Distress Dispersion and Systemic Risk in Networks*

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ABSTRACT

I develop a model of contagion that stems from endogenous risk-sharing when financial firms differ in distress levels. Firms face costly liquidation and strategically trade assets, thereby forming links. When firms are highly dispersed in financial distress, the network composition is distorted in two ways: it features too many links with distressed firms and too few risk-sharing links among non-distressed firms. The inefficiency arises from an externality when bilateral trading terms are not contingent on links faraway in the network. I also show empirical evidence that the distress dispersion across financial firms provides a novel indicator for systemic risk.

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1 Introduction

The interconnectedness of financial institutions is a key feature of the modern financial system. Linkages are formed by a diverse range of transactions and contracts that connect firms to each other. A growing literature identifies these linkages as a major source of systemic risk (e.g. Allen and Gale (2000), Duffie (2014), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015)). The insights are evident in the recent financial crisis: initial losses caused the financial distress of a few firms, which then spread via linkages to otherwise healthy firms, resulting in systemic failures. Yet, these studies analyze contagion in given network structures and do not consider firms' strategic behavior in forming links and choosing counterparties.

In this paper, I model endogenous linkage formation which allows firms to strategically build connections for profit and risk diversification. A recent literature examines linkage formation among homogeneous firms and concludes that either over- or under-connections prevail in the financial system.[1] In contrast, this paper studies the linkage formation among firms differing in financial distress levels. Such framework provides novel implications for efficiency and systemic risk by generating over- and under-connections simultaneously.

I show that the endogenously formed network features inefficiency and leads to systemic risk measured by the probability of joint failures. A link between two non-distressed firms creates value from risk-sharing, whereas a link with a distressed firm can be socially costly as it raises systemic risk through balance sheet interdependence. I find that, when firms are highly dispersed in financial distress, the *network composition* is distorted in two ways: there are too many links with distressed firms and too few risk-sharing links among non-distressed firms. The inefficiency arises as firms write bilateral contracts that are not contingent on the entire network structure. Hence, the non-distressed firms have incentives to link with distressed firms for profit, while failing to internalize negative spillovers. Such inefficient network generates contagion and loss in risk-sharing, creating excessive systemic risk. By embedding a new dimension of link heterogeneity, my model provides unique predictions on the efficiency of network composition.

[1] Castiglionesi and Navarro (2011) show that the decentralized network is under-connected when counterparty risk is high. Bramoulle and Kranton (2007) in a risk-sharing model show under-connection when links have positive externalities, but are costly for individuals to form. Farboodi (2015) illustrates over-connection in an endogenous core-periphery network. In Cabrales, Gottardi, and Vega-Redondo (2014), the coalition-proof risk-sharing network has asymmetric segmentation with over-connection.

In the model, financial firms face costly liquidation risks and strategically trade assets, thereby forming a network. There are a finite number of firms financed by short-term debt and each invests in a long-term asset. A random fraction of the asset is liquid and can be used to repay debt. If the amount of liquid asset falls short of the debt level, a costly liquidation is triggered.[2] To hedge the idiosyncratic liquidation risk, firms can strategically enter into bilateral forward contracts to trade liquid assets. A two-sided link in a network is formed when both parties trade a fraction of each other's liquid asset claims. Firms *differ ex ante* in how liquid their assets are expected to be. This generates the key feature of the model: cross-sectional heterogeneity in financial distress levels. Difference in asset liquidity also implies a price of trade in each contract. Motivated by the incomplete contract literature, I assume that prices in the bilateral trades are not contingent on the entire network structure. Specifically, I consider *local contingency*; that is, prices are contingent on which firms the two parties directly trade with. Given the network formed, the liquid asset holding of a firm depends not only on who its direct counterparties are, but rather on the entire network structure.[3] As a benchmark for efficiency, I solve for the optimal network that minimizes total bank liquidations.

The pairwise stable network formed in equilibrium can be inefficient relative to the optimal benchmark: there can be excess links with distressed firms and insufficient risk-sharing links among non-distressed firms. When distress dispersion is high across firms, the optimal network is such that the non-distressed firms form risk-sharing links and that the most distressed firm be isolated. In contrast, the equilibrium network with four or more firms shows that the distressed firm is always connected with the most liquid firm. This suboptimal link between the liquid and the distressed firm ("*distress link*" hereafter) transmits risky assets in the network and leads to systemic risk, measured by the risk of joint liquidation.

The inefficiency is caused by network externalities. Linking with a distressed firm potentially avoids liquidation, thus is *ex ante* profitable for the most liquid firm. However, when a firm is too distressed, linking with it can be socially costly because distressed assets are then shared jointly by all connected firms and so the balance sheets of other banks in the network are contaminated. This negative network externality in turn reduces risk-sharing participation

[2] A firm with a low level of liquid asset has difficulty in repaying short-term debt and hence is *distressed*.

[3] Following Cabrales, Gottardi, and Vega-Redondo (2014), I model this balance sheet interdependence as an iterative swap process which represents asset securitization.

among non-distressed firms. As such, two forces reinforce and lead to inefficiency: the transmission of distressed assets that should have been isolated and the insufficient risk-sharing among non-distressed firms.

The necessary ingredients for the mechanism are interconnectedness, heterogeneity, and local contingency. Interconnectedness transmits risky assets, thus enabling the spillover. Heterogeneity in financial distress generates different incentives to form links. When there are only two firms or multiple identical firms, there is no externality. However, when there are multiple firms differing in distress levels, the most liquid firm who acts like a gateway to link with the distressed firm can make profit while shifting risks to its direct and indirect counterparties. But interconnectedness and heterogeneity are not enough. The externalities fail to be internalized due to local contingency. Firms that bear the externalities cannot jointly give incentives to the liquid firm as long as one of the indirectly connected firms along the chain cannot make bilateral payments contingent on the distress link.

While the prior literature largely focuses on the average soundness of the financial sector,[4] my second primary result identifies a novel indicator for the level of network inefficiency: the distress dispersion across financial firms. In my model, inefficiency arises when the distress dispersion is sufficiently high and increases with the level of dispersion thereafter. This positive relation owes to the wedge between individual and social incentives to form a distress link. When distress dispersion is higher, the distressed gets more distressed and the liquid gets more liquid. It is precisely then that the most liquid firm has a stronger incentive to form the socially costly distress link.

Using insights from the model, I discuss policy implications for financial stability. The links with distressed firms in the model can be interpreted as acquisitions of distressed firms. This interpretation is reasonable because distressed financial firms are commonly acquired by healthier institutions in the same industry.[5] More than 1000 distressed financial firms were

[4] Atkeson, Eisfeldt, and Weill (2014) measure the median Distance to Insolvency of largest financial firms based on the Leland's model of credit risk. Gilchrist and Zakrajsek (2012) show that the average credit spreads on outstanding corporate bonds has predictive power for economic activity. Rampini and Viswanathan (2015) focus on how the net worth of a representative intermediary and a corporate sector jointly affect the cost of financing; a constant returns to scale assumption makes the distribution of intermediaries' net worth irrelevant.

[5] Acharya, Shin, and Yorulmazer (2010) argue that if a bank needs to restructure or be sold, the potential buyers are generally other banks. Almeida, Campello, and Hackbarth (2011) document that distressed firms are acquired by liquid firms in their industries for financial synergies. Such acquisitions are more likely when industry-level asset specificity is high and firm-level asset specificity is low, which applies to the financial sector.

acquired during 2000-2013, including Countrywide Financial and Riggs Bank.[6] Despite the fact that acquisitions are a prevailing regulatory approach to improve financial stability,[7] my findings imply that excess acquisitions may emerge precisely when more banks are distressed, thus increasing systemic risk rather than reducing failures. Based on this result, regulators can restore efficiency by supervising the acquisitions of distressed firms.

Finally, I provide empirical evidence that the distress dispersion across financial institutions provides a novel indicator for systemic risk. Following Laeven and Levine (2009), I measure distress by estimating Z-scores of financial firms. The time series of distress dispersion shows large variations over time. It also has a countercyclical pattern and appears to lead recessions. Consistent with the model predictions, the empirical dispersion series significantly comoves with future economic activities and systemic risk, bank failures, acquisitions of distressed firms, and interbank risk sharing. Moreover, I run forecasting regressions to evaluate whether the dispersion series conveys new information about aggregate indicators beyond what is contained in the average distress and existing systemic risk measures. The estimates confirm that the dispersion series has high predictive power for future systemic risk.

This paper builds on network theory and its applications in economics and finance.[8] Pioneered by Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000), a growing literature argues that certain financial network structures are prone to contagion.[9] While powerful for analyzing how risks propagate under different connection properties, this stream of research treats the network structures as given. My paper studies network formation, hence contributes to the analysis of how links evolve in response to changes in policies or aggregate conditions.

This paper studies how the interaction between distress heterogeneity and incomplete contracts affect linkage formation and systemic risk. As such, my paper belongs to the recent research on financial network formation with various frictions. Castiglionesi and Navarro (2011) illustrate network fragility when undercapitalized banks gamble with depositors' money. Gofman

[6]The asset size of these acquisitions was $2.2 trillion, about half the size of all current banking deposits.

[7]White and Yorulmazer (2014) provide a summary of resolution options for bank distress/failure. An acquisition "imposes the least cost since the franchise value is preserved, there is no disruption to the bank's customers or the payment system itself, and there are no fiscal costs." For this reason, acquisition is the primary choice by resolution authorities whenever there are willing acquirers.

[8]See surveys by Jackson (2003, 2008) and Allen and Babus (2009).

[9]See Eisenberg and Noe (2001), Dasgupta (2004), Nier, Yang, Yorulmazer, and Alentorn (2007), Gai, Haldane, and Kapadia (2011), Greenwood, Landier, and Thesmar (2015), Caballero and Simsek (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Elliott, Golub, and Jackson (2014), and Glasserman and Young (2015).

(2011) highlights that bargaining friction and intermediation lead to welfare loss. Zawadowski (2013) focuses on a type of risk shifting stemming from banks' underinsurance of counterparty risk. Di Maggio and Tahbaz-Salehi (2014) emphasize the role of secured interbank lending in overcoming moral hazard. Glode and Opp (2015) show that intermediation chains endogenously form in decentralized markets under asymmetric information.[10]

In the network formation literature, my paper is closest to Farboodi (2015) who shows that a core-periphery structure inefficiently arises from a lending constraint and the opportunity to earn intermediation rent. While my model also generates excessive systemic risk due to the formation of certain inefficient links, I differ by studying linkage formation among firms differing in financial distress. Inefficiency arises because liquid firms link with distressed firms for profit under contract incompleteness. Moreover, I model links on the asset side of the balance sheet. The asset interdependence structure can be used to regulate bank acquisitions. Finally, the finding that the distress dispersion is a critical state variable has implications in systemic risk forecast.

The key friction underlying the network inefficiency here is the failure to offer incentives contingent on the entire network structure. In this regard, my paper is related to the literature on incomplete contracts.[11] From Hart and Moore (1988), agents cannot write contracts contingent on states that cannot be clearly specified, even if the states are perfectly foreseeable. The reason is that the states written in the contracts must be verifiable in court. In my setting, given that the links entered by other firms are not specifiable or verifiable, bilateral prices are contingent only on who the two firms directly trade with. This assumption is in line with Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) who show that inefficient networks can emerge in interbank lending markets with contingency debt covenants.

Finally, this paper adds to the studies on the trade-off between diversification and contagion. Banal-Estanol, Ottaviani, and Winton (2013) evaluate the coinsurance benefit and the contagion cost in conglomerate mergers. I follow Cabrales, Gottardi, and Vega-Redondo (2014) and extend this trade-off in a network setting. Acharya (2009), Wagner (2010), and Ibragimov, Jaffee, and

[10]Related papers also include Lagunoff and Schreft (2001), Babus (2015), Blume, Easley, Kleinberg, Kleinberg, and Tardos (2013), Castiglionesi and Wagner (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014), Cabrales, Gottardi, and Vega-Redondo (2014), and Farboodi (2015).

[11]See for example Hart and Moore (1988, 1999), Tirole (1999), Maskin and Tirole (1999), and Segal (1999).

Walden (2011) show that diversification may lead to greater systemic risk as banks tend to over-diversify by holding similar portfolios. While these papers study costly joint failures among ex ante homogeneous agents, my paper complements by showing that links among heterogeneous firms can result in both *over* and *under* diversification.

2 Model

This section describes a model of network formation in which financial firms strategically trade assets via bilateral forward swap contracts.

2.1 Environment

Consider a four-date economy with a finite number of levered financial firms, denoted by $i = 1, ..., N$. All agents are risk neutral, have full information; and there is no discounting.

At date 0, each firm borrows 1 unit of short-term debt from a continuum of creditors and invests in an asset with fixed return R. Similarly to Diamond and Rajan (2011), the asset has delayed cash flow, thus generating liquidity risk. A random component a_i becomes liquid at date 2 and can be used to repay debt, whereas the rest $R - a_i$ is illiquid and matures at date 3. Given this financing structure, a maturity mismatch arises. A firm can be interpreted as a financial institution, e.g., an investment firm investing in a certain class of securities, or a commercial bank issuing an unsecured loan.

At date 1, the amount of liquid return firms expect to receive becomes public, given by exogenous vector ν. Then they simultaneously decide to enter into bilateral forward swap contracts for risk-sharing, thus forming links. Each forward swap contract promises a claim to a fraction of each other's liquid asset holdings. This is the only date a strategic decision is made.

At date 2, liquid return, given by $a_i = \nu_i + \sigma\varepsilon_i$, is available to use. The idiosyncratic shock ε_i is i.i.d. standard normal and is independent of ν_i.[12] Firms fulfill the forward swap contracts. Based on the overall linkage structure, firms obtain potentially diversified liquid asset holdings, which they use to repay short-term debt.[13] If the liquid asset holdings fall short of debt, the

[12] a_i being negative means that further liquidity input is needed in the asset investment.

[13] Introducing debt roll-over, renegotiation, or endogenous liquidation boundary do not change the qualitative features. To separate from risk-shifting due to agency conflict between shareholders and depositors (Jensen and Meckling (1976)), limited liability is not imposed to firm owners.

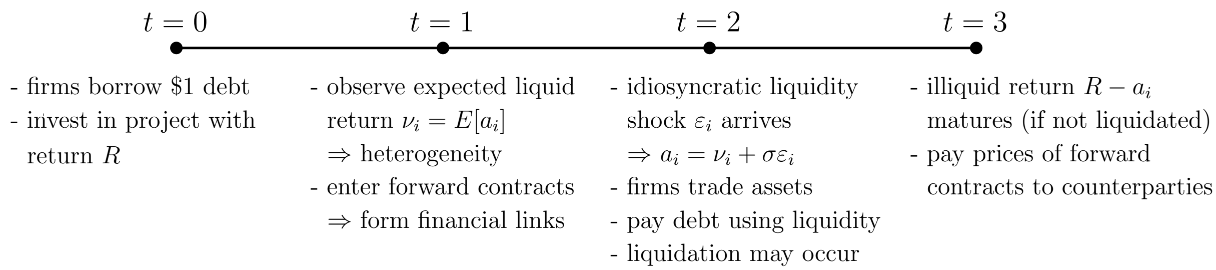


Figure 1. Model Timeline.

firm liquidates its illiquid asset at a fixed cost c, for instance due to disruption of service or by selling at a discount to industry outsiders as in Shleifer and Vishny (1992).[14]

At date 3, if not liquidated, the illiquid component of the asset return $R - a_i$ matures. Using this return, the payments associated with the forward swap contracts are paid in full.

Firms differ at date 1 in the amount of expected liquid return ν. This generates heterogeneity in financial distress. I follow Roy (1952) and define a distress statistic, z_i, as the number of standard deviations that firm i is expected to be away from liquidation ($z_i \equiv \frac{\nu_i - 1}{\sigma}$). A firm with high z_i has highly liquid asset and low financial distress, hence is *liquid*. In contrast, a firm is *distressed* if it has a low z_i. To highlight the role of heterogeneity, I adopt an equally-spaced vector ν such that z_i satisfies

$$z_i = \bar{z} + \frac{N + 1 - 2i}{2}\delta, \quad i = 1, ..., N. \tag{1}$$

z has mean $\bar{z}$, which measures the average distance from liquidation. Let $\bar{z} > 0$ so that firms invest in positive NPV projects on average. Moreover, z is equally spaced with step size $\delta \geq 0$, which proxies for the degree of distress dispersion. With such a structure for z, the first and second moments are sufficient to determine the distribution.[15]

2.2 Network Formation

At date 1, firms strategically enter into bilateral forward swap contracts. In this network formation game, each firm simultaneously announces price offerings to other firms and decides which

[14] Assume $R - c > 1$, so after liquidation, banks can always pay back their debt in full, which justifies the zero interest rate. James (1991) finds using US data 1985-1988 that substantial value is preserved if a failed bank is sold to another bank, but is lost if liquidated by the FDIC. The cost can result from deadweight loss in liquidation due to asset specificity, loss of franchise value, or disruption of credit and payment services associated with relationship banking (see White and Yorulmazer (2014)).

[15] I rank firms by z_i for expository purpose. Distress is modeled as exogenous, while in reality firms choose liquidity holding and risk-taking which endogenously determine distress levels. Acharya, Shin, and Yorulmazer (2010) argue that liquid banks hoard cash for potential gains from asset sales. This implies that an otherwise endogenous setting would generate even bigger heterogeneity during an aggregate liquidity shortage.

offers to accept. A *strategy* of firm i includes prices $p_{i\bullet} = (p_{i1}, ..., p_{i,i-1}, p_{i,i+1}, ..., p_{iN})$ and links $l_{i\bullet} = (l_{i1}, ..., l_{i,i-1}, l_{i,i+1}, ..., l_{iN})$: Firm i offers to purchase assets from j at a unit price p_{ij}, and accepts to sell $l_{ij} \in \{0, \bar{l}\}$ fraction of its liquid asset to firm j at date 2, where $\bar{l} \in (0, 1)$.[16] The bilateral prices can potentially be made contingent on other links in the network.

A contract is signed (a two-sided link is formed) when two firms decide to swap asset claims at the offered prices. Let the matrix L represent the linkage structure; its element satisfies

$$L_{ij} = L_{ji} = \min\{l_{ij}, l_{ji}\}. \tag{2}$$

Firms i and j are directly linked ($L_{ij} = \bar{l}$) only if $l_{ij} = l_{ji} = \bar{l}$. This specification ensures that no firms end up being a net asset seller or buyer so each firm still holds one unit of liquid asset. It also captures an important aspect of the OTC derivatives market: firms have large gross notional positions and small net positions. After the asset swaps, each firm holds a non-negative share of its own asset. As such, L is a symmetric, doubly stochastic matrix.[17] When $L_{ii} = 1$, firm i is isolated. The set of N firms and the links connecting them define the network. Depending on the distress levels of the two connecting firms, the network is composed of *risk-sharing links* which connect two non-distressed firms, and *distress links* which connect a liquid and a distressed firm.

2.3 Payoffs and Firm Value

Firms' liquid asset holdings, denoted by vector $h_i(a, L)$, depend on not only their direct counterparties, but rather how firms are interconnected. As such, the linkage creates interdependence on the *asset side* of firms' balance sheets. I model links via asset swaps because prior studies highlight that correlated portfolio exposures are the main source of systemic risk in the financial sector.[18] In addition, asset swaps simplify the calculation of final asset holdings and systemic risk by avoiding value discontinuities in cascades such as in Elliott, Golub, and Jackson (2014).[19]

At date 3, firms deliver payment transfers according to the forward swap contracts. Their

[16] From Lemma 1, under Assumption 2 all results would remain if instead firms have a continuum strategy space of $l_{ij} \in [0, 1)$.

[17] A square matrix is doubly stochastic if all its entries are non-negative and the sum of the entries in each of its rows or columns is 1.

[18] See for example Elsinger, Lehar, and Summer (2006) and DeYoung and Torna (2013).

[19] More generally, the asset swaps capture interregional cross-holdings of deposits in Allen and Gale (2000).

final payoffs are determined by the liquid return a, the network L, and the prices p,

$$\Pi_i(a, L, p) = \underbrace{h_i(a, L) + R - a_i - 1}_{\textit{asset net of debt}} - \underbrace{\mathbb{1}_{(h_i(a,L)<1)}c}_{\textit{liquidation cost}} - \underbrace{\sum_{j \neq i} (p_{ij} - p_{ji})\, L_{ij}}_{\textit{net payments from swaps}} . \tag{3}$$

Firm value at date 1 equals the expected value of $\Pi_i(a, L, p)$,

$$V_i(z, L, p) = \mathbb{E}_1\left[h_i(a, L)\right] + R - \nu_i - 1 - \Pr\left(h_i(a, L) < 1\right)c - \sum_{j \neq i} (p_{ij} - p_{ji})\, L_{ij}. \tag{4}$$

2.4 Bilateral Prices and Asset Swaps

The key features of a network formation game are the payoff functions and the payment transfers. To further specify these terms in my framework, I next impose assumptions on the bilateral prices and the asset swap process.

Local Contingency Who have the power to decide on a link between two firms is crucial to linkage formation. The bilateral prices allow for payments between counterparties, which in turn define the decision power to form links. Given that a link L_{ij} "alters the payoffs to others, it seems reasonable to suppose that other firms, especially the [direct counterparties of] i and j should have some say in the formation of a link between i and j" (Goyal (2009)). Following this spirit, I model contingent transfers: bilateral prices are set under local contingency.

