Capital Heterogeneity, Volatility Risk, and the Value Premium

Yong Kil Ahn*
Hong Kong University of Science and Technology

May 28, 2016

Abstract

I propose an investment-based asset pricing model augmented with intangible capital and transient volatility shock. Already-acquired intangible capital and new R&D investment are complementary inputs in knowledge production. The distinctive evolutionary dynamics of intangible capital as opposed to that of physical capital mitigate the negative impact of temporary volatility shock on output. Physical-capital-intensive value firms are thus more exposed to volatility risk and require more premium. The value premium is unconditionally positive *ex ante*, and the expected return of value firms surges conditionally upon a temporary volatility shock.

JEL classification: E22, E23, G11, G12.

Keywords: intangible capital, volatility shock, production, value premium

*Address: Department of Finance, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Email: ykahn@connect.ust.hk. I would like to appreciate K.C. John Wei for his invaluable comments. I would also like to thank Po-Hsuan Hsu, Kai Li, Peng Wang, and Chu Zhang for their valuable suggestions.

1. Introduction

This paper investigates the role of intangible capital complementarity in a firm's expected stock return in an extended neoclassical growth framework. In a seminal article, Griliches (1979) identifies that already-acquired intangible capital and new research and development (henceforth R&D) investment are complementary inputs in knowledge production. This specification has since been widely used in finance and economics, e.g., Hall and Hayashi (1989) and Romer (1990). Data also suggest that US firms have steadily increased the stock of intangible capital over the past sixty years, e.g., Hall (2001) and Falato, Kadyrzhanova, and Sim (2013). To the extent that the value of a firm measures the value of all productive capital (e.g., plants, structures, know-how, employee expertise, brands, organization capital, etc.), intangible capital is a crucial component of the value of the firm.

Here, I propose an investment-based asset pricing model augmented with heterogeneous capitals and time-varying volatility. In particular, I extend the canonical neoclassical growth model to include intangible capital and uncertainty fluctuation and use it to conclude that complementarity in intangible capital accumulation process together with time-varying volatility can generate a notably positive value spread.

My focus on the form of intangible capital builds on the growing literature on technology capital. McGrattan and Prescott (2009) have incorporated the notion of technology capital into the standard neoclassical growth theory. A firm's technology capital is the stock of the firm's unique expertise and know-how accumulated from investing in R&D, brand, organization capital, etc. Unlike other forms of capital, technology capital can be replicated without cost in multiple plants simultaneously. Technology capital incorporates the feature that a firm's unique know-how accumulated from past R&D investments can be freely employed in all of the firm's plants at the same time[1].

In linking the book-to-market ratio and the R&D effect, I adopt standard empirical

[1] The total output at the firm level exhibits constant return-to-scale, so that standard analysis can be easily applied.

evidence as given. Among others, Lev and Sougiannis (1999) show that "Low BM [book-to-market] companies have a large R&D capital, while high BM companies have low R&D investment". Ai and Kiku (2016) also point out that "high R&D spending and high Tobin's Q, low leverage, and low dividend yields ... are characteristic of growth firms"[2].

It has been well-recognized that time-varying economic uncertainty affects asset prices. The view that fluctuating macroeconomic uncertainty has significant impacts on asset returns stands mostly on the consumption-based asset pricing literature. For example, Boguth and Kuehn (2013) show that firms with cash flows more sensitive to consumption volatility require higher premium. Bansal, Kiku, Shaliastovich, and Yaron (2014) argue that an increase in aggregate volatility affects consumption and thus variation in the pricing kernel. Turning from consumption to production, aggregate uncertainty is also correlated with corporate activities in the real economy. Among others, Bloom (2009) shows that an increase in aggregate volatility is associated with an increase in the dispersion of firm profit growth, firm stock return, total factor productivity, and GDP forecast.

Optimal investment decision under conditions of fluctuating economic uncertainty, e.g., Lucas and Prescott (1971), Bloom, Bond, and Van Reenen (2007), and Bloom (2009), is a key driver of the correlation reported in the literature. It turns out that accounting for the causal link to asset returns has nonetheless proven far from simple. On both the empirical side and the theoretical side, canonical production-based asset pricing models, e.g., Cochrane (1991) and Zhang (2005), have been difficult to reconcile with the empirical phenomena that economic uncertainty fluctuation and joint uncertainty-cashflow relations determine stock return properties. Caballero (1991) also argues that the presence of asymmetric adjustment costs is not sufficient to render a negative relationship between investment and mean-preserving changes in uncertainty. A jointly dynamic uncertainty-investment structure has therefore become an important objective in the production-based asset pricing literature.

Recent literature on fluctuating economic uncertainty emphasizes that the impact of

[2] The Pearson product-moment correlation between book-to-market ratio and R&D intensity using the Compustat data from 1981 to 2014 is significantly negative at the 1% level.

temporary volatility shock appears to be salient in the production side of the real economy (Bloom (2009) and Arellano, Bai, and Kehoe (2012)). This simple fact, together with the concept of intangible capital complementarity, explains empirical regularities in corporate investment and financing activities and thus asset returns. The jointly optimal investment dynamics for physical capital and technology capital under time-varying volatility serve well to explain the cross-section of stock returns, providing a fresh insight into why physical-capital-intensive value firms require more risk premium than R&D-intensive growth firms.

The key mechanism is based on the real option theory, e.g., Bernanke (1983) and Bloom (2009). It is costly to adjust the stock of physical capital at each plant. When the real economy is bad and economic uncertainty is high, firms sit tight until economic conditions become clearer to avoid reducing the stock of physical capital. Moreover, idiosyncratic volatility is higher than aggregate volatility, e.g., Campbell, Lettau, Malkiel, and Xu (2001). Hence, a significant increase in idiosyncratic volatility at each level of disaggregation (i.e., from the macroeconomy to each plant) exacerbates the inaction on physical investment at each plant. However, it is not the case for intangible capital. The complementarity between already-acquired intangible capital and new R&D investment implies that firms have an incentive to sustain their R&D program at a certain level in order to be productive. Consequently, the real option effect is more significant for physical capital investment, leading physical-capital-intensive value firms to have cash flows which are more sensitive to the state of the real economy.

I incorporate this idea into the model by assuming the usual law of motion for physical capital stock but assuming that the stock of intangible capital evolves non-linearly in a complementary Cobb-Douglas manner. The standard production-based asset pricing framework deals with the evolution of intangible capital in the same way that physical capital develops over time. More formally, $G_{t+1} = (1-\delta_G)G_t + R_t$ where G_t is the stock of intangible capital at time t, δ_G is the rate of depreciation, and R_t is R&D investment for the period. Griliches (1979), Hall and Hayashi (1989), and Klette (1996) point out that this commonly-recognized accumulation process has unrealistic implications in two ways. First off, by assuming the same law of motion for the two capital stocks, intangible capital is supposed to be symmetric in correspondence with physical capital. The second problem is that the + sign between already-acquired intangible capital (G_t) and current R&D spending (R_t) necessarily means that G_t and R_t are substitutes. Taken together, R&D is supposed to be lumpy and intermittent, comparably to physical investment, which is not the case in the data. Unlike the usual law of motion, I have chosen a non-linear complementary knowledge-production function: $G_{t+1} = G_t^{1-\delta_G} R_t^{\delta_G}$. The new specification of intangible capital accumulation process allows firms to persistently spend on R&D because the complementarity between the existing stock of intangible capital and new R&D induces firms to continue to spend on R&D.

I derive the value premium and find that it is unconditionally positive *ex ante* in the model. Once installed, re-adjusting the stock of physical capital at a fire sale price is highly costly. Therefore, when the real economy is uncertain, firms take more precautions in physical investment to avoid selling off their tangible assets in the future, generating the real option effect. On the contrary, investments in R&D projects are immediately expensed, and current R&D investment complements the existing stock of technology capital. This complementarity between current R&D and the stock of already-acquired technology capital implies that the marginal product of R&D is decreasing in the amount of the current R&D project under standard assumptions. Firms thus have the incentive to stabilize R&D expenditures over time. As a result, physical investment responds to fluctuating economic uncertainty more negatively than R&D, implying that physical-capital-intensive firms are more exposed to volatility risk. Since stocks whose dividends are more sensitive to volatility require higher expected returns, the value premium is significantly positive.

The value spread is not only reliably positive but also responsive to transient volatility shocks. Conditional upon a volatility shock in the economy, the expected return of value firms surges temporarily, implying that the realized value premium plummets (often to negative in the data). The volatility shock is not long-lasting but short-lived, so the realized value premium tends to revert back to the unconditional mean upon resolution of the volatility shock. This channel lends support to Chen, Petkova, and Zhang (2008) in explaining the puzzlingly low performance of value strategies in the 1990s.

My model offers a new channel through which to understand the determinants of stock returns from the production side of the real economy. First, by linking the value premium to the optimal investment decision with two types of capital under fluctuating uncertainty, this paper provides a new economic explanation for the differences between glamour firms and value firms. The new volatility shock channel explored in this paper adds to the literature of the driving forces of the value premium[3]. Second, I present a real economic mechanism of the characteristic-based asset pricing models, e.g., Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003). Book-to-market ratio is a proxy for the responsiveness of corporate activities to volatility shock (or the state of the real economy) and thus has a predictive power in explaining the cross-section of stock returns. Third, this paper proposes a unified framework to analyze the determinants of corporate investment and financing decisions (Graham and Harvey (2001)) such as financial flexibility, earnings, cash flow volatility, and insufficient internal funds. Those determinants are intuitively telling, but it is hard to quantify them in a rational expectations equilibrium. My model successfully captures the three characteristics of corporate investment and financing decision.

The rest of the paper is organized as follows. Section 2 discusses the impact of time-varying volatility on the production side of the real economy to validate the use of an investment-based asset pricing framework. Section 3 extends the standard neoclassical growth model to include technology capital and uncertainty fluctuation. Section 4 discusses how to compute the model. A useful model is supposed to explain the data, therefore Section 4 also summarizes a detailed empirical assessment of the model. In Section 5, I present my key findings on the value premium through the lens of the proposed investment-based asset

[3] Several explanations have been advanced to explain the value premium. For example, rational variation by Fama and French (1993), Fama and French (1996), and Zhang (2005), investor sentiment by Bondt and Thaler (1985) and Lakonishok, Shleifer, and Vishny (1994), selection bias from Compustat by Kothari, Shanken, and Sloan (1995), and data snooping Conrad, Cooper, and Kaul (2003).

pricing model. Section 6 concludes the paper.

2. The Impact of Time-Varying Volatility

Aggregate volatility fluctuates over time. Figure 1 shows that monthly realized volatility of the S&P 500 index, a measure of economic uncertainty at the aggregate level, is not only time-varying but also clearly counter-cyclical. Volatility rises by 60% (7.96%p) on average in recessions recognized by the National Bureau of Economic Research (the gray bars in Figure 1).

[Insert Figure 1 Here]

In addition to the cyclical variation in economic uncertainty, market volatility appears to jump up after major economic events and becomes attenuated in periods following immediately after. For example, the Asian financial crisis in 1997 and the default of Long-Term Capital Management in 1998 occurred in between the 1991 recession and the 2001 recession. For the period from 1997 to 1998, market volatility surged three times higher than that of the tranquil mid-1990s.

To investigate that stock market volatility does jump, I use Barndorff-Nielsen and Shephard (2006)'s bipower variation test and find statistically significant evidence for volatility jumps[4]. The bipower variation test rejects the null hypothesis that the realized monthly average volatility data reported in Figure 1 are driven by a continuous semi-martingale process (H statistic = -1.4555 with p-value 0.07 and G statistic = -1.5366 with p-value 0.06). I further test the null of no-jumps using the VXO index from Chicago Board Options Exchange (CBOE)[5]. The test results using the monthly VXO index reveals much stronger evidence for volatility jumps (H statistic = -6.0143 with p-value $<$ 0.0001 and G statistic -8.7299 with

[4] See Appendix A for the bipower variation test for jumps.

[5] The VXO index is CBOE's volatility index. The VXO index has a longer data than the canonical VIX index. For more details, refer to CBOE's VXO website (http://www.cboe.com/micro/vxo/).

p-value < 0.0001)[6].

The theoretical literature on the impact of time-varying economic uncertainty focuses on the real option effect of uncertainty shock, e.g., Bernanke (1983), McDonald and Siegel (1986), Eberly (1994), and Bloom (2009). For instance, firms have the option to open a gold mine or undertake a new investment project but don't need to exercise the option immediately. Once the option is exercised, opportunity cost to reverse the option exercise is large. Hence, the more uncertain the real economy is, the greater the incentive to keep the irreversible option and wait for more information.

On the consumption side, when economic uncertainty increases, people's confidence in the future is shaken. For example, households may be thinking of buying a new car, but they could buy now or wait until next year. A household may be concerned about its income stability over the next few years due to high economic uncertainty. Rather than buying a new car immediately, it makes sense to defer spending on a car to avoid re-adjusting quotidian consumption items in the future[7].

