Overlapping Information Production about Asset-Backed Securitizations

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Abstract

In this paper we study competition in the private information production and trading by large strategic traders in securities backed by a pool of assets, such as mortgage-backed pass-through securities (MBS) or other types of asset-backed securities (ABS). Our model demonstrates that when the correlation in payoffs of the individual assets in the pool is low and, consequently, the risk-diversification effect of pooling is large, strategic traders optimally prefer to remain less informed in equilibrium, even though their dollar expenditures on information production may in fact increase. When there are multiple strategic traders competing to produce private information, ABS and MBS security prices become even less informative compared to a single-trader case, despite the higher aggressiveness of trading on private information. This paper justifies low levels of adverse selection in the secondary markets for securitized ABS and MBS instruments in normal times, and provides a framework to analyze how private information production about asset-backed securities responds to changes in the economic environment.

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Introduction

Adverse selection reduces amount of trade for securitizations that are information-sensitive. Costly information production may provide valuable private information about the fundamental value of security tranches and aggravate adverse selection. On the other hand, when private information production incentives are weak, information-sensitive securities by their initial design can emerge as ones having low levels of adverse selection in equilibrium. In this paper we study competition in the information production by large strategic traders in pass-through securities backed by a pool of assets, such as mortgage-backed pass-through securities (MBS) and other types of asset-backed securities (ABS). We demonstrate how the correlation of underlying assets in the pool and the degree of overlapping information across traders affect dramatically the information production incentives and market quality.

This paper presents a model in which multiple risk-neutral traders compete in private information production about a pass-through security backed by a large pool of correlated assets. Our focus on a pass-through security offers a useful simplification to the exposition, however our key results generalize naturally to more complex tranche designs. Traders have access to a particular research technology that describes a particular sequence in which individual underlying assets in the pool are reflected in the private information set of each trader. This results in several scenarios of overlapping information across traders, and affects greatly the economic incentives of private information production. We argue that these research technologies apply naturally in the context of asset-backed securitizations. Finally, the trading profits of market participants are determined by a one-period strategic trading model, as in Kyle [1985].

Dang, Gorton, and Holmstrom [2009] show that even such an information-insensitive security as debt can become information-sensitive when a bad enough shock triggers private information production. We present a model of such mechanism and how it works in the context of securitizations. In normal times, when the correlation in payoffs of the individual assets in the pool is low and the risk-diversification effect of pooling is high, traders optimally prefer to remain less informed even though their dollar expenditures on information production may increase. When multiple strategic traders compete to produce private information, ABS and MBS security prices are likely to become less informative compared to a single-trader case, despite the higher aggressiveness of trading due to such competition. In contrast, in crisis times, when assets in the pool are more exposed to systematic shocks and the risk-diversification effect of pooling is low, there are two opposite effects on market liquidity: on the one hand, cheaper production of private information tends to reduce market liquidity, on the other hand, traders face tougher competition from their peers who used to stay away from information production and now trade more aggressively. Our framework allows to analyze how the intensive margin of the private information production about asset-backed securities responds to such changes in the economic environment.

Our paper relates to several stands of literature on securitization markets and strategic information acquisition. The literature has shown that the quality of private information is an important determinant for both the security design and tranching, as well as secondary market liquidity. DeMarzo [2005] develops a model of asset securitization process in an environment with asymmetric information. Informed intermediaries prefer to construct information-insensitive security tranches backed by a pool of assets when the risk diversification effect of pooling the assets together outweighs the loss of asset-specific private information intermediaries might possess. In this paper we endogenize the asset-specific private information of intermediaries. Glaeser and Kallal [1997] study the incentives of intermediaries in the pass-through MBS markets to acquire noisy signals about the true security value, with a particular focus on the information restrictions by original security issuers. Our paper provides the analysis of the intensive margin of information production and the associated Cournot competition of producing research about the underlying assets in the pool. Subrahmanyam [1991] shows that the adverse selection is typically weaker in a composite basket security compared to individual securities because of the diversification benefit provided by informed traders and the heavier liquidity trading. Security-specific analysts have higher incentives to acquire private information when the security is heavily weighted in the basket and vice versa. Our analysis shows why adverse selection may be further weakened by the overlapping information traders acquire independently of each other.

On another side there is both theoretical and empirical literature studying how number of analysts covering stocks affects market depth (Brennan and Subrahmanyam [1995], Holden and Subrahmanyam [1992] and others). We complement this literature by studying the optimal level of information precision of each analyst when the structure of information is granular, as in the case of asset-backed pass-through securities. Dierker [2006] studies Cournot competition in information acquisition by strategic traders who make uncorrelated forecast errors about the asset fundamental value as in Verrecchia [1982]. In this paper we extend the set of research technologies available to traders, and we argue that our extension is particularly important in the context of asset-backed securities.

The structure of the paper is as follows. In section 1 we present our model of research technologies that traders use in order to acquire information about underlying assets. The key novelty is that our research technologies allow for different degrees of overlapping information traders acquire about asset fundamentals. In section 2 we present the trading environment with multiple strategic traders and derive traders' equilibrium research efforts. In section 3 we analyze the market quality implications of endogenous information production for price informativeness, market depth, price and order flow volatility. Section 4 concludes. Proofs of the propositions are in the Appendix.

1 Structure of Information and Research Technologies

1.1 Research Technologies

In the model some traders have access to a research technology that allows them to produce private information about the underlying pool of assets. The research technology describes the relationship between traders' expenditures on information production and the quality of private information they get.

Suppose there is a pass-through security with the true fundamental value v, and traders strategically produce additional signals s_i that are correlated with v. Using this notation we can define a research technology as:

Definition 1.1. *A* research technology *is a mapping from traders' expenditures on research* $\mathbf{c} = (c_1 \dots c_N)$ *to the joint distribution of their signals and the fundamental value of a security* v*:* $\mathcal{G}(v, s_1 \dots s_N)$*, where* s_i *is the signal i-th informed trader produces,* $i \in \{1 \dots N\}$*.*

Example. In Verrecchia [1982] each informed trader observes the true fundamental security value $v \sim N(0, \Sigma_0)$ perturbed by a normally-distributed forecast error ϵ_i with zero mean and precision $\gamma_i = 1/\sigma_i^2 \geq 0$. Forecast errors are independent across traders. Traders can choose different levels of precision γ of their forecast errors ϵ_i and there is an associated cost function $TC(\gamma)$. The research technology would be a mapping from traders' expenditures $(c_1 \dots c_N)$ to the distribution of signals $(v, s_1 \dots s_N) \sim N(0, \Psi)$, where:

$$\Psi = \begin{pmatrix} \Sigma_0 & \Sigma_0 & \cdots & \Sigma_0 \\ \Sigma_0 & \Sigma_0 + 1/TC^{-1}(c_1) & \cdots & \Sigma_0 \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_0 & \Sigma_0 & \cdots & \Sigma_0 + 1/TC^{-1}(c_N) \end{pmatrix}$$

1.2 Structure of Information

For the purpose of our analysis we focus on a pass-through security as it provides a good illustration of the forces that drive competition in the information production. A pass-through security derives its value from a large underlying pool of assets, which can be individual mortgages, auto loans, credit card accounts or student loans, and all cash-flows from these assets are passed directly to the holder of the security. Our methodology can be extended to more complex derivative instruments when they are backed by a pool of homogenous assets.

Suppose the true fundamental value of a pass-through security is v and there are M underlying assets in the pool. Each asset in the pool has a fundamental value e_i where $i \in \{1, \cdots, M\}$, which contributes to the fundamental value v of the pass-through security. The underlying assets in the pool are homogenous and their fundamental values e_i are identically distributed and can be positively correlated.[1]

$$v = \sum_{i=1}^{M} e_i \tag{1}$$

We assume that it is equally costly to "poke around suburbs"[2] and learn about the performance of each individual asset in the underlying pool, so the total expenditure on information production is proportional to the number of assets under study. This assumption however does not preclude diminishing marginal returns to a dollar spent on producing information. Here we want to stress the importance of the possibility that assets in the pool have positively correlated fundamental values e_i. When the fundamental values of individual assets in the pool are positively correlated, traders can use the information about some of the assets in the pool to partially predict the performance of the other assets for free. Thus the marginal value of the first dollar spent on

[1] As $M \to \infty$ our description converges to a large homogenous portfolio (LHP) and the approximation techniques used in the credit risk literature, e.g. as in Vasicek [2002]. However our focus is different, as we are not modelling the credit risk of the pass-through security. In reality, most traditional pass-through security are insured against default by government-sponsored agencies.

[2] Lewis [2010] Liar's Poker. WW Norton&Company, 2010.

information production can be substantially higher than the rest.

The informative signal s_i is defined as the partial sum of the fundamental values of individual assets in the pool. When a trader uses his research technology to reveal the performance of $\omega_i \times M$ assets in the pool, the informative signal is:

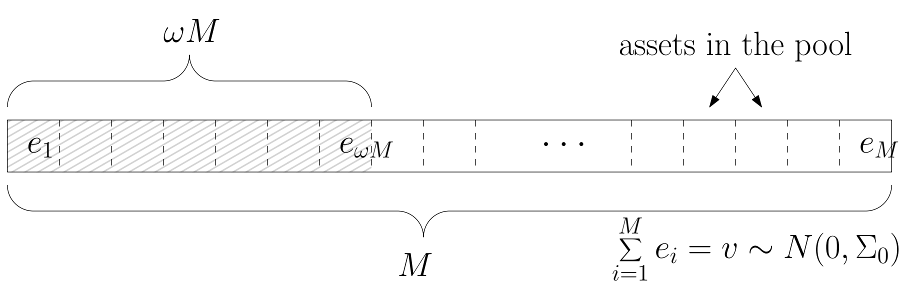
$$s_i = \sum_{i=1}^{\omega_i \times M} e_i \tag{2}$$

We will refer to ω_i as the research effort of trader i. Note that producing information about 50 mortgages out of the total $M = 100$ mortgages in the pool is the same as producing information about 100 mortgages out of the total $M = 200$ mortgages in the pool—in both cases exactly half of the underlying pool is covered by the information production efforts. The proportion ω of the total size of the pool M describes the research effort of a trader. The necessary and sufficient condition for the total size of the pool M not to affect the quality of the informative signal s_i above is the infinite-divisibility condition provided below:

Infinite-Divisibility Condition. *When M increases, the covariance of any two fundamentals e_i and e_j decreases at a rate M^2, so that $Cov(e_i, e_j) \times M^2$ is a constant for any integer M.*

Figure 1 summarizes the structure of private information about the pass-through security:

Figure 1: Private signal as a sum of asset values in the pool



In what follows we assume that all fundamental values of assets in the pool e_i have a joint-Normal distribution and may be correlated. Using the infinite-divisibility condition 1.2 we characterize the distribution of traders' private signals s_i:

Proposition 1.1. *Suppose that trader i exerts a research effort $\omega_i \in [0, 1]$ and obtains a private* *signal s_i. Assume the infinite-divisibility condition and let $\rho = Cov(e_i, e_j) \times M^2/\Sigma_0$. Then:*

$$\begin{aligned} Var(s_i) &= \omega_i \Sigma_0 (1 - (1 - \omega_i)\rho) \\ Cov(s_i, v) &= \omega_i \Sigma_0 \end{aligned} \tag{3}$$

See Appendix A.1 for proof.

1.3 Overlapping Information

When there are multiple traders who compete and produce private information about the same pool of assets, they can overlap by studying independently the performance of the same assets in the pool. The extent of such overlapping information they get will affect the covariance of their private signals s_i.

Example: Suppose $M = 6$ and there are three traders with research efforts $\omega_1 = 1/6$, $\omega_2 = 1/3$, and $\omega_3 = 1/2$. The first agent reveals one fundamental, the second agent chooses two fundamentals, and the third chooses three. It is possible that neither of the three traders has overlapping assets with others, as when agent 1 reveals $\{e_1\}$, agent 2 reveals $\{e_2, e_3\}$ and agent 3 reveals $\{e_4, e_5, e_6\}$. We refer to this case as a *non-overlapping research.* At the other extreme, all traders may reveal fundamentals in the same order, as when trader 1 reveals $\{e_1\}$, trader 2 reveals $\{e_1, e_2\}$ and trader 3 reveals $\{e_1, e_2, e_3\}$. We refer to this case as a *perfectly-overlapping research.* The third possible scenario occurs when each trader draws an independent random sample of asset to study, and we refer to this case as an *independently-overlapping research.* Figure 2 demonstrates the three scenarios of overlapping information described above.

