# The Term Structure of Currency and Bond Risk Premia[1,2]

Rui Chen

Central University of Finance and Economics

Ke Du

Southwestern University of Finance and Economics

Xiaoneng Zhu

Shanghai University of Finance and Economics

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[1]Rui Chen: School of Finance, Central University of Finance and Economics, Beijing, China, 100081, Tel.: 86 10 62288607, e-mail: r.chen@cufe.edu.cn; Ke Du: Institute of Financial Studies, Southwestern University of Finance and Economics, Chengdu, China, 610074, Tel.: 86 28 87099045, e-mail: duke20072009@yahoo.com; Xiaoneng Zhu: School of Finance, Shanghai University of Finance and Economics, Guoding Road 777, Shanghai, China, 200433, Tel.: 86 21 65908162, e-mail: xiaonengz@gmail.com.

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**Abstract**

Standard exchange rate models view high interest rate currencies as less risky, but resolving the forward premium puzzle requires high interest rate currencies to be more risky. This paradox can be explained by a decreasing curve of forward currency risk premia. This paper proposes a two-economy affine term structure model of exchange rates and interest rates with unspanned macroeconomic risks to account for the term structure of currency and bond risk premia. In our setting, we can decompose pricing kernels into two orthogonal components respectively linked to term-structure factors and common unspanned macro factors. Our estimation shows that our model can simultaneously explain the properties of currency and bond risk premia.

**JEL Classifications**: G1, E4, F3.

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# 1 Introduction

One of more puzzling facts in finance is the paradoxical implications of various exchange rate models: while the empirical failure of the uncovered interest rate parity implies that a high interest rate currency is likely more risky, standard exchange rate models (e.g., Dornbusch, 1976; Frankel, 1979) generally suggest that a high interest rate currency is less risky. According to the uncovered interest rate parity (UIP), a regression of exchange rate changes on interest rate differentials should produce a slope coefficient of one. As opposed to UIP, however, empirical work following Hansen and Hodrick (1980), Bilson (1981), and Fama (1984) often reveals a negative slope coefficient. This anomalous finding is termed as the forward premium puzzle. It suggests a tendency of high interest rate currencies to appreciate. A risk-based explanation of the forward premium puzzle requires that the Treasury bonds in the high interest rate country are more exposed to risks of exchange rate movements so that there is a negative correlation between the forward premium in the foreign exchange market and the subsequent change in the spot exchange rate. Put more simply, high interest rate bonds are more risky.

On the other hand, classic exchange rate models, such as the textbook Mundell-Fleming model or the popular Dornbusch (1976) and Frankel (1979) models, assume the UIP condition holds. They postulate that the level of the exchange rate is equal to weighted average of rational expectations of future short-term interest differentials. A natural prediction of these equilibrium models is that a high interest rate currency should have a lower price. Though this relationship is borne out in the data, but the price of the high interest currency is not low enough as predicted by the equilibrium models (see Engel, 2016). This finding constitutes the exchange rate level puzzle. Engel (2016) argues that this puzzling behavior of the level of exchange rates can be attributed to a time-varying risk premium. If a high interest currency has the lower risk premium, it would induce stronger currency. A risk-based explanation for the level puzzle therefore implies that high interest rate bonds are less risky, as opposed to the risk-based explanation of the forward premium puzzle.

This paper attempts to understand these seemingly paradoxical implications of the UIP condition and standard exchange rate models, which, to our knowledge, has not been explored before. Whereas the forward premium puzzle indicates that a high interest rate currency has higher expected returns in the short run, the exchange rate level puzzle shows that a high interest rate currency has lower expected returns at longer horizons. Combined together, the two puzzles imply a decreasing curve of forward currency risk premia along the maturity spectrum. In particular, the currency risk premium is positive initially, then it becomes negative at longer horizons. We define the forward currency risk premia

$$\rho^{(1)}_{t+j+1} = i^*_{t+j} + s_{t+j+1} - s_{t+j} - i_{t+j} \; for \; j \geq 1, \tag{1}$$

where $s_t$ denote the logarithm of the exchange rate, expressed in terms of how many units of the foreign currency with interest rate $i^*$ is needed in exchange for one unit of the home currency with interest rate $i$.[1] This decreasing curve of (forward) currency risk premia constitutes the term structure of currency risk premia. It consolidates the paradox proposed by Engel (2016) in a unified framework and is the central puzzle of this paper.

To resolve the puzzling term structure of currency risk premia, we propose a two-country affine term structure model (ATSM-X) of exchange rates and interest rates with unspanned macroeconomic risks. We build our model upon the pioneering work of Backus, Foresi, and Telmer (2001), who show that currency risk premia can be written as a function of the higher moments of foreign and domestic pricing kernels. In our setting, pricing kernels are lognormal, so currency risk premia link to the ratio of the volatilities of pricing kernels. With this ratio, we are able to identify the unique stochastic discount factor that prices both currencies and bonds in two countries ex-

[1] It is related to, but differs from the term structure of carry trade risk premia (Lustig, Stathopoulos, and Verdelhan, 2014), which is the return arising from the currency carry trade based on various bonds of different maturities.

amined. Hence, the ATSM-X model can be used to simultaneously account for the properties of currency risk premia and bond risk premia. The ATSM-X model guarantees the internal consistency across two economies without imposing some artificial constraints on the dynamics of exchange rates. Naturally, the ATSM-X model indicates that the forward exchange rate includes a time-varying risk premium for bearing both currency and interest rate risks.

The ATSM-X model has an $(m + n)$ factor structure for the pricing kernel of each economy. In the ATSM-X model, the $m$ local term-structure factors capture cross-sectional properties of interest rates. The other $n$ factors are common macro variables. In the spirit of Engel (2016), our empirical analysis uses the log difference of inflation and industrial production growth rate in two economies as macro factors. To better capture economic fundamentals in the two economies and retain parsimony, our set of macro factors includes a unspanned latent common factor. We use these macro factors to capture currency and bond risk premia in two markets. To see the point, a simple regression shows that the macro factors account for approximately 60% of currency risk premia at long horizons, though the explanatory power of the macro factors are low at short horizons.

Importantly, these macro factors are unspanned (see, Joslin, Priebsch, and Singleton, 2014; Duffee, 2011)[2] by the term structure factors because they contain additional information for future bond risk premia but do not affect the cross section of interest rates. Theoretically, we show that the pricing kernel of each economy can be decomposed into two orthogonal terms respectively composed of macro factors and term-structure factors. The martingale component of macro factors capture the independent portion of currency and bond risk premia. Because macro factors unequally affect bond risk premia, they warp the term structure of currency risk premia and play an important role in understanding the decreasing curve of currency risk premia.

[2]Numerous studies (e.g., Ang and Piazzesi, 2003; Ludvigson and Ng, 2009; Cooper and Priestley, 2009; Duffee, 2011; Bakshi and Panayotov, 2013; Zhou and Zhu, 2015) reveal that macroeconomic factors can predict time-varying risk premia in currency and bond markets.

We estimate the ATSM-X model separately for four pairs of countries: the U.S. as the home country and Canada, Germany, Japan, and the U.K. as the foreign countries. Our first set of results relates to the currency bond risk premia. We find that our model can largely account for the term structure of currency risk premia. Specifically, when we run the Fama (1984) regression and the Engel (2016) regression, our model is able to generate a slope coefficient of one, suggesting that the ATSM-X model is able to account for the term structure of currency risk premia. Consistent with the prediction of standard exchange rate models, we also generate a slope coefficient of one for the Engel regression using real interest rates. If we remove unspanned macro factors, however, the two-country term structure model cannot generate the decreasing term structure of currency risk premia and resolve the Engel (2016) paradox. To further provide insights on the importance of unspanned macro factors, we find that currency risk premia are closely linked to economic activity. In contrast, currency risk premia recovered from the joint term structure model without unspanned macro factors are largely acyclical. We also find that our model can simultaneously fit the term structure of bond risk premia. Consistent with Joslin, Priebsch, and Singleton (2014) and Jotikasthira, Le, and Lundblad (2015), we find unspanned macro factors play an important role in understanding the properties of bond risk premia.

Our model draws from several contributions in the literature that investigates the behavior of currency and bond prices using joint term structure models of exchange rates and interest rates. Some early attempts include Nielsen and Saá-Requejo (1993), Saá-Requejo (1994), Bakshi and Chen (1997), and Bansal (1997). Concentrating on affine term structure models, the important work of Backus, Foresi, and Bansal (2001) extends affine class to a multicurrency setting. They find the extended models had problems in accounting for the forward premium puzzle. Motivated by this deficiency, Han and Hammond (2003) and Leippold and Wu (2007) extend the multicountry term structure model to include independent exchange rate factors. They find that these factors are essential in simultaneously fitting exchange rates and the term structure of interest rates. Alternatively, Brandt and Santa-Clara (2002) and Anderson, Hammond, and Ramezani (2010) introduce market incompleteness into the joint term structure model and investigate exchange rate excess volatility. Moreover, a strand of literature (Dewachter, and Maes, 2001; Hodrick and Vassalou, 2002; Ahn, 2004; Inci and Lu, 2004) explores the effects of global factors on properties of currency risk premia. Based on the assumption of an integrated capital market, Brennan and Xia (2006) examine the relations between currency risk premia, exchange rate volatility, and the volatilities of pricing kernels. More recently, Sarno, Schneider, and Wagner (2012) explore the properties of foreign exchange risk premia. They develop a non-Gaussian multicurrency affine term structure model to account for the forward premium puzzle and find a tradeoff between fitting the term structure of interest rates and the forward premium. Though their global model can account for the properties of foreign exchange risk premia, but the fitting of the term structure has a relatively low accuracy. While all these papers focus on spot currency risk premia, our paper attempts to account for the term structure of currency risk premia. From a perspective of anomalies, these papers attempt to resolve the forward premium puzzle, we try to explain the Engel (2016) paradox.

Our paper is also related to the literature that attempts to resurrect the UIP condition. Earlier studies include applications of the capital asset pricing model to exchange rates (e.g., Frankel and Engel, 1984; Mark, 1988), statistical methods of modelling currency risk premia (e.g., Hansen and Hodrick, 1983; Domowitz and Hakkio, 1985; Cumby, 1988), and behavioral explanations (e.g., Froot and Thaler, 1990; Eichenbaum and Evans, 1995, Mankiw and Reis, 2002). Recent explanations includes return skewness (e.g., Brunnermeier, Nagel, and Pedersen, 2009; Chen and Gwati, 2013; Jurek and Xu, 2014; Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan, 2015), overconfidence (e.g., Burnside, Han, Hirshleifer, and Wang, 2011), habit formation (e.g., Verdelhan, 2010), rare disaster (e.g., Farhi and Gabaix, 2014), long run risks (e.g., Bansal and Shaliastovich, 2013), country size as a proxy of risk (Hassen, 2013), and infrequent portfolio decisions (e.g., Bacchetta and Wincoop, 2010). Alternatively, a new strand of literature using portfolio analysis to search risk factors for understanding currency risk premia, some important studies include Lustig and Verdelhan (2007), Lustig, Roussanov, and Verdelhan (2011), and Menkhoff, Sarno, Schmeling, and Schrimpf (2012).

We proceed as follows. In Section 2 we discuss the economic intuition behind the ATSM-X model and present some preliminary results on the importance of unspanned macro factors. We then propose the two-economy term structure model with unspanned macroeconomic risks that takes a large step toward bringing joint term structure models in line with the historical evidence. Section 3 discusses the econometric methodology for estimating the ATSM-X model and estimates the model. Section 4 discusses the implications of the ATSM-X model for the forward premium puzzle, the level of exchange rates, and term structure anomalies. Section 5 offers conclusions and suggestions for further research. All proofs are in Appendices.

# 2 The ATSM-X Model

This section presents the ATSM-X model. In Section 2.1, we discuss the economic intuition behind the relationship between exchange rates and interest rates and introduce basic notations. Section 2.2 provides some preliminary results on the importance of unspanned macro factors. Section 2.3 presents the settting of the ATSM-X model.

## 2.1 Economic Foundation

To obtain some intuition on the relationship between exchange rates and interest rates, we first introduce some basic notations. As in (1), we define the spot $n$-period excess currency return as

$$\rho_{t+n}^{(n)} = i_{t,n}^{*} + s_{t+n} - s_t - i_{t,n} \tag{2}$$

where $s_t$ denote the log of exchange rate, expressed in terms of US dollar price per unit of foreign currency, $i_{t,n}^{*}$ denote the $n$-period foreign nominal interest at time $t$ and $i_{t,n}$ denote the domestic interest rate. Our analysis focus on the term structure of currency risk premia, it also involve forward currency risk premia. Define at time $t$, the $j$-period forward 1-period excess currency return as

$$\rho_{t+j+1}^{(1)} = i_{t+j}^{*} + s_{t+j+1} - s_{t+j} - i_{t+j} \tag{3}$$

with $i_{t+j}^{*}$ and $i_{t+j}$ represents the 1-period interest rates.

The corresponding ex anti risk premiums are defined as follows. The ex anti risk premium for spot $n$-period currency return is defined as

$$v_t^{(n)} \equiv E_t^P(\rho_{t+n}^{(n)}) \tag{4}$$

where the expectation is taken under the physical measure. Furthermore, the ex anti forward premium for $j$-period forward looking 1-period excess return is defined as

$$v_{t,j}^{(n)} \equiv E_t^P(v_{t+j}^{(1)}) = E_t^P(\rho_{t+j+1}^{(1)}) \tag{5}$$

at time $t$.

By assuming that the market is rational and there is no-arbitrage opportunity, the UIP condition postulates that investment in foreign countries should generate no risk premium, or the excess return should be unpredictable. UIP also implies that the forward rate should be an unbiased predictor of the future spot exchange rate. This is termed as forward unbiased hypothesis (FUH). Empirical tests are usually facilitated via the 'Fama regression' which studies the spot excess return with

$$\begin{aligned} \rho_{t+n}^{(n)} &= \alpha + \gamma(i_{t,n}^{*} - i_{t,n}) + \varepsilon, \\ s_{t+n} - s_t &= \alpha + \beta(i_{t,n} - i_{t,n}^{*}) + \varepsilon. \end{aligned} \tag{6}$$

If the UIP is valid, we should observe zero risk premium, which implies $\alpha = 0$, $\gamma = 0$ and $\beta = 1$. However, a vast of empirical studies suggest that $\beta$ is negative rather than being 1, and $\gamma$ is greater than 1 instead of being 0. This implies that the risk premium for investing in foreign countries positively covaries with the interest difference,

$$cov_t(\rho^{(n)}_{t+n}, i^*_{t,n} - i_{t,n}) = cov_t(E^P_t(\rho^{(n)}_{t+n}), i^*_{t,n} - i_{t,n}) > 0\ . \tag{7}$$

One rationale for this stylized fact is that when $i^*$ is relatively higher, invest in foreign country becomes risker and thus it generates higher risk premium. The results of the Fama regression is also termed as the forward bias puzzle in the literature because if $\beta \neq 1$, the forward rate becomes an biased predictor of the future spot rate.

