Does Macro-Asset Pricing Matter for Corporate Finance?

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Abstract

In an asset-pricing model calibrated to match the standard asset pricing empirical properties – in particular, the time-variation in the equity premium – we calculate the value implications of sub-optimal capital budgeting decisions. Specifically, we calculate that an investment policy that ignores the time variation in the equity premium, such as would occur with a cost of capital following the CAPM, incurs a 13.3% value loss. We also document the implications for a firm's asset returns in this context.

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1 Introduction

In business practice and education, the Net Present Value (NPV) rule is widely used for a firm's capital budgeting decision. The wide spread use of NPV is one of the great successes of business education. Evaluation of the NPV, of course, entails discounting future cash flows at an appropriate cost of capital. The work-horse model used in both the classroom and industry is the Capital Asset Pricing Model. While there is much ad-hoc adjustment in use (e.g., CFO's tend to round up the cost of capital), the CAPM is the de facto standard way to determine the risk-adjusted discount rate. Graham and Harvey (2001) survey companies throughout the U.S. and Canada and find that 74.9% of respondents use the NPV for capital budgeting, and 73.5% use the CAPM.

The CAPM is, of course, a static model and is agnostic about the dynamic properties of the equity premium. Thus, the use of the CAPM in practice implies that the discount rate is constant across time or economic-state. Typically, people use a number like 5% or 6% (Welch (2000) and subsequent update). In contrast, the central feature of research in macro-asset pricing for at least the last decade has been focussed on not just explaining the level of the equity premium ("the equity premium puzzle"), but in understanding its dynamic properties. Cochrane (2011), for example, points out that the time-variation in the equity premium is on the same order of magnitude as the level. That is; the equity premium swings between 1% and 11%. Given this fluctuation, using a constant CAPM-inspired discount rate is sub-optimal. In this paper, we quantify the value loss caused by an investment policy that ignores the time-variation in risk premium.

To measure the quantitative implications, it is necessary to construct a model that is reasonably well calibrated. With this objective in mind, we build a model for the underlying economic environment and firm-level investment. In order to capture in a tractable way how firm managers' characterization of risks influences their investment decisions, we assume and calibrate a standard endowment economy. We tune the model to have time-varying risk or, counterfactually, not. Specifically, the economic environment is based on the long-run risk models of Bansal and Yaron (2004). We use the version from Backus, Routledge, and Zin (2010), where the economy is described by two state Markov process; the two states are the expected growth of endowment and the volatility of the growth. Here, the time-variation in risk premium arises from the stochastic volatility of endowment growth.

Given the setup of economic environment, we model firm-level investment as follows. At each date, a firm receives an opportunity to invest in a new project. The investment project is exposed to systematic risk in that its future cash flows are correlated with aggregate endowment. The firm evaluates the NPV of the project based on the perceived state of economy as well as project-specific characteristics. If the evaluated NPV turns out positive, the firm invests. If the firm does not invest, the opportunity vanishes. This resembles the now-or-never options in Berk, Green, and Naik (1999). Projects have a finite life. Hence, assets in place evolve as new projects come in and old ones retire. Given the firm operation, the firm value consists of the value of the existing projects as well as the value of future opportunities. Since how firm managers invest will depend on their model of the economy – is the price of risk time varying? – firm value, both assets in place and growth options, will also depend on their model. If the firm fails to correctly model the price of risk or discount rates, it will incur a value loss as a result of sub-optimal investment decisions. Here, we quantify the size of this loss.

The basic idea is to consider two economies. One economy will have constant equity premium (from a constant volatility assumption) and the other economy will feature time-variation in the equity premium. While both of these calibrations will match the usual moments of aggregate asset returns, only second economy generates a dynamic equity premium. In each of these two economies, we will consider two representative firms and their investment policies. One firm - Type 1 - will act as if the equity premium is constant. The other - Type 2 - will act as if the equity premium is dynamic. This will let us consider the optimal investment behavior (Type 1 in the constant-volatility economy and Type 2 in the stochastic-volatility economy) as well as measure the cost of a sub-optimal policy. Thus we can measure the cost of acting as a Type 1 firm (a CAPM-like cost of capital) in a world with a dynamic equity premium. We can also measure the counter-factual cost of a Type 2 firm that happens to live in a world with a static equity premium. In addition, we also look at the returns produced by firms in each of these settings.

The estimate of value loss and return differentials also depend on project-specific characteristics. We calibrate these characteristics so that the average of the book-to-market ratio in a simulated firm-panel replicates its empirical counterpart. Within the calibrated economies, the estimated value loss is as follows. In a world with dynamic equity premium, the sub-optimally investing Type 1 firm has the present value of growth options 13.3% lower than the Type 2. In contrast, if the world features constant equity premium, as implied in the CAPM, the Type 2 firm incurs only 0.8% loss in growth option by its sup-optimal investments. The asymmetry in the value loss is largely driven by the timing of sub-optimal investment. In the world with dynamic equity premium, the Type 1 firm overinvests most at the state of highest uncertainty in growth, the exact state where the marginal rate of substitution is the highest. On the contrary, the Type 2 firm in the economy with constant equity premium is not exposed to such coordination between erroneous investment and the marginal rate of substitution, thus having a lower value loss. In addition to quantification of value losses, we also document the firm types yield statistically significant return differences in each economy. The returns on firms with sub-optimal investment policy are higher than those with correct investment in both economies.

The paper is organized as follows. In the next section, we describe the economic environment and firm-level investments. In section 3, we provide the valuations of projects and then derive the investment rule and the resulting firm values. In section 4, we calibrate the model and examine differences in firm values and returns on firm caused by the firm type and investment rule. Section 5 concludes.

2 Model

We model the pricing kernel that results from a standard Bansal and Yaron (2004) endowment economy. Characterizing the pricing kernel as the product of preferences and consumption growth facilitates calibration (and comparison to other well-known models). In addition, we model investment projects that deliver cash flows exposed to consumption growth risk. This set-up gives the projects their "systematic" risk. Our model is only partial-equilibrium since we do not connect the sum of all projects in the economy back to aggregate consumption.

2.1 Economic Environment

Preferences of the representative agent are recursive as in Epstein and Zin (1989), and Weil (1989). The decision interval is one month. Preferences at date t are given by

$$U_t = [(1-\beta)c_t^{\rho} + \beta\mu\,(U_{t+1})^{\rho}]^{1/\rho} \tag{1}$$

where μ is the certainty equivalent, i.e., $\mu(U_{t+1}) = E_t\left[U_{t+1}^{\alpha}\right]^{1/\alpha}$. The marginal rate of substitution – the pricing kernel – is

$$m_{t+1} \;=\; \beta(c_{t+1}/c_t)^{\rho-1}\,[U_{t+1}/\mu(U_{t+1})]^{\alpha-\rho}$$

where $(c_{t+1}/c_t)^{\rho-1}$ accounts for the short-run consumption growth risk, and the next term $(U_{t+1}/\mu_t(U_{t+1}))^{\alpha-\rho}$ captures the effect of the agent's expectation on future utility. The derivation of the pricing kernel is provided in appendix A.1 through A.3. (It is algebra similar to Backus et al. (2010)).

We specify an exogenous stochastic process for consumption growth. The consumption growth from date $t-1$ to t, denoted by $g_t = c_t/c_{t-1}$, is described with the underlying state variable, x_t, a vector of arbitrary dimension. Specifically, the logarithm of consumption growth is assumed to be $\log g_t = g + e^T x_t$, where e is a constant vector. The dynamics of the state variable x_t features AR(1) with a stochastic volatility:

$$\begin{aligned} x_{t+1} &= A x_t + v_t^{1/2} B w_{t+1} \\ v_{t+1} &= (1-\varphi)v + \varphi v_t + b w_{t+1} \end{aligned} \tag{2}$$

where v is the unconditional mean of v_t, $\{w_t\} \sim NID(0, I)$, and $Bb^T = 0$. With this representation, x_t and v_t controls the conditional mean and volatility of consumption growth in future, respectively, and they summarize the state of economy.[1] The stochastic volatility in the growth is the source of creating time-variation in equity premium. Therefore, we can represent the different types of economy or firm - having the time-varying equity premium or not - by turning on or off the stochastic volatility channel while keeping others equal. Thus, for the description of the economy with constant equity premium, we impose that the volatility of consumption growth is constant.

With these agent preferences and consumption dynamics, we can derive the pricing kernel. The logarithm of the pricing kernel is

$$\log m_{t+1} = \delta_0 + \delta_x^T x_t + \delta_v v_t + \lambda_x^T w_{t+1} + \lambda_v^T w_{t+1} \tag{3}$$

[1] If $e = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} \phi & \theta \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \sigma_e & 0 \\ \sigma_e & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 0 & \sigma_w \end{bmatrix}$, the dynamics is an approximation of the dynamics in Bansal and Yaron (2004) with stochastic volatility. The Guassian shock to volatility is an approximation, obviously. It is straightforward to relax but makes the algebra less transparent.

where δ_0, δ_x, δ_v, λ_x, and λ_v are known functions of preference and consumption dynamics parameters. The derivation of the pricing kernel is provided in appendix A.2 and A.3. As equation (3) shows, the pricing kernel changes across the states of the economy, i.e., the conditional mean and volatility of the consumption growth, and innovations. If a firm manager does not perceive correctly the underlying economy and risks in growth, the manager's pricing kernel will be different from the other with correct understanding of risks. This discrepancy leads to different evaluations for the same cash flows, so they would invest differently from each other.

2.2 Firms

Firms operate with an infinite horizon. Individual projects are finite-lived. At each date, a new opportunity becomes available to a firm. The firm decides whether to undertake the new project or not. If a new project is not undertaken, that opportunity is gone (now-or-never option), and the firm will receive a new opportunity next date. If the firm decides to invest in a new project, the initial cash flow is negative (investment) followed by subsequent positive cash flows. Project termination is deterministic in our setting.

To undertake a project at date t requires upfront investment of Id_t, where d_t represents the size of cash flows from the project. Once undertaken, the project delivers cash flows of which growth is correlated with consumption growth, and the positive cash flows start at date $t+1$. Let d_{t+s} denote cash flow at date $t+s$ from the project. The growth in cash flow at date $t+s$ is given by

$$\frac{d_{t+s}}{d_{t+s-1}} = \exp\left(g + e^T(Ax_{t+s-1} + \beta_t v_{t+s-1}^{1/2} B w_{t+s}) - \frac{\beta_t^2 v_{t+s-1}}{2} e^T B B^T e\right). \tag{4}$$

where β_t controls the covariance between the cash flows and consumption, thereby capturing systematic risk of the project. The basic idea of the expression for cash flow is that the mean growth and volatility of the project-level cash flows are influenced by the economic state x_t and v_t, respectively, which describe consumption growth. Specifically, the growth in project cash flow is the mean-preserving spread of the consumption growth, where β_t determines the covariance between project cash flow and consumption, so it is the systematic risk of the project. Note that the systematic risk β_t is project-specific. The β_t is drawn from a distribution and known at the date of investment decision and the realized β_t is constant for the life of the project. For simplicity, we assume that systematic risk β_t is drawn from a uniform distribution over $[0, \beta_{max}]$. The project generates these cash flows for N periods and becomes obsolete afterwards. In addition, we normalize the size of investment I to be 1 and assume that all projects have the same size of initial cash flow d^0.

