Bogus Joint Liability Groups in Microfinance *

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Abstract

This paper analyzes the optimal group loan contract if the borrowers can choose to form a standard group as in the microfinance literature or a bogus group where the entire funds provided by the lender for the group are invested in only one borrower's project. We show that the borrowers are more likely to form a bogus group when the heterogeneity in productivity of the group members are sufficiently high. Furthermore, when there are separating contracts offered, for a group with different types of projects, bogus group is more likely to arise if the relative heterogeneity of productivity is high; while for a group with same types of projects, bogus group is more likely to arise if absolute level of productivity is high. We show that allowing the bogus group to exist in equilibrium, not only prevent the loss caused by the bogus group under the standard group loan contract, but also enhance the efficiency of the economy because (i) the entire loans are invested in the project with relatively higher productivity in a bogus group, and (ii) the loan size for the bogus group is always larger than that of the standard group loan contract.

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1 Introduction

Lending to the poor, especially in rural areas, is a difficult task all around the world. As documented in literature, the main reason is that the poor people cannot supply adequate collateral when borrowing. Consequently, the poor people have no handy instruments to mitigate borrowing cost due to the information asymmetry such as adverse selection, moral hazard, costly state verification etc. between them and the lending group.[1] As one of the promising methods to mitigate the borrowing cost due to the insufficient collateral, group-lending has attracted plenty of attention for the past decades.[2] In contrast to the conventional bilateral lender-borrower contracts, the group loan contracts involve a lender and a group of poor borrowers without collateral. The lender provides loans to each member of the group and requires each of them invest in one's own project. The members of the group take joint-responsibility of the group's liability: if any member of the group defaults, his liability need to be repaid by the other group members. Otherwise the entire group loses the opportunity for future refinancing (Chowdhury (2005)). Such design can allow the bank to provide loans to the borrowers without collateral at a relatively low interest rate. As a result of such risk diversification effect, more poor people potentially are affordable to borrow money from the bank.[3]

The microfinance practices in rural China recently demonstrate a seemly surprising evidence against the standard group-lending. We have interviewed by telephone with 366 clients (borrowing groups) of CFPAMF (China Foundation for Poor Alleviation Microfinance), the leading microfinance lender in China. These interviewees are in three rural villages, and the main access to finance their projects is through CFPAMF. During the interview, we ask how the group loan is used, and the finding from the interview demonstrates that nearly 70% of the borrowing groups, the entire loan is fully used by only one of group members in her project while the other members only play the role of "co-signers". This practice is inconsistent with the borrowing terms with the CFPAMF. However, CFPAMF does not take action to eliminate this practice. We call this unconventional group formation as "bogus group", a phenomenon called "Lei Da Hu" in Chinese.[4] This is in contrast to the conventional group-lending practice, where each member gets one share of the loans and invests the share in her own project, i.e., "standard groups".

It is quite interesting to observe the majority of the lending group is bogus since according to the theory, group-lending rather bogus group can help the reduce the borrowing cost, and increase the probability of obtaining the loan. Given the fact that the access to the microfinance is very precious to the borrowers, it seems surprising to observe the existence of the bogus group in practice. Our paper tries to address this issue by identifying the (possible) justifications for the existence of the bogus group.

[1]See Mosley (1986), Udry (1990), Morduch (1999), Ahlin and Waters (2011) etc.

[2]See Armendariz de Aghion (1999), Ghatak and Guinnane (1999), Ahlin and Waters (2011),Ahlin (2012) etc.

[3]In addition to the risk diversification effect mentioned above, the group-lending benefits the poor people via the monitoring cost reduction effect since the group members has comparative advantages in information revealing relative to the lenders such as banks or moneylenders. Intuitively, the members in the group can monitor each other or verify the state of each other's investment projects with relatively low cost because usually they live geographically nearby (Ghatak and Guinnane (1999), Karlan (2005)), they also have stronger enforcement power to make the members repay because they have close social links and can impose powerful social sanctions on members who default strategically (Besley and Coate (1995), Armendariz de Aghion (1999)). Therefore the lender is willing to lend to the group even though they have no collateral since the members with these comparative advantages have incentive to monitor, verify and enforce each other when they have joint-responsibility for any of the members' default. In other words, the group members' comparative advantages on information revealing helps them access to the credit and improves the efficiency of the economy.

[4] The "co-signers" are willing to participate either because the main borrower can provide the (social) benefit to them or because they might want to invite the main borrower to co-sign the contract in future.

We build a static model with a population of the borrowers who are endowed with a project, which can be of high or low productivity. The borrowers have no fund to finance their projects so they have to borrow the loans from a microfinace lender (a bank). Due to some exogenous reason, the lender only offer the loan through group-lending. After getting the loans from the lender, the group members can either operate as a standard group or a bogus group. In both cases, all the group members take the joint-responsibility of the group's liability. Therefore, in both cases, if the joint liability is not fulfilled, all the group members will be shut off from the access to future financing.

We start our investigation by a benchmark case where the borrowers can freely choose to be either standard group or bogus group and lender can observe the type of the projects. For instance, in practice, the microfinance lender will collect the information on the project the borrower will conduct conditional on the loan is issued. We find that if only one loan contract is offered, only the group of members with heterogeneity in project productivity will be bogus group. The bogus group is more likely to arise if the heterogeneity of the borrowers' productivity is sufficiently high (condition (13)). This result highlights the one important difference between the bogus group and the standard group. As documented in literature, the standard group can offer an opportunity for the group member to share the risk, i.e., the probability for the borrowers to obtain the future financing is larger than that in bogus group where only one project is undertaken. Therefore, if the group decides to switch from a standard group to a bogus group, the benefit from this risk-sharing effect is forgone. In return, the bogus group offers a chance to the group member to use the entire loan to do the project with highest productivity. This benefit of increase in the total output only exists when the group members are endowed with different projects. Consequently, in this benchmark case, only the group where members are endowed with different projects might choose to be bogus group. And if the difference in the productivity is larger, the benefit of the increase in the total output switching from standard group to the bogus group is larger. Notice that, since there is no risk-sharing effect in the bogus group, i.e, the probability of default is higher for a bogus group than that of a standard group. Therefore, if the microfinance lender does not acknowledge the possibility of the existence of the bogus group, the contract offered to the standard group will incur a loss.

We show that the microfinance lender can undertake several strategies to avoid the deficit. A naive strategy is to exclude the existence of bogus group by holding the interest rate at the level for standard group and reducing the loan size so that borrowers' incentive to form the bogus group is reduced. This strategy might be due to that the lender can not tolerate the existence of bogus group or be due to some other external reasons, for example, the lender just cannot tolerate the "cheating behavior" of operating as a bogus group; or the interest rate of the group-lending market is regulated and the lender cannot make break even under such interest rate if bogus groups exist; or the lender hope every borrower engage in ones own project to improve her long run skills and experience, or to enhance the employment rate of the community and maintain the stability of the social order etc. We show that such treatment to the bogus group problem is not optimal if we consider the joint payoff of the borrowers in the group. As in the practice, the CFPAMF does not take action to exclude the existence of the bogus group even if it acknowledge its existence. The other strategy is to explicitly allow the existence of bogus group in equilibrium and offer the optimal contract, suitable for either standard group or bogus group. We find that, no matter the productivity is observable or not for the lender, for a group with different types of projects, bogus group is more likely to arise if the relative heterogeneity of productivity is high; while for a group with same types of projects, bogus group is more likely to arise if absolute level of productivity is high. This result highlights the second important difference between the bogus group and the standard group, i.e., the way to share the total surplus is different. In the standard group, since the repayment decision is made by each member independently, each group member might have the incentive to free-ride her partner by strategic defaulting, especially when the loan size is large. Therefore, to design the optimal contract targeted to standard groups, the lender need to reduce the loan size to reduce such strategic default and free-riding incentives. In contrast, in the bogus group, the entire loan is used for a single project with the highest productivity, the surplus is shared and the repayment decision is made by the two members collectively. Therefore, there is no strategic interaction free-riding incentive in the repayment stage for the group members. Consequently, the lender might be able to offer a loan contract of larger loan size to the bogus group than that of a standard group.

Consequently, when the separating contracts are offered, the loan size of the bogus group contract is larger than that of the standard group. Furthermore, we argue that if the projects have positive NPV, this suggests that by allowing the existence of bogus group and providing the bogus group loan contract accordingly to them, we not only prevent the loss caused by the bogus group under the standard group loan contract, but also enhance the efficiency of the economy because (i) the entire loans are invested in the project with relatively higher productivity in a bogus group, and (ii) the loan size for the bogus group is always larger than that of the standard group loan contract.

Thus, in our paper, we highlights the difference between a bogus group and a standard group by focusing on three mechanism: the well-documented "risk-sharing" premium offered by the standard arrangement of group-lending, the more efficient fund allocation to the highest productivity projects and the reduced negative effect on loan size caused by strategic interaction in repayment stage by the bogus group. As to our knowledge, the latter two mechanism of the bogus group is not well studied in literature. We show that in a standard group-lending model, allowing the existence of bogus group can actually increase the welfare.

Our paper contributes to the literature in the following ways. First, motivated by the group lending practice in China, we extend the standard group lending framework to allow the possibility of the endogenous choice of the group type. Furthermore, we show that the optimal design of the group lending should take into account the existence of the bogus group explicitly. Second, we demonstrate that there are fundamental difference between standard group lending group and bogus group. In particular, forming a bogus group, while forfeit the benefit of the risk-sharing, which is one of the main benefits of the group lending, can also leads to the decrease in the lending cost due to reduced possibility of the free-riding of the group member. Furthermore, bogus group might allow all the loan to be used more efficiently since all the loan will be used in the most efficient projects. As to our knowledge, these features of the a bogus group is seldom discussed, or, equivalently, the these possible drawbacks of the standard group lending practice are seldom discussed.

Our paper also relates to the mechanism design literature when the principal have both (ex-ante) adverse selection and ex-post moral hazard. In our paper, when the types of the projects are not observable, for instance, the reported intended usage of the loan might be misleading in practice, the micro lender still would like to maximize the total welfare of the borrowers. Now, he faces the adverse selection due to unobservable type of projects and the moral hazard due to the group might optimally decide the group type (standard or bogus) after they obtain the loan. We show that in our case, the micro lender can design up to two optimal contracts which can be used to separate the groups.

The remainder of the paper is organized as follows. Section 2 introduces the institution background of group-lending in China; section 3 is the model, where standard group-lending problem, the problem brought by the possibility of bogus group, and the solution regard to this problem are discussed successively; and section 4 concludes.

2 Institution background

Microcredit is loaned to a micro-entrepreneur by a bank or other institution and can be offered, often without collateral, to a group or an individual. It came to prominence in the 1980s, though early experiments date back 30 years ago in Brazil, Bangladesh and a few other countries. This innovation empowered the poor in a new way by providing them with access to financial services that were formal and secure.

During the year of 2010, the whole scale of microcredit in China is up to \$14.1 billion, with 2.4 million active borrowers. Among the microcredit institutions all around the country, CFPAMF (China Foundation for Poor Alleviation Microfinance) has ranked the 1st in both loan scale and active members.

CFPAMF is established in 1989 as the China's largest and best-known charitable NGO ,committed to contributing to poverty alleviation and empowerment of the poor. CFPAMF is treated to be one of the most significant projects in CFPA which is offering loan support to the poor and inspiring these micro-entrepreneurs with a spirit of self-development. In 2009, CFPAMF was formally transformed into a professional and national microfinance institution. Currently, CFPAMF is one of the largest microfinance institutions in China reaching out to about 100,000 clients spread over 13 districts in China and through 53 branch offices. Since 1996, when CFPA launched its microfinance projects, the cumulative number of loans distributed is 378,322, amounting to \$2472,074,500 and with a very high overall repayment rate of over 99.87%.

The institution have divided their work between headquarter and several branches. The HQ makes standardized operation processes and management rules. The branches focus on rigorous execution of these obligations.In the operation of group-lending, all programs employ specially trained officers who introduce the regulations and expected program costs and benefits to potential members. Later, these officers also assume the responsibility for training and monitoring the groups. CFPAMF's rules for group formation include:

1. A group must consist of 2-5 self-chosen members.
2. All group members must be from the same village.
3. There shall be no more than one member from the same household in a group. It is also not desirable for close relatives to be in the same group.
4. As each group is formed, it elects its own leader among the members.