An analogous logic applies to firms. Physical investment or hiring decisions can't be easily reversed due to adjustment costs, so it is better to pause temporarily when the economy turns bad. When the economy is highly uncertain, both productive firms and unproductive firms become less sensitive to the economic conditions. Productive firms are not expanding their capacity and unproductive firms are not contracting enough to the optimal point. The caution induced by volatility risk hinders the reallocation of resources across firms. As a result, high volatility makes firms temporarily intact in corporate activities and hence they wait to evaluate the situation until the economic smoke abates as in Bloom (2009)[8].

To establish the impact of volatility shock on the production side of the real economy,

[6] Using the daily VXO data, the null hypothesis of no-jump is rejected at the 0.0001 significance level.

[7] Deferring purchases of nondurable goods is not easy, so the real option effect is more prominent on durable consumption (Bloom (2014)).

[8] Real option effects arise when corporate decisions are not easily reversible. Thus, firms may continue to hire unskilled workers even when volatility is high because it is not costly to lay off part-time employees. Real option effects also depend on the degree of competition within an industry. For example, if firms are racing for a patent or a new product, the option value to defer spending will erode (Bloom (2014) and Patnaik (2015)).

I follow Bloom (2009) and run a monthly "short-run" orthogonal structural vector autoregression (SVAR) in the order of the S&P 500 index, an indicator function for economic uncertainty shock, federal funds rates (FFR), average hourly earnings, consumer price index (CPI), weekly hours, employment, and industrial production. The idea of this order is that volatility risk propagates into the economy through the stock market level first, prices (FFR, wages, and CPI), and then quantities (weekly hours, employment, and industrial production)[9].

A key issue associated with the SVAR is how to estimate it correctly. My identification allows for straightforward "structural" interpretation of volatility shocks in the sense that those volatility shocks spread into the real economy through stock market, price variables, and quantity variables. Using a non-stationary variable (e.g., the S&P 500 index) could cause a problem. But, Sims, Stock, and Watson (1990) show that the estimated coefficients of the SVAR with non-stationary variables remain consistent and the impulse response function is also consistent in the short run.

The indicator function for high volatility states takes value 1 for each of the 17 shocks from 1972 to 2015 in Table 1. In principal, the periods of volatility shocks are chosen to be 1.65 standard deviation above the Hodrick-Prescott detrended mean with $\lambda = 129,600$ (Ravn and Uhlig (2002)) of the monthly realized volatility of the S&P 500 index[10]. I then link the economic uncertainty indicator function to economic events such as war, disaster, or policy change as in Bloom (2009). I extend Bloom (2009)'s definition to include the Loma Prieta earthquake (October 1989), the liquidity shortfall (August 2007), the global financial crisis (October 2008), the Euro-zone crisis (May 2010), and the European sovereign debt crisis (August 2011). Other volatility events overlap with Bloom (2009) and include OPEC I (December 1973), Franklin National (October 1974), OPEC II (November 1978), the Iran hostage (March 1980), the US monetary cycle change (October 1982), Black Monday

[9] Appendix B details the data used for the SVAR model.

[10] If one shock lasts for several consecutive months, I choose the first month. Bloom (2009) finds similar results using the realized S&P 500 volatility before 1990 and the VIX index since 1990.

(October 1987), Gulf War I (October 1990), the Asian financial crisis (November 1997), the Russian moratorium (September 1998), the 9/11 terrorist attack (September 2001), the WorldCom and Enron accounting scandal (September 2002), and Gulf War II (February 2003).

[Insert Table 1 Here]

To examine the impact of volatility shock on output across different industries, I choose six major manufacturing industries based on R&D intensity reported in Table 2: 1) Chemical (NAICS=325), 2) Computer (NAICS=334), 3) Medical Equipment (NAICS=3391), 4) Machinery (NAICS=333), 5) Electrical Equipment (NAICS=335), and 6) Plastics and Rubber Products (NAICS=326)[11]. I exclude highly concentrated industries measured by the Herfindahl-Hirschman index because the real option effect of volatility shock depends on the degree of competition within the chosen industry[12]. When an industry is highly concentrated, firms have more freedom to cope with the state of the real economy, e.g., Patnaik (2015). Those six industries are chosen to be highly competitive, so that the industry structure doesn't play a role in explaining the link between the magnitude of the real option effect and R&D intensity.

[Insert Table 2 Here]

The percentage impact of volatility shock on industrial production, controlling for the S&P 500 index, FFR, wage, CPI, weekly hours, and employment, is plotted in Figure 2. I emphasize that the impact of stock market level on output is already controlled when backing out the impact of volatility shock on output. The impulse response function in Figure 2

[11] North American Industrial Classification System (NAICS) codes do not seamlessly match with Standard Industrial Classification (SIC) codes. Hence, I tabulate R&D intensity for major manufacturing industries from 1999 when the U.S. Department of Commerce first reported industrial R&D as a percent of net sales by NAICS codes.

[12] The Herfindahl-Hirschman index is a measure of the degree of competition in the industry. The index is defined as $H = \sum_{i=1}^{N} s_i^2$ in which N is the number of firms in the industry and s_i is the market share of firm i in percentage. The index ranges from 0 to 10,000, moving from a competitive market to a monopolistic market.

shows that R&D-intensive industries in Panel A such as Chemical, Computer, and Medical Equipment cope well with a sudden volatility shock. Given a volatility shock, industrial production in those industries doesn't decline over the next 36 months. The ± 1 standard error bound, denoted by dotted (+1 standard error) and dashed (-1 standard error) line, highlights that the results are significant at the 5% level. On the other hand, low R&D (i.e., physical-capital-intensive) industries in Panel B such as Machinery, Electrical Equipment, and Plastics and Rubber Products are highly responsive to volatility shocks. Industrial production falls sharply by 2-3%p in three months, rebounding back to the original level in six months, and overshooting over the next two years. In sum, the data suggest that the output of R&D-intensive industries is less responsive to volatility shocks than that of physical-capital-intensive industries.

[**Insert Figure 2 Here**]

Having established that the responsiveness of industrial production to market volatility shock is dichotomous between R&D-intensive industries and physical-capital-intensive industries, it is clear that the cross-sectional dispersion of industry output growth rates is positively associated with market volatility. To examine further the cross-section of corporate performance, I compute quarterly output growth rates for 194 US manufacturing industries for the period from 1972Q1 to 2015Q4. At each point in time, the cross-sectional dispersion is defined by the standard deviation of 194 manufacturing industry output growth rates. Figure 3 shows that, when aggregate volatility is high, the cross-sectional standard deviation of industry output growth rates widens out due to the difference in sensitivities to market volatility across industries[13]. More formally, the Pearson product-moment cor-

[13] The observation that the cross-sectional dispersion of corporate performance surges when the real economy is bad is not limited to the industry data. In fact, the cross-sectional dispersion rises sharply during periods of high volatility at every disaggregation (e.g., firm, plant, or product) level. At the firm level, Campbell et al. (2001) show that individual stock returns are 50% more volatile in economic downturns than in booms. At the plant level, Kehrig et al. (2011) show that the empirical productivity distribution of US manufacturing plants are more dispersed in recessions than in booms. Even at the individual product level (e.g., beverage), the variation in item prices is almost 50% higher in recessions (Vavra (2014)). I use this evidence to incorporate a jointly dynamic investment-uncertainty structure into the standard growth

relation coefficient between cross-sectional standard deviation of quarterly industry output growth rate and aggregate volatility is 0.2846 with p-value 0.0001.

[Insert Figure 3 Here]

I proceed to investigate asset pricing implications. The results in Figure 2 and Figure 3 predict that the dividends of growth stocks tend to be less responsive to volatility shocks than those of value stocks. To check this prediction, I regress ten portfolio returns sorted on book-to-market ratio on the volatility shock indicator function defined in Table 1:

$$r_{i,t} = \alpha_i + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t} \quad (1)$$

$$r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t} \quad (2)$$

where $r_{i,t}$ is the excess return of each of the book-to-market decile portfolios at time t, $\mathbb{I}_{\text{volatility shock}}$ is the volatility shock indicator function defined in Table 1, and MKT_t is the market excess return. I also run a more general regression on the innovations of the VXO index:

$$r_{i,t} = \alpha_i + \beta_i^V \Delta VXO_t + \epsilon_{i,t} \quad (3)$$

$$r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \Delta VXO_t + \epsilon_{i,t} \quad (4)$$

[Insert Table 3 Here]

Table 3 summarizes the results from Eq.(1), Eq.(2), Eq.(3), and Eq.(4). Consistent with the prediction in Figure 2 and Figure 3, the factor loadings of the value-minus-growth portfolio are significantly negative in all of the regression specifications. The risk loading of the value-minus-growth portfolio on volatility shock is -3.50 ($t = -3.06$) in Panel A. After controlling for the market excess return, the risk loading is still significantly negative (-3.59

model at the firm level.

with $t = -3.02$) in Panel B. Using innovations in the VXO index exhibits the same results. In Panel C and D, we observe a significantly negative risk loading of the value-minus-growth portfolio on the changes in the VXO index: -0.16 with $t = -2.38$ in Panel C and -0.24 with $t = -2.59$ in Panel D. Taken together, the result implies that when there is a market volatility shock, value stocks would suffer more than growth stocks.

3. The Model

I take the canonical neoclassical growth model and incorporate technology capital (McGrattan and Prescott (2009)) and time-varying market volatility into it. As is standard in the literature, I formulate an economy populated by a continuum of *ex ante* homogeneous consumers with unit mass and heterogeneous value-maximizing firms in a competitive market. Time is monthly from 0 to ∞ throughout this paper.

As the focus of this paper is on the production side of the real economy, I don't close the economy in general equilibrium. I find it computationally more efficient to specify an exogenous stochastic discount factor[14]. As long as I discipline the stochastic discount factor to match the rich aggregate dynamics observed in the data, this approach seems reasonable. A detailed assessment of the defined stochastic discount factor is discussed in Section 4.

3.1. Firms

3.1.1. Profit

A firm consists of N identical production units. Each production unit produces a homogeneous final good which is either consumed or used for the production of another final goods. Technology capital that firm i possesses can be costlessly exploited at all plants simul-

[14] Hence, aggregate consumption is exogenous, whereas aggregate dividends are endogenous in the model. But, both are affected by the same aggregate productivity shock process in the model. To validate the pricing kernel, I match the risk-free rate and the maximal Sharpe ratio induced by the assumed pricing kernel with those observed in the data.

taneously. Following McGrattan and Prescott (2009) and McGrattan and Prescott (2010), I assume that the firm production function exhibits constant return-to-scale in technology capital and composite production input. G_i units of technology capital together with K_i units of physical capital produce:

$$F_i = \tilde{A}(G_i N)^{\phi}(K_i^{\alpha})^{1-\phi} \tag{5}$$

in which ϕ is the technology capital share, α is the physical capital portion in production input, and $\tilde{A}$ represents a stochastic productivity shock process which will be defined later.

The demand in the economy is iso-elastic with a constant elasticity ϵ and a stochastic demand shifter $\tilde{B}$:

$$Q = \tilde{B}P^{-\epsilon}. \tag{6}$$

This formulation leads to the following revenue function of firm i:

$$\begin{aligned} Y_i &= P \times F_i \\ &= \tilde{A}^{1-1/\epsilon}\tilde{B}^{1/\epsilon}(G_i N)^{\phi(1-1/\epsilon)} K_i^{\alpha(1-\phi)(1-1/\epsilon)}. \end{aligned} \tag{7}$$

Redefining $a = \phi(1-1/\epsilon)$, $b = (1-\phi)(1-1/\epsilon)$, and $\tilde{A}^{1-1/\epsilon}B^{1/\epsilon} = A_i^{1-a-b}$, I have

$$Y_i = A_i^{1-a-b}(G_i N)^a (K_i^{\alpha})^b.$$

Here, I combine the productivity shock $\tilde{A}$ and the stochastic demand shifter $\tilde{B}$ into one term A_i. Let $f_K \geq 0$ be the fixed cost of production (i.e., operating leverage). The net output produced by firm i is then:

$$Y_i = A_i^{1-a-b}(G_i N)^a (K_i)^b - f_K \tag{8}$$

3.1.2. Technology

The economic condition $A_{i,t}$ that firm i faces at time t is modeled as two different productivity shocks: aggregate productivity shock x_t and firm-level idiosyncratic productivity shock $z_{i,t}$ with $A_{i,t} = \exp(x_t + z_{i,t})$.

The aggregate productivity shock process x_t is assumed to follow an exogenous stationary Markov process:

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_t^x \epsilon_{t+1}^x, \quad \epsilon_{t+1}^x \sim N(0, 1) \tag{9}$$

which aims to capture the total factor productivity at the macroeconomy (e.g., Cooley and Prescott (1995)). The firm-level idiosyncratic productivity shock $z_{i,t}$, which is crucial to generate a non-trivial cross-section of firms (e.g., Zhang (2005)), follows a first-order autoregressive process:

$$z_{i,t+1} \;\; = \;\; \rho_z z_{i,t} + \sigma_t^z \epsilon_{t+1}^{z_i} \tag{10}$$

where $\epsilon_{t+1}^{z_i} \sim N(0, 1)$ is independent of each other for any i and t.