Assumption 1 (Local Contingency) *Bilateral prices can be written contingent on the direct links entered by the two firms. Let L_i be the i-th row of L, then*

$$p_{ij}\left(L_i, L_j, L_k\right) = p_{ij}\left(L_i, L_j, \hat{L}_k\right), \quad \forall k, \forall \hat{L}_k \neq L_k. \tag{5}$$

Firm i offers prices p_{ij} contingent on its own links L_i and its direct counterparty's links L_j (i.e. the identities of j's counterparties). Full information guarantees that firms all know z, based on which they can foresee and *ex post* confirm the equilibrium network.[20] The restriction is that, even if firm i foresees that it indirectly connects to a third firm k, the price it offers cannot vary with the links of firm k.

[20] I focus on full information so that firms fully observe z and the links L. This environment abstracts from other potential frictions in the financial markets, such as private information, uncertainty, and limited commitment.

Assumption 1 is the key friction in the model. It is motivated by an inherent feature of the financial market: While firms try to impose contingencies that directly restrict counterparties' action, e.g. credit-risk-related contingent features in derivatives, deal-contingent derivatives, and debt covenants, it is extremely rare for firms to specify in every bilateral contract detailed contingencies on every possible network structure. One reason is that institutions do not publicly disclose the identities of their counterparties. As in Hart (1993), even if the bilateral relations they form could be foreseeable by other institutions, "they might be difficult to specify in advance in an unambiguous manner. [Hence], a contract that tries to condition on these variables may not be enforceable by a court." This is essentially an example of *incomplete contracts*.[21]

Price Offering Rule In each bilateral contract, what matters for firm payoffs is the net transfer payment $(p_{ij} - p_{ji}) L_{ij}$. The same net payment can be achieved by a continuum of gross payments. To ensure a unique set of equilibrium prices, I assume that prices are proposed as take-it-or-leave-it offers. The price offered is bounded below by seller's outside option when it cannot form any links. For each contingency, at least one price within a pair is set at the lower bound whereas the other may be offered at a premium according to the incentives needed for the counterparty to form or sever links. Formally,

$$\forall \{p_{ij}, p_{ji}\}, p_{ij} = V_j (z, L_j = 0), p_{ji} \geq V_i (z, L_i = 0). \tag{6}$$

This price offering rule grants the buyer with full bargaining power. Still, the fact that each firm in a swap is simultaneously a buyer and a seller together with the pairwise stability concept (to introduce later) imply that the price offering rule is not the basis for potential inefficiency.

Asset Swap Process The interdependence of liquid asset holdings is due to an iterative swap process: firms cannot trade a specific asset directly; rather, they swap assets with their direct counterparties iteratively. Given the linkage matrix L, the asset holdings after the first round of swap is $h^{(1)} = La$. Applying L to $h^{(1)}$ gives the second round, $h^{(2)} = Lh^{(1)} = L \times La = L^2 a$. It captures the securitization process such as the origination and trades of asset-backed securities.[22]

[21] An alternative motivation relates to transaction costs à la Williamson (1975). As the size and complexity of the network builds up, it would be prohibitively costly to include every possible structure in every contract by every firm. This is consistent with the fact that we do not observe such types of contracts in practice.

[22] This iterative process is instantaneous and does not affect the payment of prices. "These exchanges of assets can be viewed as reflecting a process of repeated rounds of securitization and trade of the assets of a financial

Specifically, I assume that the iteration goes on for infinitely many rounds.

Assumption 2 (Infinite Swap) *Firms swap liquid assets according to the linkage matrix L iteratively for infinite rounds. The final liquid asset holdings h are given by*

$$h(a, L) = \lim_{K \to \infty} L^K a. \tag{7}$$

Under Assumption 2, final holdings h depend on the liquid returns of both *direct and indirect* counterparties with *no decay.* For a network with $N = 3$ and $L_{12} = L_{23} = \bar{l}$, $L_{13} = 0$. After the first round, $h_1^{(1)} = \left(1 - \bar{l}\right) a_1 + \bar{l} a_2$. After infinite rounds, $h_1 = h_2 = h_3 = \frac{1}{3} a_1 + \frac{1}{3} a_2 + \frac{1}{3} a_3$; hence, firm 1 holds $\frac{1}{3}$ shares of a_3 even if it does not directly link with firm 3. The following lemma formalizes this property of the final asset holdings.

Lemma 1 (Complete risk-sharing) *$\lim_{K \to \infty} L^K$ is doubly stochastic and coincides with complete risk-sharing among all firms connected in the same component.*[23] *I.e. the holdings of each firm are equally weighted by the liquid assets of all firms directly or indirectly connected to it.*

Lemma 1 has three key implications. First, it is the linkage structure (whether L_{ij} is 0 or positive) rather than the amount of swap that determines the final holdings of each firm. As such, the results would still hold if instead $l_{ij} \in [0, 1)$, that is, if we allow firms to make linkage decisions in a continuum space. This rationalizes the simplification that l_{ij} is a binary variable. Second, this setup has the advantage that the optimal level of risk-sharing is achieved as long as the links are optimal, which allows me to focus on linkage formation.[24] Third, the same partition of components can be achieved by various linkage structures. To ensure the uniqueness of linkage formation, I introduce the following concept of robustness.

Assumption 3 (Pairwise Robustness) *Firm i will not accept to sell to firm j if there is negative bilateral surplus from linking in absence of all other links.*

If a link generates positive surplus to the two firms only when other links are present, then it is not robust to the formation or termination of the other links. As discussed later in Section 2.6, pairwise robustness implies that a highly distressed firm will only sell to a highly liquid firm.

firm in order to diversify its risks." (Cabrales, Gottardi, and Vega-Redondo (2014)).

[23] A component of a network is a maximal connected subset: each firm can reach any other firm in the same component following one or more links. Components provide a partition of firms.

[24] More details see Section 3.1. Bramoulle and Kranton (2007) has a similar assumption: players share risk repeatedly with neighbors and achieve complete risk-sharing after infinite rounds of interactions.

Finally, the holding of own asset $L_{ii} = 1 - \sum_{j \neq i} L_{ij} \geq 0$ implies that the maximum number of links a firm can form is $1/\bar{l}$. If $1/\bar{l}$ is not bounded above, the number of possible network structures is $2^{\frac{N(N-1)}{2}}$, which increases exponentially with N. To maintain tractability, from here onwards I follow a similar assumption in Allen, Babus, and Carletti (2012) and restrict the number of links a firm can form.

Assumption 4 (Chain Networks) *Each firm can form a maximum of two links, i.e.* $\bar{l} = \frac{1}{2}$.

Possible network topologies therefore limit to an arbitrary collection of chains and cycles.[25] Chains satisfy *minimality* whereas cycles do not. This is because from Lemma 1, the asset holdings from a cycle remain if one link is deleted. Hence, assuming a epsilon-positive cost of linking would justify the minimality requirement. Hence we only solve chains and discuss other network structures in subsection 3.7. Chains of bilateral contracts are commonly observed among institutions in over-the-counter markets. Li and Schurhoff (2014) show that 23% of round-trip trades of municipal bonds involve a chain of 2 or more intermediaries. Hollifield, Neklyudov, and Spatt (2014) find that round-trip trades of collateralized mortgage obligations (CMOs) involve 1.76 dealers on average. The number of firms here can be interpreted as the longest path in an otherwise general network, such as the core-periphery structure.[26]

2.5 The Equilibrium

At $t = 1$, each firm i simultaneously proposes contingent price offerings $p_{i\,\cdot} \in \mathbb{R}^{(n-1)^2(n-2)+(n-1)}$ and makes linkage decisions $l_{i\,\cdot} \in \mathbb{R}^{(n-1)\frac{n}{2}+1}$ to maximize expected firm value.[27] Next I define the notion of *pairwise Nash equilibrium with contingent transfers*. I embed bilateral prices into pairwise Nash stability of Jackson and Wolinsky (1996) along the lines of contingent transfers in Bloch and Jackson (2007).

Definition 1 *The equilibrium of a network formed by bilateral forward swap contracts is characterized by the linkage structure* L^e *and the set of bilateral prices* p^e*, such that*

[25] A chain (cycle) is a sequence of firms and links that start with firm i and end with firm $j \neq i$ (firm i).

[26] For papers analyzing the formation of core-periphery networks among banks or dealers see e.g. Neklyudov (2014), Farboodi (2015), and Chang and Zhang (2015).

[27] The contingent price offering p_i is a vector of length $n-1$. From Assumption 1, each bilateral price p_{ij} is a function of who else j connects to and who else i connects to $2\binom{n-2}{1} + 2\binom{n-2}{2}$, as well as one scenario where neither i or j has any counterparty. The linkage decision is a vector of binary variables with length $n-1$. Given that the maximum number of links is two, $\binom{n-1}{2} + \binom{n-1}{1} + \binom{n-1}{0} = (n-1)\frac{n}{2} + 1$ gives the possible scenarios for counterparties including no counterparty.

- *Optimality: each firm i takes as given other firms' strategies $\left(l_{j\,\bullet}, p_{j\,\bullet}\right)_{j\neq i}$, and chooses its own strategy $\left(l_{i\,\bullet}, p_{i\,\bullet}\right)$ to optimize its firm value, i.e.*

$$V_i\left(z, L^e, p^e\right) = \max_{\left(l_{ij}\in\left\{0,\bar{l}\right\}, p_{ij}\right)_{j\neq i}} V_i\left(z, L, p\right), \tag{8}$$

subject to (1) - (4), the price offering rule (6), and Assumptions 1 - 4.

- *Pairwise Nash stability with contingent transfers: denote L^e_{-ij} as the matrix L^e by deleting L^e_{ij}, and similar notations apply to the prices. Then $\forall L^e_{ij} = 0$ and $\forall(\hat{p}_{ij}, \hat{p}_{ji}) \neq (p^e_{ij}, p^e_{ji})$*

$$V_i\left(z, L^e_{-ij}, L_{ij} = \bar{l}, p^e_{-\{ij,ji\}}, \hat{p}_{ij}, \hat{p}_{ji}\right) > V_i\left(z, L^e, p^e\right), \tag{9}$$

$$\Rightarrow V_j\left(z, L^e_{-ij}, L_{ij} = \bar{l}, p^e_{-\{ij,ji\}}, \hat{p}_{ij}, \hat{p}_{ji}\right) < V_j\left(z, L^e, p^e\right). \tag{10}$$

- *Feasibility:*

$$L \times \mathbb{1}_{N\times 1} = L^\top \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}. \tag{11}$$

The refinement notion of pairwise stability with contingent transfers states that given the contingent price offerings by other counterparties, two firms i and j connect if and only if they can generate positive bilateral surplus from L_{ij}. I focus on the addition of links as firms can already unilaterally sever links when optimizing. Pairwise stability with transfers naturally applies here as the goal is to understand which networks arise and remain stable when firms can share rents with local counterparties through bilateral payments.

2.6 Discussions

Synergy from links The two types of links, risk-sharing links and distress links, generate different sources of synergy. A risk-sharing link that connects two non-distressed firms always generates a positive surplus by reducing the volatility of liquid assets. For instance, a link between two *ex ante* identical non-distressed firms reduces the liquidation probability of each firm. In comparison, a distress link has an extra source of synergy from the distress heterogeneity. Take two firms with $\nu_i = 1.5$, $\nu_j = 0.8$. Even when $\sigma = 0$, there is gain as the liquidation of firm j can surely be avoided. Note that only firms with large enough ν_i are able to profit from such a link. Lemma 2 characterizes the risk-sharing surplus between a pair.

Lemma 2 *Between any pair of firms with $\{z_i, z_j\}$, the risk-sharing surplus is positive if and only if $\frac{1}{2}(z_i + z_j) > 0$; if this holds, the surplus increases monotonically with dispersion $|z_i - z_j|$.*

Payment seniority In the model, debt is paid at date 2 using liquid holdings after asset trades. Payments for the forward swap contracts are paid in full at date 3 using the illiquid return of the asset. This specification assumes that short-term creditors have seniority over the OTC derivative counterparties. The motivation is that derivatives seniority here would create additional inefficiency in risk-sharing similar to that in Bolton and Oehmke (2014). Following the example above, let instead $\nu_1 = 1.2$, $\nu_2 = 0.8$, and $\varepsilon_i = \varepsilon_2 = 0$. When net payment is transferred at date 3, both firms avoid liquidation. But whenever firm 2 has to transfer a positive net payment to firm 1 at date 2, firm 2 incurs liquidation. So my specification helps to isolate from other inefficiency channels associated with the derivatives payments.

Algorithm for linkage formation There are multiple ways to determine which network emerges given a set of contingent price offerings. I illustrate the following linking game. Under rational expectations, firms form a common belief about the equilibrium linkage structure L^b. Based on this belief, firms simultaneously submit strategies $l_i(L^b)$ and $p_{i\cdot} = \left(p_{ij}(z, L_i^b, L_j^b)\right)_{j \neq i}$. Given the strategies, the realized equilibrium network is consistent with the initial belief $L^e = L^b$. An alternative guess-and-verify approach is described in Bloch and Jackson (2007).

3 Network Inefficiency

Next I examine the efficiency of the equilibrium network relative to a benchmark that minimizes total liquidation costs. I show that the unique equilibrium network is inefficient when the dispersion of financial distress is high: there are more distress links and fewer risk-sharing links.

3.1 Optimal Network

Under the model specifications for links and the asset swap process, the optimal network is chosen to minimize total liquidation costs (i.e. to maximize total firm values).

Definition 2 *The optimal network L^* minimizes total expected liquidation costs, i.e.*

$$L^* = \arg \min_{L_{ij} \in \{0, \bar{l}\}} \sum_{i=1}^{N} \Pr(h_i < 1)\, c, \tag{P1}$$

subject to the conditions of two-sided links $L_{ij} = L_{ji}$, infinite swap (7), and feasibility (11).

I next solve P1 and characterize the properties of L^* in the two dimensional space of $\bar{z}$ and δ.

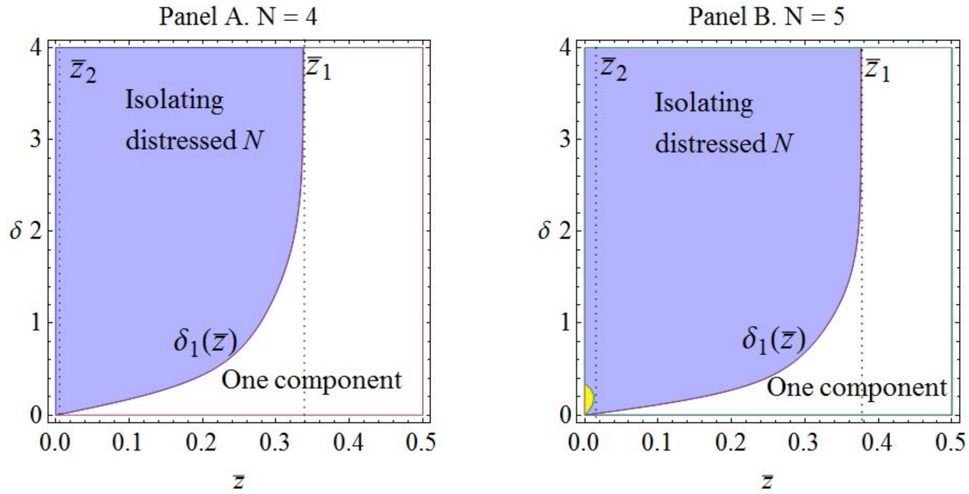


Figure 2. Optimal Network. This figure shows the optimal risk-sharing network characterized in Proposition 1 for $N = 4, 5$. The horizontal and vertical axes represent the mean and dispersion of firm distress. In the white region, all firms are linked in one component. In the dark region $(\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z}))$, firm N is isolated.

Proposition 1 (Optimal Network) $\exists \bar{z}_1, \bar{z}_2$, $\bar{z}_1 > \bar{z}_2 \geq 0$, $\exists$ *cutoff function* $\delta_1(\bar{z}) > 0$ *such that*

- $\forall \bar{z} \geq \bar{z}_1, \delta \geq 0$ *or* $\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta \in [0, \delta_1(\bar{z})]$, *all firms are connected in one component; formally, either* $L^*_{ij} > 0$ *or there exists a path between* i *and* j, *i.e.* $L^*_{ik_1}, ..., L^*_{k_m j} > 0$*;*
- $\forall \bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z})$, *the distressed firm* N *is isolated* ($L^*_{NN} = 1$)*, whereas all other firms are connected in one component.*

The optimal network for any given N is fully characterized by the first two moments of distress distribution, $\{\bar{z}, \delta\}$. The equally-spaced vector structure of z and the fixed liquidation cost are key for this result. All firms diversify maximally by linking in one component when $\bar{z}$ is high (low average distress) or when $\bar{z}$ is low and dispersion δ is low. In contrast, when $\bar{z}$ is low and dispersion δ is high, the most distressed firm N should be isolated, whereas all other firms are connected in one component. The patterns are shown in Figure 2 for $N = 4, 5$.[28] The mechanism lies in the trade-off between diversification benefit and contagion cost.[29] When $\bar{z}$

[28]The cutoff value $\bar{z}_2$ is zero for $N = 4$, and is positive for $N \geq 5$. For $\bar{z} \in [0, \bar{z}_2]$ when $N \geq 5$, there are regions when L^* isolates more than one firm: in Figure 2 Panel B, both firms 4 and 5 are isolated in the tiny hump-shaped region in the lower left corner. As δ increases further, L^* switches from isolating two firms to one firm. This is because the total expected liquidity of the first N-1 firms increases with δ which results from the assumed structure of vector z. The Appendix provides a general analysis for $N > 5$ and plots figures for $N = 6, 7, 8$. I focus attention on whether or not to isolate firm N rather than a group of distressed firms. This is constructed to highlight the role of dispersion. The result that all risk-sharing firms are connected in a single component rather than multiple segments is due to zero decay in interdependence from infinite swap.

[29]This trade-off between risk-sharing and contagion is in line with Cabrales, Gottardi, and Vega-Redondo (2014), who find that, when shock distribution has thin tails, firms should be connected in one component,

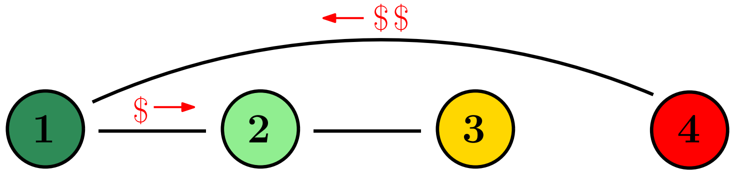


Figure 3. Equilibrium Network ($N = 4$). This figure shows the unique equilibrium four-firm chain network. Firms are ranked by the level of distress, and firm 4 is distressed. A solid line represents a link between two firms. A $ arrow indicates the direction of net payment transfers via bilateral prices.

is low and δ is high, firm N is heavily distressed. The contagion cost of linking firm N with all other firms dominates the risk-sharing benefit, which rationalizes isolating it to be socially optimal.

The model specifications on links and asset swaps do not deviate the optimal network from the best possible risk-sharing outcome. In Online Appendix A.1, I show that under the infinite asset swap, the asset holdings implied by the optimal network are equivalent to the optimal allocations if the social planner were to directly allocate asset holdings for each firm. Hence, total liquidation costs achieve the minimum as long as the network is optimal.

3.2 Excess Distress Link

The question I address next is whether the optimal network can be decentralized in the network formation, and if not, in which ways the equilibrium network is inefficient.

Proposition 2 (Excess Distress Link) *For $N = 4$, $\forall \bar{z}, \delta$, the unique equilibrium features $L^e_{12} = L^e_{23} = L^e_{14} = \bar{l}$: all firms connect in one chain with firm 4 connected to firm 1.*

For all parameter values in a four-firm chain,[30] all firms are connected in one component in equilibrium including the most distressed firm. Comparing Propositions 1 and 2, when $\bar{z}$ is low and δ is high, the optimal network has no distress link; however, the equilibrium network is inefficient and features excess distress link. Figure 3 illustrates the intuition. L^* is not stable because firm 1 deviates to link with firm 4 for profit.[31] Then firm 2 severs the $1-2$ link as the cost of indirectly holding a faction of distressed asset gets high. To prevent 2 from disconnecting, firm 1 offers a contingent premium p_{12}. This premium price equates the value of firm 2 to its outside

whereas when shock distribution has fat tails, maximum segmentation into small components is optimal.

[30]For $N \leq 3$, the equilibrium network is unique and coincides with the optimal network because local contingency contracts are complete and are sufficient to internalize network externalities (see Online Appendix A.3).

[31]The $1 - N$ link satisfies pairwise robustness; it is always profitable ($\bar{z} > 0$) and the surplus increases with δ.

option: the best it can get upon withdrawing. This leads to over-connection: the distressed firm 4 should have been isolated but is linked with others. Firm 2 cannot afford to pay a premium price to prevent 1 from connecting with 4. This is because the benefit of isolating 4 is shared by both 2 and 3, and so firm 2 would be worse-off paying the premium on its own.

Pairwise robustness explains why firm 2 would not link with 4 to compete against 1 for profit. In the inefficiency region where δ is large, the bilateral surplus between 2 and 4 is negative. This means that 2 alone does not have incentive to connect with 4 unless connected with 1. However, 1 is then better off deleting the $1-2$ link, rendering the $2-4$ link not stable. Essentially, large enough heterogeneity plays a key role in keeping the $1-N$ link profitable (and pairwise robust) by eliminating non-credible competitions.

3.3 Risk Sharing Loss

As the chain network gets longer, the excess distress link can crowd out valuable risk-sharing links, thus giving rise to an additional channel of inefficiency from the loss of risk-sharing.