Engel (2016) proposes an alternative test to analyse the 1-period excess returns in a forward looking manner,

$$\rho^{(1)}_{t+j+1} = \alpha + \beta_j(i^*_t - i_t) + \varepsilon, \tag{8}$$

where $\rho^{(1)}_{t+j+1}$ is the excess return in the future time $t + j$, $j$ varies from 0 to 10 years. Note that, the above regression is identical to the Fama regression for $j = 0$. Again, if UIP is valid, forward risk premium, $v^{(1)}_{t,j}$, should be equal to 0. This means taht $\beta_j$ equals to 0 across all horizons. However, Engel (2016) finds that $\beta_j$ starts from a positive value when $j = 0$ (Engel regression is identical with Fama regression for $j = 0$), then decreases and turns to some negative values when $j$ is large enough. The findings violate the UIP, more important, the results are suggesting that $\beta_j$ changes sign, and becomes negative. In fact, $\beta_j$ are deeper negative in term of real interest rates according to Engel (2016). The findings further suggest the summation of forward excess returns for all $j$ negatively covaries with nominal or real interest difference (to consistent with Engel (2016), we use real interest rate difference here),

$$cov_t(\sum_{j=0}^{\infty} \rho^{(1)}_{t+j+1}\ ,\ r^*_t - r_t) = cov_t(E^P_t \sum_{j=0}^{\infty} v^{(1)}_{t+j}\ ,\ r^*_t - r_t) < 0, \tag{9}$$

where $E_t \sum_{j=0}^{\infty} \rho^{(1)}_{t+j+1}$ represents the risk premium for rolling investing in foreign savings account for a long time horizon, $r^*_t$ and $r_t$ represent the real interest rates. One rationale for this patten is that investing in foreign country for a long horizon becomes less risky when interest rate in the foreign country is relatively higher. This forms a paradox with the explanation of forward bias puzzle: high interest country become risker rather than less risky. Note that, Fama regression studies a investment strategy for a short time horizon, while Engel regression investigates the investment in foreign savings account for a rather long period including short time horizon. Therefore, the property of the term structure of the forward risk premium may shed the light on the above paradox.

Brennan and Xia (2006) explain the forward bias puzzle by the time-varying risk premium $v_t^{(n)}$. If one takes the conditional expectation on the $n$-period spot excess return, one would have

$$\begin{aligned} \rho_{t+n}^{(n)} &= E_t^P(\rho_{t+n}^{(n)}) + \varepsilon \\ &= v_t^{(n)} + 0 \times (i_{t,n}^* - i_{t,n}) + \varepsilon \\ s_{t+n} - s_t &= v_t^{(n)} + 1 \times (i_{t,n} - i_{t,n}^*) + \varepsilon \end{aligned} \tag{10}$$

The above equation is similar to the Fama regression but with the $n$-period spot risk premium taking place of $\alpha$. If $v_t^{(n)}$ is stochastic and correlated with interests, $i_{t,n} - i_{t,n}^*$, it makes coefficient $\beta$ diverge from 1. Furthermore, to literature argues that to generate the patten as in Fama regression, the risk premium should has the following property

$$\begin{aligned} &var(v_t^{(n)}) > var(E_t^P(dep_t^{(n)}), \\ &cov(v_t, E_t^P(dep_t^{(n)})) > 0, \end{aligned} \tag{11}$$

where $dep_t^{(n)} \equiv s_{t+n} - s_t$ is the depreciation rate. If one models the risk premiums correctly and then, substitute into equation (10), the puzzle vanishes. Thus equation (10) is referred as the risk adjusted Fama's regression.

On the other hand, Engel (2016) argues that the level puzzle could be explained by the excess comovement of exchange rate: If iterating the excess return (2) to infinity, one gets

$$s_t^T = s_t^{IP} - E_t(\sum_{j=0}^{\infty}(\rho_{t+j+1}^{(1)} - \overline{\rho})) \tag{12}$$

with $s_t^T = s_t - \lim_{k \leftarrow \infty}(E_t s_{t+k} - k(\overline{s_{+1} - s}))$ representing the detrended exchange rate and $s_t^{IP} = E_t \sum_{i=0}^{\infty}(y_{t+i\delta}^* - y_{t+i\delta} - (\overline{y^* - y}))$ represents the interest parity part. Empirical findings directly yield,

$$cov_t(s_t^T, r_t^* - r_t) > cov_t(s_t^{IP}, r_t^* - r_t) \ , \tag{13}$$

which can be interpreted as the detrended exchange rate covaries with interest difference greater than the interest parity supposes to. If the exchange rate has excess volatility that can not be explained by the interest rates, $cov_t(s_t^T, r_t^* - r_t)$ could becomes larger than can be explained rationally. Thus, although the forward bias puzzle and level puzzle have opposite rational explanations, the proposed reasons are not exclusively contradicted with each other. We propose a novel two-country affine model to solve this paradox by incorporating all proposed properties above into the model.

We explain our model in a general framework before introducing the specific dynamic process in the next section. we start from Backus, Foresi, and Bansal (2001),

$$\frac{S_t}{S_0} = \frac{\Pi_t^*/\Pi_0^*}{\Pi_t/\Pi_0}, \tag{14}$$

where $\Pi$ and $\Pi^*$ are the domestic and foreign global pricing kernels respectively. We use the notation $\pi_t$ and $\pi_t^*$ to denote the log of corresponding pricing kernels. The relationship between depreciations and pricing kernels could be derived in the following stochastic differential equation (SDE),

$$ds_t = d(\pi_t^* - \pi_t). \tag{15}$$

The relationship (14) has been discovered by Backus, Foresi, and Bansal (2001), however, it is important to mention that the domestic and foreign pricing kernels in (14) should be the 'global' pricing kernel that prices all traded assets rather than bonds only. This indicates that the domestic and foreign pricing kernels may decompose into 2 parts: the bond market specific pricing kernel ($\Pi_t^b$ and $\Pi_t^{b*}$) and an martingale ($\Pi_t^u$ and $\Pi_t^{u*}$) which is orthogonal to the bond market specific parts, s.t.

$$\Pi_t = \Pi_t^b \Pi_t^u \quad and \quad \Pi_t^* = \Pi_t^{b*} \Pi_t^{u*} \tag{16}$$

and the short term bond yield can be expressed as

$$\begin{cases} i_t = -\log E_t^P(\Pi_{t+\delta}/\Pi_t) = -\log E_t^P(\Pi_{t+\delta}^b/\Pi_t^b) \\ i_t^* = -\log E_t^P(\Pi_{t+\delta}^*/\Pi_t^*) = -\log E_t^P(\Pi_{t+\delta}^{b*}/\Pi_t^{b*}) \end{cases}. \tag{17}$$

The SDE of exchange rates could thus be derived by,

$$ds_t = d(\pi_t^{b*} - \pi_t^b) + d(\pi_t^{u*} - \pi_t^u) \tag{18}$$

where $d(\pi_t^{u*} - \pi_t^u)$ is unspanned to the bond market by construction, which is similar to Ludvigson and Ng (2009), Duffee (2011) and Joslin, Priebsch, and Singleton (2014). In their papers, the authors link this unspanned part to macroeconomic variables via a rotation of latent factors. We find the unspanned part provides excess volatility to exchange rate that assure the excess comovement (13), which provides an potential solution to long term risk premium documented by Engel (2016).

To further explain our idea, we decompose the excess return as follows,

$$\begin{aligned} \rho_{t+1}^{(1)} &= E_t^P(\rho_{t+1}^{(1)}) + \varepsilon_t \\ &= E_t^P(s_{t+1} - s_t) + i_t^* - i_t + \varepsilon_t \\ &= \underbrace{\pi_{t+1}^{b*} - \pi_{t+1}^b + y_t^* - y_t}_{\text{spanned part } (v_t^S)} + \underbrace{\pi_{t+1}^{u*} - \pi_{t+1}^u}_{\text{unspanned part } (v_t^U)} + \varepsilon_t. \end{aligned} \tag{19}$$

Thus, we can obtain an risk adjusted excess return as

$$\rho_{t+1}^{(1)} - v_t^S - v_t^U = \varepsilon_t, \tag{20}$$

which should be independent with all the information up to time $t$. We term the following equation risk adjusted Engel regression

$$\rho_{t+j+1}^{(1)} - v_{t+j}^S - v_{t+j}^U + i_t^* - i_t = \alpha + \beta_j(i_t^* - i_t) + \eta_{t,j} \tag{21}$$

with $j$ varies from 0 to 10 years. In case we correctly estimated $v_t^U$, the risk adjusted Engel regression should yield $\beta = 1$, $\gamma = 0$. However, if $v_t^U$ is omitted, $\eta_{t,T} = \varepsilon_t + pre_t^U$, which is correlated with information at time t. This makes $\beta$ diverge from 1, which forms the level puzzle as documented in Engel (2016).

## 2.2 Importance of Unspanned Macro Risks

In this subsection, we discuss the importance of unspanned macro risks in excess currency and bond returns. Though several international no-arbitrage term structure models have been proposed to simultaneously account for the properties of interest-rate term structure and foreign exchange rates, these models generally imply that the dynamics of interest rates and exchange rates are exclusively driven by term-structure factors.[3] In contrast, our ATSM-X model has two important implications about macro risks. First, the ATSM-X model implies that macro risks are unspanned by yield-curve factors. The second implication is that exchange rates are exclusively not spanned by interest rates. Our model thus dissociates exchange rates from interest rates, without violating the fundamental pricing equations that relate exchange rates, pricing kernels, and interest rates.

With regard to the first implication, macroeconomic factors are unspanned if they

[3] A few studies (e.g., Leippold and Wu, 2007) incorporate unspanned exchange-rate factors into international no-arbitrage term structure models to account for the features of exchange rate dynamics. However, these models assume that macro factors are independent of term-structure factors.

are not related to the contemporaneous cross section of interest rates but it does help forecast future excess returns on the bonds. Duffee (2011) and Joslin, Le, and Singleton (2013) demonstrate that macro-finance models that do not nest unspanned macro risks counterfactually imply that macro factors are a transformation of bond yields. As such, macro factors do not contain additional information for predicting future excess bond returns. In light of the defficiency of traditional term structure models with macro factors, Joslin, Priebschi, and Singleton (2014) propose a term structure model with unspanned macro risks where macro factors predict bond risk premia, above and beyond the predictive power of yield-curve factors.

To obtain some intuition on the importance of unspanned macro risks in understanding the properties of bond risk premia, we conduct a simple regression analysis. In line with Cochrane and Piazzesi (2005), we use the following notation for a nominal forward rate at time $t$ for loans between time $t+n-1$ and $t+n$ is defined as

$$f_{t,n} \equiv p_{t,n-1} - p_{t,n}, \tag{22}$$

where $p_{t,n} = \log(P_{t,n})$ is the log price of an $n$-period bond at time $t$. The log holding period return from buying an $n$-year bond at time $t$ and selling it as an $n-1$ year bond at time $t+1$ is

$$y_{t+1,n} = p_{t+1,n-1} - p_{t,n}. \tag{23}$$

Naturally, the risk premium on an $n$-year discount bond over a short-term bond is the difference between the holding period return of the $n$-year bond and the 1-period interest rate

$$rx_{t+1,n} \equiv y_{t+1,n} - i_t. \tag{24}$$

Following the vast literature, we investigate whether macro risks predict excess bond returns above and beyond the information contained in the yield curve by running two regressions. The first regression uses the principle components (PCs) to predict excess bond returns. The second regression uses both the PCs and unspanned macro risks to forecast excess bond returns. Our macro factors are inflation and industrial production growth rate. Panel A of Table 1 reports the regression results for predicting annual excess bond returns. An important feature that emerges from the table is that macro factors contain plentiful information for predicting future bond risk premia. Look at the UK bond market, the regression without macro factors delivers a $R$-square of 8.32% for the two-year excess bond return, but the regression with macro factors (and their lags) generates a $R$-square of 25.9%, more than triple the predictive power of the regression without macro factors. Though to a less extent, the similar pattern shows up for other bonds and in other markets.

Another interesting finding that comes from the table is that macro factors play a more important role in predicting short-term excess bond returns than predicting long-run excess bond returns. Let us take Germany as an example. For the excess return of the two-year bond, by adding macro factors, the $R$-square of the predictive regression almost triple from 7.18% to 21.58%. In contrast, for the excess return of the ten-year bond, macro factors just increases the $R$-square of the predictive regression from 17.82% to 23.60%. It is less than double. Overall, the finding that macro factors are unspanned by yield-curve factors are largely consistent with the results presented by Ludvigson and Ng (2008), Cooper and Priestly (2009), Duffee (2011), Joslin, Le, and Singleton (2013), Joslin, Priebschi, and Singleton (2014), and Zhou and Zhu (2015).

[Insert Table 1 about Here]

Turning to the second implication, we also run a simple regression to shed light on the importance of unspanned macro risks. We investigate the exchanges rates of the major economies (Canada, Germany, Japan, and the U.K.) relative to the U.S.[4] Specifically, we first run the excess currency return, $\rho_{t+n}^{(n)}$, on the PCs of the pair of term

[4]For the other two G7 countries (France and Italy), because they use the same currency as Germany for most of our sample period, we do not separately investigate these two economies.

structures in the corresponding two markets. Our second approach adds unspanned macro factors for two economies into the regression. Panel B of Table 1 presents the results for predicting excess currency returns. It is evident from the table that macroeconomic factors have additional predictive ability for forecasting future excess currency returns. We take Canada as an example to illustrate the point. Considering the excess currency return for a 1-month holding period, when macro factors are excluded, the $R$-square of the predictive regression is 2.64%. By contrast, with macro factors included, the $R$-square of the predictive regression more than quadruples to a level of 14.78%. For all other markets, we observe a similar pattern.

We are not only interested in the excess currency return from buying/selling short-term bonds, but also that from buying/selling longer maturity bonds. This is because we investigate the term structure of currency risk premia instead of just the excess currency return of holding short-term domestic/foreign bonds. In light of this, Panel B also reports the predictive ability of excess currency returns from buying/selling longer maturity bonds. Similarly, we find macro factors have additional predictive power for future excess currency returns. Look at the exchange rate of the Japanese Yen against the US dollar, when the underlying asset is the 10-year Treasury bond for computing excess currency return, adding macro factors increases the $R$-square of the predictive regression from 2.49% to 9.65%. Panel B also presents the predictive results for the half-year holding period. We find that macro factors consistently lead to a higher $R$-square of the predictive regression, suggesting that macro factors contain additional information for predicting excess currency returns. Taken together with the results from the predictive regressions of bond risk premia, we find that macro factors contain important information for future currency and bond risk premia.

