical evidence of joint performance evaluation, a recent work by Alok and Goplan (2014) estimate that a \$1,000 increase in the other division return on assets, the pay of division manager will increase by \$0.86. They also provide evidence for the complementarity of capital, pay-for-division, and pay-for-other division performance, which is consistent with our model.

project. It does not depend on the nature of distortion for the other manager. For example, in (28), if we replace $e^{-\eta_i t}$ by 1 (as if project j has stochastic flow (15)), the convergence result for manager i still holds. Spillover effect and production complementarity at best affect the speed of convergence. Second, we do not prove the result for power of incentives when the distortion increases over time. In general, it depends on whether the increasing distortion is proportional or absolute. In the former case, as in the case of section 5.1.2, or when the type process follows a geometric Ornstein-Uhlenbeck process (24) with the initial signal being the long-run average $\bar{x}$, one can see from (25) that the stochastic flow is $\phi^i_{\bar{x}}(t, \bar{x}_i, B^{\eta_i}_{it}) = \theta_{it}(1 - e^{-\eta_i t})$, the project will be terminated in a finite deterministic time and incentives vanish. Nevertheless, with increasing absolute distortion, as θ_{it} can grow unbounded, we can make no definite prediction for the long-run incentives in this case.

6 Conclusion

In this paper we study how dynamic private information and moral hazard in team shape the capital budgeting process and managerial compensation in a firm with multiple divisions. Taking advantage of the recent development in dynamic mechanism design, we analyze a continuous-time headquarters-managers model and characterize the optimal long-term mechanism. The mechanism provides a way to design capital allocation and managerial contract in order to induce the managers to truthfully reveal their project information. Our model delivers a number of new implications concerning divisional investment and pay-performance sensitivity.

At a practical level, our analysis suggests that when the headquarters is designing its capital budgeting process, it should take into account the nature of manager's private information. Private information regarding the initial profitability or the growth rate of a project require different mechanism design. In general, the headquarters can commit to capital policy with project exclusion, termination, or dynamic investment with appropriate growth rates and sensitivity to profitability shocks. And the design of investment policy and incentive contracts needs to account for possible dependence, spillover effect, and production complementarity among the divisions.

Overall, our empirical implications can be tested against data and guide future empirical investigation into internal capital markets; and help designing capital budgeting procedures when a conglomerate firm invests in long-term projects.

Appendix

Proof of Proposition 2. Let $(\hat{\vartheta}_j, (\hat{\theta}_{js})_{s\leq t})$ be manager j's past reports up to time t. Suppose manager i with an initial signal ϑ_i reports $\hat{\vartheta}_i$ at time 0 and consistently misreports in subsequent times. At each point in time, manager i will have to hide her lies by choosing efforts in such a way that the cash flows are same as if her reported type coincides with the true one The concealment effort must satisfy

$$\alpha_o e_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + \hat{\theta}_{it} = \alpha_0 \hat{e}_{it} + \theta_{it}$$
$$\alpha e_{ijt}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + v\hat{\theta}_{it} = \alpha \hat{e}_{ijt} + v\theta_{it}$$

This is because the headquarters recommends effort $(e_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}), e_{ijt}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}))$ based on the past reports by both managers and it expects the productivity contributed by manager i to be $\alpha_o e_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + \hat{\theta}_{it}$ and $\alpha e_{ijt}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + v\hat{\theta}_{it}$ in division i and j, respectively. Otherwise, it knows that manager i has lied about her information or has not followed the recommended effort choices, and hence will impose a heavy penalty. Thus, the above pair of equation specifies possible efforts that manager i can choose to maker her reports consistent. It follows that

$$\begin{aligned}\hat{e}_{it} =& e_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + \frac{1}{\alpha_o}\left(\hat{\theta}_{it} - \theta_{it}\right)\\ \hat{e}_{ijt} =& e_{ijt}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + \frac{v}{\alpha}\left(\hat{\theta}_{it} - \theta_{it}\right)\end{aligned} \tag{29}$$

for $i = A, B$. Note that (29) implies that manager who reports truthfully at time t, $\hat{\theta}_{it} = \theta_{it}$, must follow the headquarter's recommended effort choices.