This specification of project might seem to imply too strong tie between a project payout and aggregate consumption. However, in the firm-level, the specification still produces an imperfect correlation between the firm payout and consumption, as in Bansal and Yaron (2004). This is because the firm-level payout on a date is a collection of cash flows of projects that have idiosyncratic covariances with consumption. The idiosyncracy produces a loose link between the firm payout and consumption, while the exposure to consumption growth risk captures the systematic risk.

3 Valuation

The projects we consider have cash flows across time, so we begin the valuation by pricing elementary assets that deliver cash flow at a single date. With the values of these elementary assets, we can evaluate the project and analyze the firm's investment decision. Also we evaluate the value of resulting assets in place and the value of future investment opportunities or growth options.

3.1 The Valuation of Project Payout

Consider an elementary asset that delivers risky cash flow d_{t+s} with the systematic risk β_j at a single date $t+s$. Let q_t^s denote the date t-price-payout ratio of the asset. The price at t of the asset that pays at date $t+1$ is determined by

$$q_t^1 = E_t\left[m_{t+1}\frac{d_{t+1}}{d_t}\right]. \tag{5}$$

Similar to the prices of "zero-coupon equity" in Lettau and Wachter (2007), the price is an exponential affine function of the state variables as follows:

$$q_t^1 = \exp\left(\delta_0 + g + \frac{\lambda_v^T\lambda_v}{2} + (\delta_x^T + e^T A)x_t + \left(\delta_v + \frac{\lambda_x^T\lambda_x^T}{2} + \beta_j e^T B\lambda_x\right)v_t\right). \tag{6}$$

where δ_0, δ_x, δ_v, λ_x, and λ_v are known functions of parameters for preferences and consumption dynamics. The price-payout ratio of the asset with maturity $s > 1$ is

$$q_t^s = \exp\left(D_{0,s} + D_{x,s}x_t + D_{v,s}v_t\right) \tag{7}$$

where $D_{0,s}$, $D_{x,s}$, and $D_{v,s}$ are the constants which are recursively related with the constants for the asset with maturity of $s-1$ in the following way:

$$\begin{aligned}
D_{0,s} &= \delta_0 + g + D_{0,s-1} + D_{v,s-1}(1-\varphi)v + (1/2)\left(\lambda_v^T + D_{v,s-1}b\right)\left(\lambda_v^T + D_{v,s-1}b\right)^T \qquad (8)\\
D_{x,s} &= \delta_x^T + (e^T + D_{x,s-1})A \\
D_{v,s} &= \delta_v + D_{v,s-1}\phi_v - (1/2)\beta^2 e^T BB^T e + (1/2)(\lambda_x^T + \beta e^T B + D_{x,s-1}B)(\lambda_x^T + \beta e^T B + D_{x,s-1}B)^T.
\end{aligned}$$

The derivation is presented in appendix A.5.

3.2 The Valuation of Assets in Place

A firm has the two sources of the value - assets in place and growth options. Assets in place refer to a collection of existing projects coming from past investment decisions. Growth options denote the value of future investment opportunities.

The NPV of the date-t project, say P_t, is

$$P_t = E_t \left[\sum_{s=1}^{N} m_{t,t+s} d_{t+s} \right] - I d_t \tag{9}$$

where $m_{t,t+s}$ is the marginal rate of substitution between consumption at t and at $t+s$, which is given by $m_{t,t+s} = \prod_{k=1}^{s} m_{t+k}$. The NPV normalized by payout, p_t, is

$$p_t = E_t \left[\sum_{s=1}^{N} m_{t,t+s} \frac{d_{t+s}}{d_t} \right] - I = \sum_{s=1}^{N} q^s(x_t, v_t, \beta_t) - I \tag{10}$$

where the value is expressed with the prices of the elementary assets in section 3.1. As the investment opportunity is a now-or-never option, the firm undertakes the project whenever its NPV, p_t, is positive. Thus the firm's investment decision at date t depends on both the project-specific shock, β_t, and the state of the economy, (x_t, v_t). Finally, the firm's perception of the economy and the pricing kernel may well influence the investment policy.

We represent the investment decision at date j with an indicator χ_j such that $\chi_j = 1$ if the firm invests or 0 otherwise. Then the value of assets in place, denoted by K_t, is

$$K_t = \sum_{j=t-N+1}^{t} \sum_{s=1}^{N-t+j} \chi_j q^s(x_t, v_t, \beta_j).$$

The expression simply means that the assets in place include past projects which were undertaken at date $t-N+1$ or afterwards, because projects become obsolete N periods after their inception dates.

3.3 The Valuation of Growth Options

To value the firm's growth options, we consider a single investment opportunity that will arrive at $t+1$. Because the firm will take on the project only if its NPV is positive, the payoff of the investment opportunity is similar to that of a financial option. When both firm-specific shock and economic states are realized at date $t+1$, the option value is $\max(p_{t+1}, 0)$. Let $f(x_{t+1}, v_{t+1})$ denote the value of the option conditional on states (x_{t+1}, v_{t+1}) and prior to realization of project-specific risk β_{t+1}. The option value is expressed as follows:

$$f\left(x_{t+1}, v_{t+1}\right) = \int_0^{\overline{\beta}} p(x_{t+1}, v_{t+1}, \beta) \frac{1}{\beta_{\max}} d\beta \tag{11}$$

where $\overline{\beta}$ is the investment threshold for systematic risk such that $p(x_{t+1}, v_{t+1}, \overline{\beta}) = 0$.[2]

The firm has a series of investment options which will become available from $t+1$ onwards. The date-t present value of growth options, say $S\left(x_t, v_t\right)$, can be expressed in a recursive way:[3]

$$\begin{aligned} S\left(x_t, v_t\right) &= E_t\left[\sum_{s=1}^{\infty} \frac{m_{t,t+s}}{d_t} \max(P_{t+s}, 0)\right] \\ &= E_t\left[m_{t,t+1} E_{\beta_{t+1}}\left[\max\left(p_{t+1}, 0\right)\right] + m_{t,t+1} \frac{d_{t+1}}{d_t} E_{t+1}\left[\sum_{s=1}^{\infty} \frac{m_{t+1,t+1+s}}{d_{t+1}} \max(P_{t+1+s}, 0)\right]\right] \\ &= E_t\left[m_{t,t+1} f\left(x_{t+1}, v_{t+1}\right) + m_{t,t+1} \frac{d_{t+1}}{d_t} S\left(x_{t+1}, v_{t+1}\right)\right]. \end{aligned} \tag{12}$$

In the derivation, we use the law of iterated expectation to value the investment option available at date $t+1$. From the recursive structure, the present value of growth options is solved numerically as a function of the state variables.[4]

[2] Of course, $\overline{\beta}$ is a function of the state, (x_{t+1}, v_{t+1}), but we omit the argument for simplicity.

[3] $E_{\beta_{t+1}}$ denotes the expectation over the distribution of systematic risk β_{t+1}.

[4] For the computation, we use a finite support of state variables by discretizing the support of the state based on the method of Tauchen (1985). We then solve for growth options with the value function iteration.

With the expression of growth options, we can analyze the result of using an incorrect pricing kernel. Suppose a firm has an incorrect model of the risk – say it ignores the dynamic properties of the risk premium – then investment policy is sub-optimal. Moreover, the degree of the sub-optimality can be state-dependent. For example, if the firm tends to invest particularly badly when when the marginal rate of substitution is high, the value loss will be particularly large.

4 Quantitative Analysis

We are interested in the quantitative implications of a firm's investment policy. It is important to ask this question in a sensibly calibrated model. Here, we look at two economies - the one with constant risk premium and the other with dynamic risk premium. Both are set to match features of aggregate consumption and aggregate asset returns including the equity premium. After the economic environment is quantitatively specified, the project characteristics are chosen to reproduce the empirical moments regarding firms' asset and income characteristics. With the calibrated model, we analyze firms' investment policies and value implications of incorrect investment rules. Also, we study return differential on firms' assets by simulating firm panels in each economy.

4.1 Calibration

The calibration consists of two steps. First, we calibrate each economic environment to fit the stylized facts of consumption growth and aggregate asset returns, using the empirical moments in Bansal and Yaron (2004). Following their calibration procedure, we simulate 1,000 samples of 840 month-long time series of consumption growth, given a choice of parameters characterizing consumption and preferences. In matching features of aggregate stock returns, we regard the consumption stream as the aggregate equity and compute its monthly returns. Appendix A.4 provides the analytical expression of return on equity. The realized monthly consumption growth and returns are aggregated to annual frequency, and they are compared to corresponding annual empirical moments. Note that we calibrate separately the two economies. By making the two economies differ only in terms of the dynamic property of the equity premium and have the same sensible properties otherwise, we can fairly compare the two economies and isolate the impact that dynamic equity premium has as opposed to static equity premium.

Table 1 shows the calibration results. The Dynamic Equity Premium column refers to the economy featuring the stochastic volatility in growth. This economy is characterized by time-variation both in mean and volatility of growth, x_t and v_t. In contrast, the Constant Equity Premium column is for the economy with constant volatility, where only the mean of growth is time-varying; in this economy, the expected growth in cash flow is state-dependent, while the equity premium is constant. This constant risk is along the line with the CAPM. In the calibration results, both economies match similarly the empirical moments including autocorrelations of consumption growth and the mean and the standard deviation of asset returns. Therefore, two types are isomorphic with respect to the matched moments, but a main distinction between the two is the presence of time-variation of the equity premium. As reported in the row of the standard deviation of conditional mean excess return, the Static Equity Premium economy cannot have such time-variation, which a number of recent studies support including Ludvigson and Ng (2007), Welch and Goyal (2008), Jermann (2010), and Cochrane (2011).

Now we turn to calibrating the parameters that describe projects: project lifetime N, size of initial cash flow d^0, and maximum of systematic risk β_{max}. In calibrating these parameters, we use data on manufacturing firms (SIC 2000-3999) from COMPUSTAT for years 1974 to 2012. First, to pin down the project lifetime, we use the average depreciation rate, Depreciation and Amortization (DP)/Property, Plant and Equipment (PPEGT), and assume that assets depreciate exponentially at the average rate. Then, the project lifetime is defined as the time elapsed since the inception of a project until only 1% of an asset remains. Using the empirical average 0.236 of the depreciation rate, the project lifetime is 9 years.