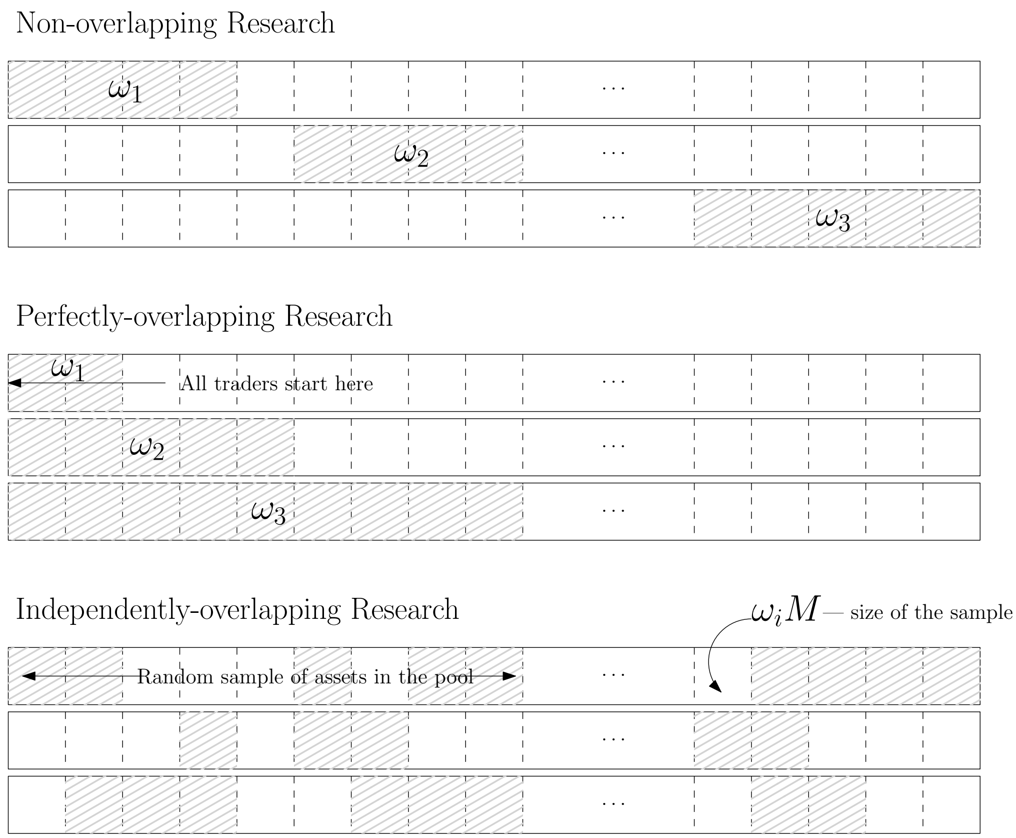
Definition 1.2. *Suppose there are N potentially informed traders on the market and ω_i is the research effort of the i-th trader. Suppose for $\forall i \in \{1 \dots N\}$ $\omega_i M$ is a Natural number. Denote by S_i the subset of M assets in the underling pool each trader decides to reveal.*

1. *Traders engage in* non-overlapping research *if $\forall i \neq j$ we have $S_i \cap S_j = \varnothing$. This can happen only if $\sum_{i=1}^{N} \omega_i \leq 1$.*

2. *Traders engage in* perfectly-overlapping research *if $\forall i \neq j$ such that $\omega_i \geq \omega_j$ we have $S_i \cap S_j = S_j$.*

3. *Traders engage in* randomly-overlapping research *if $\forall i$: S_i is a simple random sample of size $\omega_i M$.*

Figure 2: Possible Scenarios for Overlapping Information



For simplicity of exposition, consider two traders who have access to a research technology on the market, $N = 2$. Denote their research efforts by $\omega_2 \leq \omega_1 \leq 1$. The overlapping information scenario will determine the covariance of informative signals traders get, which is characterized in the following proposition:

Proposition 1.2. *Let two potentially informed traders with research efforts $\omega_1 \geq \omega_2$ obtain signals s_1 and s_2. Assume the infinite-divisibility condition holds and let $\rho = Cov(e_i, e_j) \times M^2/\Sigma_0$. Then the $Cov(s_1, s_2)$ is:*

$$Cov(s_1, s_2) = \Sigma_0 \times \rho \times \omega_1\omega_2 \leftarrow \textit{non-overlapping} \tag{4}$$

$$Cov(s_1, s_2) = \Sigma_0 \times \omega_2 \times (1 - (1 - \omega_1)\rho) \leftarrow \textit{perfectly-overlapping} \tag{5}$$

$$Cov(s_1, s_2) = \Sigma_0 \times \omega_1\omega_2 \leftarrow \textit{independently-overlapping} \tag{6}$$

See Appendix A.2 for proof.

Interestingly, we can show the equivalence of the independently-overlapping research technology and the information acquisition technology used by Verrecchia [1982] and commonly referred to in the literature on information acquisition:

Proposition 1.3. *Consider the following definition of a signal:* $\widetilde{s}_i = v + \varepsilon_i$ *where* ε_i *is trader-specific forecast error uncorrelated across traders:* $Cov(\varepsilon_i, \varepsilon_j) = 0$*. Precision of forecast error* ε_i *is proportional to the trader's expenditure on research* c_i*:* $Var(\varepsilon_i) = 1/\widetilde{TC}^{-1}(c_i)$*, where* $\widetilde{TC}(\cdot)$ *is an increasing function. This research technology is equivalent to independently-overlapping research technology defined in 1.2.*

Proof. Define w_i and $TC(w_i)$ in the following way:

$$\begin{aligned}
\omega_i &= \frac{\Sigma_0(1-\rho)\widetilde{TC}^{-1}(c_i)}{1+\Sigma_0(1-\rho)\widetilde{TC}^{-1}(c_i)} \in [0,1) \\
TC(\cdot) &: TC\left(\frac{\Sigma_0(1-\rho)\widetilde{TC}^{-1}(c_i)}{1+\Sigma_0(1-\rho)\widetilde{TC}^{-1}(c_i)}\right) = c_i
\end{aligned}$$

Then redefine each trader's signal as: $s_i = \omega_i \times \widetilde{s}_i = \omega_i \times (v + \varepsilon_i)$. Scaling by a constant preserved the information contained in trader' signal. The joint distribution of vector $(v, s_1, \cdots, s_N)$ is the same as under the independent-overlapping research technology. Thus the two research technologies are equivalent. □

The result in proposition 1.3 is an interesting interpretation of the research technology in Verrecchia [1982]. Take a pool of identically distributed fundamentals and two informed traders who choose optimally the size of the independent random samples to draw from the pool. This turns out to be equivalent to each trader observing the true value of the asset with a forecast error, independently distributed across traders. Note that our approach to information acquisition is more general and allows to study alternative scenarios for agents' overlapping information—other than *independently-overlapping*, so that the forecast errors are not independent across traders anymore.

Finally, we want to allow for the intermediate scenarios of overlapping information that occur in between the three scenarios described above. We introduce a parameter $\gamma \in [0, 1]$ that captures severity of overlapping information: $\gamma = 0$ corresponds to non-overlapping research; $\gamma = 0.5$ corresponds to independently-overlapping research; $\gamma = 1$ corresponds to perfectly-overlapping research. We use linear interpolation to describe what happens for intermediate values of γ, and the following proposition summarizes the results:

Proposition 1.4. *Let two potentially informed traders with research efforts $\omega_1 \geq \omega_2$ obtain signals s_1 and s_2. Assume the infinite-divisibility condition holds and let $\rho = Cov(e_i, e_j) \times M^2/\Sigma_0$. Let $\gamma \in [0,1]$ describe the degree of overlapping information in traders' research technologies so that for $\gamma \leq 0.5$ the linear interpolation between non-overlapping and independently-overlapping scenarios is used, while for $\gamma > 0.5$ the linear interpolation between independently-overlapping and perfectly-overlapping scenarios is used. Then the $Cov(s_1, s_2)$ is:*

$$
\begin{aligned}
Cov(s_1, s_2) &= \Sigma_0 \times \omega_1\omega_2 \times (\rho + 2\gamma(1-\rho)), \ \textit{when } \gamma \leq 0.5 \qquad (7) \\
Cov(s_1, s_2) &= \Sigma_0 \times \omega_2 \times (\rho\omega_1 + (1-\rho)(1 + 2(\gamma - 1)(1-\omega_1))), \ \textit{when } \gamma > 0.5
\end{aligned}
$$

See Appendix A.3 for proof.

1.4 The Limiting Stochastic Process

In the prior discussion we used a discrete number M of assets in the underlying pool. In this section we show that our results generalize to the limiting case when $M \to \infty$. We derive the limiting stochastic process that describes the structure of information and allows us to use any real value for trader's research effort $\omega_i \in [0,1]$.

Definition 1.3. *Suppose that the unit interval $V = [0,1]$ describes the set of individual assets in the underlying pool. Traders have research efforts $\omega_i = \mu(S_i) \in (0,1]$, where $S_i \in V$ is the subset of underlying assets trader i learns about. Let $W(t)$ be Brownian motion. Then the informative signal traders obtain is defined as:*

$$
\begin{aligned}
s_i &= \int_{S_i} dX(t), \ \textit{where:} \qquad (8) \\
dX(t) &= \frac{\rho}{(1-\rho) + \rho t} X(t) \times dt + \sqrt{\Sigma_0(1-\rho)} \times dW(t) \qquad (9)
\end{aligned}
$$

The SDE in equation 9 describes a random process $X(t)$ that captures the information structure of an infinite pool of correlated homogenous assets. The true value of a pass-through security under this notation is $v = X(1)$, which corresponds to a research effort $\omega = 1$. Now we can define the three overlapping information scenarios via the measure of intersections in traders' information sets. When there are two traders with information sets $S_i \in V$ and research efforts $\omega_i = \mu(S_i) \in (0,1]$ and $\mu(S_2) \le \mu(S_1)$, we have:

1. Non-overlapping research is defined as: $\mu(S_1 \cap S_2) = 0$

2. Perfectly-overlapping research is defined as: $\mu(S_1 \cap S_2) = \omega_2$

3. Independently-overlapping research is defined as: $\mu(S_1 \cap S_2) = \omega_1\omega_2$

The following proposition establishes equivalence between the discrete and continuous versions of informative signals defined above.[3]

Proposition 1.5. *The discrete version of an informative signal defined in equation (2) is equivalent to the continuous version defined in equation (8) in terms of its distributional properties* $Var(s_i)$, $Cov(s_i, s_j)$, *and* $Cov(s_i, v)$.
See Appendix A.4 for proof.

2 Trading Environment

In order to characterize the profitability of information production we analyze strategic trading decisions of traders on a secondary market for the pass-through security, and apply the research technologies developed above to find optimal traders' research efforts ω_i. We use Kyle [1985] model to describe the strategic trading environment with asymmetric information.

Firstly we describe traders' information production incentives in a one-period trading environment with one asset and two risk-neutral traders who have access to a research technology. We then study the robustness of our equilibrium in a setting with larger number of traders in section 3.

There is a mass of uninformed liquidity traders with no access to a research technology and a competitive price-setting mechanism, which is a market-maker who intermediates the trades.[4] The true fundamental value of the pass-through security is $v \sim N(\overline{v}_0, \Sigma_0)$. Before submitting

[3] The continuous definition in equation (8) is more general in the sense that it does not rely on $\omega_i M$ being a positive integer number. It justifies the joint-Normal distribution of signals and the true asset value v with the same covariance matrix as provided by the discrete version in propositions 1.1 and 1.2.

