On the Tradeoff between the Sensitivity Effect and the Informativeness Effect in Executive Compensation

Hui Ou-Yang and Weili Wu*

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*Ou-Yang is with Cheung Kong Graduate School of Business and Wu is with Peking University. E-mails: houyang@ckgsb.edu.cn and wuweili@pku.edu.cn. We thank Li Liu, Qi Liu, Zheng Zhang, and seminar participants at Central University of Finance and Economics and Peking University for their advice.

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Abstract

We employ an equilibrium model of agency and asset pricing under asymmetric information. We find that the optimal pay-performance sensitivity (PPS) decreases with the sensitivity of the stock price to the agent's effort but increases with the informativeness of the stock price about the agent's effort. Empirical studies have found both positive and negative relations between the PPS and the stock price informativeness, between the PPS and the stock price volatility, between the PPS and the firm value, as well as a positive relation between the PPS and the stock liquidity. The tradeoff between the sensitivity effect and the informativeness effect allows us to explain all of these empirical results in a consistent manner.

1 Introduction

Many empirical studies have tested the impact of financial markets on executive contracts. For example, Ferreira et al. (2012) find that the pay-performance sensitivity (PPS) is negatively related to the stock price informativeness, but Kang and Liu (2010) and Firth et al. (2014) document a positive relation between them. Similarly, Hartzell and Starks (2003) find a positive relation between the PPS and institutional ownership. Moreover, Jayaraman and Milbourn (2011) find a positive relation between the PPS and market liquidity. Empirical studies further find both positive and negative relations between the PPS and the stock price volatility and between the PPS and the firm performance.[1] These studies present challenges to the current theoretical literature on agency, it is thus of great importance to present a theoretical model that can address these empirical issues in a consistent manner.

To provide self-consistent explanations for these empirical findings, one must utilize an equilibrium model of agency and asset pricing under asymmetric information. Holmstrom and Tirole (HT, 1993) have pioneered such a model.[2] Because HT do not specify the cost functions for the manager's effort and the speculator's information acquisition effort, they are unable to solve the optimal contract explicitly. By adopting the commonly used quadratic cost functions, we solve for the optimal contract and other properties of the HT model. We find that the risk-neutrality of both the market maker and the speculator in the original HT model prevents us from explaining the empirical results consistently. Therefore, we extend the HT model to incorporate a risk-averse market maker.[3]

Specifically, we adopt a simplified version of the HT model in which there are three periods. At time 0, a risk-neutral entrepreneur hires a risk-averse manager to run a firm, whose effort at time 0 affects the expectation of the firm's payoff which realizes at time 2. The payoff contains two random shocks which occur at time 1 and 2, respectively. The entrepreneur sells the firm in the stock market at time 1. The stock price is determined by a Kyle-type (1985) model, in which

[1] We shall present a comprehensive list of references on these two relations in Section 4.

[2] See, for example, Ou-Yang (2005), Bolton, Scheinkman, and Xiong (2006), and Goldman and Slezak (2006) for other integrated models of agency and asset pricing.

[3] We obtain similar results using a risk-averse speculator.

the risk-neutral speculator first acquires private information about the firm's payoff by spending costly effort and then trades strategically taking the price impact of his trade into account. As in Bolton, Scheinkman, and Xiong (2006), the compensation contract is linear in both the short-term stock price at time 1 and the long-term firm's payoff at time 2, and the sum of the incentives on the stock price and the payoff is interpreted as the PPS.

We identify two effects that determine the optimal contract: the sensitivity effect and the informativeness effect. To explain these two effects, suppose that a linear contract is based on a signal about the manager's effort, then the optimal PPS on this signal depends on two factors. The first is the sensitivity of the signal to the manager's effort, which we term *the sensitivity of the signal*. The purpose of setting the incentive contract is to relate the manager's compensation to his effort. Therefore, when the sensitivity of the signal decreases, to maintain the sensitivity of the manager's compensation to his effort, the optimal PPS on the signal will increase accordingly. We term this result *the sensitivity effect*. The second is the informativeness of the signal about the manager's effort or the extent to which the signal reflects manager's effort. When the signal provides more precise information about the manager's effort, the optimal PPS on the signal increases, which we term *the informativeness effect*. This effect is in the spirit of the informativeness principle (Holmstrom, 1979; Shavell, 1979; Gjesdal, 1982; Grossman and Hart, 1983; Kim, 1995).[4] Moreover, we find that the manager's optimal effort increases with the informativeness of the signal about the manager's effort but it is independent of the sensitivity of the signal.

We illustrate that a key consequence of combining agency with equilibrium asset pricing under asymmetric information is that both the sensitivity of the stock price to and the informativeness of the stock price about the manager's effort are endogenous, and they usually move in the same direction. We find that the PPS increases with the informativeness of the stock price about the manager's effort, due to the informativeness effect, but decreases with the sensitivity of the stock price, due to the sensitivity effect. As a result, a tradeoff between the sensitivity effect and the

[4]It argues that the principal should maximize the precision of the performance measure used to evaluate the agent by including additional signals about the agent's effort in the compensation contract. See Chaigneau, Edmans, and Gottlieb (2014) for recent discussions of the informativeness principle on related issues.

informativeness effect arises.

The stock price is determined by market participants' beliefs about the firm's payoff. Market participants do not observe the manager's effort, which determines the expectation of the firm's payoff, so they have to form priori beliefs about the manager's effort. Furthermore, the market maker can obtain an estimator of the manager's effort based on the total order flow, which is the sum of the informed trading and the noise trading. The estimator based on the total order flow does not perfectly reveal the payoff, so the market maker puts some weight on the priori beliefs about the manager's effort when he sets the price. Then, the stock price can be expressed as a weighted average of market participants' priori beliefs about the manager's effort and the market maker's estimator of the manager's effort. Because market participants' priori beliefs about the manager's effort is not influenced by the manager, the sensitivity of the stock price to the manager's effort equals the weight of the market maker's estimator of the manager's effort.

When the total order flow contains more precise information about the manager's effort, the market maker usually puts a larger weight on his estimator on the manager's effort, which increases the sensitivity of the stock price. Consequently, when the change in an exogenous parameter leads to an increase in the stock price informativeness about the manager's effort, the sensitivity of the stock price to the manager's effort usually increases as well. The former increases the PPS, but the latter decreases it. Consequently, there exists a tradeoff between the sensitivity effect and the informativeness effect.

The tradeoff between these two effects allows us to derive a number of interesting results on the relations between market variables and the PPS, providing potential explanations for the aforementioned empirical findings.

We obtain both positive and negative relations between the PPS and the stock price informativeness about the stock payoff. The negative relation is seemingly in contrast with the conventional intuition. Note that the stock price informativeness about the payoff is usually proportional to the stock price informativeness about the manager's effort. Hence, when the stock price is more informative about the stock payoff, the stock price is usually more sensitive to the effort of the manager.

The former increases the PPS due to the informativeness effect, but the latter reduces the PPS due to the sensitivity effect. When the second risk of the payoff changes, the sensitivity effect dominates the informativeness effect, so the PPS decreases with the stock price informativeness about the stock payoff, which is consistent with the empirical result in Firth et al. (2014). When the information acquisition cost changes, the informativeness effect dominates the sensitivity effect, so the PPS increases with the stock price informativeness, which is consistent with Hartzell and Starks (2003), Kang and Liu (2010), and Ferreira et al. (2012). As a robustness check, we also obtain both positive and negative relations between the PPS and the fraction of informed trading.[5]

We use the Kyle λ to measure the illiquidity of the stock. Because both the PPS and the stock liquidity are endogenous in our equilibrium model, the relation between them is driven by exogenous parameters. When the second risk of the stock payoff changes, we find a positive relation between them, which is consistent with the empirical result of Jayaraman and Milbourn (2011). When the second risk of the stock payoff increases, the risk faced by the market maker increases. As a result, he increases the price impact λ or the market liquidity decreases. Meanwhile, when the second risk of the stock payoff increases, the informativeness effect dominates the sensitivity effect, so the PPS decreases with the second risk of the stock payoff. Therefore, we achieve a positive relation between the PPS and the liquidity.

In classical agency models, the compensation contract is based on the payoff of a project and the sensitivity of the payoff to the agent's effort is an exogenous constant. An increase in the risk of the payoff decreases its informativeness about the agent's effort, but does not affect the sensitivity, so there is no tradeoff between the two effect. Hence, the relation between the PPS and the risk of the payoff is negative. A higher PPS also leads to a higher expected value of the project. Empirically, however, both positive and negative relations between the PPS and the stock price volatility and between the PPS and the firm value have been obtained. The tradeoff between the

[5] We define the fraction of informed trading as the standard deviation of the speculator's demand over the standard deviation of the total order flow, which is similar to the probability of informed trading (PIN) in Easley et al. (1996). They and many others use the PIN measure as a proxy for the stock price informativeness. Bartov et al. (2000), Gibson et al. (2004), Nagel (2005), and Boehmer and Kelley (2009) find that institutional ownership improves stock price efficiency. If we interpret the informed speculator as the institutional investors, then the informed trading can be used as a proxy for the institutional trading as in Hartzell and Starks (2003).

two effects under a risk-averse market maker allows us to explain these two long standing puzzles about the relations between the PPS and the stock price volatility and between the PPS and the firm performance.

Because both the stock price volatility and the PPS are endogenous, the relation between them is driven by exogenous parameters. For example, when the cost of information acquisition changes, we can obtain a positive relation between the PPS and the stock price volatility. Specifically, when it increases, the speculator's effort decreases. As a result, both the sensitivity of the stock price and its informativeness about the manager's effort decrease. For the PPS, the informativeness effect dominates the sensitivity effect, so the PPS decreases with the cost of information acquisition. Besides, the stock price volatility increases with the sensitivity of the stock price but decreases with the informativeness of the stock price about the manager's effort. For the stock price volatility, the sensitivity effect dominates the informativeness effect, so the stock price volatility also decreases with the cost of information acquisition. Therefore, a positive relation between the PPS and the stock price volatility arises. The negative relation can also be obtained by changing other exogenous parameters.

As for the relation between the PPS and the firm performance, if the sensitivity of a signal is fixed, as assumed in traditional agency models, then an increase in the informativeness of the signal about the manager's effort will increase both the optimal PPS and the manager's optimal effort. This results in a positive relation between the expected payoff and the PPS. Because of the tradeoff between the sensitivity effect and the informativeness effect, this conclusion does not always hold. As we have stated, the manager's optimal effort depends only on the informativeness of the signal about the manager's effort, but the PPS depends on both the informativeness and the sensitivity of the signal. When the informativeness of the stock price about the manager's effort increases, which increases both the manager's effort and the PPS, the sensitivity of the stock price usually increases at the same time, which decreases the PPS. For the PPS, the sensitivity effect sometimes dominates the informativeness effect, leading to a negative relation between the PPS and the manager's effort (the expected firm value).

A few papers have attempted to interpret the mixed findings on the relation between the PPS and the stock price volatility (Jin, 2002; Prendergast, 2002; Guo and Ou-Yang, 2006; Cao and Wang, 2013; He et al., 2013) and the relation between the PPS and the firm performance (Guo and Ou-Yang, 2006). For example, Prendergast (2002) obtains a positive relation between the PPS and the payoff volatility. He accounts for an effect of uncertainty on incentives with the possibility of monitoring and delegation. The marginal returns to delegation are likely lower in more risky environments, as a principal may have little idea about the right actions to take. Therefore, higher incentives are needed to induce increased effort from an agent. In a more stable environment, a principal may be able to monitor an agent's input so that high incentives are unnecessary. All these papers do not contain an equilibrium asset pricing model, so they are unable to explain the empirically tested relations between the PPS, the stock price volatility, and other market variables.

In sum, this paper highlights the sensitivity effect and the informativeness effect in executive compensation. Because both effects are endogenously determined in our model, the tradeoff between them allows us to derive the relations between the PPS and various market variables. The paper is the first to provide consistent explanations for the empirical results on the relations between the PPS and the stock price informativeness, between the PPS and the stock liquidity, between the PPS and the stock price volatility, and between the PPS and the firm value.

2 Model

We present a three-period integrated model of principal-agent and asset pricing by simplifying the HT model but extending it to the case of a risk-averse market maker. At time 0, an entrepreneur (the principal) hires a manager (the agent) to run an all-equity firm, and specifies the manager's compensation contract, based on which the manager chooses his effort level. At time 1, the entrepreneur sells the firm in a stock market. The firm is still under operation by the manager, and its payoff remains uncertain. At time 2, the firm's payoff is realized and fully recognized by the stock market. The details of each period are described below.

2.1 Entrepreneur and manager

At time 0, a risk-neutral entrepreneur sets the compensation contract, and hires a risk-averse manager. Given the contract, the manager chooses the level of an unobservable effort e to devote. Following HT, we assume that the manager's effort e affects the firm's payoff at time 2, denoted as v, according to

$$v = e + \theta + \delta, \tag{1}$$

where $\theta \sim N(0, \sigma_\theta^2)$ and $\delta \sim N(0, \sigma_\delta^2)$ are two independent random shocks affecting the firm's payoff, which are beyond the control of the manager. Especially, the random shock θ occurs at time 1 and the random shock δ occurs at time 2. The risk of the payoff is $\mathrm{Var}(v) = \sigma_\theta^2 + \sigma_\delta^2$, where σ_θ is the first risk and σ_δ is the second risk.