Proposition 3 (Risk Sharing Loss) *For $N = 5$, the unique equilibrium chain network has excess distress link ($\sum_{i \neq N} L^e_{iN} > 0, \sum_{i \neq N} L^*_{iN} = 0$) when $\bar{z} \in [\bar{z}_2, \bar{z}_1]$, $\delta > \delta_1(\bar{z})$. In particular, $\exists$ cutoff function $\eta(\bar{z}) > \delta_1(\bar{z})$ such that*

- $\forall \delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$, *all firms are connected in one chain, so there is* over-connection*;*
- $\forall \delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\}$, *the non-distressed firms are not connected in one chain, so there is* inefficient composition *due to both excess distress link and insufficient risk-sharing.*

Proposition 3 formalizes two channels of inefficiency: over-connection from the excess distress link and under-connection from risk-sharing loss. Consistent with Cabrales, Gottardi, and Vega-Redondo (2014), the optimum network requires assortativity and complete connection in each component. However, the equilibrium is dissortative. When $\bar{z}$ is low and δ is high, the distressed firm which should be isolated, is linked by firm 1 via the excess distress link. This occurs in the colored regions in Figure 4 where $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta > \delta_1(\bar{z})$. Particularly, in the lighter region ($\delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$) where all firms are linked in one component in $[5-1-3-2-4]$, inefficiency only results from over-connection. Contingent premium prices are paid such that the outside option values of both firms 2 and 3 are matched.

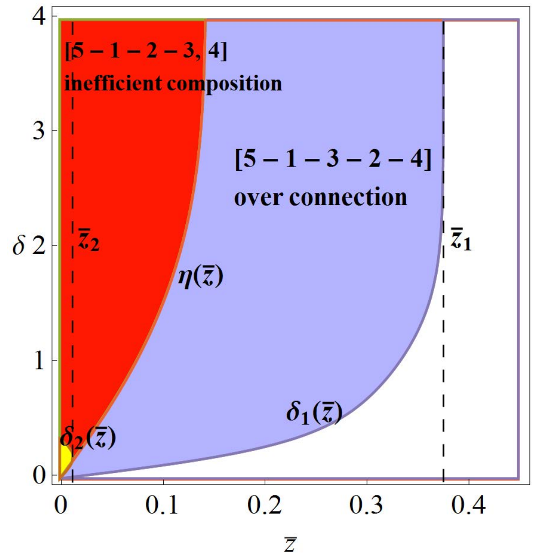


Figure 4. Equilibrium Network ($N = 5$). This figure shows the unique equilibrium for a five-firm network in $\{\bar{z}, \delta\}$. The horizontal and vertical axes represent the mean and dispersion of firm distress. In blue and red regions, the optimal network isolates firm 5. Blue (lighter) region denotes over-connection and red (darker) region denotes inefficient network composition.

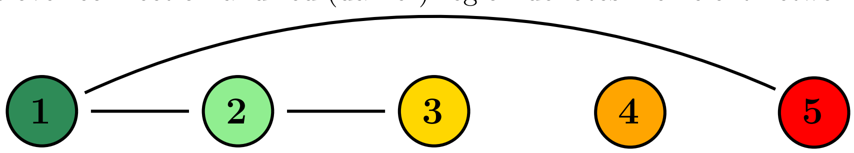


Figure 5. Inefficient Network Composition ($N = 5$). This figure shows the unique equilibrium network in the upper left corner of the parameter space in Figure 4. It has inefficient network composition that features both over- and under-connections.

As we move to the upper left darker region ($\delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\}$), it gets too expensive for 1 to match the outside option of 3. As long as the 1-3 link severs, the 2-4 link also severs and the 1-2 link forms (shown in Figure 5). As such, the externality from the distress link crowds out potential gains from risk-sharing. The equilibrium features inefficient composition with over- and under-connections simultaneously. In the proof, I show that other structures are either not stable or cannot be supported by bilateral prices under local contingency, thereby confirming that the above is the unique equilibrium network.

3.4 Generalization to Arbitrary N Length Chains

So far, the network inefficiency results only concern the cases of $N = 4, 5$. Such small number of players defines a minimal strategy space; this allows me to characterize the precise equilibrium network along with the supporting prices, thereby demonstrating the economics forces behind the network externality. A four-firm chain is the minimal structure where over-connection shows up; a five-firm chain is the minimal structure where loss of risk-sharing arises. I show how this result of network inefficiency holds for a general case with arbitrary $N \geq 5$ in Online Appendix A.2, and offer a proof based on induction.

The intuition is similar to the case of $N = 5$. Denote the chain network where all firms are connected as $[N - 1 - j - k - ...]$. Pairwise robustness requires that N is on one end of the chain linked with firm 1. When δ is small, contingent premium prices are paid such that the outside options of both firms j and k are matched. As δ increases, the outside option of j gets high so 1-j link severs. Notice that the remaining $[j - k - ...]$ fits precisely the case of $N - 2$ given the symmetric structure of z vector. If $[j - k - ...]$ is stable, $[N - 1, j - k - ...]$ gives the equilibrium network; or else, we analyze if any of the segmented components of $j - k - ...$ can connect with firm 1, until a stable structure is found. While Proposition 6 in Online Appendix A.2 does not fully characterize the equilibrium structure in terms of the orders of firms in the chain, it suffices to demonstrate that the economic forces of network externality also prevail in the general case.

3.5 Incomplete Contracts as the Key Friction

The inefficiency is caused by network externalities. Due to local contingency, firm 1 fails to internalize the negative externalities to its direct and indirect counterparties. When Assumption 1 is relaxed, bilateral prices $p_{ij}(z, L)$ induce the efficient network for all parameter values. This validates that the mere underlying friction is that bilateral prices are not contingent on the entire network structure. This result hence fits into Bloch and Jackson (2007)'s contribution on the role played by transfers payments in the formation of networks.

Recall the $N = 4$ case. When δ is high, linking with the distressed firm 4 by 1 imposes an externality to both 2 and 3. To prevent this distress link, firms 2 and 3 need to jointly offer incentives to 1. In Online Appendix A.3, I show that, if and only if $L^*_{14} = 0$, there exist unique premium prices p^*_{21} and p^*_{32} both contingent on L^e_{14} to support $L^e_{14} = 0$. Essentially, the premium price offered by firm 3 is contingent not only on the link between 2 and 3, but also on the links of its counterparty's counterparty.

3.6 Sufficient State Variables to Restore Efficiency

To restore efficiency, it is important to discuss the set of sufficient state variables that can complete the contracting space. The entire network structure L is such a state variable. But detailing contingencies for every possible network structure is not only too costly but also unenforceable. Alternatively, liquidation probability of a direct counterparty j, $\Pr(h_j < 1)$, measures the direct risk exposure of j hence should be priced. However, $\Pr(h_j < 1)$ is a function of L, thus is sensitive to the formation or termination of even a faraway link. An "instantaneous credit rating" based on the entire network structure is needed but is practically challenging.

Interestingly for chain networks, the sufficient state variables to restore efficiency include the direct links of the two firms and their *net transfers* with direct counterparties. Formally, the optimal network can be decentralized under pairwise Nash stability by bilateral prices $p_{ij}\left(z,\, L_i\left(p_{i\,\bullet} - p^T_{\bullet\, i}\right),\, L_j\left(p_{j\,\bullet} - p^T_{\bullet\, j}\right)\right)$,[32] where $(p_{j\,\bullet} - p^T_{\bullet\, j})$ is a vector of net transfers between j and j's counterparties. Such prices allow each firm to ensure that its direct counterparty passes on the right amount of incentive along the chain. In a four-firm chain, if p_{32} is contingent on $p_{21} - p_{12}$ besides local contingency, then equivalently p_{32} is contingent on L_{14} when $L_{12} > 0$. This completes the contracting space. The contracts between 2 and 3 only need to be written on all direct links and net transfers associated with these two firms, which are practically implementable.[33]

Offering incentives along chains here is analogous to offering incentives over time in a dynamic principal-agent problem. The former requires prices contingent on the entire chain of links ("structure dependence"). The latter has incentive compatibility constraints based on the entire sequence of outcomes ("history dependence"). Similar to promised utility summarizing all relevant history, here a counterparty's net transfer with its direct counterparty captures the risk exposure from the rest of the chain, hence is a sufficient state variable in bilateral prices.

[32] $p_{i\,\bullet}$ denotes the ith row of the p matrix; $p^T_{\bullet\, i}$ is the transpose of the ith column of the p matrix. $L_i\left(p_{i\,\bullet} - p^T_{\bullet\, i}\right)$ is the vector of net transfers of i with its direct counterparties. For a proof of the $N = 4$ case, see Online Appendix A.4.

[33] The sufficient state space is smaller than that of L: take the four-firm chain network, p_{32} does not need to be contingent on L_{14} when $L_{12} = 0$ and $L_{34} = 0$.

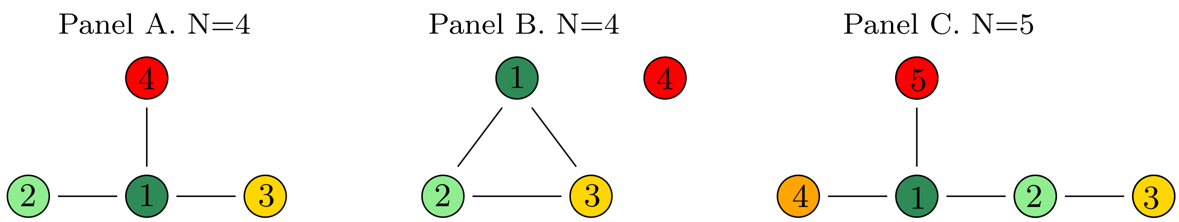


Figure 6. General Network Structures.

3.7 Other Network Structures

The main results of the paper are based on the chain assumption. A chain of four firms is the minimal structure I need to illustrate the network inefficiency. The number of firms here can be seen as the longest path of a general non-star network. Nonetheless, even with a star network, a similar type of externality occurs. Figure 6 Panel A shows a star network with $N = 4$ and firm 1 in the center. Same as before, firm 1 has incentive to link with 4 while imposing externalities to 2 and 3. To prevent this link, both 2 and 3 need to coordinate and jointly offer incentives to 1 so that the rent of firm 1 is matched. The price offered by 3 to 1 needs to be contingent on the price offered by 2 to 1. But this is not feasible under local contingency. For details see Online Appendix A.5. Panel B and Panel C illustrate similar variants.

More broadly, once we relax the chain assumption so that each firm can link to multiple others, coordination among multiple firms becomes necessary to prevent externality; however, local contingency limits the coordination.[34] This result has implications for derivatives markets regulations, for instance the effectiveness of the new mandatory clearing regime via central counterparty clearing.

4 The Distress Dispersion

In this section, I investigate factors that indicate the level of network inefficiency. While prior literature has largely focused on the first moment of financial distress, I show that the distress dispersion across firms is a critical indicator for inefficiency, measured by value loss and excess systemic risk. Using comparative statics, I validate the role of distress dispersion by associating network inefficiency to changes in the network composition.

[34]This relates to what Bloch and Jackson (2007) refer to as the "multitude of interrelated bilateral problems."

4.1 Measures of Network Inefficiency

In the model, the first measure of network inefficiency is the loss of total firm value. Define *value loss*, ΔV, as the difference in total expected firm values between the optimal and the equilibrium networks. Let $\Delta V\%$ be the *percentage value loss*, which is the percentage of *value loss* over total firm values under the optimal network. The expressions are specified as follows,

$$\Delta V = \sum_{i=1}^{N} V_i\left(z, L^*, p^*\right) - \sum_{i=1}^{N} V_i\left(z, L^e, p^e\right); \quad \Delta V\% = \frac{\Delta V}{\sum_{i=1}^{N} V_i\left(z, L^*, p^*\right)}, \tag{12}$$

where $\Delta V > 0$ whenever $L^e \neq L^*$. Since prices are bilateral transfers between firms, value loss equals the increment of total liquidation costs. This is a natural measure of the effectiveness of risk-sharing among firms.

The second measure focuses on the probability of joint liquidation event. Define *systemic risk* as the probability that the majority of firms liquidate at the same time,[35] denoted by $\Pr_{sys}^{L}$,

$$\Pr_{sys}^{L} = \Pr\left(\sum_{i=1}^{N} \mathbb{1}_{(h_i(a,L)<1)} > \tfrac{1}{2}N\right). \tag{13}$$

In a network where there exists a component with size bigger than $\frac{1}{2}N$, systemic risk equals the liquidation probability of any firm in this component because all connected firms hold exactly the same liquid assets. Define *excess systemic risk* as the difference between systemic risk under the equilibrium and the optimal network, i.e.,

$$\Delta \Pr_{sys} = \Pr_{sys}^{L^e} - \Pr_{sys}^{L^*}. \tag{14}$$

While the two measures focus on different dimensions, both are positive when the distressed firm fails to be isolated.

4.2 Inefficiency, Dispersion, and Network Composition

With the two measures defined, next I characterize the properties of value loss and excess systemic risk as functions of the two moments of firm distress distribution, $(\bar{z}, \delta)$.

Proposition 4 (Distress Dispersion) *When the equilibrium network is inefficient, value loss and excess systemic risk increase with dispersion δ, decrease with average $\bar{z}$, and decrease with $\bar{z}$*

[35] The threshold number of liquidations to be considered as joint event can be more general. Here $\frac{1}{2}N$ is taken for illustration.

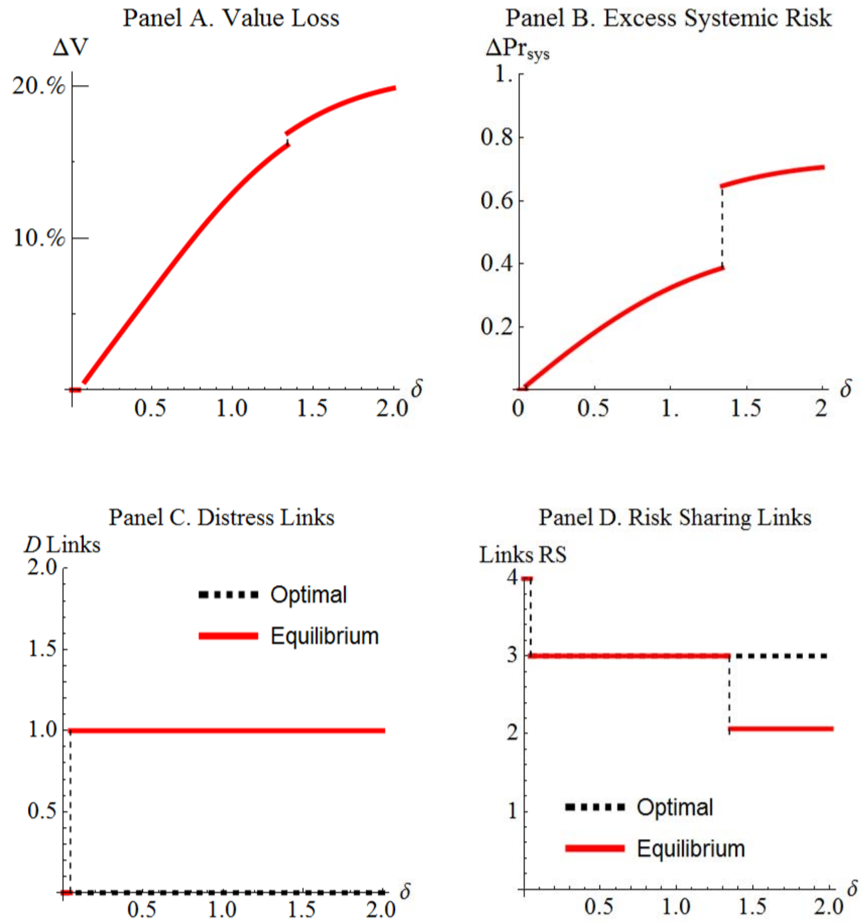


Figure 7. Increase in Dispersion. This figure shows the properties of the five-firm chain network when we raise dispersion δ. I plot the values in the equilibrium network with solid red and the optimal network with dashed black.

faster when δ is higher. Formally, for $\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z})$, $\frac{\partial \Delta V}{\partial \bar{z}} \leq 0$, $\frac{\partial \Delta V}{\partial \delta} \geq 0$, and $\frac{\partial^2 \Delta V}{\partial \bar{z} \partial \delta} \leq 0$; $\frac{\partial \Delta \Pr_{sys}}{\partial \bar{z}} \leq 0$, $\frac{\partial \Delta \Pr_{sys}}{\partial \delta} \geq 0$, and $\frac{\partial^2 \Delta \Pr_{sys}}{\partial \bar{z} \partial \delta} \leq 0$.

Proposition 4 shows that firm distress dispersion δ is a key indicator for both measures of inefficiency. First, value loss and excess systemic risk increase with dispersion δ;[36] Second, value loss and excess systemic risk decrease with average $\bar{z}$ only when dispersion δ is large enough. When the dispersion is high, firm N is so distressed that linking it with other firms generates large contagion risk and high chances of joint liquidation. Consequently, the cost from such a distress link causes higher loss in total firm values, thus increases inefficiency.

[36]With no linkages, total liquidation costs increase monotonically with dispersion because more firms are distressed. However, when firms form links optimally, total liquidation costs *decrease monotonically* with dispersion. Hence, increasing dispersion alone does not imply a worse outcome.

To further understand the role of dispersion, I analyze how the equilibrium network responds to changes in δ relative to the optimal network. Specifically, I relate the two inefficiency measures with changes in the network composition in terms of distress links and risk-sharing links. Figure 7 plots the value loss (Panel A), excess systemic risk (Panel B), distress links (Panel C), and risk-sharing links (Panel D) as functions of δ in a five-firm chain network. As δ increases, inefficiency becomes positive and increases thereafter with a jump when δ is high enough.

These patterns are due to the formation of excess distress link together with the loss of risk-sharing when dispersion gets higher. This can be seen from changes in the network composition in Panels C and D. Corresponding to where the inefficiency becomes positive, the equilibrium network has one extra distress link between 1 and 5. As δ gets higher (at where the jumps are in Panels A and B), the equilibrium network has one fewer risk-sharing link than the optimal network (dashed minus solid curves in Panel D). The risk-sharing loss from the severance of the 2-4 link creates an extra channel for inefficiency. Hence, the observed positive relation of inefficiency and δ is associated with changes in the network composition.

To summarize, the above comparative statics exercise shows that an increase in δ is associated with: *(1) higher value loss and higher systemic risk, (2) more distress links, and (3) fewer risk-sharing links.*

5 Policy Implications on the Acquisitions of Distressed Firms

In this section, I apply the model to a setting where links with distressed firms are interpreted as acquisitions. Two reasons support this particular application. First, in the data a major example of links with distressed firms is through acquisitions, such as the acquisitions of Merrill Lynch by Bank of America and HBOS by Lloyds TSB. Acharya, Shin, and Yorulmazer (2010) and Almeida, Campello, and Hackbarth (2011) show evidence that firms with enough liquidity make windfall profits by purchasing assets from distressed firms when there is liquidity scarcity. Second, compared with OTC derivative contracts which are challenging to supervise, acquisitions in the financial sector are subject to regulatory approval, making policy interventions feasible.

Acquisition is currently regarded as the primary approach to resolve firm distress because it incurs the least fiscal cost. However, my results imply that acquisitions of distressed firms should be regulated due to the externalities in financial network formation. Next I propose one such regulation. I show that a tax formula that varies with the distress distribution can induce the optimal level of acquisitions and restore the efficient network.

Proposition 5 (Acquisition Tax) *In an N-firm chain with $\bar{z} \in [\bar{z}_1, \bar{z}_2]$, the optimal network can be decentralized by a tax τ imposed to firm 1 contingent on its acquisition of firm N,*

$$\tau = (N-3)\left(\frac{1}{2}\delta\sigma + \frac{N-2}{N-3}\Phi\left[\sqrt{N}\,(-\bar{z})\right]c - \Phi\left[\sqrt{N-1}(-\bar{z}-\frac{1}{2}\delta)\right]c\right) - \frac{1}{2}P_1 c - \frac{1}{2}P_N c, \quad (15)$$

where $\Phi[.]$ is the c.d.f. of the standard normal distribution, P_1 and P_N are the liquidation probabilities for stand-alone firms 1 and N respectively. When $L^*_{NN} = 1$, τ satisfies $\tau > 0$, $\frac{\partial \tau}{\partial \delta} > 0$, and $\frac{\partial \tau}{\partial \bar{z}} < 0$.

The contingent acquisition tax τ aligns the social incentive for acquisition with that of firm 1. It accounts for the negative externalities to all *indirect counterparties* (firms $i = 3, ..., N-1$) which cannot be corrected by local contingent contracts, the positive externality to N in lowering liquidation probability of N, and the profit extracted from N by 1. The tax is a function of the distribution of distress across firms in terms of $\{N, \bar{z}, \delta\}$. When dispersion is higher, the negative externalities are bigger; hence, we require bigger tax in order to correct for the incentive mismatch. A similar argument holds for the relation with the average distress. Note that the tax is imposed contingent on the excess acquisition. So no tax is physically collected from the acquirers because the inefficient acquisition is effectively prevented *ex ante*.

The model provides theoretical guidance on how to regulate acquisitions. In particular, intervening based on firm distress distribution complements the current regulatory metrics. The concern towards "financial stability" when evaluating acquisitions was included for the first time by the Dodd-Frank Act.[37] In the recent orders approving acquisitions, for instance Capital One's acquisition of ING Bank, the Fed describes the new financial stability metrics per Dodd-Frank's mandate, covering size, substitutability, interconnectedness, complexity, and cross-border activity.[38] The discussion regarding the interconnectedness factor, however, only covers the degree of interconnectedness of the resulting firm, rather than considering the entire linkage structure and possible externalities through indirect linkages.