Next, we choose the maximum risk β_{max} of projects in the uniform distribution to match the covariance between consumption and aggregate income of firms. To obtain the aggregate income, we sum quarterly observations of Operating Income Before Depreciation (OIBDP) across entire manufacturing companies and find the quarterly growth of the aggregate operating income. The resulting ratio of covariance between the consumption and the operating income to the variance of the consumption is 3.68. In order to match the empirical ratio, $\beta_{max} = 7.36$ is chosen [5].

Finally, we find the size of initial cash flow d^0 that leads to the book-to-market ratio close to its empirical average, 0.637. Specifically, we simulate a 20,000 month-long firm panel consisting of 500 firms. Each month, the firms face a realization of the economic state

[5] The ratio of covariance between the consumption and the operating income to the variance of the consumption enables us to pin down β_{max}. Specifically, let d_t^i denote the the project i's payout at t, g_t the mean consumption growth , and β_i the systematic risk. Then, the log of sum of payouts of N projects is

$$\begin{aligned}\log(d_{t+1}^1 \cdots + d_{t+1}^N) &= \log(d_t^1 e^{(g_t+\beta_1 v_t^{1/2} Bw_{t+1} - \beta_1^2 v_t e^T BB^T e/2)} \cdots + d_t^N e^{(g_t+\beta_N v_t^{1/2} Bw_{t+1} - \beta_N^2 v_t e^T BB^T e/2)}) \\ &\approx \log(d_t^1 e^{(g_t+\overline{\beta} v_t^{1/2} Bw_{t+1} - \overline{\beta}^2 v_t e^T BB^T e/2)} \cdots + d_t^N e^{(g_t+\overline{\beta} v_t^{1/2} Bw_{t+1} - \overline{\beta}^2 v_t e^T BB^T e/2)}) \\ &+ \frac{d_t^1 (v_t^{1/2} Bw_{t+1} - 2\overline{\beta} v_t e^T BB^T e/2)}{d_t^1 \cdots + d_t^N}(\beta_1 - \overline{\beta}) + \cdots + \frac{d_t^N (v_t^{1/2} Bw_{t+1} - 2\overline{\beta} v_t e^T BB^T e/2)}{d_t^1 \cdots + d_t^N}(\beta_N - \overline{\beta})\end{aligned}$$

where the Taylor expansion with respect to β_i is used around the weighted average of β_i of N projects, $\overline{\beta} = \frac{d_t^1 \beta_1 \cdots + d_t^N \beta_N}{d_t^1 \cdots + d_t^N}$. From the definition of $\overline{\beta}$, the first order terms in the expansion cancel out, leading to

$$\log\left(\frac{d_{t+1}^1 \cdots + d_{t+1}^N}{d_t^1 \cdots + d_t^N}\right) \approx g_t - \frac{\overline{\beta}^2 v_t e^T BB^T e}{2} + \overline{\beta} v_t^{1/2} Bw_{t+1}.$$

Thus, the unconditional covariance between consumption growth and aggregate income growth is $\overline{\beta} vBB^T$, which is $\overline{\beta}$ times the variance of consumption growth. As the number of projects approach to infinity, $\overline{\beta} \to E\left[\frac{N d_t^i}{\sum_{i=1}^N d_t^i}\beta_i\right]$. Then, using the model feature that mean growth of payout is independent of β_i, we find $E\left[\frac{N d_t^i}{\sum_{i=1}^N d_t^i}\beta_i\right] = \frac{\beta_{max}}{2}$.

that is common to all firms and firm-specific new projects. Observing the realized state and the project's systematic risk, each firm makes an investment decision. As time passes, each firm builds its own assets in place as a result of past investment decisions, while old projects expire. In this simulated panel, the book-to-market ratio is defined as the fraction of assets in place in firm value, which is the sum of assets in place and growth options. To ensure for firms to stabilize in their asset composition, we exclude first 500 observations and calculate the average on the panel. The matched book-to-market ratio is 0.636 at $d^0 = 0.00993$.

4.2 Project Values and Firm-level Investment

In the model, the project is a collection of the elementary assets delivering single risky cash flow. Thus examining the state dependence of the prices of the elementary assets helps to understand how project value and investment rule should change across the states. Figure 1 plots the prices of the elementary assets against their maturity in the economy with dynamic equity premium. Generally, the price decreases with maturity of the asset - the date at which the asset pays. The price of the asset also changes across the states characterizing economic growth: the conditional mean and the volatility of the growth. To illustrate the price changes, the figure plots the prices at high and low state for each state variable by one standard deviation. If the economy is expected to have a high mean growth (high x_t), a large payoff is expected to be delivered by the assets. Therefore, the asset price increases with the expected growth; at enough high x_t, some strips with particularly short maturities have more value than a unit in spite of the time preference.

The dependence of the asset prices on the conditional volatility is two-fold. A rise in the volatility enlarges a negative covariance between the pricing kernel and the asset payoff, magnifying the systematic risk. At the same time, the agent values future payoff more at the high volatility, due to the penalty for risk. The trade-off between two opposing forces depends on the delivery date of the assets. For short-term assets, the second effect dominates, so the price increases with the volatility. However, as the delivery date becomes farther from now, the first effect takes place to a greater extent, leading to a lower value in a state of higher volatility.

In addition to the economic states, the asset's exposure to systematic risk is another determinant of the prices. Of course, the prices of assets with greater systematic risk are lower: in Figure 1, the prices of assets of $\beta = 3$ are lower than those of assets of $\beta = 1$ with corresponding maturity in all states.

Next, we examine the firm's investment in the economy with dynamic equity premium. We characterize dependence of investment behavior on economic state by looking at the investment probability before the project-specific shock is realized. Figure 2 plots the probability that $\beta_t \leq \overline{\beta}(x_t, v_t)$, where $\overline{\beta}(x_t, v_t)$ is the investment threshold. The investment threshold changes across different economic states. For example, the firm is more likely to invest in state of a large expected growth. This is intuitive because in such a state, the project is expected to generate large cash flows, raising the ex-ante value of the project prior to realization of project-specific systematic risk.

The volatility of the growth also influences the investment policy. Specifically, the firm is more likely to invest when it faces lower uncertainty of economic growth. This negative association arises from the state-dependence of prices of the elementary assets. A higher volatility magnifies the negative covariance between the pricing kernel and payouts, thus lowering the ex-ante value of project prior to realization of project-specific risk. As a result, the firm tightens investment policy and the probability of investment falls. This relation between investment and the volatility has the same direction as the prediction of the real option theory as in Dixit and Pindyck (1993). The mechanism here, however, is different from their argument. In the real option theory, the option value of waiting is higher when the underlying project value is more volatile, so a firm invests less at high volatility. In our model, in contrast, the investment is now-or-never option, so there is no value of waiting. Instead, the dependence on the volatility comes from its impact on the pricing kernel and amplifying the exposure to systematic risk. Another point worth mentioning is that if we interpret the state of high x_t and low v_t as a boom of economy and the state of low x_t and high v_t as a recession, our model replicates the procyclical behavior of aggregate investment, a stylized fact in the business cycle literature such as King and Rebelo (1999).

We now turn to the asset prices and investment behavior in the economy with static equity premium. Figure 3 demonstrates the asset prices. Dependence of the prices on x_t is similar to the economy with dynamic equity premium: the prices are high in state of a large expected growth. The main difference from the economy with dynamic equity premium is that there is no price-dependence on v_t, obviously because the volatility is assumed to be constant. Hence, the investment probability in Figure 4 only responds to changes in the expected growth.

By comparing investment probabilities in Figure 2 and 4, we can expect consequences when the firm perceives the risk premium differently from the reality. If the real economy features the stochastic volatility and a dynamic risk premium, but a firm considers the risk premium as constant, the firm invests with the rule of Figure 4, even though the correct decision should be based on Figure 2. As a result, the firm may underinvest or overinvest, because it fails to adjust the valuation according to changes in risk premium. Another sub-optimal investment rises in the opposite case that a firm considers the uncertainty of the economy as time-varying, while there is actually no such a time variation. These sub-optimal investments result in value loss and return differentials, which will be measured in the following section.

4.3 Firm Types and Growth Options

Consider the two firms, Type 1 that acts as if the equity premium is constant and Type 2 that acts as if the equity premium is dynamic. We compare these two firms in the two economies - economy with dynamic equity premium and economy with constant equity premium. A mismatch between the real economy and firm's perception of the economy, for example Type 1 firm in the economy with dynamic equity premium, leads to incorrect valuations of projects and possibility of wrong investment decisions. The value loss caused by the sub-optimal investment should be particularly pronounced in the present value of growth options, since the main determinant of growth option is the investment policy itself. In contrast, the value of assets in place depends largely on past realization of project-specific shocks as well as investment policy. Thus we focus our analysis on growth options.

Figure 6 compares the present value of growth options of the two firms in the economy with dynamic equity premium. Before comparing the two firms, the state-dependence of the Type 2's growth options, which discounts correctly, is worth mentioning. The value of growth options depends on both the investment policy and the project value in each state. When the economy expects a larger growth (high x_t) or lower uncertainty (low v_t), the value of growth options is high. This is because in such a state, projects are of higher value on average, so the probability of investment is higher.

The Type 1 firm does not consider the fluctuating uncertainty in the growth and ends up with incorrect investment rule. The sub-optimal investment is graphically illustrated in Figure 5. The top panel in the figure plots the investment threshold of project-specific systematic risk at different levels of the volatility, when x_t is fixed at 0.0343 as an example. When the realized systematic risk is lower than the threshold of each firm, the firm's evaluation of NPV is positive, so the firm invests. The correctly discounting Type 2 adjusts the threshold to changes in volatility, while Type 1 does not. As a result of ignoring the time-variation in volatility, the Type 1 firm underinvests at low volatility and overinvests at high volatility compared to the Type 2. Because the value of growth options represents the option value associated with the investment policy, the sub-optimal investment leads to the value of the Type 1 lower than that of the Type 2 in all economic states, as the bottom panel in Figure 6 shows. In evaluating growth options of Type 1, we assume the erroneous investment behavior is evaluated based on perspectives of Type 2, the correct perspectives of the economy.

The magnitude of value loss depends on the economic states. In state of a low volatility, the average value of project is higher, whereas Type 1 underinvests then. Due to Type 1's missing particularly profitable projects, its value loss is greater in such a state than other states. In addition, also in state of a low expected growth, Type 1 incurs a greater value loss. This is because with a high marginal rate of substitution driven by a negative growth, Type 1's suboptimal investments are penalized more.

Given the state-by-state value loss, we measure the time-average of the loss to the Type 1 over the simulated time-series of economic states. On average, the Type 1 firm incurs the loss of 13.3% of the firm value. This quantity represents the value loss if a firm discounts future cash flows along the line with the CAPM, when the underlying economy has the dynamic equity premium as macro-asset pricing literature finds.