At time 1, the entrepreneur sells the whole firm in the stock market at price P.[6] The mechanism, by which the stock price of the firm is determined, will be specified in the next subsection. We assume that the manager's wealth at time 2, denoted as W_m, derives solely from the compensation. Following Bolton, Scheinkman, and Xiong (2006), the compensation contract includes both the short-term stock price P, and the long-term stock price v, and takes the linear form:

$$W_m(P, v) = a + b_1 P + b_2 v, \tag{2}$$

where a, b_1, and b_2 are the parameters set by the entrepreneur at time 0. The contract can be interpreted as meaning that the entrepreneur promises the manager cash flow $a + (b_1 + b_2)v$ at time 2, in which a is the manager's fixed compensation, $b_1 v$ can be sold by the manager at time 1 but $b_2 v$ must be held till the end. Note that the price of v at time 1 is P, so the wealth of the manager at time 2 takes the form of the contract. We can interpret $(b_1 + b_2)$ as the total number of shares that the manager receives over the two periods. As a result, $(b_1 + b_2)$ corresponds to the PPS.

The manager has a negative exponential utility U_m:

$$U_m = -\exp\{-R_m[W_m - C_m(e)]\}, \quad R_m > 0, \tag{3}$$

[6] This assumption is for simplicity. It will not affect the results in our paper, even if we assume that the entrepreneur sells an arbitrary fraction of the firm.

where R_m is the absolute risk aversion coefficient of the manager, and $C_m(e) = k_m e^2/2$ is the manager's cost of exerting effort e, with k_m being a positive constant. Without loss of generality, the manager's reservation utility is assumed to be zero.

For simplicity, we assume that all contractual compensation payments made to the manager derive from the entrepreneur. Hence, after the entrepreneur pays W_m to the manager, the terminal wealth of the entrepreneur, denoted as W_p, at time 2 is given by

$$W_p = P - W_m. \tag{4}$$

In addition, traders (shareholders) in the stock market obtain the realization of v multiplying their holding proportion of the firm at time 2.

2.2 Determination of the stock price

At time 1, the entrepreneur sells the firm at price P in the stock market. Following HT, we adopt the Kyle (1985) model to determine the stock price, with a risk-averse market maker, as adopted by Subrahmanyam (1991).

Specifically, we consider a market with a speculator, a risk-averse market maker, and liquidity traders. They buy and sell the firm at price P, determined by the market maker. The demand of liquidity traders for the firm is $u \sim N(0, \sigma_u^2)$. The speculator can choose the extent to which he is informed through an endogenous information acquisition process. Following HT, we assume that after exerting an unobservable effort ρ, the speculator observes a noisy signal, s, about $e + \theta$:

$$s = e + \theta + \epsilon, \tag{5}$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ is the noise term and uncorrelated with θ, and σ_ϵ^2 is inversely related to the speculator's effort ρ, satisfying $\sigma_\epsilon^2 = \sigma_\theta^2/\rho$. We assume that the cost of exerting effort ρ for the speculator is $C_s(\rho) = k_s \rho^2/2$, where k_s is a positive constant. The speculator submits a market order, based on his private information s, and his trade, denoted by x, is a function of s.

Following Subrahmanyam (1991), we assume that there is only one market maker. The market maker has a negative exponential utility function U_k:

$$U_k(W_k) = -\exp(-R_k W_k), \quad R_k > 0, \tag{6}$$

where R_k is his absolute risk-aversion coefficient, and W_k is his wealth at time 2. The market maker takes the total order flow $(x + u)$ and sets the stock price based on this information. Because of the existence of potential competitors, the market maker sets the price according to the condition that his certainty equivalent profit is zero, or it is indifferent for him to be the market maker or not. That is,

$$\mathbb{E}[U_k(P)|x + u] = -1. \tag{7}$$

Note that all participants (the market maker and the speculator) in the stock market do not observe manager's effort e, so they have to make decisions based on their priori beliefs about it, denoted as $\widehat{e}$.[7]

2.3 Timeline and steps of solving the model

We summarize the timeline of the model as follows.

1. In Stage 1, the entrepreneur sets a linear contract $W_m(P, v) = a + b_1 P + b_2 v$ to the firm manager. The contract is publicly announced. $(t = 0)$

2. In Stage 2, the participants of the stock market believe that the firm manager's effort is $\widehat{e}$, based on the compensation contract $W_m(P, v)$. They are committed to this belief, which turns out to be correct in equilibrium (i.e., they have rational expectations). $(t = 0)$

3. In Stage 3, given the compensation contract and taking into account the beliefs held by the participants of the stock market, $\widehat{e}$, the firm manager chooses the optimal effort e^*, which is unobservable to the entrepreneur and the market participants. $(t = 0)$

4. In Stage 4, the entrepreneur sells the firm in the stock market at price P. $(t = 1)$

5. In Stage 5, the market maker believes that the speculator would exert effort ρ_m for information acquisition. He is committed to this belief, which turns out to be correct in equilibrium (i.e., he has rational expectations). $(t = 1)$

[7] They form their priori beliefs based on the same information: the compensation contract, so their priori beliefs are identical.

6. In Stage 6, taking into account the belief held by the market maker, ρ_m, the speculator exerts effort $\rho^*(\rho_m)$ and obtains a signal $s(\rho^*)$. ($t = 1$)

7. In Stage 7, the speculator chooses the optimal trading strategy x based on the realized signal s and submits his market order x to the market maker. ($t = 1$)

8. In Stage 8, the market maker determines the stock price P based on the total order flow $(x + u)$ and his beliefs about both the speculator's effort ρ_m and the firm manager's effort $\widehat{e}$. ($t = 1$)

9. In Stage 9, the firm's payoff v is realized. ($t = 2$)

We solve the model using backward induction.

1. Step 1: In Stage 8, the market maker sets the stock price according to the condition that he earns zero certainty equivalent profit. Given the total order flow $(x+u)$ and the compensation contract $W_m(P, v)$, he sets the price based on his beliefs about the speculator's effort ρ_m and the firm manger's effort $\widehat{e}$. The stock price P is determined according to:

$$\mathbb{E}[U_k(P)|x + u, \rho_m, \widehat{e}] = -1. \tag{8}$$

2. Step 2: In Stage 7, the speculator solves for the optimal trading strategy. After having exerted effort ρ^* and obtained signal s, given the market maker's belief (ρ_m), the speculator's optimal trading strategy x maximizes his expected utility, that is,

$$x(s) = \underset{x}{\operatorname{argmax}}\ \mathbb{E}[U_s(x)|s(\rho^*), \rho_m]. \tag{9}$$

3. Step 3: In Stage 6, given the optimal trading strategy $x(s)$ obtained in Step 2 and the market maker's belief ρ_m, the speculator chooses the optimal ρ^* to maximize his expected utility.

$$\rho^*(\rho_m) = \underset{\rho}{\operatorname{argmax}}\ \mathbb{E}(U_s). \tag{10}$$

4. Step 4: In Stage 5, the market maker forms rational expectations. That is, his belief ρ_m coincides with the speculator's optimal effort choice ρ^*:

$$\rho^*(\rho_m) = \rho_m. \tag{11}$$

5. Step 5: In Stage 3, given the compensation contract and the equilibrium stock price function obtained in Step 1 to Step 4, the firm manager's optimal effort e^* satisfies

$$e^*(a, b_1, b_2) = \underset{e}{\operatorname{argmax}}\ U_m[W_m(a, b_1, b_2)]. \tag{12}$$

6. Step 6: In Stage 2, the speculator and the market maker form the rational expectations about e^*, that is, their beliefs $\widehat{e}$ coincide with the firm manager's optimal effort e^*:

$$e^*(a, b_1, b_2) = \widehat{e}. \tag{13}$$

7. Step 7: In Stage 1, we solve for the optimal contract, which is designed by the entrepreneur. The optimal contract maximizes his expected wealth:

$$(a^*, b_1^*, b_2^*) = \underset{(a,b_1,b_2)}{\operatorname{argmax}}\ \mathbb{E}[W_p(a, b_1, b_2)], \tag{14}$$

subject to various constraints to be specified next.

The equilibrium is formally defined as follows.

Definition 1 *An equilibrium consists of an optimal contract (a^*, b_1^*, b_2^*), an optimal effort choice by the firm manager e^*, an optimal effort choice by the speculator ρ^*, an optimal trading strategy x^*, an optimal pricing function P, and the rational prior beliefs $\rho^* = \rho_m$ and $e^* = \widehat{e}$. The optimal contract (a^*, b_1^*, b_2^*) maximizes the entrepreneur's expected utility:*

$$(a^*, b_1^*, b_2^*) = \underset{(a,b_1,b_2)}{\operatorname{argmax}}\ \mathbb{E}[W_p(a, b_1, b_2)], \tag{15}$$

subject to the following constraints:

$$e^*(a, b_1, b_2) = \underset{e}{\operatorname{argmax}}\ U_m[W_m(a, b_1, b_2)], \tag{16}$$

$$\rho^*(\rho_m) = \underset{\rho}{\operatorname{argmax}}\ \mathbb{E}(U_s), \tag{17}$$

$$x(s) = \underset{x}{\operatorname{argmax}}\ \mathbb{E}[U_s(x)|s(\rho^*), \rho_m], \tag{18}$$

$$\rho^*(\rho_m) = \rho_m, \tag{19}$$

$$e^*(a, b_1, b_2) = \widehat{e}, \tag{20}$$

$$U_m = 0, \tag{21}$$

$$\mathbb{E}[U_k(P)|x + u, \rho_m, \widehat{e}] = -1. \tag{22}$$

In Definition 1, equation (15) determines the optimal contract, subject to the incentive compatibility constraints of the manager and the speculator in equations (16), (17), and (18), the rational expectations constraints in equations (19) and (20), the manager's participation constraint in equation (21), and the market efficiency constraint in equation (22).

3 Solution to Equilibrium

According to the solution procedure given in the last section, we solve the asset pricing problem and the principal-agent problem sequentially in this section.

3.1 Asset pricing

Proposition 1 *In Stage 8, the market maker believes that the speculator has exerted effort ρ_m, and the speculator's trading strategy is $x = \beta_m(s - \widehat{e})$. Thus, the market maker sets the pricing rule as*

$$P = \mathbb{E}(v|x+u) + \frac{1}{2}R_k(x+u)\mathrm{Var}(v|x+u) = \widehat{e} + \lambda_m[\beta_m(s-\widehat{e}) + u], \tag{23}$$

where λ_m is given by

$$\lambda_m = \frac{\beta_m}{\beta_m^2(1/\rho_m+1) + \sigma_u^2/\sigma_\theta^2} + \frac{R_k}{2}\left[\sigma_\theta^2 + \sigma_\delta^2 - \frac{\beta_m^2\sigma_\theta^2}{\beta_m^2(1/\rho_m+1) + \sigma_u^2/\sigma_\theta^2}\right]. \tag{24}$$

Note that λ_m and β_m are both functions of ρ_m. For notational ease, we omit their arguments.

In Stage 7, the speculator's optimal strategy is shown as $x = \beta(s - \widehat{e})$, with the trading intensity β given by

$$\beta^* = \frac{\rho^*}{2\lambda_m(\rho^*+1)}, \tag{25}$$

where ρ^* is the speculator's optimal effort chosen in Stage 6.

In Stage 6, the speculator's optimal effort ρ^* satisfies the first-order condition:

$$4k_s\lambda_m\rho^*(\rho^*+1)^2 - \sigma_\theta^2 = 0. \tag{26}$$

In Stage 5, the market maker has rational expectations by correctly anticipating the speculator's effort choice and trading strategy. That is,

$$\rho_m = \rho^*, \quad \beta_m = \beta^*. \tag{27}$$

According to equation (24), the consequence of introducing a risk-averse market maker is that the price impact λ is bigger than that when the market maker is risk neutral. Because the market maker is risk averse, they react to the the order flow more intensely by increasing λ. This leads to lower liquidity of the market, which reduces the marginal value of the private information for the speculator. As a result, the speculator spends less effort collecting private information and trades less aggressively than when the market maker is risk neutral. Consequently, the stock price contains less information about $e + \theta$ than when the market maker is risk neutral.

When $R_k = 0$, there exist close-form solutions to the above equilibrium of the stock market, which are presented in the following proposition.

Proposition 2 *When the market maker is risk neutral, explicit solutions to β and λ are given by*

$$\beta = \frac{\sigma_u}{\sigma_\theta}\left(\frac{\rho}{\rho+1}\right)^{1/2}, \quad \lambda = \frac{\sigma_\theta}{2\sigma_u}\left(\frac{\rho}{\rho+1}\right)^{1/2}. \tag{28}$$

Substituting λ into the speculator's FOC in equation (26) and solving it, we have

$$\rho^* = \frac{[(4\sigma_\theta\sigma_u/k_s)^{2/3}+1]^{1/2}-1}{2}, \tag{29}$$

and ρ^* increases with σ_θ or σ_u but decreases with k_s.

The intuition is as follows. More noise in the market can disguise the private information and makes it easy to earn money for the speculator, so the optimal effort of the speculator increases with σ_u. When σ_θ increases or the payoff is more volatile, the marginal value of the private information increases. As a result, the speculator spends a higher effort. Naturally, the speculator's optimal effort decreases with the cost of the information acquisition k_s.

When the stock market achieves an equilibrium, $\rho_m = \rho^*$, and according to equation (23) in Proposition 1, the price function is then given by

$$P = \widehat{e}(1-\beta\lambda) + \beta\lambda(s + u/\beta) \equiv \widehat{e}(1-\beta\lambda) + \beta\lambda\eta, \tag{30}$$

where $\eta = s + u/\beta = (e + \theta + \epsilon) + u/\beta$. Since $\mathbb{E}(\eta) = e$, η is the estimator of the manager's effort based on the total order flow. Therefore, the stock price is a weighted average of market participants' beliefs about the manager's effort and the estimator of the manager's effort based on the total order flow.

