A Theory of Conversations in Financial Markets*

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Abstract

We develop a theory based on Kyle (1985) to show that it can be incentive-compatible for competing investors to exchange information if they can subsequently coordinate trades. In our model, as information exchange turns competitors into allies, allied members enjoy the benefits of information advantage and monopolistic power, though there can also be direct costs related to search, setup, and coordination efforts, and indirect costs from market liquidity dry-ups. Such a trade-off determines the coalition structure of the economy. As allied members behave more monopolistically, communication has negative effects on price informativeness and market liquidity. We further apply our theory to obtain novel insights about the structure of the asset management industry, in which a company can be interpreted as a coalition of professional money managers.

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1 Introduction

Stein (2008) points out an interesting but puzzling phenomenon in financial markets. Contrary to conventional wisdom, competing investors often share information about trading opportunities even though this lowers their information advantage. What's more, receivers of this information may even use it when trading against those who provided it.[1] The literature on information structure in financial markets, however, often neglects this phenomenon (see, e.g., Colla and Mele, 2010; Ozsoylev and Walden, 2011; and Han and Yang, 2013, which we review later in this section).

In this article, we develop a theory to show that it can be incentive-compatible for competing investors to exchange information. Our model is based on a multi-trader generalization of Kyle (1985). The elements are a risky asset (the stock), multiple informed traders, a market maker, and a liquidity trader. The key ingredient of our model is that if informed traders choose to exchange information, then they can further coordinate trades. As competitors become allies, informed traders in the coalition not only improve their information advantage, but also achieve monopolistic power in trading. However, the coalition has two types of costs. First, there can be direct costs, such as costs related to search, setup, and coordination efforts. Second, there can be an indirect cost. Specifically, as the coalition gains monopolistic power and behaves more strategically, the market

[1] Empirical research usually demonstrates this phenomenon by testing investors' trading behavior. Regarding professional money managers, Hong, Kubik, and Stein (2005) show that mutual fund managers are more likely to buy or sell a particular stock if other managers in the same city are buying or selling that stock, which suggests that investors spread information about stocks to one another by word of mouth. Cohen, Frazzini, and Malloy (2008) show that mutual fund managers are more likely to buy stocks of firms that have board members of the same educational background.

Regarding individual investors, Grinblatt and Keloharju (2001) show that investors are more likely to hold, buy, and sell stocks of firms that are located close to them, communicate in their native tongue, and have chief executives of the same cultural background. Hong, Kubik, and Stein (2004) show that social households–those who interact with their neighbors or attend church–are more likely to invest in the stock market when their peers participate. Ivković and Weisbenner (2007) show that households are more likely to buy stocks from an industry if their neighbors are buying stocks from the same industry.

maker lowers market liquidity to break even. This dries up market liquidity.

In our model, whether to exchange information and subsequently coordinate trades is an endogenous decision that informed traders make after weighing the benefits and costs. Depending on this tradeoff, it is possible that all informed traders form a comprehensive coalition. It is also possible that some informed traders form several smaller coalitions, while other informed traders choose to remain independent.

Our study provides several novel insights that contribute to the literature on information structure in financial markets. First, we give a rational theory to show that it can be incentive compatible for competing investors to exchange information. Earlier studies in this literature examine information transmission from a sender to non-competing receivers. For example, in Admati and Pfeiderer (1986, 1988, and 1990), the sender does not trade herself, but sells her information to receivers for a fee. In Benabou and Laroque (1992) and Van Bommel (2003), the sender tips off the receivers, who then might move the price too far away from the fundamental level. This gives the sender an opportunity to make a profit.

Recent studies in this literature, including Colla and Mele (2010), Ozsoylev and Walden (2011), and Han and Yang (2013), recognize that there can be information exchange among competing investors. However, these studies are primarily interested in the implication of this communication for financial market outcomes. They typically assume that there is exogenously given information exchange among competing investors through certain connections.

Stein (2008) is probably the first to ask why competing investors exchange information in the first place, given that this information can be used against them. He provides a general model of conversations with competitors, in which future ideas depend on the truthful communication of earlier ideas. His model, which is based on the standing-on-theshoulders-of-others production complementarity, applies well to entrepreneurs in Silicon Valley. Ours makes more sense as a model of conversations among professional money managers. In our model, information exchange leads to trade coordination, which turns competitors into allies. The coalition benefits not only from information advantage, but also from monopolistic power in trading.

Second, our predictions on the effects of communication among competing investors on financial market outcomes contrast with predictions of existing theories in this literature. Specifically, in our model, exchanging information and subsequently coordinating trades allows the allied informed traders to gain monopolistic power. As they behave more strategically, stock prices become less informative. Moreover, the market maker lowers market liquidity to break even.

In Colla and Mele (2010) and Ozsoylev and Walden (2011), after exchanging information through exogenously given connections, informed traders try to preempt each other. As they trade more competitively, the informativeness of stock prices and market liquidity improve. Han and Yang (2013) take into consideration investors' incentives to acquire information. They show that investors may find it too costly to acquire information by themselves if they can free-ride peers through exogenously given connections. Thus, communication can lower the total amount of information in the financial markets and, thereafter, the informativeness of stock prices. Empirically, Han and Yang's prediction on the negative relation between communication and the informativeness of stock prices is more applicable to small stocks, young stocks, and risky stocks, for which information acquisition costs are high.

Finally, our model produces interesting predictions on the coalition structure of investors. First, we show that a coalition is likely to be formed by informed traders with the best-quality information. Second, a coalition, if formed, must be sufficiently large.

Third, as the average information quality increases, the coalition size (i.e., the number of informed traders in a coalition) increases, but the number of coalitions decreases. Fourth, a small number of large coalitions are likely to emerge from the group of informed traders with good-quality information and the group of informed traders with a large population.

These predictions have profound implications for the structure of the asset management industry, in which a company can be interpreted as a coalition of professional money managers.[2] For example, since a big asset management company is likely to include fund managers with the best-quality information, mutual funds affiliated with big asset management companies tend to outperform those affiliated with small asset management companies. This implication is consistent with the empirical findings of Chen, Hong, Huang, and Kubik (2004), Pollet and Wilson (2008), and Bhojraj, Cho, and Yehuda (2012). Moreover, as the advance of information technology expedites information processing and dissemination, which improves the average information quality, in recent decades industry consolidation has led to the emergence of super big asset management companies such as BlackRock.

To our best knowledge, we are the first to model an asset management company as a coalition of professional money managers. We may even be the first to look at the structure of the asset management industry. Existing theories on financial intermediaries, including Diamond and Dybvig (1983), Jacklin (1987), Diamond (1997), and Allen and Gale (1997), focus mainly on the banking industry. Existing theories on delegated asset management typically take the structure of asset management as a given and study how to incentivize professional money managers.[3]

[2]Consistent with this view, Bhojraj, Cho, and Yehuda (2012) show that sibling mutual funds in the same company tend to share information. Nanda, Wang, and Zheng (2004), Gaspar, Massa, and Matos (2006), Elton, Gruber, and Green (2007), and Bhattacharya, Lee, and Pool (2013) show that sibling mutual funds in the same company tend to coordinate trades.

[3]See, for example, Ou-Yang (2003), Palomino and Prat (2003), Gervais, Lynch, and Musto (2005),

We organize the rest of the article as follows. Section 2 describes our model. Section 3 solves for the equilibrium and then derives its properties. Section 4 applies the predictions of our model to the asset management industry. Section 5 concludes.

2 The Model

Our model is a multi-trader generalization of Kyle (1985).[4] Consider a simple exchange economy. There are three dates, -1, 0, and 1. There is one risky asset, a stock. The stock pays $\bar{F} + X$ at date 1. $\bar{F}$ is a positive constant. X is a random variable that follows a standard normal distribution; that is, $X \sim N(0, 1)$. The stock price at date 0 is denoted as P, which is to be determined.

There are three types of risk-neutral players. First, there are J informed traders. At period 0, each informed trader, indexed by j, observes a private signal about the payoff of the stock, $S_j = X + \epsilon_j$. ϵ_j follows a normal distribution with mean zero and variance v_j; that is, $\epsilon_j \sim N(0, v_j)$. She submits a market order, D_j, which is to be determined. We assume that $\forall i \neq j$, ϵ_i is independent of ϵ_j.

Second, there is a liquidity trader. She submits a market order, Z, which follows a normal distribution with mean zero and variance σ_z^2; that is, $Z \sim N(0, \sigma_z^2)$. We assume that Z is independent of X and ϵ_j, $\forall j$.

Third, there is a market maker. She receives the aggregate market order, $D = \sum_j D_j + Z$, and sets the stock price, P. The market maker faces perfect competition from other market makers, so she has zero expected profits. This implies that $P = \bar{F} + E\Big[X|D\Big]$.

At date -1, informed traders decide whether to exchange information with peers. If

Dybvig, Farnsworth, and Carpenter (2010), Vayanos and Woolley (2013), and He and Xiong (2013).

[4] Kyle's (1985) setup provides analytical tractability and has been used extensively in the microstructure literature (e.g., Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996; and Back, Cao, and Willard, 2000).

they do, then they also coordinate trades. A coalition, which does not necessarily include all informed traders, is indicated by a set A.

Here we assume that within a coalition, informed traders will tell each other their information truthfully. There can be different mechanisms to ensure this. For example, suppose that the coalition is formed among neighbors or alumni. In this case, there can be a severe reputation loss if a member is caught in a lie. It is also possible that allied members' payoffs are perfectly aligned, so they have no incentives to lie. This tends to happen among professional money managers in the same asset management company because their compensations are typically linked to company performance.

Coordination of trades is a key ingredient of our model. Stein (2008) points out that in general, if informed traders cannot coordinate trades, then they will not exchange information either. The intuition for this is that information exchange will lower their information advantage. What's more, receivers of this information may even use it when trading against those who provided it. Colla and Mele (2010) explain this intuition in more detail. They argue that private information gives informed traders monopolistic power. If informed traders exchange information but don't coordinate trades, then their monopolistic power weakens, so they will trade more aggressively to preempt peers. It can be shown that their payoff will decrease. Therefore, they will not exchange information in the first place.

The activity of exchanging information and subsequently coordinating trades in a coalition, A, causes direct costs, $C(A)$, due to search, setup, and coordination efforts. In general, $C(A)$ increases with the extent and complexity of the coalition, A, which can be measured using the number of informed traders in the coalition. Allied members in the coalition share the costs and benefits of the coalition based on the Shapley (1953) value, which captures their contributions to the coalition. In our model, an allied member's

Shapley value is proportional to her information precision, $1/v_j$.

We summarize the timeline of our model as follows.

- Date -1: (Coalition Formation Stage) Informed traders decide whether to form a coalition, A, which does not necessarily include all informed traders.
- Date 0: (Trading Stage) Each informed trader, j, receives her private information, S_j. Informed traders in the coalition, $j \in A$, share their private information, S_j, and coordinate trades. The coalition submits a market order, D_A. Each independent informed trader, $j \notin A$, submits a market order, D_j, based on her private information, S_j. The liquidity trader submits a market order, Z. After receiving the aggregate market order, $D = D_A + \sum_{j \notin A} D_j + Z$, the market maker sets the stock price, P.
- Date 1: (Final Stage) The stock pays $\bar{F} + X$, which is distributed to every player according to her holdings.