## 2.3 Affine Two-Country Model with Unspanned Macro Risks

In this section we propose a continuous-time, two-country dynamic affine term structure model with unspanned macro factors (ATSM-X). Guided by the pioneering work of

Backus, Foresi, and Bansal (2001), we assumes the joint dynamics of interest rates and foreign exchange rates are depended on several latent factors. Furthermore, as proposed in Joslin, Priebsch, and Singleton (2014) and Ludvigson and Ng (2009), investors must be compensated for risks associated with macroeconomic activity, which implies that macroeconomic variations explain at least part of time varying expected risk premia, both in currency and bond market. Similar to Joslin, Priebsch, and Singleton (2014), we incorporate Macro factors into our two-country affine model as an unspanned component, which means Macro factors do not affect risk-neutral dynamics, but do exist in real world dynamics.

According to Joslin, Singleton, and Zhu (2011), in an affine term structure framework with $n$ latent factors, we could rotate the factors for domestic interest rate into the first $n$ principle components (PCs), $P_t$, of annulized domestic bond yields without losing any generality, which yields $i_t = a_0 + a_1^\top P_t$. This rotation turns latent factors into observable factors, which greatly reduces the converge time to the global optimum for the model estimation. Similarly, we employ the first $n^*$ PCs of the annulized yields of the foreign interest rates, $i_t^* = a_0^* + a_1^{*\top} P_t$. Furthermore, the economy is completed by $m$ macro-economic factors $M_t$. This is because, except by the bond market, the exchange rate are also impacted by the information from the trade sector, which is highly related to macroeconomic variables and not fully spanned by the risk facts from bond markets. Assume the $2n + m \times 1$ vector for all the observed factors $Z_t = (P_t^\top, P_t^{*\top}, M_t^\top)^\top$ encompasses all risks in the economy, both domestic and foreign.

The dynamic process followed by the state variable $Z_t$ under the real world $P$ measure is,

$$d \begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix} = \left[ \begin{bmatrix} K_{0P}^P \\ K_{0*}^P \\ K_{0M}^P \end{bmatrix} + \begin{bmatrix} K_{1PP}^P & K_{1P*}^P & K_{1PM}^P \\ K_{1*P}^P & K_{1**}^P & K_{1*M}^P \\ K_{1MP}^P & K_{1M*}^P & K_{1MM}^P \end{bmatrix} \begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix} \right] dt + \Sigma_Z d \begin{bmatrix} W_P^P \\ W_*^P \\ W_M^P \end{bmatrix}, \tag{25}$$

where the volatility matrix

$$\Sigma_Z = \begin{bmatrix} \Sigma_{PP} & & \\ \Sigma_{P*} & \Sigma_{**} & \\ \Sigma_{MP} & \Sigma_{M*} & \Sigma_{MM} \end{bmatrix} \tag{26}$$

is a lower triangular matrix. Observe that the $p$-SDE implies both $P_t$, $P_t^*$, and $M_t$ are observable in the real world measure. The dynamics of $p$-SDE further assume $M_t$ is impacted by both $P_t$ and $P_t^*$, which confirms the empirical findings suggested by Joslin, Priebsch, and Singleton (2014), the projection errors of $M_t$ onto $P_t$ and $P_t^*$ have predictive power for both domestic and foreign interests. Furthermore, as indicated in Litterman and Scheinkman (1991), the cross-section of bond yield is determined by fewer number of factors, $P_t$ in our case. We therefore derive the dynamic of $Z_t$ under the domestic pricing measure $Q$ as,

$$d\begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix} = \left[\begin{bmatrix} K_{0P}^Q \\ K_{0*}^Q \\ K_{0M}^Q \end{bmatrix} + \begin{bmatrix} K_{1PP}^Q & 0 & 0 \\ K_{1*P}^Q & K_{1**}^Q & 0 \\ K_{1MP}^Q & K_{1M*}^Q & K_{1MM}^Q \end{bmatrix}\begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix}\right] dt + \Sigma_Z d\begin{bmatrix} W_P^Q \\ W_*^Q \\ W_M^Q \end{bmatrix}. \tag{27}$$

Thus, the domestic annulized yield with time to maturity $\tau$ at time $t$ is given as

$$i(t,\tau) = A(\tau) + B(\tau)^\top P_t, \tag{28}$$

where $A(\tau)$ is a length-$n$ vector, and $B(\tau)$ is a square matrix, the detail derivation of $A(\tau)$ and $B(\tau)$ is given in Appendix. In general, the equation implies $i(t,\tau)$ is only determined by an $n \times 1$ vector $P_t$, the domestic bond risk factors. Although there are three type of risk factors, $P_t$, $P_t^*$ and $M_t$ in our two-country economy, our model preserves the empirical consensus in term structure literature that zero-coupon bonds could be explained cross-sectionally by a few risk factors. Put differently, we treat the foreign bond risk factors and Macroeconomic variables as unspanned factors in terms of calculating the local bond price.

Similarly, for symmetric and parsimonious, we assume the dynamic of $Z_t$ for foreign pricing measure $Q^*$ as

$$d\begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix} = \left[\begin{bmatrix} K_{0P}^* \\ K_{0*}^* \\ K_{0M}^* \end{bmatrix} + \begin{bmatrix} K_{1PP}^* & K_{1P*}^* & 0 \\ 0 & K_{1**}^* & 0 \\ K_{1MP}^* & K_{1M*}^* & K_{1MM}^* \end{bmatrix}\begin{bmatrix} P_t \\ P_t^* \\ M_t \end{bmatrix}\right] dt + \Sigma_Z d\begin{bmatrix} W_P^* \\ W_*^* \\ W_M^* \end{bmatrix} \tag{29}$$

with annualized bond yield as

$$i^*(t,\tau) = A^*(\tau) + B^*(\tau)^\top P_t^*, \tag{30}$$

where $A^*(\tau)$ is a length-n vector, and $B^*(\tau)$ is a square matrix, the detail derivation of $A^*(\tau)$ and $B^*(\tau)$ is identical to $A(\tau)$ and $B(\tau)$. $P_t$ and $M_t$ are treated as unspanned factors in terms of pricing foreign zero-coupon bonds.

One of the most distinct feature for our model to Backus, Foresi, and Bansal (2001) and Sarno, Schneider, and Wagner (2012) is that the global pricing kernel for each country can be decomposed by a domestic bond market specific part and an orthogonal macro-economic related unspanned part. For domestic market, the bond market specific pricing kernel $\Pi_P$ is given by

$$\frac{d\Pi_P}{\Pi_P} = -i_t dt - \Lambda_P(Z_t)^\top dW_P^P \tag{31}$$

with the bond market specific market price of risk $\Lambda_P(Z_t)$

$$\Lambda_P(Z_t) = \Sigma_{pp}^{-1}(\mu_P^P(Z_t) - \mu_P^Q(Z_t)). \tag{32}$$

where $\Lambda_P(Z_t)$ is an affine function of state $Z_t$ even though the priced risk factors are the substate $P_t$ only. However, the bond market specific pricing kernel $\Pi_P$ is unable to determine the exchange rate as requested in equation (14). Since the pricing kernel in equation (14) should price all the risks rather than the bond market risk only. The general pricing kernel should be able to price entire state $Z_t$,

$$\frac{d\Pi_Z}{\Pi_Z} = -i_t dt - \Lambda_Z(Z_t)^\top dW_Z^P \tag{33}$$

with the market price of risk $\Lambda_Z(Z_t)$

$$\Lambda_Z(Z_t) = \Sigma_Z^{-1}(\mu_Z^P(Z_t) - \mu_Z^Q(Z_t)). \tag{34}$$

where $\Lambda_Z(Z_t) = (\Lambda_P(Z_t)^\top, \Lambda_{P*}(Z_t)^\top, \Lambda_M(Z_t)^\top)^\top$ with $\Lambda_{P*}$ prices the interest risks in foreign country and $\Lambda_M(Z_t)$ prices Macroeconomic risks. $\Lambda_{P*}$ and $\Lambda_M(Z_t)$ together form unspanned part of market price of risk to domestic bond market, $\Lambda_U(Z_t) = (\Lambda_{P*}(Z_t)^\top, \Lambda_M(Z_t)^\top)^\top$. The corresponding unspanned pricing kernel is

$$\frac{d\Pi_U}{\Pi_U} = -\Lambda_U(Z_t)^\top dW_U^P \tag{35}$$

where $\Pi_U$ is a martingale and orthogonal to $\Pi_P$, the global pricing kernel is therefore defined by, $\Pi_Z = \Pi_P \Pi_U$.

For the foreign country, the bond market specific pricing kernel and the global pricing kernel, $\Pi_{P*}^*$ and $\Pi_Z^*$ can be defined analogously, with $\Lambda_{P*}^*$, $\Lambda_Z^*$ to be their market price of risk respectively. The foreign market price of risk $\Lambda_Z^*$ is given by,

$$\Lambda_Z^*(Z_t) = \Sigma_Z^{-1}(\mu_Z^P(Z_t) - \mu_Z^*(Z_t)). \tag{36}$$

where $\Lambda_Z^* = (\Lambda_P^*, \Lambda_{P*}^*, \Lambda_M^*)^\top$. The unspanned part of the pricing kernel for this foreign country $\Pi_U^*$ and its corresponding unspanned market price of risk is formed by $\Lambda_P^*$ and $\Lambda_M^*$, i.e. $\Lambda_U^* = (\Lambda_P^*, \Lambda_M^*)^\top$. To make the structure affine, we further assume

$$\Lambda_Z - \Lambda_{Z*} = \Sigma \tag{37}$$

to be a vector of constants with $\Sigma = (\Sigma_P, \Sigma_{P*}, \Sigma_M)^\top$.

By substituting (33) into (14), the dynamic of exchange rate $s_t$ is given by

$$\begin{aligned} ds_t &= dm_t^* - dm_t \\ &= (r_t - r_t^* + \frac{1}{2}(\Lambda_Z^\top \Lambda_Z - \Lambda_Z^{*\top} \Lambda_Z^*))dt + (\Lambda_Z - \Lambda_{Z*})^\top dW_t \end{aligned} \tag{38}$$

which is affine to the state $Z_t$. The 1-period risk premium for foreign country, $v_t^{(1)}$, is

$$v_t \approx \underbrace{\frac{\delta}{2}(\Lambda_P^\top \Lambda_P - \Lambda_{P*}^{*}{}^\top \Lambda_{P*}^*)}_{spanned\ part} + \underbrace{\frac{\delta}{2}(\Lambda_U^\top \Lambda_U - \Lambda_U^{*\top} \Lambda_U^*))}_{unspanned\ part} \tag{39}$$

with $\delta$ to be the time interval of 1-period investment. Observe that in equation (39), both $\frac{\delta}{2}(\Lambda_P^\top \Lambda_P - \Lambda_{P*}^{*}{}^\top \Lambda_{P*}^*)$ and $\frac{\delta}{2}(\Lambda_U^\top \Lambda_U - \Lambda_U^{*\top} \Lambda_U^*)$ forms affine function of $Z_t$.

# 3 Model Estimation

## 3.1 Data Issues

We use end-of-month yield curve data for 5 countries: Canada, Germany, Japan, UK and the United States. In order to align the starting date of each series, we use maximal common divisor for 5 countries, 1986:01. The sample period is therefore from 1986:01 to 2015:02. The yield data in our study comes from smoothed curves. They are constructed by respective Central banks using Svensson method, Nelson and Siegel method or Spline methods.

[Insert Table 2 about Here]

Table 2 summarizes the detail of the data. The first column shows that the average yield curve is increasing. Look at the yield curve of US, the average yield maturating at 6 months, 1 year, 5 years and 10 years are 4.14%, 4.51%, 5.47% and 6.15%, respectively.

The third column suggests that the short end of the yield curve is more volatile than the long end. For example, the standard deviation of yield maturating at 6 months, 1 year, 5 years and 10 years in Canada are 3.27, 3.17, 2.81 and 2.62, respectively. Column six reports the minimal value for respective countries. Look at the interest rate of German, the minimal values for 6 months, 1 year and 5 years bonds are negative, which implies our Gaussian model may provide a better fitting for the term structure of interest rates.

Throughout this paper, we use the U.S. as home country, and we denote exchange rate as U.S. dollar per unit of foreign currency. We collect end-of-month exchange rate series from 1986:01 to 2015:02. These data are collected from The Federal Reserve Bank of St. Louis, which contains the following countries: Canada, Germany (Euro from 1999:01), Japan, and United Kingdom. We use inflation and industrial production index growth rate as our unspanned Macroeconomic factors, which are also collected from The Federal Reserve Bank of St. Louis for five countries, from 1986:01 to 2015:02 on monthly basis.[5]

[Insert Table 3 about Here]

Panel A of Table 3 provides the summary statistics for end-of-month exchange rates. The exchange rates for Canada and UK are more stable comparing to Japan and German according to column 6 and column 7. Panel B of Table 3 reports the summary statistics for macro variables. We observe a similar pattern for Japan and German, while U.S, U.K and Canada seem belonging to another pattern.

## 3.2 Estimation Methods

The proposed model is an affine latent variable model. In our model, we apply a small number, $2n + m$, of driving state variables to jointly fit a large number of variables:

[5]inflation rate is constructed from the log difference of seasonality adjusted consumer price index. Industrial production index growth rate is also calculated from the log difference of seasonality adjusted industrial production index.

both the interest rate pricing with different maturities and the first 2 moments of the exchange rate with different time intervals. Inspired by Joslin, Singleton, and Zhu (2011) and Joslin, Priebsch, and Singleton (2014), we rotate the state variables $P_t$ and $P_t^*$ to the first $n$ principle components of domestic and foreign bond yields. We also rotate $M_t$ to $m$ observable macro economic variables. Thus the latent state variables become observable ones so that we avoid the complicated filtering procedure or Bayesian procedure as in previous literatures, see e.g. Sarno, Schneider, and Wagner (2012). This makes our study simple and robust.

Another notable property of our model is that the $Q$ and $Q^*$ parameters relative to bond pricing, $K_{0P}^Q$, $K_{1PP}^Q$, $K_{0*}^*$ and $K_{1**}^*$ are irrelevant to the parameters that determines exchange rate moments because of unspanned interest rate market setup. This implies that our estimation procedure for matching depreciation rates and pricing bonds are two different procedures. This also implies that we have a potential to both accurately fit the exchange rate and price the bond, which is thought as a 'trade off' in previous literatures, see Sarno, Schneider, and Wagner (2012). A detailed canonic study for this scenario is given in Appendix B.