For further discussion, we define the following concepts. Suppose that $y = y_0 + \tau_y(v + \zeta)$ is a signal about the manager's effort e, where y_0 and τ_y are positive constants and ζ is a noise term with a mean of zero, which is independent of v. We define the sensitivity of y as $\tau_y \equiv \partial\mathbb{E}(y)/\partial\mathbb{E}(v)$. If $y_0 = 0$ and $\tau_y = 1$, then y is defined as a *normalized signal.* For example, the firm's payoff itself is a normalized signal. We denote $(v + \zeta)$ as Ω_y, which is termed normalized y. Then, y can be expressed as $y = y_0 + \tau_y\Omega_y$. Notice that the extent to which signal y reflects the manager's effort depends only on $\mathrm{Var}(\Omega_y)$ rather than by $\mathrm{Var}(y)$. For example, if $\tau_y = 0$, then $\mathrm{Var}(y) = 0$ but y does not contain any information about the manager's effort. If $\tau_y > 0$ and $\mathrm{Var}(\Omega_y) = 0$, then y fully reveals the manager's effort. Therefore, we define the informativeness of y about the manager's effort as $\mathrm{Var}(\Omega_y)^{-1}$, i.e., it is inversely proportional to $\mathrm{Var}(\Omega_y)$.

According to the above definitions and equation (30), we have $\tau_P = \beta\lambda$ and $\Omega_P = \eta$. That is, the sensitivity of the stock price is $\beta\lambda$ and the normalized price is η. According to equation (25), regardless of the market maker's risk aversion, we have

$$\beta\lambda = \frac{\rho^*}{2(\rho^* + 1)}. \tag{31}$$

From equation (31), we obtain that $\beta\lambda$ increases with ρ^*. Since the normalized price is η, the informativeness of the stock price about the manager's effort is $\mathrm{Var}(\eta)^{-1}$. The next proposition presents the impacts of parameters, R_k, k_s, σ_δ, σ_θ, or σ_u on ρ^* and $\mathrm{Var}(\eta)$.

Proposition 3 *1. When the market maker is risk neutral,* $\mathrm{Var}(\eta) = \sigma_\theta^2/\beta\lambda$ *and* $\mathrm{Var}(\eta)$ *decrease with σ_u but increase with σ_θ or k_s. 2. When the market maker is risk averse, we have the following results. ρ^* decreases with R_k, k_s, or σ_δ but increases with σ_θ or σ_u, and ρ^* converges to a constant, when σ_θ or σ_u goes to infinity.* $\mathrm{Var}(\eta)$ *increases with R_k, k_s, or σ_δ, and* $\mathrm{Var}(\eta)$ *first decreases and then increases, when σ_u or σ_θ increases.*

When the market maker is risk neutral, according to Proposition 2 and equation (31), $\beta\lambda$ increases with σ_u or σ_θ, but decreases with k_s. Then, $\mathrm{Var}(\eta)$ decreases with σ_u but increases with

k_s. Besides, when σ_θ increases, $\beta\lambda$ increases as well, but σ_θ increases faster. Hence, $\mathrm{Var}(\eta)$ increases with σ_θ.

When the market maker is risk averse, ρ^* increases with σ_θ or σ_u, but decreases with k_s for the same reason as the case of the risk-neutral market maker. According to equation (24), the market impact λ increases with R_k and σ_δ. Hence, the trading intensity β decreases accordingly, so does the optimal effort of the speculator. Generally, we have $\mathrm{Var}(\eta) = \sigma_\theta^2 + \sigma_\theta^2/\rho + \sigma_u^2/\beta^2$. Since both ρ^* and β decrease with R_k, k_s, or σ_δ, $\mathrm{Var}(\eta)$ increases with them. When σ_θ or σ_u goes to zero, ρ^* also goes to zero, but it goes to zero faster than σ_θ or σ_u because of the risk-averse market maker. As a result, $\mathrm{Var}(\eta)$ goes to infinity, when σ_θ or σ_u goes to zero. Therefore, $\mathrm{Var}(\eta)$ first decreases and then increases with σ_θ or σ_u.

Adapting from Kyle (1985), we define the informativeness of the stock price (about the firm's payoff) as follows.

$$\frac{\sigma_\theta^2}{\mathrm{Var}(\theta|P)} = \left[1 - \frac{\sigma_\theta^2}{\mathrm{Var}(\eta)}\right]^{-1}. \tag{32}$$

Therefore, given σ_θ, a smaller $\mathrm{Var}(\eta)$ implies higher stock price informativeness. When exogenous variables, σ_u, k_s, R_k, and σ_δ (except for σ_θ) change, the stock price informativeness is proportional to the informativeness of the stock price about the manager's effort e^*.

3.2 Principal and agent

Solving the entrepreneur's optimization problem, we have the following proposition.

Proposition 4 *In Stage 3, given the contract (a, b_1, b_2), manager's optimal choice of e^* is given by*

$$e^* = \frac{b_1\beta\lambda + b_2}{k_m}. \tag{33}$$

In Stage 1, the optimal contract b_1^* and b_2^* are given by

$$b_1^* = \frac{1}{\beta\lambda}\frac{1}{Z}\phi_1, \quad b_2^* = \frac{1}{Z}\phi_2, \tag{34}$$

where

$$\phi_1 = \frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2}, \quad \phi_2 = \frac{\sigma_\epsilon^2 + \sigma_u^2/\beta^2}{\sigma_\delta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2}, \tag{35}$$

$$Z = 1 + R_m k_m V, \quad V = \mathrm{Var}(\phi_1 \eta + \phi_2 v) = \sigma_\delta^2 + \sigma_\theta^2 - \frac{\sigma_\delta^4}{\sigma_\delta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2}. \tag{36}$$

Notice that $\phi_1 + \phi_2 = 1$. Then, the optimal manager's effort is given by

$$e^* = \frac{1}{k_m Z}. \tag{37}$$

The ratio of the weight on the stock price to that on the firm's payoff is given by

$$\frac{b_1^*}{b_2^*} = \frac{\phi_1}{\beta\lambda\phi_2} = \frac{\sigma_\delta^2}{\beta\lambda(\sigma_\epsilon^2 + \sigma_u^2/\beta^2)}. \tag{38}$$

The expected wealth of the risk-neutral entrepreneur is given by

$$\mathbb{E}(W_p) = \frac{e^*}{2}. \tag{39}$$

We next explain the properties of the optimal contract and optimal effort. Before that, we introduce the best signal of manager's effort, which can help us understand the optimal contract when the compensation contract includes multiple signals about the manager's effort.

3.2.1 Best signal of the manager's effort

ϕ_1 and ϕ_2 in equation (35) can also be obtained as follows. Note that the normalized price $\Omega_P = \eta$ and the firm's payoff v are two normalized signals. With η and v, we can construct a family of normalized signals: $\psi_1\eta + \psi_2 v$, where ψ_1 and ψ_2 are nonnegative constants and $\psi_1 + \psi_2 = 1$. In this family of signals, there exists a signal $(\psi_1^*\eta + \psi_2^* v)$, which satisfies the following condition:

$$(\psi_1^*, \psi_2^*) = \underset{\psi_1,\psi_2}{\mathrm{argmin}}\ \mathrm{Var}(\psi_1\eta + \psi_2 v) \quad \text{s.t. } \psi_1 + \psi_2 = 1. \tag{40}$$

Solving the above problem, we obtain

$$\psi_1^* = \phi_1, \quad \psi_2^* = \phi_2, \tag{41}$$

where ϕ_1 and ϕ_2 are defined in equation (35). Thus, $(\phi_1\eta + \phi_2 v)$ is defined as the best signal, in terms of its minimum variance in the family, and the variance of the best signal $V = \mathrm{Var}(\phi_1\eta + \phi_2 v)$ is given in equation (36).

The process of setting the contract can be interpreted as the procedure that the entrepreneur first combines the two signals by constructing the best signal $(\phi_1\eta + \phi_2 v)$, and then sets the linear contract based on it. Standard derivations deliver the incentive part of the optimal contract in terms of the best signal:

$$\frac{1}{1 + R_m k_m V}(\phi_1\eta + \phi_2 v), \tag{42}$$

which is equivalent to that in Proposition 4. The concept of the best signal provides a new way to understand the optimal contract, which includes multiple signals about the manger's effort. In effect, if the entrepreneur sets the contract based only on the stock price, then the best signal is η and the incentive part of the optimal contract in equation (42) is reduced to the one in the traditional agency model:

$$\frac{\eta}{1 + R_m k_m \mathrm{Var}(\eta)}. \tag{43}$$

3.2.2 An example for the sensitivity effect and the informativeness effect

There are two key effects that detemine b_1^* and b_2^*: the sensitivity effect and the informativeness effect. We explain these two effects and their impact on optimal contracting and manager's optimal effort in a simple agency model below.

A risk-neutral entrepreneur hires a risk-averse manager, who has a negative exponential utility, to run a firm. The firm's payoff is given by $\pi = he + \xi$, where h is the productivity factor, e is the manager's effort, and ξ is a zero-mean random shock to the firm's payoff. The manager's cost function is quadratic. Besides, there is another signal $y = y_0 + \tau_y(\pi + \zeta)$ that contains information about the manager's effort. Suppose that the manager's compensation contract is linear in y: $a + by$, where a and b are constants set by the entrepreneur and b is the PPS. The optimal incentive on y and the manager's optimal effort are then given as follows.

$$b^* = \frac{1}{\tau_y} \cdot \frac{1}{(1 + R_m k_m \mathrm{Var}(\Omega_y)/h^2)}, \quad e^* = \frac{h}{k_m[1 + R_m k_m \mathrm{Var}(\Omega_y)]},$$

where $\Omega_y = (\pi + \zeta)$.

We highlight three results in this model, which has not been discussed explicitly in the literature. First, when τ_y is lower, the optimal PPS is higher. We define the effect of the sensitivity τ_y on the optimal PPS as the sensitivity effect. Note that the purpose of setting the incentive contract is to relate the manager's compensation to his effort. When the sensitivity of y decreases, to maintain the sensitivity of the manager's compensation to his effort, the optimal PPS will increase accordingly. Special attention should be paid to the difference between the productivity factor h and the sensitivity of the signal τ_y. Their impacts on the optimal PPS are in the opposite directions.[8]

Second, when $\mathrm{Var}(\Omega_y)$ increases, the optimal PPS decreases. We define the effect of $\mathrm{Var}(\Omega_y)$ on the optimal PPS as the informativeness effect. Notice that the informativeness of y about the manager's effort is determined only by $\mathrm{Var}(\Omega_y)$ rather than by $\mathrm{Var}(y)$. Therefore, the informativeness effect means that when signal y provides more precise information about the manager's effort, the optimal incentive on y increases accordingly. We emphasize that the informativeness effect corresponds to the risk of the normalized y, $\mathrm{Var}(\Omega_y)$, rather than on that of the original signal y, $\mathrm{Var}(y)$, even though the contract is based on y rather than on Ω_y. This result has not been recognized in the literature.

Third, the manager's optimal effort decreases with $\mathrm{Var}(\Omega_y)$ but does not vary with τ_y. This result means that the efficiency of the compensation contract depends only on the informativeness of the signal about the manager's effort or that when the signal reflects more precise information about the manager's effort, the manager's optimal effort increases. It is worth emphasizing that the manager's optimal effort is independent of τ_y. For example, we still consider the case in which $\mathrm{Var}(y)$ is a positive constant, τ_y goes to infinity, and $\mathrm{Var}(\Omega_y)$ goes to zero. In this case, although $\mathrm{Var}(y)$ is a positive constant, because y tends to fully reveal the manager's effort, the manager's optimal effort approaches the first best solution.

Notice that $\mathrm{Var}(y) = \tau_y^2 \mathrm{Var}(\Omega_y)$. If τ_y is fixed, then $\mathrm{Var}(y)$ is proportional to $\mathrm{Var}(\Omega_y)$. Hence, the informativeness effect leads to the standard agecny prediction that a higher risk of the signal

[8]When h is higher, both the optimal PPS and the manager's optimal effort increase. When τ_y increases, however, the optimal PPS decreases and the manager's effort does not change.

(a higher Var(y)) decreases the optimal PPS. In addition, if τ_y is fixed, then both the PPS and the manger's optimal effort decrease with Var(Ω_y). These results correspond to the standard prediction that higher managerial incentives enhance firm performance. Nonetheless, as we shall show later, if y is the equilibrium stock price, then there is a tradeoff between τ_y and Var(Ω_y) and as a result, those standard predictions do not always hold.

3.2.3 Determinants of b_1^* and b_2^*

Recall that $\beta\lambda$ is the sensitivity of the stock price to the manager's effort. According to equation (34), an increase in $\beta\lambda$ decreases b_1^*. This is the sensitivity effect for b_1^*. In addition, a higher V makes the best signal noisier or the entrepreneur more difficult to estimate the effort of the manager, so an increase in V will decrease b_1^* and b_2^*. This is the informativeness effect for both b_1^* and b_2^*.