At time t, for any past reports $(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t})$, the headquarters expects to observe cash flows

$$\pi_{it}\left((\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t})\right) = \left(\alpha_o e_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + e_{jit}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}) + \hat{\theta}_{it} + v\hat{\theta}_{jt}\right) k_{it}(\hat{\vartheta}, (\hat{\theta}_s)_{s<t})$$

with division i under the concealment efforts (29). Note that this expected cash flows is independent of the true type. Now let $U_i(\vartheta_i; \hat{\vartheta}_i)$ be the payoff of manager i with an initial signal ϑ_i but reports $\hat{\vartheta}_i$ at time 0 and consistently misreports afterward. We can write it as

$$U_i(\vartheta_i; \hat{\vartheta}_i) = \mathbb{E}^{\vartheta_i}\left[\int_0^\infty e^{-rt}\left(w_{it}\left(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}, (\hat{\pi}_s)_{s\leq t}\right) - h(\hat{e}_{it}, \hat{e}_{ijt})\right) dt\right] \tag{30}$$

where $\hat{\pi}_t = \left(\pi_{At}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t}), \pi_{Bt}(\hat{\vartheta}, (\hat{\theta}_s)_{s\leq t})\right)$. Using $\theta_{it} = \phi^i(t, \vartheta_i, Z_{it})$, the partial derivative of $U_i(\vartheta_i; \hat{\vartheta}_i)$

with respect to ϑ_i is

$$\begin{aligned}\frac{\partial}{\partial\vartheta_i}U(\vartheta_i;\hat{\vartheta}_i) =&\frac{\partial}{\partial\vartheta_i}\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}\left(w_{it}\left(\hat{\vartheta},(\hat{\theta}_s)_{s\le t},(\hat{\pi}_s)_{s\le t}\right)-h(\hat{e}_{it},\hat{e}_{ijt})\right)dt\right]\\ =&\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}\left(-\frac{\partial}{\partial\vartheta_i}h(\hat{e}_{it},\hat{e}_{ijt})\right)dt\right]\\ =&\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}\phi^i_\vartheta(t,\vartheta_i,Z_{it})\left(\frac{e_{it}(\hat{\vartheta},(\hat{\theta}_s)_{s\le t})}{\alpha_o}+\frac{ve_{ijt}(\hat{\vartheta},(\hat{\theta}_s)_{s\le t})}{\alpha}\right)dt\right]\end{aligned}$$

By finite expected impact of initial signal, and with appropriate boundedness condition on e_i and e_{ij}, $\frac{\partial}{\partial\vartheta_i}U(\vartheta_i;\hat{\vartheta}_i)$ is bounded and $U_i(\vartheta_i;\hat{\vartheta}_i)$ is absolutely continuous in ϑ_i. Note that $V_i^\Gamma(\vartheta_i)=\sup_{\hat{\vartheta}_i}U_i(\vartheta_i;\hat{\vartheta}_i)$. Then application of theorem 2 in Milgrom and Segal (2002) delivers the envelope condition (6).■

Proof of Proposition 3. By definition of the value function,

$$\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}w_{it}dt\right]=V_i^\Gamma(\vartheta_i)+\mathbb{E}_{\vartheta_i}\left[\int_0^\infty e^{-rt}h(e_{it},e_{ijt})dt\right]$$

Using this, we can eliminate the wages in the headquarter's expected discounted profits (3),

$$\int_{\Theta^2}\mathbb{E}_\vartheta\left[\int_0^\infty e^{-rt}\sum_i\left(\pi_{it}-c(k_{it})-h(e_{it},e_{ijt})\right)dt\right]dG(\vartheta)-\sum_i\int_\Theta V_i^\Gamma(\vartheta_i)dG_i(\vartheta_i) \tag{31}$$

Evaluating the second term in (31),

$$\int_\Theta V_i^\Gamma(\vartheta_i)dG_i(\vartheta_i)=V_i^\Gamma(\bar{\vartheta})-\int_{\underline{\vartheta}}^{\bar{\vartheta}}\frac{\partial V_i^\Gamma(\vartheta_i)}{\partial\vartheta_i}G_i(\vartheta_i)d\vartheta_i=\int_{\underline{\vartheta}}^{\bar{\vartheta}}\frac{\partial V_i^\Gamma(\vartheta_i)}{\partial\vartheta_i}\frac{1-G_i(\vartheta_i)}{g_i(\vartheta_i)}g_i(\vartheta_i)d\vartheta_i+V_i^\Gamma(\underline{\vartheta})$$

where the first equality uses integration by parts, and the second equality uses the fundamental theorem of calculus. Substituting (6) into this expression, then (31) becomes the dynamic virtual surplus (8).