Accordingly, we could thus divide our estimation procedure into 3 steps: (1) Because the latent variables are now observable, we estimate P-measure parameters for the observable factors $Z_t$ with a standard time series approach. Because the time series analysis for observable factors $Z_t$ are rather standard, this makes our study even more simple and robust. (2) Estimate $Q$ and $Q*$ parameters that is related to interest rate pricing with a JSZ canonic representation, see Joslin, Singleton, and Zhu (2011). (3) Estimate the remaining $Q$ and $Q*$ parameters with the information of exchange rate moments. Details of the estimation procedure is given in Appendix C.

# 4 Model Performance

In this section, we investigate whether the ATSM-X model can account for the empirical failure of the UIP condition and equilibrium exchange rate models. If the ATSM-X model can generate the appropriate time variation in expected excess returns, it can provide a risk-based explanation for exchange rate anomalies. In Section 4.1, we run the Fama and Engel regressions to show the presence of exchange rate anomalies. In Section 4.2, we show that the ATSM-X model can generate a time-varying risk premium that resolves the two exchange rate anomalies. We achieve this by showing that the slope coefficient of the Fama and Engel regressions to unity, which cannot be statistically rejected. One salient feature of the ATSM-X model is the incorporation of unspanned macro factors. In Section 4.3, we examine the importance of unspanned macro factors in currency and bond risk premia by decomposing the model-implied currency and bond risk premia into two components, which are respectively driven by term-structure factors and macro factors.

## 4.1 Results from the Fama and Engel Regressions

Two of the most important anomalies regarding exchange rate dynamics are the forward premium puzzle and the empirical failure of classic equilibrium exchange rate models. According to the UIP condition, if investors are risk-neutral and have rational expectations, exchange rate changes will eliminate any gain arising from interest rate differentials across markets. That is to say, forward premium should serve as unbiased predictors of future currency depreciation. For example, if the forward exchange rate exceeds the current spot rate by 2%, the future spot rate is expected to depreciate by 2%.

The UIP condition is intuitive and economically appealing, it therefore has been investigated thoroughly using a variety of econometric methods and data. A typical approach is to regress the ex post future exchange rate changes on current forward discounts. Contrary to the prediction of the UIP, the estimated regression slope coefficient is generally found to be less than one and is often not significantly different from minus one, which implies that high interest rate currencies tend to appreciate, and low interest rate currencies tend to depreciate. This generates predictably positive excess returns for high interest rate currencies.

[Insert Table 4 about Here]

Motivated by the fact that the 2007-2008 world financial crisis has led to a crash in carry trade returns, we re-examine the validity of the UIP condition. Our empirical analysis is based on non-overlapping observations for prediction horizons of 1 month, 3 months, 6 months, 1 year, 3 years and 10 years. Our spot exchange rate data consist of the bilateral Canadian dollar, German mark/euro, Japanese yen, and pound sterling exchange rates viz-a-viz the US dollar.

Almost all empirical studies run the Fama regression (see equation (6)) using nominal interest rates. In the line of this strand of literature, we conduct the Fama regression analysis using nominal interest rates. Table 4 summarizes the empirical results for various interest rates of different maturities. The results confirm the usual finding of a strong forward rate bias for the currencies under investigation. All the currencies except for the Canadian dollar show coefficients that are usually statistically less than zero at high significance levels. For the Canadian dollar, regression coefficients are significantly less than one at short end of the maturity spectrum. At the long end ($n = 3$ and 10 years), regression coefficients are close to one, not very against the UIP condition. Taken together, we can generally reject the hypotheses that the regression coefficient is one for all currencies.

In international finance, foreign exchange rates are often linked to interest rates. Two of the best-known empirical relationships are the uncovered interest rate parity and standard exchange rate models. The uncovered interest rate parity concerns the rate of change of the exchange rate. Standard exchange rate models concern the level of the exchange rate. Dornbusch (1976) and Frankel (1979) are the original papers to draw the link between real interest differentials and the level of the exchange rate in modern, asset-market approach to exchange rates. An important prediction of these standard exchange rate models is that when a country has a higher than average relative interest rate, the price of foreign currency should be lower than average. In the data for currencies of major economies relative to the U.S., a flood of research documents this empirical relationship. However, it is generally found that the strength of the home currency tends to be greater than is warranted by rational expectations of future short-term interest differentials as model posit under interest parity.

The strength of the home currency implies excess comovement of the level of the exchange rate and the interest differential. As discussed in Section 2, we can express this excess comovement mathematically. This excess comovement means that the covariance of the stationary component of the exchange rate with the foreign less U.S. interest rate is more negative than would hold under interest rate parity: $cov(E_t \sum_0^\infty \rho_{t+j+1}, r^*_{t,1} - r_{t,1}) < 0$. Though this empirical finding of excess comovement is not very puzzling in itself, it has opposite implications with the UIP: This finding implies high interest rate currency is less risky, the empirical failure of the UIP indicates that high interest rate is riskier.

To shed light on the empirical relationship between interest rate differentials and exchange rate levels implied by standard exchange rate models, we run Engel (2016) regression (8) using the data for currencies of major economies relative to US. Our preliminary empirical analysis uses nominal interest rates. Such an analysis is comparable to the Fama regression based on nominal interest rates. Another advantage of using nominal interest rates is that we do not need to calculate inflation expectations, which are not observable. Furthermore, Engel (2016) stresses that results from the regression based on nominal interest rates are roughly consistent with those from the regression based on real interest rates.

[Insert Table 5 about Here]

We run the Engel regression using the ex post forward currency risk premium. Table 5 presents the results of the Engel regression using the data for currencies of major economies relative to US. In the Fama regression, the independent variable is exchange rate changes or the spot n-period forward risk premium $\rho_{t+1}^{(n)}$. In contrast, the regressand in Engel regression is the forward one-period currency risk premium $\rho_{t+j+i}^{(1)}$ for $j \geq 1$. If the currency risk premium is a constant over the entire spectrum, we should observe a regression slope that equals to 0. However, in the short spectrum, we typically find a large positive slope coefficient, which is significant for the US dollar against Japanese yen, the British pound, the merged Deutsch mark and euro series. For the US dollar against Canadian dollar, though the slope coefficient is insignificant, it is still positive. These results are consistent with the results from the Fama regression, suggesting that high interest rate currencies are riskier.

However, when we move to the long end of the spectrum, a striking thing that emerges from the Table is that the slope coefficient are consistently negative, which are statistically significant at the long end. These findings suggest that the term structure of currency risk premia is downward sloping. More importantly, this downward-sloping term structure of currency risk premia indicates that high interest rate currencies are less risky, as opposed to the implication of the uncovered interest rate parity. This constitutes the paradox proposed by Engel (2016). Another interesting pattern that emerges from the table is that the term structure of currency risk premia in some markets is not monotonically decreasing. For instance, the slope coefficient of the Engel regression is very volatile for the exchange rate of the Canadian dollar against the US dollar.

[Insert Table 6 about Here]

We now conduct empirical analysis using real interest rates. This is important because the standard equilibrium models of the level of the exchange rate (e.g., Dornbusch, 1976; and Frankel, 1979) link the stationary component of the exchange rate to real interest differentials. Table 6 reports the empirical results using ex post forward currency risk premia as the regressand. It is evident that the slope coefficient of the Engel regression is often negative. Compared to the evidence from the regression based on nominal interest rates, we find that even at the short spectrum, the slope coefficient is often negative. As such, the evidence is more against the evidence from the empirical test of the UIP conditions, making the paradox proposed by Engel even more puzzling. Another interesting thing that emerges from the table is that the term structure of currency risk premia is not monotonic and currency risk premia is very volatile. Overall, the results from the Fama and Engel regressions suggest the empirical failure of the UIP and the presence of the exchange rate paradox.

## 4.2 The ATSM-X Model and Puzzles

As a preliminary analysis, we examine how well the ATSM-X model fits the US and foreign term structure of interest rates and exchange rate changes born out in the data. As discussed above, the fitness of the yield curve in the ATSM-X model is determined by three local term-structure factors and two unspanned macro factors. In our exercise, local term-structure factors are the first 3 principle components of all interest rates. In Table 7 we present statistics that describe the in-sample fit. Specifically, we report the root mean-squared pricing errors (RMSE) of the domestic US yields and the respective foreign yields measured in basis points.[6] As shown in Panel A of Table 7, average yield pricing errors are small, ranging between 2.1 and 11.6 basis points.

[Insert Table 7 about Here]

Panel B summarizes the fitness of exchange rates. In the ATSM-X model, exchange rate dynamics are simultaneously driven by term-structure factors of two markets as well as unspanned macro factors. We report the results from regressing observed exchange rate changes on model-implied exchange rate changes. The regression results suggest that intercepts ($\alpha$) are virtually zero, though we reject the null hypothesis

[6] We estimate independently the US term structure of interest rates for all four pairs of exchange rates. The fitness are very similar. We report in Table 7 the results from the ASTM-X model for the exchange rate of US dollar against Mark/Euro.

that intercepts are statistically different from 0. Slope coefficients ($\beta$) are close to one, and we cannot reject the hypothesis that slope coefficients are equal to one. The results show that exchange rate dynamics implied by the ATSM-X model closely match observed exchange rate changes.

In addition to the fitness, what is the extent to which the ATSM-X model can account for the predictability of excess bond returns? A striking empirical finding in recent finance research is the predictability of excess bond returns. On the theoretical side, the predictability of excess bond returns are consistent with economic theories, which suggests that Insofar as economic variables affect future consumption and investment opportunities, they are important state variables for predicting excess bond returns. In this spirit, a number of studies (e.g., Fama and Bliss, 1987; Campbell and Shiller, 1991; Ludvigson and Ng, 2009; Joslin, Priebsch, and Singleton 2014) show that numerous macro and financial factors have important forecasting power for future bond risk premia.[7] In particular, forward rates are found to have strong predictive power for future excess bond returns. Since investors' beliefs about future bond prices determine what investors are willing to pay for bonds. This indicates that forward rates contains all information relevant to predicting bond risk premia. Along this line, Cochrane and Piazzesi (2005) find that a tent-shaped linear combination of forward rates strongly predicts excess bond returns. Toward this end, we investigate how well the ATSM-X model can account for the predictability of excess bond returns, a major anomaly in the bond pricing literature.

Given the excess bond return, $rx_{t,n}$, defined in equation (24), the average excess log return across the maturity spectrum is defined as

$$\overline{rx}_{t+1} \equiv \frac{1}{4}{}_{n=2}^{5} rx_{t+1,n}. \tag{40}$$

[7]Zhu (2015) reviews the academic literature that investigates the predictability of excess bond returns.

According to finance theories, investors' beliefs about future bond prices are impounded into current prices. As such, the time-$t$ term structure should contain substantial information about future changes in excess bond returns. In this spirit, Cochrane and Piazzesi (2005) extend the Fama and Bliss (1987) approach and run regressions of excess returns on all forward rates:

$$rx_{t+1,n} = \beta_{0,n} + \beta_{1,n} i_{t,1} + \beta_{2,n} f_{t,2} + ... + \beta_{5,n} f_{t,5} + \varepsilon_{t+1,n}. \tag{41}$$

Drawing on the fact that the same function of forward rates predicts holding period returns at all maturities, Cochrane and Piazzesi (2005) construct a tent-shaped linear combination of forward rates, namely the CP factor, to parsimoniously predict one-year ahead excess bond returns. Specifically, the CP factor is constructed by regressing the average excess return across maturities at each time $t$ on the one-year yield and four forward rates $\mathbf{f}_t \equiv [i_{t,1}\ f_{t,2}\ f_{t,3}\ f_{t,4}\ f_{t,5}]'$:

$$\overline{rx}_{t+1} = \gamma_0 + \gamma' \mathbf{f}_t + \bar{\varepsilon}_{t+1}, \tag{42}$$

where $\gamma = [\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5]'$. The CP factor is the fitted value of regression (42).

Cochrane and Piazzesi (2005, 2008) show that the predictive ability of the CP factor cannot be captured by the popular yield curve factors of level, slope and curvature. Recently, the tent-shaped linear combination of forward rates has been viewed as a stylized fact to be matched by term structure models. In particular, Kessler and Scherer (2009) and Sekkel (2011) demonstrate that show that the empirical finding of Cochrane and Piazzesi (2005) hold for other developed countries. For Germany, Japan, and UK, they find that a single-factor model captures well the predictability of international excess bond returns. This factor in these markets tend to have a tent-shape.

[Insert Figure 1 about Here]

We first check whether a single-factor can capture well the predictability of bond risk premia. Because five forward rates create a near-perfect colinearity problem, we report results for forward rates with $n = 1, 3, 5$ years (see, also, Lettau and Wachter, 2011), though these results are robust to alternative choices. As shown in the left panels of Figure 1, the tent-shape finding of Cochrane and Piazzesi (2005) is quite apparent, suggesting the robustness of the tent-shape. Next, we examine whether the ambiguity model can replicate the tent-shape. The right panels of Figure 1 plots the model-implied coefficients. It is evident that the ambiguity model replicates the tent-shape and explains the prediction capability of forward rates for excess returns.

An implication of many economic models is that high interest rate currencies tend to appreciate. However, numerous empirical studies suggest the opposite: future exchange rate changes and current interest rate differentials are negatively correlatively. Our analysis in Section 3 also indicates a tendency for high interest rate currencies to appreciate. This departure from uncovered interest parity, which is usually termed as the forward premium anomaly, has spawned a second generation of papers attempting to account for it. The ATSM-X model generates the time-varying currency risk premium, which is jointly driven by term-structure factors and unspanned macro factors. In the setting of ATSM-X, the forward exchange rate is the sum of the expected spot rate plus a time-varying risk premium which compensates both for unspanned macro risks, interest rate risk, and currency risk. As a consequence, the ATSM-X model is likely to provide a risk-based explanation for the forward premium puzzle.

[Insert Table 8 about Here]

To examine whether the ATSM-X model is able to generate the time-varying risk premium that can account for the empirical failure of the UIP condition, we run risk-adjusted Fama regression (10). If the ATSM-X model is successful in producing a risk premium with the requisite properties, we should observe a slope coefficient of one in the risk-adjusted regression. Table 8 provides an overview on how well the ATSM-X model explain the forward premium puzzle for four exchange rates. Our evidence clearly shows that the slope coefficient is significantly positive and close to one for all four currencies and for all maturities. We also statistically test the null hypothesis that the slope coefficient is equal to one. In more than half cases, we cannot reject the null hypothesis.

Compared to the Fama regression results presented in Table 4, a striking feature is the increase in regression $R$-square. We illustrate the point using the 1-month, 1-year, and 10-year regressions. For the US dollar against Japanese yen, the merged Deutsch mark and euro series, the British pound, and the Canadian dollar, $R$-square increases respectively from 0.042, 0.254, and 0.140 to 0.063, 0.468, and 0.759. For the US dollar against the merged Deutsch mark and euro series, $R$-square increases respectively from 0.001, 0.041, and 0.412 to 0.133, 0.218, and 0.485. For the US dollar against the British pound, $R$-square increases even more dramatically. The US dollar against the Canadian dollar shows a similar pattern. The change in $R$-square further confirms the importance of the time-varying risk premium in accounting for the forward premium puzzle.