When the manager chooses an effort, he cannot influence others' beliefs about his effort, so the manager believes that the sensitivity of the stock price is not one but $\beta\lambda$, by equation (30). In contrast, the sensitivity of the payoff is always one by definition. This is the key difference between the stock price and the firm's payoff as signals about the manager's effort. The existence of the sensitivity effect for b_1^* stems from the fact that the manager's effort is unobservable. If the manager's effort were observable to all participants in the stock market, then from equation (30) we would have

$$P = e(1 - \beta\lambda) + \beta\lambda\eta = e + \beta\lambda(\theta + \epsilon + u/\beta), \tag{44}$$

and the sensitivity of the stock price would always be one.

b_1^* and b_2^* also increase with ϕ_1 and ϕ_2, respectively, which we term *the relative weight effect*. For example, ϕ_1 represents how much η contributes to the best signal $(\phi_1\eta + \phi_2 v)$. Note that the firm's payoff is specified as $v = (e+\theta)+\delta$ and the normalized price is given by $\eta = (e+\theta)+(\epsilon+u/\beta)$. The common component of v and η is $(e + \theta)$. $(\epsilon + u/\beta)$ and δ are independent, so the variance of the best signal is smaller than that of η or v. To help us understand ϕ_1 and ϕ_2, we consider two extreme cases. In the first case, the stock price fully reveals $(e+\theta)$, i.e., $\mathrm{Corr}(\theta, P) = 1$. $P = \widehat{e}(1-\beta\lambda)+\beta\lambda\eta$, so $\text{Corr}(\theta, P) = 1$ is equivalent to $\sigma_\epsilon^2 + \sigma_u^2/\beta^2 = 0$. Then, according to equation (35), $\phi_2 = 0$ and $\phi_1 = 1$. In other words, knowing v does not help the entrepreneur monitor the manager's effort at all. In the second case, $\sigma_\delta = 0$, so $\phi_2 = 1$ and $\phi_1 = 0$. That is, the payoff fully reveals $(e + \theta)$, and the price does not provide extra information beyond that provided by the payoff. From these two extreme cases, we observe that ϕ_1 and ϕ_2 are determined by $(\sigma_\epsilon^2 + \sigma_u^2/\beta^2)$ and σ_δ.

For the purpose of estimating the manager's effort, $(\theta + \delta)$ is the noise in the payoff v, and $(\theta + \epsilon + u/\beta)$ is the noise in the normalized price η. Obviously, θ is the common noise of both the payoff and the normalized price, and δ and $\epsilon + u/\beta$ are their respective specific noises. It is natural that the signal with higher precision should contribute more to the best signal, i.e., a bigger relative weight. Note that θ is the common noise of the two normalized signals, so it does not influence the relative weight of them. Then, only the specific noises of the signals affect their relative weights in the best signal. Specifically, a smaller specific noise leads to a higher relative weight. For example, if σ_δ^2 is much smaller than $\sigma_\epsilon^2 + \sigma_u^2/\beta^2$, then η is much noisier than v. Therefore, the relative weight of the normalized price will be close to zero, because η cannot provide extra information about the manager's effort. Likewise, if σ_δ^2 is much bigger than $\sigma_\epsilon^2 + \sigma_u^2/\beta^2$, the relative weight of the payoff will be close to zero, because the payoff cannot provide additional information.

3.2.4 Tradeoff between the sensitivity effect and the informativeness effect

Recall that $(b_1 + b_2)$ corresponds to the PPS. From equation (34), we have

$$b_1^* + b_2^* = \frac{1}{Z}\left(\frac{\phi_1}{\beta\lambda} + \phi_2\right) = \frac{1}{Z}\left[\left(\frac{1}{\beta\lambda} - 1\right)\phi_1 + 1\right]. \tag{45}$$

Then, there are three effects that affect $(b_1^* + b_2^*)$: the sensitivity effect $\beta\lambda$, the informativeness effect V, and the relative weight effect ϕ_1.

According to Proposition 3, the sensitivity of the stock price $\beta\lambda$ and the risk of the normalized price $\text{Var}(\eta)$ usually move in the opposite directions. For instance, when k_s, σ_δ, or R_k changes, $\beta\lambda$ and $\text{Var}(\eta)$ move in the opposite directions. When σ_θ or σ_u changes, if they are small, $\beta\lambda$ and $\text{Var}(\eta)$ also move in the opposite directions.

The frequent negative relations between $\beta\lambda$ and $\text{Var}(\eta)$ are not coincident. From equation (30), the stock price is the weighted average of $\widehat{e}$ and η. Recall that $\beta\lambda = \rho^*/[2(1+\rho^*)]$ as given in equation (31). Thus, $\beta\lambda$ increases with ρ^*, the precision of the speculator's private signal. On the other hand, when ρ^* increases, the stock price informativeness usually increases, i.e., $\sigma_\theta^2/\mathrm{Var}(\eta)$ increases.[9] As we discussed earlier, the informativeness of the stock price about the manager's effort usually increases with the stock price informativeness about the firm's payoff. Therefore, when the sensitivity of the stock price $\beta\lambda$ increases, the risk of the normalized price $\mathrm{Var}(\eta)$ usually decreases.

Because the best signal is composed of both the normalized price, η, and the payoff, v, the risk of the best signal usually increases with the risk of the normalized price. As a result, the tradeoff between $\beta\lambda$ and $\mathrm{Var}(\eta)$ leads to the tradeoff between the sensitivity effect and the informativeness effect.

3.2.5 Manager's optimal effort

Because the optimal contract can be interpreted as a single-signal contract based on the best signal, the manager's optimal effort depends only on the risk of the best signal, V, according to equation (37). The best signal is composed of both the normalized price, η, and the payoff, v, so the risk of the best signal, V, usually increases with the risk of the normalized price. As a result, when the stock price is more informative about the manager's effort, the efficiency of the contract will be improved. Furthermore, the stock price informativeness about the payoff usually increases with its informativeness about the manager's effort. Therefore, the manager's optimal effort usually increases with the stock price informativeness about the payoff.

4 Implications

4.1 PPS and the stock price volatility

Classical agency models predict a negative relation between the PPS and the volatility of the project payoff but have found both positive and negative relations.[10] A few theoretical papers

[9] When the market maker is risk neutral, $\sigma_\theta^2/\mathrm{Var}(\eta) = \beta\lambda$ always increases with ρ^*. Under the risk-averse market maker, when k_s, R_k, or σ_δ decreases, ρ increases and $\mathrm{Var}(\eta)$ decreases.

[10] Core and Guay (1999) and Oyer and Shaefer (2005) find a positive relation, Aggarwal and Samwick (1999) document a negative relation, and Cao and Wang (2013) find a negative relation between the PPS and the systematic

have attempted to explain this puzzle by introducing various mechanisms. In this subsection, we provide potential explanations for these mixed results. Empirical tests use the fraction of the total shares owned by the manager (managerial ownership) as a proxy for the PPS and use stock price volatilities instead of the risk of the payoff. A key insight of our explanations is that both the PPS and the stock price volatility are endogenous variables and they are driven by five parameters, σ_θ, σ_δ, R_k, k_s, and σ_u. Because of the tradeoff between the sensitivity effect and the informativeness effect, we can obtain different relations between the PPS and the price volatility, by changing these parameters.

4.1.1 Price contract

To help understand the intuition, we first consider a simplified contract, $W_m = a + bP$, which depends only on the stock price as in Goldman and Slezak (2006). The optimal incentive on the stock price is given by the following proposition.

Proposition 5 *Given the price contract $W_m = a + bP$, the optimal PPS, b^*, is given by*

$$b^* = \frac{1}{\beta\lambda[1 + R_m k_m \mathrm{Var}(\eta)]}. \tag{46}$$

According to our earlier analyses, $\beta\lambda$ represents the sensitivity effect and $\mathrm{Var}(\eta)$ represents the informativeness effect. We first demonstrate that when the market maker is risk neutral, the relation between the PPS and the price volatility is always negative.

Case I: Risk-neutral market maker

Note that the price volatility is given by $\mathrm{Var}(P) = \beta^2\lambda^2\mathrm{Var}(\eta)$. Thus, both the PPS b^* and the price volatility are determined by the sensitivity effect $\beta\lambda$ and the informativeness effect $\mathrm{Var}(\eta)$. When we state that one effect dominates the other, it means that the moving direction of b^* or $\mathrm{Var}(P)$ is determined by the dominating effect. For example, when an exogenous variable changes, $\beta\lambda$ increases but $\mathrm{Var}(\eta)$ decreases. If $\mathrm{Var}(P)$ still increases with $\beta\lambda$, then we state that the sensitivity effect dominates the informativeness effect for the price volatility.

risk but a positive relation between the PPS and the idiosyncratic risk. Prendergast (2002) summarizes additional empirical evidence on the mixed results on this relationship.

According to equation (24), when the market maker is risk neutral, we have $\mathrm{Var}(\eta) = \sigma_\theta^2/\beta\lambda$ and thus $\mathrm{Var}(P) = \beta\lambda\sigma_\theta^2$. Obviously, when σ_θ^2 is controlled for, the sensitivity effect dominates the informativeness effect for the price volatility, i.e., the price volatility increases with $\beta\lambda$. Moreover, from equation (46), the optimal incentive b^* can be expressed as

$$b^* = \frac{1}{\beta\lambda + R_m k_m \sigma_\theta^2}. \tag{47}$$

From equation (47), when σ_θ^2 is controlled for, the sensitivity effect dominates the informativeness effect for b^* as well, i.e., b^* decreases with $\beta\lambda$. Thus, when σ_θ^2 is controlled for, the price volatility increases but b^* decreases with $\beta\lambda$. The relation between the PPS and the price volatility is thus negative.

Furthermore, according to Proposition 3 and equation (31), when σ_θ increases, both $\beta\lambda$ and $\mathrm{Var}(\eta)$ increase. Thus both the sensitivity effect and informativeness effect decrease the PPS but increase the price volatility. As a result, the relation between the PPS and the price volatility is negative. We thus obtain that when the market maker is risk neutral, the PPS always decreases with the price volatility.

Case II: Risk-averse market maker

When the market maker is risk averse, there are no explicit solutions to β, λ, and ρ^*, so we analyze this case by numerical calculations. In Figure 1, we plot the impacts of exogenous variables on the two effects: $\beta\lambda$ and $\mathrm{Var}(\eta)$, in two cases of $R_k = 0$ and $R_k = 0.1$. In Figure 2, we describe how the PPS and the price volatility vary with the five exogenous variables and plot the relations between the PPS and the price volatility driven by these exogenous variables. For example, Subplot A1 of Figure 2 presents how the PPS and the price volatility vary with σ_θ, and Subplot B1 of Figure 2 presents correspondingly the relation between the PPS and the price volatility driven by σ_θ. Based on these plots, we demonstrate that there are mixed relations between the PPS and the price volatility, driven by different exogenous parameters. Particularly, when σ_θ or k_s increases, we can have the positive or inverted U-shaped relation between the PPS and the price volatility.

According to Proposition 3 and equation (31), when σ_θ increases, $\beta\lambda$ increases, $\mathrm{Var}(\eta)$ first

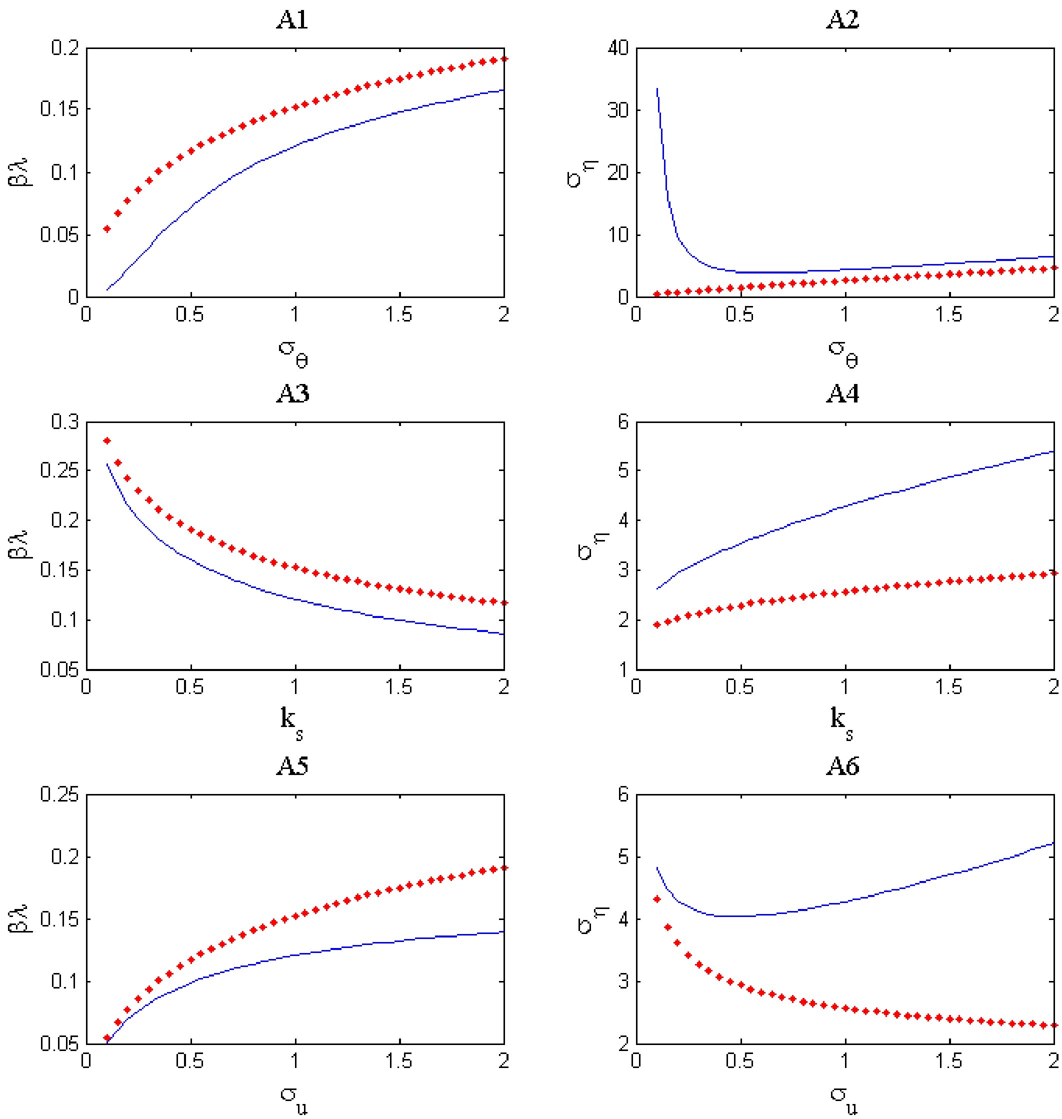

Figure 1: Risk neutral vs. risk averse

Figure 1 plots the impact of exogenous variables on the two effects: $\beta\lambda$ and $\sigma_\eta = \mathrm{Var}(\eta)^{1/2}$, under two cases of $R_k = 0$ and $R_k = 0.1$. The dotted lines correspond to the case of $R_k = 0$; the solid lines correspond to the case of $R_k = 0.1$. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.1$, when they are not varying in the horizontal axes.