CFPAMF branches advertise regularly around the county areas, so most people in the areas are aware of CFPAMF. Clients firstly form the group and then go to the CFPAMF to apply for the loans. If the group meets the basic criteria (have their business, understand the rules, and want a loan), the CFPAMF will provide a training for the clients regarding the joint liability, group operations and the importance of group solidarity, and monitoring of loan repayment by all members. In particular, the CFPAMF will make it very clear that the clients themselves are responsible for monitoring the group members in order to ensure that loan proceeds are used properly and to enforce repayment and attendance. Then each individual receives her first loan. The whole process typically takes one week.

3 The model

The economy is populated by two types of agents: lenders and borrowers. The borrowers are endowed with one investment project each, which has to be financed by taking a loan at $t = 1$. The borrowers have no capital, so the entire initial investment L required to implement an investment project must be financed by borrowing a loan from the microfinance lender. There are two types of investment projects: a "conventional" project with productivity k_L and a "high-return" project with productivity k_H, where $k_H > k_L > 0$. Both project types ($i = H, L$) succeed with probability p and the output, generated at $t = 2$, is given by

$$Y_i = \begin{cases} k_i L, & \text{with probability } p, \\ 0, & \text{with probability } 1 - p. \end{cases} \tag{1}$$

Throughout this section, we assume that the lender can observe the types of the projects, i.e., the productivity parameter k_i. One interpretation is that in practice, the microfinance lender will collect the information on the types of the project the borrowers will do before providing the loan contract. We will consider the situation with unobservable k_i in section 3.4, which shows that our main conclusion and intuition still hold.

Both lenders and borrowers are risk-neutral. We assume an environment with limited enforcement (the project return Y is non-verifiable which gives rise to the possibility of strategic default. Loan terms must therefore be such that borrowers have incentive to pay the loan back. In addition, we assume that borrowers are subject to limited liability: if the project fails, a borrower (involuntarily) defaults, in which case the lender cannot punish the borrower further. For simplicity, the borrowers' outside option (if they do not invest in their project) is normalized to zero.

In our model, a representative competitive lender only provides a group loan contract due to some exogenous reason. For instance, the microfinance lender could be delegate by a NGO who would like to finance the projects for the poor people. This group-lending loan is issued to a group consisting of two borrowing members who need investment funds.[5] A group loan contract consists of two loans of size L each and a pre-specified total repayment amount $2R$. The group loan contract has a *joint liability* clause: each member is fully responsible for the total group obligation $2R$. That is, if the lender does not receive $2R$ (either from a single borrower or from both borrowers combined) at $t = 2$, then both borrowers are cut off from access to credit in the future. Maintaining future access to credit has present discounted value $V > 0$ for a borrower. Furthermore, we assume due to free entry of the microlenders, or the microlender's not-for-profit mission, the microfinance lender makes zero profit in equilibrium.

The group of two borrowers can either operate as a "standard group" or as a "bogus group". In a standard group each member invests L into one's *own* business project, as assumed in the literature on group-lending or as required by microlenders in practice. In contrast, in a bogus group the two members invest the total amount, $2L$, into a single one of the two projects and share the surplus, which can be viewed as "cheating" behavior by the lender.

In section 3.1, we study the standard group-lending problem as in the literature without allowing the possibility of forming bogus groups. In section 3.2, we assume that the borrowers can choose their group form freely, i.e. either standard group or bogus group, then discuss the problem brought by the possibility of bogus groups arising in equilibrium as well as analyze the condition under which a group opt to operating as bogus both generally and particularly when the standard group loan contract in section 3.1 is offered. In section 3.3, we discuss how the bogus group problem can be addressed in an optimal way. In section 3.4, we consider the situation in which each borrower's investment productivity k_i is unobservable for the lender.

[5] We assume a group only consists of two members for simplicity.

3.1 When Bogus group formation is impossible

We start with a basic case where due to some exogenous reason, the formation of bogus group is not possible. That is, we are in the case which is equivalent to the standard group-lending literature. In this case, the timing is as follows:

- stage 0: Two borrowers form a group and then each borrower is randomly endowed with an investment project with productivity parameter k_i, $i \in \{H, L\}$, which is observable for the lender;
- stage 1: The lender offers a group loan contract with terms $\{L, R\}$;
- stage 2: Each borrower's investment is launched and her project output is realized on period later, each borrower only observes the output realization of her own project.[6]
- stage 3: Repayment decisions are made by the group members and all payoffs are realized.

Notice that if a borrower's project succeeds while her partner claims to default (because of the failure of her project or strategic consideration), it is never optimal to repay an amount strictly between 0 and $2R$ since either defaulting (repaying zero) and forfeiting the entire future value V, or repaying in full ($2R$) and securing the future value V is the dominant strategy. Similarly, it is never optimal to repay an amount between 0 and R if one's partner claims to repay. Given this, the lender optimally chooses the contract $\{L, R\}$ while the strategy of each group member is either "Repay" in full (R or $2R$ depending on the partner's action) or "default" (repay zero). We study Nash equilibria within the group below.

We call the type of a group ij if the productivity of the two projects are k_i and k_j respectively. There can be three types of groups: HH, LL, HL. Generally, consider the ij group's problem for $i, j \in \{H, L\}$.

Without loss of generality, we assume $k_i \geq k_j$. The group loan contract is *feasible*, that is, each member's project generates enough output when succeed to cover repayment if $\min\{k_i L, k_j L\} \geq 2R$, or equivalently

$$R \leq \frac{1}{2} k_j L. \tag{2}$$

Given the feasibility condition, the repayment decisions made by the two members in stage 3 results from a simultaneous moving game with its normal form presented in the table 1, where only the payoffs of the row player i are listed, those of the column player are symmetric around the diagonal.[7]

	Repay	Default
Repay	$k_i L - R - (1-p)R + V$	$k_i L - 2R + V$
Default	$k_i L + V$	$k_i L$

Table 1: The normal form of the game in the repayment stage.

Conditional on the type j member choosing "Repay", type i would also optimally repay when her project succeeds if his payoff from repaying R is not smaller than her payoff from strategically

[6]As modeled in Armendariz de Aghion (1999) etc.

[7]It will be shown that the strategic interaction happened in this repayment game leads to smaller loan sized offered by the lender, and such fiction is one of the driving forces of our model. In section 3.5, we rule out such strategic interaction friction by assuming that the members in a standard group make repayment decision collectively, we find that the bogus group problem still exist and the main solutions to such problem is similar.

defaulting (repaying zero), that is, $k_iL - R - (1-p)R + V \geq k_iL + pV$, or

$$(2-p)R \leq (1-p)V \tag{3}$$

The LHS of (3) is the expected repayment (the marginal cost of not defaulting), while the RHS is the marginal benefit because the probability that the borrower obtains the future credit access value V increases from p to 1. The above condition implies the following constraint on R:

$$R \leq \frac{1-p}{2-p}V. \tag{4}$$

Similarly, conditional on the type j member choosing "default", the type i member would optimally choose to "repay" if paying back $2R$ and securing V results in a larger payoff than defaulting, i.e. $k_iL - 2R + V \geq k_iL$, or

$$R \leq \frac{1}{2}V. \tag{5}$$

Since condition (4) implies (5), (Repay, Repay) is the unique Nash equilibrium ("Repay" is at least weakly dominant strategy) whenever $R \leq \frac{1-p}{2-p}V$.[8]

Given the contract $\{L, R\}$ with the condition (2) and (4), the joint payoff of the standard group ij is

$$W_{ij}(L, R|S) = p(k_i + k_j)L - 2p(2-p)R + 2p(2-p)V, \tag{6}$$

where S indicates that the group form is standard.

Forming a standard group increases the repayment probability from the success probability p to $p^2 + 2p(1-p) = p(2-p)$. Though the group repays $2p(2-p)R$ on average, which is more than $2R$, the group jointly maintains higher value from future access to finance, i.e. $2p(2-p)V$. The net benefit derived from the standard group lending is $2p(2-p)(V-R) > 0$ conditional on (4).

The lender receives the required payment $2R$ back with probability $p^2 + 2p(1-p) = p(2-p)$, and receives nothing otherwise. Thus, the lender's participation constant is $2p(2-p)R - 2L \geq 0$, or

$$R \geq \frac{L}{p(2-p)}. \tag{7}$$

Assuming zero profits for the lender because of free entry or because of the lenders' mission, **as explained above**, the optimal standard group loan contract can be defined as the loan size and repayment $\{L_S, R_S\}$ which maximize the joint payoff of a standard group, that is, the contract $\{L_S, R_S\}$ which solves:

$$\max_{L,R} \; W_{ij}(L, R|S), \tag{8}$$

$$\text{s.t. } (2),\ (4),\ (7).$$

The two constraints (2) and (7) are collectively equivalent to $\frac{L}{p(2-p)} \leq R \leq \frac{1}{2}k_jL$, which is non-empty only if $k_j \geq \frac{2}{p(2-p)}$. If $k_j < \frac{2}{p(2-p)}$, then the above constraint never hold, so the market

[8] In general, the pure strategy Nash equilibrium of this game will be (i) (Repay, Repay) if $R \leq \frac{1-p}{2-p}V$, (ii) (Repay, Default) or (Default, Repay) if $\frac{1-p}{2-p}V < R \leq \frac{1}{2}V$, or (iii) (Default, Default) if $R > \frac{1}{2}V$. It is possible for the lender to offer a contract with $\frac{1-p}{2-p}V < R \leq \frac{1}{2}V$ and make break even by setting $L = pR$. However, such a contract leads to an "asymmetric" equilibrium in which one borrower "exploits" the other and there is no risk sharing between the group members. Such equilibrium is against the initial intention of standard group-lending and may hardly survive or be implemented in reality. In order to show the basic intuition and the driving forces of our model clearly and cohesively, we first focus on the standard group loan contract in the (Repay, Repay) regime with $R \leq \frac{1-p}{2-p}V$. In the appendix, we show that even when we allow the existence of (Repay, Default) or (Default, Repay) equilibrium, our main results and intuition still hold.

collapses. Hence, we make the following assumption so that the market is active for all types of groups.

Assumption 1. *Assume $k_L \geq \frac{2}{p(2-p)}$ so that the standard group-lending market does not collapse for all types of groups.*

Then we obtain the following results.

Proposition 1. *When bogus group formation is impossible, the optimal standard group loan contract is $\mathcal{S} = \{L_S, R_S\}$ where*

$$L_S = p(1-p)V, \;\; R_S = \frac{L_S}{p(2-p)}. \tag{9}$$

Proof. See appendix A.1. □

Thanks to the risk sharing between the two borrowers, the lender can get $2R$ back with probability $p(2-p) > p$, so the interest rate of contract $\mathcal{S}$ is $r_S = \frac{R_S}{L_S} = \frac{1}{p(2-p)}$, which is lower than the capital cost of the projects $\frac{1}{p}$. Nevertheless, as mentioned above, when each member succeeds, the actual payment on average is $(2-p)R > R$. The effect of lower interest rate and the higher average repayment exactly cancel out since the effective interest rate is also equal to the capital cost $\frac{1}{p}$ since the actual payment is $(2-p)R_N^* = \frac{L_N^*}{p}$. This is also the reason that why the lender makes break even with a interest rate $\frac{1}{p(2-p)}$ lower than the capital cost $\frac{1}{p}$. However, even though the benefit of lower interest rate is actually canceled out by the higher expected repayment level, one benefit remains as the net gain from standard group lending: the probability of maintaining the future access to finance increases from the natural success probability p to $p(2-p)$. In all, the joint payoff for a ij group given contract $\mathcal{S}$ is

$$W_{ij}(L_S, R_S|S) = (p(1-p)(k_i + k_j) + 2)pV. \tag{10}$$

Notice that the contract $\mathcal{S}$ is independent of k_i and k_j, therefore, even when we assume that k_i and k_j are unobservable for the lender, we should abstain the same contract and results above.

3.2 When bogus group is possible

A standard assumption in the literature on joint-liability lending, and also standard practice in microfinance, is that each borrower is expected to invest in her own business project. However, as motivated by the evidence reviewed in the introduction, suppose that the lender is unable to enforce that each group member invests in their own project. Therefore, borrowers can choose to form either a "standard group", or a "bogus group", that is a borrowing group in which all loaned money ($2L$) is invested into a *single* business project run by one of the borrowers. Essentially, here we extend the standard group-lending framework by allowing the endogenous choice of the group form, i.e. standard or bogus.

Allowing for the endogenous choice between the standard group and the bogus group, the model's timing is as follows:

- stage 0: Two borrowers form a group and then each borrower is randomly endowed with a business project with productivity k_i, $i \in \{H, L\}$; the productivities k_i are observable to the lender;
- stage 1: The lender provides a loan contract $\{L, R\}$;
- stage 2: The group members choose to operate as a standard group or as a bogus group;

- stage 3: Investment is launched and output(s) is/are realized one period later;
- stage 4: Repayment decisions are made by the group members and all payoffs are realized.