In our model, whether to exchange information and subsequently coordinate trades in a coalition is an endogenous decision that informed traders make after weighing the benefits and costs. Depending on this tradeoff, it is possible that all informed traders form a comprehensive coalition. It is also possible that some informed traders form several smaller coalitions, while other informed traders choose to remain independent.

3 The Equilibrium

In what follows, we solve for the equilibrium using backward induction. In Section 3.1, we focus on the trading stage at date 0. We suppose that a coalition, A, has already been formed and solve for every player's optimal trading strategy. Our analysis can easily be extended to include multiple coalitions. In Section 3.2, we focus on the coalition formation stage at date -1, where we study whether informed traders form a coalition, how many coalitions can be formed, and how big a coalition can be.

3.1 Trading Stage

3.1.1 Trading with No Coalition

Before jumping to the case with coalitions, we first study a benchmark case in which there is no coalition, so all informed investors compete against each other.

Proposition 1. *Consider the case with no coalition of informed traders, which is denoted by* (NA)*. There is a linear equilibrium in which the market maker sets the stock price according to* $P(NA) = \bar{F} + \lambda(NA)D$*, and each informed trader submits a market order* $D_j(NA) = \theta_j(NA)S_j$*.* $\lambda(NA)$ *and* $\theta_j(NA)$ *are given by:*

$$\begin{aligned}\lambda(NA) &= \frac{K(NA)/\sigma_z}{1 + \sum\limits_j \dfrac{1}{1+2v_j}}, \\ \theta_j(NA) &= \frac{\sigma_z/K(NA)}{1+2v_j},\end{aligned}$$

where $K(NA) = \sqrt{\sum\limits_j \dfrac{1+v_j}{(1+2v_j)^2}}$.

Proof: See the Appendix.

To understand this equilibrium, we consider the following special case, which Holden and Subrahmanyam (1992) have studied. The intuitions we obtain in this special case help us understand the equilibrium when there are coalitions of informed traders.

A special case: Let $v_j = V$, $\forall j$, so every informed trader has the same-quality information. Denote the informativeness of the stock price as $\mathrm{Var}(X|P)$. It is straightforward to show that

$$\begin{aligned}
\mathrm{Var}(X|P(NA)) &= \frac{1}{1+J/(1+2V)}, \\
\theta_j(NA) &= \frac{\sigma_z}{\sqrt{J(1+V)}}, \\
\lambda(NA) &= \frac{\sqrt{J(1+V)}/\sigma_z}{1+2V+J}.
\end{aligned}$$

A decrease in J has three consequences. First, the informativeness of the stock price decreases (high $\mathrm{Var}(X|P)$). There are two reasons for this. One reason is that as J decreases, the sources of information decrease, which lowers the total amount of information in the market. The other reason is that there is less competition among informed traders, so they behave more monopolistically. Their trades reveal less information to the market. Second, informed traders trade more aggressively (high θ_j). This is because as the competition decreases, informed traders are less subject to the winner's curse problem. Third, market liquidity decreases (high λ) for sufficiently large J, such as $J > 1 + 2V$. This is because as informed traders behave more monopolistically, the market maker must increase λ to break even.

An increase in informed traders' information quality (i.e., V decreases) also has three consequences. First, the price informativeness increases (low $\mathrm{Var}(X|P)$). This is because informed traders reveal more precise information through trades to the stock price. Second, informed traders trade more aggressively (high θ_j). This is because they are more confident about their own information. Third, market liquidity decreases (high λ) if informed traders' information is not very precise, such as $3 + 2V > J$. This is because the market maker faces a more severe information disadvantage and must increase λ to break even.

3.1.2 Trading with a Coalition

Suppose that a coalition of some informed traders, A, has been formed. Denote

$$S_A = \frac{\sum_{j \in A} \frac{S_j}{v_j}}{\sum_{j \in A} \frac{1}{v_j}} = X + \epsilon_A.$$

$\epsilon_A = \dfrac{\sum_{j \in A} \frac{\epsilon_j}{v_j}}{\sum_{j \in A} \frac{1}{v_j}}$ follows a normal distribution with mean zero and variance $v_A = 1 \Big/ \sum_{j \in A} \frac{1}{v_j}$; that is, $\epsilon_A \sim N(0, v_A)$.

Lemma 1. *As far as X is concerned, S_A is a sufficient statistic of $\{S_j : j \in A\}$.*

Proof: See the Appendix.

An implication of this lemma is that we can treat the coalition, A, as one informed trader who observes a private signal S_A. In the following proposition, we use this idea to describe the coalition's demand for the stock.

Proposition 2. *Consider the case with a coalition of informed traders, which is denoted by (A). There is a linear equilibrium in which the market maker sets the stock price according to $P(A) = \bar{F} + \lambda(A)D$, the coalition submits a market order $D_A(A) = \theta_A(A)S_A$, and every other informed trader, $j \notin A$, submits a market order $D_j(A) = \theta_j(A)S_j$. $\lambda(A)$, $\theta_A(A)$, and $\theta_j(A)$ are given by:*

$$\lambda(A) = \frac{K(A)/\sigma_z}{1 + \frac{1}{1+2v_A} + \sum_{j \notin A} \frac{1}{1+2v_j}},$$

$$\theta_A(A) = \frac{\sigma_z/K(A)}{1+2v_A},$$
$$\theta_j(A) = \frac{\sigma_z/K(A)}{1+2v_j}, \ \forall j \notin A,$$

where $K(A) = \sqrt{\frac{1+v_A}{(1+2v_A)^2} + \sum_{j\notin A} \frac{1+v_j}{(1+2v_j)^2}}$.

Proof: See the Appendix.

The following corollary describes the monotonic properties of the equilibrium with respect to the coalition size.

Corollary 1. *Suppose that the coalition expands from* A *to* A'*, where* $A' = A \cup \{b\}$ *and* $b \notin A$*. Then,*

(i) the stock price becomes less informative; that is, $\mathrm{Var}(X|P(A')) > \mathrm{Var}(X|P(A))$*;*

(ii) market liquidity decreases; that is, $\lambda(A') > \lambda(A)$*;*

(iii) informed traders bid more aggressively; that is, $\theta_{A'}(A') > \theta_A(A)$ *and* $\theta_j(A') > \theta_j(A)$*,* $\forall j \notin A'$*.*

Proof: See the Appendix.

Intuitively, the expansion of the coalition reduces the effective number of informed traders. Then, as the special case in the last subsection suggests, this reduces the intensity of competition among informed traders. As informed traders both inside and outside the coalition behave more monopolistically, they trade more aggressively but release less information to the market. Therefore, the stock price becomes less informative. The market maker must increase λ, the sensitivity of the stock price to the market order she receives, to break even. This lowers market liquidity.

3.2 Coalition Formation Stage

To study the coalition formation, we need to look at the effects of a coalition of informed traders, A, on each player's ex ante (date -1) payoff. An immediate observation is that the coalition has no effect on the market maker's ex ante payoff because on average, her payoff is always 0.

For other players, in the case with no coalition of informed traders, denote $E\Pi_j(NA)$ and $E\Pi_{\mathrm{L}}(NA)$ as the ex ante payoffs to each informed trader and the liquidity trader. It follows from the proof of Proposition 1 that

$$\begin{aligned} E\Pi_j(NA) &= E\Big[E\Big[(\bar{F}+X-P(NA))\cdot D_j(NA)\Big|S_j\Big]\Big] = \lambda(NA)\theta_j^2(NA)(1+v_j), \\ E\Pi_{\mathrm{L}}(NA) &= E\Big[(\bar{F}+X-P(NA))\cdot Z\Big] = -\lambda(NA)\sigma_z^2. \end{aligned}$$

In the case with a coalition of informed traders, A, denote $E\Pi_A(A)$, $E\Pi_j(A)$ ($\forall j \notin A$), and $E\Pi_{\mathrm{L}}(A)$ as the ex ante payoffs to the coalition, each independent informed trader, and the liquidity trader. It is straightforward to show that

$$\begin{aligned} E\Pi_A(A) &= E\Big[E\Big[(\bar{F}+X-P(A))\cdot D_A(A)\Big|S_A\Big]\Big] = \lambda(A)\theta_A^2(A)(1+v_A), \\ E\Pi_j(A) &= E\Big[E\Big[(\bar{F}+X-P(A))\cdot D_j(A)\Big|S_j\Big]\Big] = \lambda(A)\theta_j^2(A)(1+v_j), \forall j \notin A, \\ E\Pi_{\mathrm{L}}(A) &= E\Big[(\bar{F}+X-P(A))\cdot Z\Big] = -\lambda(A)\sigma_z^2. \end{aligned}$$

Use $\Gamma_A(A)$, $\Gamma_j(A)$ ($\forall j \notin A$), and $\Gamma_{\mathrm{L}}(A)$ to describe the effects of the coalition on the ex ante payoffs to the coalition, each independent informed trader, and the liquidity trader.

$$\begin{aligned} \Gamma_A(A) &= E\Pi_A(A) - \sum_{j\in A} E\Pi_j(NA), \\ \Gamma_j(A) &= E\Pi_j(A) - E\Pi_j(NA), \;\; \forall j \notin A, \end{aligned}$$

$$\Gamma_{\rm L}(A) \;=\; E\Pi_{\rm L}(A) - E\Pi_{\rm L}(NA).$$

Corollary 2. *(i) The coalition increases the ex ante payoff to independent informed traders (if any); that is,* $\Gamma_j(A) > 0$, $j \notin A$.

(ii) The coalition decreases the ex ante payoff to the liquidity trader; that is, $\Gamma_{\rm L}(A) < 0$.

Proof: See the Appendix.

A coalition has three effects on an independent informed trader's ex ante payoff. First, it reduces competition among informed traders, which increases her payoff. Second, it represents a stronger competitor, which decreases her payoff. Third, it leads to dry-ups of market liquidity, which also decreases her payoff. Part (i) of Corollary 2 suggests that the first effect dominates the other two effects. The net effect of a coalition on an independent informed trader's payoff is positive.

Part (ii) of Corollary 2 suggests that a coalition has a negative effect on the liquidity trader's payoff. This is because the coalition causes dry-ups of market liquidity.

Does a coalition increase the ex ante payoff to allied members in the coalition? The answer to this question is not obvious. On one hand, a coalition reduces competition and improves the allied members' information advantage (relative to that of independent informed traders), which increases their payoff. On the other hand, a coalition also causes direct costs due to search, setup, and coordination efforts, which are represented by $C(A)$, and an indirect cost from dry-ups of market liquidity, which decreases their payoff. Intuitively, a coalition should gain the most if it can significantly reduce competition and improve the allied members' information advantage. However, we cannot show this explicitly except in the following two polar cases regarding $C(A)$.

Two Polar Cases: In the first polar case, it is prohibitively costly to form a coalition (i.e., $C(A) \to \infty$, $\forall A$). In this case, any possible gains from the coalition will be outweighed by the high costs; that is, $\Gamma_A(A) \leq C(A)$. Therefore, no coalition of informed traders will be formed.

In the second polar case, the cost to form a coalition is negligible (i.e., $C(A) \to 0$, $\forall A$). In this case, a coalition that includes all informed traders will be formed. We refer to this coalition as a comprehensive coalition. One may wonder whether an informed trader prefers to leave the coalition and remain independent. We can show that if all other informed traders stay in the coalition, then she will also stay in the coalition. Therefore, a comprehensive coalition is sustained as a Nash equilibrium.