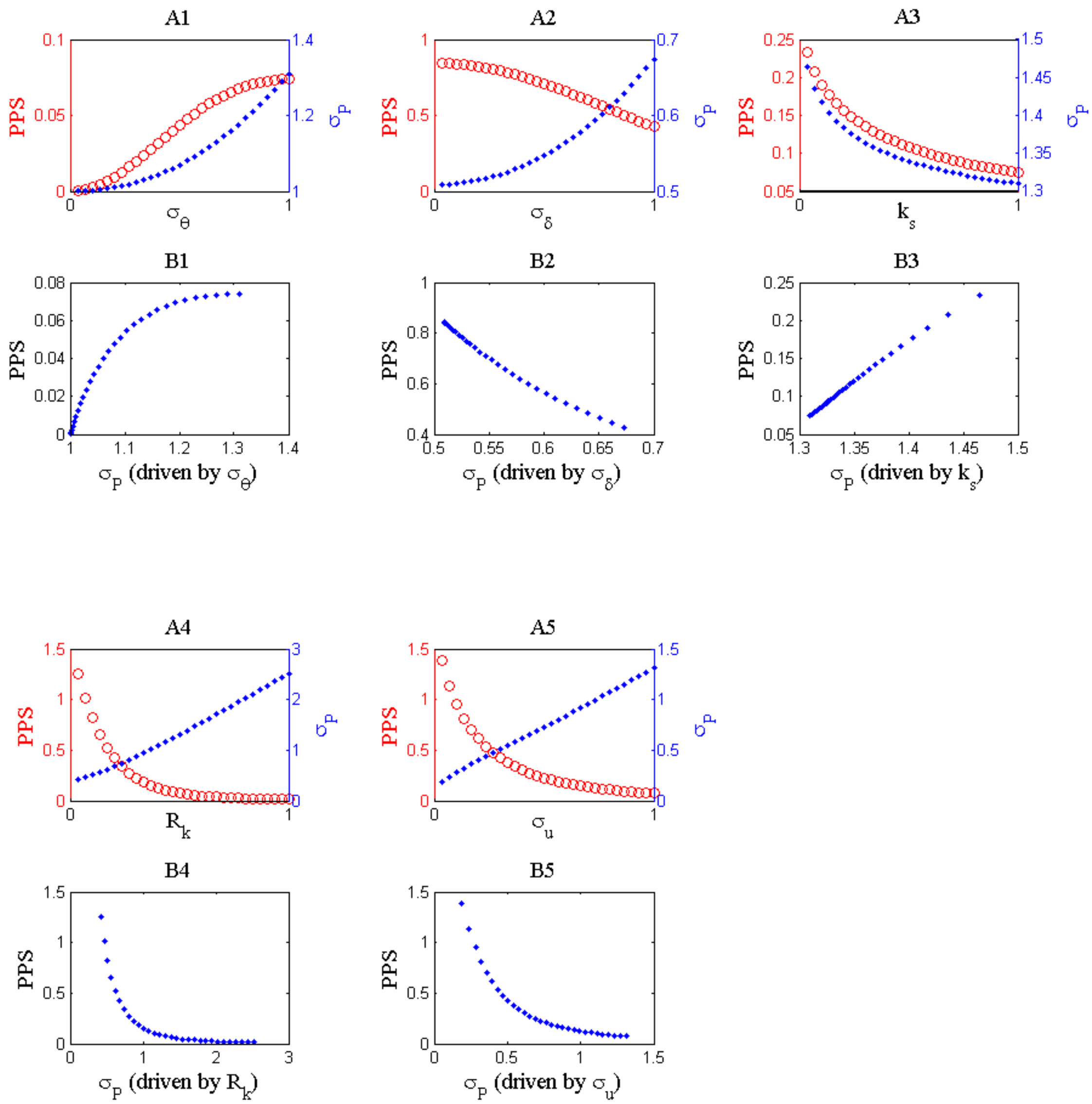

Figure 2: PPS vs. the price volatility

Figure 2 illustrates the relation between the PPS and the price volatility ($\sigma_P = \mathrm{Var}(P)^{1/2}$) under the price contract when the market maker is risk averse. Subplots A1 to A5 depict how the PPS and the price volatility vary with σ_θ, σ_δ, k_s, R_k, and σ_u. The circled lines correspond to the PPS and the dotted lines correspond to the price volatility. Subplots B1 to B5 depict the relation between the PPS and the price volatility driven by these variables. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.5$, when they are not varying in the horizontal axes.

decreases and then increases. As shown in Subplot A1 of Figure 2, the price volatility increases with σ_θ, but when σ_θ is small, the PPS increases with σ_θ. Therefore, when σ_θ is small, we obtain a positive relation between them. The reason is that when the market maker is risk averse and σ_θ is small, $\mathrm{Var}(\eta)$ decreases with σ_θ as shown in Subplot A2 of Figure 1, rather than increases with σ_θ as when the market maker is risk neutral. As a result, the PPS increases with σ_θ, rather than decreases with σ_θ, when it is small.

Similarly, according to Proposition 3 and equation (31), when k_s increases, $\beta\lambda$ decreases but $\mathrm{Var}(\eta)$ increases. For the price volatility, similar to the case of $R_k = 0$, the sensitivity effect dominates the informativeness effect, and as a result, the price volatility decreases with k_s. For the PPS, however, in contrast to the case of $R_k = 0$, the informativeness effect dominates the sensitivity effect. Therefore, the PPS decreases with k_s. Consequently, there exists a positive relation between the PPS and the price volatility, when the cost of information acquisition k_s changes. The reason for this positive relation is that when the market maker is risk averse, $\mathrm{Var}(\eta)$ increases with k_s faster than when the market maker is risk neutral as shown in Subplot A4 of Figure 1. Thus, the informativeness effect can dominate the sensitivity effect for the PPS.

The tradeoff between the informativeness effect and the sensitivity effect is a necessary condition for the mixed results. When an exogenous variable changes, if $\beta\lambda$ and $\mathrm{Var}(\eta)$ move in the same direction, then the PPS will decrease with the price volatility. For example, if both $\beta\lambda$ and $\mathrm{Var}(\eta)$ increase, then the PPS decreases but the price volatility increases, leading to a negative relation between the two. Therefore, a necessary condition for the positive relation is that $\beta\lambda$ and $\mathrm{Var}(\eta)$ move in the opposite directions. If we do not consider the asset pricing aspect as in a traditional principal-agent model, in which the compensation contract is in terms of the payoff and the sensitivity of the payoff is an exogenously given constant, then the PPS always decreases with the risk of the payoff.

4.1.2 Price-payoff contract

To build on the results of the price contract, we now consider the price-payoff contract, in which $W_m = a + b_1 P + b_2 v$. Under this contract, the PPS corresponds to $(b_1^* + b_2^*)$. Note that there are three effects that affect $(b_1^* + b_2^*)$: the sensitivity effect $\beta\lambda$, the informativeness effect V, and the relative weight effect ϕ_1. These effects and factors are tangled together and influence the relation between the PPS and the price volatility in a complicated manner. For example, when k_s increases, ρ^* decreases but $\text{Var}(\eta)$ increases according to Proposition 3. Therefore, $\beta\lambda$ and ϕ_1 decrease but Z increases. Thus, the informativeness effect and the relative weight effect decrease the PPS, but the sensitivity effect increases it. The competition among the three effects makes the results subtle.

Case I: Risk-neutral market maker

We first consider the case of the risk-neutral market maker. We can obtain the closed-form solution to the optimal PPS as

$$b_1^* + b_2^* = \left[R_m k_m \sigma_\theta^2 + 1 - \frac{1}{1/(1-\beta\lambda) + \sigma_\theta^2/\sigma_\delta^2} \right]^{-1}. \tag{48}$$

Similar to the case of the price contract, we find that a positive relation does not exist in this case.

Proposition 6 *When the market maker is risk neutral, the relations between $(b_1^* + b_2^*)$ and the price volatility are nonpositive.*

Proof: When the market maker is risk neutral, the price volatility $\text{Var}(P) = \beta\lambda\sigma_\theta^2$ and therefore, it varies only with k_s, σ_u, or σ_θ. When σ_θ and σ_δ are controlled for, the price volatility increases with $\beta\lambda$, but $b_1^* + b_2^*$ decreases with $\beta\lambda$ according to equation (48), leading to a negative relation between the PPS and the price volatility. When σ_δ increases, the price volatility is unchanged but $b_1^* + b_2^*$ increases. When σ_θ increases, $\beta\lambda$ increases, leading to an increase in the price volatility, but $b_1^* + b_2^*$ decreases. Therefore, there is no positive relation between $b_1^* + b_2^*$ and the price volatility.

Case II: Risk-averse market maker

Again, when the market maker is risk averse, there are no explicit solutions to β, λ, and ρ^*, so we analyze this case by numerical calculations. In Figure 3, we plot the impacts of exogenous variables on the three effects: the sensitivity effect $\beta\lambda$, the informativeness effect V, and relative weight effect ϕ_1, in two cases of $R_k = 0$ and $R_k = 0.1$. In Figure 4, we describe how the PPS and the price volatility vary with the five exogenous variables and plot the relations between the PPS and the price volatility driven by the five exogenous variables. Based on these plots, we demonstrate that there are mixed relations between the PPS and the price volatility, driven by different exogenous parameters. Particularly, when σ_θ, σ_δ, or k_s increases, we have positive or inverted U-shaped relations between the PPS and the price volatility.

According to Subplot A1 of Figure 4, the price volatility increases with σ_θ. According to Subplots A1 to A3 of Figure 3, when the market maker is risk averse and σ_θ is small, the informativeness effect and the relative weight effect increase the PPS and the sensitivity effect decreases it. As a result, the PPS first increases and then decreases with σ_θ, leading to a positive relation when σ_θ is small.

According to Subplot A2 of Figure 4, the price volatility increases with σ_δ. As shown in Subplots A4 to A6 of Figure 3, the sensitivity effect and the relative weight effect increase the PPS, but the informativeness effect decreases it. As a result, the PPS first increases and then decreases with σ_δ. Therefore, when σ_δ is small, the PPS increases with the price volatility.

According to Subplot A3 of Figure 4, the price volatility decreases with k_s. As shown in Subplots A7 to A9 of Figure 3, the sensitivity effect increases the PPS, but the informativeness effect and the relative weight effect decrease it. As a result, the PPS decreases with k_s. Therefore, the relation between the PPS and the price volatility is positive.

Although the situation is more complicated under the price-payoff contract, the mechanism is still the tradeoff between the informativeness effect and the sensitivity effect. From Figure 3, the informativeness effect and the relative weight effect usually influence the PPS in the same direction.

In summary, we show that under both the price contract and the price-payoff contract, the relation between the PPS and the stock price volatility can be negative or positive, when different parameters change. Therefore, our results offer a potential explanation for the mixed empirical findings on this relation.