Next, we turn to the economy with static equity premium and compare the two firms in Figure 7. Contrary to the previous economy, now the Type 1 firm evaluates projects correctly. The Type 2 firm acts as if the volatility of growth is time-varying, even though the economy actually features constant volatility. Hence the Type 2 firm has an incorrect investment rule, as shown in the bottom panel of Figure 5. This results in a value loss to Type 2, as shown in Figure 7. However, the magnitude of the loss of the Type 2 is much smaller than that of the mismatching counterpart, the Type 1 firm in the economy with dynamic equity premium. The Type 2 firm in the economy with constant equity premium is exposed to the average loss of only 0.8% of growth options. The asymmetry in the value loss of the two mismatch cases comes from the timing of sub-optimal investment. In the economy with dynamic equity premium, the Type 1 overinvests most at the highest level of the volatility, the exact state when the marginal rate of substitution is high. On the other hand, the Type 2's incorrect investment decisions do not have such a coordination between the sub-optimal investment and the pricing kernel. As a result, the failure to correctly characterize the risk property causes a larger value loss in the economy with dynamic equity premium.

As to our main question, whether the time-variation in equity premium matters to a firm's capital budgeting, our answer is yes. If the economy features the time-varying equity premium, whether to consider the variation or not results in a sizable difference in growth options of 13.3%.

4.3.1 Project Characteristics and Sub-Optimal Investment

Obviously, the value loss due to the sub-optimal investment depends on the project characteristics - lifetime of projects, N, and size of required initial investment, I. In this section, we focus on the economy with dynamic equity premium and study how the value loss to the Type 1 changes across different lifetimes of project.

If we simply change the project lifetime and keep initial investment unchanged, the average value of project also changes. For example, if the project delivers cash flows for a longer period for the same investment, the average value of project before the realization of project-specific shock increases and so does the investment probability. Thus, to focus on exposure to risk of different project lifetimes and its effect on the value loss, we control for the investment probability; we adjust the amount of initial investment so that the investment probability is the same across different lifetimes. We can interpret this comparative statics as a benchmark for industry-specific value implications of investment policy; industries are different in terms of average project duration.

Figure 8 plots value loss to growth options of Type 1 against project lifetime. We find that value loss increases considerably from near 8% to 22%, as the project lifetime increases from 5 to 15 years, indicating that when Type 1 sub-optimally invests in longer-term projects, Type 1 incurs a larger value loss. This is because for later cash flows in a longer-term project, there is a greater uncertainty due to a longer time interval between now and delivery date of the cash flow. Thus, a precision of characterizing risk of cash flows becomes more important, and Type 1's ignoring dynamic risk premium leads to a larger value loss.

4.4 Firm Types and Returns

In this section, we look at the model's implication on the relation between firm types and returns on firms. Specifically, we study how using the correct or incorrect investment rule influences the returns in each economy. As the firm value, assets in place in particular, depends on a long history of past investment decisions, it is difficult to study the returns analytically, so we use a simulation. In the simulation, we have a group of firms with the optimal policy for investment, for example, Type 2 in the economy with dynamic equity premium, and the other group of firms with sub-optimal policy, Type 1 in the example. Then we allow firms to build their project portfolios over time by employing the given investment rules and investigate effects on the firm values and returns.

The simulation procedure is as follows. For each underlying economy, we generate 200,000 month-long history of economic states. We then let 500 Type 1 firms and 500 Type 2 firms operate in the economy. At the beginning, all firms have no project in place. As time passes, each firm faces the economic state and the investment opportunity with project-specific systematic risk, and each decides whether to invest or not in the new project, based on its evaluation of NPV. To control the effect of different realization of project shocks, we assume that each firm from Type 1 has the counterpart from Type 2 with the identical history of project shocks. With this setting, any difference in collective firm assets between the two groups is attributable to their investment policies. In this firm panel, we compute realized returns on the portfolios - one consisting of Type 1 firms and the other consisting of Type 2 firms.

Table 2 reports the return differences between the firm types in each economy. In both economies, firm's risk characterization and the resulting investment rule generate statistically significant return differences. In economy with dynamic equity premium, the monthly return on Type 1 portfolio is on average 0.24% higher than that on Type 2 portfolio. In order to understand what drives such return differences, we decompose the firm value into growth options, $V_{G,t}$, and assets in place, $V_{A,t}$, plus cash, H_t. Then, returns on firms can be decomposed as follows:

$$\begin{aligned} R_{t+1} &= \frac{V_{t+1} + H_{t+1}}{V_t} \\ &= \frac{V_{G,t+1}/d_{t+1}}{V_t/d_t}\frac{d_{t+1}}{d_t} + \frac{V_{A,t+1}/d_{t+1} + H_{t+1}/d_{t+1}}{V_t/d_t}\frac{d_{t+1}}{d_t} \\ &= \frac{S_{t+1}}{S_t}\frac{d_{t+1}}{d_t}(1 - \mathrm{BM}_t) + \frac{K_{t+1} + h_{t+1}}{K_t}\frac{d_{t+1}}{d_t}\mathrm{BM}_t \\ &= R_{G,t+1}\frac{d_{t+1}}{d_t}(1 - \mathrm{BM}_t) + R_{A,t+1}\frac{d_{t+1}}{d_t}\mathrm{BM}_t \end{aligned} \tag{13}$$

where S_t denotes growth options normalized by current payout, K_t denotes normalized assets in place, h_t normalized cash, $R_{G,t+1}$ return on growth options, $R_{A,t+1}$ return on assets in place, and $BM_t(= K_t/(S_t + K_t))$ a pseudo book-to-market ratio - the fraction of assets in place in the firm value. The expression shows that return on firm is a weighted average of returns on growth options and return on assets in place, where the weight is the book-to-market ratio. Given this decomposition, we study the return differences by examining how the two firm types differ in each component of the returns.

The second row of Table 2 shows that in the economy with dynamic equity premium, returns on growth options of firms with incorrect investment rule, Type 1, are 0.79% higher than those returns of optimally investing Type 2. We can identify the source of this return differences by looking at how the magnitude of value loss in Type 1's growth options changes across the states. Figure 6 shows the Type 1's value loss is particularly large when the expected growth is low and the loss decreases as the expected growth approaches to the mean level. Combined with the mean-reverting property of the economic state, the state dependence of value loss implies that the expected return on growth options of Type 1 is large at the state of low expected growth, surpassing the return of Type 2. The panel (a) in Figure 9 depicts the differences in returns on growth options and documents difference of as large as 3.5% when the economy features a low mean and a low volatility in growth.

Type 1 also has higher returns on assets in place, but the difference is not statistically significant. This is not surprising in that assets in place is largely determined by history of project-specific shocks, so its dependence on economic state is not as clear as we observe for growth options. As a result of the weighted average of these two returns, returns on Type 1 firms are statistically significantly higher than those on Type 2, mainly due to higher return on growth options.

In economy with constant equity premium, the sub-optimally investing Type 2 portfolio has higher returns than Type 1 by 0.01%. The panel (b) of Figure 9 compares return on growth options between the two firm types. Although Type 2 firm has higher return than Type 1 in some states of high expected growth, Type 2 firm has lower return in other states including states near mean, which firms frequently face. Consequently, Type 2's average return on growth options is lower by 0.0009%, as documented in Table 2. Quantitatively, the major determinant of return differentials in this economy is the covariance between return on assets in place and the book-to-market ratio. Type 2's returns on assets in place are higher than Type 1 in states of high mean growth, when the book-to-market ratio appears high, thus imposing more weight on returns on assets in place. As a result of the coordination effect, Type 2 has higher return on firm, as Table 2 reports, even though both of the expected return components are lower for Type 2.

In short, whether a firm's discounting correctly reflects the underlying economy or not changes risk characteristics including exposure to sub-optimal investment and sensitivity to the risk measured by the book-to-market ratio. As a result, it produces statically significant return differences, as well as firm value differences.

5 Conclusion

Capital investment decisions in practice are typically hard business problems. They involve difficult and long-horizon forecasts, cut across many functional business areas, and are often strategic. The NPV rule and framework is a powerful tool for structuring this complex decision. The framework is, of course, not without many assumptions that do not strictly hold. In this paper, we look at one specific common practice in capital budgeting – discounting cash flows at a constant cost of capital. In our calibrated model, the implication is large. We estimate a 13.3% value loss from this decision. This loss is much larger that the 0.8% loss for the (counter-factual) scenario of a firm that is in a constant risk-premium economy but investing according to a dynamic-risk-premium model. The dramatic difference is that the over-investment in the first case is correlated with bad states of the economy. It is quite possible that firms do take account of time variation in equity premium without direct calculation. Capital budgets are procyclical in part, it seems from casual observations, as capital budgeting projects receive more scrutiny in recessions. One way we might infer if firms are indeed investing without regard for the time-variation in the equity premium is through subsequent return behavior. In our calibrated model, monthly returns on the incorrectly discounting firms are 0.24% higher on average than those on correctly discounting firms. We leave an empirical exploration of this question to future research.

References

Backus, David K., Bryan R. Routledge, and Stanely E. Zin, 2010, The cyclical component of us asset returns, *Working Paper* .

Bansal, Ravi, and Amir Yaron, 2004, Risk for the long run: a potential resolution of asseet pricing puzzles, *Journal of Finance* 59, 1481–1509.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Cochrane, John H., 2011, Presidentail address: Discount rates, *Journal of Finance* 66, 1047–1108.

Dixit, Avinash K., and Robert S. Pindyck, 1993, *Investment under Uncertainty* (Princeton University Press).

Epstein, Larry G., and Stanely E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57, 937–969.

Graham, John R., and Campbell R. Harvey, 2001, The theory and practice of corporate finance: evidence from the field, *Journal of Financial Economics* 60, 187–243.

Jermann, Urban J., 2010, The equity premium implied by production, *Journal of Financial Economics* 98, 279–296.

King, Robert G., and Sergio T. Rebelo, 1999, Resuciating real business cycle, *Handbook of Macroeconomics* 927–1007.

Lettau, Martin, and Jessica A. Wachter, 2007, Why is long-horizon equity less rrisk? a duration-based explanation of the value premium, *Journal of Finance* 62, 55–92.

Ludvigson, Sydney C., and Serena Ng, 2007, The empirical risk-return relation: a factor analysis approach, *Journal of Financial Economics* 2007, 171–222.

Tauchen, George, 1985, Finite state markov-chain approximations to univariate and vector autoregressions, *Economic Letters 20* 20, 177–181.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journalf of Moneytary Economics* 24, 401–421.

Welch, Ivo, 2000, Vpuzzle financial economists on the equity premium and other issues, *Journalf of Business* 73, 501–537.

Welch, Ivo, and Amit Goyal, 2008, A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies* 21, 1455–1508.