In Section 3, using both nominal and real interest rates, we document that the covariance of the stationary component of the exchange rate with the foreign less U.S. interest rate is more negative than would hold under interest parity,

$$cov(E_t \sum_{0}^{\infty} \rho_{t+j+1}^{(1)}, r_{t,1}^{*} - r_{t,1}) < 0. \tag{43}$$

We thus need to explain why it is more negative. In particular, when this fact is combined with the forward premium puzzle, it constitutes a puzzling paradox: the forward premium puzzle requires high interest currencies to be riskier, but this fact implies currencies with higher interest rate to be less risky.

The paradox implies a decreasing curve of forward currency risk premia along the maturity spectrum. We already show that the time-varying risk premium generated by the ATSM-X model is able to account for the forward premium puzzle. In the same spirit, we also run risk-adjusted Engel regression (21). If the risk premium delivered by the ATSM-X model can resolve the puzzle proposed by Engel (2016), it means that the ATSM-X can generate the decreasing curve of forward currency risk premia. As a consequence, it can simultaneously account for the forward premium puzzle and the Engel puzzle.

[Insert Table 9 about Here]

As a preliminary analysis, we run the risk-adjusted Engel regression using nominal interest rates. Table 9 provides an overview on how well the ATSM-X model can resolve the Engel puzzle. It is evident from the table that the slope coefficients are consistently close to one for all currencies and for all maturities, as suggested by standard exchange rate models. Statistically, these coefficients are significant at the 1 percent level. To provide insights on the confidence level of the slope coefficient of one, we test the null hypothesis that the slope coefficient is equal to one. We find that the null hypothesis is rarely rejected, suggesting the fitness of the ATSM-X model. A more salient feature of the risk-adjusted Engel regression that emerges from the table is explanatory power. Compared with the Engel regression results reported in Table 5, for the US dollar against Japanese yen, the merged Deutsch mark and euro series, the British pound, and the Canadian dollar, the average $R$-square increases respectively from 3%, 1%, 1%, and less than 1% to 37%, 52%, 43%, and 52%. The dramatic increase in the average $R$-square suggests the importance of the time-varying risk premium in understanding the link between exchange rate changes and interest rate differentials.

[Insert Table 10 about Here]

We also run the risk-adjusted Engel regression using real interest rates. Since standard exchange rate models are typically focus on the link between real interest rate differentials and exchange rate dynamics, it seems such an analysis is very important. Table 10 summarizes the risk-adjusted Engel regression results using real interest rate differentials. For all currencies and all forward maturities, we consistently find that the slope coefficient is statistically significant and is close to one, which is the theoretical value of the slope coefficient. In most cases, we cannot reject the null hypothesis that the slope coefficient is equal to one. More notably, the average $R$-square increases respectively from 1%, 1%, less than 1%, and 2% to 98%, 96%, 98%, and 98% for the US dollar against Japanese yen, the merged Deutsch mark and euro series, the British pound, and the Canadian dollar. Taken together with the findings from the risk-adjusted Fama regression and the risk-adjusted Engel regression based on nominal interest rates, our analysis suggests that the ATSM-X model can generate the decreasing term structure of forward currency risk premia and can resolve the paradox stressed by Engel (2015).

## 4.3 Decomposition of Currency and Bond Risk Premia

In Section 4.2, we show that the time-varying risk premium implied by the ATSM-X model can account for exchange rate puzzles and the predictability of bond risk premia. The ATSM-X model thus provides a risk-based explanation for asset pricing puzzles. In this section we analyze the nature of the time variation in expected excess returns implied by the ATSM-X model.

In a complete market, the percentage change in the exchange rate reflects the difference between the log of the domestic and the foreign pricing kernels. In this spirit, we decompose currency and bond risk premia into a macro-factor-specific component and a yield-factor-specific component. In our setting, macro factors are unspanned by term-structure factors. So, macro factors affect contemporaneous bond yields and exchange rates only through their correlation with term-structure factors, but they are allowed to independently predict future currency and bond risk premia. As a consequence, expected excess currency and bond returns implied by macro factors and by term-structure factors are independent. Our way of examining the nature of expected excess returns is to check how the two components of currency and bond risk premia are related to global risk measures and to economic activity.

In line with Sarno, Schneider, and Wagner (2012) as well as Lustig, Roussanov, and Verdelhan (2014), our proxy for global risk is based on the $VIX$ S&P500 implied volatility index traded at the Chicago Board Options Exchange (CEOE), which is highly correlated with similar volatility indexes in other countries. In addition to being a proxy for global risk, the $VIX$ index can also be viewed as a proxy for funding liquidity constraints (see, Brunnermeier, Nagel, and Pedersen, 2008). As such, "flight-to-quality" and "flight-to-liquidity" arguments, which posit that investors demand a higher risk premium in bad times, suggest that short-horizon expected currency risk premia should be negatively correlated with the $VIX$ multiplied by the sign of the current yield differential: in times of global market uncertainty and higher funding liquidity, market participants require higer risk premia on high yield currencies but accept lower risk premia on low yield currencies.

For the US dollar against Japanese yen, the merged Deutsch mark and euro series, the British pound, and the Canadian dollar, the contemporaneous correlations of 1-month-ahead expected currency risk premia with the VIX multiplied by the sign of the interest rate differential, $sVIX \equiv VIX \times sign[i_{t,1} - i^*_{t,1}]$, are respectively $-0.40$, $-0.57$, $-0.18$, and $-0.24$. Using block-bootstrapped methods, we find these contemporaneous correlations are consistently significant at the 1% level. The singificantly negative correlations provide the supporting evidence on our priors that foreigen exchange risk premia are driven by global risk perception in a way that is consistent with economic intuition.

[Insert Figure 2 about Here]

Figure 2 illustrates economically large effects of unspanned macro factors on risk premiums in currency markets. The first striking fact that emerges from the figure is that a large portion of currency risk premia is attributable to unspanned macro risks. In contrast, only a small portion of risk premia is driven by term-structure factors. The second fact that emerges from the figure is that currency risk premia driven by unspanned macro factors are very volatile. Two resolve the forward premium puzzle from a risk-based perspective, Fama (1984) Fama shows that implied risk premium on a currency must be negatively correlated with its expected rate of depreciation and has greater variance. The volatile risk premium implied by unspanned macro factors might account for why the ATSM-X model can resolve the forward premium puzzle. As currency risk premia implied by macro factors vary along the sprectrum, they deliver a decreasing term structure of currency risk premia and account for the Engel paradox.

Numerous recent studies (e.g., Lustig and Verdelhan, 2007; De Santis and Fornari, 2008; and Lustig, Roussanov, and Verdelhan, 2014) demonstrate that risk premia on US exchange rates are countercyclical to the US economy. In this spirit, we use industrial production as a measure of the state of the US economy and examine how currency risk premia are related to IP growth rate. If the model-implied risk premium is countercyclical, the relation between expected excess currency returns and output growth should be negative. It is evident from the figure that macro-factor-specific risk premia are countercyclical. In contrast, yield-factor-specific currency risk premia appear to be acyclical. These results thus suggest that foreign exchange risk premia are driven by macroeconomic fundamentals in a way that is consistent with economic intuition.

[Insert Figure 3 about Here]

We also document economically large effects of the unspanned macro factors on risk premiums in international Treasury bond markets. Figure 3 respectively plots the "in-two-years-for-one-year" forward term premia for five markets. We can attribute a large portion of movements in bond risk premia to unspanned macro factors. These results are consistent with those from Joslin, Priebsch, and Singleton (2014) and Jotikasthira, Le, and Lundblad (2015). It is also consistent with the findings from the analysis of currency risk premia. Overall, these findings indicate the importance of unspanned macro factors in understanding the behavior of currency and bond risk premia.

Economic theories suggest that rational, utility-maximizing investors must be compensated for bearing macroeconomic risks. In economic recessions, investors are reluctant to take on risk. Heightened risk aversion during economic downturns thus pushes up the risk premium. In light of this, we examine fluctuations in bond risk premia over the business cycle. Notably, the risk premia from the ATSM-X model show a pronounced cyclical pattern with peaks during recessions (the shaded areas). For the purpose of comparison, we also estimate a two-economy term structure model without unspanned macro factors. Figure 3 displays the "in-two-years-for-one-year" forward term premia imlied by the model without macro factors. "in-two-years-for-one-year" forward term premia for five markets. The figure indicates that there are systematic differences between the two "in-two-years-for-one-year" forward term premia. Indeed, the risk premia from the model without macro factors appears to be acyclical.

# 5 Conclusions

In this paper, we propose a two-economy affine term structure model with unspanned macroeconomic risks (ATSM-X). The ATSM-X model has an $(m + n)$ factor structure and can simultaneously model the term structure of interest rates from different countries, as well as the exchange rates between them. Importantly, the pricing kernel implied by the ATSM-X model has two orthogonal components: a component driven by $m$ term structure factors and a component driven by $n$ unspanned macro factors. As such, both currency risk premia and bond risk premia are driven by $m$ term structure factors and $n$ unspanned macro factors. The affine model enables us to maintain internal consistency, but it explicitly accounts for the fact that a predominant portion of the exchange rate movement and bond excess return dynamics is independent of the movements in the cross section of interest rates in either country.

Our empirical analysis shows that the ATSM-X model can account for the paradoxical implications of the empirical failure of UIP and standard exchange rate models. Specifically, based on risk premia implied by the ATSM-X model, we show that the risk-adjusted Fama regression is able to generate a slope coefficient of one. In addition, the risk-adjusted Engel regression can recover a slope coefficient of zero, both for nominal and real interest rates and real interest rates. On the other hand, the ATSM-X model fits the term structure of interest rates in each country well. It also recovers the tent-shape of bond return predictability. Overall, these findings indicates the ATSM-X model generates a downward-sloping term structure forward currency risk premia and an appropriate term structure of bond risk premia.

We show the important role of unspanned risks in explaining the links between global macroeconomic fundamentals and the cross section of international interest rates and exchange rates. Indeed, currency and bond risk premia driven by term-structure factors are acyclical. In sharp contrast, currency and bond risk premia driven by unspanned macro factors are somewhat countercyclical, as most economic theories suggest. We view our results as suggestive for further research on the links between macroeconomic variables and exchange rates using modern asset pricing methods.

# Appendix A: Bond Pricing

The price of an zero coupon bond with maturity $T$ is given by

$$P(t,T) = E_t^Q\left(e^{-\int_t^T i_s ds}\right) = e^{\phi_0(T-t)+\phi_1(T-t)^\top P_t}\ , \tag{44}$$

where $\phi_0$ and $\phi_1$ satisfy the following system of Riccati equations

$$\begin{aligned}\partial\phi_0(\tau) &= \frac{1}{2}\phi_1^\top \Sigma_{PP}\Sigma_{PP}^\top \phi_1 + {K_{0P}^{Q}}^\top \phi_1 - a_0 \\ \partial\phi_1(\tau) &= {K_{1PP}^{Q}}^\top \phi_1 - a_1\end{aligned} \tag{45}$$

with the initial condition $\phi_0(0) = 0$, $\phi_1(0) = 0$. The annualized bond yield is thus given by

$$i(t, \tau) = A(\tau) + B(\tau)^\top P_t \tag{46}$$

with $A = -\phi_0(\tau)/\tau$ and $B = -\phi_0(\tau)/\tau$.

# Appendix B: A Canonical Study to ATSM-X

In this appendix, we study how many parameters can be identified at most by our model. And we will also show that the parameters for bonding pricing is irrelevant to the exchange rate estimation, which forms the crucial distinctiveness for our empirical research.

At first, let us show that given all the P-parameters and Q-measure parameters that are related to bond pricing, $K_{0P}^Q$, $K_{1PP}^Q$, $K_{0*}^*$ and $K_{1**}^*$, the exchange rate are still free to estimate. According to the SDE of exchange rates, we find that its dynamic is determined by the drift

$$\frac{1}{2}(\Lambda_Z^\top \Lambda_Z - \Lambda_Z^{*\top} \Lambda_Z^*) = (\Lambda_Z - \Lambda_Z^*)^\top \Lambda_Z - \frac{1}{2}(\Lambda_Z - \Lambda_Z^*)^\top (\Lambda_Z - \Lambda_Z^*) \tag{47}$$

and the diffusion parameters

$$(\Lambda_Z - \Lambda_Z^*)^\top \tag{48}$$

with $Z_t$ to be an $2 * n + m$ vector. According to our simplification setup, we assume $\Lambda_Z - \Lambda_Z^*$ to be a vector of constants, thus $\Lambda_Z - \Lambda_Z^*$ can be written as follows

$$\Lambda_Z - \Lambda_Z^* = \Sigma_Z^{-1} \begin{bmatrix} K_{0P}^* - K_{0P}^Q \\ K_{0*}^* - K_{0*}^Q \\ K_{0M}^* - K_{0M}^Q \end{bmatrix}. \tag{49}$$

One can immediately finds that even $K_{0P}^Q$ and $K_{0*}^*$ is given, diffusion parameters $\Lambda_Z -$

$\Lambda_Z^*$ are still free to estimate because $K_{0P}^*$ and $K_{0*}^Q$ are not restricted. For the diffusion term of the exchange rate, it can be split into a constant

$$(\Lambda_Z - \Lambda_Z^*)^\top \Sigma_Z^{-1}(\mu^P - \mu^Q) - (\Lambda_Z - \Lambda_Z^*)^\top (\Lambda_Z - \Lambda_Z^*) \tag{50}$$

and a linear combination of the latent factor $Z_t$

$$(\Lambda_Z - \Lambda_Z^*)^\top \Sigma_Z^{-1} \begin{bmatrix} K_{1PP}^Q & 0 & 0 \\ K_{1*P}^Q & K_{1**}^Q & 0 \\ K_{1MP}^Q & K_{1M*}^Q & K_{1MM}^Q \end{bmatrix} Z_t. \tag{51}$$

Since the constant (50) is determined by 2 additional constants parameters $K_{0M}^*$ and $K_{0M}^Q$, even assuming $\Lambda_Z - \Lambda_Z^*$ is given, the constant (50) is still free to estimate. Furthermore, for the linear combination of $Z_t$, since parameters $K_{1MP}^Q$, $K_{1M*}^Q$ and $K_{1MM}^Q$ are unrestricted, this linear combination is also free to estimate. Thus all the P-parameters and Q-measure parameters that are related to bond pricing, $K_{0P}^Q$, $K_{1PP}^Q$, $K_{0*}^*$ and $K_{1**}^*$ are irrelevant to exchange rate distribution. Furthermore, since the exchange rate SDE in our setup is affine, we can simply rewrite it as

$$ds_t = (i_t - i_t^* + a + b^\top Z_t)dt + \Sigma_s^\top dW_t \tag{52}$$

with $a$ to be a scalar, $b$ to be a $(2n + m) \times 1$ vector of constants and $\Sigma_s$ to be a $(2n + m) \times 1$ vector of constants. $a$ $b$ and $\Sigma_s$ are all the parameters we can identify except the P-parameters and $K_{0P}^Q$, $K_{1PP}^Q$, $K_{0*}^*$ and $K_{1**}^*$.