In what follows, we assume that $C(A)$ is prohibitively high for a comprehensive coalition, but reasonably low for a partial coalition. This allows us to focus on the partial coalition(s). We use a numerical analysis to examine under what conditions a partial coalition can improve the payoff of allied members, so they will form a partial coalition. We consider cases with different levels of information quality, V, and/or different numbers of informed traders, J. This allows us to examine the impacts of information and competition on our results.

3.2.1 Who Joins a Partial Coalition?

Consider a simple case in which there are only $J = 3$ informed traders. Let $v_1 \leq v_2 \leq v_3$, so they are ordered by their information quality. Consider a partial coalition of two informed traders. Who will join the coalition?

Our above discussion suggests that a partial coalition increases the payoff to the allied informed players when it significantly reduces competition and improves the allied informed traders' information advantage. In this simple case, all feasible partial coalitions include two informed traders, so they reduce competition to the same degree. Therefore, the optimal partial coalition hinges solely on its information advantage. This implies that the partial coalition should include informed traders with the best-quality information. Informed traders with the worst-quality information should remain independent. The following corollary presents this intuition formally.

Corollary 3. *Consider a simple economy with* $J = 3$ *and* $v_1 \leq v_2 \leq v_3$.

(i) A coalition between information traders 2 and 3 produces negative synergy; that is, $\Gamma_A(A = \{2, 3\})) \leq 0$.

(ii) A coalition between informed traders 1 and 2 produces higher synergy than a coalition between informed traders 1 and 3; that is, $\Gamma_A(A = \{1, 2\}) \geq \Gamma_A(A = \{1, 3\})$.

(iii) Informed trader 1 receives more benefit from a coalition with informed trader 2 than from a coalition with informed trader 3; that is, $\Gamma_1(A = \{1, 2\}) = \dfrac{1/v_1}{1/v_1 + 1/v_2} \Gamma_A(A = \{1, 2\}) \geq \dfrac{1/v_1}{1/v_1 + 1/v_3} \Gamma_A(A = \{1, 3\}) = \Gamma_1(A = \{1, 3\})$.

Proof: See the Appendix.

Part (i) of Corollary 3 implies that a partial coalition between informed traders with the poorest-quality information produces negative synergy, so it is never formed. Parts (ii) implies that a partial coalition between the informed traders with better-quality information produces higher synergy. Part (iii) implies that the informed trader with the best-quality information quality prefers to ally with the informed trader with the second-best-quality information. Here an informed trader, if she joins a coalition, shares the coalition synergy based on the Shapley value, which can be measured using her information precision $(1/v_j)$.

Taken together, Corollary 3 suggests that a feasible partial coalition is $A = \{1, 2\}$, which can be formed if

$$\Gamma_A(A=\{1,2\}) \geq \max(0, \Gamma_A(A=\{1,3\}), \Gamma_A(A=\{2,3\})). \tag{1}$$

Figure 1 depicts the feasible coalition, $A = \{1, 2\}$, using a numerical analysis. We let $1 = v_1 \leq v_2 \leq v_3 \leq 40$. In the shaded area, Eq. (1) holds, so the partial coalition, $A = \{1, 2\}$, can be formed. Note that this area features a high value of v_3 (≥ 3.5). Therefore, a partial coalition exists only if independent informed traders do not have very precise information. This confirms our above discussion that a partial coalition increases the payoff to the allied informed players when it significantly improves the allied informed traders' information advantage.

[Insert Figure 1 here.]

For the partial coalition, $A = \{1, 2\}$, to be stable, informed traders 1 and 2 should not leave the coalition. Corollary 3 shows that the feasible partial coalition is either $\{1, 2\}$ or $\{1, 3\}$, so informed trader 1 will always stay in the coalition. To ensure that informed trader 2 won't leave the coalition, we follow Jackson and Wolinsky (1996) to add another condition to Eq. (1) to ensure this.

$$\Gamma_2(A=\{1,2\}) = \frac{1/v_2}{1/v_1 + 1/v_2}\Gamma_A(A=\{1,2\}) \geq \Gamma_2(A=\{1,3\}). \tag{2}$$

This equation ensures that informed trader 2 gains more from staying in the coalition than from becoming independent while letting informed traders 1 and 3 form a coalition.

[Insert Figure 2 here.]

Figure 2 shows that in the dark-shaded area, the coalition, $A = \{1, 2\}$, is stable. In the light-shaded area, the coalition, $A = \{1, 2\}$, is fragile, so informed trader 2 may leave the coalition. There are two notable observations. First, the stable area features a higher v_3 than the fragile area does. This suggests that to be stable, a coalition needs to have a significant information advantage (relative to informed trader 3). Second, the stable area also features a much smaller v_2. This suggests that to be stable, the two informed traders, 1 and 2, in the coalition must have similar-quality information, so they have a similar share of the coalition synergy.

[Insert Figure 3 here.]

Figure 3 considers a more general case. There are $J = 10$ informed traders. Their information qualities satisfy $1 = v_1 \leq v_2 \leq v_3 \leq 20$ and $v_j = 50$, $\forall j > 3$. The shaded area, which features a higher value of v_3 (≥ 6.5), satisfies Eq. (1), so the coalition $A = \{1, 2\}$ can be formed in this area. An interesting observation from comparing Figures 3 and 1 is that as J increases, independent informed trader 3 must have poorer information quality for the partial coalition $A = \{1, 2\}$ to be formed. This is because as there are more independent informed traders, independent informed trader 3 must have poorer information quality, so that the coalition $A = \{1, 2\}$ can maintain sufficient information advantage to make a profit.

[Insert Figure 4 here.]

In Figure 4, $v_j = 35$, $\forall j > 3$, so these informed traders have relatively better information quality than in Figure 3. Compared with Figure 3, the shaded area, in which the coalition $A = \{1, 2\}$ can be formed, shifts farther upwards. This area features an even higher value of v_3 (≥ 9.5). This is because as independent informed traders $j > 3$ have better information quality, independent informed trader 3 must have even poorer information quality, so the coalition $A = \{1, 2\}$ can maintain sufficient information advantage to make a profit.

Prediction 1. *A coalition tends to be formed by informed traders with the best-quality information.*

3.2.2 Minimum Coalition Size

Consider a symmetric case in which $v_j = V$, $\forall j$, so all informed traders have the same-quality information. Figures 1 to 4 suggest that in this symmetric case, a small partial coalition $A = \{1, 2\}$ won't be formed because it does not significantly reduce competition and gain information advantage. Figure 5 verifies this. Panel (a) shows that if formed, the partial coalition $A = \{1, 2\}$ will have negative synergy (i.e., $\Gamma_A(A = \{1, 2\}) < 0$). Panel (b) shows that consistent with Corollary 2, independent informed traders generally gain from this coalition (i.e., $\Gamma_j(A = \{1, 2\}) > 0$, $\forall j \geq 3$).

[Insert Figure 5 here.]

For a partial coalition to be formed, it needs to be sufficiently large because only a large coalition can significantly reduce competition and obtain information advantage. The minimum size of a coalition, $\underline{m}$, can be obtained as follows:

$$\begin{aligned} \min_{m} \quad & m \\ \text{s.t.} \quad & \Gamma_A(A = \{1, 2, ..., m\}) \geq 0. \end{aligned}$$

Figure 6 plots $\underline{m}$ depending on the information quality. We assume that there are $J = 40$ informed traders. One observation is that $\underline{m}$ increases with the information quality, V. For example, $\underline{m}$ for a single coalition is 27 when $V = 10$, and it increases to 30 when $V = 5$. This is because as other informed traders have good-quality information, the coalition must be sufficiently large to reduce competition and gain information advantage. Figure 6 also considers the possibility that there are multiple coalitions. $\underline{m}$ decreases with the number of coalitions. This suggests that coalitions can have an externality effect on one another because each coalition lowers the intensity of competition in the market, benefiting other coalitions. This lowers the minimum size of a coalition.

[Insert Figure 6 here.]

Figure 7 plots $\underline{m}$ depending on the number of coalitions. We assume that there can be $J = 20$, 30, and 40 informed traders. Consistent with Figure 6, $\underline{m}$ decreases in the number of coalitions. Moreover, $\underline{m}$ increases in the total number of informed traders. This is because when there are many informed traders, a coalition must be sufficiently large to reduce competition and gain information advantage.

[Insert Figure 7 here.]

Prediction 2. *A coalition, if formed, must be sufficiently large.*

3.2.3 Optimal Coalition Structure

Now we study the optimal coalition structure. We continue to let $v_j = V$, $\forall j$, so all informed traders have the same-quality information. We also follow Dessein and Santos (2006) to assume a quadratic cost function for a coalition:

$$C(A(m)) = \gamma(m-1)^2.$$

In this specification, m represents the number of allied informed traders in the coalition. γ is a constant.

By symmetry, there can be n coalitions, each of which has m informed traders. Allied members have the same Shapley value, so they share the coalition synergy equally.

The optimal coalition structure of the economy, $\{m, n\}$, maximizes the average coalition member's net gain as follows:

$$\max_{m,n} \quad \frac{1}{m} \left[\Gamma_A(A = \{1, 2, ..., m\}) - C(A(m)) \right].$$

In Figure 8, we assume that there can be $J = 30$ or 40 informed traders. We let $C(A(m)) = 10^{-4}(m-1)^2$. Panels (a) and (b) show that as the information quality improves (low V), the optimal coalition size, m, increases, and the optimal number of coalitions, n, decreases. The intuition for these results is as follows. As informed traders have good-quality information, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, the optimal coalition size should increase. This also leads to a smaller number of coalitions because in this simple example, the population of informed traders is constant.

Panels (a) and (b) also show that as the number of informed traders increases (high J), the optimal coalition size may or may not increase, but the number of coalitions generally increases. Intuitively, a larger number of coalitions implies that the competition is significantly reduced, so the coalitions can make a profit.

[Insert Figure 8 here.]

Prediction 3. *As information quality improves, the optimal coalition size increases and the number of coalitions decreases.*

Next, we consider the case in which there are two groups of informed traders, indexed by the subscripts 1 and 2, in the economy. Informed traders from the same group have the same-quality information. Specifically, if informed trader j is in group 1 (group 2), then $v_j = V_1$ ($v_j = V_2$). We follow the social-network literature (e.g., Jackson, 2008) to assume that the two groups of informed traders live in two separate "islands," so they can form coalitions only within each group. In our framework, this requires that the costs of maintaining an across-group coalition, which can be related to monitoring and enforcing, be prohibitively high.

By symmetry, in group 1 (2), there can be n_1 (n_2) coalitions, each of which has m_1 (m_2) informed traders. For simplicity, we assume that the two groups are equally weighted in the sense that the optimal coalition structure m_1, m_2, n_1, n_2 maximizes the net gains of the average coalition member from each group:

$$\max_{m_1,m_2,n_1,n_2} \frac{1}{m_1}\Big[\Gamma_A(A=\{1,...,m_1\}) - C(A(m_1))\Big] + \frac{1}{m_2}\Big[\Gamma_A(A=\{1,...,m_2\}) - C(A(m_2))\Big].$$

[Insert Figure 9 here.]