3.1.3. Economic Uncertainty

For simplicity, the stochastic processes for volatility are defined as a two-state Markov chain with transition probability $\pi_{i,j}$:

$$\sigma_t^x \;\; \in \;\; \{\sigma_L^x,\, \sigma_H^x\} \tag{11}$$

$$\sigma_t^z \;\; \in \;\; \{\sigma_L^z,\, \sigma_H^z\} \tag{12}$$

$$Pr\Big[\sigma_{t+1} = \sigma_j | \sigma_t = \sigma_i\Big] = \pi_{i,j} \tag{13}$$

Eq.(11) and Eq.(12) are parsimonious but the process has been proven to be powerful enough to capture the data (Bloom (2009)). Eq.(13) implies that the two stochastic conditional volatility processes are based on the same Markov process. Hence, high aggregate volatility states are associated with high idiosyncratic volatility states as in Figure 3. This formulation enables the technology distribution at the firm level to jointly depend on conditional idiosyncratic volatility and in turn aggregate states of the economy, creating a mixed non-linear structure for the technology distribution[15].

3.1.4. Heterogeneous Capital

In constructing the model where firms optimally choose the stock of technology capital and physical capital, a key empirical fact that I want to incorporate into the model is that physical investment is intermittent and lumpy whereas R&D spending is highly persistent. The first-order auto-correlation coefficient for yearly R&D spending of Compustat manufacturing firms in the United States for the period of 1981 to 2014 is 0.7693, while that of yearly physical investment is 0.4818. The second-order auto-correlation coefficients are 0.4818 for R&D and 0.3367 for physical investment. Although it may be because the adjustment cost associated with physical investment is much higher than that of R&D investment, it may also be due to a difference in the way the two types of capital are accumulated and exploited.

In short, a factory is different from an engineer. Both are capital goods but they serve different purposes. Firms install fixed assets such as equipment, machinery, or plants to produce final goods immediately. Those fixed assets are specific for each plant. In contrast, technology capital (e.g., know-how, blueprints, or brand) is firm-specific in the sense that it can be used for production at every plant where the firm operates.

Besides, the accumulation process of technology capital through R&D possesses unique characteristics which make technology capital different from physical capital. First, more

[15] Finite-state Markov chains evolve over time in a discrete way, so that Markov regime shifting models are more suitable for modeling discontinuous changes (i.e., jumps) in volatility. GARCH models are also simple enough but it is impossible to decouple productivity and volatility using GARCH. To illustrate this, let us assume that productivity (z_t) follows an AR(1) process: $z_{t+1} = \rho_z z_t + \sigma_t \epsilon_{t+1}$. σ_t is assumed to follow a GARCH process, say $\sigma_{t+1}^2 = \omega + \alpha(\sigma_t \epsilon_{t+1})^2 + \beta\sigma_t^2$. It is then impossible to disentangle a volatility shock from a level shock since both the level and the conditional volatility of idiosyncratic productivity are driven by one shock (ϵ_{t+1}). For more details, refer to Fernández-Villaverde and Rubio-Ramírez (2010).

than half of R&D spending is the wages and salaries paid by firms to highly educated engineers and scientists whose efforts create firm-specific intangible capital (Hall (2002)). To the extent that the accumulated technology capital is impossible to fully codify, firms have an incentive to smooth out their R&D spending in order to avoid frequently hiring and firing talented engineers and key persons. Second, the gestation period for R&D is longer than the conventional time-to-build period for ordinary physical investment. R&D is often carried out for a prolonged period of time to aim to yield long-term profit. Third, the marginal product of R&D may not be equal to unity. For example, two-year commitment of five R&D engineers may not have the same productivity as five-year commitment of two R&D engineers. Fourth, it is hard for outsiders to evaluate a firm's tacit technology capital stock. The more tacit an asset is, the lower its redeployability is (Williamson (1988)). That is, there is no outside competitive market for technology capital. The resale value of firm-specific technology capital is thus nearly zero.

I formulate this idea by assuming the usual law of motion for the stock of physical capital:

$$K_{i,t+1} = (1 - \delta_K)K_{i,t} + I_{i,t} \tag{14}$$

but assuming that technology capital is accumulated non-linearly in a complementary Cobb-Douglas manner[16]:

$$\begin{aligned} G_{i,t+1} &= \big(G_{i,t}\big)^{1-\delta_G}\big(R_{i,t}\big)^{\delta_G}, \quad \delta_G \in (0,1) && (15) \\ R_{i,t} &\geq 0 && (16) \end{aligned}$$

where $K_{i,t}$ is the stock of physical capital at time t, δ_K is the rate of physical capital depreciation, $I_{i,t}$ denotes physical investment, $G_{i,t}$ is the stock of technology capital at time t,

[16] Hall and Hayashi (1989), Klette (1996), and Hall (2002) discuss how the stock of knowledge capital evolves over time. Doraszelski and Jaumandreu (2013) investigate the impact of R&D on productivity and provide empirical support for non-linearity in the knowledge capital accumulation process.

δ_G is the portion of R&D in knowledge production[17], and $R_{i,t}$ indicates R&D expenditure. Also, R&D spending is assumed to have no resale value and accordingly can't be reversed as in Eq.(16)[18].

Both Eq.(14) and Eq.(15) are linearly homogeneous, but the marginal product of physical investment is in sharp contrast to that of R&D. The marginal product of physical investment is unity in Eq.(14), whereas the marginal return to R&D expenditure is decreasing in $R_{i,t}$ in Eq.(15) if $\delta_G \in (0,1)$:

$$\frac{\partial K_{i,t+1}}{\partial I_{i,t}} = 1 \tag{17}$$

$$\frac{\partial^2 K_{i,t+1}}{\partial I_{i,t}^2} = 0 \tag{18}$$

$$\frac{\partial G_{i,t+1}}{\partial R_{i,t}} = \delta_G \big(G_{i,t}\big)^{1-\delta_G} \big(R_{i,t}\big)^{\delta_G - 1} \tag{19}$$

$$\frac{\partial^2 G_{i,t+1}}{\partial R_{i,t}^2} = \big(\delta_G - 1\big)\delta_G \big(G_{i,t}\big)^{1-\delta_G} \big(R_{i,t}\big)^{\delta_G - 2} < 0. \tag{20}$$

Consequently, Eq.(14) and Eq.(15) can successfully capture the observed empirical pattern that physical investment is lumpy and intermittent while R&D is spread out over time. Physical investment involves large and lump-sum expenditures which can't be easily reversed. The unit marginal product of physical investment implies that firms time their business conditions for physical investment to avoid costly reversing the physical capital stock back to the previous level. On the contrary, lumpy R&D investment decreases the marginal product of R&D due to Eq.(19), generating incentive to smooth out R&D spending over time.

[17] This can be interpreted as the rate of technology capital depreciation. Since δ_G is defined in a non-arithmetic way, δ_G is not directly comparable with δ_K.

[18] In computation, I use a normalizing constant ρ_0 to keep the stock of technology capital from vanishing if the firm doesn't spend on R&D in a period: $G_{i,t+1} = G_{i,t}^{1-\delta_G}(\rho_0 + R_{i,t})^{\delta_G}$. The results are not affected by the choice of ρ_0.

3.1.5. Adjustment Cost

Adjusting the stock of capital goods is subject to non-convex (or fixed) disruption cost (Cooper and Haltiwanger (2006)) and convex adjustment cost (Hall (2001) and Zhang (2005)). Following the literature, I specify a piece-wise quadratic adjustment cost function for physical investment:

$$C^K_{i,t}(I_{i,t}, K_{i,t}) = \begin{cases} b^+_K K_{i,t} + \frac{c^+_K}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t} & \text{if } I_{i,t} > 0, \\ 0 & \text{if } I_{i,t} = 0, \\ b^-_K K_{i,t} + \frac{c^-_K}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t} & \text{if } I_{i,t} < 0. \end{cases} \tag{21}$$

in which $b^-_K > b^+_K > 0$ and $c^-_K > c^+_K > 0$ capture the fact that it is more costly to adjust tangible capital downward. Changing the stock of technology capital through R&D investment also entails adjustment costs:

$$C^G_{i,t}(R_{i,t}, G_{i,t}) = \begin{cases} 0 & \text{if } R_{i,t} = 0, \\ b_G G^i_t + \frac{c_G}{2}\left(\frac{R_{i,t}}{G_{i,t}}\right)^2 G_{i,t} & \text{if } R_{i,t} > 0. \end{cases} \tag{22}$$

Here, I follow Hayashi (1982) and introduce a separate adjustment cost function. The incurred adjustment costs are directly cashed out from internal funds. The evolution of physical and technology capital (i.e., Eq. (14) and Eq. (15)) is therefore left intact[19].

The adjustment cost associated with changing the stock of technology capital through R&D, together with the complementarity in the technology capital accumulation process defined in Eq.(15), may explain why differences in R&D intensity among firms persist over time. That is, even within the same industry, some firms do R&D to accumulate technology capital, while other firms don't spend on R&D at all. Because it is costly to initiate an R&D project, marginal firms don't spend on R&D. Once a firm kicks off an R&D project to boost

[19] Another way to introduce adjustment cost can be found in Uzawa (1969) where adjustment costs are directly incorporated into the law of motion for capital.

its production technology, the technology capital stock that the firm has acquired already through past R&D investments makes future R&D investments more productive due to the complementarity in Eq.(15), leading the firm to continue to spend on R&D.

3.1.6. Financing

Firms can issue equity at any point in time. I denote the net payout to equity holders by $E_{i,t}$. Equity flotation is costly (e.g., Hennessy and Whited (2007)) and can be described by

$$\Gamma(E_{i,t}) = \big(\gamma_0 + \gamma_1 |E_{i,t}|\big)\mathbb{I}_{\{E_{i,t}<0\}}. \tag{23}$$

Denoting distributions to equity holders by $D_{i,t} = E_{i,t} - \Gamma(E_{i,t})$, corporate financing and investment decisions must satisfy the following resource constraint:

$$D_{i,t} = Y_{i,t} - I_{i,t} - C^K_{i,t} - R_{i,t} - C^G_{i,t} \tag{24}$$

3.1.7. Equity Value Maximization

Equity holders optimally choose physical investment $I_{i,t}$ and R&D investment $R_{i,t}$ to maximize the following Bellman equation:

$$V_{i,t}(K_{i,t}, G_{i,t}, \sigma_t, x_t, z_{i,t}) = \max\left[0, \max_{I_{i,t},R_{i,t}} [D_{i,t} + \mathbb{E}_t(M_{t,t+1}V_{i,t+1})]\right] \tag{25}$$

subject to Eq.(14), Eq.(15), Eq.(16), and Eq.(24).

3.2. Households

Following Berk et al. (1999), Zhang (2005), and Kuehn and Schmid (2014), I don't close the model in general equilibrium. Rather, I take advantage of a parametric stochastic discount factor because the focus of this paper is on the production of the economy. As far as the assumed stochastic discount factor matches the aggregate dynamics observed in the data, this approach seems plausible. A detailed assessment of the assumed pricing kernel is discussed in Section 4.

In particular, the stochastic discount factor is given by:

$$\log(M_{t,t+1}) = \log\beta + \nu_t(x_t - x_{t+1}) \tag{26}$$

$$\nu_t = \nu_0 + \nu_1(x_t - \bar{x}), \quad \nu_1 < 0 \tag{27}$$

in which β is the representative consumer's subjective discount factor, x_t is the aggregate productivity shock process defined in Eq.(9), and both ν_0 and ν_1 are constant.

The rationale behind Eq.(26) is the following. The pricing kernel implied by a power-utility representative agent is given by:

$$\log(M_{t,t+1}) = \log\beta + RRA(c_t - c_{t+1}) \tag{28}$$

where c_t denotes the logarithm of aggregate consumption and RRA is the coefficient of relative risk aversion. As in Zhang (2005), I tie c_t with the aggregate state variable x_t by assuming $c_t = a_c + b_c x_t$ for some constant a_c and b_c and defining $\nu_t = RRA \times b_c$. The result follows. The process $\nu_t = \nu_0 + \nu_1(x_t - \bar{x})$ with $\nu_1 < 0$ is clearly decreasing in $(x_t - \bar{x})$, therefore, unlike the constant price of risk of power utility, the process ν_t with $\nu_1 < 0$ captures the fact that the price of risk is counter-cyclical.

3.3. Rational Expectations Equilibrium

In a rational expectation equilibrium, the evolution T_μ of the cross-section of firms over time, together with the mapping T_p from the aggregate state to the inter-temporal marginal rate of substitution of the representative household-owner p, characterize the economy from an individual firm's perspective. Letters with a prime denote next period's values.

$$\mu' = T_\mu(\mu, \sigma, x) \tag{29}$$

$$p' = T_p(\mu, \sigma, x) \tag{30}$$

That is, given optimal policies, the cross-sectional distribution μ of firms at time t satisfies:

$$\mu(\sigma', x', z') = P\Big[\sigma_{t+1} = \sigma', x_{t+1} \leq x', z_{t+1} \leq z'\Big] \tag{31}$$

4. Computing the Model

A unique solution for continuous, concave, and bounded value function is guaranteed thanks to Stokey and Lucas (1989). This means that numerical results will converge to the unique solution due to the fixed point theorem. Hence, I adopt numerical dynamic programming to approximate the competitive equilibrium.

I use iterative procedure to maximize the value function. I begin with an initial guess for the value function and solve for the firm's optimization problem in Eq.(25) on discrete state space[20]. I iterate the procedure until the value function converges.