[37] The Dodd-Frank Act in Section 604(d) amends Section 3(c) of the Bank Holding Company Act of 1956 and requires the Fed to consider "the extent to which a proposed acquisition, merger or consolidation would result in greater or more concentrated risks to the stability of the United States banking or financial system."

[38] www.federalreserve.gov/newsevents/press/orders/2012orders.htm

The key issue is how to implement such a tax. The regulators need to account for the distribution of financial distress. One feasible approach detailed in Section 6 is to estimate quarterly Z-scores of financial firms. Among the limitations of this measure are the low frequency and the opacity of balance sheets. Using exclusive regulatory data, the banking supervisors can potentially achieve better estimates by using higher frequency data or alternative models such as CAMELS ratings. If excess acquisitions are prevented, alternative resolution methods in case of failure include liquidation or Purchase and Assumption (P&A) transactions.[39]

Several acquisition cases observed during the recent financial crisis render the baseline model counterfactual, e.g. the acquisitions of Bear Stearns. To rationalize such observed interventions, in Online Appendix A.6, I study an extension of the baseline model that allows for analyzing government interventions both before and after the linkage formation. Results indicate that, if the excess acquisition is not effectively prevented *ex ante*, the too-connected-to-fail problem arises: liquidating the distressed firm is too costly due to spillovers to its existing counterparties. I argue that government interventions such as bailout and subsidies are *ex post* optimal, thereby rationalizing these actions observed during the recent financial crisis.

6 Empirical Evidence

The model predicts that network inefficiency increases with distress dispersion. In this section, I document empirical evidence that the distribution of distress across financial institutions provides a novel measure for systemic risk. I establish this result by first examining how the cross-sectional mean and dispersion of distress correlate with indicators for aggregate systemic risk, liquidation costs, distress links through acquisitions, and interbank risk-sharing. I then confirm the findings using predictive regressions.

[39] In a P&A transaction, a healthy institution assumes some or all of the obligations, and purchases some or all of the assets, of the failed institution. The Federal Deposit Insurance Corporation Improvement Act of 1991 mandates the FDIC to choose the resolution method least costly to the Deposit Insurance Fund. To comply with this mandate, the FDIC chose P&A transactions as the resolution method for a great majority of failing banks. For detailed institutional background on bank failures see White and Yorulmazer (2014), Granja, Matvos, and Seru (2015), and the the Guidance for Developing Effective Deposit Insurance Systems from FDIC, at http://www.fdic.gov/deposit/deposits/international/guidance/guidance/FailedResolution.pdf.

6.1 Measurement and Data

The quarterly accounting data provide the basis for the measurement of financial distress. The sample of financial institutions includes bank holding companies and all Federal Deposit Insurance Corporation (FDIC) insured commercial banks and savings institutions. Data of bank holding companies for the period of 1986-2013 are taken from FR Y-9C filings provided by the Chicago Fed. Data for commercial banks (Call Reports) and savings institutions (Thrift Financial Reports) are taken from the FDIC's Statistics on Depository Institutions, available for 1976-2013. Next, I describe the measurement of financial distress.

6.1.1 Z-score

I measure financial distress by estimating the Z-score, which has been widely used in the literature as an indicator for a institution's distance from insolvency (e.g. Roy (1952), Stiroh (2004), Boyd and De Nicolo (2005) and Laeven and Levine (2009)). The Z-score combines accounting measures of profitability, leverage and volatility, and is defined as the return on assets (ROA) plus the capital-asset ratio divided by the standard deviation of ROA. Simply put, it equals the number of standard deviations that an institution's ROA has to drop below the expected value before equity is depleted. For this reason, the Z-score provides a good proxy for financial distress (the state variable z_i in the model). In particular, the Z-score is estimated according to

$$\text{Z-score}_{i,t} = \frac{\frac{1}{T}\sum_{\tau=0}^{T-1} ROA_{i,t-\tau} + \frac{1}{T}\sum_{\tau=0}^{T-1} CAR_{i,t-\tau}}{\sigma_{t-T+1}^{t}(ROA_i)}, \tag{16}$$

where $ROA_{i,t}$ and $CAR_{i,t}$ are respectively the ROA (net income over total assets) and capital asset ratio (total equity capital over total assets) for firm i in quarter t. Here I consider a rolling window of eight observations. The estimated Z-score is highly skewed; hence, I follow Laeven and Levine (2009) and Houston, Lin, Lin, and Ma (2010) and adopt the natural logarithm of the Z-score as the distress measure.

The time series of the mean and dispersion of log Z-score are estimated by taking the average and standard deviation across all financial firms in each quarter. Figure 8 plots the quarterly series of dispersion, mean, and the 10-90 percentile range of log Z-score over the period of 1978-2013. From Figure 8, we can make the following observations. First, relative to the cross-sectional mean, the dispersion of log Z-score displays a fair amount of variation and has an

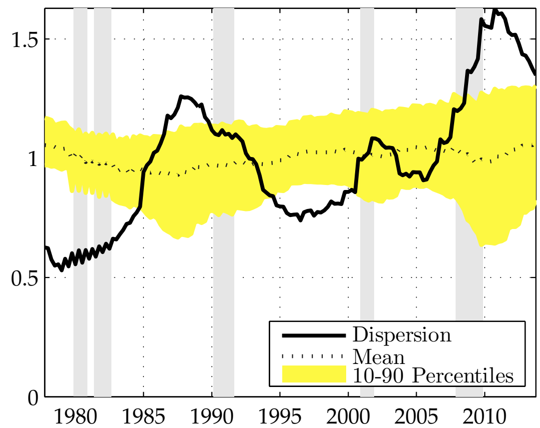


Figure 8. Log Z-score Moments across Financial Institutions. This figure plots the quarterly time series of dispersion, mean, and the 10-90 percentile range of log Z-score across all financial institutions over the period of 1978-2013. The series are normalized such that both the dispersion and the mean are centered around one. Shaded bars indicate NBER recessions.

increasing trend. Second, the dispersion series demonstrates a countercyclical pattern: it increases during the Savings and Loan crisis in the late-1980s, the Dot-com crash and the recession afterwards, as well as during the 2007-2009 financial crisis. Based on the comparative statics in Section 4.2, precisely during the crises spell, high dispersion leads to high network inefficiency, which potentially increases systemic risk and aggravates the crises. Finally, the dispersion series appears to lead recessions. Take the most recent crisis for instance, the dispersion starts to increase since 2006, and by the time financial firms enter the crisis in the 3rd quarter of 2007, they already show significant dispersion in financial distress. These features combined suggest that the time series of dispersion can potentially signal economic changes and systemic risk, which I will test at the end of this section.

While the Z-score provides a quantitative measure for distress, it is worth noting a few limitations. The first limitation is that, the Z-scores are an endogenous outcome of certain degrees of risk diversification, thus are not exogenous to firms as assumed in my model. Nonetheless, the Z-score gives the best available proxy for distress in a static framework because it is estimated using past data, which are taken as given by firms to make decisions onwards. Furthermore, as shown by Acharya, Shin, and Yorulmazer (2010), initially liquid firms tend to hoard liquidity or deleverage for potential gains from asset sales, whereas risk management tools for an initially distressed firm are limited. Hence, the ranks of the estimated Z-score across firms can generally reflect the ranks of initial distress. The second limitation pertains to the usage of book value based on accounting data. While I acknowledge that the Z-score omits off-balance sheet activities and might give a biased assessment of risk-taking, off-balance sheet usages are only relevant for a few institutions.

6.1.2 Acquisitions of Distressed Firms

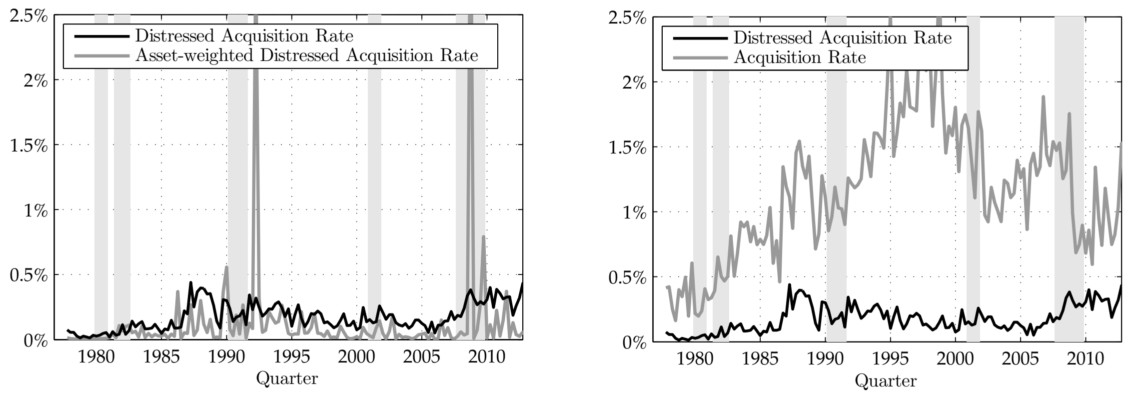
An acquisition of a distressed firm occurs when the target has a low Z-score. This allows us to account for the links with the distressed firms in the model. The acquisition transactions are taken from the Chicago Fed Mergers and Acquisitions dataset. The dataset records all the acquisitions of banks and bank holding companies since 1976, keeping track of both the target and acquirer entities at the merger completion date. I drop the observations that are failures or restructurings.[40] I then use RSSD ID of the target firm and match the dataset with quarterly accounting data two quarters ahead.[41] Around 86% (17,930) of the observations are matched. I identify a *distressed acquisition* if the target firm reports a negative net income two quarters prior to the acquisition completion date, or if the target firm has a log Z-score of below 2.35 (two standard deviations below the sample mean) at least once for within two to four quarters before the acquisition completes.

Using this strategy, around 20% (3,153) of the matched acquisitions are classified as distressed acquisitions. The rest mostly occurred during the merger wave in the 2000s after the Gramm-Leach-Bliley Act, which enabled mergers among investment banks, commercial banks, and insurance companies. Among the distressed acquisitions, notable examples include Countrywide by Bank of America, Riggs and Sterling by PNC, and Wachovia by Wells Fargo.

Figure 9(a) plots the quarterly percentage of distressed acquisitions over total number of financial institutions as well as the distressed acquisition rate weighted by the asset size of the targets. From the plots, distressed acquisition rates are countercyclical. Two periods with clustered acquisitions are the Savings and Loan crisis and the 2007-2009 financial crisis. The

[40]Failures refer to transactions with Termination Reason Code = 5. Restructurings occur when the target entities and the acquirer entities have exactly the same entity name but different Federal Reserve RSSD IDs.

[41]I match the quarterly accounting dataset two quarters ahead because the merger date in Chicago Fed M&A dataset represents the completion date and is usually later than the last quarter when the non-survivor firm files quarterly report. To facilitate the match, I include the FR Y-9LP and FR Y-9SP fillings for bank holding companies. However, since these non-consolidated parent banks only report semiannually, I do not include them when computing the Z-score distributions.



(a) Distressed Acquisitions Rate

(b) Distressed vs. Total Acquisition Rate

Figure 9. Distressed Acquisitions Rate. This figure plots the quarterly (asset-weighted) distressed acquisition rate for 1978-2013 (left) and compares the distressed acquisition rate to the total acquisition rate (right). Shaded bars indicate NBER recessions.

asset-weighted acquisition rate displays significant spikes.[42] Panel 9(b) compares the distressed acquisition rate to the total acquisition rate. The insignificant comovement between the two series shows that variations in distressed acquisitions are unlikely driven by merger waves.

6.2 Model Predictions

As shown in the comparative statics in Section 4.2, an increase in dispersion (together with a decrease in average Z-score) is associated with *higher systemic risk, more liquidations, more (excess) distress links through acquisitions, and fewer risk-sharing links.* Next, I illustrate that patterns in the data provide suggestive evidence for these model-predicted relations.

6.2.1 Aggregate Indicators

The goal is to provide aggregate level evidence that distress dispersion is indicative of economic activity and financial stability. To measure macroeconomic activity, I use the *Chicago Fed National Activity Index (CFNAI)*,[43] which is adopted in Giglio, Kelly, and Pruitt (2015) to evaluate the predictive power of various systemic risk measures. As an indicator for systemic risk, I take the Chicago Fed's *National Financial Conditions Index (NFCI).*

[42]Some spikes reach as high as 3%, while the plots are trimmed at 2.5%. The spikes include one in the 2nd quarter of 1992 due to the acquisition of Security Pacific, one in 2007-2008 mostly due to the acquisitions of Lasalle bank (10/01/2007), Countrywide (01/11/2008), National City (10/24/2008), and Wachovia (12/31/2008).

[43]The CFNAI is designed to gauge overall economic activity and related inflationary pressure. It includes the following subcomponents: production and income (P&I), sales, orders, and inventories (SO&I), employment, unemployment, and hours (EU&H), and personal consumption and housing (C&H).

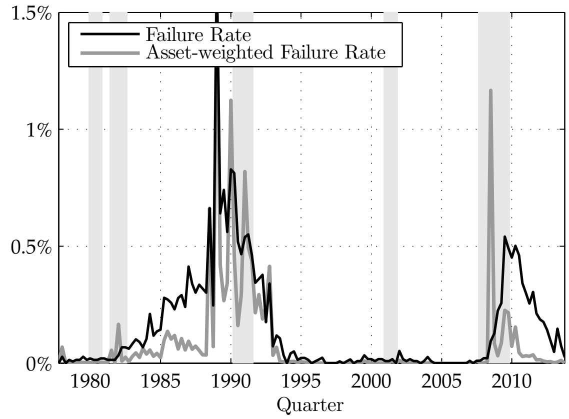


Figure 10. Bank Failure Rates. This figure plots the quarterly failure rate and asset-weighted failure rate of commercial banks and savings institutions for 1978-2013. Shaded bars indicate NBER recession dates.

I measure the failure rates of financial institutions to proxy for liquidations. The sample are aggregated from all failures of commercial banks and savings institutions in 1976-2013 based on the FDIC Failure and Assistance Transaction Reports, as well as the bank holding companies failures from the Chicago Fed Mergers and Acquisitions dataset (with Termination Code = failure). In total, I obtain 3,473 failures with an asset value of 1.84 trillion in 2010 dollars. I construct the quarterly failure rates (numbers of failures over the numbers of total financial institutions) as well as the failure rates weighted by the failing institution's asset size. As depicted in Figure 10, failure rates are strongly countercyclical: the majority of bank failures took place during the Savings and Loan crisis and the 2007-2009 crisis.

Regarding the linkage composition, the model predicts that non-distressed firms that do not engage in distressed acquisitions withdraw from risk-sharing contracts as a consequence of network externalities. Direct evidence on this prediction would be obtained if full information on individual level linkage were available. Instead, I consider the lending and interbank lending behavior of small to medium-sized commercial banks as proxies for risk-sharing contracts since these institutions are more likely to be the non-distressed and non-acquirer firms in the model. In particular, using data from the Fed's H.8 release, I construct the fractions of *bank credit and Fed funds and reverse Repos with banks* over total assets for small to medium-sized (beyond top 25) commercial banks.

Table 1. Summary Statistics and Univariate Correlations

	Mean	StDev	Sacf	Correlations w/ log Z-score	
				Mean	Dispersion
Mean of Log Z-score	1.00	0.03	0.90		
Dispersion of Log Z-score	1.00	0.22	0.97	-0.13	
A. Economic activity and systemic risk					
Chicago Fed National Activity Index (CFNAI)	-0.11	0.72	0.80	-0.03	-0.30**
National Financial Conditions Index (NFCI)	-0.34	0.54	0.84	-0.25**	0.37***
B. Bank failures					
Failure Rate (%)	0.18	0.25	0.72	-0.60***	0.45***
Asset-weighted Failure Rate (%)	0.11	0.25	0.34	-0.38***	0.17*
C. Distressed acquisitions					
Distressed Acquisition Rate (%)	0.21	0.09	0.64	-0.41***	0.60***
Distressed over Total Acquisition Rate	0.19	0.13	0.71	-0.44***	0.68***
D. Lending and interbank lending					
Small Comm. Bk Credit over Assets	0.88	0.02	0.94	-0.26**	-0.73***
Small Comm. Bk Fed Funds Loan over Assets	0.02	0.01	0.85	-0.09	-0.53***

Notes: This table reports summary statistics for the quarterly cross-sectional mean and dispersion of log Z-score, indicators for economic activity and systemic risk (A), bank failures (B), distressed acquisitions (C), and lending and interbank lending (D). Group A series are from FRED. Series in groups B and C are aggregated based on data from the FDIC and the Chicago Fed. Group D series are constructed from the Fed's Z.1 and H.8 release. Data availability on bank holding companies restricts the analysis to 1986-2013. Sacf is the first-order sample autocorrelation coefficient. The last two columns report the correlation coefficients between cross-sectional mean and dispersion of log Z-score and each series in groups A-D. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

6.2.2 Univariate Correlations

Table 1 provides the summary statistics of the above series as well as their univariate correlation coefficients with the mean and dispersion of financials' log Z-scores. Both the mean and dispersion series are rescaled such that the two series are centered around one. The distress dispersion displays higher variation over time and does not significantly correlate with the mean of distress, thereby corroborating the argument that dispersion provides new information not captured by the mean.

Well aligned with the theoretical findings, dispersion series correlate negatively with the economic activity index CFNAI and positively with the systemic risk index NFCI. In other words, high dispersions relate to bad economic times and low financial stability. As the model predicts, the failure rates and distressed acquisition rates are significantly higher when the dispersion is higher or when the average Z-score is lower. Additionally, the distressed acquisitions as a fraction of total acquisitions correlate even more significantly with the log Z-score moments, ruling out the possibility that the variations in distressed acquisitions are due to changes in total acquisition rates. These patterns all corroborate that high dispersion is associated with more distressed acquisitions and consequently, more failures. Last but not least, indicators for lending and interbank lending have negatively significant correlation with dispersion. Small and medium-sized commercial banks reduce interbank lending and exposures with other banks in the Fed Funds and Reverse Repos market, with significance at the 0.001 level. This finding supports that certain risk-sharing contracts terminate as dispersion increases.

6.2.3 Predictive Regressions

Evidence from the univariate correlations provides a strong indication that the distress dispersion comoves with aggregate indicators. However, contemporaneous correlations do not necessarily imply that the distress dispersion is able to forecast systemic risk. Hence, the next goal is to evaluate whether the distress dispersion has predictive power for future aggregate indicators by providing additional information beyond what is contained in the average distress and existing systemic risk measures.

To this end, I run forecasting regressions of the above introduced aggregate indicators on the dispersion and mean of log Z-score controlling for moments including the term spread used in Giglio, Kelly, and Pruitt (2015), the leverage of both financial business and the security broker-dealers as in Adrian, Etula, and Muir (2014), and the growth rate of non-financial corporate liability as a measure of aggregate credit creation. The forecasting horizons range from one to four quarters and the data cover the years of 1986-2013. To overcome correlation and autocorrelations in the time series, I calculate Newey-West standard errors.

Table 2 reports the coefficient estimates on the dispersion and mean of log Z-score, the values of R^2 when I run the regressions with and without the dispersion series. The regression results echo the findings from the univariate correlations and indicate striking predictive power of the dispersion series to forecast economic activity and systemic risk, failures, distressed acquisitions, and interbank lending. The predictive power is evidenced by both the economic significance of the regression coefficients and the differences in the R^2s with and without dispersion in

Table 2. Predictive Regressions using Distress Dispersion

Quarters	1	2	3	4	1	2	3	4
Forecasting	**A. CFNAI**				**NFCI**			
Dispersion	-2.09***	-4.04***	-5.85***	-7.50***	1.52**	2.77**	3.83**	4.72**
Mean	2.75	6.74	8.66	7.92	-8.95***	-17.80***	-25.73***	-32.32***
R^2	44.85	52.03	54.05	52.48	53.22	53.40	52.24	50.56
R^2 w/o disp	28.15	34.78	36.47	34.74	37.54	39.28	39.42	38.86
Forecasting	**B. Failure Rate(%)**				**Asset-weighted Failure Rate(%)**			
Dispersion	0.53***	1.03***	1.56***	2.07***	0.24*	0.48*	0.77*	1.04*
Mean	-3.81***	-7.91***	-12.21***	-17.12***	-2.68**	-5.41**	-7.92**	-11.29**
R^2	58.98	68.10	70.03	71.31	16.97	26.79	32.53	37.64
R^2 w/o disp	50.16	58.46	59.91	60.99	11.07	18.68	22.29	26.14
Forecasting	**C. Acquisition Rate(%)**				**Distressed over Total Acquisition Rate**			
Dispersion	0.16*	0.33*	0.50*	0.68*	0.29***	0.63***	1.00***	1.34***
Mean	-1.45**	-2.66**	-3.81**	-4.43**	-1.19*	-2.16*	-3.24**	-3.90*
R^2	47.31	57.25	64.90	67.04	53.90	63.06	72.20	75.26
R^2 w/o disp	41.24	49.52	56.12	57.55	43.64	48.92	54.41	56.11
Forecasting	**D. Sml Bk Credit over Assets**				**Sml Bk Fed Funds over Assets**			
Dispersion	-0.04**	-0.08**	-0.12**	-0.16**	-0.01*	-0.02*	-0.03**	-0.04***
Mean	-0.02	-0.01	-0.01	-0.04	0.05	0.08	0.09	0.07
R^2	69.85	70.68	71.18	71.44	57.07	63.49	64.71	63.93
R^2 w/o disp	62.69	63.39	63.83	63.76	54.04	59.60	59.48	56.94

Notes: This table summarizes the ability of distress dispersion to forecast future economic activity, systemic risk, failure rates, distressed acquisition rates, and bank lending behavior. Aggregate indicators in groups A-D are regressed respectively on the cross-sectional dispersion and mean of log Z-score controlling for the term spread, the leverage of financial business and security broker-dealers, and the growth rate of real non-financial corporate liability. Forecasting horizons range from one to four quarters and the data cover the years of 1986-2013. The table reports the regression coefficients of the dispersion and mean of log Z-score, the R^2, as well as the R^2 when the regressions are run without the dispersion series. *, **, *** denote statistical significance (based on Newey-West standard errors) at the 5%, 1%, and 0.1% level.

the regressors. For example, the estimates in the forecasting regression of CFNAI imply that (holding the *mean* fixed) a one-standard-deviation increase in *Dispersion* (=0.22) relates to a 0.46 ($= 0.22 \times 2.09$) decrease in CFNAI. Notably, the national activity index CFNAI, the credit and loans and the interbank lending of small and medium-sized commercial banks all respond negatively to an increase in distress dispersion, but not to changes in the mean of distress. Overall, these results paint a clear picture: the second moment of the cross-sectional distress distribution conveys new information about future activities in the financial sector in terms of systemic risk, failures, acquisitions, as well as interbank lending behavior.