Point-wise maximization of (8) leads to the following system of first-order conditions with respect to capital, own effort, and help effort:

$$\alpha_o e_{it}+\alpha e_{ijt}+\theta_{it}+v\theta_{jt}=k_{it}$$

$$\alpha_o k_{it}=e_{it}+\frac{1}{\alpha_o}\phi^i_\vartheta(t,\vartheta_i,Z_{it})\psi_i(\vartheta_i)$$

$$\alpha k_{jt}=e_{ijt}+\frac{v}{\alpha}\phi^i_\vartheta(t,\vartheta_i,Z_{it})\psi_i(\vartheta_i)$$

for both $i=A,B$. The solution to the system is the optimal allocation.■

Proof of Proposition 5. Fix an optimal allocation $(k_{it}(\vartheta,\theta_t),e_{it}(\vartheta,\theta_t),e_{ijt}(\vartheta,\theta_t))_{t\ge0}$. Since the allocation depends only on the initial signals ϑ and time-t types θ_t. The proof consists of two steps.

Step 1. *Incentives to report θ_{it} for all $t > 0$.* Let the flow wages be linear in cash flows, that is, let $\bar{w}_{it}(\vartheta, \theta_t, \pi_t) = b_{it}(\vartheta, \theta_t) + S_{it}(\vartheta, \theta_t)\pi_{it} + S_{ijt}(\vartheta, \theta_t)\pi_{jt}$, where for both managers, the share terms are defined as

$$S_{it}(\vartheta, \theta_t) = \begin{cases} \frac{e_{it}(\vartheta,\theta_t)}{\alpha_o k_{it}(\vartheta,\theta_t)} & \text{if } k_{it}(\vartheta, \theta_t) > 0 \\ 0 & \text{otherwise} \end{cases}; \; S_{ijt}(\vartheta, \theta_t) = \begin{cases} \frac{e_{ijt}(\vartheta,\theta_t)}{\alpha k_{jt}(\vartheta,\theta_t)} & \text{if } k_{jt}(\vartheta, \theta_t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

and choose $b_{it}(\vartheta, \theta_t)$ such that division manager i's time-t payoff equals her time-t information rent under the optimal allocation:

$$\begin{aligned} b_{it}(\vartheta, \theta_t) =& h\left(e_{it}(\vartheta, \theta_t), e_{ijt}(\vartheta, \theta_t)\right) - S_{it}(\vartheta, \theta_t)\pi_{it}(\theta_t; \theta_t) - S_{ijt}(\vartheta, \theta_t)\pi_{jt}(\theta_t; \theta_t) \\ &+ \int_{\underline{\theta}}^{\hat{\theta}_{it}} \left(\frac{e_{it}(\vartheta, \tilde{\theta}_{it}, \theta_{jt})}{\alpha_o} + \frac{v e_{ijt}(\vartheta, \tilde{\theta}_{it}, \theta_{jt})}{\alpha} \right) d\tilde{\theta}_{it} \end{aligned} \tag{33}$$

where $\pi_{it}(\theta_t; \hat{\theta}_t) = \left(\alpha_o e_{it}(\vartheta, \hat{\theta}_t) + \alpha e_{ijt}(\vartheta, \hat{\theta}_t) + \theta_{it} + v\theta_{jt}\right) k_{it}(\vartheta, \hat{\theta}_t)$ is the time-t cash flows when the managers report $\hat{\theta}_t$, given the true project type is θ_t. In what follows, we drop the notation ϑ. Let $u_{it}(\theta_{it}; \hat{\theta}_{it})$ be a type-θ_{it} manager i's time-t payoff if she reports $\hat{\theta}_{it}$. Given the above linear contract and allocation,

$$u_{it}(\theta_{it}; \hat{\theta}_{it}) = b_{it}(\hat{\theta}_t) + S_{it}(\hat{\theta}_t)\pi_{it}(\theta_{it}, \hat{\theta}_{jt}; \hat{\theta}_t) + S_{ijt}(\hat{\theta}_t)\pi_{jt}(\theta_{it}, \hat{\theta}_{jt}; \hat{\theta}_t) \tag{34}$$

when a type-$\hat{\theta}_{jt}$ manager j reports truthfully at time t. Fix manager j's type $\hat{\theta}_{jt}$, incentive compatibility therefore requires that for all manager i and $t > 0$, and $\theta_{it}, \hat{\theta}_{it}$,

$$u_{it}(\theta_{it}; \theta_{it}) \geq u_{it}(\theta_{it}; \hat{\theta}_{it}) \tag{35}$$

when the other manager j is truthful. Now we verify (35) under the linear wage contract.