4.1. Approximating AR(1) Processes

I follow Rouwenhorst (1995) to approximate the aggregate productivity process (x_t) defined in Eq.(9)[21]. As the same procedure can be applied to the idiosyncratic productivity shock process, I focus on how to discretize the aggregate productivity process below.

I assume that x_t can take n_x values, say $\{x_1, \ldots, x_{n_x}\}$ over the interval $[-\bar{\epsilon}, \bar{\epsilon}]$ with

[20] I first confine the state space to a closed and bounded (hence, compact) set (e.g., $[0, \bar{K}]$) to have a unique solution in the dynamic programming. The state space is discretized using the McGrattan et al. (1998) method.

[21] Tauchen (1986), Tauchen and Hussey (1991), and Adda and Cooper (2003) are also available. Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more robust than other approximation methods when the Markov process is highly persistent.

$x_1 = -\bar{\epsilon}$ and $x_{n_x} = \bar{\epsilon}$. The variance of x is then given by

$$Var[x] = \frac{\bar{\epsilon}^2}{n_x - 1} \tag{32}$$

For any $n_x \geq 2$, two parameters $p,\ q \in (0,1)$ govern the transition matrix of the n_x-state Markov chain. Moreover, it can be shown that the first-order auto-correlation is equal to $p + q - 1$. Hence, the auto-correlation of Eq.(9) and its variance can be matched by setting

$$\frac{\bar{\epsilon}^2}{n_x - 1} = \frac{\sigma_x^2}{1 - \rho_x^2} \tag{33}$$

$$p = q = \frac{1 + \rho_x}{2} \tag{34}$$

For $n_x = 2$, the two-state Markov chain transition matrix is defined as:

$$\Theta_2 = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

Working forward recursively, the n_x-state Markov chain transition matrix for any $n_x \geq 3$ is defined as:

$$\Theta_{n_x} = p \begin{bmatrix} \Theta_{n_x-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0} & \Theta_{n_x-1} \\ 0 & \mathbf{0}' \end{bmatrix} + (1-q) \begin{bmatrix} \mathbf{0}' & 0 \\ \Theta_{n_x-1} & \mathbf{0} \end{bmatrix} + q \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \Theta_{n_x-1} \end{bmatrix}$$

in which $\mathbf{0}$ is a $(n_x - 1) \times 1$ column vector of zeros. Because the conditional probability is supposed to sum to one, the middle rows besides the top and bottom one are divided by two.

4.2. Calibration

The model requires me to calibrate 27 parameters: nine for business conditions, seven for production function, three for pricing kernel, six for investment adjustment costs, and two for external equity financing cost. I list all the parameters in Table 4.

[Insert Table 4 Here]

I begin with technology parameters on which we have strong prior beliefs. The aggregate state variable x_t follows a stationary auto-regressive process. The autoregressive coefficient of the process is assumed to be 0.9933, and the conditional volatility is set to be 0.0040 for low volatility state and 0.0080 for high volatility state, respectively. $\rho_x = 0.9933$ and $\sigma_L^x = 0.0040$ at the monthly frequency are consistent with $0.98 = (0.9933)^3$ and $0.0070 = 0.0040 \times \sqrt{1 + (0.9933)^2 + (0.9933)^4}$ at the quarterly frequency in the real business cycle literature (e.g., Cooley and Prescott (1995)). Following Bloom (2009), I assume that conditional volatility becomes twice during periods of high volatility states, i.e., $\sigma_H^x = 2 \times \sigma_L^x$. The long-run mean of the aggregate productivity process, $\bar{x} = -4.4$, is calibrated to match steady-state capital stock.

The parameters $\rho_z = 0.9800$ and $\sigma_L^z = 0.20$ denote the degree of persistence and cross-sectional dispersion in the idiosyncratic productivity process. I thus restrict these two parameters to be able to unconditionally match the cross-sectional dispersion reported in Gomes (2001), Campbell et al. (2001), and Pástor and Pietro (2003). Also, consistent with the real business cycle literature (e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)), I assume that micro uncertainty fluctuates more than aggregate uncertainty.

The calibrated idiosyncratic productivity shock is about fifty times as volatile as the aggregated productivity shock. Such a high idiosyncratic volatility is necessary to generate a non-trivial cross-section of firms. Nonetheless, I stress that the value of firm is not very responsive to the idiosyncratic productivity shock process z_t because z_t affects only cash flow. On the contrary, the aggregate productivity shock process x_t affects both cash flow and discount rate simultaneously. When the economy is in recessions, a firm's cash flow is low, and the representative consumer's marginal utility will be high. This two effects lead firm value to be more sensitive to the aggregate productivity shock.

Volatility shock occurs once every three years ($\pi_{L,H} = 1/36$) in the model, and its half life is assumed to be two months ($\pi_{H,H} = \sqrt{2}$). I borrow this numbers directly from Bloom (2009) who figures out 17 uncertainty shocks in 46 years. The assumed transition probability from low volatility state to high volatility state, $\pi_{L,H} = 1/36$, is also close to 0.0278 reported in Bloom et al. (2012).

The second set of parameters is related to the production side of the real economy. Hence, I base my calibration on the macro-finance literature. The share of physical capital (α) in the production input is 0.70. I set the technology capital share in output (ϕ) to be 0.07 following McGrattan and Prescott (2009) and McGrattan and Prescott (2010). The constant demand elasticity is assumed to be 4 as in Bloom (2009). This means that markup is 1/3[22].

The monthly rate of physical capital depreciation (δ_K) is set to be 0.01 following Cooper and Haltiwanger (2006) and Zhang (2005), implying 12% per year as is usual in the literature. The share of current R&D expenditure in generating technology capital is set at 0.05 at the monthly frequency, largely consistent with Hall, Jaffe, and Trajtenberg (2000) and Falato et al. (2013). The fixed cost of production $f = 0.4580$ is chosen to match the average book-to-market of the model with that observed the data (e.g., Gomes (2001)). The number of plants per firm (N) is assumed to be 10. I also try $N = 5$ and $N = 20$. The results are not sensitive to the number of plants.

Given the parametric form of the stochastic discount factor, the gross risk-free rate can

[22] $\dfrac{dPQ}{dQ} = \left(1 - \dfrac{1}{\epsilon}\right) Q^{-1/\epsilon} \tilde{B}^{1/\epsilon}$. Hence, $P = \dfrac{\epsilon}{1-\epsilon} \dfrac{dPQ}{dQ}$.

be written in a closed-form as well:

$$
\begin{aligned}
R^f_{t,t+1} &= \frac{1}{\mathbb{E}_t[M_{t,t+1}]} \\
&= \frac{1}{\beta}\exp\left[-[\nu_0+\nu_1(x_t-\bar{x})](1-\rho_x)(x_t-\bar{x})-0.5\big[\sigma_x\big(\nu_0+\nu_1(x_t-\bar{x})\big)\big]^2\right] \quad (35)
\end{aligned}
$$

Any excess return R^e_{t+1} is supposed to obey:

$$
\mathbb{E}_t[M_{t,t+1}R^e_{t+1}] = \mathbb{E}_t[M_{t,t+1}]\mathbb{E}_t[R^e_{t+1}] + \rho_{M,R}\sigma_t[M_{t,t+1}]\sigma_t[R^e_{t+1}] = 0 \quad (36)
$$

where $|\rho_{M,R}| \leq 1$ is the correlation coefficient. The maximal Sharpe ratio of the pricing kernel, i.e., the Hansen and Jagannathan (1991) bound is thus:

$$
\begin{aligned}
\frac{\sigma_t[M_{t,t+1}]}{\mathbb{E}_t[M_{t,t+1}]} &= \sqrt{e^{\sigma_x^2\left(\nu_0+\nu_1(x_t-\bar{x})\right)^2}-1} \quad (37) \\
&\geq \frac{|\mathbb{E}_t[R^e_{t+1}]|}{\sigma_t[R^e_{t+1}]} \quad (38)
\end{aligned}
$$

The pricing kernel is supposed to be volatile enough to match the observed Sharpe ratio. I therefore discipline the stochastic discount factor to match the Sharpe ratio and the first and second moments of risk free rate observed in the data. This can be accomplished by setting $\beta = 0.9933$, $\nu_0 = 26$, and $\nu_1 = -250$. Table 5 compares model-generated key aggregate moments with those observed the data. The data source is Campbell and Cochrane (1999). The first and second moment of real interest rate generated under the benchmark calibration are 0.0237 and 0.0356, respectively. Campbell and Cochrane (1999) also report similar numbers using the postwar US data.

[Insert Table 5 Here]

Table 5 also reports Sharpe ratios computed on the aggregate productivity shock space. The average Sharpe ratio is 0.4116 throughout the business cycle in the benchmark model. The variation in Sharpe ratios is substantial. Sharpe ratios range between 0.0292 and 0.8014 on the grid of the aggregate productivity shock. The model-generated average Sharpe ratio is 0.6334 in recessions and 0.2271 in booms. The numbers are largely consistent with Lustig and Verdelhan (2012) who report that the Sharpe ratio ranges from 0.14 to 0.85 and the average Sharpe ratio is 0.66 in bad times and 0.35 in expansions. Having pinned down a set of key aggregate moments using the benchmark calibration, I emphasize that the stochastic discount factor has no more room to match the cross-section of stock returns, which is the focus of this paper.

To calibrate the parameters of adjustment costs, I follow Cooper and Haltiwanger (2006), Zhang (2005), Bloom (2009), and Belo, Lin, and Bazdresch (2014). There is no consistent calibration in the literature, but the key idea is that it is more costly to adjust the stock of capital downward. This goal can be accomplished by assuming $b_K^+ << b_K^-$ and $c_K^+ << c_K^-$. Hence, I set b_K^+, b_K^-, c_K^+, and c_K^- to be 0.02, 0.03, 3, and 30, respectively. The adjustment cost of technology capital is not well-studied in the literature. I set $b_G = 0.03$ and $c_G = 3$ because it is presumably the case that R&D projects are more costly to initiate but less costly to continue, compared to physical investment.

I am left with those parameters related to external equity issuance cost. Firms can raise fund by issuing new equity whenever the cash flow is short of required physical and R&D investment in return for equity flotation costs. Following Hennessy and Whited (2007) and Kuehn and Schmid (2014), I assume that equity issuance incurs both fixed cost ($\gamma_0 = 0.0032$) and variable cost ($\gamma_1 = 0.01$).

5. Quantitative Implications

5.1. The Value Premium

I now investigate the cross-section of stock returns. To compare the benchmark model with the data, I first summarize the characteristics of book-to-market decile portfolios observed in the data from 1962 to 2015. Panel A of Table 6 shows that the value premium is unconditionally positive (0.47% per month) in the data. Also, the alpha from the Capital Asset Pricing Model (CAPM) for the high-minus-low book-to-market portfolio is significantly positive (0.47% with $t = 2.60$).

[Insert Table 6 Here]

To see how the benchmark model matches the data, I simulate 100 artificial samples, each with 1000 firms, from the benchmark model and compute means, standard deviations, alphas, and betas from the CAPM. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French (1993)'s timing convention and run the CAPM regression. Panel B of Table 6 shows that the benchmark model successfully generates a notably positive value spread. The model-generated value spread closely matches that of the data. The value premium in the benchmark model is 0.45% per month unconditionally, close to the observed value premium of 0.47% per month in the data.

Even though the benchmark model does a reasonable job in explaining the value spread, the model fails to capture the failure of the CAPM. The estimated CAPM alpha of the high-minus-low book-to-market portfolio is not significantly different from zero in Panel B. It is because the economy is modeled in a dynamic factor framework where exogenous state variables perfectly capture the model economy and the cross-section of firms generated by idiosyncratic shocks will be eventually integrated out. Given that I parameterize the benchmark model in a dynamic factor structure, this failure suggests that the pricing kernel should be more complex in the model. In fact, this problem is prevalent in most investment-based asset pricing models. For example, Panel C, borrowed from Lin and Zhang (2013), shows that the CAPM perfectly holds in Zhang (2005) because he models his economy in a dynamic one-factor framework.

5.2. The Impact of Volatility Shock on Stock Returns

I define an indicator function for model-implied volatility shock to compare the empirical evidence reported in Table 3 with the model-generated data. The stochastic processes for volatility are assumed to be a two-state Markov in Eq.(11) and Eq.(12). Hence, the model-implied indicator function for volatility shock equals one when the two-state Markov chain is in the high volatility state. I then run the same regression as in Eq.(1) and Eq.(2) using simulated data to check the validity of the model.

[**Insert Table 7 Here**]

Table 7 summarizes the results from Eq.(1) and Eq.(2) using the model-generated data. Consistent with the results from the real data in Table 3, the factor loading of the value-minus-growth portfolio on volatility shock is significantly negative: -3.59 with $t = -3.02$ in Panel A and -3.82 with $t = -6.95$ in Panel B in Table 7. Those numbers closely match the results from the real data: -3.50 with $t = -3.06$ in Panel A and -3.59 with $t = -3.02$ in Panel B of Table 3. This implies that growth stocks do better than value stocks during periods of high volatility because the output of R&D-intensive growth firms are less responsive to that of physical-capital-intensive value firms. Taken together, the benchmark model works well to establish a causal link between volatility shock and the cross-section of stock returns.