In Figure 9, we assume that each group has 40 informed traders. Informed traders in group 1 have better-quality information than those in group 2; specifically, $V_1 = 1$ and $V_2 > 1$. We also let $C(A(m)) = 10^{-4}(m-1)^2$. Panels (a) and (b) show that group 1 has a larger optimal coalition size but a smaller number of coalitions than group 2 does. The intuition for these results is as follows. As informed traders in group 1 have relatively good-quality information, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, group 1 should have a larger coalition size, which also leads to a smaller number of coalitions. By significantly reducing competition in the whole economy, group 1's coalition structure, large m and small n, also has a spillover effect on group 2's coalition structure. Group 2's optimal coalition size doesn't need to be very large, but the coalitions can still make a profit.

[Insert Figure 10 here.]

In Figure 10, we assume that there are more informed traders in group 1 than in group 2; that is, $J_1 > 100$ and $J_2 = 100$. We let all informed traders have the same-quality information, $V_1 = V_2 = 5$. Also, $C(A(m)) = 10^{-5}(m-1)^2$. Panels (a) and (b) show that group 1 has a large optimal coalition size but a smaller number of coalitions than group 2 does.[5] The intuition for these results is similar to that for the case in Figure 9. As there are more informed traders in group 1, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, group 1 should have a larger coalition size, which also leads to a smaller number of coalitions. By significantly reducing competition in the whole economy, group 1's coalition structure, large m and small n, also has a spillover effect on group 2's coalition structure. Group 2's optimal coalition size doesn't need to be very large, but the coalitions can still make a profit.

Prediction 4. *A small number of large coalitions are likely to originate from the group of informed traders who possess good-quality information and the group of informed traders with a large population.*

4 Applications to the Asset Management Industry

In this section, we apply our theory to understand the structure of the asset management industry. In this industry, a company can be interpreted as a coalition of professional money managers. This interpretation is broadly consistent with empirical findings on this industry (see Footnote 2 for a discussion of the empirical findings).

[5] Note that there are some bumps in the plots, due to the nature of integer programming.

4.1 Firm Size and Performance

Predictions 1 and 2 imply that a big asset management company is likely to include fund managers with the best-quality information. This implication is consistent with empirical findings by Chen, Hong, Huang, and Kubik (2004) and Pollet and Wilson (2008). They show that mutual funds affiliated with big asset management companies tend to outperform those affiliated with small asset management companies. Bhojraj, Cho, and Yehuda (2012) further show that mutual fund managers affiliated with big asset management companies indeed possess an information advantage over those affiliated with small asset management companies.

4.2 Technology and Firm Size

Prediction 3 implies that information environment improves, the consolidation of the asset management industry will lead to the emergence of a few big asset management companies. Recent decades have seen fast growth in information technology, which improves information quality. During the same period, the asset management industry experienced a wave of mergers, which gave rise to a few gigantic firms. For example, BlackRock acquired State Street Research in 2005, Merrill Lynch Investment Managers in 2006, Quellos Group in 2007, and Barclays Global Investors in 2009. As of September 2014, Blackrock is the biggest asset management company.

5 Conclusions

In this article, we try to answer an interesting question raised by Stein (2008): why, contrary to conventional wisdom, do competing investors in financial markets often share their information about trading opportunities even though this information could be used against them later?

Our model is based on Kyle's (1985) framework. A novel feature of our model is that we allow informed traders who exchange information to coordinate trades. As information exchange turns competitors into allies, allied members enjoy benefits of information advantage and monopolistic power, though there can also be direct costs related to search, setup, and coordination efforts, and indirect costs from market liquidity dry-ups. Therefore, it can be incentive-compatible for competing investors to exchange information.

Our model also gives interesting predictions about the effects of communications on financial market outcomes and about the coalition structure of the economy. We use these predictions to obtain numerous insights about the structure of the asset management industry, in which a company can be interpreted as a coalition of professional money managers.

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Appendix

Proof of Proposition 1: Our proof has two steps. In Step 1, we study how an informed trader chooses her demand, conditional on the price and all the other informed traders' demands being given by Proposition 1. In Step 2, we study how the market maker sets up the price, conditional on all informed investors' demands being given by Proposition 1.

Step 1: Consider an informed trader j. Suppose that the price, $P(NA)$, and the demands of all other informed investors, $D_i(NA)$, $\forall i \neq j$, are given by Proposition 1. Then, her payoff from trading can be expressed as:

$$\begin{aligned}
\Pi_j(S_j) &= E\Big[D_j\Big(\bar{F} + X - P(NA)\Big)|S_j\Big] \\
&= E\Big[D_j\Big(X - \lambda(NA)(D_j + \sum_{i\neq j} D_i(NA) + Z)\Big)|S_j\Big] \\
&= E\Big[D_j\Big(1 - \lambda(NA)\sum_{i\neq j}\theta_i(NA)\Big)X - \lambda(NA)D_j^2|S_j\Big] \\
&= D_j\Big(1 - \lambda(NA)\sum_{i\neq j}\theta_i(NA)\Big)E\Big[X|S_j\Big] - \lambda(NA)D_j^2 \\
&= D_j\Big(1 - \lambda(NA)\sum_{i\neq j}\theta_i(NA)\Big)\frac{S_j}{1+v_j} - \lambda(NA)D_j^2.
\end{aligned}$$

The first order condition implies

$$D_j(NA) = \frac{1 - \lambda(NA)\sum_{i\neq j}\theta_i(NA)}{2\lambda(NA)}\frac{S_j}{1+v_j} = \frac{\sigma_z/K(NA)}{1+2v_j}S_j,$$

where the last equality follows by substituting the expressions of $\lambda(NA)$ and $\theta_i(NA)$, $\forall i \neq j$ from Proposition 1. The second order condition holds obviously. Therefore, we can write $D_j(NA) = \theta_j(NA)S_j$, where $\theta_j(NA) = \dfrac{\sigma_z/K(NA)}{1+2v_j}$.

Step 2: Consider the market maker. Suppose that the demands of all informed traders, $D_j(NA)$, $\forall j$, are given by Proposition 1. Then, the total demand can be expressed as:

$$D = \sum_j D_j(NA) + Z = \sum_j \Big(\theta_j(NA)S_j\Big) + Z = X\sum_j \theta_j(NA) + \sum_j \Big(\theta_j(NA)\epsilon_j\Big) + Z.$$

Substituting into the price equation gives

$$\begin{aligned} P &= \bar{F} + E\Big[X|D = X\sum_j \theta_j(NA) + \sum_j \Big(\theta_j(NA)\epsilon_j\Big) + Z\Big] \\ &= \bar{F} + \frac{\sum_j \theta_j(NA)}{\Big(\sum_j \theta_j(NA)\Big)^2 + \sum_j \Big(\theta_j(NA)^2 v_j\Big) + \sigma_z^2} D. \end{aligned}$$

Therefore, we can write $P(NA) = \bar{F} + \lambda(NA)D$, and

$$\begin{aligned} \lambda(NA) &= \frac{\sum_j \theta_j(NA)}{(\sum_j \theta_j(NA))^2 + \sum_j \Big(\theta_j(NA)^2 v_j\Big) + \sigma_z^2} \\ &= \frac{\sum_j \frac{\sigma_z/K(NA)}{1+2v_j}}{\Big(\sum_j \frac{\sigma_z/K(NA)}{1+2v_j}\Big)^2 + \sum_j \Big((\frac{\sigma_z/K(NA)}{1+2v_j})^2 v_j\Big) + \sigma_z^2} \\ &= \frac{K(NA)/\sigma_z}{1 + \sum_j \frac{1}{1+2v_j}}, \end{aligned}$$

where the last equality follows by substituting into the expression of $K(NA)$.

Q.E.D.

Proof of Lemma 1: It suffices to show that the distribution of X conditional on S_j, $\forall j \in A$, is identical to the distribution of X conditional on S_A. Note that X, S_j ($\forall j \in A$), and S_A are normally distributed. So the conditional distribution of X must be a normal distribution. We just need to show that

$$
\begin{aligned}
E\Big[X|S_j, \forall j \in A\Big] = \frac{S_A}{1+v_A} &= E\Big[X|S_A\Big], \\
\mathrm{Var}\Big[X|S_j, \forall j \in A\Big] = \frac{v_A}{1+v_A} &= \mathrm{Var}\Big[X|S_A\Big],
\end{aligned}
$$

which follows immediately after we write down the joint distribution of X, S_j ($\forall j \in A$), and S_A.

Q.E.D.

Proof of Proposition 2: Consider the coalition as a hypothetical informed trader, indicated by A, who observes a private signal S_A. The remaining proof is identical to the proof of Proposition 1.

Q.E.D.

Proof of Corollary 1: (i) It follows from Proposition 2 that

$$
\begin{aligned}
\mathrm{Var}(X|P(A)) &= \mathrm{Var}\Big[X|D = D_A(A) + \sum_{j \notin A} D_j(A) + Z\Big] \\
&= \mathrm{Var}(X) - \lambda(A)\mathrm{Cov}(X, D) \\
&= 1 - \lambda(A)\Big(\theta_A(A) + \sum_{j \notin A} \theta_j(A)\Big) \\
&= 1 - \frac{K(A)/\sigma_z}{1 + \frac{1}{1+2v_A} + \sum_{j \notin A} \frac{1}{1+2v_j}} \Big(\frac{\sigma_z/K(A)}{1+2v_A} + \sum_{j \notin A} \frac{\sigma_z/K(A)}{1+2v_j}\Big) \\
&= \frac{1}{1 + \frac{1}{1+2v_A} + \sum_{j \notin A} \frac{1}{1+2v_j}}.
\end{aligned}
$$

Similarly,

$$\mathrm{Var}(X|P(A')) = \frac{1}{1+\dfrac{1}{1+2v'_A}+\sum_{j\notin A'}\dfrac{1}{1+2v_j}}.$$

Note that $v_{A'} = 1\Big/(\dfrac{1}{v_A}+\dfrac{1}{v_b})$ and

$$\begin{aligned}
& \frac{1}{1+2v_{A'}} - (\frac{1}{1+2v_A}+\frac{1}{1+2v_b}) \\
= & -\frac{v_A(1+2v_b)+v_b(1+2v_A)}{(v_A+v_b+2v_Av_b)(1+2v_A)(1+2v_b)} \\
< & \; 0. \qquad (3)
\end{aligned}$$

Therefore,

$$\begin{aligned}
& (\frac{1}{1+2v_{A'}}+\sum_{j\notin A'}\frac{1}{1+2v_j}) - (\frac{1}{1+2v_A}+\sum_{j\notin A}\frac{1}{1+2v_j}) \\
= & \frac{1}{1+2v_{A'}} - (\frac{1}{1+2v_A}+\frac{1}{1+2v_b}) \\
< & \; 0,
\end{aligned}$$

which implies $\mathrm{Var}(X|P(A')) > \mathrm{Var}(X|P(A))$.

(ii) Denote $G \equiv \sum_{j\notin A'} \dfrac{1+v_j}{(1+2v_j)^2}$ and $Q \equiv \sum_{j\notin A'} \dfrac{1}{1+2v_j}$. It is straightforward to show that $G > Q/2$.