[4] ABS and MBS pass-through securities are traded on OTC markets with a search friction, described in Duffie, Gârleanu, and Pedersen [2005], Hugonnier, Lester, and Weill [2014] and others. However the standard pass-through securities are one of the most liquid instruments among a richer set of securitizations, so the magnitude of the search friction is not too high. Here we assume that a competitive market-maker is a reduced-form representation of a price-formation process with relatively small OTC friction, so the market structure has only second-order effects on the information production results.

trades each trader learns the value of his informative signal s_i. Then traders submit simultaneously their trades $x_i = X_i(s_i)$ to the market-maker, where $X_i(\cdot)$ is a function of s_i. Liquidity traders add their aggregate order flow, which is $u \sim N(0, \sigma_u^2)$ and is independent of all other variables. The total order flow is $\Sigma x + u = \sum_{i=1}^{2} x_i + u$ and the price is $p = P(\Sigma x + u)$, where $P(\cdot)$ is a function of the total order flow. The expected trading profits for each trader are $E(\pi_i|s_i) = E((v-p)x_i|s_i) = E\Pi_i(X_i, X_j, P)$. As in Kyle [1985], we assume that the true value of a pass-through security and the informative signals (v, s_1, s_2) are jointly-Normally distributed and we restrict our analysis to the linear pricing rule $P(\Sigma x + u)$ and linear trading strategies $X_i(s_i)$. We define the trading equilibrium:

Definition 2.1. *A* trading equilibrium *is a set of functions X_1, X_2, P that satisfies two conditions:* Profit Maximization (10): *For any trader $\forall i \in \{1, 2\}$, any alternative trading strategy X' and any realization of i-th signal s_i (the opponent's trading strategy is denoted by X_{-i}) does not yield higher expected trading profit; and* Market Efficiency (11): *Function P satisfies the fair-pricing condition below.*

$$E\Pi_i(X_i, X_{-i}, P) \geq E\Pi_i(X', X_{-i}, P) \ \forall i \in \{1, 2\} \tag{10}$$

$$P(\Sigma x + u) = E(v|\Sigma x + u) \tag{11}$$

In our information production model, traders' research efforts ω_i describe the proportion of the underlying pool of assets, traders have accurate information about. As we noted, having private information about the same proportion of the underlying pool of assets may imply different levels of traders' informativeness when assets in the pool are correlated. The amount of information each trader has depends both on the research effort ω_i and asset correlation ρ. We use the α_i notation for the amount of information each informed agent has:

$$\alpha_i = \frac{\Sigma_0 - Var(v|s_i)}{\Sigma_0} \in [0, 1] \tag{12}$$

The proposition below characterizes the trading decisions of informed traders in terms of the amount of information α_i each of the two traders has:

Proposition 2.1. *Let ρ_{12} denote the correlation between the signals s_1 and s_2 two traders obtain. There exists a unique linear trading equilibrium defined by $X_i = (\beta_i/\lambda)s_i$ for each $i \in \{1, 2\}$ and*

$P = \lambda(\Sigma x + u)$ *where constants* β_i *and* λ *are:*

$$\beta_i = \sqrt{\frac{\Sigma_0}{Var(s_i)}} \times \frac{\left(2\sqrt{\alpha_i} - \rho_{12}\sqrt{\alpha_j}\right)}{4 - (\rho_{12})^2} \tag{13}$$

$$\lambda = \sqrt{\frac{\Sigma_0}{\sigma_u^2}} \times \frac{\sqrt{\left(2\sqrt{\alpha_1} - \rho_{12}\sqrt{\alpha_2}\right)^2 + \left(2\sqrt{\alpha_2} - \rho_{12}\sqrt{\alpha_1}\right)^2}}{4 - (\rho_{12})^2} \tag{14}$$

See Appendix B.1 for proof.

2.1 Optimal Research Efforts

Above we have established that for any research efforts ω_i chosen by traders there is a unique linear trading equilibrium that arises, and is described in Proposition 2.1. In this section we assume that both agents choose their research efforts $\omega_i \in [0, 1]$ simultaneously in the beginning of the trading game. These choices are simultaneous, so the deviations are not directly observable to competitors as in the Cournot game, such as when a trader chooses a different level of research effort and obtains a signal with different properties.

Definition 2.2. *Denote by* $\Psi^\omega = \Psi(\omega_1, \omega_2)$ *the covariance matrix of signals* $s_1(\omega_1)$*,* $s_2(\omega_2)$*, and the true value of the pass-through security* v*. Denote by* $(X_1^\omega, X_2^\omega, P^\omega)$ *the unique linear trading equilibrium for* Ψ^ω*. Then research efforts* (ω_1, ω_2) *constitute* a Nash equilibrium *if* $\forall \hat{\omega} \in [0, 1]$ *and* $\forall i \in \{1, 2\}$ *the following condition holds:*

$$\begin{aligned} &\max\nolimits_{X_i} \left[E\Pi_i(X_i, X_{-i}^\omega, P^\omega)|s_i(\hat{\omega})\right] - c(\hat{\omega}) \leq \\ &\leq E\Pi_i(X_i^\omega, X_{-i}^\omega, P^\omega|s_i(\omega_i)) - c(\omega_i) \end{aligned} \tag{15}$$

Condition 15 highlights two noteworthy ideas: 1) when one informed trader chooses an off-equilibrium research effort $\hat{\omega}$, all other market participants do not observe such deviation; 2) the trader is able to reoptimize his trading strategy X_i given the new properties of the informative signal $s_i(\hat{\omega})$. In the following lemma we derive the reoptimized trading strategy for the trader who deviates and the associated expected profits.

Lemma 2.1. *Suppose that trader* i *enters the trading game with an arbitrary signal* s_i^{new}*. The opponent and the market-maker do not observe the quality of the new signal, instead they follow a given trading equilibrium for some covariance matrix* Ψ *(their belief about* s_i *is no longer consistent with the true properties of* s_i^{new}*). Under these circumstances, given* β_{-i} *and* λ *determined* *according to Proposition 2.1, trader i's optimal trading strategy is* $X_i^{\text{new}} = (\beta_i^{\text{new}}/\lambda)s_i^{\text{new}}$ *where:*

$$\beta_i^{\text{new}} = \frac{1}{2\sqrt{Var(s_i^{\text{new}})}}\left(\sqrt{\Sigma_0 \times \alpha_1^{\text{new}}} - \rho_{12}^{\text{new}} \times \sqrt{Var(s_{-i}) \times \beta_{-i}}\right) \tag{16}$$

$$E(\pi_i^{\text{new}}) = \frac{1}{4\lambda}\left(\sqrt{\Sigma_0 \times \alpha_1^{\text{new}}} - \rho_{12}^{\text{new}} \times \sqrt{Var(s_{-i}) \times \beta_{-i}}\right)^2 \tag{17}$$

See Appendix B.2 for proof.

Now we present our results on the existence and uniqueness of Nash equilibria under different research technologies and the scenarios of overlapping information. For convenience, we impose restriction on the cost function $TC(\omega)$ so that a single trader with access to a research technology never finds it optimal to acquire information about the entire pool of assets, which would correspond to $\omega = 1$. We refer to such condition as the *single-agent-interior condition*:

Single-Agent-Interior Condition. *In a model with a single trader with access to a research technology and a weakly convex cost function* $TC(\omega)$ *the trader's optimal research effort is* $\omega_1 \in (0, 1)$*. This is ensured by the following restriction on the cost function:*

$$\frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega = 1) > \frac{(1-\rho)\sqrt{\Sigma_0 \times \sigma_u^2}}{2} \tag{18}$$

We study interior equilibria with $\omega_1, \omega_2 \in (0, 1)$ as well as corner equilibria with $\omega_1 > \omega_2 = 0$. For our discussion of the model with two traders who have access to a research technology we assume $\omega_1 \geq \omega_2$. We also assume that research is equally costly to traders.

Firstly, we find that asymmetric interior Nash equilibria do not exist when research technology is characterized by low degree of overlapping information $\gamma \leq 1/2$:

Lemma 2.2. *Under a research technology characterized by low degree of overlapping information* $\gamma \leq 1/2$ *and linear trading and pricing rules, if for some linear cost function* $TC(\omega_i)$ *an interior Nash equilibrium exists with* $\omega_i \in (0, 1), \forall i \in \{1, 2\}$*, and* $\gamma \neq \frac{\rho}{2(1-\rho)}$*, then this equilibrium is symmetric with* $\omega_1 = \omega_2$*.*

See Appendix B.3 for proof.

We also study corner Nash equilibria, where one of the two traders optimally decides not to participate neither in research nor trading. These cases can represent informational barriers to entry created by the costly information production. It turns out that these barriers are particularly strong when the correlation of assets in the underlying pool is weak. The next lemma allows to characterize such cases in our model:

Lemma 2.3. *Assume a cost function $TC(\omega_i)$ that satisfies the single-agent-interior condition. Under a research technology with the degree of overlapping information γ and linear trading and pricing rules, there exists a corner Nash equilibrium with $\omega_1 = \omega^*, \omega_2 = 0$ if and only if the following holds:*

$$\rho \leq \frac{2\gamma}{1+2\gamma}, \text{ when } \gamma \leq 1/2 \tag{19}$$

$$\rho \leq \frac{(2-2\gamma)\omega^* + 2\gamma - 1}{(3-2\gamma)\omega^* + 2\gamma - 1}, \text{ when } \gamma > 1/2 \tag{20}$$

See Appendix B.4 for proof.

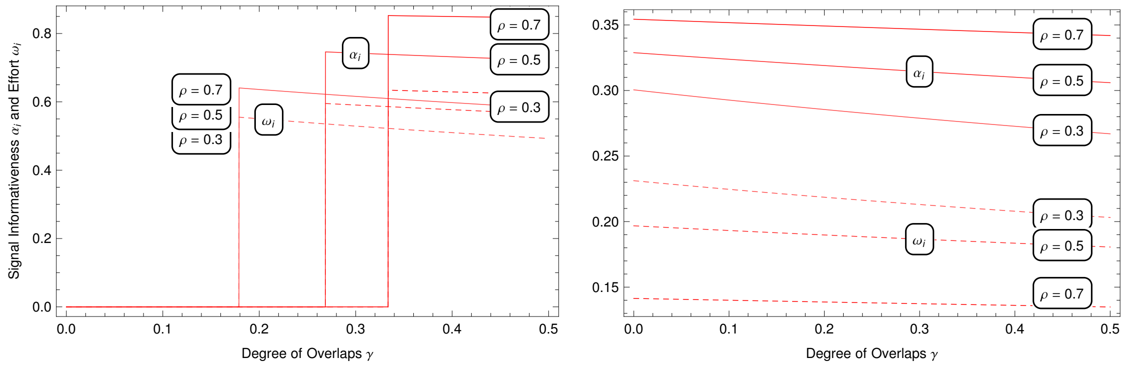
It should be noted that the condition 20 depends on the optimal research effort ω^* of a single trader. Intuitively, the less advanced trader performs a catch-up research when his research effort is lower than of his competitor, and the more advanced is the leading trader, the more assets in the underlying pool the first trader has to evaluate. This effect is especially strong when one considers the perfectly-overlapping research technology with $\gamma = 1$.

The following proposition completes the set of results on the existence and uniqueness of Nash equilibria in our information production model:

Proposition 2.2. *Assume a linear cost function $TC(\omega_i)$ that satisfies the single-agent-interior condition. Under a research technology characterized by the degree of overlapping information γ and linear trading and pricing rules: 1) There exists a Nash equilibrium when $\gamma \leq 1/2$ that is symmetric: $\omega_1 = \omega_2 \in (0, 1)$. 2) There exists a unique asymmetric Nash equilibrium when $\gamma = 1$. See Appendix B.5 for proof.*

Figures 3 and 4 illustrate traders' optimal choices of research efforts ω_i and the informational content of signals α_i. Each figure has two panels: the left panel corresponds to relatively low marginal cost of research, so that $\omega^* = 1$; the right panel corresponds to relatively high marginal costs of research, so that $\omega^* = 0.5$. Recall that ω^* is the optimal research effort of a single trader with access to a research technology.