5.3. Sensitivity Analysis

To examine further the robustness of the results, I conduct comparative sensitivity analysis by varying 1) volatility shock, 2) adjustment cost, 3) price of risk, and 4) equity financing cost. This experiments help demonstrate which channels contribute more significantly to the value premium. Table 8 reports the results from comparative analysis.

[**Insert Table 8 Here**]

First off, I shut the volatility shock channel down. If the conditional volatility for both productivity shock processes is constant, the value premium significantly shrinks down to 0.09% per month (equivalent to 1.12% per annum) in Table 8. Secondly, I set the adjustment cost function to be symmetric. The value spread is still notably positive and comparable to that from the data and the benchmark model. Thirdly, the counter-cyclical price of risk doesn't significantly contribute to the value spread. Unlike the Zhang (2005) model where the counter-cyclical price of risk and highly asymmetric adjustment cost are key drivers of the value spread, the proposed investment-based model in this paper generates a significantly positive value spread (0.40% per month) even with symmetric adjustment cost and constant price of risk. Lastly, the existence of equity financing cost also marginally affects the value premium. Given the external financing cost structure where the fixed cost ($\gamma_0 = 0.0032$) is smaller than the variable cost ($\gamma_1 = 0.01$), value firms that issue equity in lump-sum bear more equity flotation cost. Taken together, the value spread is reliably positive and doesn't vary much in all of the specifications in Table 8 except for the case of constant conditional volatility, emphasizing the importance of the real option channel of time-varying volatility shock.

5.4. Demystifying the Book-to-Market Ratio

Although firm characteristics are not directly linked to the aggregate productivity shock process in Eq.(9), book-to-market ratio has a predictive power in explaining the cross-section of equity returns. To see this, let's consider the first-order condition of the value function posited by Eq.(25) with respect to I_t:

$$-1 - \frac{\partial C_t^K}{\partial I_t} + \mathbb{E}_t\Big[M_{t,t+1}\frac{\partial V_{t+1}}{\partial I_t}\Big] = 0 \tag{39}$$

The law of motion for K_t implies that $\frac{\partial K_{t+1}}{\partial I_t} = 1$. Hence, we can replace $\frac{\partial V_{t+1}}{\partial I_t}$ with $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ in Eq.(39). Also, the gross investment return $R^I_{t,t+1}$ from I_t is supposed to satisfy the following

Euler equation:

$$\mathbb{E}_t[M_{t,t+1}R^I_{t,t+1}] = 1 \tag{40}$$

Combining Eq.(39) and Eq.(40) yields:

$$R^I_{t,t+1} = \frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{1 + \frac{\partial C^K_t}{\partial I_t}} \tag{41}$$

where $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ is the marginal benefit of physical capital and $1 + \frac{\partial C^K_t}{\partial I_t}$ is the marginal cost of investment. The marginal cost of investment is nothing but the marginal q. Putting it differently,

$$1 + \frac{\partial C^K_t}{\partial I_t} = q = \mathbb{E}[M_{t,t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}] \tag{42}$$

The construction of the marginal benefit of investment on the right hand side of Eq.(42) has been the focus of the literature so far. However, whatever structure we explore, the left-hand side of Eq.(42) links the market-to-ratio to the assumed factor model[23]. In other words,

$$\mathbb{E}\big[M_{t,t+1}\frac{K_t}{V_t}\frac{\partial V_{t+1}}{\partial K_{t+1}}\big] = 1 = \mathbb{E}_t[M_{t,t+1}R^I_{t,t+1}]$$

Taken together, book-to-market ratio captures the optimal investment decision under fluctuating economic uncertainty. This result is in line with Berk et al. (1999) and Gomes et al. (2003) who link the book-to-market ratio directly to the stock return.

6. Conclusions

I have developed a parsimonious investment-based asset pricing model in a dynamic factor structure. By linking the cross-sectional dispersion of firms to the state of the real economy, I create a non-orthogonal space-time continuum of firms. Technology capital accu-

[23] Marginal q is equal to average q when production function is constant return-to-scale and adjustment cost function is linearly homogeneous. The proof is in Appendix C. See Hayashi (1982) for more details.

mulated from persistent R&D investment is central to explaining the dichotomous impact of fluctuating economic uncertainty on production between R&D-intensive firms and physical-capital-intensive firms. The key mechanism is that complementarity between past R&D and current R&D incentivizes firms to continue to spend on R&D, leading R&D-intensive firms to be less sensitive to the state of the real economy.

The benchmark model puts forth the canonical neoclassical production models in that my model works well to explain the optimal investment decision under time-varying economic uncertainty. The value spread arises naturally due to the difference in sensitivities to the state of the economy. Value firms are more exposed to volatility risk and thus require higher premium. Unlike other production models, the benchmark model generates significantly positive abnormal return on portfolios sorted on book-to-market ratio. Moreover, the factor loading of volatility shock on the return differential between high and low book-to-market portfolio is reliably negative, implying that value firms are more negatively affected by sudden volatility shocks than growth firms. Finally, I also show how characteristic-based asset pricing models suggested by Berk et al. (1999) and Gomes et al. (2003) can be interpreted in an investment-based asset pricing framework.

Future research is certainly necessary. The mechanism explored in this paper is asset complementarity and uncertainty fluctuation. A natural extension of the model is to include a separate stochastic uncertainty process and link the process to the productivity processes conditionally. A study at a more disaggregated level (e.g., plant) is also called for.

Appendix A. Bipower Variation Test for Jump

Let λ_t denote the log price of an asset. For continuous time $t \geq 0$ and for any partition $0 = t_t < ... < t_n = t$ with $\sup_j[t_{j+1} - t_j] \to 0$ as $n \to 0$, the quadratic variation process of a semi-martingale λ_t is:

$$[\lambda_t] = p\text{-}\lim_{n\to\infty} \sum_{j=0}^{n-1} (\lambda_{t_{j+1}} - \lambda_{t_j})^2 \tag{43}$$

For some interval δ of time t, we define the $\delta-$returns on a discretized λ_t on intervals with length δ as:

$$y_j = \lambda_{j\delta} - \lambda_{(j-1)\delta}, \quad j = 1, ..., \lfloor t/\delta \rfloor \tag{44}$$

in which $\lfloor \cdot \rfloor$ is the integer part operator. The realized quadratic variation of the discretized log price process can be defined as:

$$[\lambda_\delta]_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2 \tag{45}$$

The 1,1-order bipower variation, if it exists, is:

$$\{\lambda_t\} = p\text{-}\lim_{\delta\downarrow 0} \sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}||y_j| \tag{46}$$

Similar to the realized quadratic variation process of the discretized log price process, the realized bipower variation process can be defined as:

$$\{\lambda_\delta\}_t^{[1,1]} = \sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}||y_j| \tag{47}$$

Also, the realized quadpower variation process is:

$$\{\lambda_\delta\}_t^{[1,1,1,1]} = \delta^{-1} \sum_{j=4}^{\lfloor t/\delta \rfloor} |y_{j-3}||y_{j-2}||y_{j-1}||y_j| \tag{48}$$

If there is no jump, the realized quadratic variation is supposed to converge to the realized bipower variation because a continuous semi-martingale process drives the two variations. If the underlying process does jump, the realized quadratic variation would be greater than the realize bipower variation because the magnitude of jumps is squared in the realized quadratic variation process. Based on this intuition, Barndorff-Nielsen and Shephard (2006) propose two feasible test statistics for jump. The linear jump test statistic has the asymptotic distribution:

$$\hat{G} = \frac{\delta^{-1/2}\big(\mu^{-2}\{\lambda_\delta\}_t^{[1,1]} - [\lambda_\delta]_t\big)}{\sqrt{\theta\mu^{-4}\{\lambda_\delta\}_t^{[1,1,1,1]}}} \to N(0,1) \tag{49}$$

The ratio jump test rejects the null of no jumps if the following statistic is significantly negative:

$$\hat{H} = \frac{\delta^{-1/2}}{\sqrt{\theta\{\lambda_\delta\}_t^{[1,1,1,1]}/\big(\{\lambda_\delta\}_t^{[1,1]}\big)^2}}\Big[\frac{\mu^{-2}\{\lambda_\delta\}_t^{[1,1]}}{[\lambda_\delta]_t} - 1\Big] \to N(0,1) \tag{50}$$

in which $\mu = \mathbb{E}|u|$ with u $N(0,1)$ and $\theta = \frac{\pi^2}{4} + \pi - 5 \approx 0.6090$.

Appendix B. Data for Vector Auto-Regression

I detail the data used for the structural vector auto-regression (SVAR) in Section 2. The SVAR model is in the order of the S&P 500 index from the Center for Research in Security Prices, an indicator function for volatility shocks defined in Table 1, effective federal funds rates (series ID: FEDFUNDS from the Federal Reserve Research Database), average hourly earnings (Series ID: CES3000000008 from the Bureau of Labor Statistics), consumer price index (Series ID: CUSR0000SA0 from the Bureau of Labor Statistics), weekly hours (Series ID: CES3000000007 from the Bureau of Labor Statistics), employment (Series ID: CES3000000001 from the Bureau of Labor Statistics), and industrial production (Series ID: IP.G325.S for Chemical, IP.G334.S for Computer, IP.N3391.S for Medical Equipment, IP.G333.S for Machinery, IP.G335.S for Electrical Equipment, and IP.G326.S for Plastics and Rubber Products from the Bureau of Labor Statistics). The data are monthly from 1972 to 2015. All variables are Hodrick-Prescott detrended with $\lambda = 129,600$ to remove a slow-moving cyclical component (Ravn and Uhlig (2002)).

Appendix C. Marginal q and Average q

I summarize Hayashi (1982). I intend to be brief as the proof is standard in the literature. Following Hayashi (1982), I use Newton's dot notation (e.g., $\dot{K}_t$) for differentiation below.

A price-taking firm maximizes the present value of the expected future stream of cash flows

$$\max_{I_t, L_t} V_t = \int_t^{\infty} e^{-r(s-t)} \big[Y_s - w_s L_s - I_s - A(I_s, K_s) \big] ds \tag{51}$$

subject to $I_t - \delta K_t = \dot{K}_t$ where Y_t is output, A_t is the capital adjustment cost, K_t is physical capital stock, L_t are hours worked, w_t is the wage, δ is the depreciation ratio.

The current value Hamiltonian is

$$H = Y_t - w_t L_t - I_t - A(I_t, K_t) + q_t(I_t - \delta K_t) \tag{52}$$

where q_t is the shadow price of the capital accumulation, (i.e., marginal q). Let me suppress time subscript t. The first-order conditions are

$$\begin{aligned} \frac{\partial H}{\partial L} &= Y_L - w = 0 & (53) \\ \frac{\partial H}{\partial I} &= -1 - A_I + q = 0 & (54) \\ \dot{q} &= rq - (Y_K - A_K - \delta q) & (55) \end{aligned}$$

And, the transversality condition is given by:

$$\lim_{t \to \infty} e^{-rt} q_t K_t = 0. \tag{56}$$

We assume that $Y(K_t, L_t)$ is constant return-to-scale (CRS) and $A(I_t, K_t)$ is linearly homogeneous. From the first-order conditions, we have:

$$\dot{q}K = (r + \delta)qK - Y_K K + A_K K$$

The differential equation for qK is

$$
\begin{aligned}
\dot{(qK)} &= q\dot{K} + \dot{q}K \\
&= q(I - \delta K) + (r + \delta)qK - Y_K K + A_K K \\
&= (A_I + 1)I + rqK - (Y - Y_L L) + A_K K \\
&= rqK - (Y - wL - I - A)
\end{aligned}
$$

The third and last equality use the CRS assumption and the linearly-homogeneous adjustment cost assumption, respectively. w is the labor-market-clearing wage. This leads to the following equation:

$$
\begin{aligned}
-e^{-rt}(Y_t - w_t L_t - I_t - A(I_t, K_t)) &= e^{-rt}\Big(\frac{d(q_t K_t)}{dt} - rq_t K_t\Big) \\
&= \frac{d(e^{-rt} q_t K_t)}{dt}
\end{aligned}
$$

Integrating both sides yields

$$
\begin{aligned}
-V_t &= -\int_t^{\infty} e^{-r(s-t)}(Y_s - w_s L_s - I_s - A_s)ds \\
&= \int_t^{\infty} d[e^{-r(s-t)} q_s K_s] \\
&= -q_t K_t
\end{aligned}
$$

Finally, marginal q is equal to average q (i.e., market-to-book ratio):

$$
q_t = \frac{V_t}{K_t}
$$

References

Adda, J., Cooper, R. W., 2003. Dynamic economics: quantitative methods and applications. MIT press.

Ai, H., Kiku, D., 2016. Volatility risks and growth options. Management Science 62, 741–763.

Arellano, C., Bai, Y., Kehoe, P. J., 2012. Financial frictions and fluctuations in volatility. Federal Reserve Bank of Minneapolis Research Department Staff Report 466 .

Bansal, R., Kiku, D., Shaliastovich, I., Yaron, A., 2014. Volatility, the macroeconomy, and asset prices. The Journal of Finance 69, 2471–2511.