$$\begin{aligned} u_{it}(\theta_{it}; \theta_{it}) \geq& u_{it}(\theta_{it}; \theta_{it}) + \int_{\underline{\theta}}^{\theta_{it}} \left[\frac{1}{\alpha_o} \left(e_{it}(\hat{\theta}_t) - e_{it}(\tilde{\theta}_{it}, \hat{\theta}_{jt}) \right) + \frac{v}{\alpha} \left(e_{ijt}(\hat{\theta}_t) - e_{ijt}(\tilde{\theta}_{it}, \hat{\theta}_{jt}) \right) \right] d\tilde{\theta}_{it} \\ =& \int_{\hat{\theta}_{it}}^{\theta_{it}} \left(\frac{e_{it}(\hat{\theta}_t)}{\alpha_o} + \frac{v e_{ijt}(\hat{\theta}_t)}{\alpha} \right) d\tilde{\theta}_{it} + \int_{\underline{\theta}}^{\hat{\theta}_{it}} \left(\frac{e_{it}(\tilde{\theta}_{it}, \hat{\theta}_{jt})}{\alpha_o} + \frac{v e_{ijt}(\tilde{\theta}_{it}, \hat{\theta}_{jt})}{\alpha} \right) d\tilde{\theta}_{it} \\ =& \left(\frac{e_{it}(\hat{\theta}_t)}{\alpha_o} + \frac{v e_{ijt}(\hat{\theta}_t)}{\alpha} \right) (\theta_{it} - \hat{\theta}_{it}) + u_{it}(\hat{\theta}_{it}; \hat{\theta}_{it}) \\ =& S_{it}(\hat{\theta}_t)(\theta_{it} - \hat{\theta}_{it}) k_{it}(\hat{\theta}_{it}) + v S_{ijt}(\hat{\theta}_{it})(\theta_{it} - \hat{\theta}_{it}) + u_{it}(\hat{\theta}_{it}; \hat{\theta}_{it}) \\ =& u_{it}(\theta_{it}; \hat{\theta}_{it}) \end{aligned}$$

In the first line, the second term on the right-hand side is negative by the monotonicity of the allocation in $\hat{\theta}_{it}$. This implies the first inequality. The second and third equalities use the definition of the bonus payment (33) and the time-t payoff (34). The second last line uses the definition of shares (32), and observe that using (34) again, the last line follows.

Step 2. *Incentives to report the initial signal ϑ_i.* Define a lump sum payment $\beta_i(\vartheta)$ to be spread out time, so that $B_{it}(\vartheta,\theta_t) \equiv b_{it}(\vartheta,\theta_t) + \beta_i(\vartheta)$, and $w_{it}(\vartheta,\theta_t,\pi_t) \equiv \bar{w}_{it}(\vartheta,\theta_t,\pi_t) + \beta_i(\vartheta)$. Specifically,

$$\begin{aligned}\beta_i(\vartheta) = &\int_{\underline{\vartheta}}^{\vartheta_i} \mathbb{E}\left[\int_0^\infty re^{-rt}\phi^i_\vartheta(t,\tilde{\vartheta}_i,Z_{it})\left(\frac{e_{it}(\tilde{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha_o} + \frac{ve_{ijt}(\tilde{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha}\right)dt\right]d\tilde{\vartheta}_i \\ &- \mathbb{E}\left[\int_0^\infty re^{-rt}\left(\int_{\underline{\theta}}^{\theta_{it}}\left(\frac{e_{it}(\vartheta,\tilde{\theta}_{it},\theta_{jt})}{\alpha_o} + \frac{ve_{ijt}(\vartheta,\tilde{\theta}_{it},\theta_{jt})}{\alpha}\right)d\tilde{\theta}_{it}\right)dt\right] \end{aligned} \tag{36}$$