Figure 3: Optimal Choices of Research Efforts ω_i and Signal Informativeness α_i $(\gamma \leq 1/2)$

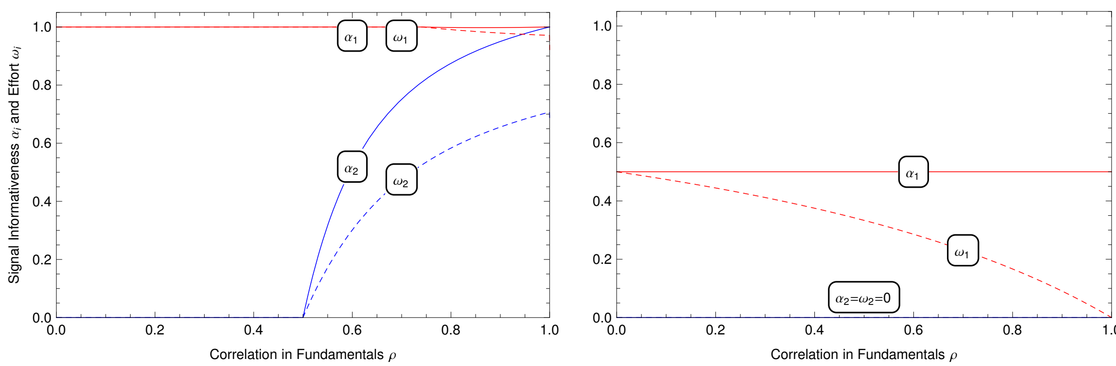


Panel A: Single-agent $\omega^* = 1$

Panel B: Single-agent $\omega^* = 0.5$

The figures illustrate that more overlapping information reduce incentives of private information production. In the case of a perfectly-overlapping research presented in figure 4 when $\rho\alpha_1 \leq 1/2$ the second agent optimally decides to stay away from producing information and trading (see appendix B.5 for details).

Figure 4: Optimal Choices of Research Efforts ω_i and Signal Informativeness α_i $(\gamma = 1)$



Panel A: Single-agent $\omega^* = 1$

Panel B: Single-agent $\omega^* = 0.5$

In the following section we study the market quality: the equilibrium price informativeness, market depth, price and volume volatility for the pass-through security.

3 The Market Quality

There are two forces that drive competition between traders who have access to a research technology. The first force is the trading aggressiveness that feeds information into prices and partially reveals private signals of traders to the general public through price movements. It can be demonstrated that two traders with identically distributed signals s_1 and s_2 reveal more information through price movements compared to a single trader with a signal s_3 such that $Var(v|s_3) = Var(v|s_1, s_2)$. The second force is the competition in information production and the resulting overlapping information traders have about the underlying pool of individual assets, which is the primary focus of our study.

Consider first the case with a single trader with access to a research technology, $N = 1$. The trading equilibrium is characterized by $\beta_1 = \frac{\sqrt{\Sigma_0}\sqrt{\alpha_1}}{2\sqrt{Var(s_i)}}$ and $\lambda = \frac{\sqrt{\Sigma_0}\sqrt{\alpha_1}}{2\sqrt{\sigma_u^2}}$. The equilibrium trading strategy is $(\beta_1/\lambda)s_1 = \sigma_u \frac{s_1}{\sqrt{Var(s_1)}}$ and its variance does not depend on neither research effort ω_1 nor informativeness of the signal α_1. The trader scales the signal so that the variance of the trade amount is equal to the variance of liquidity traders' order flow u, as described in Dierker [2006]. Also note that the price impact λ increases in α_1.

3.1 Price Informativeness

Once the trading is over, an outside person can learn information about the true value of the pass-through security by observing the price, or the total trading volume which provides the same information in the model. The informativeness of prices is measured by $\mathcal{L}$:

$$\mathcal{L} = \frac{\Sigma_0 - Var(v|\Sigma x + u)}{\Sigma_0} \in [0, 1] \tag{21}$$

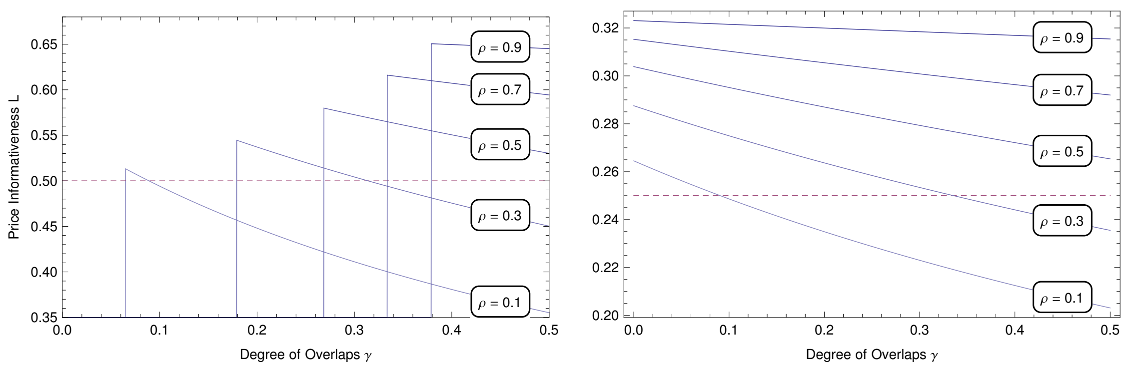
When there is a single trader with access to a research technology on the market, the equilibrium price informativeness is $\mathcal{L} = \alpha_1/2$. As described in Kyle [1985] exactly half of the insider's information is revealed through prices. The following lemma describes price informativeness $\mathcal{L}$ when two traders have access to a research technology, $N = 2$:

Lemma 3.1. *In the trading equilibrium outlined in Proposition 2.1 the equilibrium price informativeness is:*

$$\mathcal{L} = \frac{2(\alpha_1 + \alpha_2) - \rho_{12}\sqrt{\alpha_1}\sqrt{\alpha_2}}{4 - (\rho_{12})^2} \tag{22}$$

When α_1 and α_2 are sufficiently close to each other, price informativeness is higher the more information α_1 traders produce and the less correlated traders' signals ρ_{12} are. We need the model to tell us how traders' adjust their research efforts in response to a tougher competition. As it follows from our results in section 2.1 the two sets of possible overlapping information scenarios—relatively low overlapping information with $\gamma \leq 1/2$ and relatively severe overlapping information with $\gamma > 1/2$ must be considered separately given the way we specify the correlation in traders' signals in Proposition 1.4.

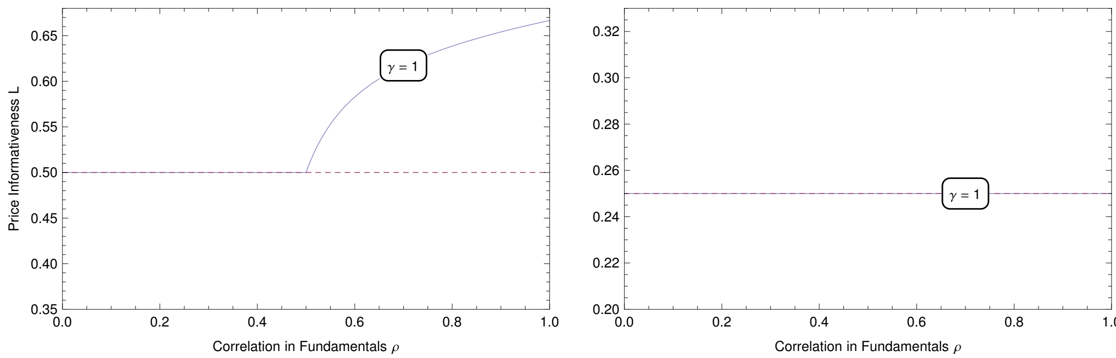
Figure 5: Effect of Overlapping Information on Price Informativeness $\mathcal{L}$ ($\gamma \leq 1/2$)



Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

Figure 5 shows effect of overlapping information on price informativeness when $\gamma \leq 1/2$: The left panel corresponds to a relatively low cost of doing research so that $\omega^* = 1$, the right panel corresponds to relatively high cost of research so that $\omega^* = 1/2$. We find that when individual assets in the pool are not highly correlated and thus the degree of diminishing returns to research is small (small values of ρ), higher degree of overlapping information results in lower equilibrium price informativeness (the curve is below the dashed line, which is the single agent benchmark). Low correlation in underlying assets make information production a harder task, as more assets need to be covered by research efforts to reduce posterior variance of true pass-through value v by the same amount.

Figure 6: Effect of Overlapping Information on Price Informativeness $\mathcal{L}$ ($\gamma = 1$)



Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

Figure 6 presents our analysis of the perfectly-overlapping research technology ($\gamma = 1$ case). We find that for relatively low correlation in underlying assets in the pool (low values of ρ) the second trader decides not to use his research technology and withdraw from trading. However, when ρ becomes high and the second trader decides to compete in the information production, the price informativeness increases.

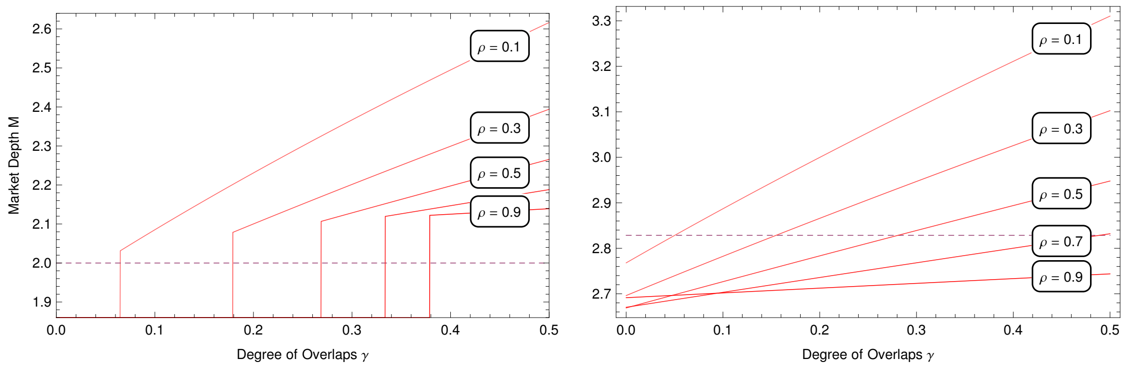
3.2 Market Depth

Another measure of market quality is market depth, which is the inverse of price sensitivity to the order flow. Market depth is particularly important for large institutional traders with big orders and arbitrage seekers, which fits naturally our story of strategic information production. We denote this measure by $\mathcal{M}$.

When there is a single trader with access to a research technology on the market, the market depth is inversely proportional to the amount of information α_1 this trader has: $\mathcal{M} = \frac{2\sigma_u}{\sqrt{\Sigma_0}\sqrt{\alpha_1}}$. The following result describes the market depth $\mathcal{M}$ when two traders have access to a research technology, $N = 2$:

$$\mathcal{M} = \sqrt{\frac{\sigma_u^2}{\Sigma_0}} \times \frac{4 - (\rho_{12})^2}{\sqrt{\left(2\sqrt{\alpha_1} - \rho_{12}\sqrt{\alpha_2}\right)^2 + \left(2\sqrt{\alpha_2} - \rho_{12}\sqrt{\alpha_1}\right)^2}} \tag{23}$$

Figure 7: Effect of Overlapping Information on Market Depth $\mathcal{M}$ ($\gamma \leq 1/2$)

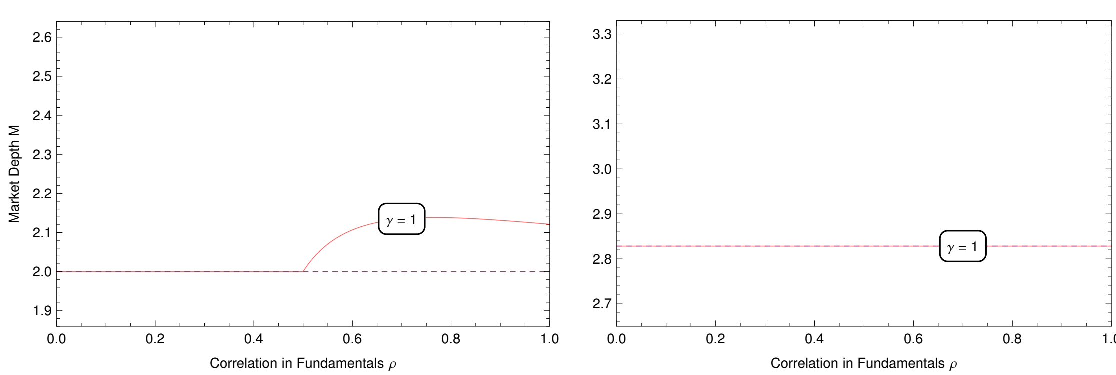


Panel A: Single-agent $\alpha^* = 1$

Panel B: Single-agent $\alpha^* = 0.5$

Figure 7 illustrates the market depth for different degrees of overlapping information when $\gamma \leq 1/2$. We find that market depth increases as there is more overlapping information generated by the research technology traders use. It turns out that more overlapping information reduces the degree of adverse selection the market faces and allows for a deeper market in equilibrium, as the correlation in traders' signals and their orders goes up. Figure 8 presents our analysis of market depth for the perfectly-overlapping research $\gamma = 1$. It turns out that as assets in the underlying pool become more correlated, this reduces informational barriers to entry for the additional traders with access to a research technology. However, the highest market depth is achieved for the level of such asset correlation below $\rho < 0.8$. For the levels of asset correlation that are too high, information production becomes cheap and this aggravates the adverse selection problem.