Note that

$$\lambda(A')^2 - \lambda(A)^2 = \frac{K(A')^2/\sigma_z^2}{(1+\dfrac{1}{1+2v_{A'}}+Q)^2} - \frac{K(A)^2/\sigma_z^2}{(1+\dfrac{1}{1+2v_A}+\dfrac{1}{1+2v_b}+Q)^2},$$

$$
\begin{aligned}
&\propto \frac{\frac{1+v_{A'}}{(1+2v_{A'})^2}+G}{(1+\frac{1}{1+2v_{A'}}+Q)^2} - \frac{\frac{1+v_A}{(1+2v_A)^2}+\frac{1+v_b}{(1+2v_b)^2}+G}{(1+\frac{1}{1+2v_A}+\frac{1}{1+2v_b}+Q)^2} \\
&> \frac{\frac{1+v_{A'}}{(1+2v_{A'})^2}+Q/2}{(1+\frac{1}{1+2v_{A'}}+Q)^2} - \frac{\frac{1+v_A}{(1+2v_A)^2}+\frac{1+v_b}{(1+2v_b)^2}+Q/2}{(1+\frac{1}{1+2v_A}+\frac{1}{1+2v_b}+Q)^2},
\end{aligned}
$$

where the inequality follows from $G > Q/2$ and Eq. (3).

For $\lambda(A') > \lambda(A)$, it suffices to show that

$$
\begin{aligned}
\eta &\equiv \Big[\frac{1+v_{A'}}{(1+2v_{A'})^2}+Q/2\Big](1+\frac{1}{1+2v_A}+\frac{1}{1+2v_b}+Q)^2 \\
&\quad -\Big[\frac{1+v_A}{(1+2v_A)^2}+\frac{1+v_b}{(1+2v_b)^2}+Q/2\Big](1+\frac{1}{1+2v_{A'}}+Q)^2 \\
&> 0.
\end{aligned}
$$

After substituting the expression of $v_{A'} = 1\Big/(\frac{1}{v_A}+\frac{1}{v_b})$, write

$$
\eta = \eta_1 + 2Q\eta_2 + Q^2\eta_3,
$$

where

$$
\begin{aligned}
\eta_1 &\propto (v_A+v_b+v_Av_b)\Big[4(v_b+1)v_A^2+(4v_b^2+1)v_A+4v_b^2+v_b\Big], \\
\eta_2 &\propto (16v_b^3+24v_b^2+10v_b+2)v_A^3+(24v_b^3+20v_b2+4v_b+\frac{1}{4})v_A^2 \\
&\quad +(10v_b^3+4v_b^2+\frac{1}{2}v_b)v_A+2v_b^3+\frac{1}{4}v_b^2, \\
\eta_3 &\propto (32v_b^3+28v_b^2+8v_b+1)v_A^3+(28v_b^3+16v_b^2+2v_b)v_A^2+(8v_b^3+2v_b^2)v_A+v_b^3.
\end{aligned}
$$

It is obvious that $\eta_1, \eta_2, \eta_3 > 0$. Therefore, $\eta > 0$ and $\lambda(A') > \lambda(A)$.

(iii) Note that $\forall \alpha, \beta > 0$. We have the following inequality:

$$\begin{aligned} f(\alpha,\beta) &\equiv \beta(\alpha+\beta+\alpha\beta)(1+2\alpha)^2 - (1+\alpha)(\alpha+\beta+2\alpha\beta)^2 \\ &= -\alpha^3(4+3\beta) - \alpha^2(1+5\beta) - \alpha\beta \\ &< 0. \end{aligned}$$

Substituting $v_{A'} = 1\Big/(\frac{1}{v_A} + \frac{1}{v_b})$ yields

$$\begin{aligned} K(A')^2 - K(A)^2 &= \frac{1+v_{A'}}{(1+2v_{A'})^2} - \frac{1+v_A}{(1+2v_A)^2} - \frac{1+v_b}{(1+2v_b)^2} \\ &\propto f(v_A, v_b)(1+2v_A)^2 + f(v_b, v_A)(1+2v_b)^2 \\ &< 0. \end{aligned}$$

It follows immediately that $\forall j \notin A'$,

$$\theta_j(A') = \frac{\sigma_z/K(A')}{1+2v_j} > \frac{\sigma_z/K(A)}{1+2v_j} = \theta_j(A).$$

Also, since $v_{A'} < v_A$, we have

$$\theta_{A'}(A') = \frac{\sigma_z/K(A')}{1+2v_{A'}} > \frac{\sigma_z/K(A)}{1+2v_A} = \theta_A(A).$$

Q.E.D.

Proof of Corollary 2: (i) Step 1 of the Proof of Proposition 1 implies that $\forall j \notin A$,

$$E\Pi_j(NA) = E\Big[\lambda(NA)D_j^2\Big] = E\Big[\lambda(NA)\theta_j^2(NA)S_j^2\Big] = \lambda(NA)\theta_j^2(NA)(1+v_j).$$

Similarly,

$$E\Pi_j(A) = \lambda(A)\theta_j^2(A)(1 + v_j).$$

It follows from Corollary 1 that $\theta_j(A) > \theta_j(NA)$ and $\lambda(A) > \lambda(NA)$. Therefore,

$$\Gamma_j(A) = E\Pi_j(A) - E\Pi_j(NA) = \lambda(A)\theta_j^2(A)(1 + v_j) - \lambda(NA)\theta_j^2(NA)(1 + v_j) > 0.$$

(ii) Note that

$$E\Pi_{\mathrm{L}}(NA) = E\Big[(\bar{F} + X - P(NA)) \cdot Z\Big] = E\Big[(X - \lambda(NA)D) \cdot Z\Big] = -\lambda(NA) \cdot \sigma_z^2.$$

Similarly,

$$E\Pi_{\mathrm{L}}(A) = -\lambda(A) \cdot \sigma_z^2.$$

It follows from Corollary 1 that $\lambda(A) > \lambda(NA)$, so

$$\Gamma_{\mathrm{L}}(A) = E\Pi_{\mathrm{L}}(A) - E\Pi_{\mathrm{L}}(NA) = (\lambda(NA) - \lambda(A)) \cdot \sigma_z^2 < 0.$$

Q.E.D.

Proof of Corollary 3: (i) Recalling step 1 of the Proof of Proposition 1, we have

$$E\Pi_j(NA) = E\Big[\lambda(NA)D_j^2\Big] = E\Big[\lambda(NA)\theta_j^2(NA)S_j^2\Big] = \lambda(NA)\theta_j^2(NA)(1+v_j).$$

Similarly, it follows from Proposition 2 that

$$E\Pi_A(A) = \lambda(A)\theta_A^2(A)(1+v_A).$$

Denote $v_{23} = 1\Big/(\frac{1}{v_2}+\frac{1}{v_3})$. Write

$$\begin{aligned}
& \Gamma_A(A=\{2,3\}) \\
= \;& E\Pi_A(A=\{2,3\}) - \Big[E\Pi_2(NA)+E\Pi_3(NA)\Big] \\
= \;& \frac{\sigma_z}{\sqrt{\frac{1+v_{23}}{(1+2v_{23})^2}+\frac{1+v_1}{(1+2v_1)^2}}}\frac{\frac{1+v_{23}}{(1+2v_{23})^2}}{1+\frac{1}{1+2v_{23}}+\frac{1}{1+2v_1}} \\
& -\frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2}+\frac{1+v_2}{(1+2v_2)^2}+\frac{1+v_3}{(1+2v_3)^2}}}\frac{\frac{1+v_2}{(1+2v_2)^2}+\frac{1+v_3}{(1+2v_3)^2}}{1+\frac{1}{1+2v_1}+\frac{1}{1+2v_2}+\frac{1}{1+2v_3}}.
\end{aligned}$$

Denote $v_2 = v_1 + a$ and $v_3 = v_1 + b$ where $b \geq a \geq 0$. We can show that $\Gamma_A(A=\{2,3\})$ is proportional to a polynomial of v_1, a, and b, which is non-positive. (We derive this polynomial using Mathematica. It is six pages long and available upon request from the authors.) Therefore, $\Gamma_A(A=\{2,3\}) \leq 0$.

(ii) Denote $v_{12} = 1\Big/(\frac{1}{v_1}+\frac{1}{v_2})$, and $v_{13} = 1\Big/(\frac{1}{v_1}+\frac{1}{v_3})$. Write

$$\begin{aligned}
& \Gamma_A(A=\{1,2\}) \\
= \;& E\Pi_A(A=\{1,2\}) - \Big[E\Pi_1(NA)+E\Pi_2(NA)\Big]
\end{aligned}$$

$$
\begin{aligned}
= & \frac{\sigma_z}{\sqrt{\frac{1+v_{12}}{(1+2v_{12})^2}+\frac{1+v_3}{(1+2v_3)^2}}}\frac{\frac{1+v_{12}}{(1+2v_{12})^2}}{1+\frac{1}{1+2v_{12}}+\frac{1}{1+2v_3}} \\
& -\frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2}+\frac{1+v_2}{(1+2v_2)^2}+\frac{1+v_3}{(1+2v_3)^2}}}\frac{\frac{1+v_1}{(1+2v_1)^2}+\frac{1+v_2}{(1+2v_2)^2}}{1+\frac{1}{1+2v_1}+\frac{1}{1+2v_2}+\frac{1}{1+2v_3}},
\end{aligned}
$$

and

$$
\begin{aligned}
& \Gamma_A(A=\{1,3\}) \\
= & E\Pi_A(A=\{1,3\})-\Big[E\Pi_1(NA)+E\Pi_3(NA)\Big] \\
= & \frac{\sigma_z}{\sqrt{\frac{1+v_{13}}{(1+2v_{13})^2}+\frac{1+v_2}{(1+2v_2)^2}}}\frac{\frac{1+v_{13}}{(1+2v_{13})^2}}{1+\frac{1}{1+2v_{13}}+\frac{1}{1+2v_2}} \\
& -\frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2}+\frac{1+v_2}{(1+2v_2)^2}+\frac{1+v_3}{(1+2v_3)^2}}}\frac{\frac{1+v_1}{(1+2v_1)^2}+\frac{1+v_3}{(1+2v_3)^2}}{1+\frac{1}{1+2v_1}+\frac{1}{1+2v_2}+\frac{1}{1+2v_3}}.
\end{aligned}
$$

We can show using Mathematica Symbolic Computing that $\Gamma_A(A=\{1,2\}) \geq \Gamma_A(A=\{1,3\})$.

(iii) Similarly to the proof for Part (ii), we can also show using Mathematica Symbolic Computing that

$$
\begin{aligned}
\Gamma_1(A=\{1,2\}) &= \frac{1/v_1}{1/v_1+1/v_2}\Gamma_A(A=\{1,2\}) \\
&\geq \frac{1/v_1}{1/v_1+1/v_3}\Gamma_A(A=\{1,3\}) = \Gamma_1(A=\{1,3\}).
\end{aligned}
$$

Q.E.D.