We first consider the endogenous decision of the group form which is made in stage 2, that is, whether the borrowers choose to form a standard group or a bogus group. If the group chooses to be a bogus group, the two group members collectively decide which project to invest and whether or not to repay when succeed. The surplus is allocated between the two members by Nash-bargaining.

Proposition 2. *Given any group-lending loan contract $\{L, R\}$, it is optimal for the ij group to form a bogus group if*

$$p(k_i - k_j)L > 2p(1-p)(V-R). \tag{11}$$

Proof. See appendix A.2. □

Forming a standard group instead of a bogus group resembles buying an insurance. For a borrower in a standard group, when she succeeds (with probability p), she can always obtain the future value V as long as she repays. However, conditional on project success, in expectation she needs to repay $(2-p)R$ in a standard group (pay $2R$ in case of partner project failure and R in case of partner project success) instead of $2R/2 = R$ (per person) in a bogus group. The difference $(2-p)R - R = (1-p)R$ can be thought of as the insurance premium when a borrower's project succeeds (with probability p). If a borrower's project fails on the other hand, due to limited liability she pays nothing but would receive pV in expectation in a standard group as opposed to 0 in a bogus group. Thus, pV is the risk sharing benefit of being in a standard group when a borrower's project fails (with probability $1-p$). The ex-ante total net benefit (insurance value) from being in a standard group for both borrowers is hence

$$\mathcal{I} = 2(1-p)pV - 2p(1-p)R = 2p(1-p)(V-R). \tag{12}$$

which is positive since we have $V > R$ based on the no-default condition (4).

If the borrowers form a bogus group instead of a standard group, they will optimally invest all the loaned funds ($2L$) into the project with higher productivity. Without loss of generality assume $k_i \geq k_j$. Total group output increases by $p(k_i - k_j)L$ compared to forming a standard group under the same loan contract. However the group members forgo the insurance value (or risk sharing benifit) of being in a standard group. We thus obtain the Lemma (2) result.

Notice that, for a homogeneous ii (HH or LL) group, the LHS of (11) in Lemma (2) is zero and so this condition is never satisfied for such groups – a bogus group does not offer any benefit in terms of additional project return while still requires foregoing the risk-sharing value inherent in a standard joint liability group. We therefore see that condition (11) holds only for heterogeneous (HL) groups and only if the productivity difference $k_H - k_L$ is sufficiently large so that the extra output benefit of forming a bogus group overwhelms the loss of insurance (risk sharing) value in a standard group.

The consequence of the possibility of bogus group forming if the lender (mistakenly) offers the optimal standard group contract $\mathcal{S}$ is thus as follows:

Proposition 3. *If the standard group loan contract $\mathcal{S}$ is offered, any HL groups with*

$$k_H - k_L > \frac{2}{p(2-p)}, \tag{13}$$

will optimally operate as bogus groups, which consequently incur a loss to the lender.

Proof. See appendix A.3. □

The above proposition demonstrates the main problem with not taking into account the possibility of endogenously forming bogus groups – lenders would no longer break even using the standard group loan contract $\mathcal{S}$.

If condition (13) holds, the lender must offer alternative contract(s) to avoid the loss caused by the bogus groups. As motivated by our empirical observation mentioned in the induction part, we propose the following assumption:

Assumption 2. *Assume $k_H - k_L > \frac{2}{p(2-p)}$ so that the bogus group will arise when the standard standard group loan contract $\mathcal{S}$ is offered.*

For any contract designed for standard group, we have $R = \frac{L}{p(2-p)}$, substitute it into condition (11) when $ij = HL$, it becomes

$$p(k_H - k_L)L > 2p(1-p)\left(V - \frac{L}{p(2-p)}\right). \tag{14}$$

Given k_H and k_L, if we increase the loan size L, the LHS of the above condition (productivity benefit of bogus group) will be amplified while the RHS of it (insurance/risk-sharing benefit of standard group) will be dampened (because the "insurance premium" payed by the borrowers in good state becomes cheaper), therefore the above condition is more likely to hold and bogus group is more likely to arise. On the contrary, only by reducing the loan size, can the lender mitigate the borrowers' incentive to operate as bogus group. We will discuss how can the lenders address the bogus group problem when they acknowledge the possibility of bogus groups forming in next section and the above intuition will help explain our results.

3.3 Solutions to the bogus group problem

In this section, we consider two possible cases and propose strategies accordingly for the lender to solve the bogus group problem in each case. The first case is that the lender, for some exogenous reasons, cannot allow the existence of bogus group and would like to design an appropriate group loan contract that eliminates the incentive for any group to operate as a bogus one.[9] The second case, on the contrary, is that the lender allows the existence of bogus group and is free to design appropriate contract for the group by taking into account the group's self-selection behavior for its group type (standard or bogus), which is a hidden action for the lender.

3.3.1 Exclude the incentive of forming bogus group

In this section, we first consider the case in which the lender cannot allow the existence of bogus group in any situation, so that she wants to design a contract $\{L^*_{ij}, R^*_{ij}\}$ for any ij group such that using it the borrower group would optimally choose to be a standard group and each member will not default strategically. The timing of the game is identical to the form discussed in section 3.2. The strategy of the lender is the contract $\{L^*_{ij}, R^*_{ij}\}$, while the strategy of each borrower consists of the repay-default choice and, collectively, the group type choice. The contract $\{L^*_{ij}, R^*_{ij}\}$ will be

[9] For example, as discusses in the introduction, the lender just cannot tolerate the "cheating behavior" of operating as a bogus group; or the interest rate of the group-lending market is regulated and the lender cannot make break even under such interest rate if bogus groups exist; or the lender hope every borrower engage in ones own project to improve her long run skills and experience, or to enhance the employment rate of the community and maintain the stability of the social order etc.

designed so that: (i) any borrower group will be standard, (ii) each group member will optimally repay when her own project succeeds (iii) the lender breaks even and (iv) the group's joint payoff is maximized.

From the previous section we know that contract $\{L_{ij}^*, R_{ij}^*\}$ must ensure that standard groups will repay (condition (2) and (4)) and that bogus group will not arise. To exclude bogus groups we thus have to ensure that the converse of condition (11) is true, that is,

$$(k_i - k_j)L \leq 2(1-p)(V-R) \tag{15}$$

holds at $\{L_{ij}^*, R_{ij}^*\}$.

The contract $\{L_{ij}^*, R_{ij}^*\}$ solves:

$$\max_{L,R} W_{ij}(L, R|S) = p(k_i + k_j)L - 2p(2-p)R + 2p(2-p)V \tag{16}$$

$$\text{s.t. (2), (4), (15), and } R = \frac{L}{p(2-p)} \tag{17}$$

The constraint (15) for ii group is always true, and thus redundant. For HL group, it is equivalent to

$$(k_H - k_L)L + 2(1-p)R \leq 2(1-p)V. \tag{18}$$

At the optimum the lender's break-even constraint, $p(2-p)R = L$ implies that R is an increasing function of L, so the LHS of (18) increasing in L. For given parameter values, the no-bogus constraint (18) would thus hold only if the loan size L is sufficiently small. Since the standard group loan contract $\mathcal{S}$ does not meet this condition for HL group (as shown in Proposition 3), this implies that to rule out bogus groups the loan size for HL group must be reduced relative to L_S, as described in the following proposition.

Proposition 4. *Suppose the the lender observes the productivity of each borrower but cannot allow the existence of bogus group, then*

(i) the optimal contract for ii group $\{L_{ii}^*, R_{ii}^*\}$ is still $\mathcal{S} = \{L_S, R_S\}$;

(ii) the optimal contract for HL group $\{L_{HL}^*, R_{HL}^*\}$ is $\mathcal{E} = \{L_E, R_E\}$ where

$$L_E = \frac{p(2-p)V}{1 + \frac{p(2-p)}{2(1-p)}(k_H - k_L)} < L_S \text{ and } R_E = \frac{L_E}{p(2-p)}. \tag{19}$$

Proof. See appendix A.4. □

The the contract for standard group-lending $\mathcal{S}$ is still optimal for ii group because under this contract, such group has no incentive to be bogus group. Since the HL group opt to operate as bogus group under contract $\mathcal{S}$, only by reducing the loan size to the level of contract $\mathcal{E}$ can the lender eliminate the borrower's incentive to be bogus group. The interest rate in contract $\mathcal{E}$ is the same as in contract $\mathcal{S}$ since in both cases all groups are standard in equilibrium and hence the lender's break-even condition is the same per dollar lent. In order to rule out bogus groups, HL group's welfare is reduced under contract $\mathcal{E}$ because the loan size provided to is smaller than that in contract $\mathcal{S}$ derived in section 3.1 in which bogus groups were assumed away either exogenously or because of enforcement by the lender.

3.3.2 Allow the existence of bogus group

Suppose now the lender can allow the existence of both standard groups and bogus groups and therefore wishes to design the optimal contract contingent on the group type ij, such that the lender can make break even granted that the group can self-select to be either standard or bogus group. The timing in this case is as follows.

- stage 0: Two borrowers form a group and then each borrower is randomly endowed with a project with productivity k_i, $i \in \{H, L\}$, which is observable to the lender;
- stage 1: The lender offers the contract;
- stage 2: Given the contract, the borrower group chooses to be either standard or bogus;
- stage 3: Investment is launched and output is realized one period later;
- stage 4: Repayment decisions are made by the group members and all payoffs are realized.

For group ij, the lender's objective is to find the optimal contract $\{L_{ij}^{\#}, R_{ij}^{\#}\}$ that maximizes the group's joint payoff, subject to the no strategic default condition and the lender's breaks even condition. The last requirement comes from the free-entry assumption which rules out cross-subsidization across contracts or group types. The optimal contract could implements the endogenous group type to be either standard ($\tau = 1$) or bogus ($\tau = 0$), thus the no strategic default condition and the lender's breaks even condition should adapt accordingly, i.e. the lender solves the following problem:

$$W_{ij}^{\#} = \max_{L,R,\tau\in\{0,1\}} W_{ij}(L, R, \tau), \tag{20}$$

subject to

$$R \leq \tau\frac{1-p}{2-p}V + (1-\tau)V, \tag{21}$$

$$R = \tau\frac{L}{p(2-p)} + (1-\tau)\frac{L}{p}, \tag{22}$$

$$\tau W_{ij}(L, R|S) + (1-\tau)W_{ij}(L, R|B) \geq \tau W_{ij}(L, R|B) + (1-\tau)W_{ij}(L, R|S), \tag{23}$$

where $W(L, R, \tau) = \tau W_{ij}(L, R|S) + (1-\tau)W_{ij}(L, R|B)$, and the joint payoff function $W_{ij}(\cdot, \cdot|S)$ and $W_{ij}(\cdot, \cdot|B)$ are given as follows:

$$W_{ij}(L, R|S) = \begin{cases} p(k_i + k_j)L - 2p(2-p)R + 2p(2-p)V, & R \leq \frac{1-p}{2-p}V \text{ (Repay,Repay)} \\ p((k_i + k_j)L - 2R + 2V), & \frac{1-p}{2-p}V < R \leq \frac{V}{2} \text{ (Repay,Default)} \\ p(k_i + k_j)L, & R > \frac{V}{2} \text{ (Default,Default)} \end{cases}, \tag{24}$$

$$W_{ij}(L, R|B) = \begin{cases} 2pk_iL - 2pR + 2pV, & R \leq V \text{ (Repay)} \\ 2pk_iL, & R > V \text{ (Default)} \end{cases}. \tag{25}$$

The last constraint (23) is the self-selection constraints stating that whenever a contract that implements a particular group type (standard or bogus) is offered, it is then optimal for that group itself to operate as the desired type. The contracts are feasible given the assumption 1.

Then we attain the following proposition.