7 Conclusion

Given the importance of financial interconnectedness, policies on financial stability should not analyze institutions in isolation. This paper develops a network formation model to highlight a novel channel of systemic risk due to externalities via financial links.

Adding to the recent literature on financial network formation, this paper embeds heterogeneity in financial distress and examines the efficiency of the equilibrium risk-sharing network. I have shown that, when firms display high distress dispersion, the equilibrium features inefficiency in network composition: there are too many links with the distressed firms and too few risk-sharing links among liquid firms. When prices in the bilateral contracts cannot be contingent on the overall network structure, the liquid firm fails to internalize the negative externalities. Hence, it has incentives to connect with distressed firms for profit while shifting risks away to their direct and indirect counterparties. This extra link with the distressed firm not only generates contagion risk but also crowds out valuable risk-sharing links, thereby increasing systemic risk. Notably, this inefficiency is more severe when institutions are more dispersed in financial distress.

My model provides new policy insights for financial stability. Regulators should eliminate network inefficiencies by overseeing the composition of financial linkages. The links with distressed firms in the model can be interpreted as acquisitions. In such context, regulators can restore efficiency by supervising the acquisitions of distressed firms based on the distress dispersion across financial firms.

While detailed data on the precise linkages among financial institutions are limited,[44] this paper draws a relation between the degree of network inefficiency and the cross-sectional distribution of firm fundamentals, thus contributing to the measurement of systemic risk. The test can be extended along the lines of Giglio, Kelly, and Pruitt (2015) by comparing the distress dispersion measure to existing systemic risk measures such as CoVaR (Brunnermeier and Adrian (2014)) and Marginal and Systemic Expected Shortfall (Acharya, Pedersen, Philippon,

[44] For current challenges in measuring linkages and systemic risk, see for example Bisias, Flood, Lo, and Valavanis (2012), Hansen (2013), and Yellen (2013).

and Richardson (2010)). Also, my model uniquely predicts that links connecting firms with different distress levels respond differently to an increase in dispersion. The qualitative predictions on network composition can be tested with possibly better data access going forward.

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8 Appendix: Proofs and Additional Lemmas

Proof of Lemma 1

Before showing the properties of the asset composition matrix $\underset{K\to\infty}{lim} L^K$, we first analyze the features of the matrix L.

Claim 1 *The linkage matrix L has all real eigenvalues: the largest is 1 and all others lie within the unit circle.*

Proof L is symmetric so all its eigenvalues are real. L is doubly stochastic, so $L \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$ and thus $\lambda = 1$ is its eigenvalue with eigenvector $\mathbb{1}_{N\times 1}$. Suppose for contradiction that there exists an eigenvalue $\lambda > 1$. Then there exists a non-zero vector x such that $Lx = \lambda x > x$. Given the rows of L are non-negative and sum to 1, each element of vector Lx is a convex combination of the components of x. This implies that $\max[Lx] \leq \max[x]$, which contradicts with $\max[\lambda x] > \max[x]$. Hence all eigenvalues cannot exceed 1 in absolute value.

Lastly, we show that $\lambda = -1$ is not an eigenvalue of L. It is equivalent to show that the matrix $L + I$ is non-singular. All the off-diagonal elements of $L + I$ are within 0 and 1, and all the diagonal elements are within 1 and 2. The largest element for any column or row is on the diagonal, so there are no columns or rows that are zero or linearly dependent. Therefore $det(L + I) > 0$, and $\lambda = -1$ cannot be an eigenvalue. This concludes the proof of Claim 1. □

Next we apply Claim 1 to show the limiting properties of $L^\infty = \underset{K\to\infty}{lim} L^K$. Given L is a doubly stochastic matrix, $L \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$, $L^\top \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$. Then $L^K \times \mathbb{1}_{N\times 1} = L^{K-1} \times L \times \mathbb{1}_{N\times 1} = L^{K-1} \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$. Similarly ${L^\top}^K \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$, so L^∞ is also a doubly stochastic matrix.

Since the eigenvalues of L, denoted by $\{\lambda_1, \lambda_2, ..., \lambda_M\}$, are real, there exists an orthogonal matrix Q with $Q' = Q^{-1}$ such that $L^\infty = Q\Lambda Q^{-1}$, $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_M)$ and the columns of Q are eigenvectors of unit length corresponding to λ_1, λ_2,..., λ_M. Without loss of generality, we rank the eigenvalues $\lambda_i \geq \lambda_{i+1}$, then

$$L^\infty = Q\Lambda Q^{-1}...Q\Lambda Q^{-1} = Q\Lambda^\infty Q^{-1} = Q \begin{bmatrix} \lambda_1^\infty & 0 & .. & 0 \\ 0 & \lambda_2^\infty & .. & .. \\ .. & .. & .. & .. \\ 0 & .. & .. & \lambda_M^\infty \end{bmatrix} Q^{-1} = Q \begin{bmatrix} 1 & 0 & .. & 0 \\ 0 & 0 & .. & .. \\ .. & .. & .. & .. \\ 0 & .. & .. & 0 \end{bmatrix} Q^{-1},$$

where the last step follows from $\lambda_1 = 1$ and $\lambda_i < 1, \forall i \neq 1$. Let the first column of Q, which is the unit length eigenvector corresponding to $\lambda_1 = 1$ be x_1, then

$$L^\infty x_1 = x_1, \quad x_1^\top x_1 = 1.$$

Since each entry of L^∞ is positive, the above relations imply that the unit length eigenvectors satisfy $x_{11} = x_{12} = .. = x_{1M} = \frac{1}{\sqrt{M}}$. We have

$$L^\infty = Q \begin{bmatrix} 1 & 0 & .. & 0 \\ 0 & 0 & .. & .. \\ .. & .. & .. & .. \\ 0 & .. & .. & 0 \end{bmatrix} Q^{-1} = \begin{bmatrix} x_{11}^2 & x_{11}x_{12} & .. & x_{11}x_{1M} \\ x_{12}x_{11} & x_{12}^2 & .. & .. \\ .. & .. & .. & .. \\ x_{1M}x_{11} & .. & .. & x_{1M}^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{M} & \frac{1}{M} & .. & \frac{1}{M} \\ \frac{1}{M} & \frac{1}{M} & .. & .. \\ .. & .. & .. & .. \\ \frac{1}{M} & .. & .. & \frac{1}{M} \end{bmatrix}.$$

Hence L^∞ coincides with complete risk-sharing regardless of the initial entries of L_{ij} in L. Q.E.D.

Proof of Lemma 2

The bilateral risk-sharing surplus is the reduction in liquidation cost, i.e. $\Pr(z_i < 0)c + \Pr(z_j < 0)c - 2\Pr(\frac{z_i+z_j}{2} < 0)c = \Phi(-z_i)c + \Phi(-z_j)c - 2\Phi(-\sqrt{2}\frac{z_i+z_j}{2})c = \Phi(-\frac{z_i+z_j}{2} - |\frac{z_i-z_j}{2}|)c + \Phi(-\frac{z_i+z_j}{2} + |\frac{z_i-z_j}{2}|)c - 2\Phi(-\sqrt{2}\frac{z_i+z_j}{2})c$. Function $\Phi(x)$, $\forall x < 0$ is monotonically increasing and convex with x. Hence $\frac{z_i+z_j}{2} > 0 \iff \Phi(-\frac{z_i+z_j}{2} - |\frac{z_i-z_j}{2}|)c + \Phi(-\frac{z_i+z_j}{2} + |\frac{z_i-z_j}{2}|)c > 2\Phi(-\frac{z_i+z_j}{2})c > 2\Phi(-\sqrt{2}\frac{z_i+z_j}{2})c$. When $\frac{z_i+z_j}{2} > 0$, holding $z_i + z_j$ fixed, the first derivative of the bilateral surplus with respect to $|\frac{z_i-z_j}{2}|$ is $\Phi'(-\frac{z_i+z_j}{2} + |\frac{z_i-z_j}{2}|)c - \Phi'(-\frac{z_i+z_j}{2} - |\frac{z_i-z_j}{2}|)c > 0$. Q.E.D.

Proof of Proposition 1

Total liquidation probability in a full risk-sharing network with all N firms is

$$\sum_{i=1}^{N} \Pr(h_i < 1) = N\Phi[\sqrt{N}(-\bar{z})]. \tag{17}$$

Total liquidation probability when the first $N-k$ firms fully share risk and firms $N-k+1$ to N each stays separate is

$$\sum_{1}^{N-k} \Pr(h_i < 1) + \sum_{N-k+1}^{N} \Pr(a_i < 1) = (N-k)\Phi\left[\sqrt{N-k}(-\bar{z} - \frac{N-k}{2}\delta)\right] + \sum_{N-k+1}^{N} \Phi\left[-\bar{z} - \frac{N+1-2i}{2}\delta\right]. \tag{18}$$

Take the limit when $\delta \to \infty$, equations (17) and (18) become respectively $N\Phi\left[-\sqrt{N}\bar{z}\right]$ and k. This shows that when δ is very large, full risk-sharing is optimal with high values of $\bar{z}$, and isolating firm N is optimal with low values of $\bar{z}$.

Next we focus on the regions when the tradeoff is between full risk-sharing and isolating firm N. Equating (17) and (18) for $k = 1$ defines a cutoff function $\delta_1(\bar{z})$. Applying the implicit function theorem, the curve is well-defined for $\bar{z} < \bar{z}_1$ and $\frac{\partial \delta_1(\bar{z})}{\partial \bar{z}} \geq 0$. $\forall \bar{z} \geq \bar{z}_1$ and $\forall \delta > 0$, $\sum_{i=1}^{N} \Pr(h_i < 1) < \sum_{i=1}^{N-1} \Pr(h_i < 1) + \Pr(a_N < 1)$, so full risk-sharing is optimal.

Equating (18) for $k = 1$ and $k = 2$ gives the curve $\delta_2(\bar{z})$ which divides the regions between optimally isolating firm N only and isolating both firms $N-1$ and N. We can show that $\exists \bar{z}_2$ so that $\bar{z}(\delta_2) < \bar{z}_2$, i.e. $\forall \bar{z} > \bar{z}_2$, isolating firm N is always preferred to isolating two firms. Q.E.D.

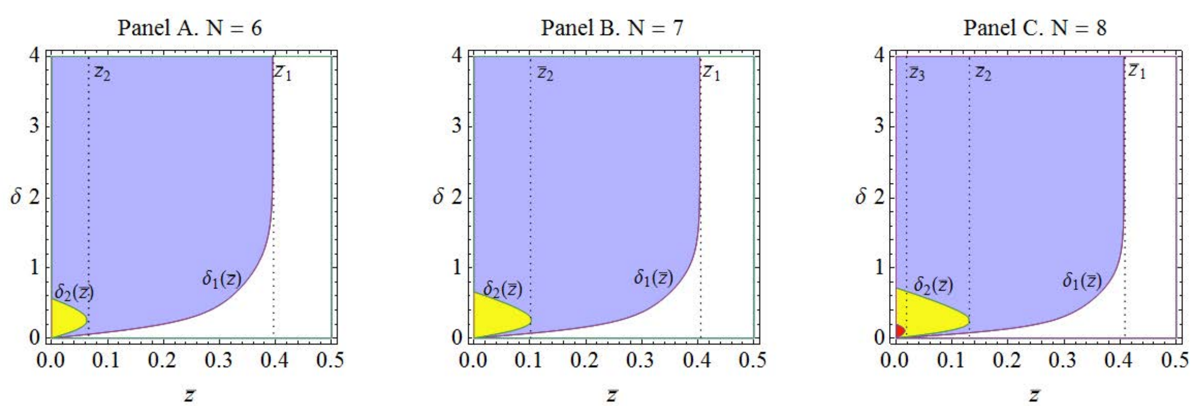


Figure 11. Optimal Network for $N = 6, 7, 8$. The white, blue, yellow and red regions indicate respectively isolating 0,1,2,3 firms.

Proof of Proposition 2

I will first show that $[4-1-2-3]$ is a stable network supported by bilateral prices. In what follows, denote V_i^a, $\bar{V}_i^L$, and V_i^L, respectively as the autarky value of firm i without any linkages, the value of firm i in network L before paying bilateral prices, and after paying bilateral prices.

For $L=[4-1-2-3]$, the bilateral prices with local contingency are denoted as follows, $(p_{41}-p_{14})_{|L_{12}}$, $(p_{21}-p_{12})_{|L_{14},L_{23}}$, $(p_{32}-p_{23})_{|L_{12}}$. To decentralize $[4-1-2-3]$, prices satisfy

$$V_1^{4123}=\bar{V}_1^{4123}+\bar{l}\,(p_{21}-p_{12})_{|L_{14}=L_{23}=\bar{l}}+\bar{l}\,(p_{41}-p_{14})_{|L_{12}=\bar{l}}\geq \max\{V_1^a,V_1^{14}\}; \tag{19}$$

$$V_2^{4123}=\bar{V}_2^{4123}-\bar{l}\,(p_{21}-p_{12})_{|L_{14}=L_{23}=\bar{l}}+\bar{l}\,(p_{32}-p_{23})_{|L_{12}=\bar{l}}\geq \max\{V_2^{23},V_2^a\}; \tag{20}$$

$$V_3^{4123}=\bar{V}_3^{4123}-\bar{l}\,(p_{32}-p_{23})_{|L_{12}=\bar{l}}\geq V_3^a; \tag{21}$$

$$V_4^{4123}=\bar{V}_4^{4123}-\bar{l}\,(p_{41}-p_{14})_{|L_{12}=\bar{l}}\geq V_4^a. \tag{22}$$

At the lower bound of each bilateral price (without contingency premiums), only (20) is not satisfied. Hence firm 1 pays firm 2 the minimum required premium price $p_{12|L_{14}=L_{23}=\bar{l}}$ such that (20) binds. Paying this premium, we still have $V_1^{4123}\geq \max\{V_1^a,V_1^{14}\}$, implying that paying the premium to prevent 2 from withdrawing is a dominating strategy for firm 1. Therefore, $L^e=[4-1-2-3]$.

Next I will show that other structures are either not stable or cannot be supported by bilateral prices, thus confirming that $[4-1-2-3]$ is the unique equilibrium. Among the links connecting 4, only $1-4$ satisfies pairwise robustness. Hence other potential candidates for equilibrium include $[4,1-2-3]$, $[4-1-3-2]$, and $[4,2-1-3]$.

In region $(\bar{z}\in[\bar{z}_2,\bar{z}_1],\delta>\delta_1(\bar{z}))$, $L^*=[1-2-3,4]$ and $\sum_1^4\bar{V}_i^{4123}<\sum_1^3\bar{V}_i^{123}+V_4^a$. To decentralize $L^*=[4,1-2-3]$, we require that firm 2 pays the premium price $p_{21|L_{14}=0,L_{23}=\bar{l}}$ to prevent 1 from linking with 4. Suppose such prices exist, we require

$$V_1^{123}=\bar{V}_1^{123}+\bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}}\geq V_1^{4123}; \tag{23}$$

$$V_2^{123}=\bar{V}_2^{123}-\bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}}+\bar{l}\,(p_{32}-p_{23})_{L_{12}=\bar{l}}\geq V_2^{4123}. \tag{24}$$

Given $\bar{l}\,(p_{32}-p_{23})_{L_{12}=\bar{l}}$ fixed under local contingency, we can show that $V_1^{123}+V_2^{123}<V_1^{4123}+V_2^{4123}$; hence, there do not exist bilateral prices that satisfy the inequalities in (23) and (24), confirming that$L^*=[4,1-2-3]$ cannot be decentralized.

In $[4-1-3-2]$, firm 3 needs to pay prices to both 1 and 2 to be connected in the risk-sharing component. Hence 3 prefers $[4-1-2-3]$ in which only one price needs to be paid. Firm 2 also prefers $[4-1-2-3]$ where its outside option is $\max\{V_2^{23},V_2^a\}$, bigger than the outside option in $[4-1-3-2]$. Therefore, $[4-1-3-2]$ is not stable.

For $[4,2-1-3]$, it is not stable in region $(\bar{z}>\bar{z}_1,\delta<\delta_1(\bar{z}))$ because both 1 and 3 obtain higher value when $1-3$ link severs and $1-4$, $2-3$ links form. In region $(\bar{z}\in[\bar{z}_2,\bar{z}_1],\delta>\delta_1(\bar{z}))$, $\sum_1^4\bar{V}_i^{4123}<\sum_1^3\bar{V}_i^{123}+V_4^a$. For $[4,2-1-3]$ to be stable, we require that both firms 2 and 3 pay premium prices $p_{21|L_{14}=0,L_{13}=\bar{l}}$ and $p_{31|L_{14}=0,L_{12}=\bar{l}}$ such that

$$V_1^{213}=\bar{V}_1^{123}+\bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}}+\bar{l}\,(p_{31}-p_{13})_{L_{14}=0,L_{12}=\bar{l}}\geq V_1^{4123}; \tag{25}$$

$$V_2^{213}=\bar{V}_2^{123}-\bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}}\geq V_2^a. \tag{26}$$

$$V_3^{213}=\bar{V}_3^{123}-\bar{l}\,(p_{31}-p_{13})_{L_{14}=0,L_{12}=\bar{l}}\geq V_3^a \tag{27}$$

(25) binding gives an lower bound for the premium prices to firm 1. However, $p_{21|L_{14}=0,L_{13}=\bar{l}}$ and $p_{31|L_{14}=0,L_{12}=\bar{l}}$ cannot be determined separately; rather, the premium price offered by 3 depends on the strategy of 2 (its counterparty's counterparty), which is not feasible under local contingency. Therefore, $[4,2-1-3]$ is not stable. Q.E.D.

Proof of Proposition 3

I will first show that $L^e = \begin{cases} [5-1-3-2-4], & \forall\delta \in [\delta_1(\bar{z}), \eta(\bar{z})] \\ [5-1-2-3,4], & \forall\delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\} \end{cases}$ is a stable network supported by bilateral prices. Notations are as in the proof of Proposition 2. To decentralize $[5-1-3-2-4]$, the bilateral prices with local contingency $(p_{51}-p_{15})_{|L_{13}}$, $(p_{31}-p_{13})_{|L_{15},L_{23}}$, $(p_{32}-p_{23})_{|L_{13},L_{24}}$, $(p_{42}-p_{24})_{|L_{23}}$ satisfy,

$$V_1^{51324} = \bar{V}_1^{51324} + \bar{l}\,(p_{31}-p_{13})_{|L_{15}=L_{23}=\bar{l}} + \bar{l}\,(p_{51}-p_{15})_{|L_{13}=\bar{l}} \geq \max\left\{V_1^a, V_1^{15}\right\}; \tag{28}$$

$$V_2^{51324} = \bar{V}_2^{51324} + \bar{l}\,(p_{32}-p_{23})_{|L_{13}=L_{24}=\bar{l}} - \bar{l}\,(p_{24}-p_{42})_{|L_{23}=\bar{l}} \geq \max\left\{V_2^a, V_2^{234}, V_2^{23}\right\}; \tag{29}$$

$$V_3^{51324} = \bar{V}_3^{51324} - \bar{l}\,(p_{31}-p_{13})_{|L_{15}=L_{23}=\bar{l}} - \bar{l}\,(p_{32}-p_{23})_{|L_{13}=L_{24}=\bar{l}} \geq \max\left\{V_3^a, V_3^{234}\right\}; \tag{30}$$

$$V_4^{51324} = \bar{V}_4^{51324} + \bar{l}\,(p_{24}-p_{42})_{|L_{23}=\bar{l}} \geq \max\left\{V_4^a, V_4^{24}\right\}; \tag{31}$$

$$V_5^{51324} = \bar{V}_5^{51324} - \bar{l}\,(p_{51}-p_{15})_{|L_{13}=\bar{l}} \geq \max\left\{V_5^a, V_5^{15}\right\}. \tag{32}$$

Here the outside options of firms 2 and 3 are derived based on the values they would obtain in a 3-firm chain network with firm 4. This directly follows from the analysis of $N=3$. If $\sum_2^4 V_i^{324} \geq \sum_2^3 V_i^{23} + V_4^a$, $[3-2-4]$ is the unique efficient equilibrium with 2 offering premium price to 3 such that $V_3^{324} = V_3^a$; otherwise, $[2-3,4]$ is the unique efficient equilibrium with 3 offering contingent premium price to 2 such that $V_2^{23} = V_2^{324}$ and $V_3^{23} > V_3^a$.