Figure 8: Effect of Overlapping Information on Market Depth $\mathcal{M}$ ($\gamma = 1$)



Panel A: Single-agent $\alpha^* = 1$

Panel B: Single-agent $\alpha^* = 0.5$

3.3 Volatility of Prices and Total Order Flow

Here we analyze the variability of prices and order flow on the secondary market for the pass-through security. When there is a single trader with access to a research technology on the market, the volatility of the price level is $Var(p) = \Sigma_0 \alpha_1 / 2$. The price volatility is proportional to the informativeness of the trader's signal α_1. It turns out that the price volatility is equal to the price informativeness measure scaled by the variance of pass-through security value v, as captured in the following Lemma:

Lemma 3.2. *In the trading equilibrium outlined in Proposition 2.1 the equilibrium volatility of prices is equal to the product of equilibrium price informativeness and the volatility of pass-through security value v:*

$$Var(p) = \Sigma_0 - Var(v|\Sigma x + u) = \Sigma_0 \times \mathcal{L} \tag{24}$$

Proof.

$$\Sigma_0 - Var(v|\Sigma x + u) = \frac{Cov(v, \Sigma x + u)^2}{Var(\Sigma x + u)} = \left(\frac{Cov(v, \Sigma x + u)}{Var(\Sigma x + u)}\right)^2 \times Var(\Sigma x + u)$$
$$= (\lambda^2) \times Var(\Sigma x + u)$$

□

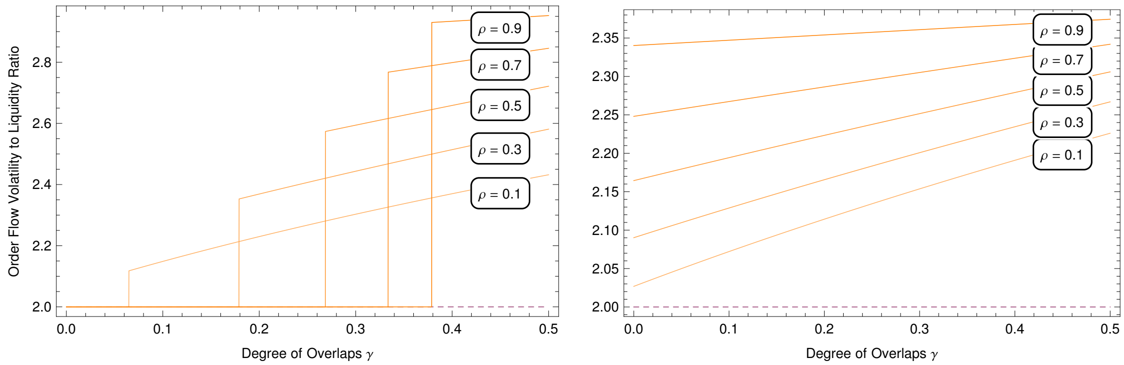
The total order flow volatility is defined as $Var(\Sigma x + u)$. When there is a single trader with access to a research technology on the market, the total order flow volatility is $2\sigma_u^2$. The following lemma describes the volatility of the order flow when two traders have access to a research technology, $N = 2$:

Lemma 3.3. *In the trading equilibrium outlined in Proposition 2.1 the equilibrium volatility of total order flow is:*

$$Var(\Sigma x + u) = \sigma_u^2 \times (4 - (\rho_{12})^2) \times \frac{2((\alpha_1 + \alpha_2) - \rho_{12}(\sqrt{\alpha_1}\sqrt{\alpha_2}))}{(2(\sqrt{\alpha_1}) - \rho_{12}(\sqrt{\alpha_2}))^2 + (2(\sqrt{\alpha_2}) - \rho_{12}(\sqrt{\alpha_1}))^2} \tag{25}$$

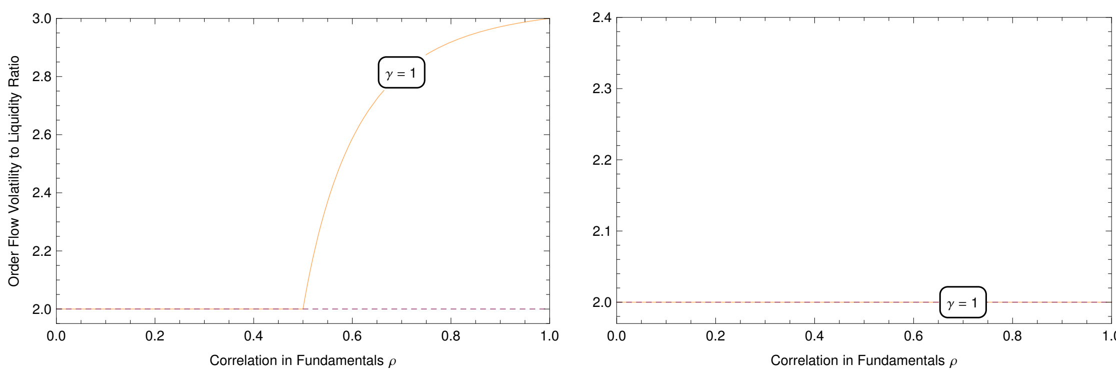
Figure 9 illustrates the order flow volatility for different degrees of overlapping information when $\gamma \leq 1/2$. We find that the total order flow volatility generally increases with the degree of overlapping information in the research technology used by traders. Figure 10 illustrates the order flow volatility for the perfectly-overlapping research $\gamma = 1$.

Figure 9: Effect of Overlapping Information on Total Order Flow Volatility $Var(\Sigma x + u)$ ($\gamma \leq 1/2$)



Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

Figure 10: Effect of Overlapping Information on Total Order Flow Volatility $Var(\Sigma x + u)$ ($\gamma = 1$)



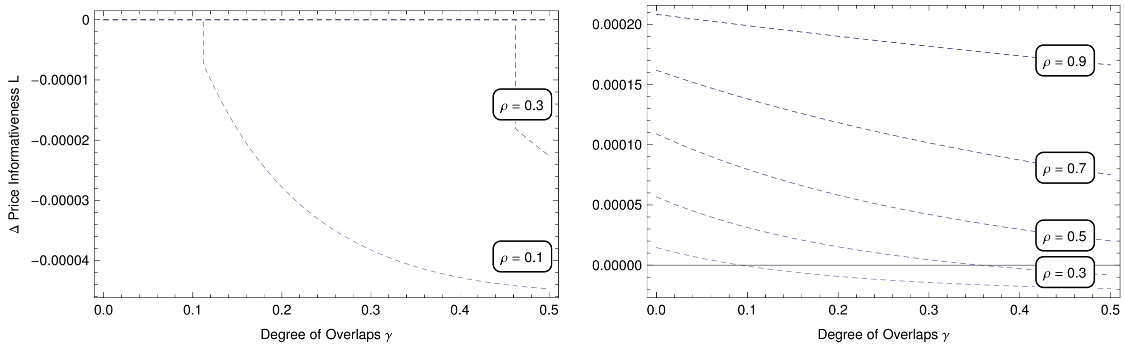
Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

3.4 Model with More Informed Traders

In this section we study the robustness of our equilibrium in a setting with larger number of traders. We present our results for the information production model with N informed agents and symmetric equilibrium research efforts. We solve numerically for a symmetric Nash equilibrium in a model with $N = 50$ traders with access to a research technology and then compare it to the equilibrium with one more trader, $N = 51$. We find that our findings in section 3.1 are robust to adding more traders to the analysis.

Figure 11 illustrates the effect of an additional 51^{th} trader on the price informativeness for different degrees of overlapping information in the research technology. We focus on symmetric equilibria only, and we limit our analysis to cases when $\gamma \leq 1/2$.

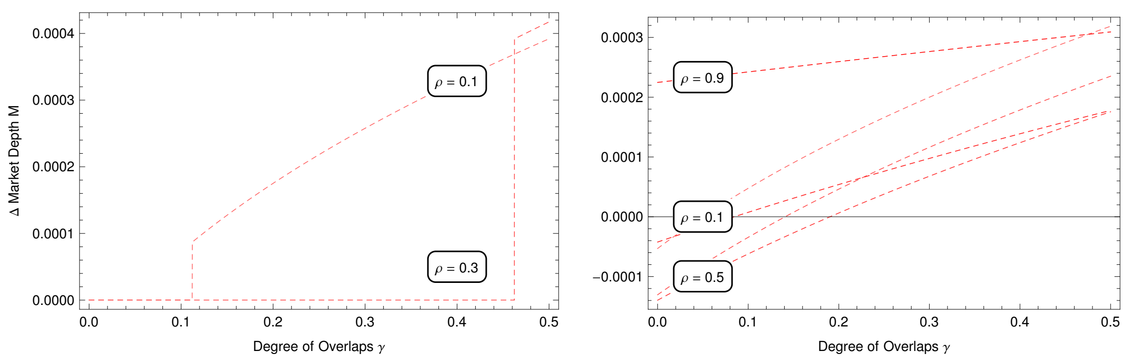
Figure 11: Effect of Overlapping Information on Marginal Change in Price Informativeness $\mathcal{L}$ when $N = 50$



Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

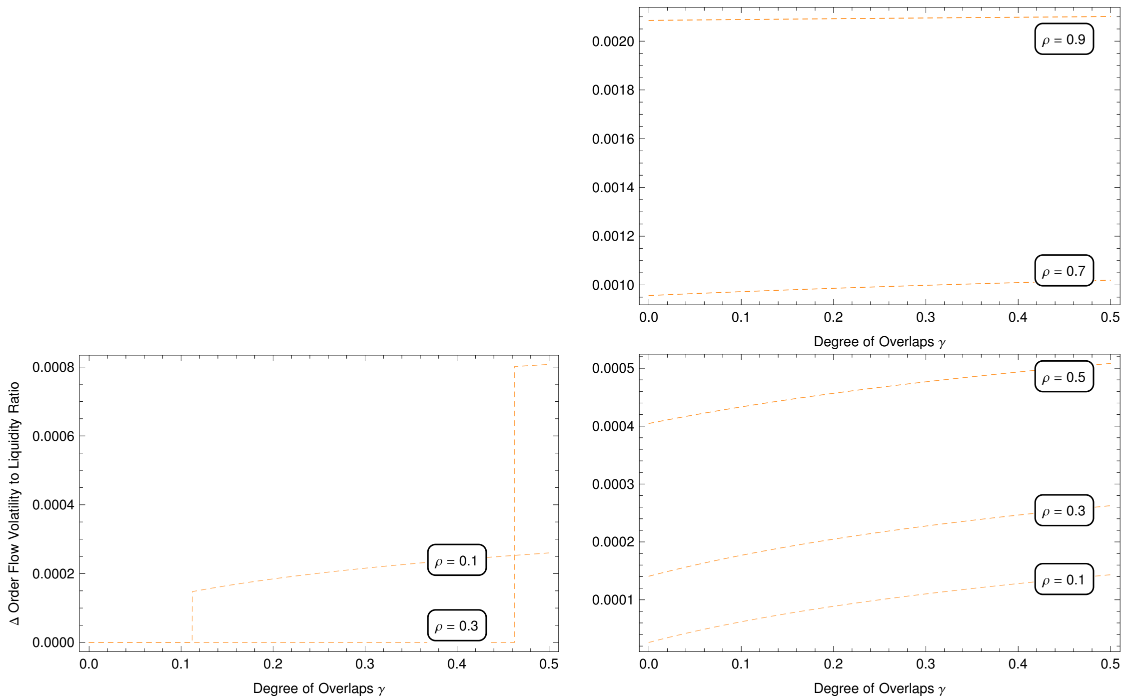
Figures 12 and 13 illustrate the effect of an additional 51^{th} trader on the market depth and the total order flow volatility. We find that the market depth and the total order flow volatility increase when more traders participate in information production.