Proposition 5. *Suppose the lender observes each borrower's productivity and allows the existence of bogus group, then*

(i) the optimal contract for ii group $\{L_{ii}^{\#}, R_{ii}^{\#}\}$ *is* $\mathcal{S} = \{L_S, R_S\}$ *if* $k_i \leq \frac{1}{p^2}$ *or* $\mathcal{B} = \{L_B, R_B\}$ *if* $k_i > \frac{1}{p^2}$ *where* $L_B = pV$ *and* $R_B = \frac{L_B}{p}$*, ii group operates as standard group given contract* $\mathcal{S}$ *while operates as bogus group given contract* $\mathcal{B}$*;*

(ii) the optimal contract for HL group $\{L_{HL}^{\#}, R_{HL}^{\#}\}$ *is* $\mathcal{E} = \{L_E, R_E\}$ *if* $k_H \leq f(k_L)$ *or* $\mathcal{B} = \{L_B, R_B\}$ *if* $k_H > f(k_L)$ *where* $f(k_L) = \frac{1}{2}(k_L + c + \sqrt{(k_L + c)^2 - 4k_L/p})$ *and* $c = \frac{2p^2-5p+4}{p(2-p)}$*, HL group operates as standard group given contract* $\mathcal{E}$ *while operates as bogus group given contract* $\mathcal{B}$*.*

Proof. see appendix A.5. □

Recalled that according to proposition 2, given a standard group loan contract, any homogeneous ii group, unlike the HL group, has no incentive to be bogus group because such group, when operating as bogus group, gains nothing in terms of output but forfeits the risk sharing benefit of standard group and thus suffers net loss. However, when the lender allows the existence of bogus group and offers the appropriate contract $\mathcal{M}$ for bogus groups, a seemingly surprising result revealed by proposition 5 is that it is optimal for even homogeneous ii group to be bogus group under contract $\mathcal{M}$ when the absolute level of the productivity k_i is sufficiently high. The reason is that homogeneous ii group can benefit from the larger loan size of contract $\mathcal{M}$ especially when the productivity k_i is sufficiently high.

The interest rate of contract $\mathcal{M}$, designed for bogus groups, is equal to the capital cost $\frac{1}{p}$. While the contract designed for standard group, such as $\mathcal{S}$ (or $\mathcal{E}$), requires lower interest rate $\frac{1}{p(2-p)}$ due to the risk sharing between the two members in a standard group. As we mentioned in section 3.1, the lender breaks even with lower interest rate $\frac{1}{p(2-p)}$ because it actually requires each borrower to repay $(2-p)R_S > R_S$, thus the effective interest rate is also equal to the capital cost $\frac{1}{p}$ since the actual payment is $(2-p)R_S = \frac{L_S}{p}$.

Though the effective interests are the same under two types of contracts, the loan size designed for standard group is smaller than that for bogus group because of the possibility of strategic default consideration that only exist in a standard group. When the group operates as a standard group, the two borrowers invest in two independent projects, and each member chooses her own repayment decision independently, thus providing each borrower an opportunity to default strategically when her own project indeed succeeds and "free ride" on the partner's repayment action. In specific, the repayment decisions of the two borrowers comes as a Nash-equilibrium of a simultaneous game shown in table 1. Such strategic interaction between the two members reduces the loan size provided for them since only when the loan size is sufficiently small will they play (Repay, Repay) in equilibrium, and such factor negatively affects the welfare of the group members. We call the such inefficiency as the "strategic interaction cost" for standard group lending. However, when the borrowers form a bogus group, they act collectively in the repayment stage and there is no possibility of strategic default by one borrower, thus no one can "free ride" on other's repayment action. Given this fact, the loan sizes for a bogus group could be larger and such factor positively affects the welfare of the group members.

The above analysis can also be interpreted by the following two formulas: we can rewrite repayment condition (3) (for the best standard group contract $\mathcal{S}$) and (21) (for contract $\mathcal{M}$) as

$$\frac{L_S}{p} \leq V - pV \tag{26}$$

$$\frac{L_M}{p} \leq V - 0 \tag{27}$$

As stated before, the LHS are the marginal payments and the RHS are the marginal increase in the future credit value. The above two conditions clearly show that the loan size of standard group contract is smaller because the marginal gain in the future credit value is less, not due to the terminal value V is lower, but due to the initial level $pV > 0$ is higher. This reveals the fact the there is some "free riding" problem and the possibility of strategic interaction between the two members that cuts down the incentive to repay in the standard group: if the loan size is too high so that the borrower has to repay too much, she would rather free ride on her peer's repayment behavior than repay herself, thus, in equilibrium the loan size must be small enough so that (Repay, Repay) becomes the Nash equilibrium. The loan size of contract $\mathcal{E}$ is even smaller than that of contract $\mathcal{S}$ because of the additional bogus group problem stated in proposition 3.

Compared with homogeneous ii group, HL group has even higher incentive to operating under the contract designed for bogus group: when doing so, it only forfeits the risk sharing benefit of standard group, whereas gains not only from the enlarged loan size of contract $\mathcal{M}$ but also from the increasing of average productivity form $\frac{1}{2}(k_H + k_L)$ to k_H. Thus, as long as the k_H is high enough relative to k_L so that the increasing in output could be quite significant, the HL group should optimally operating as bogus group under contract $\mathcal{M}$.

The optimal contract in this section, which allows the existence of bogus group, compared with that in section 3.3.1, enhances the not only welfare of HL group, but also that of homogenous ii group.

3.4 When the productivity k_i is unobservable for the lender

In this section, we consider the situation in which the borrower's exogenous type, the productivity parameter $k_i, i \in \{H, L\}$, is unobservable for the lender. As shown below, the main properties of the model with observable productivity generalize to the model in this section.

Notice that the standard group loan contract $\{L_S, R_S\}$ is independent of k_i and k_j, therefore it is still optimal to offer the contract $\mathcal{S}$ defined in Proposition **1** when bogus groups were assumed away either exogenously or because of enforcement by the lender. Therefore proposition 2 and 3 still apply to this section. In particular, if the group can freely choose to form either standard group or bogus group, the condition under which bogus group arises is also the same with that depicted in proposition 2.

Thus, we only need to reconsider the optimal strategies for the lender to address the bogus group problem in the following.

3.4.1 Exclude the incentive of forming bogus group

We first consider the case in which the lender cannot allow the existence of bogus group in any situation. Different from the above previous analysis, since the lender is unable to observe the group type ij, she cannot design a contract contingent on ij. On the contrary, the lender can only offer a common contract $\{L^*, R^*\}$ independent from ij for the entire population so that using it any borrower group would optimally choose to be a standard group and each member will not default strategically. The timing of the game is identical to the form discussed in section 3.2. except that the lender does not observe k_i. The strategy of the lender is the contract $\{L^*, R^*\}$, while the strategy of each borrower consists of the repay-default choice and, collectively, the group type choice. The contract $\{L^*, R^*\}$ will be designed so that: (i) any borrower group will be standard, (ii) each group member will optimally repay when her own project succeeds (iii) the lender breaks even and (iv) the **ex-ante expected** joint payoff of the group (or equivalently, the total payoff of the population) is maximized. The last requirement indicates that the lender, who is unable to observe the group type knows the distribution of the group type ij cares about the aggregate welfare of the the economy.

From the previous section we know that contract $\{L^*_{ij}, R^*_{ij}\}$ must ensure that standard groups will repay (condition (2) and (4)) and that bogus group will not arise. To exclude bogus groups we thus have to ensure that the converse of condition (11) is true, that is, holds at $\{L^*_{ij}, R^*_{ij}\}$.

So the contract $\{L^*, R^*\}$ solves:

$$\max_{L,R} W_{ij}(L, R|S) = p(k_i + k_j)L - 2p(2-p)R + 2p(2-p)V \tag{28}$$

$$\text{s.t. (2), (4), } R = \frac{L}{p(2-p)}$$

and

$$(k_i - k_j)L \leq 2(1-p)(V - R), \ \forall ij \in \{HH, HL, LL\} \tag{29}$$

where the no-bogus condition (29) is the IC constraint that must hold for all types of groups. It turn out that it is binding for HL type, i.e.

$$(k_H - k_L)L + 2(1-p)R \leq 2(1-p)V. \tag{30}$$

Then similar to the analysis of appendix A.4, we obtain the following proposition.

Proposition 6. *Suppose the the lender cannot observe the productivity of each borrower and cannot allow the existence of bogus group, then the optimal contract is* $\mathcal{E} = \{L_E, R_E\}$.

In this case, the lender cannot provide contracts contingent on group type ij. In order to rule out bogus groups, borrowers' welfare is reduced under contract $\mathcal{E}$ because the loan size provided to any group type is smaller than that in contract $\mathcal{S}$ derived in section 3.1 in which bogus groups were assumed away either exogenously or because of enforcement by the lender. While only borrowers in HL groups would choose to operate as bogus, all borrowers obtain smaller loans. This is inefficient.

3.4.2 Allow the existence of bogus group

Similarly, optimal contract in Proposition (5) is no longer optimal, since the lender cannot observe the group's productivity parameter k_i and k_j and cannot provide contracts contingent on ij. Since the lender allows the existence of both standard group and bogus group in equilibrium, therefore, based on the break even conditions for two types of group form, two types of contracts in terms of interest rate should be offered: one type, designed for standard groups, has gross interest rate $\frac{1}{p(2-p)}$, and the other type, designed for bogus groups, has gross interest rate $\frac{1}{p}$. According to the previous analysis, the joint payoff of a group is always increasing in the loan size whatever the productivity types of the group members are. Hence, within each contract type in terms of interest rate, only one contract can be offered.

Above analysis implies that the lender can offer two contracts $\mathcal{N} = \{L^\#_N, R^\#_N\}$ and $\mathcal{M} = \{L^\#_M, R^\#_M\}$ targeted at standard group and bogus group respectively, such that (a) any group chooses contract $\mathcal{N}$ self-selects to be standard form and any group chooses contract $\mathcal{M}$ self-selects to be bogus form, (b) no borrower default strategically and (c) the ex-ante expected joint payoff of the group (or equivalently, the total payoff of the population) is maximized. Suppose the proportion of the ij group in the population is q_{ij}, and $\sum_{ij} q_{ij} = 1$, then $L^\#_N, R^\#_N, L^\#_M, R^\#_M$ solve the problem[10]

$$\max_{L_N, R_N, L_M, R_M} \sum_{ij} q_{ij} W_{ij}(L_N, R_N, L_M, R_M) \tag{31}$$

[10] We can interpret the problem as a optimal mechanism design problem in which the lender, who does not observe the type of the group ij, maximizes the expected payoff of the group or equivalently the ex-ante aggregate welfare of all the borrowers. The lender makes break even per contract because of the free entry of lenders, no cross-subsidization among different types of groups exist given our optimal solution derived below.

subject to

$$R_M \leq V \tag{32}$$

$$R_M = \frac{L_M}{p} \tag{33}$$

$$R_N \leq \frac{1-p}{2-p} V \tag{34}$$

$$R_N = \frac{L_N}{p(2-p)} \tag{35}$$

$$\begin{array}{l} \max\{W_{ij}(L_N, R_N|S), W_{ij}(L_M, R_M|B)\} \\ \geq \max\{W_{ij}(L_N, R_N|B), W_{ij}(L_M, R_M|S)\} \end{array}, \forall ij \in \{HH, HL, LL\} \tag{36}$$

where $W_{ij}(L_N, R_N, L_M, R_M) \equiv \max\{W_{ij}(L_N, R_N|S), W_{ij}(L_M, R_M|B)\}$, and the joint payoff function $W_{ij}(\cdot,\cdot|S)$ and $W_{ij}(\cdot,\cdot|B)$ are defined by equation (24) and (25) respectively. The contracts are feasible given the assumption 1.

Notice that the contract $\mathcal{N}$, designed for standard group, will never be in the (Repay, Default) regime ($\frac{1-p}{2-p}V < R \leq \frac{V}{2}$). In this regime, the break even condition of the lender implies that the interest rate should be equal to that of contract $\mathcal{M}$, thus the functional form of $W_{ij}(\cdot,\cdot|S)$ is the same with $W_{ij}(\cdot,\cdot|B)$, but the constraint for the repayment (or equivalently for loan size) is tighter for such contract $\mathcal{N}$ than for contract $\mathcal{M}$, that is, any contract in this regime is dominated by contract $\mathcal{M}$. That is why we only seek the optimal contract $\mathcal{N}$ in the (Repay, Repay) regime defined by the constraint (34).

The last constraint (23) is the self-selection constraints stating that no group would "cheat" by operating as bogus group under contract $\mathcal{N}$, the one designed for standard groups, and vise versa.