Let $u_{i0}(\vartheta_i;\hat{\vartheta}_i)$ be the expected discounted payoff of a manager, under the contract in step 1, who has an initial signal ϑ_i but reports $\hat{\vartheta}_i$ at time 0 and continue to report truthfully her type θ_{it} in the future times. Now we show that under the linear contract, it is incentive compatible for manager i to report truthfully her initial signal ϑ_i. To do this, we use a variant of proposition 4 in Bergemann and Strack (2015):

Proposition (Bergemann and Strack, 2015). *Let $\Theta \subset \mathbb{R}$ and $u_{i0} : \Theta \times \Theta \to \mathbb{R}$ be absolutely continuous in the first variable with weak derivative $\frac{\partial u_{i0}(\vartheta_i,\hat{\vartheta}_i)}{\partial \vartheta_i}$. Also, let $\frac{\partial u_{i0}(\vartheta_i,\hat{\vartheta}_i)}{\partial \vartheta_i}$ be increasing in the second variable. Then the payment*

$$P_i(\vartheta) = \int_{\underline{\vartheta}}^{\vartheta_i} \frac{\partial u_{i0}(\tilde{\vartheta}_i,\tilde{\vartheta})}{\partial \vartheta_i} d\tilde{\vartheta}_i - u_{i0}(\vartheta_i,\vartheta_i)$$

ensures that truth-telling is optimal for any ϑ_j.

Given the flow wages w_{it} in step 1, the managers have incentives to report truthfully their project types θ_t at all times. By definition of $u_{i0}(\vartheta_i;\hat{\vartheta}_i)$, we have

$$u_{i0}(\vartheta_i;\hat{\vartheta}_i) = \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\underbrace{\bar{w}_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t,\pi_{it}) - h(e_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t),e_{ijt}(\hat{\vartheta}_i,\vartheta_j,\theta_t)}_{=u_{it}(\phi^i(t,\vartheta_i,Z_{it});\theta_{it})}\right)dt\right] \tag{37}$$

As the manager reports truthfully for all $t > 0$, Envelope theorem applies to $u_{it}(\phi^i(t,\vartheta_i,Z_{it});\theta_{it})$ and allows us to compute $\frac{\partial u_{it}}{\partial \theta_{it}}$, so

$$\begin{aligned}\frac{\partial u_{it}}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial \vartheta_i} &= \Big(S_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t)k_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t) + vS_{ijt}(\hat{\vartheta}_i,\vartheta_j,\theta_t)k_{jt}(\hat{\vartheta}_i,\vartheta_j,\theta_t)\Big)\phi^i_\vartheta(t,\vartheta_i,Z_{it}) \\ &= \phi^i_\vartheta(t,\vartheta_i,Z_{it})\left(\frac{e_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha_o} + \frac{ve_{ijt}(\hat{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha}\right)\end{aligned} \tag{38}$$

where the first equality holds because a change in ϑ_i affects θ_{it} through its effect on the aggregator ϕ^i, which in turns affect the time-t cash flows. The second equality uses the shares (32). Differentiating (37) with respect to ϑ_i and applies (38), we have

$$\frac{\partial u_{i0}(\vartheta_i;\hat{\vartheta}_i)}{\partial \vartheta_i} = \mathbb{E}\left[\int_0^\infty e^{-rt}\phi^i_\vartheta(t,\vartheta_i,Z_{it})\left(\frac{e_{it}(\hat{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha_o}+\frac{ve_{ijt}(\hat{\vartheta}_i,\vartheta_j,\theta_t)}{\alpha}\right)dt\right]$$

By ex-post monotonicity, the term inside the expectation operator is increasing in $\hat{\vartheta}_i$, as a result $\frac{\partial u_{i0}(\vartheta_i;\hat{\vartheta}_i)}{\partial \vartheta_i}$ is increasing in the second variable. Note that $u_{it}(\phi^i(t,\vartheta_i,Z_{it});\theta_{it}) = \int_{\underline{\theta}}^{\theta_{it}}\left(\frac{e_{it}(\vartheta,\tilde{\theta}_{it},\theta_{jt})}{\alpha_o}+\frac{ve_{ijt}(\vartheta,\tilde{\theta}_{it},\theta_{jt})}{\alpha}\right)d\tilde{\theta}_{it}$ by (33) and (34). Then proposition (BS, 2015) implies that the payment $P_i(\vartheta)=\frac{\beta_i(\vartheta)}{r}$ will ensure the incentive for the manager to truthfully report the initial signal, regardless of ϑ_j. Observe that $\beta_i(\vartheta)$ does not depend on the future types θ_t, thus it does not affect manager i's incentive to report her type at all subsequent times.■