Barndorff-Nielsen, O. E., Shephard, N., 2006. Econometrics of testing for jumps in financial economics using bipower variation. Journal of financial Econometrics 4, 1–30.

Belo, F., Lin, X., Bazdresch, S., 2014. Labor hiring, investment, and stock return predictability in the cross section. Journal of Political Economy 122, 129–177.

Berk, J. B., Green, R. C., Naik, V., 1999. Optimal investment, growth options, and security returns. The Journal of Finance 54, 1553–1607.

Bernanke, B. S., 1983. Irreversibility, uncertainty, and cyclical investment. The Quarterly Journal of Economics pp. 85–106.

Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77, 623–685.

Bloom, N., 2014. Fluctuations in uncertainty. The Journal of Economic Perspectives pp. 153–175.

Bloom, N., Bond, S., Van Reenen, J., 2007. Uncertainty and investment dynamics. The review of economic studies 74, 391–415.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., Terry, S. J., 2012. Really uncertain business cycles. Tech. rep., National Bureau of Economic Research.

Boguth, O., Kuehn, L.-A., 2013. Consumption volatility risk. The Journal of Finance 68, 2589–2615.

Bondt, W. F., Thaler, R., 1985. Does the stock market overreact? The Journal of finance 40, 793–805.

Caballero, R. J., 1991. On the sign of the investment-uncertainty relationship. The American Economic Review 81, 279–288.

Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

Campbell, J. Y., Lettau, M., Malkiel, B. G., Xu, Y., 2001. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. The Journal of Finance 56, 1–43.

Chen, L., Petkova, R., Zhang, L., 2008. The expected value premium. Journal of Financial Economics 87, 269–280.

Cochrane, J. H., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. The Journal of Finance 46, 209–237.

Conrad, J., Cooper, M., Kaul, G., 2003. Value versus glamour. The Journal of Finance 58, 1969–1996.

Cooley, T. F., Prescott, E. C., 1995. Economic growth and business cycles. Frontiers of business cycle research 39, 64.

Cooper, R. W., Haltiwanger, J. C., 2006. On the nature of capital adjustment costs. The Review of Economic Studies 73, 611–633.

Doraszelski, U., Jaumandreu, J., 2013. R&d and productivity: Estimating endogenous productivity. The Review of Economic Studies p. rdt011.

Eberly, J. C., 1994. Adjustment of consumers' durables stocks: Evidence from automobile purchases. Journal of Political Economy pp. 403–436.

Falato, A., Kadyrzhanova, D., Sim, J., 2013. Rising intangible capital, shrinking debt capacity, and the us corporate savings glut .

Fama, E. F., French, K. R., 1992. The cross-section of expected stock returns. the Journal of Finance 47, 427–465.

Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. Journal of financial economics 33, 3–56.

Fama, E. F., French, K. R., 1996. Multifactor explanations of asset pricing anomalies. The journal of finance 51, 55–84.

Fernández-Villaverde, J., Rubio-Ramírez, J., 2010. Macroeconomics and volatility: Data, models, and estimation. Tech. rep., National Bureau of Economic Research.

Gomes, J., Kogan, L., Zhang, L., 2003. Equilibrium cross section of returns. Journal of Political Economy 111.

Gomes, J. F., 2001. Financing investment. American Economic Review pp. 1263–1285.

Graham, J. R., Harvey, C. R., 2001. The theory and practice of corporate finance: Evidence from the field. Journal of financial economics 60, 187–243.

Griliches, Z., 1979. Issues in assessing the contribution of research and development to productivity growth. The Bell Journal of Economics pp. 92–116.

Hall, B., Hayashi, F., 1989. Research and development as an investment. Tech. rep., National Bureau of Economic Research.

Hall, B. H., 2002. The financing of research and development. Oxford review of economic policy 18, 35–51.

Hall, B. H., Jaffe, A. B., Trajtenberg, M., 2000. Market value and patent citations: A first look. Tech. rep., national bureau of economic research.

Hall, R. E., 2001. The stock market and capital accumulation. American Economic Review 91, 1185–1202.

Hansen, L. P., Jagannathan, R., 1991. Implications of security market data for models of dynamic economies. Journal of Political Economy pp. 225–262.

Hayashi, F., 1982. Tobin's marginal q and average q: A neoclassical interpretation. Econometrica: Journal of the Econometric Society pp. 213–224.

Hennessy, C. A., Whited, T. M., 2007. How costly is external financing? evidence from a structural estimation. The Journal of Finance 62, 1705–1745.

Kehrig, M., et al., 2011. The cyclicality of productivity dispersion .

Klette, T. J., 1996. R&d, scope economies, and plant performance. The RAND Journal of Economics pp. 502–522.

Kopecky, K. A., Suen, R. M., 2010. Finite state markov-chain approximations to highly persistent processes. Review of Economic Dynamics 13, 701–714.

Kothari, S. P., Shanken, J., Sloan, R. G., 1995. Another look at the cross-section of expected stock returns. The Journal of Finance 50, 185–224.

Kuehn, L.-A., Schmid, L., 2014. Investment-based corporate bond pricing. The Journal of Finance 69, 2741–2776.

Lakonishok, J., Shleifer, A., Vishny, R. W., 1994. Contrarian investment, extrapolation, and risk. The journal of finance 49, 1541–1578.

Lev, B., Sougiannis, T., 1999. Penetrating the book-to-market black box: the r&d effect. Journal of Business Finance & Accounting 26, 419–449.

Lin, X., Zhang, L., 2013. The investment manifesto. Journal of Monetary Economics 60, 351–366.

Lucas, R. E. J., Prescott, E. C., 1971. Investment under uncertainty. Econometrica: Journal of the Econometric Society pp. 659–681.

Lustig, H., Verdelhan, A., 2012. Business cycle variation in the risk-return trade-off. Journal of Monetary Economics 59, S35–S49.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. The Quarterly Journal of Economics pp. 707–728.

McGrattan, E. R., Prescott, E. C., 2009. Openness, technology capital, and development. Journal of Economic Theory 144, 2454–2476.

McGrattan, E. R., Prescott, E. C., 2010. Technology capital and the us current account. American Economic Review 100, 1493–1522.

McGrattan, E. R., et al., 1998. Application of weighted residual methods to dynamic economic models. Federal Reserve Bank of Minneapolis, Research Department.

Pástor, L., Pietro, V., 2003. Stock valuation and learning about profitability. The Journal of Finance 58, 1749–1790.

Patnaik, R., 2015. Competition and the real effects of uncertainty .

Ravn, M. O., Uhlig, H., 2002. On adjusting the hodrick-prescott filter for the frequency of observations. Review of economics and statistics 84, 371–376.

Romer, P. M., 1990. Endogenous technological change. Journal of Political Economy pp. S71–S102.

Rouwenhorst, K. G., 1995. Asset pricing implications of equilibrium business cycle models. Frontiers of business cycle research .

Sims, C. A., Stock, J. H., Watson, M. W., 1990. Inference in linear time series models with some unit roots. Econometrica: Journal of the Econometric Society pp. 113–144.

Stokey, N., Lucas, R., 1989. Recursive methods in economic dynamics. Harvard University Press.

Tauchen, G., 1986. Finite state markov-chain approximations to univariate and vector autoregressions. Economics letters 20, 177–181.

Tauchen, G., Hussey, R., 1991. Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. Econometrica: Journal of the Econometric Society pp. 371–396.

Uzawa, H., 1969. Time preference and the penrose effect in a two-class model of economic growth. The Journal of Political Economy pp. 628–652.

Vavra, J., 2014. Inflation dynamics and time-varying volatility: New evidence and an ss interpretation*. Quarterly Journal of Economics 129.

Williamson, O. E., 1988. Corporate finance and corporate governance. The journal of finance 43, 567–591.

Zhang, L., 2005. The value premium. The Journal of Finance 60, 67–103.

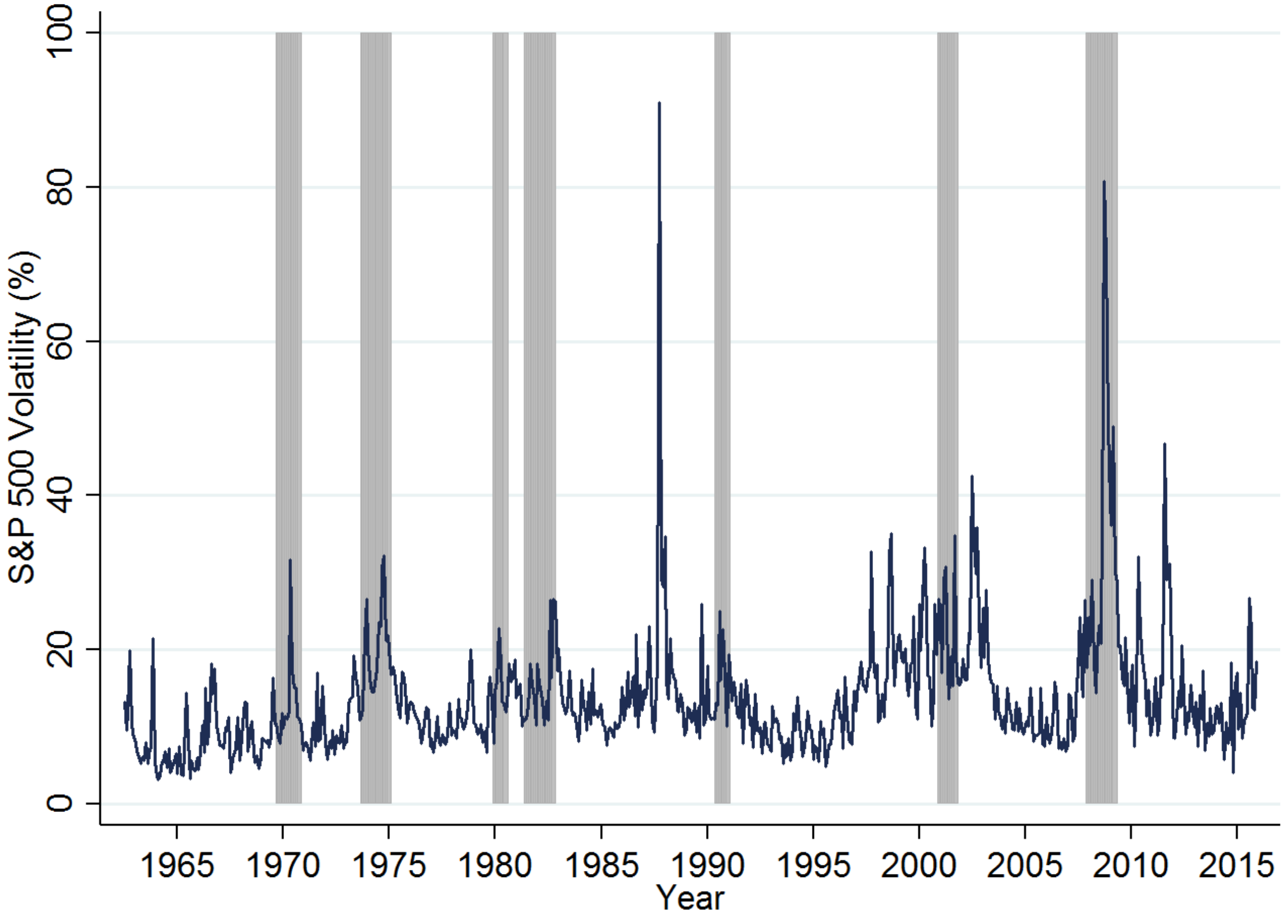
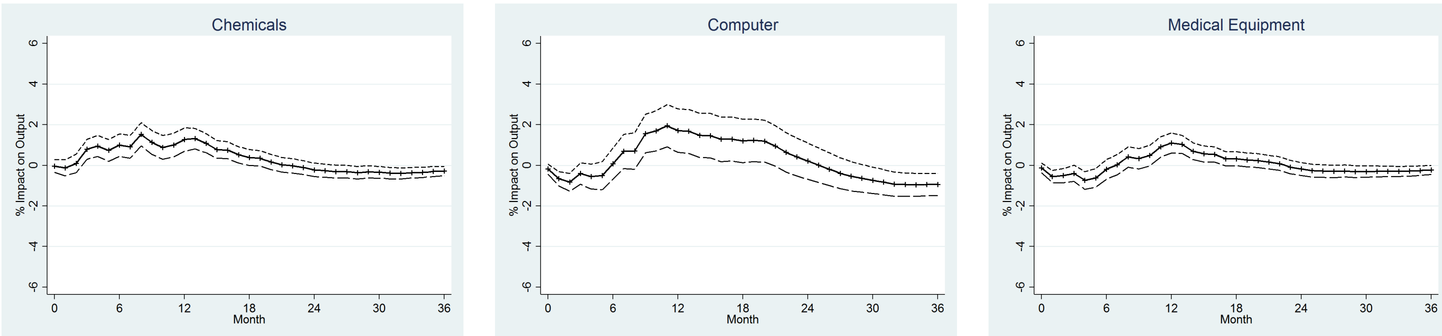


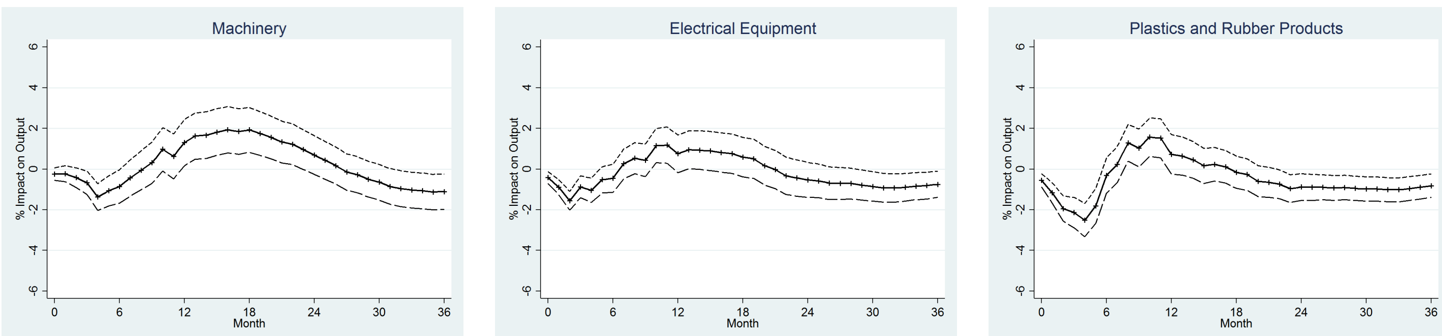
Fig. 1. Monthly S&P 500 Volatility. This figure plots the monthly realized volatility of the S&P 500 index from July 1962 to December 2015. Gray bars represent recessions as recognized by the National Bureau of Economic Research.