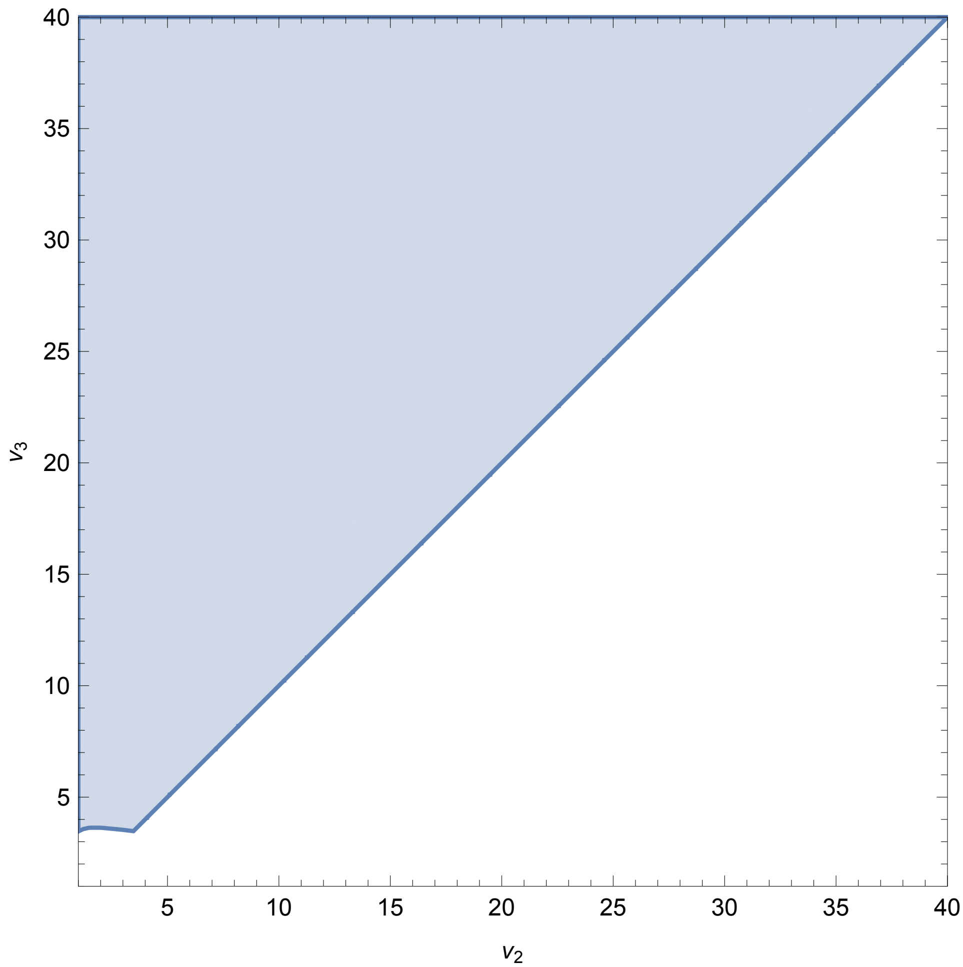


Figure 1: A Partial Coalition When $J = 3$

There are $J = 3$ informed traders. Their information qualities satisfy $1 = v_1 \leq v_2 \leq v_3 \leq 40$. In the shaded area,

$$\Gamma_A(A = \{1, 2\}) \geq \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})),$$

so a partial coalition, $A = \{1, 2\}$, can be formed.

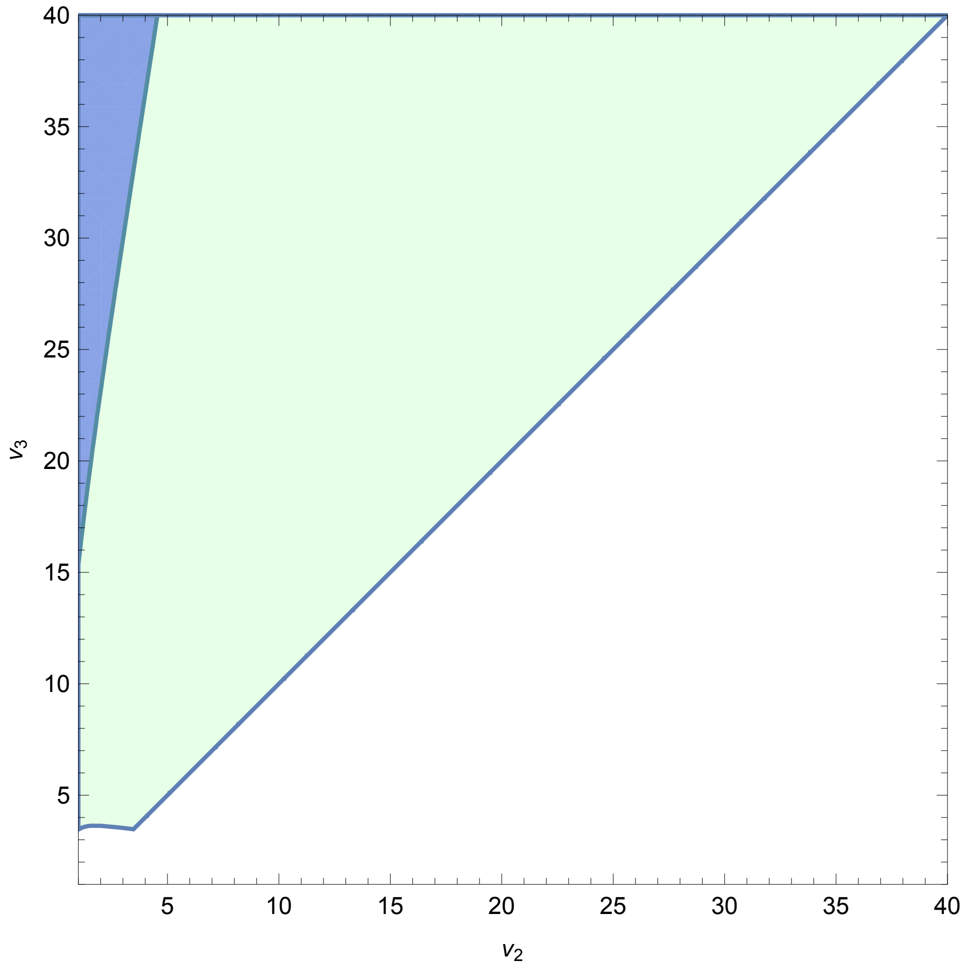


Figure 2: A Partial Coalition When $J = 3$: Stability

There are $J = 3$ informed traders. Their information qualities satisfy $1 = v_1 \leq v_2 \leq v_3 \leq 40$. The shaded area satisfies

$$\Gamma_A(A = \{1,2\}) \geq \max(0, \Gamma_A(A = \{1,3\}), \Gamma_A(A = \{2,3\})).$$

The dark-shaded area further satisfies

$$\Gamma_2(A = \{1,2\}) = \frac{1/v_2}{1/v_1 + 1/v_2} \Gamma_A(A = \{1,2\}) \geq \Gamma_2(A = \{1,3\}).$$

A partial coalition, $A = \{1,2\}$, is stable in the dark-shaded area, but fragile in the light-shaded area.

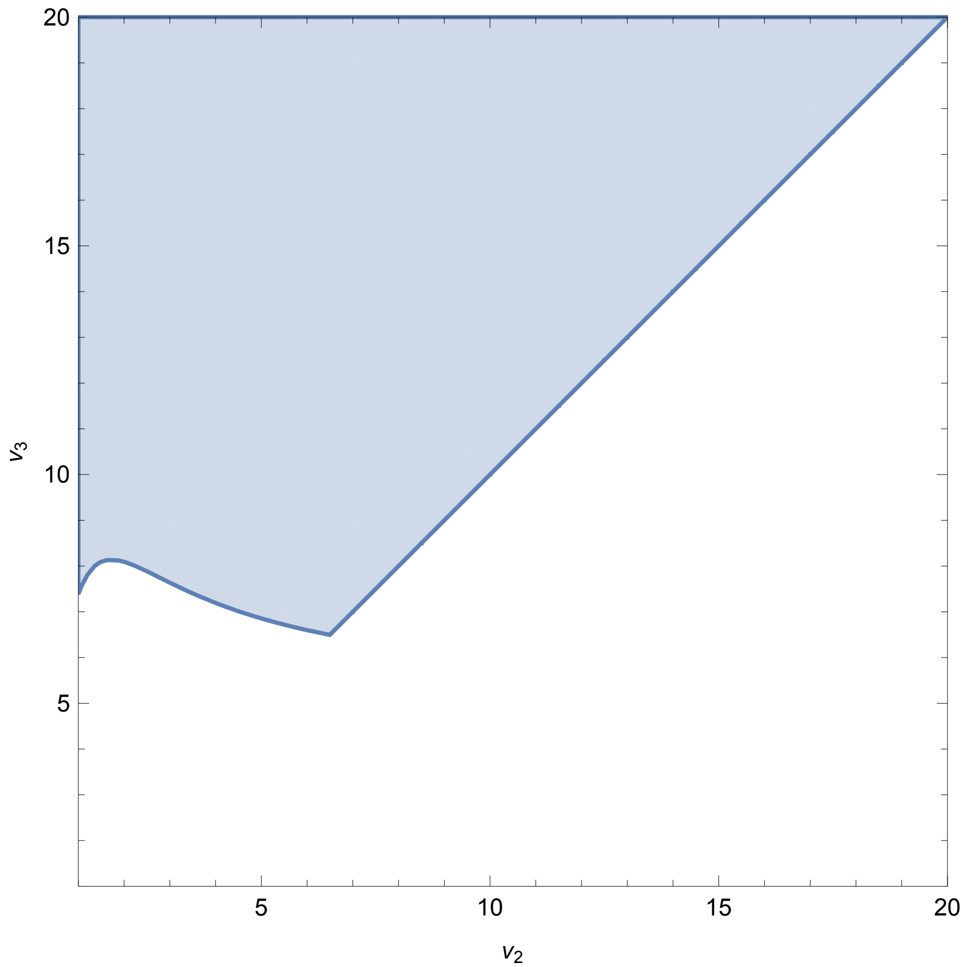


Figure 3: A Partial Coalition When $J = 10$: Case 1

There are $J = 10$ informed traders. Their information qualities satisfy $1 = v_1 \leq v_2 \leq v_3 \leq 20$ and $v_j = 50$, $\forall j > 3$. In the shaded area,

$$\Gamma_A(A = \{1, 2\}) \geq \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})),$$

so a partial coalition, $A = \{1, 2\}$, can be formed.