Conditional on (32), (33), (34) and (35), the above problem can be simplified as:

$$\max_{L_N, L_M} \sum_{ij} q_{ij} W_{ij} \left(L_N, \frac{L_N}{p(2-p)}, L_M, \frac{L_M}{p} \right) \tag{37}$$

subject to

$$L_M \leq pV \tag{38}$$

$$L_N \leq p(1-p)V \tag{39}$$

$$\begin{array}{l} \max\left\{ W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|S\right), W_{ij}\left(L_M, \frac{L_M}{p}|B\right) \right\} \\ \geq \max\left\{ W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|B\right), W_{ij}\left(L_M, \frac{L_M}{p}|S\right) \right\} \end{array}, \ \forall ij \in \{HH, HL, LL\}, \tag{40}$$

where

$$W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|S\right) = (p(k_i+k_j)-2)L_N + 2p(2-p)V \ \text{(Repay,Repay)}, \tag{41}$$

$$W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|B\right) = 2\left(pk_i - \frac{1}{2-p}\right) L_N + 2pV \ \text{(Repay)} \tag{42}$$

$$W_{ij}\left(L_M, \frac{L_M}{p}|B\right) = 2(pk_i - 1)L_M + 2pV \ \text{(Repay)} \tag{43}$$

$$W_{ij}\left(L_M, \frac{L_M}{p}|S\right) = \begin{cases} (p(k_i+k_j) - 2(2-p))L_M + 2p(2-p)V, & L_M \leq \frac{p(1-p)}{2-p}V \ \text{(Repay,Repay)}; \\ (p(k_i+k_j)-2)L_M + 2pV, & \frac{p(1-p)}{2-p}V < L_M \leq \frac{pV}{2}; \ \text{(Repay,Default)} \\ p(k_i+k_j)L_M, & L_M > \frac{pV}{2}. \ \text{(Default,Default)} \end{cases} \tag{44}$$

Lemma 1. *The constraint* (38) *is binding, i.e.*

$$L_M = pV. \tag{45}$$

Proof. see appendix A.6. □

Given the above facts, the constraint (40) can be simplified as:

$$\max\{W_{ij}(\mathcal{N}|S), W_{ij}(\mathcal{M}|B)\} \geq W_{ij}(\mathcal{N}|B),\ \forall ij \in \{HH, HL, LL\}, \tag{46}$$

where

$$W_{ij}(\mathcal{N}|S) \equiv W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|S\right) = (p(k_i+k_j)-2)L_N + 2p(2-p)V \text{ (Repay, Repay)} \tag{47}$$

$$W_{ij}(\mathcal{N}|B) \equiv W_{ij}\left(L_N, \frac{L_N}{p(2-p)}|B\right) = 2\left(pk_i - \frac{1}{2-p}\right)L_N + 2pV \text{ (Repay)} \tag{48}$$

$$W_{ij}(\mathcal{M}|B) \equiv W_{ij}(L_M, V|B) = 2k_i p^2 V \text{ (Repay)} \tag{49}$$

The condition (46) means that the HL group (i) either finds that it is better to operate as a standard group under contract $\mathcal{N}$ ($W_{ij}(\mathcal{N}|S) \geq W_{ij}(\mathcal{N}|B)$), or (ii) finds that it is better to operate as a bogus group under contract $\mathcal{M}$ rather than operate as a bogus group under contract $\mathcal{N}$ ($W_{ij}(\mathcal{N}|S) < W_{ij}(\mathcal{N}|B)$ even though it is better to operate as a bogus group under contract $\mathcal{N}$ ($W_{ij}(\mathcal{M}|B) \geq W_{ij}(\mathcal{N}|B)$). Besides, the group choose one of the two contracts that yields higher joint payoff, so the group choose contract $\mathcal{M}$ if

$$W_{ij}(\mathcal{N}|S) < W_{ij}(\mathcal{M}|B) \Leftrightarrow L_N < \frac{2(pk_i - (2-p))}{p(k_i+k_j)-2}pV \tag{50}$$

while choose contract $\mathcal{N}$ otherwise.

Now we are ready to prove the following proposition.

Proposition 7. *Suppose the lender cannot observe the productivity of each borrower but allows the existence of both standard and bogus group in equilibrium, then the optimal contract menu that maximizes the expected payoff of a group consists of two contracts* $\mathcal{N} \equiv \{L_N^{\#}, R_N^{\#}\}$ *and* $\mathcal{M} \equiv \{L_M^{\#}, R_M^{\#}\}$. *Contract* $\mathcal{M}$, *designed for bogus group, is always* $\mathcal{B} = \{L_B, R_B\}$, *while contract* $\mathcal{N}$, *designed for standard group is (i)* $\mathcal{S} = \{L_S, R_S\}$ *if* $k_H > \frac{1}{p^2(2-p)}$, *(ii)* $\mathcal{E} = \{L_E, R_E\}$ *if* $k_H < f(k_L)$, *or (iii)* $\mathcal{F} = \{L_F, R_F\}$ *if* $f(k_L) \leq k_H \leq \frac{1}{p^2(2-p)}$, *where* $L_F = \frac{pk_H - 1}{pk_H - \frac{1}{2-p}}pV$ *and* $R_F = \frac{L_F}{p(2-p)}$. *Homogenous ii group choose contract* $\mathcal{M}$ *iff* $k_i \geq \frac{1}{p^2}$, *while HL group choose contract* $\mathcal{M}$ *iff* $k_H \geq f(k_L)$. *Any group choose contract* $\mathcal{N}$ *operates as standard group and any group choose contract* $\mathcal{M}$ *operates as bogus group.*

Proof. see appendix A.7. □

According to proposition 7, when $k_L \geq \frac{1}{p^2} > \frac{1}{p^2(2-p)}$ and $k_H \geq f(k_L)$, it is optimal for the lender to provide contract menu $\{\mathcal{S}, \mathcal{M}\}$ and actually all types of the groups will choose contract $\mathcal{M}$. This in consistent with the results suggested by proposition 5 in section 3.3.2 where k_i is observable for the lender, since contract $\mathcal{M}$ is optimal for all types of groups given these parameter values no matter k_i is observable or unobservable for the lender.

Furthermore, as suggested by proposition 7, the contract menu $\{\mathcal{S}, \mathcal{M}\}$ is optimal as long as a looser condition with $k_H > \frac{1}{p^2(2-p)}$ is satisfied, because though HL group has incentive to be a bogus group under contract $\mathcal{S}$, it will not choose contract $\mathcal{S}$ since it is still better for it to choose contract $\mathcal{M}$.

However, when $k_H \leq \frac{1}{p^2(2-p)}$, HL group will find that being bogus group under contract $\mathcal{S}$ is more attractive than being bogus group under contract $\mathcal{M}$, which will consequently cause loss to the lender. Thus, in this case, the loan size of the contract designed for the groups which prefer to be standard (ii group) should be reduced to the level of L_F so that HL group will no longer choosing this contract and operating as bogus. The lower bound of the loan size for contract $\mathcal{N}$ is L_E because no group is willing to be bogus given that level, and there is no need to reduce the loan size lower than L_E.

In the following, we visualize the above results in the figures. The assumptions 1 and 2 restricts the parameter value space to the blue area displayed in Figure 1.

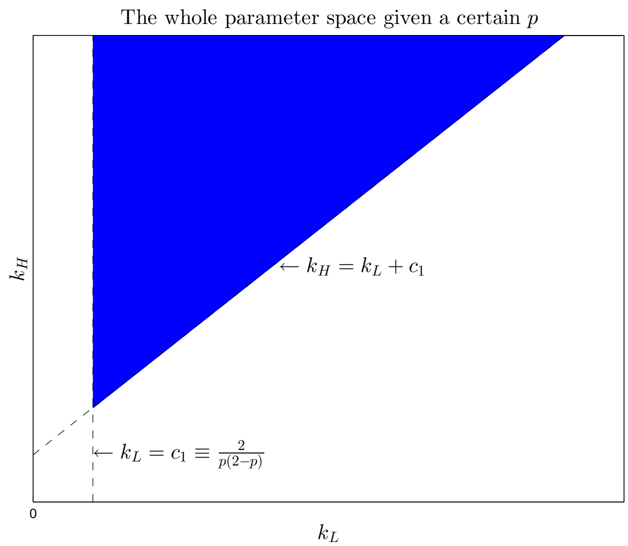


Figure 1: the whole parameter value space (blue area)

Define the group's choice function as

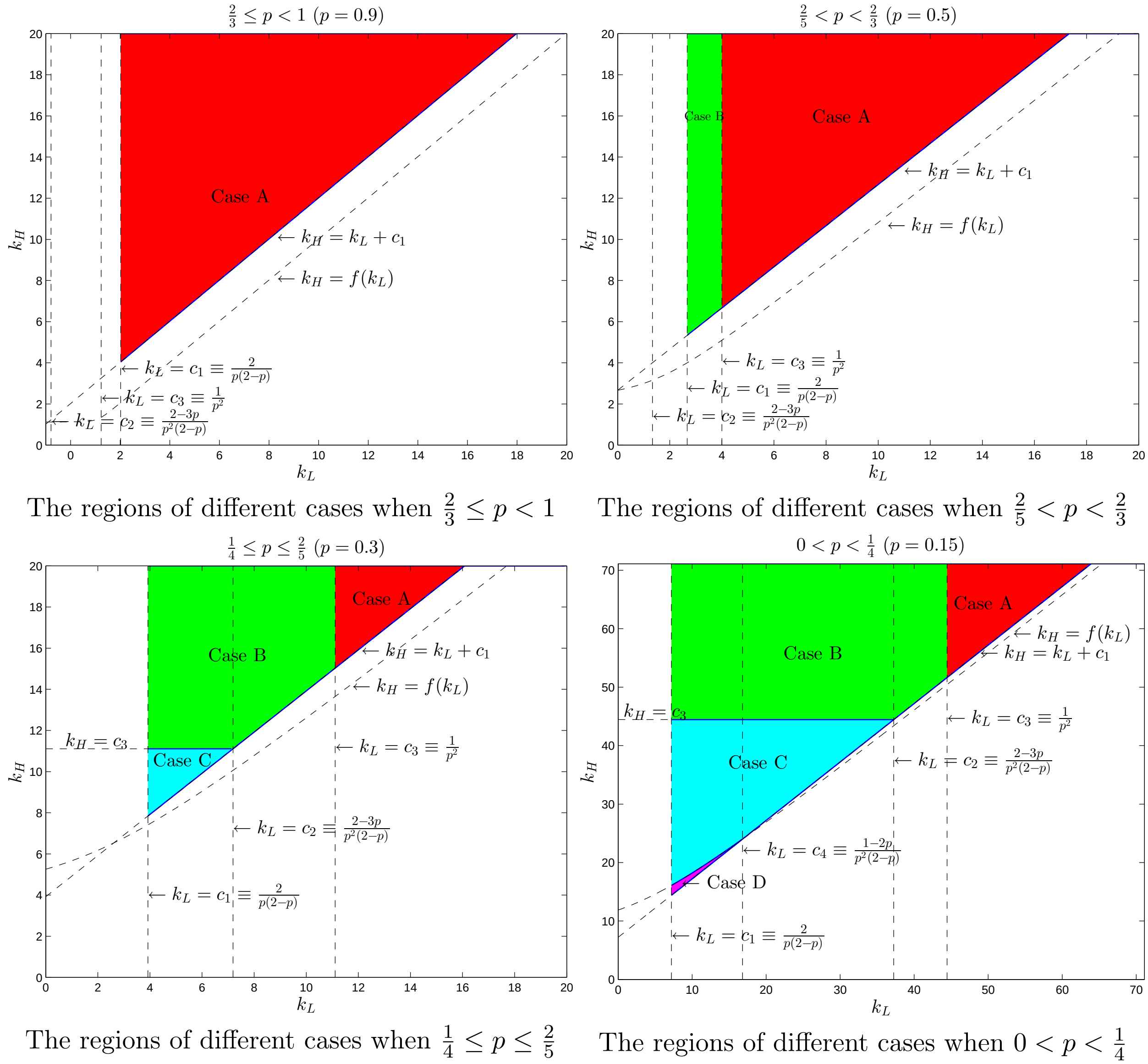
$$\tau(k_i, k_j) = \begin{cases} 1, & \text{if } W_{ij}(\mathcal{N}|S) \geq W_{ij}(\mathcal{M}|B) \\ 0 & \text{otherwise} \end{cases} \tag{51}$$

Let $\mathcal{C} = \{HL, HH, LL\}$ be the complete set of the types, $\emptyset$ be the empty set, and $\mathcal{C}_{\mathcal{M}|B}$ be the set of the types that choose contract $\mathcal{M}$ and operating as bogus group, i.e. $\mathcal{C}_{\mathcal{M}|B} = \{ij|\tau_{ij} = 0\}$. Let $\mathcal{C}_{\mathcal{N}|B} = \overline{\mathcal{C}_{\mathcal{M}|B}} = \{ij|\tau_{ij} = 1\}$ be the set of types that choose contract $\mathcal{N}$. Then we define Case A, B, C, D as the case where $\mathcal{C}_{\mathcal{M}|B} = \mathcal{C} = \{HL, HH, LL\}$, $\mathcal{C}_{\mathcal{M}|B} = \{HL, HH\}$, $\mathcal{C}_{\mathcal{M}|B} = \{HL\}$, and $\mathcal{C}_{\mathcal{B}} = \emptyset$ respectively. Then we have Corollary 1, which is visualized in Figure 2.