Panel A: High R&D Industries



Chemical Computer Medical Equip.

Panel B: Low R&D Industries



Machinery Electrical Equip. Plastics

Fig. 2. Impulse Response Function. This figure plots the impulse response function from the structural vector auto-regression (SVAR) for six major manufacturing industries: 1) Chemical (NAICS=325), 2) Computer (NAICS=334), 3) Medical Equipment (NAICS=3391), 4) Machinery (NAICS=333), 5) Electrical Equipment (NAICS=335), and 6) Plastics and Rubber Products (NAICS=326). The SVAR is in the order of the S&P 500 index from the Center for Research in Security Prices, an indicator function for volatility shocks defined in Table 1, effective federal funds rates (series ID: FEDFUNDS from the Federal Reserve Research Database), average hourly earnings (Series ID: CES3000000008 from the Bureau of Labor Statistics), consumer price index (Series ID: CUSR0000SA0 from the Bureau of Labor Statistics), weekly hours (Series ID: CES3000000007 from the Bureau of Labor Statistics), employment (Series ID: CES3000000001 from the Bureau of Labor Statistics), and industrial production (Series ID: IP.G325.S for Chemical, IP.G334.S for Computer, IP.N3391.S for Medical Equipment, IP.G333.S for Machinery, IP.G335.S for Electrical Equipment, and IP.G326.S for Plastics and Rubber Products from the Bureau of Labor Statistics). The data are monthly from 1972 to 2015. All variables are Hodrick-Prescott detrended with $\lambda = 129,600$ to remove a slow-moving cyclical component (Ravn and Uhlig (2002)). Dotted and dashed lines represent the ± 1 standard error bound, respectively.

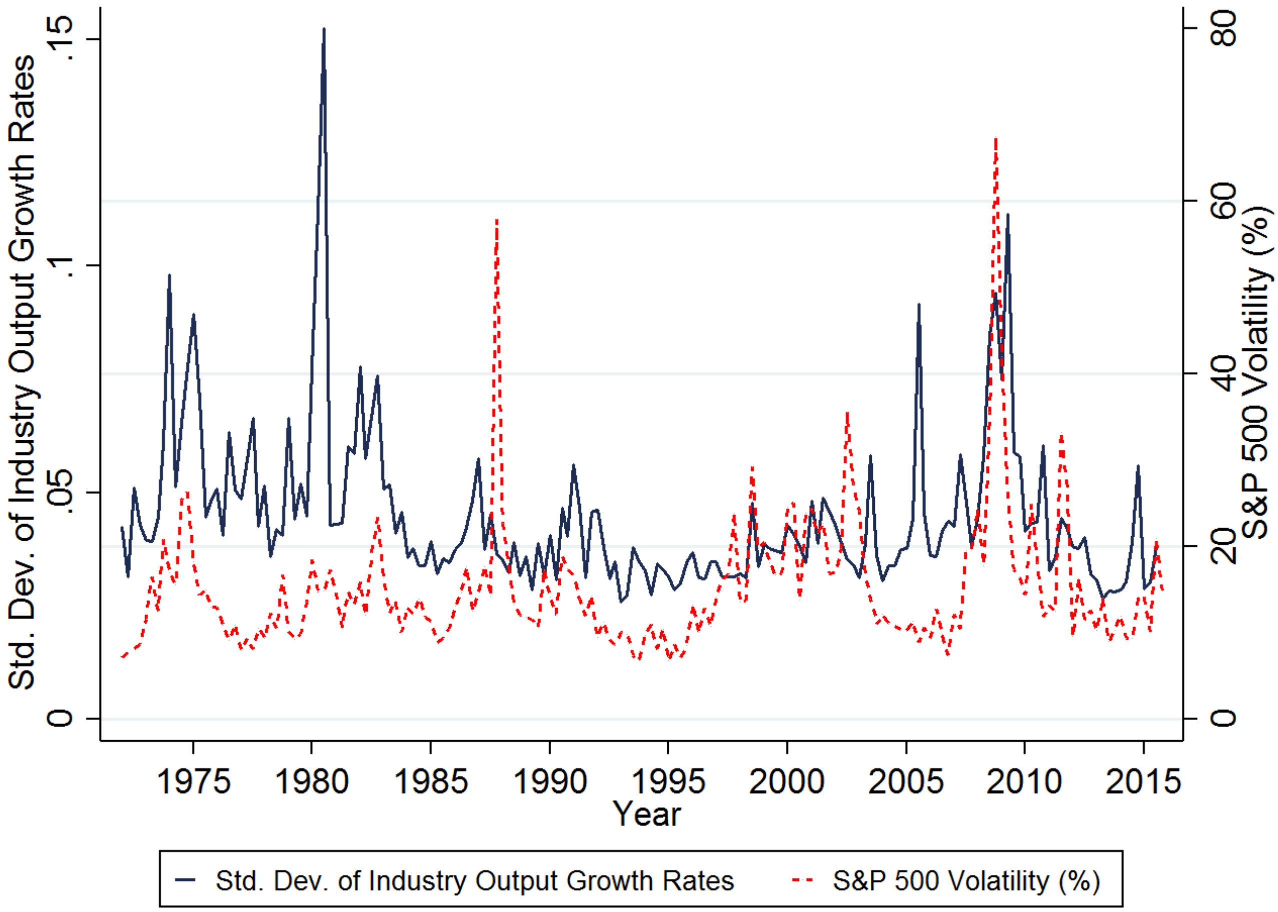


Fig. 3. The Cross-Sectional Standard Deviation of Industry Output Growth Rates. The solid line denotes the cross-sectional standard deviation of quarterly industry growth rates. For each quarter from 1972Q1 to 2015Q4, I compute the standard deviation of 194 US manufacturing industries. The dotted line represents the S&P 500 index volatility for the corresponding period.

Event	Time	Type
Cuban Missile Crisis	October 1962	Political
Assassination of JFK	November 1963	Terror
Vietnam War	August 1966	War
Cambodian Campaign	May 1970	War
OPEC I	December 1973	Economic
Franklin National	October 1974	Economic
OPEC II	November 1978	Economic
Iran Hostage	March 1980	War
US Monetary Cycle Change	October 1982	Economic
Black Monday	October 1987	Economic
Loma Prieta Earthquake	October 1989	Natural Disaster
Gulf War I	August 1990	War
Asian Financial Crisis	November 1997	Economic
Russia Moratorium and LTCM	September 1998	Economic
911 Terrorist Attack	September 2001	Terror
WorldCom and Enron	July 2002	Economic
Gulf War II	March 2003	War
Liquidity Shortfall	August 2007	Economic
Global Financial Crisis	October 2008	Economic
Euro-zone Crisis	May 2010	Economic
European Sovereign Debt	August 2011	Economic

Table 1: Major Volatility Shocks Since 1962. This table shows major volatility shocks since 1962. I detrend the monthly S&P 500 volatility data in Figure 1 using the Hodrick-Prescott filter with $\lambda = 129,600$ to remove a slow-moving cyclical component. Those shocks are chosen to be 1.65 standard deviation above the Hodrick-Prescott detrended mean. If one shock lasts for several consecutive months, I choose the first month.

Industry	NAICS	HHI	R&D Intensity (%)								
			1999	2000	2001	2002	2003	2004	2005	2006	2007
Manufacturing	31-33		3.8	3.6	4.1	3.7	3.5	3.8	4.0	3.6	4.1
Food	311	94	0.4	D	0.5	D	D	0.6	0.7	0.7	D
Beverage	312	797	D	0.7	0.4	0.5	0.5	1.3	1.4	1.3	0.4
Textile	313-6	N/A	0.7	D	D	D	D	1.2	1.6	1.4	1.7
Wood	321	32	0.5	0.8	1.1	D	D	D	D	0.7	D
Paper	322-3	N/A	D	D	D	D	D	D	D	1.2	D
Petroleum	324	693	0.4	D	D	D	D	0.4	D	0.3	D
Chemicals	325	139	5.2	5.9	4.9	6.0	5.7	D	6.9	7.5	D
Plastics	326	27	1.9	D	D	D	2.1	D	2.0	1.9	D
Mineral	327	104	D	1.8	2.4	D	1.0	1.8	1.8	1.9	1.8
Primary Metal	331	213	0.4	0.5	0.7	0.7	0.7	0.7	0.6	0.5	0.6
Fabricated Metal	332	7	1.5	1.4	1.7	1.5	1.6	1.5	0.8	1.4	1.7
Machinery	333	56	3.5	3.9	4.3	4.4	4.2	3.7	3.7	3.6	3.7
Computer	334	183	10.8	9.4	13.1	9.4	11.5	9.5	D	9.2	9.9
Electrical Equip.	335	99	D	D	3.1	2.8	2.2	2.8	2.4	2.5	3.1
Transportation	336	371	4.2	4.0	4.2	3.5	D	D	D	2.9	D
Furniture	337	95	0.7	0.8	0.9	0.8	D	0.8	0.8	0.7	1.2
Medical Equip.	3391	69	D	D	D	D	6.3	5.9	7.7	5.6	3.0
Others	3392-9	186	D	D	D	D	3.3	3.2	2.9	3.5	3.7

Table 2: R&D Intensity by Industry from 1999 to 2007. HHI denotes the Herfindahl-Hirschman index for 50 largest firms as of 2007 reported by the Economic Census. NAICS is the North American Industry Classification System. D means the case that data are withheld to avoid disclosing operations of individual firms. The R&D intensity data are from the National Center for Science and Engineering Statistics. NAICS codes do not seamlessly match with Standard Industrial Classification (SIC) codes. Hence, I tabulate R&D intensity for major manufacturing industries from 1999 when the U.S. Department of Commerce first reported industrial R&D as a percent of net sales by NAICS codes.

	Portfolios Sorted on Book-to-Market Ratio										
	Growth	2	3	4	5	6	7	8	9	Value	V-G
	Panel A: $r_{i,t} = \alpha_i + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$										
α_i	0.97	1.09	1.13	1.15	1.10	1.16	1.21	1.26	1.39	1.56	0.58
t_{α_i}	[5.01]	[6.31]	[6.66]	[6.52]	[6.63]	[6.93]	[7.19]	[7.46]	[7.68]	[7.09]	[3.25]
β_i^V	-3.18	-4.68	-4.77	-5.67	-4.69	-4.66	-4.19	-5.22	-5.80	-6.68	-3.50
$t_{\beta_i^V}$	[-1.48]	[-2.28]	[-2.34]	[-3.14]	[-2.54]	[-2.44]	[-2.50]	[-2.89]	[-3.42]	[-2.91]	[-3.06]
	Panel B: $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$										
α_i	0.24	0.42	0.47	0.50	0.49	0.54	0.62	0.68	0.77	0.84	0.60
t_{α_i}	[3.32]	[7.51]	[8.44]	[6.95]	[6.54]	[7.43]	[7.20]	[7.22]	[8.01]	[6.36]	[3.31]
β_i^M	1.07	1.00	0.97	0.96	0.89	0.92	0.87	-0.87	0.92	1.05	-0.01
$t_{\beta_i^M}$	[53.24]	[62.19]	[58.13]	[44.27]	[41.40]	[39.42]	[31.01]	[28.05]	[28.97]	[23.13]	[-0.34]
β_i^V	1.59	-0.23	-0.43	-1.37	-0.70	-0.56	-0.31	-1.34	-1.70	-1.99	-3.59
$t_{\beta_i^V}$	[3.72]	[-0.64]	[-1.07]	[-3.30]	[-1.66]	[-1.01]	[-0.58]	[-2.44]	[-2.36]	[-2.36]	[-3.02]
	Panel C: $r_{i,t} = \alpha_i + \beta_i^V \Delta VXO_t + \epsilon_{i,t}$										
α_i	0.94	0.97	1.04	1.00	0.97	0.92	1.01	0.89	1.05	1.14	0.20
t_{α_i}	[4.11]	[4.82]	[5.11]	[4.75]	[4.95]	[4.58]	[5.04]	[4.43]	[4.98]	[4.37]	[0.83]
β_i^V	-0.59	-0.61	-0.58	-0.61	-0.60	-0.62	-0.52	-0.56	-0.59	-0.76	-0.16
$t_{\beta_i^V}$	[-10.94]	[-11.59]	[-8.39]	[-10.37]	[-9.91]	[-11.52]	[-9.51]	[-8.44]	[-10.27]	[-10.14]	[-2.38]
	Panel D: $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \Delta VXO_t + \epsilon_{i,t}$										
α_i	0.25	0.35	0.42	0.39	0.42	0.33	0.47	0.37	0.48	0.53	0.27
t_{α_i}	[2.64]	[4.67]	[5.32]	[3.70]	[3.83]	[3.32]	[3.78]	[2.55]	[3.59]	[2.75]	[1.08]
β_i^M	1.07	0.97	0.96	0.95	0.85	0.91	0.85	0.81	0.89	0.96	-0.11
$t_{\beta_i^M}$	[32.84]	[37.43]	[37.58]	[22.67]	[22.28]	[24.02]	[19.53]	[12.91]	[18.12]	[13.52]	[-1.25]
β_i^V	0.06	-0.02	0.00	-0.03	-0.07	-0.06	-0.00	-0.06	-0.04	-0.17	-0.24
$t_{\beta_i^V}$	[2.20]	[-0.93]	[0.09]	[-0.86]	[-2.20]	[-1.97]	[-0.02]	[-0.92]	[-1.04]	[-2.46]	[-2.59]