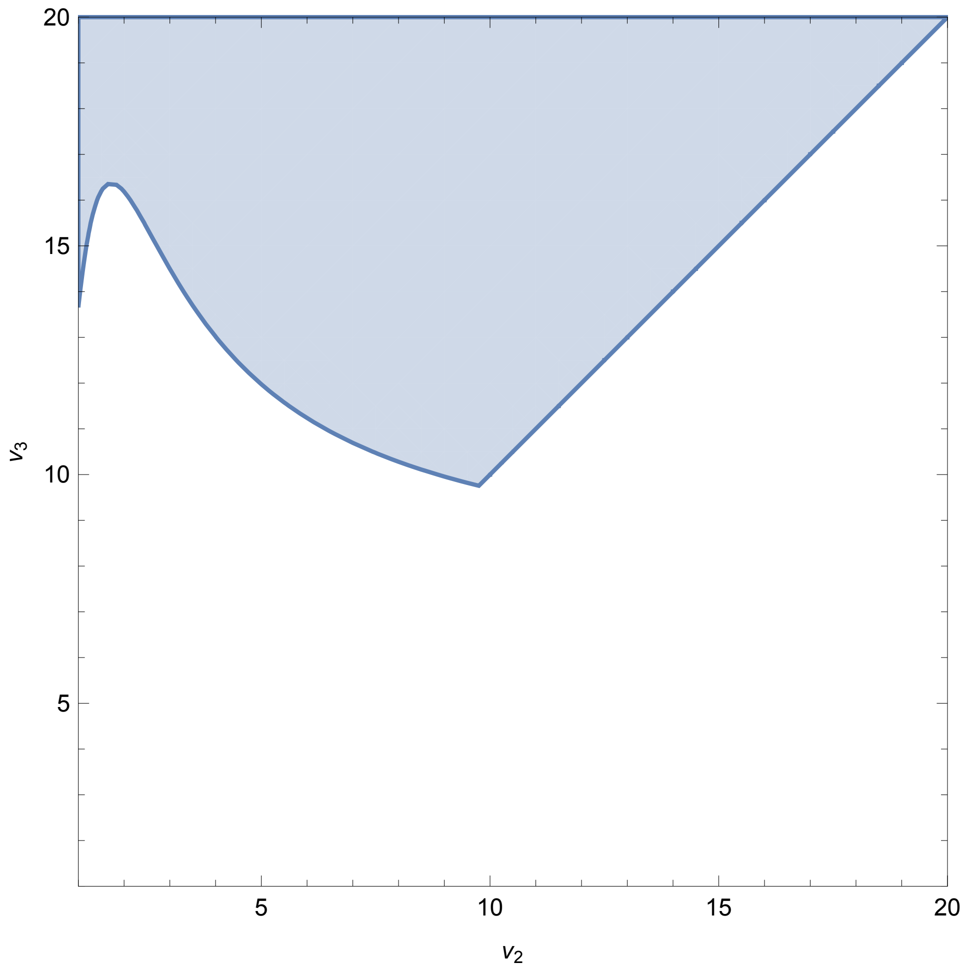
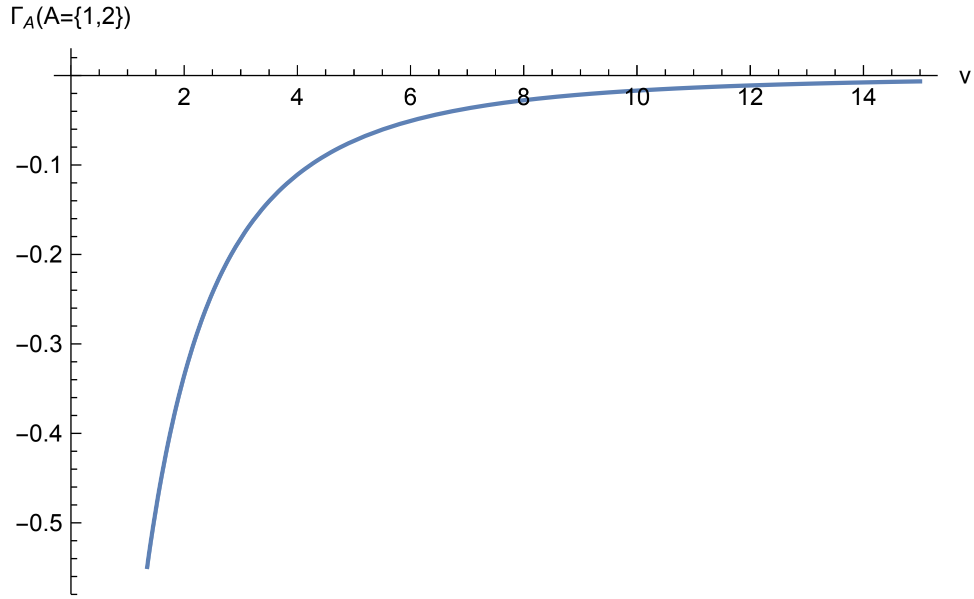


Figure 4: A Partial Coalition When $J = 10$: Case 2

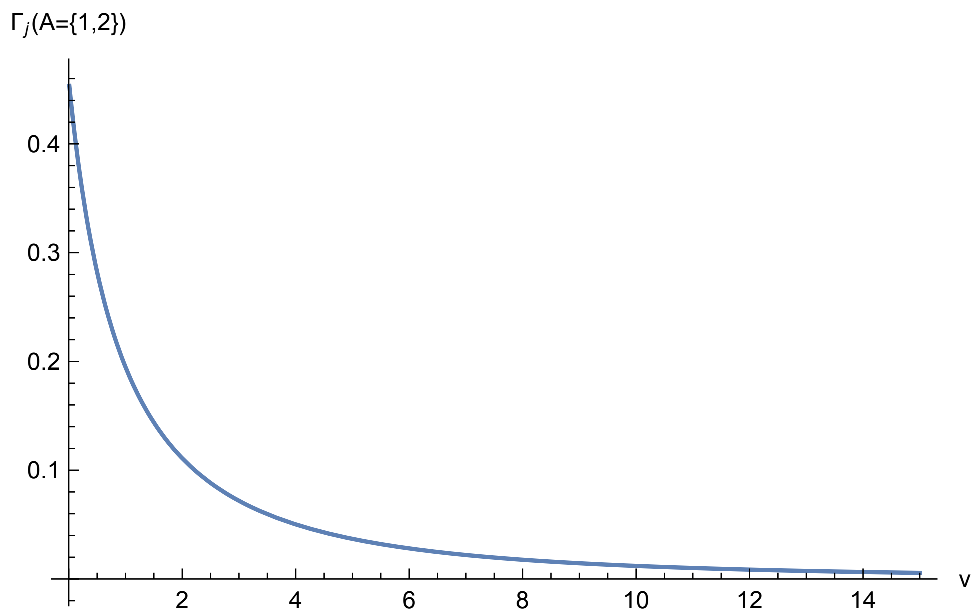
There are $J = 10$ informed traders. Their information qualities satisfy $1 = v_1 \leq v_2 \leq v_3 \leq 20$ and $v_j = 35$, $\forall j > 3$. In the shaded area,

$$\Gamma_A(A = \{1,2\}) \geq \max(0, \Gamma_A(A = \{1,3\}), \Gamma_A(A = \{2,3\})),$$

so a partial coalition, $A = \{1, 2\}$, can be formed.



(a): Synergy of Hypothetical Partial Coalition $A = \{1, 2\}$, $\Gamma_A(A = \{1, 2\})$



(b) Profit Change of an Independent Informed Trader, $\Gamma_j(A = \{1, 2\})$, $\forall j \geq 3$

Figure 5: Welfare Effects of Hypothetical Partial Coalition

There are $J = 10$ informed traders, each of whom has the same-quality information, $v_j = V$, $\forall j$. Let $\sigma_z^2 = 100$. Panel (a) plots the synergy for a hypothetical partial coalition $A = \{1, 2\}$, $\Gamma_A(A = \{1, 2\})$, depending on V. Panel (b) plots the profit change of an independent informed trader due to the coalition, $\Gamma_j(A = \{1, 2\})$, $\forall j \geq 3$, depending on V.

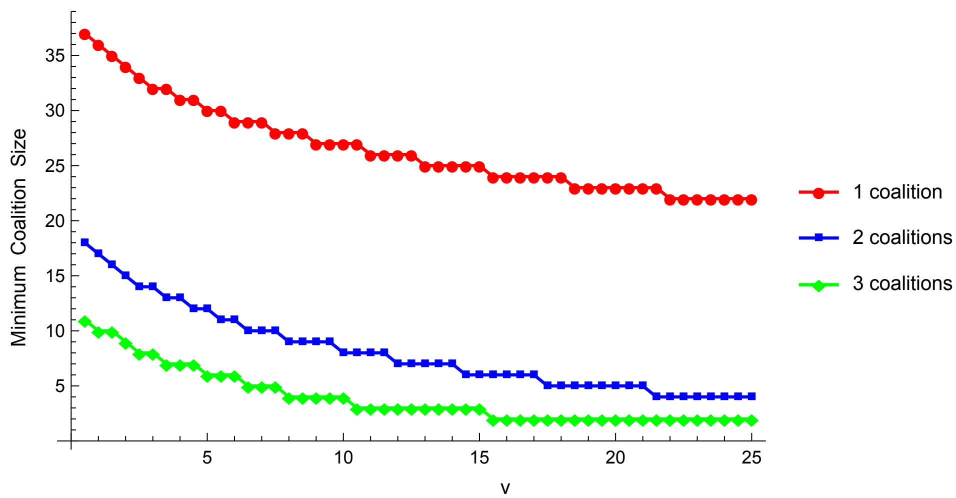


Figure 6: Minimum Coalition Size: Case 1

There are $J = 40$ informed traders, each of whom has the same-quality information, $v_j = V$, $\forall j$. We assume that there can be 1, 2, or 3 coalitions. The minimum size of a coalition, $\underline{m}$, is given by:

$$\begin{aligned} \min_{m} \quad & m \\ s.t. \quad & \Gamma_A(A = \{1, 2, ..., m\}) \geq 0. \end{aligned}$$

This figure plots the minimum coalition size, $\underline{m}$, depending on V.

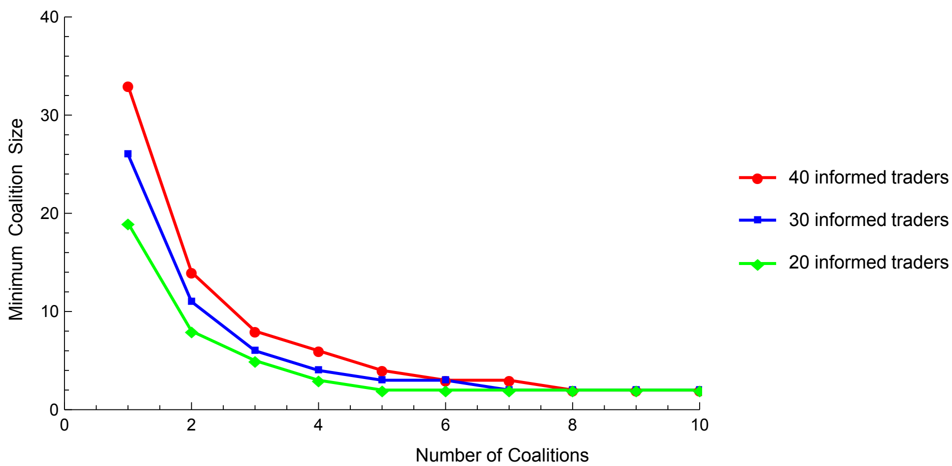
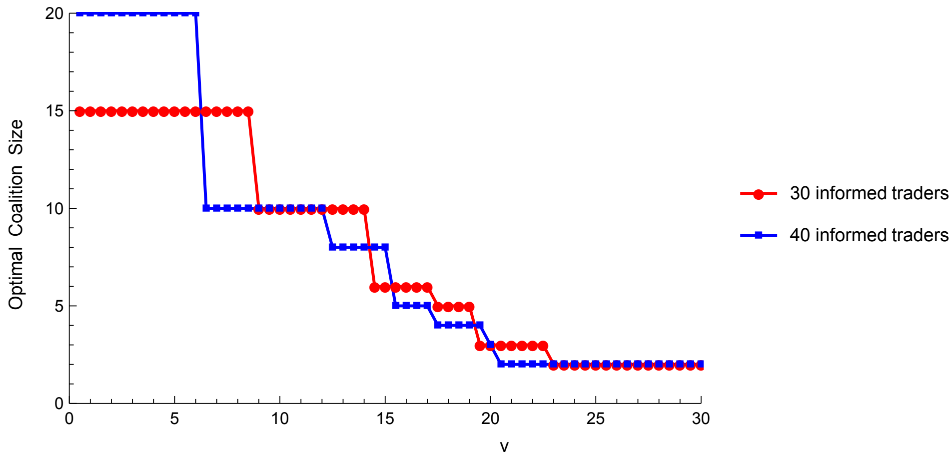