A Appendix

A.1 to A.4 are from Backus, Routledge, and Zin (2010).

A.1 The Kreps-Porteus Pricing Kernel

The pricing kernel in a representative agent model is the marginal rate of substitution between consumption at date t and consumption in state s at $t+1$. Define $\pi(s)$ as the probability of state s at $t+1$. Then the certainty equivalent is

$$\mu_t\left(U_{t+1}\right) = \left[\sum_s \pi(s) U^{\alpha}_{t+1(s)}\right]^{1/\alpha} \tag{A.1}$$

where $U_{t+1}(s)$ is continuation utility. Some derivatives of equation 1 and equation A.1 are :

$$\begin{aligned}
\frac{\partial U_t}{\partial c_t} &= U_t^{1-\rho}(1-\beta)c_t^{\rho-1} \\
\frac{\partial U_t}{\partial \mu_t(U_{t+1})} &= U_t^{1-\rho}\beta\mu_t(U_{t+1})^{\rho-1} \\
\frac{\partial \mu_t(U_{t+1})}{\partial U_{t+1}(s)} &= \pi(s)U_{t+1}(s)^{\alpha-1}\mu_t(U_{t+1})^{1-\alpha}.
\end{aligned} \tag{A.2}$$

The marginal rate of the substitution between consumption at t and consumption in state s at $t+1$ is

$$\begin{aligned}
\frac{\partial U_t/\partial c_{t+1}(s)}{\partial U_t/\partial c_t} &= \frac{\left[\partial U_t/\partial \mu_t(U_{t+1})\right]\left[\partial \mu_t(U_{t+1})/\partial U_{t+1}(s)\right]\left[\partial U_{t+1}(s)/\partial c_{t+1}(s)\right]}{\partial U_t/\partial c_t} \\
&= \pi(s)\beta\left(\frac{c_{t+1}(s)}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}(s)}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}.
\end{aligned} \tag{A.3}$$

The rate of substitution without the probability is the pricing kernel used in the model.

A.2 Loglinear Approximation and Solution for the Scaled Utility

Note that the utility can be scaled by dividing current consumption with the use of homogeneity of both the time aggregator and the certainty equivalent function. If we define scaled utility $u_t = U_t/c_t$, then the equation can be scaled to

$$u_t = \left[(1-\beta) + \beta\mu\left(g_{t+1}u_{t+1}\right)^{\rho}\right]^{1/\rho} \tag{A.4}$$

where $g_{t+1} = c_{t+1}/c_t$ is the growth rate of consumption. A first-order approximation of $\log u_t$ around $\log u$ is

$$\begin{aligned}
\log u_t &= \rho^{-1}\log\left[(1-\beta) + \beta\mu_t(g_{t+1}u_{t+1})^{\rho}\right] \\
&= \rho^{-1}\log\left[(1-\beta) + \beta e^{\log\mu_t(g_{t+1}u_{t+1})^{\rho}}\right] \\
\left[(1-\beta) + \beta e^{\log\mu_t(g_{t+1}u_{t+1})^{\rho}}\right] &\approx \kappa_0 + \kappa_1\log\mu_t(g_{t+1}u_{t+1})
\end{aligned} \tag{A.5}$$

where

$$\begin{aligned}
\kappa_1 &= \frac{\beta e^{\log\mu}}{(1-\beta) + \beta e^{\rho\log\mu}} \\
\kappa_0 &= \rho^{-1}\log\left[(1-\beta) + \beta e^{\rho\log\mu}\right] - \kappa_1\log\mu.
\end{aligned} \tag{A.6}$$

To get the solution for the scaled utility, guess the utility as function of x_t and v_t , in specific,

$$\log u_t = u + p_x^T x_t + p_v v_t. \tag{A.7}$$

Then, verify the function by plug the form into equation A.5 and compute for the coefficients $\{u, p_x, p_v\}$. First, compute the certainty equivalent:

$$\begin{aligned}
\mu_t(g_{t+1}u_{t+1})^\alpha &= E_t\left[(g_{t+1}u_{t+1})^\alpha\right] \\
&= E_t\left[e^{\alpha(g+e^T x_{t+1}+u+p_x^T x_{t+1}+p_v v_{t+1})}\right] \\
&= E_t\left[e^{\alpha\left(g+u+(e^T+p_x^T)(Ax_t+v_t^{1/2}Bw_{t+1})+p_v((1-\varphi)v+\varphi v_t+bw_{t+1})\right)}\right] \\
&= e^{\alpha\left(g+u+p_v(1-\varphi)v+\frac{1}{2}\alpha p_v^2 bb^T\right)+\alpha(e^T+p_x^T)Ax_t+\alpha\left(p_v\varphi+\frac{1}{2}\alpha\left(e^T+p_x^T\right)BB^T(e+p_x)\right)v_t} \qquad \text{(A.8)}
\end{aligned}$$

Here I used $Bb^T = 0$, and $E\left[e^x\right] = e^{a+\frac{1}{2}b}$ for $x \sim N(a, b)$. Plugging this to equation A.5 leads to

$$\begin{aligned}
u + p_x^T x_t + p_v v_t &= \kappa_0 + \kappa_1\Big[g + u + p_v(1-\varphi)v + \frac{1}{2}\alpha p_v^2 bb^T + (e^T + p_x^T)Ax_t \\
&\quad + \left(p_v\varphi + \frac{1}{2}\alpha\left(e^T + p_x^T\right)BB^T(e + p_x)\right)v_t\Big] \qquad \text{(A.9)}
\end{aligned}$$

The coefficients can be solved for as follows:

$$\begin{aligned}
u &= \kappa_0 + \kappa_1\left[u + g + p_v(1-\varphi)v + \frac{\alpha}{2}p_v^2 bb^T\right] \\
p_x^T &= e^T(\kappa_1 A)(I - \kappa_1 A)^{-1} \\
p_v &= \frac{\alpha}{2}\kappa_1(1-\kappa_1\varphi)^{-1}(e+p_x)^T BB^T(e+p_x). \qquad \text{(A.10)}
\end{aligned}$$

A.3 Derivation of the Pricing Kernel

We can substitute the scaled utility into equation 2. The pricing kernel has the term

$$\begin{aligned}
\log(g_{t+1}u_{t+1}) - \log\mu_t(g_{t+1}u_{t+1}) &= g + e^T x_{t+1} + u + p_x^T x_{t+1} + p_v v_{t+1} \\
&\quad - \left(g + u + p_v(1-\varphi)v + \frac{1}{2}\alpha p_v^2 bb^T\right) - (e^T + p_x^T)Ax_t \\
&\quad - \left(p_v\varphi + \frac{1}{2}\alpha\left(e^T + p_x^T\right)BB^T\left(e + p_x\right)\right)v_t \\
&= v_t^{1/2}(e_t^T + p_x^T)Bw_{t+1} + p_v^T bw_{t+1} \\
&\quad -\frac{\alpha}{2}p_v^2 bb^T - \frac{\alpha}{2}\left(e^T + p_x^T\right)BB^T\left(e + p_x\right)v_t \qquad \text{(A.11)}
\end{aligned}$$

The pricing kernel follows as

$$\begin{aligned}
\log m_{t+1} &= \log\beta + (\rho - 1)\log g_{t+1} + (\alpha - \rho)\left[\log(g_{t+1}u_{t+1}) - \log\mu_t\left(g_{t+1}u_{t+1}\right)\right] \\
&= \log\beta + (\rho - 1)g - (\alpha - \rho)(\alpha/2)p_v^2 bb^T \\
&\quad +(\rho - 1)e^T Ax_t - \left[(\alpha - \rho)(\alpha/2)(e + p_x)^T BB^T(e + p_x)\right]v_t \\
&\quad +v_t^{1/2}\left[(\rho - 1)e + (\alpha - \rho)(e + p_x)\right]^T Bw_{t+1} + (\alpha - \rho)p_v w_{t+1.} \\
&\equiv \delta_0 + \delta_x^T x_t + \delta_v v_t + \lambda_x^T w_{t+1} + \lambda_v^T w_{t+1} \qquad \text{(A.12)}
\end{aligned}$$

A.4 Equity Returns

We define equity as the consumption stream. The return is the ratio of its value at $t+1$, measured in units of $t+1$ consumption, to the value at t, measured in units of t consumption. The value at $t+1$ is U_{t+1} expressed in c_{t+1} units:

$$\begin{aligned}
U_{t+1}/\left(\partial U_{t+1}/\partial c_{t+1}\right) &= U_{t+1}/\left[(1-\beta)U_{t+1}^{(}1 - \rho)c_{t+1}^{\rho-1}\right] \\
&= (1-\beta)^{-1}u_{t+1}^{\rho}c_{t+1} \qquad \text{(A.13)}
\end{aligned}$$

The value at t is the certainty equivalent expressed in c_t units:

$$\begin{aligned} q_t^c c_t &= \frac{\partial U_t/\partial \mu_t(U_{t+1})}{\partial U_t/\partial c_t}\mu_t(U_{t+1}) = \frac{\beta \mu_t(U_{t+1})^\rho}{(1-\beta)c_t^\rho} c_t \\ &= \beta(1-\beta)^{-1}\mu_t(g_{t+1}u_{t+1})^\rho c_t. \end{aligned} \tag{A.14}$$

The return is the ratio:

$$\begin{aligned} r_{t+1}^c &= \beta^{-1}\left[u_{t+1}/\mu_{t+1}(g_{t+1}u_{t+1})\right]^\rho g_{t+1} \\ &= \beta^{-1}\left[g_{t+1}u_{t+1}/\mu_{t+1}(g_{t+1}u_{t+1})\right]^\rho g_{t+1}^{1-\rho} \end{aligned} \tag{A.15}$$

The log of the return is

$$\begin{aligned} \log r_{t+1}^c &= -\log\beta + (1-\rho)g - (\rho\alpha/2)p_v bb^T \\ &\quad +(1-\rho)e^T A x_t - (\rho\alpha/2)(e+p_x)^T BB^T(e+p_x)v_t \\ &\quad +v_t^{1/2}(e+\rho p_x)^T B w_{t+1} + \rho p_v b w_{t+1}. \end{aligned} \tag{A.16}$$

The price of default-free bond which delivers 1 unit of consumption is $b_t^1 = E_t[M_{t+1}]$ and the return is $r_{t+1}^1 = \frac{1}{b_t^1}$. Thus the risk-free rate is

$$\log r_{t+1}^1 = -(\delta_0 + \lambda_v^T\lambda_v/2) - \delta_x^T x_t - (\delta_v + \lambda_x^T\lambda_x/2)v_t. \tag{A.17}$$