Table 3: Market Volatility Shock and Stock Returns. Panel A reports the regression of ten portfolio returns sorted on book-to-market ratio on the volatility shock indicator function defined in Table 1: $r_{i,t} = \alpha_i + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$. Panel B reports the same regression controlling for the market excess return: $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$. In Panel C and D, I use innovations of the VXO index instead of the volatility shock indicator function: $r_{i,t} = \alpha_i + \beta_i^V \Delta VXO_t + \epsilon_{i,t}$ and $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \Delta VXO_t + \epsilon_{i,t}$. $r_{i,t}$ is the excess return of each of the book-to-market decile portfolios at time t. MKT_t is the the market excess return. The data are from Ken French's website and span from July 1962 to December 2015. The VXO index is from Chicago Board Options Exchanges and starts in 1986. Newey-West t-statistic is reported in square bracket.

Parameter	Value	Economic Rationale
Economic Environments		
ρ_x	0.9933	persistence of the aggregate productivity process
σ_L^x, σ_H^x	0.0040, 0.0080	conditional volatility of the aggregate productivity process
$\bar{x}$	-4.40	constant term of the aggregate productivity process
ρ_z	0.9800	persistence of the idiosyncratic productivity process
σ_L^z, σ_H^z	0.20, 0.40	conditional volatility of the idiosyncratic productivity process
$\pi_{L,H}$	1/36	one volatility shock per three years
$\pi_{H,H}$	$\sqrt{2}$	two-month half life
Production		
α	0.70	physical capital share in the composite inputs of production
N	10	the number of plants per firm
ϵ	4	demand elasticity (i.e, markup=33%)
ϕ	0.07	technology capital share in output
δ_K	0.01	the rate of physical capital depreciation per month
δ_G	0.05	the share of current R&D on technology capital
f	0.4580	fixed cost of production
Pricing Kernel		
β	0.9932	the representative consumer's subjective discount factor
ν_0	26	constant price of risk
ν_1	-250	time-varying price of risk
Adjustment Cost		
b_K^+, b_K^-	0.02, 0.03	non-convex parameter of physical capital adjustment cost
c_K^+, c_K^-	3, 30	quadratic parameter of physical capital adjustment cost
b_G	0.03	non-convex parameter of technology capital adjustment cost
c_G	3	quadratic parameter of technology capital adjustment cost
External Financing Cost		
γ_0	0.0032	fixed equity flotation cost
γ_1	0.01	proportional equity flotation cost

Table 4: Benchmark Calibration. The model is required to calibrate 27 parameters: nine for economic environments, seven for production function, three for pricing kernel, six for investment adjustment costs, and two for external equity financing cost. Economic environments parameters are chosen to match the real business cycle literature (Cooley and Prescott (1995)). Production function parameters are set to match the macro finance literature (Zhang (2005) and McGrattan and Prescott (2009)). I discipline the pricing kernel parameters to match the risk-free rate and the Hansen-Jagannathan bound (Lustig and Verdelhan (2012)), so that there is no more degree of freedom for the pricing kernel to capture the cross-section of firms, which is the focus of this paper. Adjustment cost parameters are from Cooper and Haltiwanger (2006) and Bloom (2009). Finally, external equity financing cost parameters are from Hennessy and Whited (2007) and Kuehn and Schmid (2014).

Moment	Data	Model
average annual real interest rate	0.02	0.0237
annual real interest rate volatility	0.03	0.0356
average Sharpe ratio through the business cycle	0.50	0.4116
average Sharpe ratio in expansions	0.38	0.2271
average Sharpe ratio in recessions	0.66	0.6334
maximum Sharpe ratio	0.82	0.8014
minimum Sharpe ratio	0.14	0.0292

Table 5: Key Aggregate Moments. This table compares model-generated key aggregate moments with those observed in the data. The data are from Campbell and Cochrane (1999) and Lustig and Verdelhan (2012). I generate a path for Eq.(9) for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. I then compute the mean and standard deviation of real interest rate. To compute the Hansen-Jagannathan bound, I discretize the aggregate productivity shock process using the Rouwenhorst (1995) method into 15 discrete intervals. Expansions are from the median to the largest x_t over the discretized intervals. Recessions make up the remaining values.

	Portfolios Sorted on Book-to-Market Ratio										
	Growth	2	3	4	5	6	7	8	9	Value	V-G
Panel A: Data (1962-2015)											
Mean	0.47	0.54	0.58	0.57	0.55	0.61	0.67	0.70	0.80	0.94	0.47
SD	5.11	4.67	4.59	4.70	4.43	4.49	4.42	4.52	4.77	5.84	5.84
α	-0.09	0.02	0.06	0.05	0.07	0.12	0.21	0.23	0.31	0.37	0.47
t_α	[-1.26]	[0.30]	[1.10]	[0.75]	[0.95]	[1.74]	[2.56]	[2.55]	[3.36]	[2.86]	[2.60]
β	1.06	1.01	0.98	0.98	0.91	0.93	0.88	0.89	0.94	1.07	0.01
t_β	[51.25]	[65.28]	[56.12]	[45.91]	[42.29]	[41.24]	[32.53]	[28.46]	[30.74]	[23.32]	[0.09]
Panel B: Benchmark Model											
Mean	0.70	0.71	0.71	0.72	0.75	0.75	0.78	0.84	0.90	1.16	0.45
SD	3.58	3.85	4.07	4.16	4.56	4.61	4.88	5.34	5.67	7.04	4.01
α	0.10	0.05	0.01	0.01	-0.04	-0.04	-0.06	-0.07	-0.07	-0.02	-0.12
t_α	[2.46]	[1.47]	[0.52]	[0.36]	[-0.98]	[-1.06]	[-1.42]	[-1.48]	[-1.42]	[-0.72]	[-1.39]
β	0.79	0.86	0.91	0.93	1.02	1.03	1.09	1.19	1.26	1.54	0.75
t_β	[60.94]	[83.07]	[98.89]	[108.41]	[109.64]	[104.55]	[80.82]	[59.05]	[53.04]	[30.40]	[12.09]
Panel C: Zhang (2005)											
Mean	0.62	0.66	0.69	0.70	0.77	0.76	0.81	0.86	0.92	1.12	0.50
SD	5.9	6.3	6.5	6.6	7.1	7.0	7.4	7.8	8.2	9.5	3.9
α	-0.02	-0.01	-0.01	-0.01	0.00	0.00	0.01	0.02	0.04	0.10	0.11
t_α	[-0.80]	[-0.60]	[-0.50]	[-0.40]	[0.00]	[-0.10]	[0.50]	[0.60]	[1.00]	[1.50]	[1.40]
β	0.86	0.91	0.95	0.96	1.03	1.02	1.07	1.13	1.17	1.36	0.50
t_β	[123.2]	[164.4]	[219.8]	[162.5]	[123.9]	[227.4]	[127.3]	[112.2]	[79.60]	[42.2]	[12.4]

Table 6: Properties of Portfolios Sorted on Book-to-Market. I simulate 100 artificial samples, each with 1000 firms, from the model. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French (1993)'s timing convention and run the regression. Panel A summarizes the results from the Capital Asset Pricing Model using the real data from 1962 to 2015. Panel B reports the results from the benchmark model of this paper. Panel C is from Lin and Zhang (2013) and summarizes the Zhang (2005) model. Mean and SD are the mean and standard deviation of monthly excess returns in each portfolio. α and β are the CAPM alpha and beta. t_α and t_β are the Newey-West t-statistics for the CAPM alpha and beta.

	Portfolios Sorted on Book-to-Market Ratio										
	Growth	2	3	4	5	6	7	8	9	Value	V-G
	Panel A: $r_{i,t} = \alpha_i + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$										
α_i	0.24	0.42	0.47	0.50	0.49	0.54	0.62	0.68	0.77	0.84	0.60
t_{α_i}	[3.32]	[7.51]	[8.44]	[6.95]	[6.54]	[7.43]	[7.20]	[7.22]	[8.01]	[6.36]	[3.31]
β_i^V	1.59	-0.23	-0.43	-1.37	-0.70	-0.56	-0.31	-1.34	-1.70	-1.99	-3.59
$t_{\beta_i^V}$	[3.72]	[-0.64]	[-1.07]	[-3.30]	[-1.66]	[-1.01]	[-0.58]	[-2.44]	[-2.36]	[-2.36]	[-3.02]
	Panel B: $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$										
α_i	0.25	0.35	0.42	0.39	0.42	0.33	0.47	0.37	0.48	0.53	0.27
t_{α_i}	[2.64]	[4.67]	[5.32]	[3.70]	[3.83]	[3.32]	[3.78]	[2.55]	[3.59]	[2.75]	[1.08]
β_i^M	1.07	0.97	0.96	0.95	0.85	0.91	0.85	0.81	0.89	0.96	-0.11
$t_{\beta_i^M}$	[32.84]	[37.43]	[37.58]	[22.67]	[22.28]	[24.02]	[19.53]	[12.91]	[18.12]	[13.52]	[-1.25]
β_i^V	1.30	0.77	0.38	0.27	-0.22	-0.32	-0.55	-1.07	-1.34	-2.58	-3.87
$t_{\beta_i^V}$	[7.96]	[5.69]	[3.28]	[2.23]	[-1.82]	[-2.44]	[-3.91]	[-5.07]	[-5.61]	[-5.91]	[-6.95]

Table 7: Volatility Shock and Value Premium from the Benchmark Model. This table reports the results from the regression of 10 portfolio returns sorted on book-to-market ratio on the market excess return and the volatility shock indicator function using simulated data. Panel A reports the baseline regression: $r_{i,t} = \alpha_i + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$. Panel B shows the same regression controlling for the excess market return: $r_{i,t} = \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{I}_{\text{volatility shock}} + \epsilon_{i,t}$. $r_{i,t}$. The indicator function for volatility shock takes value 1 when the two-state Markov chain defined in Eq.(11) and Eq.(12) is in the high volatility state. I simulate 100 artificial samples, each with 1000 firms, from the model. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French (1993)'s timing convention and run the regression. Newey-West t-statistic is reported in square bracket.

	Monthly Excess Return of Portfolios Sorted on Book-to-Market Ratio										
	Growth	2	3	4	5	6	7	8	9	Value	V-G
(1) Data	0.47	0.54	0.58	0.57	0.55	0.61	0.67	0.70	0.80	0.94	0.47
(2) Benchmark Model	0.70	0.71	0.71	0.72	0.75	0.75	0.78	0.84	0.90	1.16	0.45
(3) No Volatility Shock	0.47	0.47	0.47	0.48	0.49	0.49	0.50	0.52	0.53	0.57	0.09
(4) Symmetric Adj. Cost	0.74	0.73	0.73	0.74	0.76	0.76	0.79	0.86	0.90	1.15	0.41
(5) Constant Price of Risk	0.89	0.88	0.88	0.89	0.91	0.91	0.95	1.01	1.05	1.28	0.40
(6) No Financing Cost	0.70	0.70	0.70	0.72	0.73	0.73	0.75	0.81	0.86	1.09	0.39
(7) (4)+(5)	0.83	0.82	0.83	0.84	0.86	0.87	0.89	0.94	1.01	1.22	0.40
(8) (4)+(5)+(6)	0.86	0.86	0.86	0.88	0.90	0.89	0.91	0.96	1.00	1.22	0.35

Table 8: Sensitivity Analysis. This table reports the results from comparative sensitivity analysis by varying volatility shock, adjustment cost, price of risk, and equity financing cost. The numbers are monthly average excess return (%) for each of the book-to-market portfolios.