Corollary 1. *According to proposition 7, given the optimal separating contract pair* $(\mathcal{N}, \mathcal{M})$, *(i)* HL *group choose contract* $\mathcal{M}$ *if* $k_H > f(k_L)$ *while choose contract* $\mathcal{N}$ *otherwise, (ii)* HH *group choose contract* $\mathcal{M}$ *if* $k_H > 1/p^2$ *while choose contract* $\mathcal{N}$ *otherwise, (iii)* LL *group choose contract* $\mathcal{M}$ *if* $k_L > 1/p^2$ *while choose contract* $\mathcal{N}$ *otherwise. The following four cases are possible:*

- *If* $2/3 \leq p < 1$, *we always have* $k_H > f(k_L)$, $k_H > 1/p^2$ *and* $k_L > 1/p^2$, *thus all the three types choose contract* $\mathcal{M}$ *(top-left panel of figure 2);*

Figure 2: THE WHOLE PARAMETER SPACE AND DIFFERENT CASES.



The regions of different cases when $\frac{2}{3} \leq p < 1$ The regions of different cases when $\frac{2}{5} < p < \frac{2}{3}$

The regions of different cases when $\frac{1}{4} \leq p \leq \frac{2}{5}$ The regions of different cases when $0 < p < \frac{1}{4}$

- *If $2/5 < p < 2/3$, we always have $k_H > f(k_L)$ and $k_H > 1/p^2$, thus both HL and HH groups always choose contract $\mathcal{M}$, while both choices are possible for LL groups (top-right panel of figure 2);*
- *If $1/4 \leq p \leq 2/5$, we always have $k_H > f(k_L)$, thus only HL groups always choose contract $\mathcal{M}$, while both choices are possible for HH and LL groups (bottom-left panel of figure 2);*
- *If $0 < p \leq 1/4$, both choices are possible for HL, HH and LL groups (bottom-right panel of figure 2).*

In other words,

- *HL groups choose the bogus group contract $\mathcal{M}$ in Case A, B, C, while they choose the standard group contract $\mathcal{N}$ in Case D;*

- *HH groups choose the bogus group contract $\mathcal{M}$ in Case A, B, while they choose the standard group contract $\mathcal{N}$ in Case C, D;*

- *LL groups choose the bogus group contract $\mathcal{M}$ in Case A, while they choose the standard group contract $\mathcal{N}$ in Case B, C, D;*

Corollary 1 says that when the success probability p is sufficiently high ($\frac{2}{3} \leq p < 1$), all groups will always choose contract $\mathcal{M}$ and operate as bogus; when the success probability p is moderate ($\frac{2}{5} \leq p < \frac{2}{3}$), both HL and HH group will always choose contract $\mathcal{M}$ and operate as bogus group; when the success probability p is relatively low ($0 < p \leq \frac{2}{5}$), only HL group will always choose contract $\mathcal{M}$ and operate as bogus group.

In general, for a group with different types of projects, bogus group is more likely to arise if the relative heterogeneity of productivity is high; while for a group with same types of projects, bogus group is more likely to arise if absolute level of productivity is high. Obviously, the basic intuition of the trade-off among three factors, i.e. (i) the risk sharing benefit, (ii) strategic interaction cost and (iii) the productivity improvement in bogus group, still apply when the each borrower's productivity is unobservable for the lender: for ii group, the strategic interaction cost in the repayment stage may overwhelm the risk sharing benefit in standard group when the absolute level of productivity k_i is sufficiently high so that operating as bogus group under contract $\mathcal{M}$ is preferred; for HL group, besides the first two trade-offs, when the relative heterogeneity in productivity is sufficiently high, the extra benefit in increased joint productivity level will further strengthen the incentive for it to operate as bogus group under contract $\mathcal{M}$.

Similar to the finding in the case of observable types of project, the welfare can be enhanced if the bogus group is not excluded but is offered with an appropriate contract. To see this, let us examine the Figure 2. As in Figure 2 we can see that when the success probability of the projects is high, all types of groups (HH, HL, LL) tend to prefer being bogus groups under the "right" contract designed for bogus groups. Such result implies that by allowing the bogus group to exist in equilibrium and providing the "right" contract to them accordingly, we can enhance the welfare of the economy. Thus, bogus group is a bad news for the economy if we ignore the possibility of its existence or does not tolerate its existence, but is a good news if we can provide the proper contract for them to lead them to the efficient investment actions.

3.5 When standard group members make repayment decision collectively

In the previous sections, we assume that in a standard group, each borrower make the repayment decision individually, leaving there a possibility of strategic interaction and free riding incentive for each group member. This friction leads to a tight restriction on the loan size of standard group loan contract, and becomes one important factor that influences the contract design problem for the lender and the contract choice problem for the group. In this section, we extend our basic model by ruling out such friction in standard group, by assuming that the repayment decision in a standard group is made by the group collectively, not by each individual independently. We can interpret it as a scenario that each borrower invest into one's own project independently but the two members pool together all the outputs together one period later and they act collectively to decide whether repay or not, then they split the surplus according to preestablished allocations rules. Such repayment decision pattern and surplus allocation rule in a standard group are now the same with those of a bogus group. Thus, the unique difference between the standard group and a bogus group comes from the investment stage: the borrowers invest in two projects in a standard group while invest in one projects in a bogus group.

We still start with a basic case where due to some exogenous reason, the formation of bogus group is not possible.

In the repayment stage, the group, makes a binary choice between repaying the entire group liability $2R$ or repaying zero, similar to the previous analysis, any repayment strictly between 0 and $2R$ is not optimal. At the repayment stage, the group maximizes the joint payoff of a group, thus it will choose to repay $2R$ iff $2R \leq 2V$, or

$$R \leq V. \tag{52}$$

Given assumption 1 and the above condition, the break even condition of the lender is

$$R = \frac{L}{p(2-p)}. \tag{53}$$

Thus the lender's problem is

$$\max_{L} W_{ij}\left(L, \frac{L}{p(2-p)}|S\right) = (p(k_i + k_j) - 1)L + p(2-p)V \tag{54}$$

$$\text{s.t. } L \leq p(2-p)V \tag{55}$$

Since the objective function is increasing in L, the constraint (55) is binding, thus we obtain:

Proposition 8. *When bogus group formation is impossible and the standard group members make repayment decision collectively, the optimal standard group loan contract is $\mathcal{S}' = \{L'_S, R'_S\}$ where*

$$L'_S = p(2-p)V, \ R'_S = V. \tag{56}$$

We can see that, since the condition (52) is less restrictive than condition (4), the standard group can get larger loan size in this case.

In all, the joint payoff for a ij group given contract $\mathcal{S}'$ is

$$W_{ij}(L'_S, R'_S|S) = p^2(2-p)(k_i + k_j)V > W_{ij}(L_S, R_S|S). \tag{57}$$

Notice that the contract $\mathcal{S}'$ is independent of k_i and k_j, therefore, it does not matter whether k_i and k_j are observable for the lender or not.

By acknowledging the possibility of bogus group formation, proposition 2, which stats about a general contract $\{L, R\}$, still applies here. Notice that the RHS of condition (11) is the the ex-ante total net benefit (insurance value) from being in a standard group for both borrowers, i.e.

$$\mathcal{I} = 2p(1-p)(V-R). \tag{58}$$

It turns out that, at contract $\mathcal{S}'$, since $R'_S = V$, the insurance value of standard group relative to bogus group is zero. Such result indicates that compered with bogus group, standard group's relative advantage (insurance/risk-sharing benefit) and relative disadvantage (strategic interaction cost) accompany with each other: when we shut down the strategic interaction cost by assuming that standard group members act collectively in repayment stage, the net insurance benefit of standard group relative to bogus group also disappears. However, the productivity improvement benefit of bogus group still exists for heterogeneous group HL. Straightforwardly, we obtain the following proposition, which shows the consequence of the possibility of bogus group forming if the lender (mistakenly) offers the group loan contract $\mathcal{S}'$:

Proposition 9. *If borrowers make repayment decision collectively in a standard group, and the group loan contract $\mathcal{S}'$ is offered, any ij group with*

$$k_i - k_j > 0, \tag{59}$$

or equivalently HL group, will optimally operates as bogus group, which consequently incur a loss to the lender.

We proceed by discussing how can the lenders address the bogus group problem when they acknowledge the possibility of bogus groups forming in the following.

We first consider the case in which the lender cannot allow the existence of bogus group in any situation. To be consistent with section 3.4, we still assume that each borrower's productivity is unobservable for the lender. So the lender can offer a common contract $\{L^{*'}, R^{*'}\}$ independent from ij for the entire population so that using it any borrower group would optimally choose to be a standard group and each member will not default strategically. Similarly, the contract $\{L^{*'}, R^{*'}\}$ solves:

$$\max_{L,R} W_{ij}(L, R|S) = p(k_i + k_j)L - 2p(2-p)R + 2p(2-p)V \tag{60}$$

$$\text{s.t. } (2), (29), (52), (53),$$

Similar to the analysis of appendix A.4, it turns out that contract $\mathcal{E} = \{L_E, R_E\}$ is also the optimal solution in this case. We obtain the following proposition.

Proposition 10. *If borrowers make repayment decision collectively in a standard group, the lender cannot observe the productivity of each borrower and cannot allow the existence of bogus group, then the optimal contract is $\mathcal{E} = \{L_E, R_E\}$.*

Suppose now the lender allows the existence of both standard group and bogus group in equilibrium, similar to the analysis in previous section, the lender can offer two contracts $\mathcal{N}' = \{L_N^{\#'}, R_N^{\#'}\}$ and $\mathcal{M} = \{L_M^{\#'}, R_M^{\#'}\}$ targeted at standard group and bogus group respectively, such that (a) any group chooses contract $\mathcal{N}'$ self-selects to be standard form and any group chooses contract $\mathcal{M}'$ self-selects to be bogus form, (b) no borrower default strategically and (c) the ex-ante expected joint payoff of the group (or equivalently, the total payoff of the population) is maximized. So $L_N^{\#'}, R_N^{\#'}, L_M^{\#'}, R_M^{\#'}$ solve the problem

$$\max_{L_N, R_N, L_M, R_M} \sum_{ij} q_{ij} W_{ij}(L_N, R_N, L_M, R_M) \tag{61}$$

subject to

$$R_M \leq V \tag{62}$$

$$R_M = \frac{L_M}{p} \tag{63}$$

$$R_N \leq V \tag{64}$$

$$R_N = \frac{L_N}{p(2-p)} \tag{65}$$

$$\begin{array}{l} \max\{W_{ij}(L_N, R_N|S), W_{ij}(L_M, R_M|B)\} \\ \geq \max\{W_{ij}(L_N, R_N|B), W_{ij}(L_M, R_M|S)\} \end{array}, \forall ij \in \{HH, HL, LL\} \tag{66}$$

where $W_{ij}(L_N, R_N, L_M, R_M) \equiv \max\{W_{ij}(L_N, R_N|S), W_{ij}(L_M, R_M|B)\}$, and the joint payoff function $W_{ij}(\cdot, \cdot|S)$ and $W_{ij}(\cdot, \cdot|B)$ are defined by equation (24) and (25) respectively. The contracts are feasible given the assumption 1. Note that constraint (64) is different from (34) now.

Similarly, we attain the following proposition.

Proposition 11. *Suppose borrowers make repayment decision collectively in a standard group, the lender cannot observe the productivity of each borrower but allows the existence of both standard and bogus group in equilibrium, then the optimal contract menu that maximizes the expected payoff of a group consists of two contracts* $\mathcal{N}' \equiv \{L_N^{\#'}, R_N^{\#'}\}$ *and* $\mathcal{M}' \equiv \{L_M^{\#'}, R_M^{\#'}\}$*. Contract* $\mathcal{M}'$*, designed for bogus group, is always* $\mathcal{B} = \{L_B, R_B\}$*, while contract* $\mathcal{N}'$*, designed for standard group is (i)* $\mathcal{E} = \{L_E, R_E\}$ *if* $k_H < f(k_L)$*, or (ii)* $\mathcal{F} = \{L_F, R_F\}$ *if* $k_H \geq f(k_L)$*. Homogenous* ii *group always choose contract* $\mathcal{M}'$ *(i.e.* $\mathcal{B}$*), while* HL *group choose contract* $\mathcal{M}'$ *iff* $k_H \geq f(k_L)$*. Any group choose contract* $\mathcal{N}'$ *operates as standard group and any group choose contract* $\mathcal{M}'$ *operates as bogus group.*

The above proposition shows that, when we assume that the borrowers in a standard group make repayment decision collectively so that there is no strategic interaction cost, the loan size of the contract designed for standard group is enlarged. Consequently, even when proper bogus group loan contract $\mathcal{M}'$ (i.e. $\mathcal{B}$) is provided, the homogenous ii group still prefer to be standard group. However, since heterogeneous HL group has an extra benefit (improvement in productivity) when operating as bogus group, such group prefer the one designed for bogus group as long as the heterogeneity in productivity is sufficiently high.