The identification of the bond pricing part of our model is similar to Joslin, Priebsch, and Singleton (2014). This includes the identification of all P-measure parameters and Q-measure parameters that are related to bond pricing. According to Joslin, Singleton, and Zhu (2011) and Joslin, Priebsch, and Singleton (2014), all P parameters, including

$\Sigma_{PP}$ are identifiable. The Q-SDE related to domestic bond pricing is given by,

$$dP_t = (K_{0P}^Q + K_{1PP}^Q P_t)dt + \Sigma_{PP} dW_P^Q. \tag{53}$$

There are at most $n+1$ parameters are identifiable in the above Q-SDE: $k_\infty^Q$ for the long term mean and $\lambda^Q$ for the eigenvalues of $K_{1PP}^Q$. For simplicity, we assume the eigenvalues are real and distinct, therefore, $\lambda^Q$ is an $n \times 1$ vector of real numbers. For $Q^*$-parameters that price foreign bonds, the $Q^*$-SDE for foreign bonds is given by,

$$dP_t^* = (K_{0*}^* + K_{1**}^* P_t^*)dt + \Sigma_{**} dW_*^* \tag{54}$$

and except for $\Sigma_{**}$ (which is obtained when estimating P-SDE for the whole system), we can further identify the long term mean $k_\infty^*$ and a real and distinct vector $\lambda^*$ for the eigenvalues of $K_{1**}^*$. And these are all the parameters our model is able to identify.

# Appendix C: Estimation Details

In this section, we describe the details of our 3-step estimation.

At first, we estimate the P-parameters that determine the distribution of our factors $Z_t$. We use the notation $\theta_P$ to denote all P-parameters, $\phi(y; u, \Omega)$ to denote the density of the multi-variate normal distribution. Given the observable factors $Z_t$, the conditional density is given by,

$$p(Z|\theta_P) = \prod_{i=1}^{N} \phi\left(Z_n; E_{n-1}^P(Z_n|Z_{n-1}), V_{n-1}^P(Z_n|Z_{n-1})\right). \tag{55}$$

By maximizing the log likelihood function $log(p(Z|\theta_P))$, we estimate all the P-parameters.

The second step is to fit the government bond yield. Denote by $i^m(t,\tau)$ the model implied bond yield. We assume that the annulized interest rate yields $i(t,\tau)/\tau$ are observed with cross-sectionally i.i.d errors $\eta_t \sim \phi(0, \sigma_\eta)$,

$$i(t,\tau)/\tau = i^m(t,\tau)/\tau + \eta_t. \tag{56}$$

We further assume an invariant $\sigma_\eta$ across all the maturities with $\Sigma_\eta = diag(\sigma_\eta, \cdots, \sigma_\eta)$. The conditional density is therefore given by,

$$p(y|\Sigma_Z; k_\infty^Q, \lambda^Q, k_\infty^*, \lambda^*, \sigma_\eta) = \prod_{i=1}^{N} \phi\left(y_n; y_n^m, \Sigma_\eta\right). \tag{57}$$

By maximizing the log likelihood function $log(p(y|\Sigma_Z; k_\infty^Q, \lambda^Q, k_\infty^*, \lambda^*, \sigma_\eta))$, we estimate the bond pricing relevant parameters $K_{0P}^Q$, $K_{1PP}^Q$, $K_{0*}^*$ and $K_{1**}^*$.

The third step fits the model-implied depreciation rates to the exchange rate data. Given $\varepsilon_t = (\varepsilon_{1m}, \varepsilon_{3m}, \varepsilon_{6m}, \varepsilon_{1Y}, \varepsilon_{3Y}, \varepsilon_{10})^\top$ the depreciation rates of exchange rate such that

$$\varepsilon_T = s_{t+T} - s_t \tag{58}$$

with $T$ varies from 1 month upto 10 years. The expectation and variance of the depreciation rate is denoted by $E_t^P(\varepsilon_t)$ and $V_t^P(\varepsilon_t)$, which is derived from our affine term structure currency model, ASTM-X. The likelihood function can be written as,

$$p(s_+|s_-, Z; a, b, \Sigma_s) = \prod_{i=1}^{N} \phi\left(\varepsilon_n; E_t^P(\varepsilon_t), V_t^P(\varepsilon_t)\right). \tag{59}$$

Thus, parameters $a$ $b$ and $\Sigma_s$ can be estimated by maximizing the log of this likelihood function. Observe that since we divide our estimation procedure in 3 steps, the parameters are naturally divided into 3 groups. In each step, the parameters need to be estimated become even fewer, which again makes our estimation process parsimonious and robust.

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Table 1: Importance of Unspanned Macro Risks

| | US | | German | | | Japan | | | UK | | | Canada | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| maturities | PC | PC+M | PC | PC+M | PC+M+USPC | PC | PC+M | PC+M+USPC | PC | PC+M | PC+M+USPC | PC | PC+M | PC+M+USPC |
| Panel A | | | | | | | $R^2$ of one-year bond excess returns | | | | | | | |
| 2y | 23.89% | 31.69% | 7.18% | 21.58% | 40.45% | 30.43% | 35.31% | 48.74% | 8.32% | 25.90% | 41.80% | 16.64% | 32.35% | 40.24% |
| 5y | 17.66% | 24.40% | 12.37% | 22.21% | 47.39% | 32.55% | 36.45% | 47.14% | 19.53% | 31.62% | 39.54% | 14.95% | 20.87% | 23.83% |
| 7y | 21.47% | 27.35% | 14.82% | 22.57% | 48.33% | 28.87% | 32.16% | 41.75% | 19.30% | 29.23% | 34.44% | 14.98% | 19.18% | 20.44% |
| 10y | 27.61% | 33.25% | 17.82% | 23.60% | 47.46% | 21.29% | 23.57% | 31.91% | 13.75% | 22.50% | 26.17% | 12.93% | 17.95% | 18.64% |
| Panel B | | | | | | | $R^2$ of one-month currency excess returns | | | | | | | |
| | | | PC+dPC | | PC+dPC+M+dM | PC+dPC | | PC+dPC+M+dM | PC+dPC | | PC+dPC+M+dM | PC+dPC | | PC+dPC+M+dM |
| 2y | | | 1.76% | | 6.89% | 3.50% | | 7.21% | 4.29% | | 13.80% | 2.64% | | 14.78% |
| 5y | | | 6.18% | | 11.80% | 0.48% | | 2.14% | 4.95% | | 13.08% | 1.91% | | 15.59% |
| 7y | | | 4.63% | | 8.79% | 10.35% | | 15.87% | 7.54% | | 11.03% | 7.47% | | 20.91% |
| 10y | | | 3.24% | | 4.68% | 2.49% | | 9.65% | 7.00% | | 11.85% | 7.65% | | 11.32% |
| | | | | | | | $R^2$ of six-months currency excess returns | | | | | | | |
| 2y | | | 8.04% | | 16.80% | 15.46% | | 18.85% | 19.06% | | 23.23% | 36.24% | | 59.14% |
| 5y | | | 8.42% | | 17.53% | 12.72% | | 15.61% | 17.36% | | 32.88% | 24.57% | | 46.69% |
| 7y | | | 15.83% | | 21.07% | 25.97% | | 31.92% | 33.51% | | 40.95% | 40.01% | | 60.14% |
| 10y | | | 17.65% | | 24.29% | 9.73% | | 20.90% | 41.17% | | 44.84% | 11.19% | | 33.55% |
| | | | | | | | $R^2$ of one-year currency excess returns | | | | | | | |
| 2y | | | 10.87% | | 20.97% | 38.41% | | 46.59% | 19.92% | | 22.79% | 67.33% | | 74.17% |
| 5y | | | 17.61% | | 31.84% | 31.95% | | 34.67% | 29.28% | | 36.32% | 51.76% | | 69.14% |
| 7y | | | 27.26% | | 30.47% | 36.95% | | 41.28% | 47.91% | | 57.77% | 57.73% | | 69.38% |
| 10y | | | 34.40% | | 42.57% | 16.42% | | 27.22% | 63.74% | | 67.07% | 9.21% | | 23.45% |
| Panel C | | | | | | | $R^2$ of one-month exchange rate excess returns | | | | | | | |
| 2y | | | 1.31% | | 6.08% | 2.43% | | 6.04% | 4.75% | | 14.11% | 1.50% | | 13.66% |
| 5y | | | 6.27% | | 11.94% | 0.40% | | 2.09% | 4.82% | | 13.60% | 1.78% | | 15.59% |
| 7y | | | 4.29% | | 8.64% | 9.31% | | 14.64% | 6.63% | | 10.31% | 7.76% | | 20.96% |
| 10y | | | 2.42% | | 3.88% | 2.61% | | 10.07% | 6.22% | | 11.22% | 7.96% | | 11.58% |
| | | | | | | | $R^2$ of six-month exchange rate excess returns | | | | | | | |
| 2y | | | 5.08% | | 12.45% | 10.87% | | 14.08% | 20.40% | | 24.80% | 20.40% | | 24.80% |
| 5y | | | 9.09% | | 18.22% | 11.81% | | 14.66% | 16.30% | | 33.37% | 16.30% | | 33.37% |
| 7y | | | 14.10% | | 19.51% | 23.73% | | 29.23% | 30.13% | | 37.28% | 30.13% | | 37.28% |
| 10y | | | 14.21% | | 20.95% | 10.36% | | 21.99% | 35.88% | | 39.36% | 35.88% | | 39.36% |
| | | | | | | | $R^2$ of one-year currency excess returns | | | | | | | |
| 2y | | | 4.98% | | 15.32% | 30.65% | | 40.05% | 22.45% | | 25.54% | 67.49% | | 73.44% |
| 5y | | | 20.29% | | 34.34% | 31.01% | | 33.61% | 30.42% | | 39.48% | 51.46% | | 68.91% |
| 7y | | | 23.70% | | 27.57% | 36.10% | | 39.60% | 45.95% | | 56.06% | 57.59% | | 67.34% |
| 10y | | | 28.81% | | 37.52% | 16.87% | | 28.76% | 56.27% | | 59.59% | 10.95% | | 24.71% |

Notes: The table reports $R^2$ from regressing bond excess returns (Panel A), currency excess returns (Panel B and Panel C) on the principal components (PC), changes in principal components (dPC), macro variables (M) and changes in macro variables (dM). Bond excess return is calculated over one month. Currency excess return is calculated for one month, six months and one year. The PC group of variables contain first three principal components of yield for each country. Macro variables are inflation and industrial production growth rate.

Table 2: Summary statistics of interest rates

| | Mean | Median | Std Dev | Skew | Kurt | Min | Max |
|---|---|---|---|---|---|---|---|
| U.S. | | | | | | | |
| 6m | 4.1371 | 4.7200 | 2.7859 | 0.0570 | -0.8953 | 0.0400 | 10.6100 |
| 1y | 4.5101 | 4.8797 | 2.9896 | 0.1046 | -0.8220 | 0.0994 | 11.9854 |
| 5y | 5.4740 | 5.4064 | 2.8544 | 0.3313 | -0.3930 | 0.6273 | 13.2869 |
| 10y | 6.1544 | 5.8634 | 2.5498 | 0.4931 | -0.1686 | 1.5522 | 13.5680 |
| Canada | | | | | | | |
| 6m | 4.6386 | 4.1677 | 3.2726 | 0.6947 | -0.3504 | 0.2536 | 13.0732 |
| 1y | 4.7355 | 4.1802 | 3.1714 | 0.5674 | -0.5936 | 0.4107 | 12.8582 |
| 5y | 5.4043 | 5.1311 | 2.8078 | 0.2456 | -0.9992 | 0.6482 | 11.8613 |
| 10y | 5.8611 | 5.4296 | 2.6201 | 0.1875 | -1.1487 | 1.2577 | 11.0873 |
| German | | | | | | | |
| 6m | 3.8014 | 3.6450 | 2.5449 | 0.3103 | -0.4705 | -0.2600 | 9.6300 |
| 1y | 3.8761 | 3.7900 | 2.5188 | 0.2341 | -0.5358 | -0.2700 | 9.4700 |
| 5y | 4.6722 | 4.7800 | 2.3980 | -0.1443 | -0.6994 | -0.1200 | 9.2400 |
| 10y | 5.2463 | 5.2900 | 2.1529 | -0.2940 | -0.7189 | 0.2200 | 9.2300 |
| Japan | | | | | | | |
| 1y | 2.0019 | 0.4905 | 2.5069 | 0.9693 | -0.6164 | -0.0200 | 8.5540 |
| 5y | 2.5329 | 1.2155 | 2.4261 | 0.7992 | -0.8040 | 0.0260 | 8.2490 |
| 10y | 3.0008 | 1.7405 | 2.2895 | 0.6814 | -0.9567 | 0.2250 | 8.0590 |
| UK | | | | | | | |
| 6m | 5.8443 | 5.4967 | 3.8098 | 0.1975 | -0.7781 | 0.1951 | 14.0365 |
| 1y | 5.8295 | 5.5627 | 3.7118 | 0.1135 | -0.8599 | 0.1725 | 14.3113 |
| 5y | 6.2310 | 5.7003 | 3.2555 | 0.0697 | -1.1114 | 0.5721 | 12.9316 |
| 10y | 6.4676 | 5.2305 | 2.9579 | 0.2126 | -1.2420 | 1.3867 | 12.3681 |

Notes: The table presents descriptive statistics at different maturities for the U.S., Canada, German, Japan and UK, including mean, median, standard deviation etc. The sample period is from 1986:01 to 2015:02. The frequency is monthly.