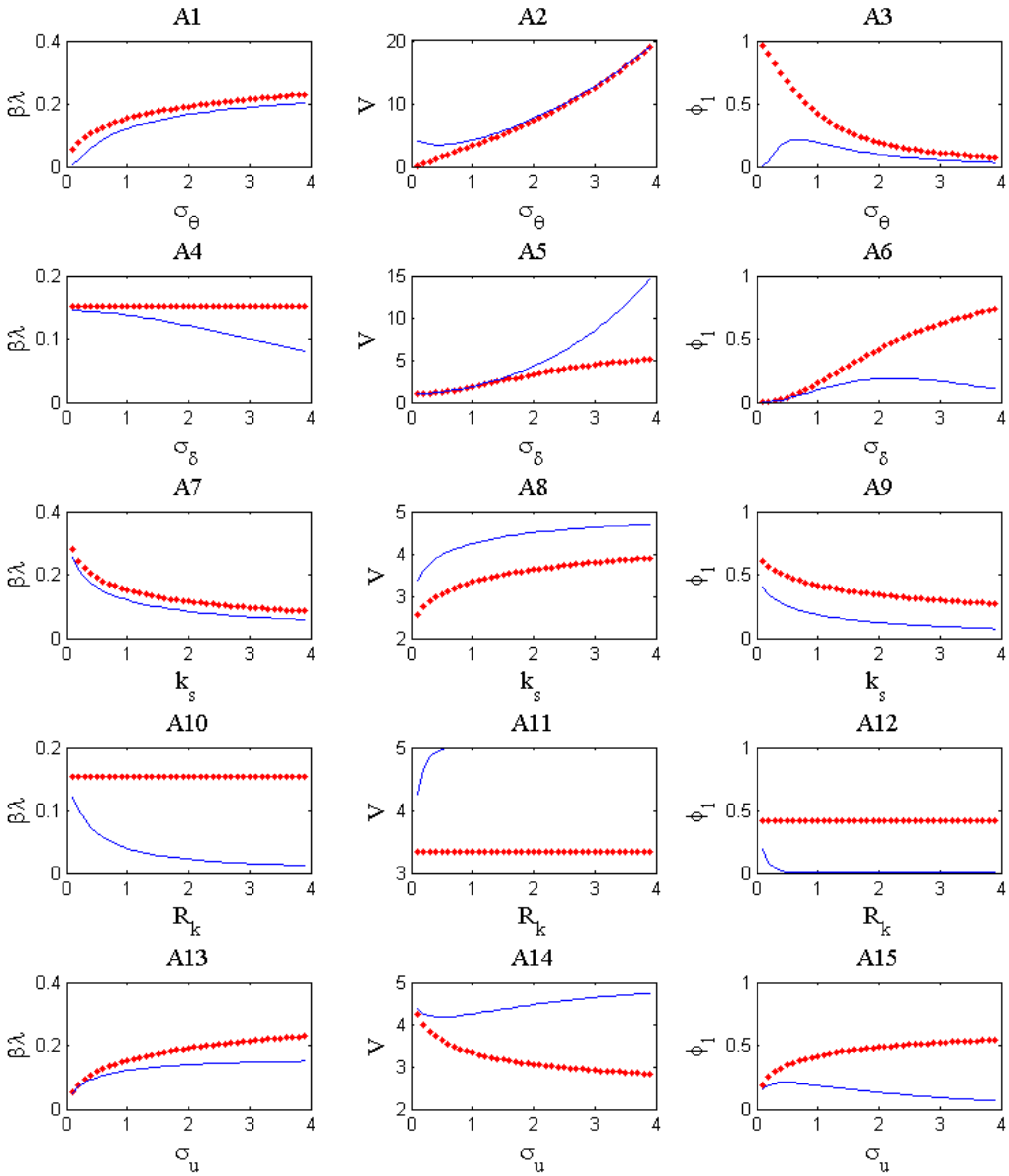

Figure 3: Risk neutral vs. risk averse

Figure 3 plots the impact of exogenous variables on the three effects: the sensitivity effect $\beta\lambda$, the informativeness effect V, and relative weight effect ϕ_1, under $R_k = 0$ and $R_k = 0.1$. The dotted lines correspond to the case of $R_k = 0$; the solid lines correspond to the case of $R_k = 0.1$. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.1$, when they are not varying in the horizontal axes.

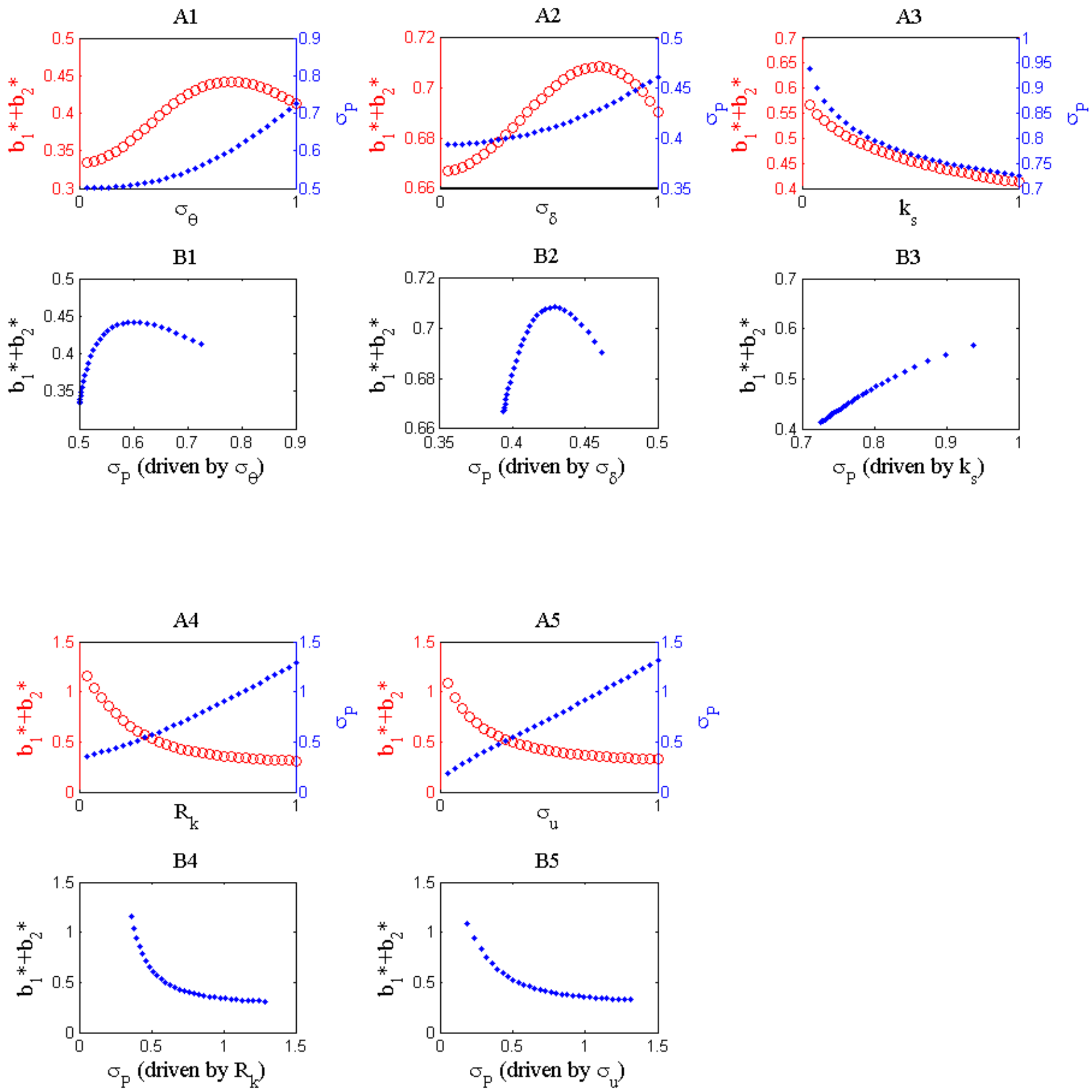

Figure 4: The PPS vs. the price volatility

Figure 4 illustrates the relation between the PPS ($b_1^* + b_2^*$) and the price volatility ($\sigma_P = \text{Var}(P)^{1/2}$) under the price-payoff contract when the market maker is risk averse. Subplots A1 to A5 depict how the PPS and the price volatility vary with σ_θ, σ_δ, k_s, R_k, and σ_u. The circled lines correspond to the PPS and the dotted lines correspond to the price volatility. Subplots B1 to B5 depict the relation between the PPS and the price volatility driven by these variables. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.5$, when they are not varying in the horizontal axes.

4.2 Expected firm value and PPS

The traditional agency model predicts that higher PPS improves the manager's optimal effort. There is a voluminous literature testing this relation, but their findings are mixed.[11] In this subsection, we show that the tradeoff between the sensitivity effect and the informativeness effect leads to the result that the manager's optimal effort does not always increase with the PPS $(b_1^* + b_2^*)$, which is consistent with the mixed empirical results.

As we show, the manager's optimal effort depends only on the risk of the best signal, but the PPS depends on both the risk of the best signal and the sensitivity of the stock price. When the risk of the best signal decreases, the manager's effort increases, so the expected firm value increases. In addition, a decrease in the risk of the best signal increases the PPS, but the sensitivity of the stock price usually increases at the same time, which decreases the PPS. The sensitivity effect sometimes dominates the informativeness effect, leading to negative relations between the manager's optimal effort or the expected firm value and the PPS.

For simplicity, we consider the case of the risk-neutral market maker. According to equation (36), the risk of the best signal V increases with k_s, σ_θ, or σ_δ but decreases with σ_u.[12] Besides, according to the proof of Proposition 6, $(b_1^* + b_2^*)$ decreases with σ_u or σ_θ, but increases with k_s or σ_δ. Therefore, when σ_θ changes, the manager's effort increases with the PPS, but when k_s, σ_δ, or σ_u changes, the manager's effort decreases with the PPS. Consequently, only when the relation between the expected firm value and the PPS is driven by σ_θ, will the relation be positive. Otherwise, it is negative.

As we can see, the relation between the PPS and the expected firm value can be positive or negative, driven by different exogenous parameters. Therefore, the mixed empirical findings on this relation are natural.

[11] Morck et al. (1988), Hubbard and Palia (1995), McConnell and Servaes (1995), Mehran (1995), Core and Larcker (2002), Anderson and Reeb (2003), Holderness et al. (2003), and Adams and Santos (2006) find a positive relation between managerial ownership and firm performance, but Demsetz and Lehn (1985), Agrawal and Knoeber (1996), Loderer and Martin (1997), Cho (1998), Himmelberg et al. (1999), Palia (2001), and Coles et al. (2012) find no relation between managerial ownership and firm performance. In addition, Benson and Davidson (2009) find an inverted U-shaped relation between managerial ownership and firm performance.

[12] From equations (28) and (29), we can obtain that β decreases with σ_θ but σ_ϵ^2 increases with σ_θ.

4.3 PPS and the stock price informativeness

The tradeoff between the two effects also allows us to obtain both positive and negative relations between the PPS and the stock price informativeness about the stock payoff or between the PPS and the stock price informativeness about the manager's effort. Intuitively, one may think that when the price is more informative about the manager's effort or the payoff, the PPS will increase. This is indeed true in classical agency models where an equilibrium asset pricing model or an endogenous sensitivity effect is absent. This result does not hold true, however, in our model in which both the sensitivity effect and the informativeness effect are endogenous.

Recall that the stock price informativeness usually decreases with the risk of the normalized price. Hence, when the stock price is more informative, the stock price is usually more sensitive to the effort of the manager. Then, the informativeness effect increases the PPS, but the sensitivity effect reduces it. If the sensitivity effect dominates the informativeness effect, then the PPS will decrease with the stock price informativeness.

When the market maker is risk neutral, we have $\mathrm{Var}(\eta) = \sigma_\theta^2/\beta\lambda$, and the stock price informativeness is determined by $\sigma_\theta^2/\mathrm{Var}(\eta) = \beta\lambda$ according to equation (32). From the proof of Proposition 6, we show that the PPS always decreases with $\beta\lambda$. Then, when the market maker is risk neutral, the relation between the PPS and stock price informativeness is always negative.

When the market maker is risk averse, however, we find both positive and negative relations between the PPS and the stock price informativeness, as obtained empirically by Hartzell and Starks (2003), Kang and Liu (2010), Ferreira et al. (2012), and Firth et al. (2014). Because there are no explicit solutions to β, λ, and ρ^*, we analyze this case by numerical calculations. From Figure 5, when k_s or R_k changes, the optimal PPS increases with the stock price informativeness, and when σ_δ changes, the PPS decreases with the stock price informativeness. For example, when k_s increases, the speculator reduces his effort to collect private information, so the informativeness of the stock price decreases, which decreases the PPS. Meanwhile, when k_s changes, the informativeness effect dominates the sensitivity effect, so the PPS decreases with k_s. Therefore, we achieve a positive relation between the PPS and the informativeness of the stock price, which is consistent with Kang and Liu (2010) and Firth et al. (2014). Similarly, when σ_δ changes, we obtain a negative relation between the PPS and the informativeness of the stock price, which is consistent with Ferreira et al. (2012).

As a robustness check, we also study the relation between the PPS and the fraction of informed trading. Recall that the optimal demand of the speculator is given by $x = \beta(s - \widehat{e})$. We measure the fraction of informed trading as $[\mathrm{Var}(x)/\mathrm{Var}(x+u)]^{1/2}$. Again, both the fraction of informed trading and the PPS are driven by other exogenous parameters.

From Figure 6, the relation between the PPS and the fraction of informed trading is positive driven by σ_u, k_s, or R_k. For example, when R_k increases, the informed trading $\mathrm{Var}(x)$ decreases. Because the noise trading remains the same, the fraction of informed trading then decreases. Both the sensitivity of the stock price to and the informativeness of the stock price about the manager's effort decrease, but the informativeness effect dominates the sensitivity effect. As a result, the PPS decreases with R_k, leading to a positive relation between the PPS and the fraction of informed trading, which is consistent with the empirical result of Hartzell and Starks (2003).

4.4 PPS and liquidity

In this section, we explore the relation between the market liquidity and the optimal PPS. We use $1/\lambda$ to measure the liquidity of the market. Note that both PPS and $1/\lambda$ are endogenous in equilibrium, so the relation between them is driven by exogenous parameters.

From Figure 7, when the relation is driven by σ_θ, σ_δ, or R_k, the relation between the liquidity and the PPS is positive; when it is driven by k_s or σ_u, the relation is negative. For example, when σ_δ increases, the market maker is faced with a higher risk, so the market maker increases the price impact λ. On the other hand, when σ_δ increases, the informativeness effect dominates the sensitivity effect, so the PPS decreases with σ_δ. Therefore, we achieve a positive relation between the PPS and the liquidity, which is consistent with the empirical result of Jayaraman and Milbourn (2011). Empirically, if we control for σ_θ, k_s, σ_u, and R_k, then we expect to find a positive relation between the PPS and the market liquidity.