4 Conclusion

In this paper, we study the group-lending problem by considering the possibility of bogus group formation. There are three driving forces that influence the contract design and group type formation in our model.

The first one is the risk sharing benefit in a standard group: the probability that the group fulfills the joint liability is higher due to risk sharing in a standard group than that that in a bogus group, thus the mechanism of the standard group lending resembles a insurance that reduces the probability of losing the future credit value for each borrower.

The second one is the strategic interaction cost in a standard group: each member of a standard group makes the repayment decision independently so that each has the opportunity to default strategically while free ride on the partner's repayment action, thus the loans size for a standard group is reduced so that both borrowers will not default strategically; however, bogus group members act collectively and there is no such strategic interaction, thus the loan size of bogus loan contract is always larger.

The third one is the productivity improvement that exist in a bogus group with heterogeneous productivity: the average productivity of a HL group will increase form $\frac{1}{2}(k_H + k_L)$ to k_H if the group deviate from being a standard group to being a bogus group, since the bogus group will invest the entire loaned fund into the project with higher productivity k_H.

Given the trade-off among the above three factors, it is straightforward to understand the findings of our model. We find that if the the borrowers can freely choose to be either standard group or bogus group, bogus group will arise under the standard group loan contract when the heterogeneity of the borrowers' productivity is sufficiently high (condition (13)), and such bogus group will cause loss to the lender. This results from the trade-off between the risk sharing benefit of standard group and productivity improvement of bogus group.

We proposed optimal contracts to solve the bogus group problem in two complementary cases: either bogus group is allowed or not allowed to exist in equilibrium for some exogenous reasons discussed in the paper. The lender can exclude the existence of bogus group only by holding the interest rate at the level for standard group and reducing the loan size so that borrowers' incentive constraint to form bogus group does not hold. This method takes effect though the channel by holding the risk sharing benefit of standard group constant while reducing the benefit from improved productivity in bogus group. Of course, it is not optimal if we only consider the joint payoff of the borrowers in the group. If bogus group is allowed to exist, the lender provides appropriate contract that targets to either standard or bogus group, depending on the group's joint payoff under two forms of groups. We find that the welfare for each type of group can be enhanced if bogus group is allowed to exist because the loan size in a bogus group can be enlarged significantly given that bogus group removes the strategic interaction cost arise in the repayment stage.

We also considered the situation in which the lender is unable to observe the productivity of each borrower. It turns out that the mains findings are similar and basic intuition still hold. The welfare of each type of group increases if bogus group is allowed and addressed properly. And homogeneous ii group prefer to be bogus group if the absolute level of productivity is sufficiently high, while HL group prefer to be bogus group if the relative heterogeneity of productivity is sufficiently high, which also reflects the trade-off the three factors summarized above.

In addition, we also show that when we assume that the group members in a standard group make repayment decision collectively, so that the strategic interaction friction is ruled out, then the loan size for standard group becomes larger. However, the bogus group problem still exists and we show that the solutions to it is similar to those we proposed above. The only main difference is that, homogeneous ii group will never choose to be bogus group even when the lender provides the proper contract for bogus group. However, HL group may optimally choose to accept the contract designed for bogus group and consequently, the welfare of such group is enhanced compared with the case where bogus group is not allowed to exist.

Such results imply that by allowing the bogus group to exist in equilibrium and providing the "right" contract to them accordingly, we can enhance the welfare of the economy. Thus, bogus group is a bad news for the economy if we ignore the possibility of its existence or does not tolerate its existence, but is a good news if we can provide the proper contract for them to lead them to the efficient investment actions.

A Appendix

A.1 Proof of Proposition 1

Since the objective function (8) is decreasing in R, lender's participation constraint must be binding (the lender breaks even), i.e. $R = \frac{L}{p(2-p)}$. Therefore the optimization problem is reduced to be

$$\max_{L} \ (p(k_i + k_j) - 2)L + 2p(2-p)V, \tag{67}$$

$$\text{s.t. } L \leq p(1-p)V. \tag{68}$$

Since the objective function is increasing in L, the constraint (68) is binding. Thus the contract $\mathcal{S} = \{L_S, R_S\}$ given in the proposition 1 is optimal.

A.2 Proof of Proposition 2

We only need to consider the joint payoff of the group when being standard group and bogus group, because as long as the total surplus increases, the Nash bargaining will yield a Pareto improvement. Since $k_i \geq k_j$, the entire loans ($2L$) will be invested in the i type project. The group loan contract $\{L, R\}$ must satisfy the feasibility condition (2), then the bogus group has enough output $2k_iL$ when succeed to cover repayment $2R$ since $2k_iL > 2k_jL \geq 4R > 2R$. The bogus group will repay upon success if $2k_iL - 2R + 2V \geq 2k_iL$, or

$$R \leq V, \tag{69}$$

while default strategically otherwise. The LHS of (21) is the repayment conditional on project success (i.e., the marginal cost of not defaulting), while the RHS is the marginal benefit from future credit access, which increases from 0 to V due to choosing to repay. Therefore the joint payoff if they form a bogus group given the contract $\{L, R\}$ is

$$W_{ij}(L, R|B) = \begin{cases} 2pk_iL - 2pR + 2pV, & R \leq V \text{ (Repay);} \\ 2pk_iL, & R > V \text{ (Default).} \end{cases} \tag{70}$$

The group loan contract $\{L, R\}$ must satisfy the condition 4, given this the ij group will form a bogus group if and only if $W_{ij}(L, R|B) \geq W_{ij}(L, R|S)$, which is exactly the condition (11).

A.3 Proof of Corollary 3

Given the contract $\{L_S, R_S\}$, condition (11) is equivalent to $L_S > \frac{p(2-p)V}{1 + \frac{p(2-p)}{2(1-p)}(k_i - k_j)}$, or

$$k_i > k_j + \frac{2}{p(2-p)}. \tag{71}$$

If $i = j$, (71) is impossible to hold. Therefore, given the standard group loan contract $\{L_S, R_S\}$, bogus group will arise only if $i = H$ and $j = L$ and and the condition (13) holds. The joint payoff of the bogus group under contract S is

$$\begin{aligned} W_{HL}(L_S, R_S|B) =& 2p(k_HL_S - R_S + V) \\ =& 2\left(pk_H - \frac{1}{2-p}\right)p(1-p)V + 2pV \\ =& 2\left(\left(pk_H - \frac{1}{2-p}\right)(1-p) + 1\right)pV. \end{aligned}$$

However, the profit of the lender is

$$2pR_S - 2L_S = 2\left(p\frac{L_S}{p(2-p)} - L_S\right) = -\frac{2(1-p)}{2-p}L_S < 0,$$

so the bogus group causes loss to the lender.

A.4 Proof of Proposition 4

In this case, the break-even condition for the lender is

$$R = \frac{L}{p(2-p)}.$$

Then the constraint (4), (2) and (18) imply

$$L \leq p(1-p)V, \tag{72}$$

and

$$L \leq \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_i - k_j)}. \tag{73}$$

The above two constraints are equivalent to

$$L \leq \min\left\{p(1-p)V, \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_i - k_j)}\right\} = \begin{cases} p(1-p)V, & \text{if } i = j \\ \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_H - k_L)}, & \text{if } i = H, j = L \end{cases}. \tag{74}$$

Since objective function increases with L, the lender should provide the largest possible loan size, the optimal contract for ii group is still the contract $\mathcal{S}$, whereas that for HL group is given by (19).

A.5 Proof of Proposition 5

The maximization problem can be equivalently broken down into a two stage maximization problem. In the first stage, given $\tau \in \{0, 1\}$, we solve

$$W_{ij}(\tau) = \max_{L,R} W(L, R, \tau), \tag{75}$$

$$\text{s.t. } (21), (22), (23). \tag{76}$$

In the second stage, we solve

$$W_{ij}^{\#} = \max_{\tau} W_{ij}(\tau). \tag{77}$$

Consider the first stage problem. Given $\tau = 1$, we have

$$W_{ij}(1) = \max_{L,R} W_{ij}\left(L, \frac{L}{p(2-p)}|S\right), \tag{78}$$

subject to

$$L \leq p(1-p)V, \tag{79}$$

$$W_{ij}\left(L, \frac{L}{p(2-p)}|S\right) \geq W_{ij}\left(L, \frac{L}{p(2-p)}|B\right), \tag{80}$$

where

$$W_{ij}\left(L, \frac{L}{p(2-p)}|S\right) = (p(k_i+k_j)-2)L + 2p(2-p)V,$$
$$W_{ij}\left(L, \frac{L}{p(2-p)}|B\right) = 2\left(pk_i - \frac{1}{2-p}\right)L + 2pV.$$

The constraint (80) is equivalent to

$$L \leq \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_i-k_j)}, \tag{81}$$

Since the objective is increasing in L, we have

$$L_{ij}^{\#}(1) = \min\left\{p(1-p)V, \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_i-k_j)}\right\} = \begin{cases} p(1-p)V, & \text{if } i=j \\ \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_H-k_L)}, & \text{if } i=H, j=L \end{cases} \tag{82}$$

and

$$\begin{aligned} W_{ij}(1) =& (p(k_i+k_j)-2)L_{ij}^*(1) + 2p(2-p)V \\ =& \begin{cases} 2(p(1-p)k_i+1)pV, & \text{if } i=j \\ \frac{k_H+k_L+\frac{(2-p)}{(1-p)}(k_H-k_L)}{1+\frac{p(2-p)}{2(1-p)}(k_H-k_L)}p^2(2-p)V, & \text{if } i=H, j=L \end{cases} \end{aligned} \tag{83}$$

Given $\tau=0$, we have

$$W_{ij}(0) = \max_{L,R} W_{ij}\left(L, \frac{L}{p}|B\right), \tag{84}$$

subject to

$$L \leq pV, \tag{85}$$
$$W_{ij}\left(L, \frac{L}{p}|B\right) \geq W_{ij}\left(L, \frac{L}{p}|S\right), \tag{86}$$

where

$$W_{ij}\left(L, \frac{L}{p}|B\right) = 2(pk_i-1)L + 2pV,$$

$$W_{ij}\left(L, \frac{L}{p}|S\right) = \begin{cases} (p(k_i+k_j)-2(2-p))L + 2p(2-p)V, & L \leq \frac{p(1-p)}{2-p}V \text{ (Repay,Repay)}; \\ (p(k_i+k_j)-2)L + 2pV, & \frac{p(1-p)}{2-p}V < L \leq \frac{pV}{2} \text{ (Repay,Default)}; \\ p(k_i+k_j)L, & L > \frac{pV}{2}. \text{ (Default,Default)} \end{cases} \tag{87}$$

Since the objective is increasing in L, we have

$$L_{ij}^*(0) = pV, \tag{88}$$

and

$$W_{ij}(0) = 2p^2k_iV. \tag{89}$$

Then consider the second stage problem (77), we have$\tau^*=0$ iff

$$W_{ij}(0) > W_{ij}(1) \Leftrightarrow \begin{cases} k_i > \frac{1}{p^2}, & \text{if } i=j \\ k_H > f(k_L), & \text{if } i=H, j=L \end{cases}, \tag{90}$$

while $\tau^*=1$ otherwise.

A.6 Proof of Lemma 1

Before proving Lemma 1, we first prove the following lemma.

Lemma 2. *Conditional on the constraint (38), (39) and (36), we can increase L_M holding L_N constant so that the constraint (36) still holds.*

Proof. Conditional on the constraint (38), (39), we have $W\left(L_M, \frac{L_M}{p}|S\right) \geq W\left(L_M, \frac{L_M}{p}|B\right)$ if and only if $L_M \leq L_M^* \equiv \min\left\{\frac{2p(1-p)V}{p(k_i-k_j)+2(1-p)}, \frac{p(1-p)V}{2-p}\right\}$.

In the regime where $L_M \leq L_M^*$, we have

$$W\left(L_M, \frac{L_M}{p}|S\right) = (p(k_i+k_j) - 2(2-p))L_M + 2p(2-p)V \tag{91}$$

Consequently,

$$\frac{\mathrm{d}W\left(L_M, \frac{L_M}{p}|B\right)}{\mathrm{d}L_M} = 2(pk_i - 1) > p(k_i+k_j) - 2(2-p) = \frac{\mathrm{d}W\left(L_M, \frac{L_M}{p}|S\right)}{\mathrm{d}L_M} \tag{92}$$

Therefore, by increasing L_M holding L_N constant, the LHS of the constraint (36) increases weakly faster than the RHS of it, so the constraint (36) will still hold.