Figure 12: Effect of Overlapping information on Marginal Change in Market Depth $\mathcal{M}$ when $N = 50$



Panel A: Single-agent $\alpha^* = 1$ Panel B: Single-agent $\alpha^* = 0.5$

Figure 13: Effect of Overlapping information on Marginal Change in Order Flow Volatility Ratio when $N = 50$



Panel A: Single-agent $\alpha^* = 1$

Panel A: Single-agent $\alpha^* = 0.5$

4 Concluding Remarks

When we talk about the levels of adverse selection in markets for information-sensitive and information-insensitive security tranches, it is important to take into account the strength of private information production incentives. The private information production incentives are affected by changes in the economic environment, such as changes to the correlation of the underlying assets in the pool, which affect the extent of diminishing returns to doing research, and the degree of overlapping information in traders' research technology.

An aggregate shock as described in Dang, Gorton, and Holmstrom [2009] can affect not only the information production incentives of active traders, but also reduce information barriers to entry and activate strategic traders who used to stay away from trading on information. These effects will change substantially the landscape of adverse selection on the securitization markets, and it is important to take these effects into account.

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A Information Structure and Research Technologies

A.1 Proposition 1.1

Proof. Equation (1) together with the $Var(v) = \Sigma_0$ implies:

$$MVar(e_i) + M(M-1)Cov(e_i, e_j) = \Sigma_0$$
$$Var(e_i) = (1/M)(\Sigma_0 - M(M-1)Cov(e_i, e_j)) \tag{26}$$

Now use definition of a signal (2) together with the above equation (26) to calculate $Var(s_i)$ and $Cov(s_i, v)$:

$$Var(s_i) = \omega\Sigma_0 - \omega(1-\omega)M^2 Cov(e_i, e_j) \tag{27}$$
$$Cov(s_i, v) = Var(s_i) + \omega M(M - \omega M)Cov(e_i, e_j) = \omega\Sigma_0 \tag{28}$$

Equation (28) is the result we need. In order to simplify equation (27) we note that $Var(s_i)$ must not depend on M according to the infinite-divisibility condition presented in Section 1. Thus as M increases, covariance of information bits e_i and e_j must decay at the rate M^2, in other words $Cov(e_i, e_j) = const/M^2$. We use the following normalization: $\rho = Cov(e_i, e_j) \times M^2/\Sigma_0$. Plugging this expression in equation (27) obtains the result. □

A.2 Proposition 1.2

Proof. We assume the infinite-divisibility condition holds and use the following normalization: $\rho = Cov(e_i, e_j) \times M^2/\Sigma_0$. There are two traders doing research, $\omega_1 \geq \omega_2$, and assume both $\omega_1 M$ and $\omega_2 M$ are Natural numbers. For non-overlapping research there are no common underlying assets reflected in both traders' signals, thus:

$$Cov(s_1, s_2) = (\omega_1 M)(\omega_2 M)Cov(e_i, e_j) = \Sigma_0 \times \rho \times \omega_1\omega_2 \tag{29}$$

For perfectly-overlapping research we can rewrite the signal first trader obtains as:

$$s_1 = s_2 + \sum_{j=1}^{(\omega_1-\omega_2)M} e_j$$

And thus we have:

$$Cov(s_1, s_2) = Var(s_2) + (\omega_2 M)(\omega_1 M - \omega_2 M)Cov(e_i, e_j)$$

Using the first equation in (1.1) to substitute for $Var(s_2)$ we obtain the result:

$$Cov(s_1, s_2) = \Sigma_0 \times \omega_2 \times (1 - (1 - \omega_1)\rho) \tag{30}$$

To derive the expression for randomly-overlapping research let $\overline{\omega}M$ underlying assets constitute the overlap of the two signals, $\overline{\omega} \leq \omega_2$. Similarly to perfectly-overlapping case, we can rewrite traders' signals as the sum of overlapping and non-overlapping parts, where $\overline{\omega}M$ is the size of the overlapping part. We denote the overlapping part of both signals by $\overline{s}$. Holding $\overline{\omega}$ fixed we express $Cov(s_i, s_j)$ in terms of $\overline{\omega}$:

$$Cov(s_1, s_2|\overline{\omega}) = Var(\overline{s}) + (\overline{\omega}(\omega_1 - \overline{\omega}) + \overline{\omega}(\omega_2 - \overline{\omega}) + (\omega_1 - \overline{\omega})(\omega_2 - \overline{\omega}))M^2 Cov(e_i, e_j)$$

The above expression simplifies to:

$$Cov(s_1, s_2|\overline{\omega}) = \overline{\omega}\Sigma_0(1 - \rho) + \omega_1\omega_2\Sigma_0\rho \tag{31}$$

When each trader makes independent random draws of size $\omega_i M$, the expected size of overlap is $E(\overline{\omega}) = \omega_1\omega_2$. Imagine trader 1 moves first and selects $\omega_1 M$ underlying assets, and assume all numbers are Natural in the following discussion. Then agent 2 draws randomly $\omega_2 M$ assets without replacement, thus the number of bits drawn by both traders follows hypergeometric distribution with parameters $N = N, m = \omega_1 N, n = \omega_2 M$. The expected value of hypergeometric distribution is mn/N, which is equal to $\omega_1\omega_2 N$ in our case. Thus the unconditional covariance of signals is:

$$Cov(s_1, s_2) = E(\overline{\omega})\Sigma_0(1 - \rho) + \omega_1\omega_2\Sigma_0\rho = \Sigma_0 \times \omega_1\omega_2 \tag{32}$$

□

A.3 Proposition 1.4

Proof. Use equation (31) expressing covariance of the two signals in terms of the size of overlap (from the proof of proposition 1.2):

$$Cov(s_1, s_2|\overline{\omega}) = \overline{\omega}\Sigma_0(1-\rho) + \omega_1\omega_2\Sigma_0\rho$$

It follows from the definition of the three research technologies that the corresponding sizes of overlap under the three scenarios are:

1. Non-overlapping research ($\gamma = 0$): $\overline{\omega} = 0$
2. Independently-overlapping research ($\gamma = 1/2$): $\overline{\omega} = \omega_1\omega_2$
3. Perfectly-overlapping research ($\gamma = 1$): $\overline{\omega} = \min(\omega_1, \omega_2)$

We take linear interpolations for intermediate overlap scenarios in the following way:

$$\overline{\omega} = \begin{cases} 2\gamma \times \omega_1\omega_2, \text{ when } 0 \leq \gamma \leq 0.5 \\ 2(1-\gamma)\omega_1\omega_2 + (2\gamma - 1)\min(\omega_1, \omega_2), \text{ when } 0.5 < \gamma \leq 1 \end{cases}$$

Without loss of generality we assume $\omega_1 \geq \omega_2$. Plugging in expression for $\overline{\omega}$ in the above equation obtains the result. □

A.4 Proposition 1.5

Proof. It is straightforward to show that results in propositions 1.1 and 1.2 go through with the continuous definition of a signal given the process X_t satisfies the given SDE (9). Here we will present the derivation of this SDE for X_t by starting with the discrete version of a signal and taking the limiting case as the number of underlying assets goes to infinity $M \to \infty$.

Start with M identical and *independent* jointly-Normally distributed random innovations, denote by a vector $\mathbf{u}_M$. Let vector $\mathbf{e}_M$ denote M identical jointly-Normal random variables with a given covariance matrix Ψ_M (all elements of $\mathbf{e}_M$ are identically distributed, thus all off-diagonal elements of Ψ_M are the same). Denote by $Var_e = Var(e_i)$ and $Cov_e = Cov(e_i, e_j)$. Vector $\mathbf{e}_M$ captures all underlying assets or fundamentals within the pass-through security available to a trader. As a first step, we express a generic element e_i of vector $\mathbf{e}_M$ as a function of previous elements $e_1, \cdots, e_{i-1}$ and innovation u_i. Then we take the limit of this expression as $M \to \infty$ to obtain an SDE for a continuous time stochastic process that we refer to as X_t. Let I_m denote an $m \times m$ identity matrix, and J_m denote $m \times 1$ vector of ones:

$$
\begin{aligned}
\mathbf{e}_M &= \Psi_M^{1/2} \times \mathbf{u}_M \\
\Psi_M &= (Var_e - Cov_e) \times I_M + Cov_e \times J_M J_M^T
\end{aligned} \tag{33}
$$

We can use Cholesky decomposition and rewrite Ψ_M as (there exist a vector F and number E so that the following holds):

$$
\Psi_M = \begin{bmatrix} \Psi_{M-1} & Cov_e \times J_{M-1} \\ Cov_e \times J_{M-1}^T & Var_e \end{bmatrix} = \begin{bmatrix} \Psi_{M-1}^{1/2} & 0 \\ F^T & E \end{bmatrix} \times \begin{bmatrix} (\Psi_{M-1}^{1/2})^T & F \\ 0 & E \end{bmatrix},
$$

where:

$$
\begin{aligned}
F &= Cov_e \times (\Psi_{M-1}^{1/2})^{-1} \times J_{M-1} \\
E &= \sqrt{Var_e - Cov_e^2 \times J_{M-1}^T \Psi_{M-1}^{-1} J_{M-1}}
\end{aligned} \tag{34}
$$

Combining equations 33 and 34 we obtain:

$$
\begin{aligned}
\begin{pmatrix} \vec{\mathbf{e}}_{i-1} \\ e_i \end{pmatrix} &= \begin{bmatrix} \Psi_{i-1}^{1/2} & 0 \\ Cov_e \times J_{i-1}^T \Psi_{i-1}^{-1} \Psi_{i-1}^{1/2} & \sqrt{Var_e - Cov_e^2 \times J_{i-1}^T \Psi_{i-1}^{-1} J_{i-1}} \end{bmatrix} \times \begin{pmatrix} \vec{\mathbf{u}}_{i-1} \\ u_i \end{pmatrix} \\
e_i &= Cov_e \times J_{i-1}^T \Psi_{i-1}^{-1} \times \vec{\mathbf{e}}_{i-1} + u_i \sqrt{Var_e - Cov_e^2 \times J_{i-1}^T \Psi_{i-1}^{-1} J_{i-1}}
\end{aligned}
$$

The last equation can be simplified by noting that:

$$
\begin{aligned}
\Psi_{i-1}^{-1} &= \frac{1}{(Var_e - Cov_e)} \times I_{i-1} - \frac{Cov_e}{(Var_e - Cov_e)(Var_e + Cov_e(i-2))} J_{i-1} J_{i-1}^T \\
J_{i-1}^T \Psi_{i-1}^{-1} &= \frac{1}{(Var_e - Cov_e)} \times J_{i-1}^T - \frac{Cov_e(i-1)}{(Var_e - Cov_e)(Var_e + Cov_e(i-2))} J_{i-1}^T \\
&= \frac{1}{Var_e + Cov_e(i-2)} J_{i-1}^T
\end{aligned}
$$

We obtain the following result:

$$
e_i = \frac{Cov_e}{Var_e + Cov_e(i-2)} J_{i-1}^T \times \vec{\mathbf{e}}_{i-1} + u_i \sqrt{Var_e - \frac{Cov_e^2(i-1)}{Var_e + Cov_e(i-2)}}
$$

We use infinite-divisibility condition and substitute the covariance terms with $Cov_e = \Sigma_0 \rho / M^2$.