At the lower bound of each bilateral price (without premiums), only (30) is not satisfied. Pairwise stability is supported by the following: 5 pays 1 premium price $p_{51|L_{13}=\bar{l}}$ such that (32) binds; 2 pays 3 premium price $p_{23|L_{13}=L_{24}=\bar{l}}$ such that (29) binds; 1 pays 3 premium price $p_{13|L_{15}=L_{23}=\bar{l}}$ such that (30) binds.

Under these required premium prices, $\forall\delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$ we have that $V_1^{51324} > V_1^{15}$, i.e. (28) is satisfied. The curve $\eta(\bar{z})$ is defined implicitly by $V_1^{51324}\left(p_{51|L_{13}=\bar{l}}, p_{13|L_{15}=L_{23}=\bar{l}}\right) = V_1^{15}$. In sum, $L^e = [5-1-3-2-4], \forall\delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$.

In the region of $\delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\}$ we have $V_1^{51324} < V_1^{15}$, thus $1-3$ link severs: when dispersion is high, it gets too expensive for 1 to match the outside option of 3. In this region, as long as $1-3$ link severs, $2-4$ link also severs (based on the analysis of $N=3$), and $1-2$ link forms. To support this structure, premium prices include $p_{51|L_{12}=\bar{l}}$, $p_{12|L_{15}=L_{23}=\bar{l}}$, and $p_{23|L_{12}=\bar{l}}$, rendering a stable network $L^e = [5-1-2-3,4]$, $\forall\delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\}$.

The above is the unique equilibrium network because other structures are either not stable or cannot be supported by bilateral prices under local contingency. Among the links connecting 5, only $1-5$ satisfies pairwise robustness. Hence we will rule out the following structures, (a) $[5,1-3-2-4]$, (b) $[5-1-3-2,4]$, (c) $[5-1-2-3-4]$, (d) $[5-1,2-3,4]$.

(a) $[5,1-3-2-4]$ cannot be supported by bilateral prices under local contingency. Similar to the proof of Proposition 8, fixing $\bar{l}\,(p_{32}-p_{23})_{|L_{13}=L_{24}=\bar{l}}$ and $\bar{l}\,(p_{24}-p_{42})_{|L_{23}=\bar{l}}$ under local contingency, there does not exist contingent price $p_{31|L_{23}=\bar{l},L_{15}=0}$ such that $V_1^{1324} \geq V_1^{51324}$ and $V_3^{1324} \geq V_3^{51324}$.

(b) $[5-1-3-2,4]$ cannot be supported by bilateral prices under local contingency. Fixing $\bar{l}\,(p_{51}-p_{15})_{|L_{13}=\bar{l}}$ and $\bar{l}\,(p_{31}-p_{13})_{|L_{15}=L_{23}=\bar{l}}$ under local contingency, there does not exist premium price $p_{32|L_{13}=\bar{l},L_{24}=0}$ such that $V_2^{5132} \geq V_2^{23}$ and $V_3^{5132} \geq V_3^{23}$.

(c) $[5-1-2-3-4]$ is not stable. Besides that 3-4 does not satisfy pairwise robustness, we can show that 1 is not able to match the outside option of firm 2 for all parameter values of $\bar{z}$ and δ. The key is that compared to 2, firm 3 is not able to extract enough net transfer from 4.

(d) $[5-1, 2-3, 4]$ is not stable. This is because $1-2$ link here always generates positive surplus and hence will form under pairwise stability. Q.E.D.

Proof of Proposition 4

I demonstrate the result in a four-firm network setting. The case of a five-firm network is demonstrated in Figure 7 in subsection 4.2.

The inefficiency occurs in the region $(\bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z}))$, where $L^* = [4, 1-2-3]$ and $L^e = [4-1-2-3]$. The value loss equals the difference of the firm values at L^* from L^e,

$$\Delta V_{(N=4)} = \sum_1^3 V_i^{123} + V_4^a - \sum_1^4 V_i^{4123} = 4\Phi\left[2(-\bar{z})\right] - 3\Phi\left[\sqrt{3}(-\bar{z} - \frac{1}{2}\delta)\right] - \Phi\left[-\bar{z} + \frac{3}{2}\delta\right].$$

The value loss has the following properties. First, directly from proposition 2, $\Delta V > 0, \quad \forall \bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z})$. Second, the value loss increases monotonically with δ

$$\frac{\partial \Delta V}{\partial \delta} = \frac{3}{2}\frac{1}{\sqrt{2\pi}}\left(\sqrt{3}e^{-\frac{3}{2}(-\bar{z}-\frac{1}{2}\delta)^2} - e^{-\frac{1}{2}(-\bar{z}+\frac{3}{2}\delta)^2}\right) > 0, \quad \forall \bar{z} \in [0, \bar{z}_1], \delta > \delta_1(\bar{z}).$$

Third, the value loss decreases monotonically with $\bar{z}$,

$$\frac{\partial \Delta V}{\partial \bar{z}} = -8\Phi'\left[2(-\bar{z})\right] + 3\sqrt{3}\Phi'\left[\sqrt{3}(-\bar{z} - \frac{1}{2}\delta)\right] + \Phi'\left[-\bar{z} + \frac{3}{2}\delta\right] < 0.$$

Finally, the cross-derivative of value loss with respect to $\bar{z}$ and δ is negative

$$\frac{\partial^2 \Delta V}{\partial \delta \partial \bar{z}} = \frac{3}{2}\Phi''\left[-\bar{z} + \frac{3}{2}\delta\right] - \frac{9}{2}\Phi''\left[\sqrt{3}(-\bar{z} - \frac{1}{2}\delta)\right] < 0.$$

For excess systemic risk, in region $(\bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z}))$,

$$\Delta \Pr_{sys} = \Pr_{sys}^{4123} - \Pr_{sys}^{123} = \Phi\left[-2\bar{z}\right] - \Phi\left[-\sqrt{3}(\bar{z} + \tfrac{1}{2}\delta)\right].$$

First, $\forall \bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z})$, $\Delta \Pr_{sys} > 0$. Second, the excess systemic risk increases with δ in the inefficiency region.

$$\frac{\partial \Delta \Pr_{sys}}{\partial \delta} = \frac{1}{2}\sqrt{\frac{3}{2\pi}}e^{-\frac{3}{8}(2\bar{z}+\delta)^2} > 0.$$

Third, it decreases with $\bar{z}$ and decreases faster when δ is larger,

$$\frac{\partial \Delta \Pr_{sys}}{\partial \bar{z}} = \frac{\sqrt{3}e^{-\frac{3}{8}(2\bar{z}+\delta)^2} - 2e^{-2\bar{z}^2}}{\sqrt{2\pi}} < 0, \quad \forall \bar{z} \in [0, \bar{z}_1], \delta > \delta_1(\bar{z});$$

$$\frac{\partial^2 \Delta \Pr_{sys}}{\partial \delta \partial \bar{z}} = -\frac{3}{4}\sqrt{\frac{3}{2\pi}}e^{-\frac{3}{8}(2\bar{z}+\delta)^2}(2\bar{z} + \delta) < 0.$$

Q.E.D.

Proof of Proposition 5

Suppose the chain network in which firm 1 acquires firm N is $N - 1 - j - k - h - ....$ Denote G the part without N and 1 ($j - k - h - ...$). I show that contingent acquisition tax

$$\tau = (N-3)\left(\bar{V}_i^{1-G} - \bar{V}_i^{N-1-G}\right) - \left(\bar{V}_i^{N-1-G} - V_N^a\right) + \bar{l}\left(p_{N1} - p_{1N}\right) \tag{33}$$

aligns the social incentive for acquisition with that of firm 1.

Under contingent τ, in an N-firm chain network where firm 1 links with firm N, the values satisfy

$$V_1^{N-1-G}(\tau) = \bar{V}_1^{N-1-G} + \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=L_{jk}=\bar{l}} + \bar{l}(p_{N1} - p_{1N})_{|L_{1j}=\bar{l}} - \tau; \quad (34)$$

$$V_j^{N-1-G} = \bar{V}_j^{N-1-G} - \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=L_{jk}=\bar{l}} + \bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}} \geq V_j^{out}; \quad (35)$$

...

$$V_N^{N-1-G} = \bar{V}_N^{N-1-G} - \bar{l}(p_{N1} - p_{1N})_{|L_{1j}=\bar{l}} \geq V_N^{out}, \quad (36)$$

where V_i^{out} denotes the outside option for firm i under $[N-1-G]$. To keep j connected, firm 1 offers premium price $p_{1j|L_{1N}=L_{jk}=\bar{l}}$ to firm j such that (35) binds. Therefore we have

$$V_1^{N-1-G}(\tau) = \bar{V}_1^{N-1-G} + \bar{V}_j^{N-1-G} + \bar{l}(p_{N1} - p_{1N})_{|L_{1j}=\bar{l}} - \tau + \bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}} - V_j^{out}. \quad (37)$$

Absence of the acquisition of firm N, the values satisfy

$$V_1^{1-G} = \bar{V}_1^{1-G} + \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=0,L_{jk}=\bar{l}}; \quad (38)$$

$$V_j^{1-G} = \bar{V}_j^{1-G} - \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=0,L_{jk}=\bar{l}} + \bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}} \geq V_j^{out}; \quad (39)$$

...

$$V_N^{1-G} = V_N^a. \quad (40)$$

In the network $[1-G]$ with firm N isolated, firm j offers maximum premium price $p_{j1|L_{1N}=0,L_{jk}=\bar{l}}$ until (39) binds. Note that $\bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}}$ is the same as in network $[N-1-G]$ due to local contingency. We have

$$V_1^{1-G} = \bar{V}_1^{1-G} + \bar{V}_j^{1-G} + \bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}} - V_j^{out}. \quad (41)$$

Comparing (37) and (41), firm 1 chooses not to link with N if and only if $V_1^{N-1-G}(\tau)$ is lower than the value of not linking V_1^{1-G}. Plugging the value of τ in (33) the following holds.

$$L_{NN}^* = 1 \iff \sum_1^N \bar{V}_i^{N-1-G} \leq \sum_1^{N-1} \bar{V}_i^{1-G} + V_N^a \iff V_1^{N-1-G}(\tau) \leq V_1^{1-G} \iff L_{NN}^e(\tau) = 1.$$

Hence, the contingent tax τ aligns the social incentive for acquisition with that of firm 1. From the expression, the tax accounts for the negative externalities to all indirect neighbors, the positive externality to N in lowering liquidation probability of N, and the profit extracted from N by 1.

$$\tau = \underbrace{(N-3)\left(\bar{V}_i^{1-G} - \bar{V}_i^{N-1-G}\right)}_{\text{negative externalities to indirect neighbors}} - \underbrace{\left(\bar{V}_i^{N-1-G} - V_N^a\right)}_{\text{positive externality to N}} + \underbrace{\bar{l}\,(p_{N1} - p_{1N})}_{\text{profit extracted from N}}.$$

Plugging in the values, we see that τ is characterized by the cross-sectional distribution of firm distress levels $\{N, \bar{z}, \delta\}$.

$$\tau = (N-3)\left(\frac{1}{2}\delta\sigma + \frac{N-2}{N-3}\Phi\left[\sqrt{N}\,(-\bar{z})\right]c - \Phi\left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta)\right]c\right) - \frac{1}{2}P_1 c - \frac{1}{2}P_N c.$$

When $L_{NN}^* = 1$, we have $\tau > 0$. Further more, $\frac{\partial\tau}{\partial\bar{z}} < 0$, $\quad \frac{\partial\tau}{\partial\delta} > 0$. Hence, τ decreases with mean $\bar{z}$ and increases with dispersion δ due to the properties of the derivatives. Q.E.D.

Online Appendix to Distress Dispersion and Systemic Risk in Networks

In this Online Appendix, I provide technical results on the optimal risk sharing allocation, the full contingent contracts, and a model extended with government interventions. I also provide additional empirical results.

A Technical Appendix

A.1 Optimal Risk Sharing Allocation

This section provides technical results for subsection 3.1. I show that the asset holdings implied by the optimal network L^* are equivalent to if the social planner were to choose asset allocations directly.

Definition 3 *Let $\mathcal{H}$ be an asset holding matrix such that firms' liquid asset holdings are $h = \mathcal{H}a$. The optimal asset allocation $\mathcal{H}^*$ is feasible and minimizes total expected liquidation costs,*

$$\mathcal{H}^* = \arg\min_{\mathcal{H}} \sum_{1}^{N} \Pr\left(h_i < 1\right) c, \tag{P2}$$

subject to the feasibility constraint $\mathcal{H} \times \mathbb{1}_{N\times 1} = \mathcal{H}^\top \times \mathbb{1}_{N\times 1} = \mathbb{1}_{N\times 1}$.

$\mathcal{H}$ being a doubly stochastic matrix ensures that no assets are created or lost from asset pooling and that each firm holds one unit of asset. The following lemma characterizes the optimal asset allocation.

Lemma 3 *If $\nexists i$ with $h_i^* = a_i$, then $h_i^* = \frac{1}{N}\sum_{j=1}^{N} a_j, \forall i$. If $\exists i$ with $h_i^* = a_i$ and $h_{i-1}^* \neq a_{i-1}$, then $h_j^* = a_j$, $\forall j \geq i$ and $h_j^* = \frac{1}{i-1}\sum_{k=1}^{i-1} a_k$, $\forall j \leq i-1$.*

Proof Let M be the number of firms that participate in risk-sharing (hold diversified assets). The total expected liquidation costs equal

$$\sum_{i=1}^{M} \Pr(h_i \leq 1) = \sum_{i=1}^{M} \Phi\left(\frac{-\mathcal{H}_{ii} z_i - \sum_j \mathcal{H}_{ij} z_j}{\sqrt{\mathcal{H}_{ii}^2 + \sum_j \mathcal{H}_{ij}^2}}\right). \tag{42}$$

Take the first order condition with respect to $\mathcal{H}_{ij}$,

$$\frac{\partial \sum_{i=1}^{M} \Pr(h_i \leq 1)}{\partial \mathcal{H}_{ij}} = \frac{\partial \Pr(h_i \leq 1)c}{\partial \mathcal{H}_{ij}} + \frac{\partial \Pr(h_j \leq 1)c}{\partial \mathcal{H}_{ji}}. \tag{43}$$

Notice $\mathcal{H}_{ii} = 1 - \sum_j \mathcal{H}_{ij}$ so the first term equals

$$\frac{\partial \Pr(h_i \leq 1)c}{\partial \mathcal{H}_{ij}} = \Phi'\left(\frac{-\mathcal{H}_{ii} z_i - \sum_j \mathcal{H}_{ij} z_j}{\sqrt{\mathcal{H}_{ii}^2 + \sum_j \mathcal{H}_{ij}^2}}\right) c \times$$

$$\frac{(z_i - z_j)\sqrt{\mathcal{H}_{ii}^2 + \sum_j \mathcal{H}_{ij}^2} - \left(-\mathcal{H}_{ii} z_i - \sum_j \mathcal{H}_{ij} z_j\right)\left(\mathcal{H}_{ii}^2 + \sum_j \mathcal{H}_{ij}^2\right)^{-\frac{1}{2}} (\mathcal{H}_{ij} - \mathcal{H}_{ii})}{\mathcal{H}_{ii}^2 + \sum_j \mathcal{H}_{ij}^2}.$$

Similarly, write out the term for firm j take derivative with respect to $\mathcal{H}_{ji} = \mathcal{H}_{ij}$, and plug $\frac{\partial \Pr(h_j \leq 1)c}{\partial \mathcal{H}_{ji}}$ into equation (43), we have

$$\frac{\partial \sum_{i=1}^{M} \Pr(h_i \leq 1)}{\partial \mathcal{H}_{ij}} |_{\mathcal{H}_{ii}=\mathcal{H}_{ij}=\frac{1}{M}} = 0, \quad \forall i \neq j.$$

The first order conditions with respect to asset holdings equal zero when each element of $\mathcal{H}$ is evaluated at $\frac{1}{M}$, thus achieving the optimal allocation. $\mathcal{H}^*_{ij} = \frac{1}{M}$ indicates full risk-sharing. It is worth noting that the only condition required for above results is that ε_i is independently distributed across firms. So the above result holds if we relabel Φ as a rather general distribution function. Q.E.D.

Lemma 3 states that if all firms diversify, they each hold the equally weighted asset $\frac{1}{N}\sum_{j=1}^{N} a_j$; if there are firms not diversifying, then more distressed firms stay separate and liquid firms diversify holding the equally weighted assets composed of all firms diversifying. As such, the optimal asset holdings boil down to determining who should diversify.

Recall from Lemma 1, the asset holdings implied by the optimal network $\underset{K\to\infty}{lim} (L^*)^K$ coincides with full risk-sharing among all connected firms. In this regard, under the infinite iterative asset swap, the optimal network L^* in (P1) achieves the best asset allocation matrix $\mathcal{H}^*$ in (P2). Hence, the network itself does not deviate risk sharing from the optimal outcome.

A.2 Towards Generalizing to Arbitrary N

This section provides technical results for subsection 3.4. The following proposition shows that the result of network inefficiency holds for a general case with arbitrary $N \geq 4$. This result extends Proposition 3 showing that as dispersion increases, the chain network has initially over-connection and then inefficient composition.

Proposition 6 *For $N \geq 5$, $\exists\ \eta(\bar{z}, N)$ such that the equilibrium chain network features over-connection $\forall \bar{z} \in [\bar{z}_2, \bar{z}_1], \delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$, and inefficient composition $\forall \bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \max\{\delta_2(\bar{z}), \eta(\bar{z})\}$, where $\bar{z}_1, \bar{z}_2, \delta_1(\bar{z}), \delta_2(\bar{z})$ are defined in Proposition 1.*

Proof Suppose the chain network where all firms are connected is $[N-1-G] = [N-1-j-k-...]$. Here firm N is on one end of the chain linked with firm 1 because of pairwise robustness. The proof is based on induction.

First, we have shown in Proposition 3 that the result holds for $N = 5$.

Second, suppose the result holds for $N-2$, where $N-2 > 5$, next we want to show that it also holds for N. In the N-firm chain network, the prices satisfy

$$V_1^{N-1-G} = \bar{V}_1^{N-1-G} + \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=L_{jk}=\bar{l}} + \bar{l}(p_{N1} - p_{1N})_{|L_{1j}=\bar{l}};$$
$$V_j^{N-1-G} = \bar{V}_j^{N-1-G} - \bar{l}(p_{j1} - p_{1j})_{|L_{1N}=L_{jk}=\bar{l}} + \bar{l}(p_{kj} - p_{jk})_{|L_{1j}=L_{kh}=\bar{l}} \geq V_j^{out};$$
$$...$$
$$V_N^{N-1-G} = \bar{V}_N^{N-1-G} - \bar{l}(p_{N1} - p_{1N})_{|L_{1j}=\bar{l}} \geq V_N^{out}.$$

When δ is small, contingent premium prices are paid such that the outside options of both firms j and k are matched. As δ increases, the outside option of j gets high (following the result of $N-2$) so 1-j link severs. Notice that the remaining ($[j-k-...]$) fits precisely the case of $N-2$ given the equally-spaced structure of the z vector. (For example, in the case of N firms, $z_2 = \bar{z} + \frac{N-3}{2}\delta$; in the case of $N-2$ firms, $z_1 = \bar{z} + \frac{N-3}{2}$.) If $[j-k-...]$ is stable, $[N-1, j-k-...]$ gives the equilibrium network; or else, we analyze if any of the segmented components of $[j-k-...]$ can connect with firm 1, until a stable structure is found. In either case, there is both over-connection and loss of risk-sharing. Q.E.D.

A.3 Full Contingent Contracts

This section provides technical results for subsection 3.5. A set of contracts that are contingent on the entire network structure decentralizes the efficient network.

Proposition 7 *The optimal network is decentralized by a set of bilateral prices contingent on the entire network structure* $p(z, L)$*.*

Proof For $N = 2$, the pairwise Nash stable network under bilateral prices is by construction the efficient network.

For $N = 3$, the pairwise Nash stable network under bilateral local contingent prices is unique and coincides with the optimal network. I solve for $\{p_{ij}|_{L_{ik},L_{jk}}, p_{ji}|_{L_{ik},L_{jk}}\}$, which has dimension $2\times\binom{N}{2}\times 2^{N-2}\times 2^{N-2}$. These prices support the efficient network but not the inefficient networks. In particular, when $L^* = [3-1-2]$, firm 1 extracts profit from 3 and offers contingent premium price to 2; when $L^* = [1-2, 3]$, firm 2 offers premium price to 1 contingent on $L_{13} = 0$.