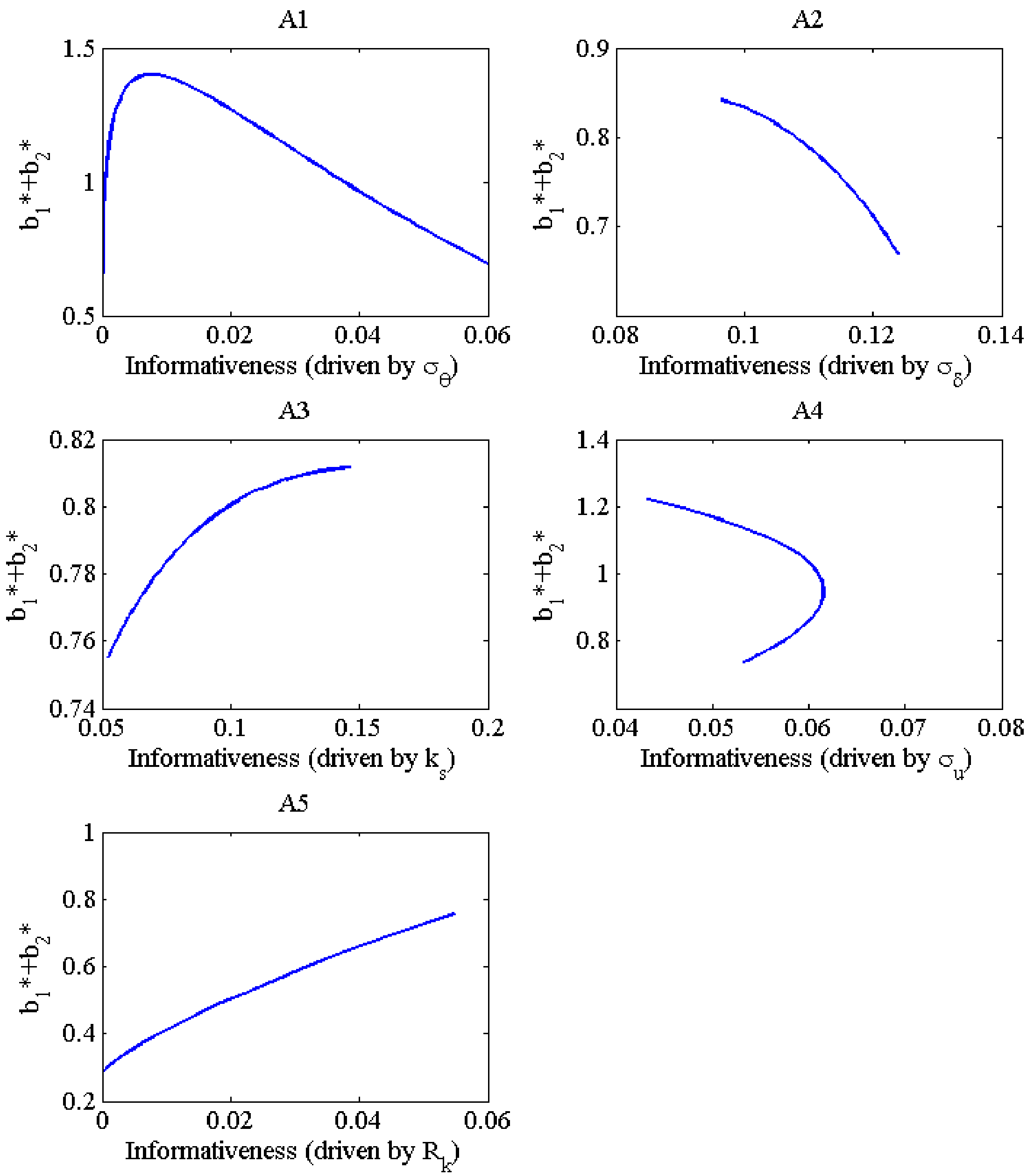

Figure 5: The PPS vs. the price informativeness

Figure 5 illustrates the relation between the PPS $(b_1^* + b_2^*)$ and the price informativeness $\sigma_\theta^2/\text{Var}(\eta)$ under the price-payoff contract when the market maker is risk averse. Subplots A1 to A4 depict the relation between the PPS and the price informativeness driven by σ_θ, σ_δ, k_s, and σ_u. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.1$, when they are not varying in the horizontal axes.

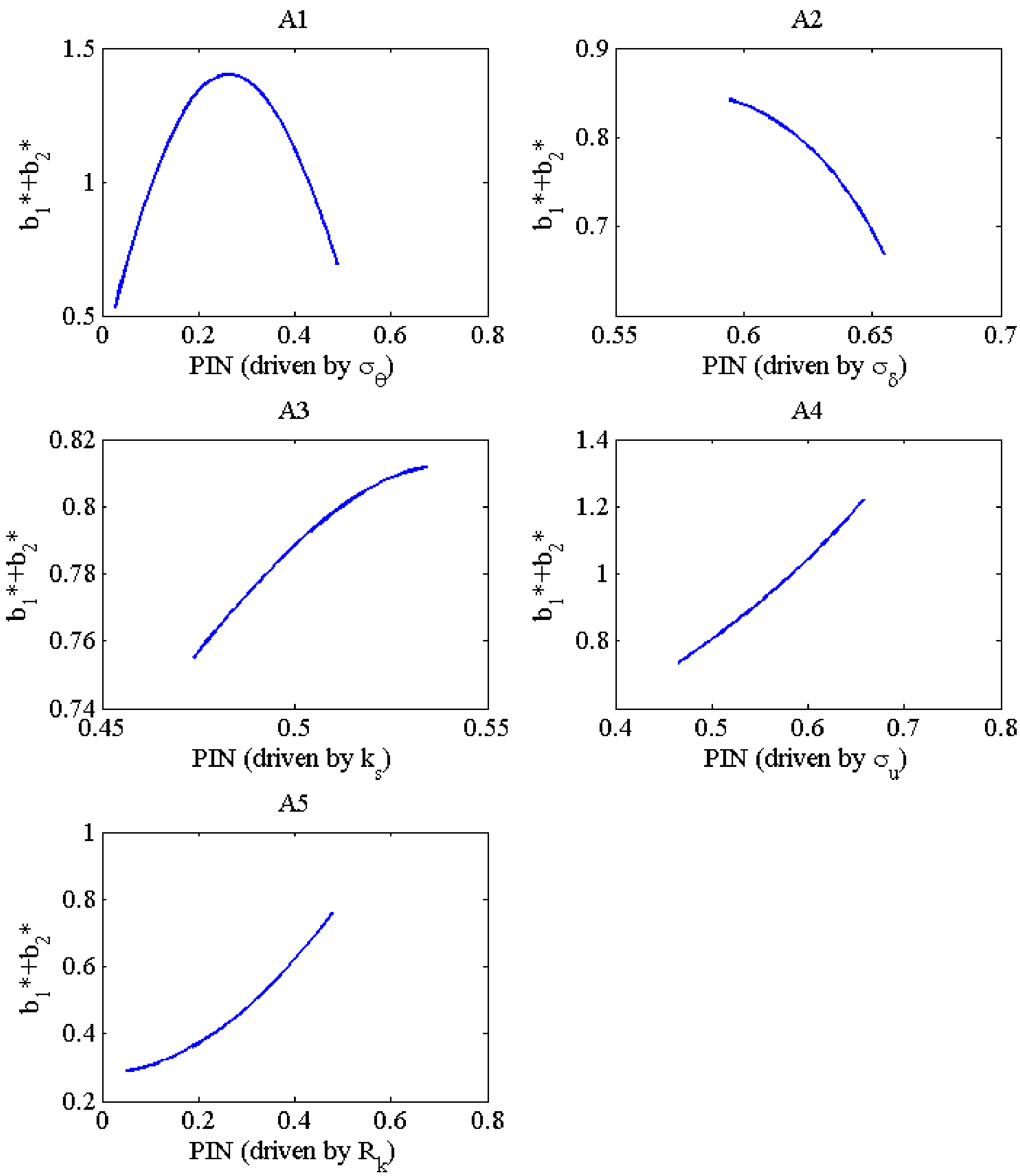

Figure 6: The PPS vs. the fraction of informed trading

Figure 6 illustrates the relation between the PPS $(b_1^* + b_2^*)$ and the fraction of informed trading under the price-payoff contract when the market maker is risk averse. Subplots A1 to A5 depict the relation between the PPS and the fraction of informed trading driven by σ_θ, σ_δ, k_s, σ_u, and R_k. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 0.1$, when they are not varying in the horizontal axes.

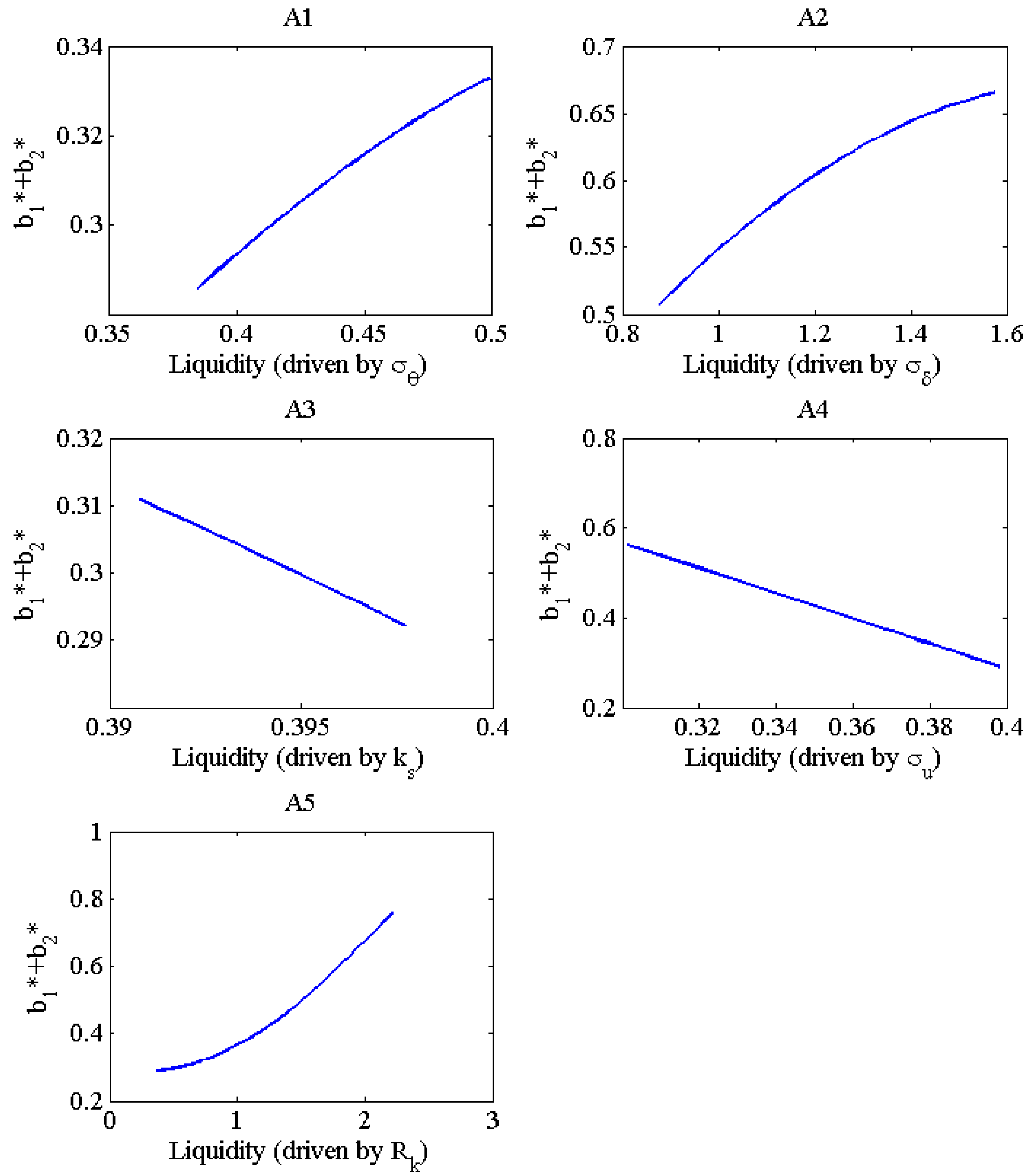

Figure 7: The PPS vs. Liquidity

Figure 7 illustrates the relation between the PPS ($b_1^* + b_2^*$) and the market liquidity $1/\lambda$ under the price-payoff contract when the market maker is risk averse. Subplots A1 to A5 depict the relation between the PPS and the market liquidity driven by σ_θ, σ_δ, k_s, σ_u, and R_k. Other parameters are $R_m = 0.5$, $k_m = 1$, $k_s = 1$, $\sigma_u = 1$, $\sigma_\theta = 1$, $\sigma_\delta = 2$, $R_k = 1$, when they are not varying in the horizontal axes.

5 Concluding remarks

In this paper, we employ a simplified version of the Holmstrom-Tirole (1993) model with one extension in which the market maker is risk averse rather than risk neutral. We identify two effects that determine the optimal contract and the manager's optimal effort: the sensitivity effect and the informativeness effect. We find that both effects are endogenous and often move in the opposite directions.[13] As a result, there is a tradeoff between them. This tradeoff allows us to derive many interesting results, such as both the positive and the negative relations between the PPS and the stock price informativeness, between the PPS and the stock price volatility, between the PPS and the expected firm value, as well as a positive relation between the PPS and the stock liquidity. These results shed light on a number of empirical findings.

[13]The tradeoff between the sensitivity effect and the informativeness effect also exists when we combine a model of agency with a model of competitive asset pricing under asymmetric information, such as Grossman and Stiglitz (1980) or Hellwig (1980).