Then, the excess return of equity is

$$\begin{aligned} \log r_{t+1}^c - \log r_{t+1}^1 &= (1/2)[(\alpha-\rho)^2 - \alpha^2]p_v^2 bb^T \\ &\quad + \left[\lambda_x^T\lambda_x/2 - (\alpha^2/2)(e+p_x)^T BB^T(e+p_x)\right]v_t \\ &\quad +v_t^{1/2}(e^T + \rho p_x^T)B w_{t+1} + \rho p_v b w_{t+1} \end{aligned} \tag{A.18}$$

A.5 Price of Elementary Assets

First, the date-t price-payout ratio of the asset that matures on the next date is

$$\begin{aligned}
q_t^1 &= E_t\left[m_{t+1}\frac{d_{t+1}}{d_t}\right] \\
&= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+v_t^{1/2}\lambda_x^T w_{t+1}+\lambda_v^T w_{t+1}} e^{g+e^T(Ax_t+\beta v_t^{1/2}Bw_{t+1})-\frac{\beta^2 v_t}{2}e^T BB^T e}\right] \\
&= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} e^{v_t^{1/2}(\lambda_x^T+e^T B)w_{t+1}+\lambda_v^T w_{t+1}}\right] \\
&= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} E_{t+1}\left[e^{v_t^{1/2}(\lambda_x^T+e^T B)w_{t+1}+\lambda_v^T w_{t+1}}\right]\right] \\
&= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} e^{\frac{v_t}{2}(\lambda_x^T+e^T B)(\lambda_x^T+e^T B)^T+\frac{\lambda_v^T\lambda_v}{2}}\right] \\
&= e^{\delta_0+g+\frac{\lambda_v^T\lambda_v}{2}+(\delta_x^T+e^T A)x_t+\left(\delta_v+\frac{\lambda_x^T\lambda_x}{2}+\beta e^T B\lambda_x\right)v_t} \\
&\equiv e^{D_{0,1}+D_{x,1}x_t+D_{v,1}v_t} \qquad \text{(A.19)}
\end{aligned}$$

Here we use the law of iterated expectation, and $Bb^T=0$, $B\lambda_v=0$, $b\lambda_x=0$, and $E\left[e^x\right]=e^{a+\frac{1}{2}b}$ for $x\sim N(a,b)$.

Next, we compute the price-payout ratio of the elementary asset with maturity $s>1$. Suppose that the price for the asset with maturity $s-1$ is given by

$$q_t^{s-1}=e^{D_{0,s-1}+D_{x,s-1}x_t+D_{v,s-1}v_t}. \qquad \text{(A.20)}$$

Then,

$$
\begin{aligned}
q_t^s &= E_t\left[m_{t,t+s}\frac{d_{t+s}}{d_t}\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}m_{t+1,t+s}\frac{d_{t+s}}{d_{t+1}}\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}E_{t+1}\left[m_{t+1,t+s}\frac{d_{t+s}}{d_{t+1}}\right]\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}e^{D_{0,s-1}+D_{x,s-1}x_{t+1}+D_{v,s-1}v_{t+1}}\right] \\
&= E_t\Big[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+v_t^{1/2}\lambda_x^T w_{t+1}+\lambda_v^T w_{t+1}+g+e^T(Ax_t+\beta v_t^{1/2}Bw_{t+1})-\frac{\beta^2 v_t}{2}e^T BB^T e} \\
&\quad \times e^{D_{0,s-1}+D_{x,s-1}(Ax_t+v_t^{1/2}Bw_{t+1})+D_{v,s-1}((1-\varphi)v+\varphi v_t+bw_{t+1})}\Big] \\
&= e^{\delta_0+g+D_{0,s-1}+D_{v,s-1}(1-\varphi)v+\frac{(\lambda_v^T+D_{v,s-1}b)(\lambda_v^T+D_{v,s-1}b)^T}{2}+(\delta_x^T+e^T A+D_{x,s-1}A)x_t} \\
&\quad \times e^{\left(\delta_v+D_{v,s-1}\phi_v-\frac{\beta^2}{2}e^T BB^T e+\frac{(\lambda_x^T+\beta e^T B+D_{x,s-1}B)(\lambda_x^T+\beta e^T B+D_{x,s-1}B)^T}{2}\right)v_t} \\
&\equiv e^{D_{0,s}+D_{x,s}x_t+D_{v,s}v_t}
\end{aligned}
\tag{A.21}
$$

In this way, we determine coefficients $\{D_{0,s}, D_{x,s}, D_{v,s}\}$ for any $s > 1$ recursively from $s = 1$.

Table 1: Calibration Results

		Data	Dynamic Equity Premium	Constant Equity Premium
State variables			x_t, v_t	x_t
			Preference Parameters	
Risk parameter	α		-7	-7.5
IES parameter	ρ		0.9	0.85
			Endowment Parameters	
AR(1)	ϕ		0.90	0.90
MA(1)	θ		-0.70	-0.70
Volatility autocorrelation	φ		0.987	1
			Implications for Consumption Dynamics	
$AC(1)$		0.49	0.43	0.43
$AC(2)$		0.15	0.17	0.18
$AC(5)$		-0.08	0.09	0.09
$AC(10)$		0.05	0.09	0.09
			Implications for Asset Returns	
$E[r_f]$		0.86	1.03	1.00
$\sigma(r_f)$		0.97	0.56	0.38
$E[r_e - r_f]$		6.33	5.02	5.02
$\sigma\left(E_t[r_e - r_f]\right)$			1.18	0
$\sigma(r_e)$		19.42	7.82	7.59

The model is calibrated to match the time series of consumption growth and aggregate stock returns. We match the autocorrelations of consumption growth, return statistics of equity and the risk-free bond. Data statistics are from Bansal and Yaron (2004). $AC(i)$ is ith autocorrelation of yearly consumption. r_e and r_f are yearly returns on equity and the risk-free bond, respectively. Other parameter values are $g = 0.0015$, $v = 0.008^2$, $\sigma_e = 1$, $\sigma_w = 0.23 \times 10^{-5}$, and $\kappa_1 = \beta = 0.997$.

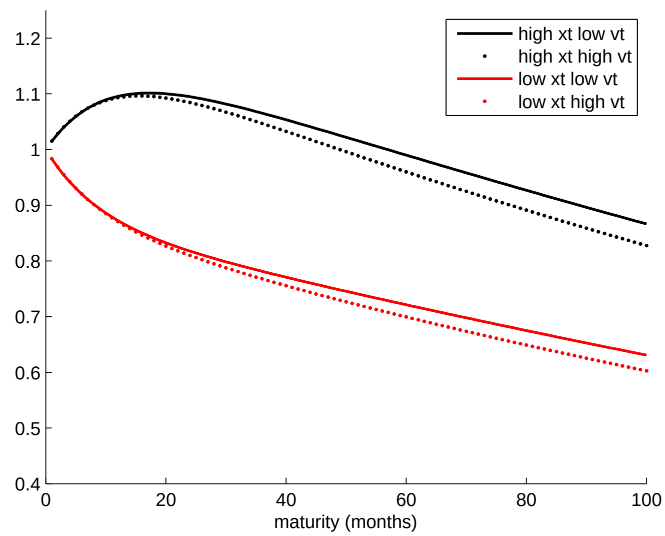
Table 2: Return Differences between Type 1 and Type 2

Economy		Dynamic Equity Premium	Constant Equity Premium
r_t (type 1) $-$ r_t (type 2)	mean	0.24%	-0.01%
	t-stat	6.36	-14.72
$r_{G,t}$ (type 1) $-$ $r_{G,t}$ (type 2)	mean	0.79%	0.0009%
	t-stat	37.27	2.83
$r_{A,t}$ (type 1) $-$ $r_{A,t}$ (type 2)	mean	0.02%	0.01%
	t-stat	1.81	4.30

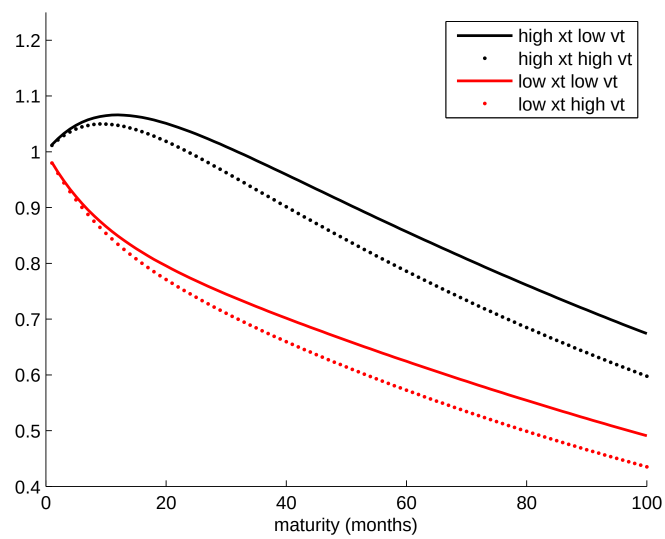
The table reports return differences between the two portfolios, one consisting of Type 1 firms and the other consisting of Type 2 firms in each underlying economy. r_t denotes realized rate of returns on firms, $r_{G,t}$ does returns on growth options, and $r_{A,t}$ returns on assets in place in the simulated panel.