We also use equation (26) from the proof of proposition 1.1 to substitute the variance terms with $Var_e = (1/M)(\Sigma_0 - M(M-1)Cov_e) = (\Sigma_0/M)(1 - \rho(M-1)/M)$:

$$e_i = \frac{\rho}{(1-\rho) + \rho(i-1)/M}\left(\frac{J_{i-1}^T \times \overrightarrow{\mathbf{e}}_{i-1}}{M}\right) + \left(\frac{u_i}{\sqrt{M}}\right)\sqrt{\Sigma_0(1-\rho) + \frac{\Sigma_0\rho(1-\rho)}{M(1-\rho) + \rho(i-1)}}$$

Now we let $i = t \times M$ for some $t \in (0,1)$, $\lim_{M\to\infty} 1/M = dt$, and $\lim_{M\to\infty}(J_{i-1}^T \times \overrightarrow{\mathbf{e}}_{i-1}) = X(t)$. Taking the limit as $M \to \infty$ we obtain the following SDE for $X(t)$:

$$dX(t) = \frac{\rho}{(1-\rho) + \rho t} X(t) \times dt + \sqrt{\Sigma_0(1-\rho)} \times dW(t)$$

□

B Trading Environment

B.1 Proposition 2.1

Proof. Conjecture linear trading strategies $X_i = (\beta_i/\lambda)s_i$ for the two traders and linear pricing rule of the form $P = \lambda(\Sigma x + u)$. Start with the profit maximization condition (10):

$$\max_{X_i}\{E\Pi_i(X_i, X_{-i}, P)\} = \max_{x}\{E((v - \lambda(x + (\beta_{-i}/\lambda)s_{-i} + u)) \times x | s_i = s)\}$$

We follow the approach of Bernhardt and Taub [2008] and rewrite the i-th trader problem as unconditional maximization with respect to trading intensity β_i:

$$\max_{x}\{E((v - \lambda(x + (\beta_{-i}/\lambda)s_{-i} + u)) \times x | s_i = s)\} =$$
$$\max_{\beta}\left\{E\left[\left(v - \lambda\left(\left(\frac{\beta}{\lambda}\right)s_i + \left(\frac{\beta_{-i}}{\lambda}\right)s_{-i} + u\right)\right)\left(\frac{\beta}{\lambda}\right)s_i\right]\right\}$$

Using the joint-normality of signals and the true asset value, plus independence of liquidity trading, the unconditional problem simplifies to:

$$\max_{\beta}\left\{\left(\frac{\beta}{\lambda}\right)Cov(s_i, v) - \left(\frac{\beta^2}{\lambda}\right)Var(s_i) - \left(\frac{\beta\beta_{-i}}{\lambda}\right)Cov(s_i, s_{-i})\right\} \tag{35}$$

Assuming λ is positive, the second-order condition for the above maximization is satisfied. Thus, it is sufficient to consider the system of two first-order conditions for two informed traders:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 2 \times Var(s_1) & Cov(s_1, s_2) \\ Cov(s_1, s_2) & 2 \times Var(s_2) \end{pmatrix}^{-1} \begin{pmatrix} Cov(s_1, v) \\ Cov(s_2, v) \end{pmatrix}$$

From the above we obtain the expression (13) for β_i presented in the proposition. Note that given conjectured linear trading and pricing rules the β_i are determined uniquely.

Now we use the market efficiency condition (11) to determine λ. Note that if λ is determined uniquely, the initial trading intensities β_i/λ will be unique. Under the conjectured linear trading strategies the total order flow is:

$$\Sigma x + u = (\beta_1/\lambda)s_1 + (\beta_2/\lambda)s_2 + u$$

and is jointly normally distributed with v. This implies linearity of pricing rule and the following result (market efficiency condition (11) represents a linear regression of v on $\Sigma x + u$):

$$\lambda = \frac{Cov(v, \Sigma x + u)}{Var(\Sigma x + u)} = \frac{(\beta_1/\lambda)Cov(s_1, v) + (\beta_2/\lambda)Cov(s_2, v)}{(\beta_1/\lambda)^2 Var(s_1) + (\beta_2/\lambda)^2 Var(s_2) + 2(\beta_1/\lambda)(\beta_2/\lambda)Cov(s_1, s_2) + \sigma_u^2}$$

$$(\lambda\sigma_u)^2 = \sum_{i=1}^{2} \beta_i(Cov(s_i, v) - \beta_i Var(s_i) - \beta_2 Cov(s_i, s_{-i}))$$

The first order condition for informed trader's problem 35 implies:

$$Cov(s_i, v) - \beta_i Var(s_i) - \beta_2 Cov(s_i, s_{-i}) = Var(s_i)\beta_i$$

Plugging this result in the above equation and simplifying we obtain expression (14) for λ. The linear trading equilibrium is unique. □

B.2 Lemma 2.1

Proof. Suppose trader-i's opponent and the market-maker follow equilibrium strategies β_{-i} and λ given by Proposition 2.1. Then trader-i's profit maximization problem is:

$$\max_x \left\{ E\left[\left(v - \lambda\left(x + \left(\frac{\beta_{-i}}{\lambda}\right) s_{-i} + u \right) \right) \times x \middle| s_i^{\text{new}} \right] \right\}$$

Following the approach in Bernhardt and Taub [2008] we rewrite the above as an equivalent unconditional maximization problem:

$$\max_{\beta}\left\{\left(\frac{\beta}{\lambda}\right)Cov(s_i^{\text{new}},v)-\left(\frac{\beta^2}{\lambda}\right)Var(s_i^{\text{new}})-\left(\frac{\beta\beta_{-i}}{\lambda}\right)Cov(s_i^{\text{new}},s_{-i})\right\}$$

The first-order condition is sufficient and gives the expression (16) for β_i^{new}:

$$\begin{aligned}\beta_i^{\text{new}} &= \frac{Cov(s_i^{\text{new}},v)-\beta_{-i}Cov(s_i^{\text{new}},s_{-i})}{2Var(s_i^{\text{new}})}\\ &= \frac{1}{2\sqrt{Var(s_i^{\text{new}})}}\left(\sqrt{\Sigma_0\times\alpha_1^{\text{new}}}-\rho_{12}^{\text{new}}\times\sqrt{Var(s_{-i})\times\beta_{-i}}\right)\end{aligned}$$

Plugging the solution for β_i^{new} into the unconditional maximization problem we obtain the expression (17) for expected profit of trader i:

$$\begin{aligned}E(\pi_i^{\text{new}}) &= \frac{1}{4\lambda\times Var(s_i^{\text{new}})}\left(Cov(s_i^{\text{new}},v)-Cov(s_i^{\text{new}},s_{-i})\times\beta_{-i}\right)^2\\ &= \frac{1}{4\lambda}\left(\sqrt{\Sigma_0\times\alpha_i^{\text{new}}}-\rho_{12}^{\text{new}}\times\sqrt{Var(s_{-i})\times\beta_{-i}}\right)^2\end{aligned}$$

□

B.3 Lemma 2.2

Proof. Both traders have access to the same research technology characterized by overlapping information parameter $\gamma\le 1/2$ and cost function $TC(\omega)$ that is assumed to be linear in ω (all identically distributed underlying assets are equally costly to reveal). In order to show the result we express the expected trading profits of informed traders in terms of their research efforts ω_1 and ω_2. We use equation (17) in Lemma 2.1 to derive the first order optimality condition necessary for a Nash equilibrium:

$$\forall i\in\{1,2\}:\frac{\Sigma_0(1-\rho)}{4\lambda}\times\left(\frac{1-(2\gamma(1-\rho)+\rho)\times\omega_j\beta_j}{1-(1-\omega_i)\rho}\right)^2=\frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_i)\tag{36}$$

The right-hand side of the equation above is the marginal cost of information acquisition, which we assume does not depend on the research effort ω. We equate marginal trading benefits of doing research for the two traders and plug in the resulting expression equilibrium values of β_1 and β_2 (λ cancels out). We also reexpress the result in terms of α_1 and α_2 to simplify exposition (there is a one-to-one mapping between research effort ω_i and %-conditional variance reduction measure α_i):

$$\alpha_i = \frac{\omega_i}{1-(1-\omega_i)\rho} \tag{37}$$

$$\alpha_2 \times \left(\frac{2(\rho - 2\gamma(1-\rho))}{4-(2\gamma(1-\rho)+\rho)^2\alpha_1\alpha_2}\right) = \alpha_1 \times \left(\frac{2(\rho - 2\gamma(1-\rho))}{4-(2\gamma(1-\rho)+\rho)^2\alpha_1\alpha_2}\right) \tag{38}$$

The above equation implies $\alpha_1 = \alpha_2$ when $\gamma \neq \frac{\rho}{2(1-\rho)}$. It is worth noting that the knife-edge case $\gamma = \frac{\rho}{2(1-\rho)}$ results in multiplicity of possible equilibria—for a given linear cost function $TC(\omega)$ continuum of Nash equilibria exists (one symmetric and continuum of asymmetric for any given linear cost function satisfying single-agent-interior condition). The marginal trading benefit of research efforts for each agent in this case is a symmetric function of α_1 and α_2.

We establish for the game with two informed traders that if an interior Nash equilibrium exists for $\gamma < 1/2$ and $\gamma \neq \frac{\rho}{2(1-\rho)}$, then it is symmetric, that is $\omega_1 = \omega_2$. $\square$

B.4 Lemma 2.3

Proof. We prove the lemma by conjecturing existence of a corner equilibrium and then verifying it is a Nash equilibrium of the game. Without loss of generality suppose $\omega_1 > \omega_2 = 0$ in the corner equilibrium. We use equation (17) in Lemma 2.1 and express expected trading profits of informed traders in terms of their research efforts ω_1 and ω_2. It can be verified that trading equilibrium for a game with one informed trader is equivalent to a trading equilibrium in a game with two informed traders when one trader's research effort is zero. The two cases $\gamma \leq 1/2$ and $\gamma > 1/2$ result in the following expressions for correlation of two signals ρ_{12}:

$$\rho_{12} = \begin{cases} (2\gamma(1-\rho)+\rho)\sqrt{\alpha_1\alpha_2}, & \text{when } 0 \leq \gamma \leq 1/2 \\ \frac{(1-2(1-\gamma)(1-\alpha_1))\sqrt{\alpha_2}}{\sqrt{\alpha_1}}, & \text{when } 1/2 < \gamma \leq 1, \end{cases} \tag{39}$$
$$\text{where: } \alpha_i = \frac{\omega_i}{1-(1-\omega_i)\rho}, \forall i \in \{1,2\}$$

Consider the case $\gamma \leq 1/2$ first. The first order optimality condition necessary for a corner Nash equilibrium is:

$$\frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1-(2\gamma(1-\rho)+\rho)\times\omega_1\beta_1}{1-\rho}\right)^2 \leq \frac{\mathrm{d}TC}{\mathrm{d}\omega}(0) = \tag{40}$$
$$= \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_1) = \frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1}{1-(1-\omega_1)\rho}\right)^2$$

We plug in the above expression the trading equilibrium values of β_1 and λ that correspond to single trader doing research on the market (Proposition 2.1):

$$\begin{aligned}\beta_1 &= \frac{1}{2}\left(\frac{1}{1-(1-\omega^*)\rho}\right)\\ \lambda &= \frac{\sqrt{2\Sigma_0}}{2\sigma_u} \times \sqrt{\frac{\omega^*}{1-(1-\omega^*)\rho}}\end{aligned}$$

The above condition 40 simplifies to the following expression:

$$\left(\frac{1}{1-(1-\omega^*)\rho} + \frac{(\rho-2\gamma(1-\rho))\times\omega^*}{2(1-\rho)(1-(1-\omega^*)\rho)}\right)^2 \leq \left(\frac{1}{1-(1-\omega^*)\rho}\right)^2$$

Using the single-agent-interior condition that implies $\omega^* < 1$ and also restrictions on model parameters $\rho \in [0,1)$ and $\gamma \leq 1/2$, the first order condition for the corner equilibrium is satisfied if and only if:

$$\rho - 2\gamma(1-\rho) \leq 0 \equiv \rho \leq \frac{2\gamma}{1+2\gamma}$$

It remains to check that the necessary first order condition above is sufficient for the corner Nash equilibrium. We show that the objective functions in traders' profit maximization problems are concave (strategies of other market participants held fixed and using equation (36) above):

$$\begin{aligned}\frac{\partial(E\pi_i - TC(\omega_i))}{\partial\omega_i} &= \frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1-(2\gamma(1-\rho)+\rho)\times\omega_j\beta_j}{1-(1-\omega_i)\rho}\right)^2 - \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_i)\\ \frac{\partial^2(E\pi_i - TC(\omega_i))}{\partial\omega_i^2} &= -\frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{2\rho(1-(2\gamma(1-\rho)+\rho)\times\omega_j\beta_j)^2}{(1-(1-\omega_i)\rho)^3}\right) < 0\end{aligned}$$

This concludes the proof for low degree of overlapping information $\gamma \leq 1/2$. Now we repeat similar steps for $\gamma > 1/2$ and use corresponding functional form for the correlation between signals ρ_{12}.