Table 3: Summary statistics of Exchange rates and macro variables

| Panel A: Exchange rates | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Median | Std Dev | Skew | Kurt | Min | Max |
| Canada | 1.2726 | 1.3019 | 0.1878 | -0.0686 | -1.2239 | 0.9553 | 1.5997 |
| German | 1.8440 | 1.7901 | 0.3852 | 1.3839 | 2.3968 | 1.2411 | 3.3025 |
| Japan | 0.0081 | 0.0084 | 0.0018 | -0.5962 | 0.2775 | 0.0038 | 0.0122 |
| U.K. | 1.6298 | 1.6084 | 0.1794 | 0.1764 | 0.0208 | 1.0931 | 2.0701 |

| Panel B: Macro variables | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Median | Std Dev | Skew | Kurt | Min | Max |
| U.S. | | | | | | | |
| Inflation % | 0.0010 | 0.0010 | 0.0011 | -1.4871 | 11.5979 | -0.0078 | 0.0059 |
| IPI % | 0.0009 | 0.0011 | 0.0027 | -1.4838 | 8.8299 | -0.0189 | 0.0088 |
| | | | | | | | |
| Canada | | | | | | | |
| Inflation % | 0.0013 | 0.0014 | 0.0058 | -0.9459 | 2.4412 | -0.0238 | 0.0155 |
| IPI % | 0.0015 | 0.0015 | 0.0021 | -0.0905 | 1.5155 | -0.0070 | 0.0082 |
| | | | | | | | |
| German | | | | | | | |
| Inflation % | 0.0007 | 0.0005 | 0.0013 | 0.9537 | 3.1431 | -0.0033 | 0.0075 |
| IPI % | 0.0007 | 0.0010 | 0.0077 | -0.1576 | 8.9077 | -0.0432 | 0.0504 |
| | | | | | | | |
| Japan | | | | | | | |
| Inflation % | 0.0002 | 0.0000 | 0.0018 | 0.8647 | 2.4210 | -0.0047 | 0.0089 |
| IPI % | 0.0005 | 0.0012 | 0.0070 | -1.5149 | 7.1652 | -0.0380 | 0.0185 |
| | | | | | | | |
| U.K. | | | | | | | |
| Inflation % | 0.0011 | 0.0013 | 0.0019 | 1.1499 | 8.3257 | -0.0042 | 0.0144 |
| IPI % | 0.0003 | 0.0005 | 0.0045 | -0.5526 | 2.0651 | -0.0215 | 0.0140 |

Notes: The table reports exchange rates (using the U.S. as the home country) and macro variables including inflation rate and industrial production growth rate. The sample period is from 1986:01 to 2015:02. The frequency is monthly. Panel A reports the exchange rates for Japan, Canada, German and UK. Panel B reports the inflation rate and industrial production growth rate for the U.S., Japan, Canada, German and UK.

Table 4: Fama regressions

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | 0.0104 | 0.0324 | 0.0700 | 0.1364 | 0.2087 | -0.1290 |
| se($\alpha$) | (0.0028) | (0.0059) | (0.0091) | (0.0127) | (0.0327) | (0.0597) |
| $\beta$ | -3.5192*** | -3.5839*** | -3.5169*** | -3.3396*** | -1.0743*** | 1.1249*** |
| se($\beta$) | (1.0575) | (0.7300) | (0.5146) | (0.3593) | (0.3019) | (0.1747) |
| t[$\beta = 1$] | -4.2736 | -6.2793 | -8.778 | -12.0779 | -6.8697 | 0.7149 |
| $R^2$ | 0.0418 | 0.0867 | 0.1553 | 0.2538 | 0.0475 | 0.1403 |
| German | | | | | | |
| $\alpha$ | -0.0011 | -0.0032 | -0.0051 | -0.0076 | -0.003 | 0.1169 |
| se($\alpha$) | (0.0017) | (0.0036) | (0.0054) | (0.0084) | (0.0133) | (0.0180) |
| $\beta$ | -0.4216 | -0.4553 | -0.6588 | -1.1244*** | -2.0889*** | -1.4841*** |
| se($\beta$) | (0.8247) | (0.5800) | (0.4217) | (0.3427) | (0.2136) | (0.1113) |
| t[$\beta = 1$] | -1.7238 | -2.5090 | -3.9334 | -6.199 | -14.4642 | -22.3217 |
| $R^2$ | 0.0010 | 0.0024 | 0.0095 | 0.0407 | 0.2736 | 0.4118 |
| UK | | | | | | |
| $\alpha$ | -0.0045 | -0.0124 | -0.0111 | -0.0033 | 0.1048 | 0.1029 |
| se($\alpha$) | (0.0036) | (0.0072) | (0.0093) | (0.0129) | (0.0152) | (0.0147) |
| $\beta$ | -2.1194* | -2.0384** | -1.1397* | -0.4496 | 1.5149*** | 0.5991*** |
| se($\beta$) | (1.2702) | (0.8593) | (0.5836) | (0.4513) | (0.2350) | (0.1008) |
| t[$\beta = 1$] | -2.4558 | -3.536 | -3.6663 | -3.2123 | 2.1916 | -3.9754 |
| $R^2$ | 0.0153 | 0.0305 | 0.0209 | 0.0055 | 0.1885 | 0.1647 |
| Canada | | | | | | |
| $\alpha$ | -0.0023 | -0.0072 | -0.014 | -0.0223 | -0.0127 | 0.2288 |
| se($\alpha$) | (0.0010) | (0.0019) | (0.0024) | (0.0041) | (0.0098) | (0.0172) |
| $\beta$ | 0.0901 | -0.0875 | 0.1897 | 0.1702 | 0.9438*** | 1.5685*** |
| se($\beta$) | (0.7706) | (0.4972) | (0.3300) | (0.2963) | (0.2795) | (0.1581) |
| t[$\beta = 1$] | -1.1808 | -2.187 | -2.4554 | -2.8005 | -0.2012 | 3.5957 |
| $R^2$ | 0.0001 | 0.0002 | 0.0025 | 0.0025 | 0.0795 | 0.4272 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $s_{t+T} - s_t = \alpha + \beta(y(t,T) - y^*(t,T)) + \varepsilon_{t,T}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 5: Engel regressions using nominal interest rates

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | 0.0111 | 0.0117 | 0.0110 | 0.0077 | -0.0028 | -0.0061 |
| se($\alpha$) | (0.0028) | (0.0028) | (0.0028) | (0.0029) | (0.0028) | (0.0027) |
| $\beta$ | 4.7983*** | 5.1463*** | 4.6031*** | 3.223*** | -1.0163 | -2.2274** |
| se($\beta$) | (1.0523) | (1.0457) | (1.0567) | (1.0764) | (1.0526) | (1.0053) |
| $R^2$ | 0.0757 | 0.0870 | 0.0695 | 0.0341 | 0.0037 | 0.019 |
| German | | | | | | |
| $\alpha$ | -0.0010 | -0.0011 | -0.0009 | -0.0012 | -0.0005 | 0.0012 |
| se($\alpha$) | (0.0017) | (0.0017) | (0.0017) | (0.0017) | (0.0016) | (0.0015) |
| $\beta$ | 1.4479* | 1.3781* | 1.9152** | 2.0231** | 0.7567 | 1.5158* |
| se($\beta$) | (0.8247) | (0.8248) | (0.8232) | (0.8195) | (0.7753) | (0.7276) |
| $R^2$ | 0.0120 | 0.0109 | 0.0209 | 0.0234 | 0.0037 | 0.0168 |
| UK | | | | | | |
| $\alpha$ | -0.0040 | -0.0029 | 0.0003 | 0.0027 | 0.0051 | 0.0055 |
| se($\alpha$) | (0.0036) | (0.0036) | (0.0037) | (0.0037) | (0.0033) | (0.0023) |
| $\beta$ | 2.9711** | 2.326* | 1.1296 | 0.1569 | -1.174 | -1.0846 |
| se($\beta$) | (1.2684) | (1.2721) | (1.2812) | (1.2783) | (1.1643) | (0.8123) |
| $R^2$ | 0.0297 | 0.0183 | 0.0043 | 0.0001 | 0.0056 | 0.0099 |
| Canada | | | | | | |
| $\alpha$ | -0.0023 | -0.0024 | -0.0019 | -0.0012 | 0.0012 | 0.0036 |
| se($\alpha$) | (0.0010) | (0.0010) | (0.0010) | (0.0011) | (0.0013) | (0.0018) |
| $\beta$ | 0.9258 | 0.8301 | -0.0004 | 0.7287 | -0.6131 | -0.3701 |
| se($\beta$) | (0.7705) | (0.7679) | (0.7847) | (0.8813) | (1.0013) | (1.4308) |
| $R^2$ | 0.0108 | 0.0088 | 0.0000 | 0.0052 | 0.0028 | 0.0005 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $\rho_{t+T+1} = \alpha + \beta(y_t^* - y_t) + \varepsilon_{t,T+1}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 6: Engel regressions using real interest rates

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | 0.0006 | 0.0014 | 0.0009 | 0.0006 | -0.0009 | -0.0013 |
| se($\alpha$) | (0.0018) | (0.0018) | (0.0018) | (0.0018) | (0.0017) | (0.0017) |
| $\beta$ | -0.1371 | 0.2353** | -0.1351 | -0.105 | -0.0851 | 0.0431 |
| se($\beta$) | (0.1171) | (0.1165) | (0.1172) | (0.1173) | (0.1130) | (0.1089) |
| $R^2$ | 0.0054 | 0.0158 | 0.0052 | 0.0031 | 0.0022 | 0.0006 |
| German | | | | | | |
| $\alpha$ | -0.0015 | -0.0016 | -0.0014 | -0.0018 | -0.0007 | 0.0007 |
| se($\alpha$) | (0.0017) | (0.0017) | (0.0017) | (0.0017) | (0.0016) | (0.0015) |
| $\beta$ | 0.1309 | 0.374** | -0.3549** | -0.0372 | -0.1759 | 0.0526 |
| se($\beta$) | (0.1674) | (0.1659) | (0.1666) | (0.1675) | (0.1565) | (0.1482) |
| $R^2$ | 0.0024 | 0.0196 | 0.0176 | 0.0002 | 0.0049 | 0.0005 |
| UK | | | | | | |
| $\alpha$ | 0.0028 | 0.0027 | 0.0031 | 0.0028 | 0.0025 | 0.0030 |
| se($\alpha$) | (0.0021) | (0.0021) | (0.0021) | (0.0021) | (0.0019) | (0.0013) |
| $\beta$ | 0.0967 | -0.0322 | -0.0567 | 0.1124 | -0.0879 | -0.0122 |
| se($\beta$) | (0.1220) | (0.1218) | (0.1218) | (0.1210) | (0.1106) | (0.0775) |
| $R^2$ | 0.0035 | 0.0004 | 0.0012 | 0.0048 | 0.0035 | 0.0001 |
| Canada | | | | | | |
| $\alpha$ | -0.0025 | -0.0023 | -0.0017 | -0.0012 | 0.0009 | 0.0033 |
| se($\alpha$) | (0.0009) | (0.0010) | (0.0010) | (0.0011) | (0.0013) | (0.0018) |
| $\beta$ | 0.4186*** | 0.0687 | -0.1496 | 0.244* | 0.1323 | 0.2663 |
| se($\beta$) | (0.1216) | (0.1262) | (0.1279) | (0.1432) | (0.1639) | (0.2333) |
| $R^2$ | 0.0824 | 0.0022 | 0.0103 | 0.0215 | 0.0049 | 0.0098 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $\rho_{t+T+1} = \alpha + \beta(r_t^* - r_t) + \varepsilon_{t,T+1}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 7: Yield fitting and matching depreciation rates

Panel A: Yield fitting

| | 1m | 3m | 6m | 1Y | 2Y | 3Y | 5Y | 10Y |
|---|---|---|---|---|---|---|---|---|
| US | 4.3453 | 2.1827 | 0.6583 | 3.8098 | 5.3072 | 3.3456 | 4.0298 | 5.6131 |
| Japan | - | - | - | 4.4908 | 4.2038 | 5.8243 | 4.1309 | 6.6907 |
| German | - | - | 3.0320 | 4.4474 | 2.6058 | 2.1050 | 3.4222 | 3.5110 |
| UK | 11.5954 | 8.6758 | 17.9456 | 7.9036 | 5.8675 | 4.0877 | 7.2056 | 10.6627 |
| Canada | 5.1180 | 3.0693 | 0.5773 | 3.5534 | 6.8897 | 6.0244 | 2.7867 | 5.8242 |

Panel B: Matching depreciation rates

| | $\alpha$ | se($\alpha$) | $\beta$ | se($\beta$) | $R^2$ |
|---|---|---|---|---|---|
| Japan | -0.0005 | 0.0002 | 0.9794 | 0.0088 | 0.9743 |
| German | -0.0005 | 0.0002 | 0.9805 | 0.0083 | 0.9768 |
| UK | -0.0007 | 0.0004 | 0.9345 | 0.0149 | 0.9389 |
| Canada | -0.0002 | 0.0002 | 0.9458 | 0.0129 | 0.9625 |

Notes: The table reports yield fittings for five counties as well as results for how well model-implied depreciation rate match observed rates. Panel A reports annualized root mean-squared errors in basis points for different yield maturities indicated in the header. Panel B reports correlations of model-implied and observed rates, while $\alpha$ denotes the intercept, $\beta$ denotes the slope coefficient, and se(.) denotes the standard errors in parentheses. $R^2$ is the in-sample coefficient of determination.