Figure 7: Minimum Coalition Size: Case 2

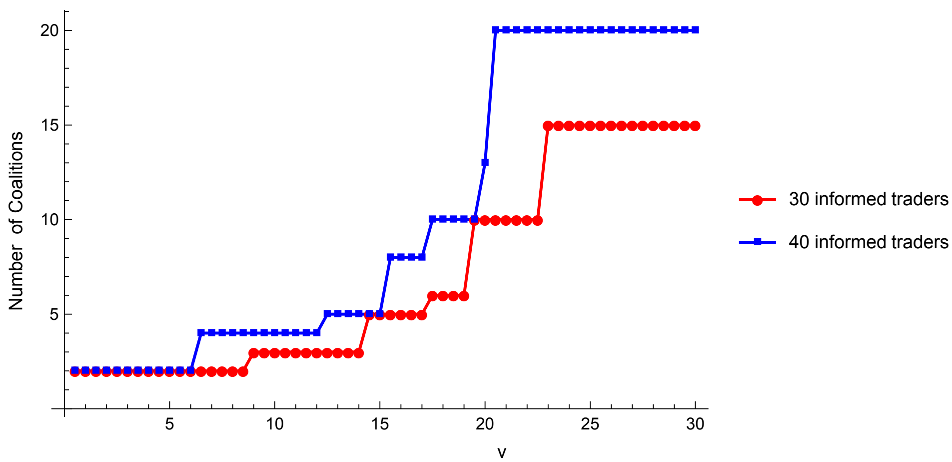
There can be $J = 20$, 30, or 40 informed traders, each of whom has the same-quality information, $v_j = 2.5$, $\forall j$. We assume that there can be several coalitions. The minimum size of a coalition, $\underline{m}$, is given by:

$$\begin{aligned} \min_{m} \quad & m \\ s.t. \quad & \Gamma_A(A = \{1, 2, ..., m\}) \geq 0. \end{aligned}$$

This figure plots the minimum coalition size, $\underline{m}$, depending on the number of coalitions.



(a) Optimal Coalition Size



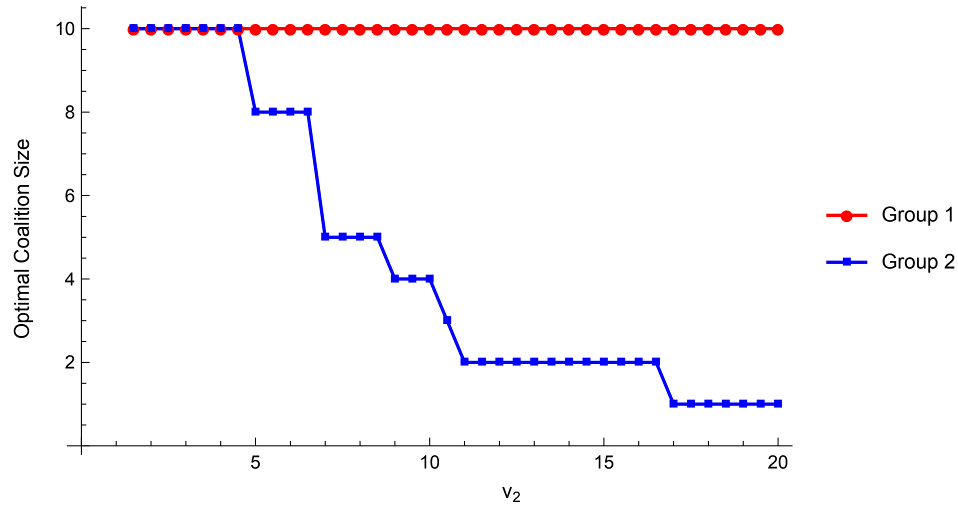
(b) Optimal Number of Coalitions

Figure 8: Optimal Coalition Structure

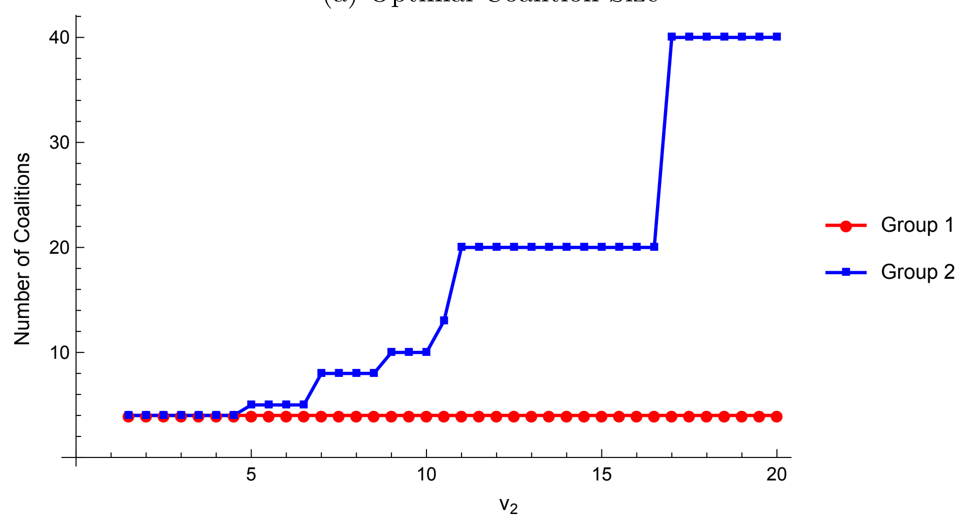
There can be $J = 30$ or 40 informed traders, each of whom has the same-quality information, $v_j = V$, $\forall j$. By symmetry, there can be n coalitions, each of which has m informed traders. The optimal $\{m, n\}$ are given by:

$$\max_{m,n} \quad \frac{1}{m} \left[\Gamma_A(A = \{1, 2, ..., m\}) - C(A(m)) \right],$$

where $C(A(m)) = 10^{-4}(m-1)^2$. This figure plots the optimal $\{m, n\}$ depending on V.



(a) Optimal Coalition Size



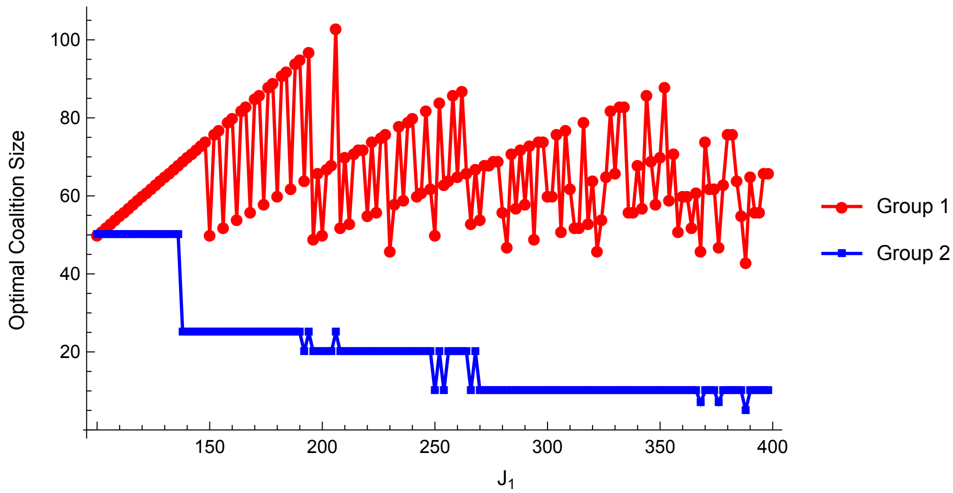
(b) Optimal Number of Coalitions

Figure 9: Optimal Coalition Structure with Two Isolated Groups of Informed Traders: Case 1

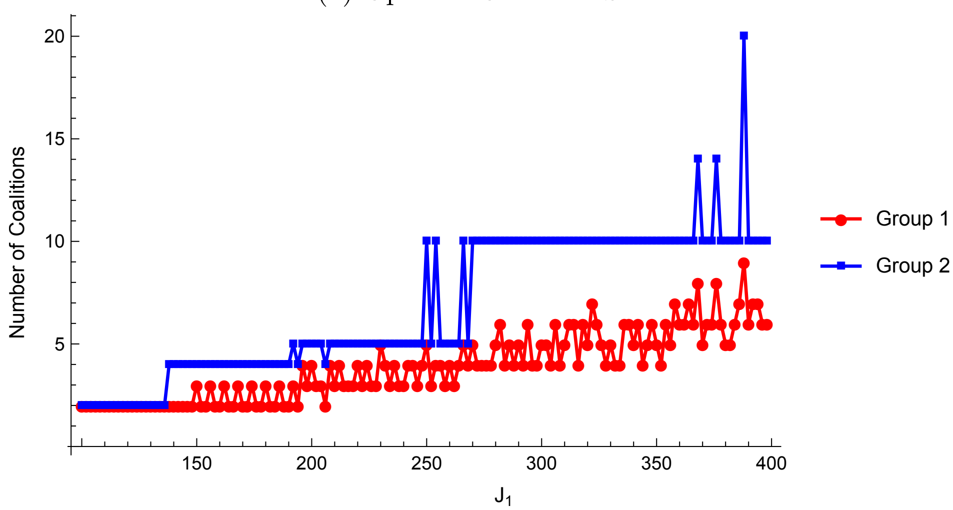
There are two isolated groups. Each group has 40 informed traders with the same-quality information, represented by V_1 and V_2. Let $V_1 = 1$ and $V_2 > 1$. By symmetry, in group 1 (2), there can be n_1 (n_2) coalitions, each of which has m_1 (m_2) informed traders. The optimal $\{m_1, m_2, n_1, n_2\}$ are given by:

$$\max_{m_1,m_2,n_1,n_2} \frac{1}{m_1}\Big[\Gamma_A(A = \{1, ..., m_1\}) - C(A(m_1))\Big] + \frac{1}{m_2}\Big[\Gamma_A(A = \{1, ..., m_2\}) - C(A(m_2))\Big],$$

where $C(A(m)) = 10^{-4}(m-1)^2$. This figure plots the optimal $\{m_1, m_2, n_1, n_2\}$ depending on V_2.



(a) Optimal Coalition Size



(b) Optimal Number of Coalitions

Figure 10: Optimal Coalition Structure with Two Isolated Groups of Informed Traders: Case 2

There are two isolated groups. Group 1 (2) has J_1 (J_2) informed traders. Let $J_1 > 100$ and $J_2 = 100$. Informed traders in both groups have the same-quality information, $V_1 = V_2 = 5$. By symmetry, in group 1 (2), there can be n_1 (n_2) coalitions, each of which has m_1 (m_2) informed traders. The optimal $\{m_1, m_2, n_1, n_2\}$ are given by:

$$\max_{m_1, m_2, n_1, n_2} \frac{1}{m_1}\Big[\Gamma_A(A = \{1, ..., m_1\}) - C(A(m_1))\Big] + \frac{1}{m_2}\Big[\Gamma_A(A = \{1, ..., m_2\}) - C(A(m_2))\Big],$$

where $C(A(m)) = 10^{-5}(m-1)^2$. This figure plots the optimal $\{m_1, m_2, n_1, n_2\}$ depending on J_1.