Figure 1: Prices of Elementary Assets in the Economy with Dynamic Equity Premium

(a) Prices of Elementary Assets with $\beta = 1$

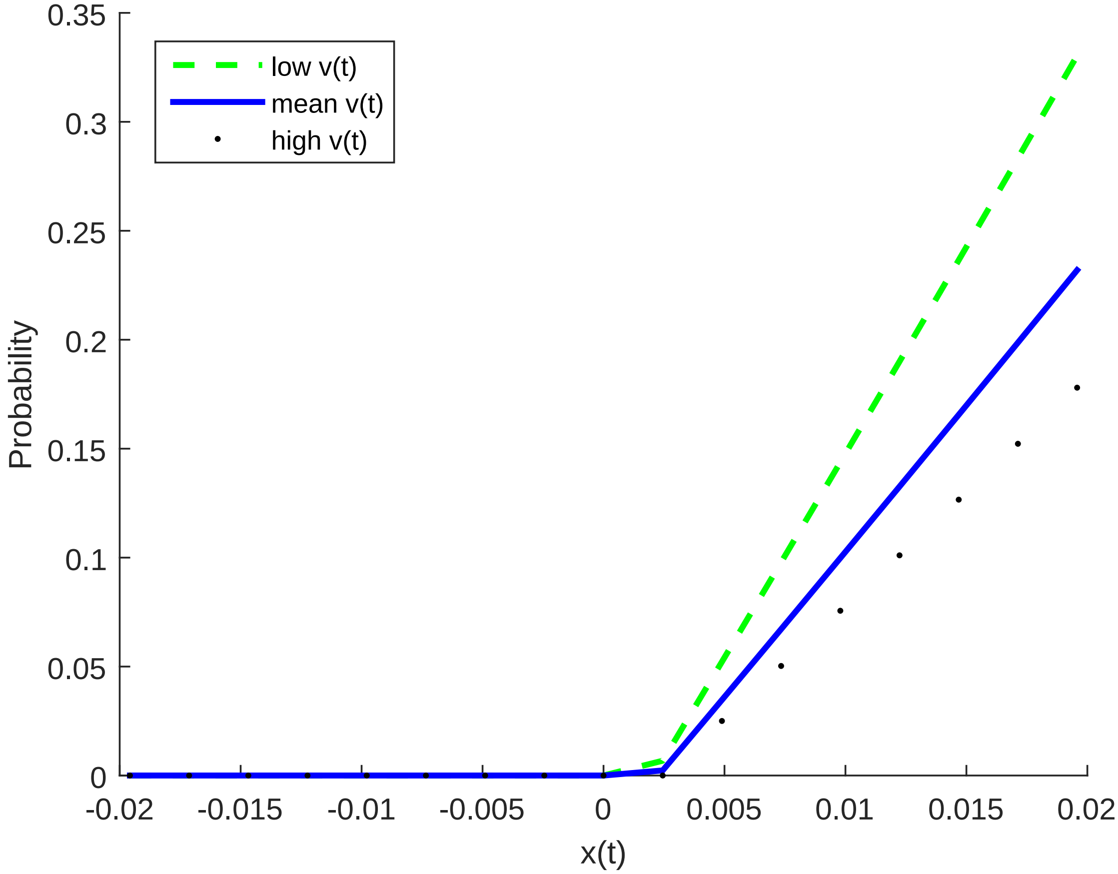


(b) Prices of Elementary Assets with $\beta = 3$



The figures plot the prices of the elementary assets in the economy featuring dynamic equity premium. The panel (a) depicts the prices in different states of the economy against delivery date, when the systematic risk is $\beta = 1$. The panel (b) depicts the prices when $\beta = 3$.

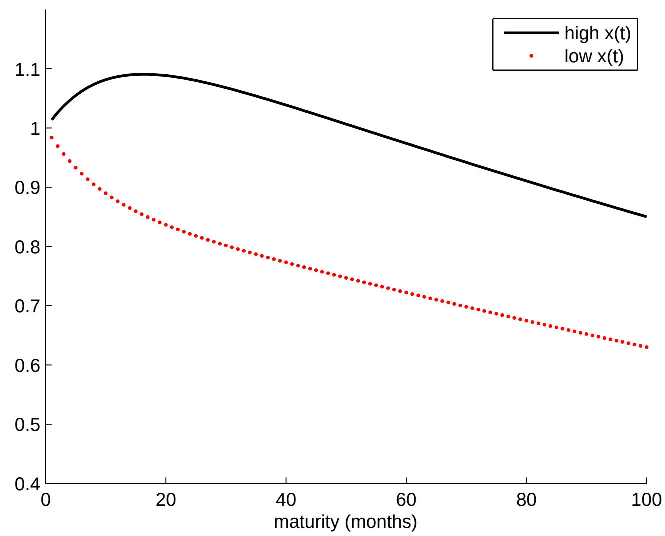
Figure 2: Investment Probability in the Economy with Dynamic Equity Premium



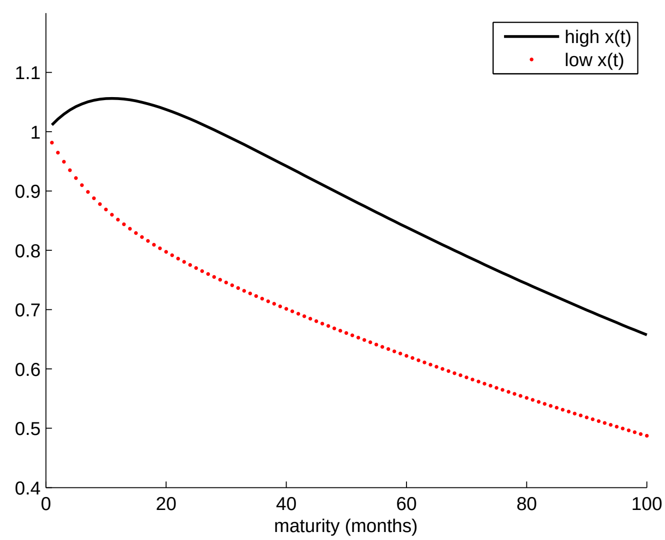
The figure plots the probability of investment before project-specific systematic risk is realized in the economy with dynamic equity premium. The project characteristics are $N = 108$ months, $d^0 = 0.00993$, and $\beta_{max} = 7.36$.

Figure 3: Prices of Elementary Assets in the Economy with Constant Equity Premium

(a) Prices of Elementary Assets with $\beta = 1$

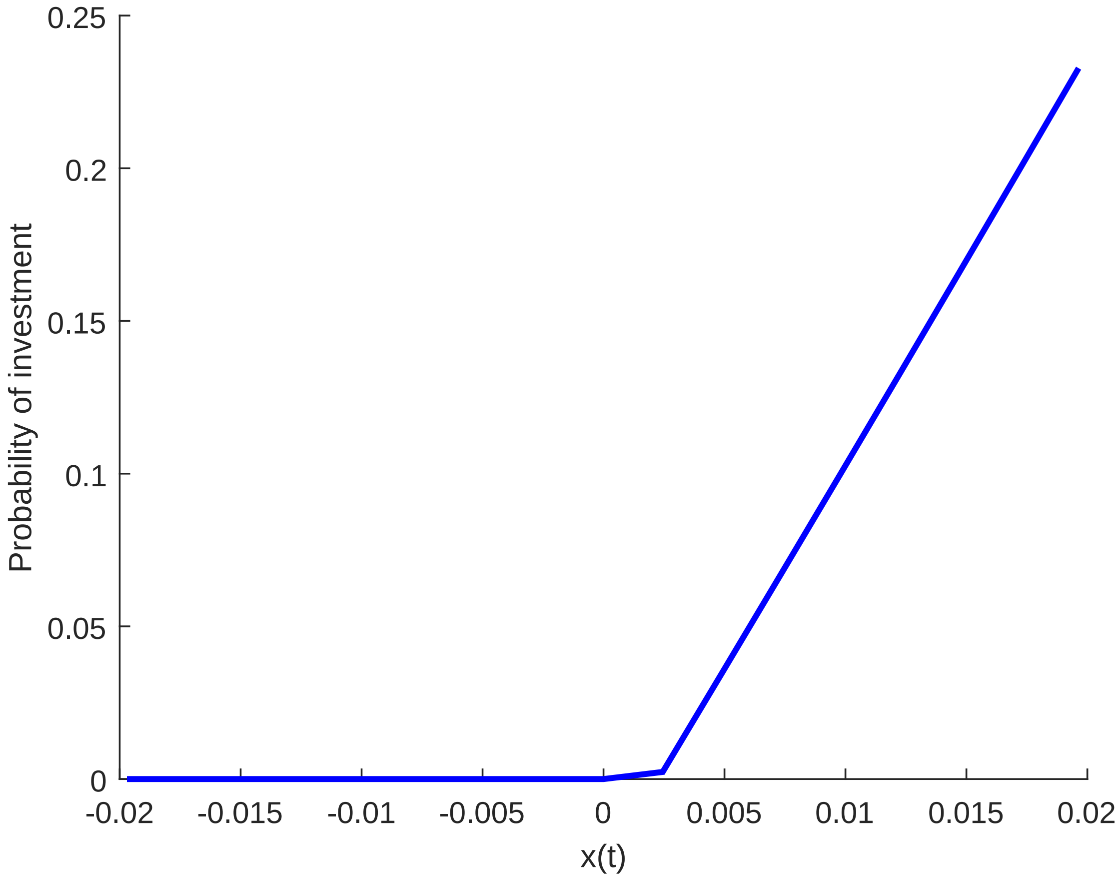


(b) Prices of Elementary Assets with $\beta = 3$



The figures plot the prices of the elementary assets in the economy featuring constant equity premium. The panel (a) depicts the prices in different states of the economy against delivery date, when the systematic risk is $\beta = 1$. The panel (b) depicts the prices when $\beta = 3$.

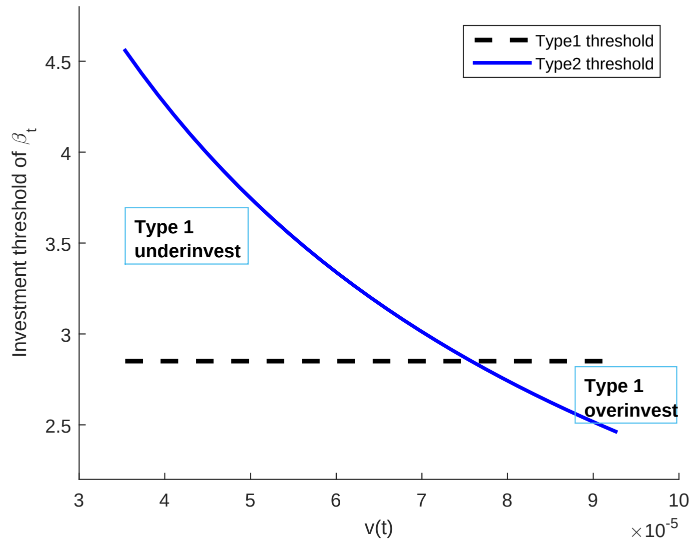
Figure 4: Investment Probability in the Economy with Constant Equity Premium



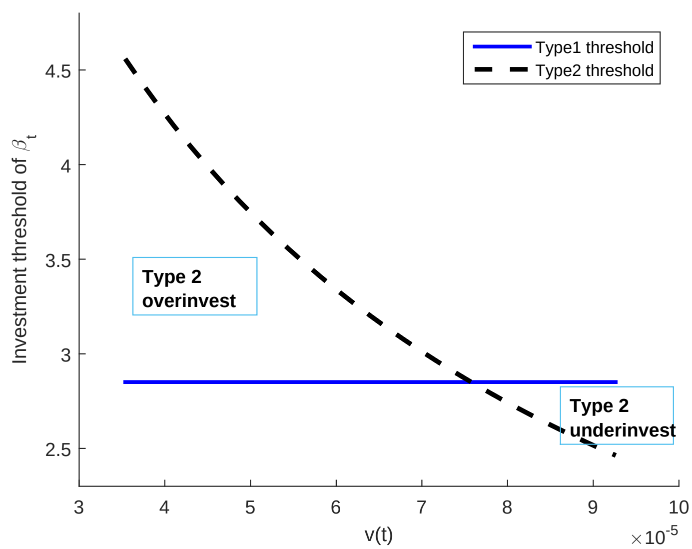
The figure plots the probability of investment before project-specific systematic risk is realized in the economy with constant equity premium. The project characteristics are $N = 108$ months, $d^0 = 0.00993$, and $\beta_{max} = 7.36$.