Table 8: Adjusted Fama regressions

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | -0.0011 | -0.0041 | -0.0094 | -0.0173 | -0.0173 | -0.0299 |
| se($\alpha$) | (0.0020) | (0.0041) | (0.0058) | (0.0072) | (0.0113) | (0.0133) |
| $\beta$ | 0.8734*** | 0.972*** | 1.0818*** | 1.1676*** | 1.0972*** | 1.1275*** |
| se($\beta$) | (0.2121) | (0.1630) | (0.1244) | (0.0847) | (0.0583) | (0.0399) |
| t[$\beta = 1$] | -0.5972 | -0.1717 | 0.658 | 1.979 | 1.6683 | 3.1968 |
| $R^2$ | 0.0626 | 0.1228 | 0.2295 | 0.4279 | 0.5826 | 0.7588 |
| German | | | | | | |
| $\alpha$ | 0.0004 | 0.0002 | 0.0005 | 0.0072 | 0.0029 | 0.0109 |
| se($\alpha$) | (0.0016) | (0.0035) | (0.0053) | (0.0078) | (0.0111) | (0.0141) |
| $\beta$ | 1.2934*** | 1.0089*** | 1.0198*** | 1.4715*** | 1.6016*** | 1.2643*** |
| se($\beta$) | (0.2068) | (0.2158) | (0.2044) | (0.1750) | (0.1079) | (0.0817) |
| t[$\beta = 1$] | 1.4185 | 0.0414 | 0.0966 | 2.6936 | 5.5746 | 3.2335 |
| $R^2$ | 0.1334 | 0.0792 | 0.0892 | 0.2177 | 0.4644 | 0.4851 |
| UK | | | | | | |
| $\alpha$ | -0.0007 | -0.0046 | -0.0122 | -0.0173 | 0.0063 | 0.0008 |
| se($\alpha$) | (0.0021) | (0.0038) | (0.0046) | (0.0051) | (0.0067) | (0.0071) |
| $\beta$ | 0.715*** | 0.956*** | 1.1120*** | 1.1719*** | 1.0418*** | 0.9696*** |
| se($\beta$) | (0.1421) | (0.1012) | (0.0752) | (0.0562) | (0.0545) | (0.0358) |
| t[$\beta = 1$] | -2.0053 | -0.4342 | 1.4894 | 3.0575 | 0.7679 | -0.8489 |
| $R^2$ | 0.1239 | 0.3325 | 0.5501 | 0.7082 | 0.6716 | 0.8041 |
| Canada | | | | | | |
| $\alpha$ | -0.0012 | -0.0037 | -0.0081 | -0.0067 | 0.0072 | -0.0182 |
| se($\alpha$) | (0.0011) | (0.0019) | (0.0023) | (0.0030) | (0.0043) | (0.0084) |
| $\beta$ | 0.6110** | 0.6117*** | 0.5654*** | 0.8149*** | 0.9944*** | 1.0770*** |
| se($\beta$) | (0.2454) | (0.154) | (0.0969) | (0.0676) | (0.0401) | (0.0317) |
| t[$\beta = 1$] | -1.5847 | -2.5208 | -4.4849 | -2.7374 | -0.1402 | 2.4291 |
| $R^2$ | 0.0448 | 0.1067 | 0.205 | 0.5239 | 0.8233 | 0.8973 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $rx_{t,T} = \alpha + \beta E_t(rx_{t,T}) + \varepsilon_{t,T}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 9: Adjusted Engel regressions using nominal interest rates

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | 0.0007 | 0.002 | 0.0025 | 0.0018 | 0.0018 | -0.0051 |
| se($\alpha$) | (0.0028) | (0.0028) | (0.0028) | (0.0028) | (0.0027) | (0.0026) |
| $\beta$ | 1.0913*** | 1.1384*** | 1.1366*** | 1.1065*** | 1.0803*** | 0.7627*** |
| se($\beta$) | (0.0870) | (0.0867) | (0.0866) | (0.0868) | (0.0849) | (0.0823) |
| t[$\beta = 1$] | 1.0495 | 1.596 | 1.5766 | 1.2275 | 0.9463 | -2.8839 |
| $R^2$ | 0.3823 | 0.4041 | 0.4039 | 0.3904 | 0.3893 | 0.2527 |
| German | | | | | | |
| $\alpha$ | 0.0002 | 0.0000 | 0.0002 | -0.0001 | 0.0006 | -0.0001 |
| se($\alpha$) | (0.0016) | (0.0016) | (0.0016) | (0.0016) | (0.0015) | (0.0014) |
| $\beta$ | 1.0119*** | 1.009*** | 1.0442*** | 1.0587*** | 1.0228*** | 1.0611*** |
| se($\beta$) | 0.0639 | 0.0639 | 0.0639 | 0.0637 | 0.0599 | 0.0579 |
| t[$\beta = 1$] | 0.1859 | 0.1408 | 0.6917 | 0.9216 | 0.3812 | 1.0549 |
| $R^2$ | 0.4964 | 0.4956 | 0.5126 | 0.5207 | 0.5342 | 0.5690 |
| UK | | | | | | |
| $\alpha$ | -0.0062 | -0.0078 | -0.0080 | -0.0075 | 0.0016 | -0.0036 |
| se($\alpha$) | (0.0035) | (0.0035) | (0.0035) | (0.0035) | (0.0033) | (0.0024) |
| $\beta$ | 1.1465*** | 1.1852*** | 1.2018*** | 1.1814*** | 0.9192*** | 0.9401*** |
| se($\beta$) | (0.1023) | (0.1014) | (0.1018) | (0.1014) | (0.0951) | (0.0685) |
| t[$\beta = 1$] | 1.4313 | 1.8268 | 1.9825 | 1.7883 | -0.8499 | -0.8745 |
| $R^2$ | 0.4121 | 0.4329 | 0.4378 | 0.4311 | 0.3428 | 0.5131 |
| Canada | | | | | | |
| $\alpha$ | -0.0005 | -0.0008 | -0.0006 | -0.0006 | 0.0004 | 0.0024 |
| se($\alpha$) | (0.0010) | (0.0010) | (0.0010) | (0.0011) | (0.0012) | (0.0018) |
| $\beta$ | 0.9657*** | 0.9715*** | 0.9153*** | 1.0166*** | 1.0322*** | 0.7438*** |
| se($\beta$) | (0.0635) | (0.0633) | (0.0625) | (0.0686) | (0.0777) | (0.1150) |
| t[$\beta = 1$] | -0.5403 | -0.4507 | -1.3554 | 0.2416 | 0.4146 | -2.2283 |
| $R^2$ | 0.6365 | 0.6405 | 0.6188 | 0.6248 | 0.5718 | 0.2407 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $\rho_{t+T+1} = \alpha + \beta(y_t^* - y_t) + \varepsilon_{t,T+1}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 10: Adjusted Engel regressions real nominal interest rates

| | 1M | 3M | 6M | 1Y | 3Y | 10Y |
|---|---|---|---|---|---|---|
| Japan | | | | | | |
| $\alpha$ | -0.0016 | -0.0015 | -0.001 | -0.0008 | 0.0001 | 0.0013 |
| se($\alpha$) | 0.0017 | 0.0017 | 0.0017 | 0.0017 | 0.0017 | 0.0017 |
| $\beta$ | 0.9995*** | 1.0001*** | 0.9963*** | 1.0049*** | 1.0133*** | 1.0146*** |
| se($\beta$) | 0.0094 | 0.0094 | 0.0093 | 0.0093 | 0.0091 | 0.0089 |
| t[$\beta = 1$] | -0.0586 | 0.007 | -0.4007 | 0.5249 | 1.4644 | 1.6327 |
| $R^2$ | 0.9782 | 0.9783 | 0.9782 | 0.9786 | 0.98 | 0.9807 |
| German | | | | | | |
| $\alpha$ | 0.0001 | 0 | 0.0001 | -0.0003 | 0.0005 | -0.0003 |
| se($\alpha$) | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0015 | 0.0014 |
| $\beta$ | 0.9995*** | 1.0126*** | 0.9693*** | 0.9965*** | 0.9875*** | 0.9985*** |
| se($\beta$) | 0.0129 | 0.0129 | 0.0128 | 0.0129 | 0.0121 | 0.0117 |
| t[$\beta = 1$] | -0.0363 | 0.9816 | -2.4033 | -0.2749 | -1.0308 | -0.1241 |
| $R^2$ | 0.9593 | 0.9605 | 0.9577 | 0.9592 | 0.9634 | 0.9661 |
| UK | | | | | | |
| $\alpha$ | -0.0019 | -0.0029 | -0.0022 | -0.0024 | 0.0000 | -0.0048 |
| se($\alpha$) | 0.002 | 0.002 | 0.002 | 0.002 | 0.0018 | 0.0013 |
| $\beta$ | 0.9929*** | 1.0116*** | 0.9958*** | 0.9982*** | 0.9754*** | 0.9834*** |
| se($\beta$) | 0.0098 | 0.0097 | 0.0098 | 0.0097 | 0.0089 | 0.0064 |
| t[$\beta = 1$] | -0.7258 | 1.1943 | -0.4305 | -0.1825 | -2.7829 | -2.6049 |
| $R^2$ | 0.983 | 0.9839 | 0.9831 | 0.9833 | 0.9855 | 0.9925 |
| Canada | | | | | | |
| $\alpha$ | -0.0008 | -0.0009 | -0.0007 | -0.0007 | 0.0005 | 0.0021 |
| se($\alpha$) | 0.0009 | 0.001 | 0.0009 | 0.001 | 0.0012 | 0.0018 |
| $\beta$ | 1.0231*** | 1.0029*** | 0.9836*** | 1.0177*** | 1.003*** | 1.0152*** |
| se($\beta$) | 0.0102 | 0.0104 | 0.0102 | 0.0111 | 0.0127 | 0.0192 |
| t[$\beta = 1$] | 2.2634 | 0.2814 | -1.6017 | 1.5874 | 0.2363 | 0.7916 |
| $R^2$ | 0.987 | 0.9861 | 0.986 | 0.9844 | 0.9791 | 0.9551 |

Notes: The table shows the results from estimating, by ordinary least squares, the regression $\rho_{t+T+1} = \alpha + \beta(r_t^* - r_t) + \varepsilon_{t,T+1}$, for the horizons indicated in the column headers. Values in parentheses are standard errors. $t[\beta = 1]$ is the $t$-statistic for testing $\beta = 1$. $R^2$ is the in-sample coefficient of determination. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Figure 1: Tent-shape of Cochrane and Piazzesi (2005) factor

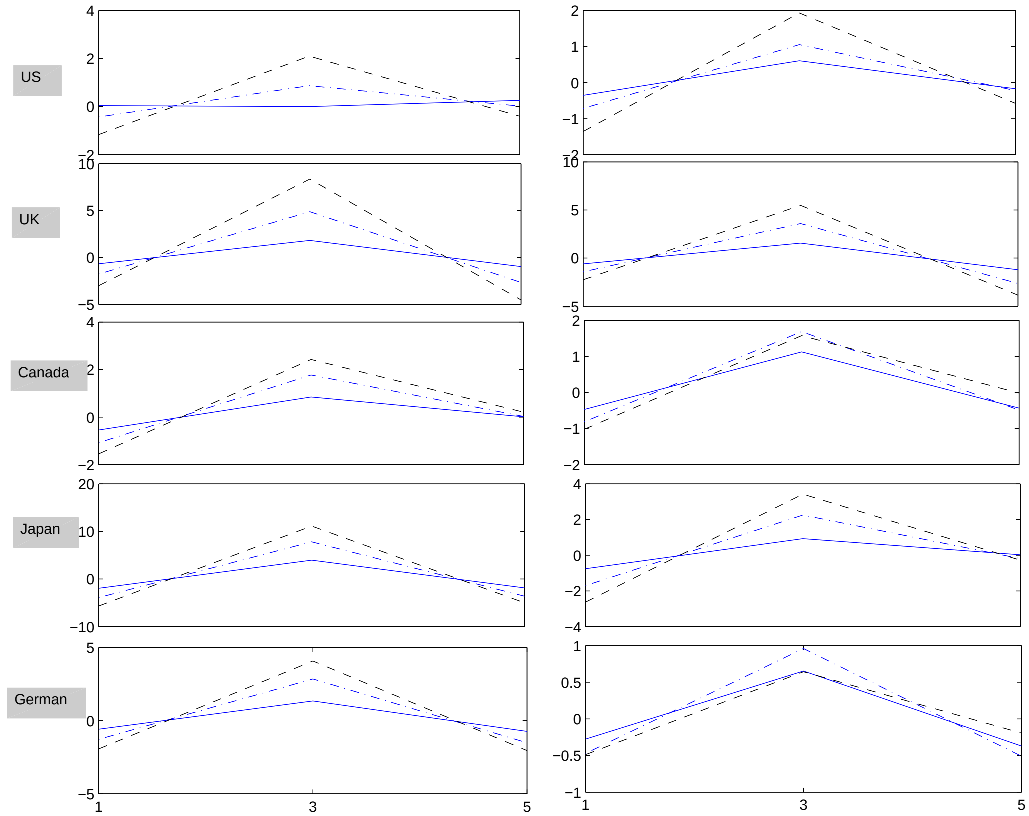

Figure 2: The decomposition of currency risk premium

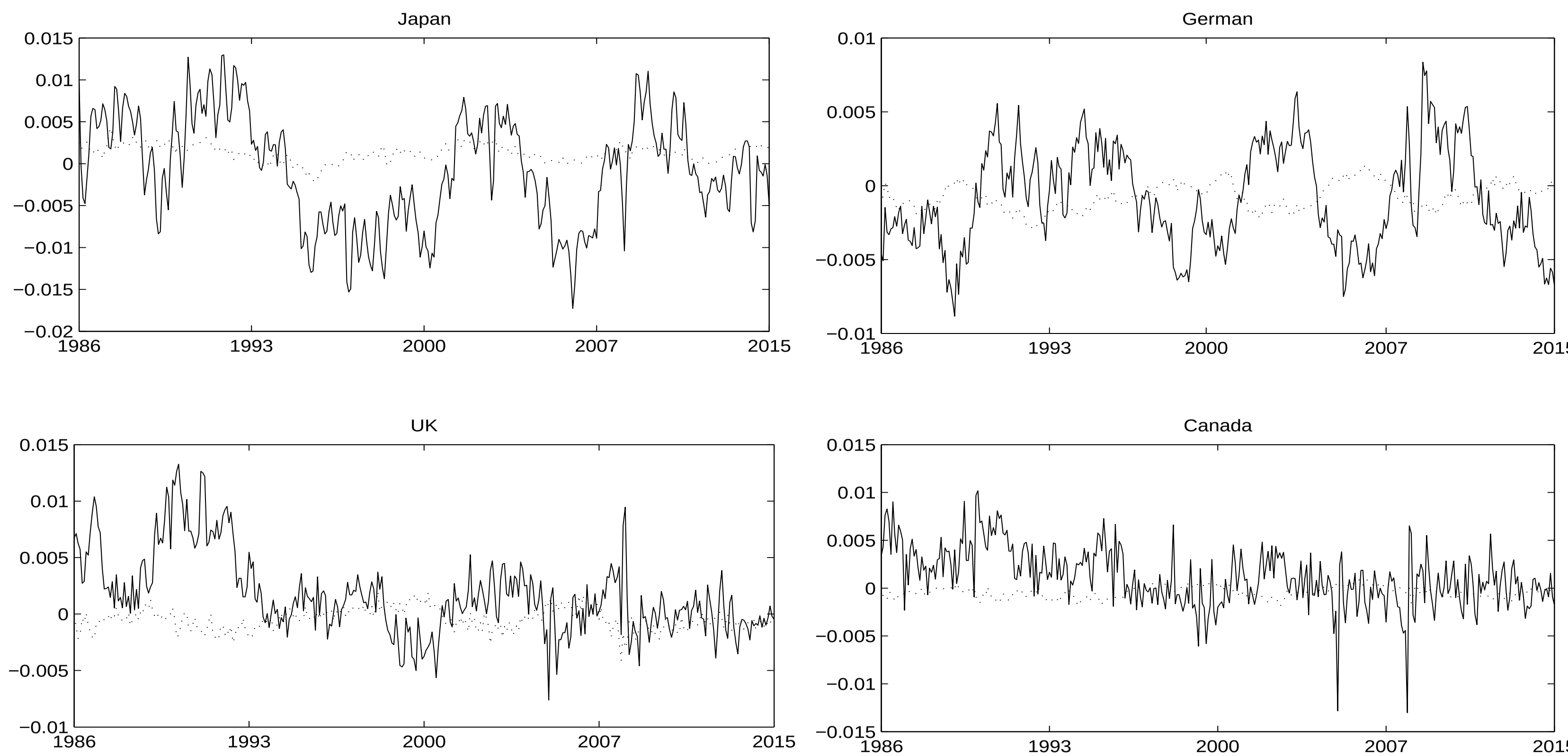

Figure 3: The decomposition of bond risk premium