Next we focus on $N = 4$. It is equivalent to show that the bilateral prices decentralize $L^e = L^* = [4-1-2-3]$ in region $(\bar{z} > \bar{z}_1, \delta < \delta_1(\bar{z}))$, and $L^e = L^* = [1-2-3, 4]$ in region $(\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z}))$. We follow the same notation as in the proof of Proposition 2. Different than in Proposition 2, the bilateral prices with contingency on faraway links are denoted as follows. (Here we only solve for those relevant for the pairwise stability and abstract from price contingency on L_{34} which does not satisfy pairwise robustness.)

$$(p_{41} - p_{14})_{|L_{12}=L_{23}=0}\,, (p_{41} - p_{14})_{|L_{12}=\bar{l},L_{23}=0}\,, (p_{41} - p_{14})_{|L_{12}=0,L_{23}=\bar{l}}\,, (p_{41} - p_{14})_{|L_{12}=L_{23}=\bar{l}}\,;$$
$$(p_{21} - p_{12})_{|L_{14}=L_{23}=0}\,, (p_{21} - p_{12})_{|L_{14}=0,L_{23}=\bar{l}}\,, (p_{21} - p_{12})_{|L_{14}=\bar{l},L_{23}=0}\,, (p_{21} - p_{12})_{|L_{14}=L_{23}=\bar{l}}\,;$$
$$(p_{32} - p_{23})_{|L_{14}=L_{12}=0}\,, (p_{32} - p_{23})_{|L_{14}=0,L_{12}=\bar{l}}\,, (p_{32} - p_{23})_{|L_{14}=\bar{l},L_{12}=0}\,, (p_{32} - p_{23})_{|L_{14}=L_{12}=\bar{l}}\,.$$

In region $(\bar{z} > \bar{z}_1, \delta < \delta_1(\bar{z}))$, $L^* = [4-1-2-3]$ and $\sum_1^4 \bar{V}_i^{4123} \geq \sum_1^3 \bar{V}_i^{123} + V_4^a$. In order to decentralize the full risk sharing network, prices satisfy

$$V_1^{4123} = \bar{V}_1^{4123} + \bar{l}\,(p_{21} - p_{12})_{|L_{14}=L_{23}=\bar{l}} + \bar{l}\,(p_{41} - p_{14})_{|L_{12}=L_{23}=\bar{l}} \geq \max\{V_1^{14}, V_1^a\}; \tag{44}$$
$$V_2^{4123} = \bar{V}_2^{4123} - \bar{l}\,(p_{21} - p_{12})_{|L_{14}=L_{23}=\bar{l}} + \bar{l}\,(p_{32} - p_{23})_{|L_{14}=L_{12}=\bar{l}} \geq \max\{V_2^{23}, V_2^a\}; \tag{45}$$
$$V_3^{4123} = \bar{V}_3^{4123} - \bar{l}\,(p_{32} - p_{23})_{|L_{14}=L_{12}=\bar{l}} \geq \max\{V_3^a, V_3^{34}\}; \tag{46}$$
$$V_4^{4123} = \bar{V}_4^{4123} - \bar{l}\,(p_{41} - p_{14})_{|L_{12}=L_{23}=\bar{l}} \geq \max\{V_4^a, V_4^{34}\}. \tag{47}$$

Contingent premium prices that support $L^e = L^* = [4-1-2-3]$ include $p_{41|L_{12}=L_{23}=\bar{l}}$ such that (47) binds, $p_{32|L_{14}=L_{12}=\bar{l}}$ such that (46) binds, $p_{12|L_{14}=L_{23}=\bar{l}}$ such that (45) binds.

In region $(\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z}))$, $L^* = [1-2-3, 4]$ and $\sum_1^4 \bar{V}_i^{4123} < \sum_1^3 \bar{V}_i^{123} + V_4^a$. To decentralize $L^* = [4, 1-2-3]$, we require that firm 2 pays a premium price $p_{21|L_{14}=0,L_{23}=\bar{l}}$ and firm 3 pays a premium price $p_{32|L_{14}=0,L_{12}=\bar{l}}$ such that

$$V_1^{123} = \bar{V}_1^{123} + \bar{l}\,(p_{21} - p_{12})_{L_{14}=0,L_{23}=\bar{l}} \geq V_1^{4123}; \tag{48}$$
$$V_2^{123} = \bar{V}_2^{123} - \bar{l}\,(p_{21} - p_{12})_{L_{14}=0,L_{23}=\bar{l}} + \bar{l}\,(p_{32} - p_{23})_{|L_{14}=0,L_{12}=\bar{l}} \geq V_2^{4123}. \tag{49}$$

Paying the premium, firm 3 has value,

$$V_3^{123} = \bar{V}_3^{123} - \bar{l}\,(p_{32} - p_{23})_{|L_{14}=0,L_{12}=\bar{l}} = V_3^{4123} + \sum_1^3 \bar{V}_i^{123} + V_4^a - \sum_1^4 \bar{V}_i^{4123}.$$

Hence, $V_3^{123} \geq V_3^{4123} \iff \sum_1^3 \bar{V}_i^{123} + V_4^a \geq \sum_1^4 \bar{V}_i^{4123}$ ($L^* = [1-2-3,4]$). Only then 3 offers the contingent prices and firm 1 disconnects L_{14}, i.e. the equilibrium switches from $[4-1-2-3]$ to $[1-2-3,4]$.

Finally, we can confirm that $[4-1-2,3]$ is not a potential deviation in either region for all possible $(p_{21}-p_{12})_{|L_{14}=\bar{l},L_{23}=0}$ and $(p_{41}-p_{14})_{|L_{12}=\bar{l},L_{23}=0}$ because $V_1^{4123}+V_2^{4123}+V_4^{4123} \geq V_1^{412}+V_2^{412}+V_4^{412}$ for $(\bar{z}>\bar{z}_1, \delta<\delta_1(\bar{z}))$, and $V_1^{123}+V_2^{123}+V_4^a \geq V_1^{412}+V_2^{412}+V_4^{412}$ for $(\bar{z}\in[\bar{z}_2,\bar{z}_1], \delta>\delta_1(\bar{z}))$.

When the length of the chain increases, prices have more dimensions of contingencies and can be solved in a similar fashion. Q.E.D.

A.4 Sufficient State Variables

This section supports subsection 3.6. For the case of four-firm chain, the optimal network can be decentralized under pairwise Nash stability by bilateral prices contingent on local links and local net transfers, $p_{ij}\left(z,\, L_i\left(p_{i\cdot}-p_{\cdot i}^T\right),\, L_j\left(p_{j\cdot}-p_{\cdot j}^T\right)\right)$.

We follow the earlier notations. It is equivalent to show that the bilateral prices decentralize $L^e = L^* = [4-1-2-3]$ in region $(\bar{z}>\bar{z}_1, \delta<\delta_1(\bar{z}))$, and $L^e = L^* = [1-2-3,4]$ in region $(\bar{z}\in[\bar{z}_2,\bar{z}_1], \delta>\delta_1(\bar{z}))$. The bilateral prices are

$$(p_{41}-p_{14})_{|L_{12}=0},\, (p_{41}-p_{14})_{|\bar{l}(p_{21}-p_{12})},\, (p_{32}-p_{23})_{|L_{12}=0},\, (p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})}$$
$$(p_{21}-p_{12})_{|L_{14}=L_{23}=0},\, (p_{21}-p_{12})_{|L_{14}=0,\bar{l}(p_{32}-p_{23})},\, (p_{21}-p_{12})_{|\bar{l}(p_{41}-p_{14}),L_{23}=0},\, (p_{21}-p_{12})_{|\bar{l}(p_{32}-p_{23}),\bar{l}(p_{41}-p_{14})}.$$

In region $(\bar{z}>\bar{z}_1, \delta<\delta_1(\bar{z}))$, to decentralize $L^* = [4-1-2-3]$, prices satisfy

$$V_1^{4123} = \bar{V}_1^{4123} + \bar{l}\,(p_{21}-p_{12})_{|\bar{l}(p_{32}-p_{23}),\bar{l}(p_{41}-p_{14})} + \bar{l}\,(p_{41}-p_{14})_{|\bar{l}(p_{21}-p_{12})} \geq \max\{V_1^{14}, V_1^a\}; \tag{50}$$
$$V_2^{4123} = \bar{V}_2^{4123} - \bar{l}\,(p_{21}-p_{12})_{|\bar{l}(p_{32}-p_{23}),\bar{l}(p_{41}-p_{14})} + \bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} \geq \max\{V_2^{23}, V_2^a\}; \tag{51}$$
$$V_3^{4123} = \bar{V}_3^{4123} - \bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} \geq \max\{V_3^a, V_3^{34}\}; \tag{52}$$
$$V_4^{4123} = \bar{V}_4^{4123} - \bar{l}\,(p_{41}-p_{14})_{|\bar{l}(p_{21}-p_{12})} \geq \max\{V_4^a, V_4^{34}\}. \tag{53}$$

(52) and (53) binding gives $\bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} = \bar{V}_3^{4123} - V_3^a$ and $\bar{l}\,(p_{41}-p_{14})_{|\bar{l}(p_{21}-p_{12})} = \bar{V}_4^{4123} - V_4^a$. Plugging these two into the binding equation in (51) (and combining with $V_2^{23} = \max[V_2^{23}, V_2^a, V_2^{234}]$) gives $\bar{l}\,(p_{21}-p_{12})_{|\bar{l}(p_{32}-p_{23}),\bar{l}(p_{41}-p_{14})} = \bar{V}_2^{4123} + \bar{V}_3^{4123} - V_2^{23} - V_3^a$.

In region$(\bar{z}\in[\bar{z}_2,\bar{z}_1], \delta>\delta_1(\bar{z}))$, to decentralize $L^* = [1-2-3,4]$, we require that firm 2 and 3 both pay premium prices such that

$$V_1^{123} = \bar{V}_1^{123} + \bar{l}\,(p_{21}-p_{12})_{|L_{14}=0,\bar{l}(p_{32}-p_{23})} \geq V_1^{4123}; \tag{54}$$
$$V_2^{123} = \bar{V}_2^{123} - \bar{l}\,(p_{21}-p_{12})_{|L_{14}=0,\bar{l}(p_{32}-p_{23})} + \bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} \geq V_2^{4123}. \tag{55}$$

(54) and (55) binding give $\bar{l}\,(p_{21}-p_{12})_{|L_{14}=0,\bar{l}(p_{32}-p_{23})} = V_1^{4123} - \bar{V}_1^{123} = \sum_1^4 \bar{V}_i^{4123} - V_2^{23} - V_3^a - V_4^a - \bar{V}_1^{123}$ and $\bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} = V_1^{4123} + V_2^{4123} - \bar{V}_1^{123} - \bar{V}_2^{123} = \sum_1^4 \bar{V}_i^{4123} - V_3^a - V_4^a - \bar{V}_1^{123} - \bar{V}_2^{123}$. Firm 3 has value,

$$V_3^{123} = \bar{V}_3^{123} - \bar{l}\,(p_{32}-p_{23})_{|\bar{l}(p_{21}-p_{12})} = \bar{V}_3^{123} - V_1^{4123} - V_2^{4123} + \bar{V}_1^{123} + \bar{V}_2^{123} = V_3^{4123} + \sum_1^3 \bar{V}_i^{123} + V_4^a - \sum_1^4 \bar{V}_i^{4123}.$$

Hence, $V_3^{123} \geq V_3^{4123} \iff \sum_1^3 \bar{V}_i^{123} + V_4^a \geq \sum_1^4 \bar{V}_i^{4123}$ ($L^* = [1-2-3,4]$). Only under this condition, firm 3 offers the contingent prices and firm 1 disconnects L_{14}. Thereby, the equilibrium switches from $[4-1-2-3]$ to $[1-2-3,4]$. In summary the critical bilateral price is $p_{32}-p_{23}$ and is given by,

$$\bar{l}\,(p_{32}-p_{23})_{|L_{12}(p_{21}-p_{12})} = \begin{cases} V_2^a - V_3^a, & L_{12}=0 \\ \bar{V}_3^{4123} - V_3^a, & L_{12}=\bar{l}, \bar{l}\,(p_{21}-p_{12}) = \sum_2^3 \bar{V}_2^{4123} - V_2^{23} - V_3^a \\ \sum_1^4 \bar{V}_i^{4123} - V_3^a - V_4^a - \sum_1^2 \bar{V}_i^{123}, & L_{12}=\bar{l}, \bar{l}\,(p_{21}-p_{12}) = \sum_1^4 \bar{V}_i^{4123} - V_4^a - V_2^{23}. \\ & \quad -V_3^a - \bar{V}_1^{123} \end{cases}$$

A.5 Contingent Contracts in Star Networks

Suppose that full risk sharing is decentralized in a star network, as shown in Figure 6 Panel A, we next show that when dispersion is high, isolating the distressed firm 4 still may not be decentralized under local contingency prices.

Based on the star network, the bilateral prices under local contingency include $(p_{41}-p_{14})_{|L_{12},L_{13}}$, $(p_{21}-p_{12})_{|L_{14},L_{13}}$, and $(p_{31}-p_{13})_{|L_{14},L_{12}}$. In region $(\bar{z} > \bar{z}_1, \delta < \delta_1(\bar{z}))$, to decentralize the full risk sharing network, prices satisfy

$$V_1^{star} = \bar{V}_1^{star} + \bar{l}\,(p_{21}-p_{12})_{|L_{14}=L_{13}=\bar{l}} + \bar{l}\,(p_{31}-p_{13})_{|L_{12}=L_{14}=\bar{l}} + \bar{l}\,(p_{41}-p_{14})_{|L_{12}=L_{13}=\bar{l}} \quad (56)$$

$$V_2^{star} = \bar{V}_2^{star} - \bar{l}\,(p_{21}-p_{12})_{|L_{14}=L_{13}=\bar{l}} \geq \max\{V_2^{23}, V_2^a, V_2^{234}\} \quad (57)$$

$$V_3^{star} = \bar{V}_3^{star} - \bar{l}\,(p_{31}-p_{13})_{|L_{12}=L_{14}=\bar{l}} \geq \max\{V_3^a, V_3^{34}\} \quad (58)$$

$$V_4^{star} = \bar{V}_4^{star} - \bar{l}\,(p_{41}-p_{14})_{|L_{12}=L_{13}=\bar{l}} \geq \max\{V_4^a, V_4^{34}\} \quad (59)$$

(57), (58), and (59) binding give $(p_{21}-p_{12})_{|L_{14}=L_{13}=\bar{l}}$, $(p_{31}-p_{13})_{|L_{12}=L_{14}=\bar{l}}$, and $(p_{41}-p_{14})_{|L_{12}=L_{13}=\bar{l}}$. The equilibrium replicates the optimal connection.

However, in region $(\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z}))$, $L^* = [2-1-3,4]$ and $\sum_1^4 \bar{V}_i^{4123} < \sum_1^3 \bar{V}_i^{123} + V_4^a$. To decentralize L^*, we require that both firms 2 and 3 pay premium price $p_{21|L_{14}=0,L_{13}=\bar{l}}$ and $p_{31|L_{14}=0,L_{12}=\bar{l}}$ such that

$$V_1^{213} = \bar{V}_1^{123} + \bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}} + \bar{l}\,(p_{31}-p_{13})_{L_{14}=0,L_{12}=\bar{l}} \geq V_1^{star}; \quad (60)$$

$$V_2^{213} = \bar{V}_2^{123} - \bar{l}\,(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}} \geq V_2^{star}. \quad (61)$$

$$V_3^{213} = \bar{V}_3^{123} - \bar{l}\,(p_{31}-p_{13})_{L_{14}=0,L_{12}=\bar{l}} \geq V_3^{star} \quad (62)$$

(60) binding gives an lower bound for $p_{21|L_{14}=0,L_{13}=\bar{l}} + p_{31|L_{14}=0,L_{12}=\bar{l}}$. However, $(p_{21}-p_{12})_{L_{14}=0,L_{23}=\bar{l}}$ and $(p_{31}-p_{13})_{L_{14}=0,L_{12}=\bar{l}}$ cannot be determined separately; rather, the premium price offered by 3 depends on the strategy of 2 who is not a direct neighbor and thus is not feasible under local contingency.

A.6 Extension with *Ex Post* Government Interventions

Several acquisition cases observed during the recent financial crisis render the baseline model counterfactual, e.g. the acquisitions of Bear Stearns, Merrill Lynch, and National City. These cases differ from the baseline setting in several dimensions. First, links with the target institutions were formed before the distress conditions fully realized. Second, government interventions such as bailout or subsidies took place. For instance, when two of Bear Stearns' funds failed in 2007, it already had many counterparties, most of whom remained in the counterparty relationship. When Bear Stearns suffered severe financial distress on March 2008, the Fed provided assistance in the form of a $29 billion non-recourse loan to JP Morgan to make the acquisition. To rationalize such observed government interventions, I next extend the baseline model, and the key deviation is that the timing of the network formation does not coincide with the observation of distress.

Suppose links cannot be severed once formed at $t = 1$ when ν is learned. Further, assume that the value of liquid asset is,

$$a_i = \nu_i + \theta_i + \sigma\varepsilon_i,\ i = 1, ..., N, \tag{63}$$

where the additional term θ_i is realized at $t = 1\frac{1}{2}$ after links are formed. Hence, ν_i and θ_i jointly determine the chances of liquidation at $t = 2$. We focus on the over-connection region where $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta \in [\delta_1(\bar{z}), \eta(\bar{z})]$ such that firm N should be isolated; nonetheless, all firms are connected at equilibrium in the absence of a tax (Propositions 1 and 2). Now, assume firm N receives a second negative liquidity shock. Let θ be a vector with $\theta_i = 0$, $\forall i = 1, ..., N-1$, and $\theta_N = -k\bar{z}\sigma$,[45] where $k > N$ such that it drags the average firm distress down below zero. In this case, links do not create positive surplus from risk-sharing any more.

A.6.1 Government Bailout

Next I analyze conditions when government bailout is *ex post* ($t = 1\frac{1}{2}$) optimal and how total costs compare to those under the *ex ante* optimal policies (imposing acquisition tax at $t = 1$). To this end, we allow for the option of government bailout in the form of costly liquidity injection. Specifically, let $B\sigma$ denote the amount of government liquidity injection to the heavily distressed firm N. Since all firms are connected and each holds the same diversified assets, they share the same probability of liquidation $\Phi\left[\sqrt{N}(-\bar{z} + \frac{k\bar{z}-B}{N})\right]$. Total costs here include expenses in both liquidation and bailout.

Government liquidity injection that covers at least total liquidity shortfall ($B^* > (k-N)\bar{z}$) is *ex post* optimal in an over-connected network as long as the liquidation cost is not very small, $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$. (See this Appendix A.6, Proposition 8.) This lower bound is smaller when the distressed firm has more counterparties or when asset volatility is lower. Now, if θ_N is not sufficiently bad so $k \leq N$, the lower bound of liquidation cost to justify government bailout is higher. In other words, the worse shock the connected banking system gets, the more likely government bailout is *ex post* optimal. This relation is consistent with the empirical observation that bailout only occurs in rare occasions with severe distress.

Even though government bailout can be *ex post* optimal, I show that it is generally more costly (as long as the cost of bailout is not very low) than *ex ante* preventing the acquisition (see Proposition 9). This result captures one critical issue in the current policy-making: the time-inconsistency problem.[46] A liquid firm makes profit by acquiring a distressed target while generating externalities. Precisely owing to the excess acquisition link, liquidation of the distressed firm gets too costly. In consequence, government bailout becomes *ex post efficient* and *ex ante inefficient*.

A.6.2 Government Subsidized Acquisition

Back to the Bear Stearns case, instead of directly injecting liquidity, the Fed extended a non-recourse loan to the acquirer JP Morgan.[47] With a slight variation, the extended model can

[45] In practice, distress signals are released gradually. The negative θ_N captures persistence in liquidity conditions.

[46] For discussions on the time-inconsistency issue, see Acharya and Yorulmazer (2007), Spatt (2009), and Chari and Kehoe (2015).

[47] The New York Fed initially agreed to provide a $25 billion collateralized loan to Bear Stearns for up to 28 days, but later decided that the loan was unavailable to them. This evidence showed that government bailout was not preferred.

rationalize this behavior. I show that, when there exist healthier institutions currently not connected with the distressed firm, government subsidized acquisition is *ex post* optimal.

Consider another group of firms connected among themselves but are separate from the existing ones. They are N firms $i = N+1, ..., 2N$ with the same average $\bar{z} > 0$ and dispersion $\delta = 0$ (without loss of generality), such that a complete risk-sharing network optimally emerges. Let the additional signal be $\theta_{N+1} = \hat{k}\bar{z}\sigma$ and $\theta_i = 0$, $\forall i = N+2, ..., 2N$, so firm $N+1$ gets a positive shock in the liquid return. Next I analyze whether firm $N+1$ has incentive to acquire the distressed firm N after learning θ, and whether the acquisition is *ex post* socially optimal.

The answer depends on how the liquidity surplus of firm $N+1$ compares with the liquidity shortage of firm N. Shown in Corollary 1 of this Appendix A.6, the acquisition is *ex post* socially optimal and occurs at equilibrium if and only if the average distress is above zero ($\hat{k} + 2N > k$). Otherwise, the acquisition has negative surplus, and firm $N+1$ does not have incentive to link with N. In this case, subsidized acquisition via liquidity injection to the acquirer is optimal if the liquidation cost is not very small ($c > \frac{\sqrt{\pi}\sigma}{\sqrt{N}}$). The intuition is that risk-sharing among the two groups of firms can reduce liquidation costs only when total expected liquidity is positive. Government subsidy can push the average liquidity above zero. The required government subsidy is lower when the positive liquidity shock of the potential acquirer is higher. This result rationalizes the observation that subsidized acquirers during the financial crisis are relatively liquid firms, such as JP Morgan and PNC (respectively acquirers of Bear Stearns and National City).

Comparing the two *ex post* policy remedies, subsidized acquisition is less costly than bailout, thus is always preferred. Nonetheless, if the excess link with the distressed firm has been prevented in the first place, liquidation would not be as expensive; hence, neither subsidized acquisition nor bailout would be necessary.