The first order optimality condition necessary for a corner Nash equilibrium when $\gamma > 1/2$ is:

$$\frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1-((2\gamma-1)(1-(1-\omega_1)\rho)+2(1-\gamma)\omega_1)\beta_1}{1-\rho}\right)^2 \leq \frac{\mathrm{d}TC}{\mathrm{d}\omega}(0) = \tag{41}$$

$$= \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_1) = \frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1}{1-(1-\omega_1)\rho}\right)^2$$

The trading equilibrium values of β_1 and λ remain unaffected by γ because only one informed trader does research. The above condition simplifies to:

$$\left(\frac{(3-2\gamma)(1-\rho)+(2\gamma(1-\rho)+3\rho-2)\omega^*}{2(1-\rho)(1-(1-\omega^*)\rho)}\right)^2 \leq \left(\frac{1}{1-(1-\omega^*)\rho}\right)^2$$

$$\rho \leq \frac{(2-2\gamma)\omega^*+2\gamma-1}{(3-2\gamma)\omega^*+2\gamma-1}, \text{ when } \gamma > 1/2$$

It remains to check the sufficiency of the above condition. The objective function of the single informed trader doing research on the market is concave:

$$\frac{\partial(E\pi_1 - TC(\omega_1))}{\partial\omega_1} = \frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1}{1-(1-\omega_1)\rho}\right)^2 - \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_1)$$

$$\frac{\partial^2(E\pi_1 - TC(\omega_1))}{\partial\omega_1^2} = -\frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{2\rho}{(1-(1-\omega_1)\rho)^3}\right) < 0$$

The objective function for the second informed trader that is at the corner consist of two parts: the catch-up part $\omega_2 < \omega^*$ and leading part $\omega_2 > \omega^*$. Instead of doing piecewise marginal analysis we use equation (17) in Lemma 2.1 to show that the total trading profits second informed trader obtains is strictly less than research costs for any research effort $\omega_2 > 0$ when the above condition holds.

$$E(\pi_2^{new}) = \frac{1}{4\lambda}\left(\sqrt{\Sigma_0 \times \alpha_2^{\text{new}}} - \rho_{12}^{\text{new}} \times \sqrt{Var(s_1) \times \beta_1}\right)^2 \leq TC(\omega_2) \tag{42}$$

When one informed trader acts on the market in equilibrium, we use its first-order condition to express the relationship between research effort and total variable cost of research for the second trader in case it decides to deviate from $\omega_2 = 0$ (under linear cost assumption and single-agent interior condition):

$$TC(\omega_2) = \frac{(1-\rho)\Sigma_0}{4\lambda}\left(\frac{1}{1-(1-\omega_1)\rho}\right)^2 \times \omega_2$$

When the second informed trader exerts lower research effort than ω_1, his marginal trading profit is decreasing in ω_2, thus trading profits cannot turn positive at $0 < \omega_2 < \omega_1$ provided that marginal trading profit is already below marginal cost at zero. However once $\omega_2 \geq \omega_1$ there are potential benefits of doing break through research, and this is the case we analyze below. Plugging in corner equilibrium values of λ and β_1 in equation (42), taking $\omega_2 \geq \omega_1$ and simplifying yields an equivalent inequality:

$$(2\omega_2(1-\rho+\rho\omega_1) - \omega_1((2\gamma-1)(1-\rho)(1-\omega_2)+\omega_2))^2 < 4\omega_2^2 \times (1-\rho)(1-\rho+\rho\omega_2)$$

Note that the expression above is quadratic in ρ. When $\rho = 0$ the expression simplifies to $\omega_1((2\gamma-1)(1-\omega_2)+\omega_2)(4\omega_2 - \omega_1((2\gamma-1)(1-\omega_2)+\omega_2)) > 0$ and is true when $\omega_2 > \omega_1$ and $\gamma > 1/2$. When $\rho = 1$ the same expression simplifies to $\omega_1^2\omega_2^2 > 0$. Now if we show that when $\rho = \frac{(2-2\gamma)\omega^*+2\gamma-1}{(3-2\gamma)\omega^*+2\gamma-1}$ (the threshold in the Lemma) the above expression is negative, our result follows—for all $\rho \in [0, \frac{(2-2\gamma)\omega^*+2\gamma-1}{(3-2\gamma)\omega^*+2\gamma-1}]$ the needed inequality holds. It turns out that this holds, when ρ is equal to our threshold, the above expression is negative. It simplifies to:

$$-4\gamma^2\omega_1(\omega_2-\omega_1)^2 + 4\gamma(\omega_1^3 + 2\omega_2^3 - \omega_1\omega_2^2(1+2\omega_2)) - \omega_1^3 - 2\omega_1^2\omega_2 - 4\omega_2^3 + \omega_1\omega_2^2(3+8\omega_2) > 0$$

The above expression is a concave quadratic polynomial in γ, the relevant range for which is $\gamma \in [0.5, 1]$. We use the region of research efforts such that $\omega_2 > \omega_1$. For convenience, let $\omega_2 = \nu\omega_1$, where $\nu > 1$. Using this reformulation for $\gamma = 0.5$ the above simplifies to $4\nu^3\omega_1^4 > 0$. For $\gamma = 1$ it simplifies to $(\nu(6+\nu(4\nu-5))-1)\omega_1^3$, which is increasing function in ν and positive when $\nu = 1$. These two facts together with concavity of the polynomial above establish the fact.

This leads us to conclude that the first order condition above is both necessary and sufficient.

□

B.5 Proposition 2.2

Proof. Firstly, consider research technologies with low degree of overlapping information $\gamma \leq 1/2$. Use equation (17) in Lemma 2.1 to rewrite informed trader's expected profits function $E(\pi_i)$ in terms of its research effort ω_i and take the first order condition of the profit maximization problem (holding ω_{-i}, λ and β_{-i} constant):

$$\frac{\Sigma_0(1-\rho)}{4\lambda} \times \left(\frac{1-(2\gamma(1-\rho)+\rho)\times\omega_{-i}\beta_{-i}}{1-(1-\omega_i)\rho}\right)^2 = \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_i)$$

Using result in Lemma 2.2 that in the class of interior Nash equilibria under $\gamma \leq 1/2$ only symmetric equilibria can exist, we substitute $\omega_i = \omega_j = \omega$ in the above equation. Plugging equilibrium values of λ and β_i, the above first order condition simplifies to:

$$\frac{(1-\rho)\sqrt{\Sigma_0 \times \sigma_u^2}(2(1-\rho)+(2\gamma(1-\rho)+3\rho)\omega)}{\sqrt{2}(2+2\gamma(1-\rho)+\rho)^2\sqrt{\omega}(1-(1-\omega)\rho)^{5/2}} = \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega)$$

Our first observation is as $\omega \to 0$ the LHS of the above expression limits to $+\infty$. Thus for any finite marginal cost of doing research $\omega = 0$ is never an equibrium. When $\omega = 1$ the LHS simplifies to $\frac{(1-\rho)\sqrt{\Sigma_0 \times \sigma_u^2}}{\sqrt{2}(2+2\gamma(1-\rho)+\rho)} < \frac{(1-\rho)\sqrt{\Sigma_0 \times \sigma_u^2}}{2} < \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega = 1)$, the latter implied by the single-agent-interior condition. Thus $\omega = 1$ is never an equilibrium. It remains to show that for any given constant RHS there is a unique solution for $\omega \in (0,1)$. Below we show that LHS is a decreasing function of ω, which establishes the result.

We show this by differentiating the LHS with respect to ω. The denominator is always positive, while the numerator is a concave quadratic polynomial in ω that needs to be negative for our result:

$$-4(2\gamma(1-\rho)+3\rho)\rho \times \omega^2 + (1-\rho)(2\gamma(1-\rho)-9\rho) \times \omega - 2(1-\rho)^2 < 0$$

The polynomial is negative both when $\omega = 0$ and $\omega = 1$. It attains its optimal value when $\omega = \frac{(1-\rho)(2\gamma(1-\rho)-9\rho)}{8(2\gamma(1-\rho)+3\rho)\rho}$. When the optimal value is attained outside the $[0,1]$ domain, the two checks at endpoints above are sufficient for the result. When $\gamma \geq \frac{9\rho}{2(1-\rho)}$ and $\gamma \leq \frac{3\rho(3+5\rho)}{2(1-\rho)(1-9\rho)}$ the optimal value is inside the $[0,1]$ domain, so we check the value of polynomial at the optimum. It is negative whenever $4\gamma^2(1-\rho)^2 - 100\gamma(1-\rho)\rho - 15\rho^2 < 0$, which holds under the above restrictions on γ (verified numerically). This completes the proof of the $\gamma \leq 1/2$ case.

Now consider the perfectly-overlapping research technology $\gamma = 1$ in the second part of the Proposition. Again, we use equation (17) in Lemma 2.1 to rewrite informed trader's expected trading profits function $E(\pi_i)$ in terms of its research effort ω_i and take the first order condition of the profit maximization problem (holding ω_{-i}, λ and β_{-i} constant). We use the formula for correlation of informed traders' signals ρ_{12} that corresponds to the perfectly-overlapping research technology $\gamma = 1$ and without loss of generality we assume $\omega_1 \geq \omega_2$. The first order condition for the two informed traders is:

$\omega_1 \geq \omega_2:$

$$\frac{\partial(E\pi_1)}{\partial\omega_1} = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1}{1-(1-\omega_1)\rho} - \frac{\omega_2}{\omega_1}\beta_2\right)\left(\frac{1}{1-(1-\omega_1)\rho} + \frac{\omega_2}{\omega_1}\beta_2\right) = \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_1)$$
$$\frac{\partial(E\pi_2)}{\partial\omega_2} = \frac{\Sigma_0(1-\rho)}{4\lambda}\left(\frac{1-\beta_1+\beta_1(1-\omega_1)\rho}{1-(1-\omega_2)\rho}\right)^2 = \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_2)$$

It can be shown that $\frac{\partial(E\pi_1)}{\partial\omega_1}(\omega) > \frac{\partial(E\pi_2)}{\partial\omega_2}(\omega)$, while $\frac{\partial(E\pi_1)}{\partial\omega_1}$ has at most one point where it changes direction. Although the first informed trader's maximization problem is not concave, the first order condition is still sufficient. Any candidate Nash equilibrium with positive $\omega_1 > 0$ and $\omega_2 > 0$ must satisfy both first order conditions above. We assume linear variable costs of research $\frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_1) = \frac{\mathrm{d}TC}{\mathrm{d}\omega}(\omega_2)$, thus we can equate marginal trading profits of two agents and obtain relationship between ω_1 and ω_2 in equilibrium:

$$\begin{aligned} & (-2\rho(1-\rho(1+\omega_1))) \times \omega_2^2 + 2(1-\rho)(3\rho\omega_1 - (1-\rho)) \times \omega_2 + \\ + \quad & \omega_1(4(1-\rho)^2 - (1-\rho(1-\omega_1))^2) = 0 \end{aligned}$$

We solve the above equation for ω_2. It turns out that when $\omega_1\rho > 1-\rho$ we have positive $0 < \omega_2 < \omega_1$. When one of the conditions is not satisfied, we have $\forall\omega_2 \in [0,1]$ the first trader with higher research effort $\omega_1 > \omega_2$ has $\frac{\partial(E\pi_1)}{\partial\omega_1}(\omega_1) > \frac{\partial(E\pi_2)}{\partial\omega_2}(\omega_2)$, thus only corner solution is possible for ω_2:

$$\begin{aligned} \text{when } \omega_1\rho &> 1-\rho: \\ \omega_2 &= \frac{-(1-\rho)(3\omega_1\rho - 1 + \rho) + (\omega_1\rho + 1 - \rho)\sqrt{(\omega_1\rho - 1 + \rho)^2 + \rho^2\omega_1^2}}{2\rho(\omega_1\rho - 1 + \rho)} \qquad (43) \\ \text{when } \omega_1\rho &\leq 1-\rho: \\ \omega_2 &= 0 \qquad (44) \end{aligned}$$

The latter case when $\omega_1\rho \leq 1-\rho$ is consistent with the result in Lemma 2.3 after plugging in $\gamma = 1$. It is interesting that when this condition does not hold, the level of first trader's research effort ω_1 uniquely determines equilibrium level of second trader's effort ω_2 according to equation (43). When $\omega_1\rho \leq 1-\rho$ the existence and uniqueness of the corner equilibria is established. When $\omega_1\rho > 1-\rho$ we establish existence and uniqueness using numerical methods. The underlying idea is to pick any linear cost function satisfying single-agent-interior condition and demonstrate that equation (43) pins down ω_1 uniquely. Single-agent interior condition ensures that the equilibrium is interior $\omega_1 < 1$.

This concludes the proof of the proposition. $\square$