6 Appendix

6.1 Proof of Proposition 1

6.1.1 Market maker

Note that participants in the stock market do not observe the manager's effort, e, so they have to make decisions based on their beliefs about it, $\widehat{e}$. Besides, the market maker believes that the speculator's trading strategy is

$$x = \beta_m (s - \widehat{e}). \tag{49}$$

The speculator chooses the optimal demand, x, based on his signal, $s(\rho^*)$, the market maker makes decisions based on his belief about it, ρ_m. Likewise, β depends on ρ^*, so the market maker also has his belief about it, $\beta_m = \beta(\rho_m)$.

According to equation (7), the market maker sets the stock price so that his certainty equivalent profit is equal to zero:

$$\begin{aligned} -1 &= \mathbb{E}(U_k|x+u) = \mathbb{E}[-\exp(-R_k W_k)|x+u] && (50)\\ &= -\exp\left[\mathbb{E}(-R_k W_k|x+u) + \frac{1}{2}\mathrm{Var}(-R_k W_k|x+u)\right] && (51)\\ &= -\exp\left\{-R_k\left[\mathbb{E}(W_k|x+u) - \frac{1}{2}R_k\mathrm{Var}(W_k|x+u)\right]\right\}, && (52) \end{aligned}$$

where $W_k = (x+u)(P-v)$. Since the market maker observes only the total order flow, P is a function of $(x+u)$. Therefore, conditional on $(x+u)$, W_k follows a normal distribution, so equation (50) and equation (51) are equivalent. From equation (52), we have that

$$\begin{aligned} 0 &= \mathbb{E}(W_k|x+u) - \frac{1}{2}R_k\mathrm{Var}(W_k|x+u)\\ &= \mathbb{E}[(x+u)(P-v)|x+u] - \frac{1}{2}R_k\mathrm{Var}[(x+u)(P-v)|x+u]\\ &= (x+u)\left[\mathbb{E}[P-v|x+u] - \frac{1}{2}R_k(x+u)\mathrm{Var}(P-v|x+u)\right]\\ &= (x+u)\left[P - \mathbb{E}\left(v|x+u\right) - \frac{1}{2}R_k(x+u)\mathrm{Var}(v|x+u)\right]. \end{aligned}$$

It follows immediately that

$$P = \mathbb{E}(v|x+u) + \frac{1}{2}R_k(x+u)\mathrm{Var}(v|x+u) = \widehat{e} + \lambda_m(x+u),$$

where λ_m is given by

$$\lambda_m = \frac{\beta_m \sigma_\theta^2}{\beta_m^2 \sigma_\theta^2 (1/\rho_m + 1) + \sigma_u^2} + \frac{R_k}{2}\left[\sigma_\theta^2 + \sigma_\delta^2 - \frac{\beta_m^2 \sigma_\theta^4}{\beta_m^2 \sigma_\theta^2 (1/\rho_m + 1) + \sigma_u^2}\right]. \tag{53}$$

6.1.2 Speculator's optimal demand given ρ

Recall that the speculator's trading strategy is

$$x = \beta(s - \widehat{e}). \tag{54}$$

The speculator's profit is then given by

$$\pi = x(v - P) - C(\rho) = x[v - \widehat{e} - \lambda_m(x + u)] - C_s(\rho).$$

His conditional expected profit can be obtained:

$$\mathbb{E}(\pi|s) = x[\mathbb{E}(v|s) - \widehat{e} - \lambda_m x] - C_s(\rho), \tag{55}$$

where

$$\mathbb{E}(v|s) = \widehat{e} + \frac{\rho}{1+\rho}(s - \widehat{e}). \tag{56}$$

By maximizing the speculator's conditional expected profit in equation (55), his optimal trading strategy is obtained as follows:

$$x = \frac{\mathbb{E}(v|s) - \widehat{e}}{2\lambda_m} = \frac{\rho}{2\lambda_m(1+\rho)}(s - \widehat{e}). \tag{57}$$

From equation (57), we obtain the optimal trading intensity:

$$\beta = \frac{\rho}{2\lambda_m(\rho + 1)}. \tag{58}$$

6.1.3 Optimal ρ^*

From equations (54), (55) and (56), the speculator's expected profit is given by

$$\mathbb{E}(\pi) = \beta\sigma_\theta^2 - \beta^2 \lambda_m \sigma_\theta^2 (1 + 1/\rho) - C_s(\rho). \tag{59}$$

Substituting β in equation (58) into equation (59) and taking the derivative of equation (59) with respect to ρ, the FOC is given by:

$$\sigma_\theta^2 - 4k_s\lambda_m\rho^*(\rho^* + 1)^2 = 0. \tag{60}$$

6.1.4 Uniqueness of the equilibrium

Note that $\rho_m = \rho^*$, $\beta_m = \beta$, and $\lambda_m = \lambda$ in equilibrium. The optimal β, λ, and ρ^* are determined by the simultaneous equations of (53), (58), and (60). Based on equation (60), we have

$$\lambda = \frac{\sigma_\theta^2}{4k_s\rho(\rho+1)^2}. \tag{61}$$

Substituting the above equation into β in equation (58), we then have

$$\beta = \frac{2k_s\rho^2(\rho+1)}{\sigma_\theta^2}. \tag{62}$$

Substituting equations (61) and (62) into equation (53), we have

$$\begin{aligned}&\frac{2R_k k_s\rho(\rho+1)^2\sigma_\delta^2}{\sigma_\theta^2} + \frac{k_s^2\rho^3(\rho+1)^3(8R_k k_s\rho^2 + 8R_k k_s\rho + 4)}{\sigma_\theta^2\sigma_u^2} \\ &+\frac{8R_k k_s^3\rho^4\sigma_\delta^2(\rho+1)^5}{\sigma_\theta^4\sigma_u^2} + 2R_k k_s\rho(\rho+1)^2 = 1.\end{aligned} \tag{63}$$

Therefore, the optimal ρ^* is determined by equation (63). The left-hand side of equation (63) is a monotonically increasing function of ρ, so there is a unique solution to the optimal ρ^*. Given the uniqueness of ρ^*, it is easy to prove the uniqueness of β and λ.

6.2 Proof of Proposition 3

Based on equation (63), we can show that ρ^* decreases with R_k, k_s, or σ_δ but increases with σ_θ or σ_u. For example, when σ_u increases, the denominators in the second and the third terms of equation (63) increase. Then, ρ^* must increase accordingly, otherwise the equation does not hold any more.

From equation (63), when σ_u or σ_θ goes to infinity, ρ^* converges to a constant. For example, when σ_u goes to infinity, if ρ^* were to go to infinity as well, then the first term and the fourth term of equation (63) would go to infinity and the equation would not hold. Therefore, the optimal ρ^* must converge to a constant.

Note that $\eta = s + u/\beta$. Substituting equation (62) into η, we have

$$\mathrm{Var}(\eta) = \frac{\sigma_\theta^2(\rho+1)}{\rho} + \frac{\sigma_\theta^4\sigma_u^2}{4k_s^2\rho^4(\rho+1)^2}. \tag{64}$$

Since ρ^* decreases with R_k or σ_δ, according to equation (64), we have that $\mathrm{Var}(\eta)$ increases with R_k or σ_δ.

From equation (62), we obtain

$$k_s = \frac{\beta \sigma_\theta^2}{2\rho^2(\rho+1)}. \tag{65}$$

Note that the optimal ρ^* decreases with k_s. Substituting equation (65) into equation (63), it can be shown that β increases with ρ^* and thus decreases with k_s. Hence, both $\mathrm{Var}(s)$ and $\mathrm{Var}(u/\beta)$ increase with k_s. Therefore, $\mathrm{Var}(\eta)$ increases with k_s.

From equation (63), when σ_u goes to zero, ρ goes to zero. Then, the first term in equation (64) goes to infinity. When σ_u goes to infinity, ρ goes to a constant, so β goes to a constant from equation (62). Then the second term in equation (64) goes to infinity. Therefore, when σ_u goes to zero or infinity, $\mathrm{Var}(\eta)$ goes to infinity. Considering the second term in equation (64) and letting $n = \sigma_\theta^4 \sigma_u^2/[4k_s^2\rho^4(\rho+1)^2]$, we have that σ_u is a function of n, $\sigma_u(n)$. Substitute $\sigma_u(n)$ into equation (63), solve for n, and substitute n into equation (64). Taking the derivative of $\mathrm{Var}(\eta)$ with respect to ρ, we find that $\mathrm{Var}(\eta)$ first decreases and then increases with σ_u.

Considering the first term in equation (64) and letting $n = \sigma_\theta^2(\rho+1)/\rho$, we have that ρ is a function of n, $\rho(n)$. Substituting $\rho(n)$ into equation (63), we have that the first term in equation (64) increases with σ_θ and that it is a convex function of σ_θ. Using similar procedures, we can show that the second term in equation (64) first decreases and then increases with σ_θ and that it is also a convex function of σ_θ. Therefore, when σ_θ goes to zero or infinity, $\mathrm{Var}(\eta)$ goes to infinity. In addition, $\mathrm{Var}(\eta)$ first decreases and then increases with σ_θ.

6.3 Proof of Proposition 4

Given the compensation contract W_m, which follows a normal distribution, the maximization problem of the manager's expected utility is equivalent to

$$\max_e \ \mathbb{E}(W_m) - \frac{1}{2}R_m \mathrm{Var}(W_m) - \frac{1}{2}k_m e^2.$$

Note that

$$
\begin{aligned}
& \mathbb{E}(W_m) - \frac{1}{2}R_m \mathrm{Var}(W_m) - \frac{1}{2}k_m e^2 \\
= \quad & a + b_1[\widehat{e}(1-\beta\lambda) + \beta\lambda e] + b_2 e \\
& -\frac{1}{2}R_m[b_1^2 \mathrm{Var}(P) + b_2^2 \mathrm{Var}(v) + 2b_1 b_2 \mathrm{Cov}(P,v)] - \frac{1}{2}k_m e^2. \qquad (66)
\end{aligned}
$$

Taking the derivative of the above equation with respect to e, we obtain the manager's optimal effort:

$$
e^* = (b_2 + b_1\beta\lambda)/k_m. \qquad (67)
$$

Since we assume the reservation utility of the manager to be zero, the participation constraint of the manager is given by

$$
\begin{aligned}
& a + b_1[\widehat{e}(1-\beta\lambda) + \beta\lambda e] + b_2 e \\
& -\frac{1}{2}R_m[b_1^2 \mathrm{Var}(P) + b_2^2 \mathrm{Var}(v) + 2b_1 b_2 \mathrm{Cov}(P,v)] - \frac{1}{2}k_m e^2 = 0. \qquad (68)
\end{aligned}
$$

Then, the expected wealth of the entrepreneur is given by

$$
e - \frac{1}{2}R_m[b_1^2 \mathrm{Var}(P) + b_2^2 \mathrm{Var}(v) + 2b_1 b_2 \mathrm{Cov}(P,v)] - \frac{1}{2}k_m e^2.
$$

Note that $\mathrm{Var}(P) = \beta^2\lambda^2(\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2)$, $\mathrm{Var}(v) = \sigma_\theta^2 + \sigma_\delta^2$, and $\mathrm{Cov}(P,v) = \beta\lambda\sigma_\theta^2$. The optimal b_1 and b_2, which maximize the expected wealth of the entrepreneur, are obtained as follows:

$$
\begin{aligned}
b_1 &= \frac{1}{\beta\lambda} \frac{1}{1 + R_m k_m \left(\sigma_\delta^2 + \sigma_\theta^2 - \dfrac{\sigma_\delta^4}{\sigma_\delta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2}\right)} \frac{\sigma_\delta^2}{\sigma_\epsilon^2 + \sigma_\delta^2 + \sigma_u^2/\beta^2}, \\
b_2 &= \frac{1}{1 + R_m k_m \left(\sigma_\delta^2 + \sigma_\theta^2 - \dfrac{\sigma_\delta^4}{\sigma_\delta^2 + \sigma_\epsilon^2 + \sigma_u^2/\beta^2}\right)} \frac{\sigma_\delta^2}{\sigma_\epsilon^2 + \sigma_\delta^2 + \sigma_u^2/\beta^2}.
\end{aligned}
$$

6.4 Proof of Proposition 5

Similar to the case of the price-payoff contract, the maximization of the manager's expected utility is equivalent to

$$
\max_e \ \mathbb{E}(W_m) - \frac{1}{2}R_m \mathrm{Var}(W_m) - \frac{1}{2}k_m e^2.
$$

Note that

$$\begin{aligned}
& \mathbb{E}(W_m) - \frac{1}{2}R_m \mathrm{Var}(W_m) - \frac{1}{2}k_m e^2 \\
= \quad & a + b[\widehat{e} + \lambda\beta(e - \widehat{e})] - \frac{1}{2}R_m b^2 \mathrm{Var}(P) - \frac{1}{2}k_m e^2.
\end{aligned}$$

Taking the derivative of the above equation with respect to e yields the manager's optimal effort:

$$e^* = b\beta\lambda/k_m.$$

The participation constraint of the manager is given by

$$a + b[\widehat{e} + \lambda\beta(e - \widehat{e})] - \frac{1}{2}R_m b^2 \mathrm{Var}(P) - \frac{1}{2}k_m e^2 = 0.$$

The expected wealth of the entrepreneur is then given by

$$e - \frac{1}{2}R_m b^2 \mathrm{Var}(P) - \frac{1}{2}k_m e^2.$$

Note that $\mathrm{Var}(P) = \beta^2\lambda^2\mathrm{Var}(\eta)$. Considering the FOC of the above equation, the optimal b^* is obtained as follows:

$$b^* = \frac{1}{\beta\lambda[1 + R_m k_m \mathrm{Var}(\eta)]}.$$

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