Proof of Corollary. Limited liability requires $w_{it}(\vartheta,\theta_t)\geq 0$ for all ϑ,θ_t. Under the linear contracts of proposition 5, at $\vartheta_i=\underline{\vartheta}$ and $\theta_{it}=\underline{\theta}$ and if efforts are zero at time t, then

$$w_{it} = -\mathbb{E}_{\underline{\vartheta}}\left[\int_0^\infty re^{-rt}\int_{\underline{\theta}}^{\theta_{it}}\left(\frac{e_{it}(\underline{\vartheta},\vartheta_j,\tilde{\theta}_t)}{\alpha_o}+\frac{ve_{ijt}(\underline{\vartheta},\vartheta_j,\tilde{\theta}_t)}{\alpha}\right)d\tilde{\theta}_{it}dt\right]\leq 0$$

A sufficient condition for limited liability to hold is to assume there is a flow of fixed effort cost c not too low:

$$c\geq \mathbb{E}_{\bar{\vartheta}}\left[\int_0^\infty re^{-rt}\int_{\underline{\theta}}^{\theta_{it}}\left(\frac{e_{it}(\bar{\vartheta},\vartheta_j,\tilde{\theta}_t)}{\alpha_o}+\frac{ve_{ijt}(\bar{\vartheta},\vartheta_j,\tilde{\theta}_t)}{\alpha}\right)d\tilde{\theta}_{it}dt\right]$$

To satisfy the managers' participation constraint, the headquarters needs to shift w_{it} in Proposition 5 upward uniformly by c.■

Proof of Implication 2. For simplicity consider $v=0$. Suppose with proportional distortion, the generalized stochastic flow can be written as

$$\phi^i_\vartheta(t,\vartheta_i,z(t,\vartheta_i,\theta_{it})) = \theta_{it}\Phi^i(t,\vartheta_i)$$

for some differentiable function $\Phi^i(t,\vartheta_i)$ weakly decreasing in ϑ_i. Increasing (decreasing) distortion is defined by $\Phi^i_t(t,\vartheta_i)>0$ (<0). It follows that $k_{it}(\vartheta_i,\theta_{it})=\frac{\theta_{it}}{H}\left[1-\Phi^i(t,\vartheta_i)\psi_i(\vartheta_i)\right]$ and its dynamics satisfies

$$dk_{it}(\vartheta_i,\theta_{it}) = \frac{\mu(\theta_{it})}{H}\left[1-\Phi^i(t,\vartheta_i)\psi_i(\vartheta_i)\right]dt+\frac{\sigma(\theta_{it})}{H}\left[1-\Phi^i(t,\vartheta_i)\psi_i(\vartheta_i)\right]dZ_{it}$$

where $\mu(\theta_{it})$ and $\sigma(\theta_{it})$ is the drift and volatility of a general diffusion process θ_{it} that admits proportional distortion. Therefore, the fact that $\Phi^i(t,\vartheta_i)$ weakly decreases in ϑ_i and the regularity on the inverse hazard rate implies a lower drift and volatility with a higher initial signal ϑ_i.■

Proof of Implication 5. Using (9) and (10) and the definition of $S_{it}(\vartheta,\theta_t)$, at any states without project exclusion, we have

$$1 - S_{it}(\vartheta, \theta_t) = \frac{\phi^i_\vartheta(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)}{\alpha_o k_{it}(\vartheta, \theta_t)}$$

Note that $k_{it}(\vartheta, \theta_t)$ needs not converge, but $\phi^i_\vartheta(t, \vartheta_i, Z_{it})$ converges to 0 uniformly. Therefore, vanishing distortion implies $S_{it}(\vartheta, \theta_t)$ converges to 1 uniformly. Similarly, using (11) and (9) imply

$$1 - S_{ijt}(\vartheta, \theta_t) = \frac{\phi^i_\vartheta(t, \vartheta_i, Z_{it})\psi_i(\vartheta_i)}{\alpha k_{jt}(\vartheta, \theta_t)}$$

Therefore, vanishing distortion implies $S_{ijt}(\vartheta, \theta_t)$ converges to 1 uniformly as well.■

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