In the regime where $L_M > L_M^*$, the constraint (36) is equivalent to

$$\max\left\{W\left(L_N, \frac{L_N}{p(2-p)}|S\right), W\left(L_M, \frac{L_M}{p}|B\right)\right\} \geq W\left(L_N, \frac{L_N}{p(2-p)}|B\right) \tag{93}$$

Notice that $W\left(L_M, \frac{L_M}{p}|B\right)$ is increasing in L_M, hence, by increasing L_M holding L_N constant, the LHS of (93) weakly increases while the RHS of it remains constant, i.e., both the precondition $L_M \geq L_M^*$ and the constraint (93) will still hold, which means that the constraint (36) will still hold.

Combining the analysis above, we achieve the argument as desired. □

According to Lemma 1, suppose $L_M < pV$, we can increase L_M holding L_N constant, so that the constraint (36) still holds. However, since the $W\left(L_M, \frac{L_M}{p}|B\right)$ is increasing in L_M, the objective function weakly increases, therefore $L_M = pV = L_B$ is optimal. Thus the optimal contract $\mathcal{M}$ should be $\mathcal{B} = \{L_B, R_B\}$.

A.7 Proof of Proposition 7

Since

$$W_{ij}(\mathcal{N}|S) \geq W_{ij}(\mathcal{N}|B) \Leftrightarrow L_N \leq L_1(ij) \equiv \frac{p(2-p)V}{1 + \frac{p(2-p)}{2(1-p)}(k_i - k_j)} \tag{94}$$

and

$$W_{ij}(\mathcal{M}|B) \geq W_{ij}(\mathcal{N}|B) > W_{ij}(\mathcal{N}|S) \Leftrightarrow L_1(ij) < L_N \leq L_2(ij) \equiv \frac{pk_i - 1}{pk_i - \frac{1}{2-p}}pV, \tag{95}$$

the condition (46) is equivalent to

$$L_N \leq \max\{L_1(ij), L_2(ij)\}, \forall ij \in \{HH, HL, LL\} \tag{96}$$

Notice that the above condition should hold for all $ij \in \{HH, HL, LL\}$, thus it is equivalent to

$$L_N \leq \max\{L_E, L_F\} \tag{97}$$

where $L_E = L_1(HL) = \frac{p(2-p)V}{1+\frac{p(2-p)}{2(1-p)}(k_H-k_L)}$ and $L_F = L_2(HL) \equiv \frac{pk_H-1}{pk_H-\frac{1}{2-p}}pV$.

Combining the condition (39) and (96) together yields

$$L_N \leq \min\{L_S, \max\{L_E, L_F\}\} = \begin{cases} L_S, & k_H > \frac{1}{p^2(2-p)} \text{ (case 1)} \\ L_F, & f(k_L) \leq k_H \leq \frac{1}{p^2(2-p)} \text{ (case 2)} \\ L_E, & k_H < f(k_L) \text{ (case 3)} \end{cases} \tag{98}$$

Notice that the objective function is weakly increasing in L_N, so the optimal values will be the largest possible while satisfying all constraints, i.e. $L_N^* = \min\{L_S, \max\{L_E, L_F\}\}$ is optimal. So the optimal contract $\mathcal{N}$, designed for standard group is (i) $\mathcal{S} = \{L_S, R_S\}$ if $k_H > \frac{1}{p^2(2-p)}$, (ii) $\mathcal{E} = \{L_E, R_E\}$ if $k_H < f(k_L)$, or (iii) $\mathcal{F} = \{L_F, R_F\}$ if $f(k_L) \leq k_H \leq \frac{1}{p^2(2-p)}$.

For HL group, by deviating from contract $\mathcal{N}$ to contract $\mathcal{M}$ (i.e. $\mathcal{B}$), the productivity and refinancing probability remains the same, the group faces the trade-off between larger loan size of contract $\mathcal{M}$ and lower interest rate of contract $\mathcal{N}$. From (95), we know that L_F is the loan size such that the HL group is indifferent between the two contracts, hence, HL group strictly prefer contract $\mathcal{M}$ in case 1 and weakly prefer contract $\mathcal{M}$ in case 2, but strictly prefer contract $\mathcal{N}$ in case 3. In summary, HL group prefer contract $\mathcal{M}$ if $L_F \geq L_E$, which is equivalent to the condition $k_H \geq f(k_L)$ in proposition 7.

According to (50), the ii group prefer contract $\mathcal{M}$ if

$$L_N < \frac{pk_i - (2-p)}{pk_i - 1}pV \tag{99}$$

where L_N is given as in equation (98). If $L_F \geq L_S$ (i,e, $\frac{pk_H-1}{pk_H-\frac{1}{2-p}} > 1-p$), then $L_N^* = L_S$, the ii group prefer contract $\mathcal{M}$ only if $1-p < \frac{pk_i-(2-p)}{pk_i-1} \Leftrightarrow k_i \geq \frac{1}{p^2}$.

If $L_E \leq L_F < L_S$, then $L_N^* = L_F$, the condition (99) becomes $k_i \geq (2-p)k_H$, which is impossible to hold. Therefore, ii group must strictly prefer contract $\mathcal{N}$ in this case. The intuition is as follows. Given the contract pair, if a ii group deviate from contract $\mathcal{N}$, under which it will be standard group, to contract $\mathcal{M}$, besides the trade-off between larger loan size and lower interest rate, such group also suffer the decrease in the probability of getting the refinancing opportunity, which deceases from $p(2-p)$ to p, that is, the net gain of such a deviation for ii group is always less than that for HL group. Notice that L_F is the threshold that makes HL feel indifferent between the two contracts, then ii group must strictly prefer contract $\mathcal{N}$.

If $L_F < L_E$, then $L_N^* = L_F$, the condition (99) is also impossible to hold. The intuition above also applies: in this case even HL group prefer contract $\mathcal{N}$, then based on the augment above, ii group also prefer contract $\mathcal{N}$.

In conclusion, ii group prefer contract $\mathcal{M}$ if and only if $L_N^* = L_S$ and $k_i \geq \frac{1}{p^2}$, or equivalently,

$$\left(\frac{pk_H - 1}{pk_H - \frac{1}{2-p}} > 1-p\right) \wedge \left(k_i \geq \frac{1}{p^2}\right) \Leftrightarrow \left(k_H > \frac{1}{p^2(2-p)}\right) \wedge \left(k_i \geq \frac{1}{p^2}\right) \Leftrightarrow k_i \geq \frac{1}{p^2} \tag{100}$$

A.8 Allow the (Repay,Default) equilibrium for standard group

In this section, we suppose that the lender is allowed to provide a group loan contract in the (Repay,Default) regime for the standard group. The joint payoff $W_{ij}(L,R|S)$ of the standard group given the contract $\{L,R\}$ is

$$W_{ij}(L,R|S)=\begin{cases} p(k_i+k_j)L-2p(2-p)R+2p(2-p)V, & R\le \frac{1-p}{2-p}V \text{ (Repay,Repay);}\\ p((k_i+k_j)L-2R+2V), & \frac{1-p}{2-p}V<R\le\frac{V}{2}; \text{ (Repay,Default)}\\ p(k_i+k_j)L, & R>\frac{V}{2}. \text{ (Default,Default)}\end{cases} \tag{101}$$

The ex-ante probability $\phi(L,R)$ that the lender receives $2R$ is

$$\phi(L,R)=\begin{cases} p(2-p), & R\le \frac{1-p}{2-p}V \text{ (Repay,Repay);}\\ p, & \frac{1-p}{2-p}V<R\le\frac{V}{2}; \text{ (Repay,Default)}\\ 0, & R>\frac{V}{2}. \text{ (Default,Default)}\end{cases} \tag{102}$$

Thus, the lender's participation constant is

$$2\phi(L,R)R-2L\ge 0 \tag{103}$$

$$\Leftrightarrow R\ge\frac{L}{\phi(L,R)}=\begin{cases}\frac{L}{p(2-p)}, & R\le\frac{1-p}{2-p}V \text{ (Repay,Repay);}\\ \frac{L}{p}, & \frac{1-p}{2-p}V<R\le\frac{V}{2}; \text{ (Repay,Default)}\end{cases} \tag{104}$$

Thus, the lender can offer a contract either in the (Repay, Repay) regime or in the (Repay, Default) regime. Assuming zero profits for the lender because of free entry or because of the lenders' mission, as explained above, the optimal standard group loan contract can be defined as the loan size and repayment $\{L_G,R_G\}$ which maximize the joint payoff of a standard group, that is, the contract $\{L_G,R_G\}$ which solves:

$$\max_{L,R}\ W(L,R|S) \tag{105}$$

$$\text{s.t. (2) and (104)}$$

The two constraints (2) and (104) can be reduced as

$$\frac{L}{\phi(L,R)}\le R\le\frac{1}{2}k_jL \tag{106}$$

which is non-empty only if

$$k_j\ge\frac{2}{\phi(L,R)}=\begin{cases}\frac{2}{p(2-p)}, & R\le\frac{1-p}{2-p}V \text{ (Repay,Repay);}\\ \frac{2}{p}, & \frac{1-p}{2-p}V<R\le\frac{V}{2}; \text{ (Repay,Default)}\end{cases} \tag{107}$$

Then we obtain the following proposition.

Proposition 12. *The optimal standard group loan contract $\mathcal{G}=\{L_G,R_G\}$ is $\mathcal{S}=(L_S,R_S)$ and the Nash-equilibrium in repayment stage is (Repay, Repay) if (i) $\frac{2}{p(2-p)}\le k_j<\frac{2}{p}$, or (ii) $k_L\ge\frac{2}{p}$, $p\le\frac{1}{2}$, or (iii) $k_L\ge\frac{2}{p}$, $\frac{1}{2}<p\le\frac{3}{4}$, $p(p-\frac{1}{2})(k_i+k_j)\le 1$; the optimal standard group loan contract $\mathcal{G}=\{L_G,R_G\}$ is is $\mathcal{H}=\{L_H,R_H\}$ and the Nash-equilibrium in repayment stage is (Repay, Default) if (d) $k_L\ge\frac{2}{p}$, $\frac{1}{2}<p\le\frac{3}{4}$, $p(p-\frac{1}{2})(k_i+k_j)>1$, or (e) $k_L\ge\frac{2}{p}$, $p>\frac{3}{4}$, where$L_H=\frac{pV}{2}$ and $R_H=\frac{V}{2}$.*

Intuitively, in the (Repay, Default) regime ($\frac{1-p}{2-p}V < R \leq \frac{V}{2}$), deviating from forming a standard group to forming a bogus group does not change the repayment probability (equal to p in both cases), thus does not change the expected joint refinancing value (equal to $2pV$ in both cases), but it increases the expected joint output from $p(k_i + k_j)L$ to $2pk_iL$, thus it is always better for the borrowers to be a bogus group in this regime.

Even though we show here that contract $\mathcal{H}$ may arise in the standard group lending practice when the bogus group problem is assumed away, it will never arise when we allow the existence of bogus group and design optimal contract(s) because any such contract will be dominated by the contract designed for the bogus group, see section 3.3.2 and 3.4.2.

References

Ahlin, C. (2012). Group lending under adverse selection with local information: The role of group size. *Mimeo*. 2

Ahlin, C. and Waters, B. (2011). Dynamic lending vs. group lending in the ghatak/stiglitz/weiss adverse selection model. *Mimeo*. 1, 2

Armendariz de Aghion, B. (1999). On the design of a credit agreement with peer monitoring. *Journal of Development Economics*, 60(1):79–104. 2, 3, 6

Besley, T. and Coate, S. (1995). Group lending, repayment incentives and social collateral. *Journal of Development Economics*, 46:1–18. 3

Chowdhury, P. R. (2005). Group-lending: Sequential financing, lender monitoring and joint liability. *Journal of Development Economics*, 77:415–439. 1

Ghatak, M. and Guinnane, T. (1999). The economics of lending with joint liability: theory and practice. *Journal of development economics*, 60(1):195–228. 2, 3

Karlan, D. S. (2005). Social connection and group banking. Working Paper. 3

Morduch, J. (1999). The microfinance promise. *Journal of Economic Literature*, 37:1569–1614. 1

Mosley, P. (1986). Risk, insurance and small farm credit in developing countries: A policy proposal. *Public Administration and Development*, 6(3):309–319. 1

Udry, C. (1990). Credit markets in northern nigeria: Credit as insurance in a rural economy. *The World Bank Economic Review*, 4(3):251–269. 1