Figure 5: Investment Rules of the Two Firm Types

(a) Economy with Dynamic Equity Premium



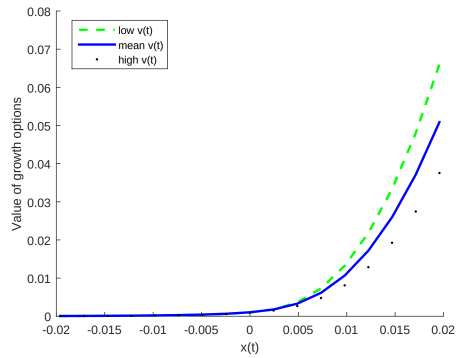
(b) Economy with Constant Equity Premium



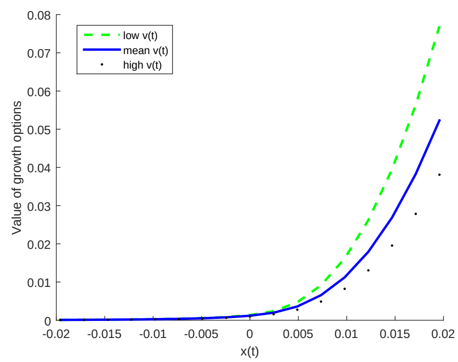
The figure plots the threshold of project-specific shock for investment of the two firm types in each economy. The solid line depicts the threshold of the correct investment policy, while the dotted line depicts the threshold of the incorrect policy.

Figure 6: Growth Options of Firms in the Economy with Dynamic Equity Premium

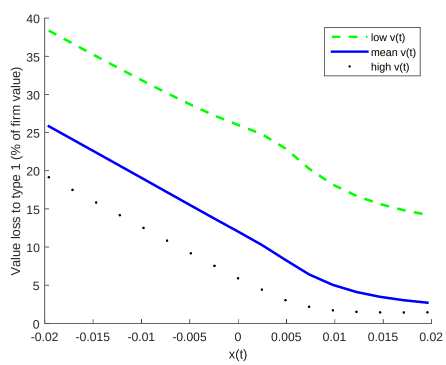
(a) Type 1 Firm



(b) Type 2 Firm



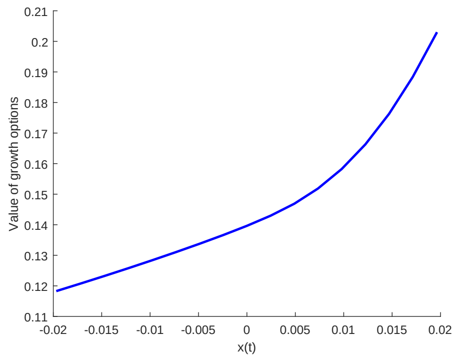
(c) Value loss to Type 1



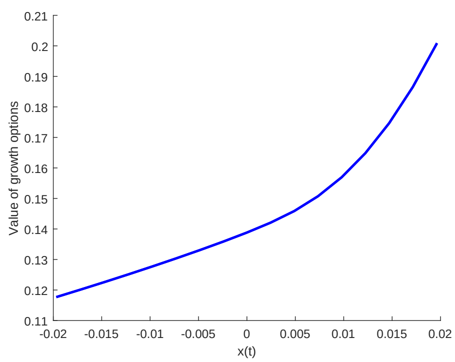
The top two figures depict the values of the growth options of the two firms at different states in the economy with dynamic equity premium. The bottom figure shows the value loss to Type 1. The value loss is defined as 100×(1- growth options of Type 1/growth options of Type 2)(%).

Figure 7: Growth Options of Firms in Economy with Constant Equity Premium

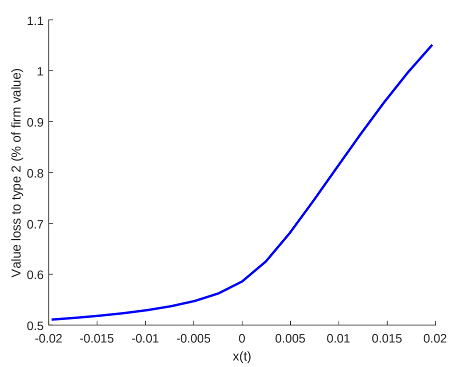
(a) Type 1 Firm



(b) Type 2 Firm

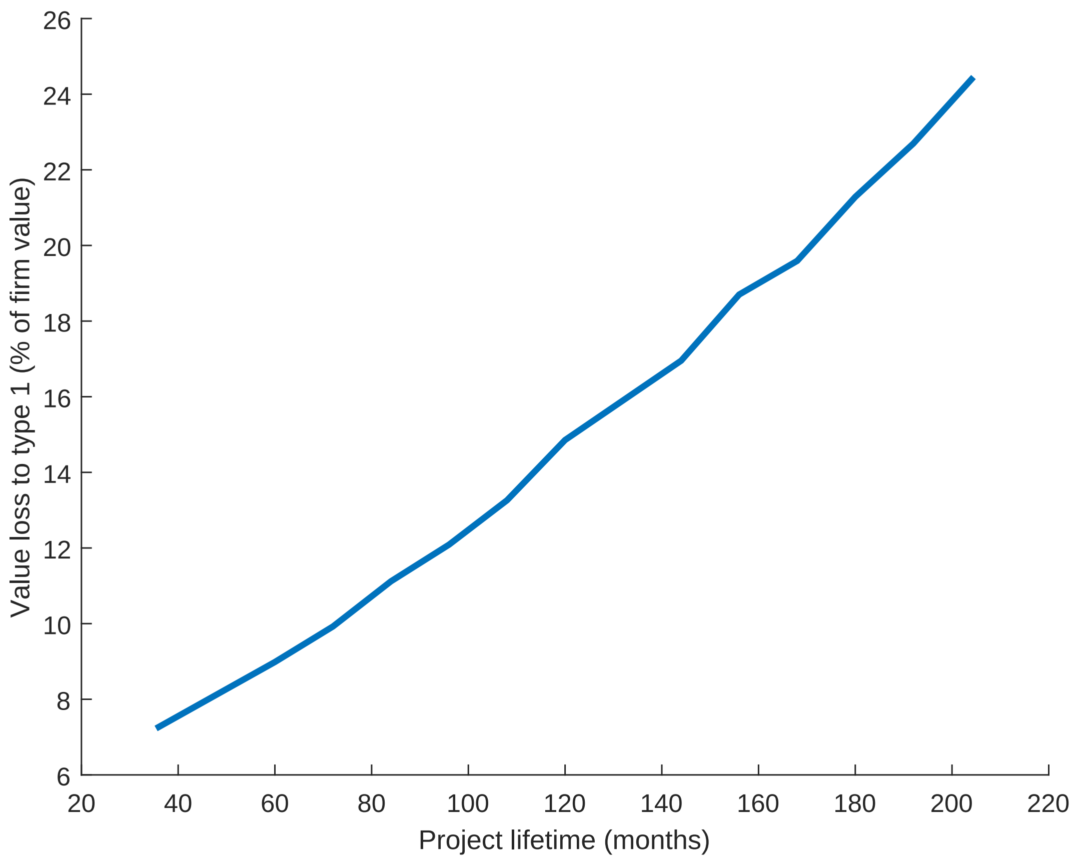


(c) Value loss to Type 2



The top two figures depict the values of the growth options of the two firms at different states in the economy with dynamic equity premium. The bottom figure shows the value loss to Type 2. The value loss is defined as 100×(1- growth options of Type 2/growth options of Type 1)(%)

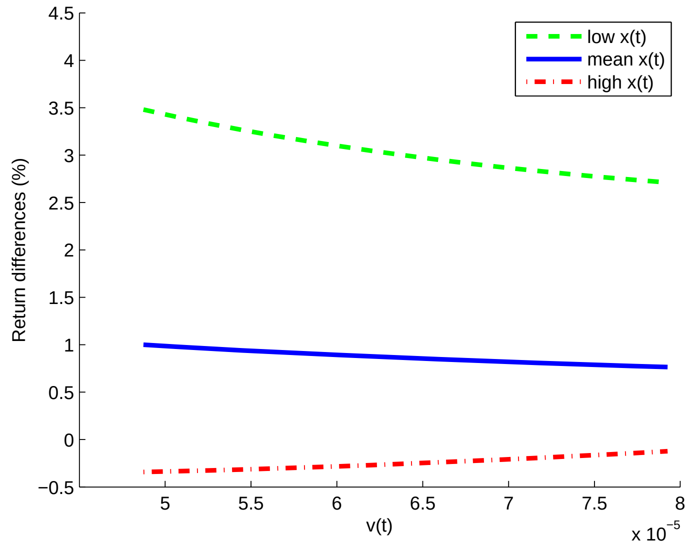
Figure 8: Project Lifetime and Value Loss to Type 1 in Economy with Dynamic Equity Premium



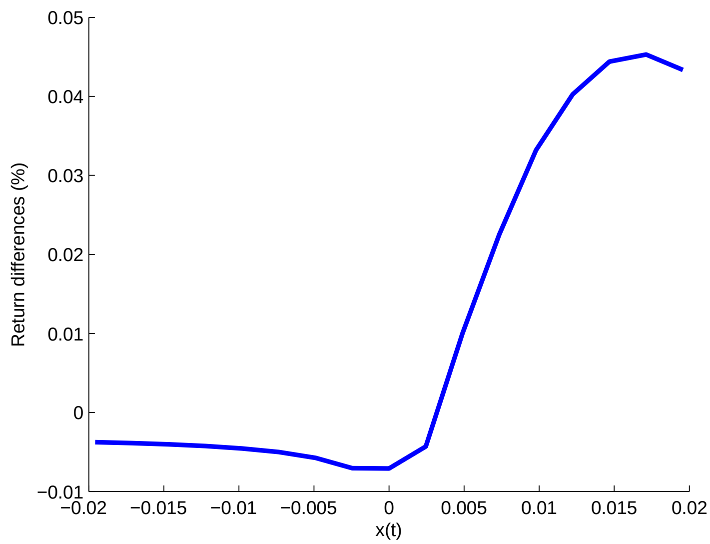
The figure depicts the value loss to Type 1 in the economy with dynamic equity premium across different lifetimes of project, when $d^0 = 0.00993$. The value loss is defined as $100\times$(1-growth options of Type 1/growth options of Type 2)(%).

Figure 9: Difference in Returns on Growth Options between Two Firm Types

(a) $E_t\left[r_G(\text{Type 1})\right] - E_t\left[r_G(\text{Type 2})\right]$
in Economy with Dynamic Equity Premium



(b) $E_t\left[r_G(\text{Type 2})\right] - E_t\left[r_G(\text{Type 1})\right]$
in Economy with Constant Equity Premium



The figure plots differences in returns on growth options between the two firm types. The panel (a) depicts the differences in economy with dynamic equity premium. Panel (b) depicts the differences in economy with constant equity premium.