A.6.3 Government Pushed Acquisition

I have shown that when the two groups have the same cardinality, the acquisition link forms at equilibrium if and only if it generates positive social surplus. However, this condition does not hold when the two groups differ in cardinality. Essentially, the relative cardinality of the two groups determines the sign of the bilateral surplus which further implies whether or not the *ex post* acquisition occurs at equilibrium. When the potential acquirer firm in the second group has more counterparties, there are more firms to share the cost of acquisition than firms in the first group to share the benefit. The bilateral surplus from the acquisition is greater than the social surplus; hence, the acquisition link forms *ex post* whenever it is socially valuable. When the potential acquirer firm in the second group has fewer counterparties, the bilateral surplus is smaller than the social surplus. The bilateral surplus can be negative when the social surplus is positive; thus, the acquisition might not occur at equilibrium even if it is *ex post* socially optimal. In such circumstances, government pushed acquisition is recommended (see Proposition 10 of this Appendix A.6).

There are many ways in which a government intervention can take place. One approach is by exerting pressure to candidate acquirers. Examples include the Fed pressuring Bank of America to acquire the distressed Merrill Lynch.[48] The regulators can also subsidize the acquirer using funds collected from the counterparties of the distressed firm. Alternatively, the regulators can provide a coordination device for collective decision-making: let the potential acquirer and all

[48] As discussed in Spatt (2010), "secretary of the Treasury Henry Paulson indicated to [Bank of America CEO] Lewis that banking supervisors would question his suitability to lead Bank of America if BoA backed out of the merger and then needed more federal support, while federal authorities agreed to provide 'ring-fencing' of difficult to value Merrill Lynch assets if Bank of America went ahead with the merger."

the counterparties of the distressed firm bargain over the payments. One such example is the initiation of collective bailout of LTCM by the New York Fed in 1998.[49]

A.6.4 Technical Results for the Above Extension

This section contains the technical results for the above extended model. Under the set up of the extended model, if the regulators had optimally isolated the distressed firm N at $t = 1$, total liquidation costs are

$$C_{\text{iso-N}} = (N-1)\,\Phi\left[\sqrt{N-1}\left(-\bar{z} - \frac{1}{2}\delta\right)\right] c + \Phi\left[(k-1)\,\bar{z} + \frac{N-1}{2}\delta\right] c. \tag{64}$$

In the absence of the acquisition tax, all firms are connected and the liquidation costs are

$$C = \sum_{i=1}^{N} \Pr\left(\tilde{h}_i < 1\right) c = N\Phi\left[\sqrt{N}\left(-\bar{z} + \frac{k\bar{z}}{N}\right)\right] c. \tag{65}$$

When we enable *ex post* government bailout as in Section A.6.1, the costs include expenses from both liquidation and bailout,

$$C_{GB} = \sum_{i=1}^{N} \Pr\left(\tilde{h}_i < 1\right) c + B\sigma = N\Phi\left[\sqrt{N}(-\bar{z} + \frac{k\bar{z} - B}{N})\right] c + B\sigma. \tag{66}$$

Notice that $C = C_{GB}\,(B = 0)$. The net gain from government bailout is $C - C_{GB}$. The next proposition shows that as long as the liquidation cost c is not very small, a positive government bailout that at least covers the total expected liquidity shortfall is *ex post* optimal.

Proposition 8 *If $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, $k > N$, government bailout in the form of liquidity injection of $B^*\sigma$ generates positive surplus, where*

$$B^* = (k-N)\bar{z} + \sqrt{N}\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}. \tag{67}$$

Proof The optimal liquidity injection B^* minimizes total costs C_{GB} and thus satifies the first order condition $\frac{\partial C_{GB}}{\partial B} = 0$, i.e. $N\Phi'\left[\sqrt{N}(-\bar{z} + \frac{k\bar{z}-B}{N})\right] c\left(-\frac{1}{\sqrt{N}}\right) + \sigma = 0$. This gives

$$\Phi'\left[\sqrt{N}(-\bar{z} + \frac{k\bar{z} - B^*}{N})\right] = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(k-N)\bar{z} - B^*}{\sqrt{N}}\right]^2} = \frac{\sigma}{\sqrt{N}c}. \tag{68}$$

Solving for B^* gives (67) which naturally implies $B^* > (k-N)\,\bar{z}$. Given $e^{-\frac{1}{2}\left[\frac{(k-N)\bar{a}_0 - B^*}{\sqrt{N}}\right]^2} \le 1$, (68) implies $\frac{\sigma}{\sqrt{N}c} \le \frac{1}{\sqrt{2\pi}} \Rightarrow c \ge \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$ That is, for B^* to be an interior solution, the liquidation cost cannot be very small.

Further, in order that B^* archives the global minimum of $C_{GB}(B)$, we take the second derivative of B,

$$\frac{\partial^2 C_{GB}}{\partial B^2} = c\Phi''\left[\sqrt{N}(-\bar{z} + \frac{k\bar{z} - B^*}{N})\right] \ge 0, \quad \forall\,(k-N)\,\bar{z} - B^* \le 0.$$

The second derivative is positive which ensures that B^* archives the global minimum of $C_{GB}(B)$, so the bailout surplus is positive, i.e., $C - C_{GB} = C_{GB}(B = 0) - C_{GB}(B = B^*) > 0$. Q.E.D.

[49] On Sept 23 1998, the New York Fed arranged a meeting for a group of LCTM's major creditors at one of its conference rooms. During this historic meeting, the creditors worked out a restructuring deal that recapitalized LTCM and avoided its bankruptcy.

A few comments on the above result. First, from equation (67), $B^*\sigma > (k-N)\bar{z}\sigma$, so $B^*\sigma$ at least matches the total expected liquidity shortfall. The extra liquidity injection, $\sqrt{N}\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}$, depends on the trade-off between cost and uncertainty. $\frac{\partial B^*}{\partial c} > 0$ implies that the bigger the liquidation cost is, the higher the optimal government bailout is; from $\frac{\partial B^*}{\partial \sigma} < 0$, optimal government bailout decreases with asset uncertainty.

Second, if instead $0 \leq k \leq N$, the average distress after the θ shock remains positive. From equation (67), a positive government bailout requires that $c \geq \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} e^{\frac{(N-k)^2\bar{z}^2}{2N}} > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$.

Third, plugging equation (67) into (66), the total costs under optimal bailout policy B^* is

$$C^*_{GB} = (k-N)\bar{z}\sigma + N\Phi\left[-\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}\right]c + \sqrt{N}\sigma\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}. \tag{69}$$

Although C^*_{GB} improves upon C, it is important to compare C^*_{GB} with the cost when the acquisition link had been prevented *ex ante*.

Proposition 9 *There exists* $\bar{c} > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$*, such that* $C^*_{GB} > C_{isoN}$ *for* $c \in [\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}, \bar{c}]$ *and for all* $\delta \geq 0$*, where* $\bar{c} = \frac{\sigma B^*}{(N-1)\Phi\left[-\sqrt{N-1}\bar{z}\right]+\Phi[(k-1)\bar{z}]-N\Phi\left[\frac{(k-N)\bar{z}-B^*}{\sqrt{N}}\right]}$*, and* B^* *is given by equation (67).*

Proof I first show that C_{isoN} decreases monotonically with δ, hence it achieves maximum at $C_{isoN}(\delta = 0)$. Then I show that C^*_{GB} is a concave function of liquidation cost c. $C^*_{GB} > C_{isoN}(\delta = 0)$ when cost $c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, and C^*_{GB} crosses the linear function $C_{isoN}(\delta = 0)$ at $\bar{c}$. Accordingly, C^*_{GB} is greater than C_{isoN} in region $c \in [\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}, \bar{c})$.

Step 1: C_{isoN} decreases with δ. Take the derivative of C_{isoN} with respect to δ,

$$\frac{\partial C_{isoN}}{\partial \delta} = \frac{(N-1)\,c}{2}\left(\Phi'\left[(k-1)\,\bar{z} + \frac{N-1}{2}\delta\right] - \sqrt{N-1}\Phi'\left[\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right)\right]\right). \tag{70}$$

Notice that $(k-1)\,\bar{z} + \frac{N-1}{2}\delta > 0$, $\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right) < 0$, and we can also show that $(k-1)\,\bar{z} + \frac{N-1}{2}\delta > -\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right)$.Accordingly, $\Phi'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ implies that

$$\Phi'\left[(k-1)\,\bar{z} + \frac{N-1}{2}\delta\right] < \Phi'\left[\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right)\right] < \sqrt{N-1}\Phi'\left[\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right)\right].$$

Plugging into equation (70), we have $\frac{\partial C_{isoN}}{\partial \delta} < 0$. Evaluate C_{isoN} at $\delta = 0$, we obtain a linear function of c,

$$C_{isoN}(\delta = 0) = (N-1)\Phi\left[\sqrt{N-1}(-\bar{z})\right]c + \Phi\left[(k-1)\,\bar{z}\right]c.$$

Step 2: C^*_{GB} is a concave function of c. Denote $J = \sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]} > 0$, then $\frac{\partial J}{\partial c} = \frac{1}{Jc} > 0$. From (68), $\Phi'(-J) = \frac{\sigma}{\sqrt{N}c}$. Sub J into equation (69), we have $C^*_{GB} = N\Phi\left[-J\right]c + (k-N)\bar{z}\sigma + \sqrt{N}\sigma J$. Take the first derivative of c, we have

$$\frac{\partial C^*_{GB}}{\partial c} = N\Phi\left[-J\right] + \frac{\sqrt{N}\sigma}{Jc} - \frac{N\Phi'\left[-J\right]}{J} = N\Phi\left[-J\right] = N\Phi\left[-\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}\right] > 0.$$

Since $\frac{\partial J}{\partial c} = \frac{1}{Jc} > 0$, $\frac{\partial C^*_{GB}}{\partial c}$ decreases with c, i.e. $\frac{\partial^2 C^*_{GB}}{\partial c^2} < 0$.

Step 3: Establish $C_{GB}^*\left(c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right) > C_{isoN}\left(\delta=0,\ c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right)$. Plugging in $c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$,

$$C_{GB}^*\left(c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right) = \frac{\sqrt{2\pi}\sigma}{2}\sqrt{N}+(k-N)\bar{z}\sigma,$$

$$C_{isoN}\left(\delta=0,\ c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right) = (N-1)\Phi\left[-\sqrt{N-1}\bar{z}\right]\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}+\Phi\left[(k-1)\,\bar{z}\right]\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}.$$

Since $k>N$, $\Phi<1$,

$$C_{isoN}\left(\delta=0,\ c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right) < N\frac{\sqrt{2\pi}\sigma}{\sqrt{N}} = \frac{\sqrt{2\pi}\sigma}{2}\sqrt{N} < C_{GB}^*\left(c=\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right).$$

Step 4: Solve for cross point $\bar{c}$. Equating $C_{GB}^*=C_{isoN}\,(\delta=0)$ and solve for c gives $\bar{c}$.

To sum up, Steps 1 - 4 establish that $C_{GB}^*>C_{isoN}$, $\forall c\in[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}},\bar{c}]$. Therefore, when liquidation cost is bounded by $\bar{c}$, C_{GB}^* is more costly than C_{isoN}. Q.E.D.

Next I consider the optimal policy when there exist healthier institutions currently not connected with the distressed firm. The set up corresponds to subsection A.6.2. The next corollary examines whether firm $N+1$ has incentive to acquire the distressed firm N after learning θ, and whether the acquisition is *ex post* socially optimal.

Corollary 1 *With no subsidy, the liquid firm $N+1$ acquires the heavily distressed N only if $\hat{k}+2N\geq k$. When $\hat{k}+2N<k$, government subsidized acquisition is ex post optimal if $c>\frac{\sqrt{\pi}\sigma}{\sqrt{N}}$; the optimal subsidy to the acquirer firm $N+1$ upon acquisition is $B_A^*\sigma$, where*

$$B_A^* = \left(k-\hat{k}-2N\right)\bar{z}+\sqrt{2N}\sqrt{-2\log\left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c}\right]}. \tag{71}$$

When there exist healthier institutions, ex post subsidized acquisition is always preferred to ex post government bailout.

Proof I first analyze conditions for the acquisition link to be *ex post* optimal. Then I examine whether the acquisition link forms at equilibrium, and then move to conditions for the positive subsidy to be optimal. Finally, I conclude that subsidized acquisition is cheaper than government bailout.

Step 1: Condition for the acquisition link to be *ex post* optimal. Without acquisition link, total liquidation costs of group one and group two are respectively

$$C_{g1}=N\Phi\left[\sqrt{N}\left(-1+\frac{k}{N}\right)\bar{z}\right]c,\quad C_{g2}=N\Phi\left[\sqrt{N}\left(-1-\frac{\hat{k}}{N}\right)\bar{z}\right]c. \tag{72}$$

With the acquisition link, total liquidation costs of the two groups become

$$C_{\text{total}}=\sum_{i=1}^{2N}\Pr\left(\tilde{h}_i<1\right)c=2N\Phi\left[\sqrt{2N}\left(-1-\frac{\hat{k}-k}{2N}\right)\bar{z}\right]c. \tag{73}$$

The acquisition link generates positive surplus if and only if $C_{g1}+C_{g2}>C_{\text{total}}$. Applying Lemma 2, we get

$$N\Phi\left[\frac{-N+k}{\sqrt{N}}\bar{z}\right]c+N\Phi\left[\frac{-N-\hat{k}}{\sqrt{N}}\bar{z}\right]c>2N\Phi\left[\frac{-2N-\hat{k}+k}{\sqrt{2N}}\bar{z}\right]c\iff \hat{k}+2N>k.$$

Step 2: Condition for the acquisition link to emerge at equilibrium. I next show that as long as the acquisition link is socially optimal, it emerges *ex post* at equilibrium.

Since prices are already set between other firms, only bilateral prices between firms N and $N+1$ are to be set. Hence whether the acquisition link can form at equilibrium is equivalent to whether the bilateral surplus between N and $N+1$ is positive. The value of firm N without the acquisition link is

$$V_N^{12\ldots N}=\left(1-\frac{N-1}{2}\delta-\frac{k}{N}\right)\bar{z}\sigma-\Phi\left[\frac{k-N}{\sqrt{N}}\bar{z}\right]c-\Phi\left(-\bar{z}+\frac{N-1}{2}\delta\right)c+\Phi\left(-\sqrt{N}\bar{z}\right)c.$$

Notice that when $k=0$, $V_N^{12\ldots N}=V_N^a$, which matches the outside option of firm N. The value of firm $N+1$ without the acquisition link is

$$V_{N+1}^{(N+1)(N+2)\ldots 2N}=\frac{\hat{k}+N}{N}\bar{z}\sigma-\Phi\left[\frac{-N-\hat{k}}{\sqrt{N}}\bar{z}\right]c.$$

For the bilateral surplus to be positive

$$\Phi\left[\frac{-N-\hat{k}}{\sqrt{N}}\bar{z}\right]c+\Phi\left[\frac{k-N}{\sqrt{N}}\bar{z}\right]c>2\Phi\left[\frac{-2N-\hat{k}+k}{\sqrt{2N}}\bar{z}\right]c\iff \hat{k}+2N>k,$$

which recovers precisely the condition for positive social surplus. This shows that if and only if $\hat{k}+2N>k$, the acquisition link is efficient and forms in equilibrium after θ realizes.

Step 3: optimal acquisition subsidy. When $\hat{k}+2N\le k$, I next show that a positive acquisition subsidy is optimal if the liquidation cost is not very small. Let the positive government subsidy be $B_A\sigma$ given to the acquirer firm $N+1$ upon forming link with firm N. Total cost with subsidized acquisition becomes

$$C_{subA}=\sum_{i=1}^{2N}\Pr\left(\tilde{h}<1\right)c+B_A\sigma=2N\Phi\left[\frac{\left(k-\hat{k}-2N\right)\bar{z}-B_A}{\sqrt{2N}}\right]c+B_A\sigma.$$

B_A^* satisfies the first order condition

$$\Phi'\left[\frac{\left(k-\hat{k}-2N\right)\bar{z}-B_A^*}{\sqrt{2N}}\right]=\frac{\sigma}{\sqrt{2N}c}. \tag{74}$$

Solving for B_A^* gives (71), and we require that $c>\frac{\sqrt{\pi}\sigma}{\sqrt{N}}$ and $\hat{k}\le k-2N$.

Step 4: subsidized acquisition is preferred to government bailout. I show that the subsidized acquisition is less costly than government bailout. From Proposition 8, for $c\in\left(\frac{\sqrt{\pi}\sigma}{\sqrt{N}},\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}\right)$, subsidized acquisition is the only option with positive surplus. For $c>\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, costs under government bailout are

$$C_{GB}^*=N\Phi\left[-\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}\right]c+(k-N)\bar{z}\sigma+\sqrt{N}\sigma\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}+N\Phi\left[\frac{-N-\hat{k}}{\sqrt{N}}\bar{z}\right]c.$$

Costs under subsidized acquisition are

$$C^*_{subA} = \left(k - \hat{k} - 2N\right)\bar{z}\sigma + \sqrt{2N}\sqrt{-2\log\left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c}\right]}\sigma + 2N\Phi\left[-\sqrt{-2\log\left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c}\right]}\right]c.$$

Denote $J = \sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]} > 0$, $H = \frac{\hat{k}+N}{\sqrt{N}}\bar{z} > 0$, then $C^*_{GB} = \sqrt{N}\sigma J + N\Phi[-J]c + \sqrt{N}\sigma H + N\Phi[-H]c + \left(k - \hat{k} - 2N\right)\bar{z}\sigma$. From (68), $\Phi'(-J) = \frac{\sigma}{\sqrt{N}c}$. Function $f(x) = \sqrt{N}\sigma x + N\Phi[-x]c$, satisfies $f'(J) = 0$, $f''(x) > 0, \forall x > 0$. This implies $C^*_{GB} > 2\sqrt{N}\sigma J + 2N\Phi[-J]c + \left(k - \hat{k} - 2N\right)\bar{z}\sigma > \sqrt{2N}\sigma J + 2N\Phi[-J]c + \left(k - \hat{k} - 2N\right)\bar{z}\sigma > \sqrt{2N}\sigma J + 2N\Phi[-J]c$.

Similarly, denote $G = \sqrt{-2\log\left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c}\right]} > 0$, then $C^*_{subA} = \sqrt{2N}G\sigma + 2N\Phi[-G]c + \left(k - \hat{k} - 2N\right)\bar{z}\sigma$. From (74), $\Phi'[-G] = \frac{\sigma}{\sqrt{2N}c}$. Function $f(x) = \sqrt{2N}\sigma x + 2N\Phi[-x]c, \quad x > 0$, achieves global minimum at $x = G$. This implies that $C^*_{GB} > C^*_{subA}$. Q.E.D.

When the two groups differ in cardinality, pushed acquisition could be *ex post* optimal. Denote N_1 (instead of N) the number of the group one firms including the heavily distressed $\theta_{N_1} = -k\bar{z}\sigma$. Denote N_2 the number of group two firms, with the same $\bar{z} > 0$, but $\delta = 0$ for simplicity. *Ex ante* an optimal complete risk-sharing network is formed among N_2 firms. The additional signal is $\theta_{N_1+1} = \hat{k}\bar{z}\sigma$, $\theta_i = 0$, $\forall i = N_1+2, ...N_1+N_2$. Hence firm $i = N_1+1$ has the highest liquid value *ex post*. Suppose after $t = 1$ when links within each group are formed and prices are set, firm N_1+1 can acquire the heavily distressed N_1.

Proposition 10 *The acquisition generates positive social surplus when the liquidity shocks*

$$\hat{k} > \max\left[\frac{\sqrt{N_1+N_2} - \sqrt{N_1}}{\sqrt{N_1+N_2} - \sqrt{N_2}}(k - N_1) - N_2,\ k - N_1 - N_2\right]. \tag{75}$$

- *when $N_2 \geq N_1$ the bilateral surplus is positive so acquisition forms at equilibrium;*
- *when $N_2 < N_1$ the bilateral surplus is negative when*

$$2\Phi\left[\frac{k - \hat{k} - (N_1+N_2)}{\sqrt{N_1+N_2}}\bar{z}\right]c > \Phi\left[\frac{k - N_1}{\sqrt{N_1}}\bar{z}\right]c + \Phi\left[\frac{-\hat{k} - N_2}{\sqrt{N_2}}\bar{z}\right]c + \frac{(N_2 - N_1)\left(N_2k + N_1\hat{k}\right)}{N_1N_2(N_1+N_2)}\bar{z}\sigma. \tag{76}$$

Proof I first show that condition (75) implies positive social surplus from the acquisition link between the liquid N_1+1 and the distressed firm N_1. Without acquisition link, total liquidation costs of group one and group two are respectively

$$C_{g1} = N_1\Phi\left[\sqrt{N_1}\left(-1 + \frac{k}{N_1}\right)\bar{z}\right]c, \quad C_{g2} = N_2\Phi\left[\sqrt{N_2}\left(-1 - \frac{\hat{k}}{N_2}\right)\bar{z}\right]c.$$

With the acquisition link, the total liquidation costs of the two groups become

$$C_{\text{total}} = \sum_{i=1}^{N_1+N_2}\Pr\left(\tilde{h}_i < 1\right)c = (N_1+N_2)\Phi\left[\sqrt{N_1+N_2}\left(-1 + \frac{k - \hat{k}}{N_1+N_2}\right)\bar{z}\right]c.$$

The acquisition link generates positive surplus if and only if $C_{g1} + C_{g2} > C_{\text{total}}$, i.e.

$$\frac{N_1}{N_1+N_2}\Phi\left[\frac{k - N_1}{\sqrt{N_1}}\bar{z}\right] + \frac{N_2}{N_1+N_2}\Phi\left[\frac{-\hat{k} - N_2}{\sqrt{N_2}}\bar{z}\right] > \Phi\left[\frac{k - \hat{k} - (N_1+N_2)}{\sqrt{N_1+N_2}}\bar{z}\right]. \tag{77}$$

Given $\Phi(.)$ is convex in the negative domain,

$$\frac{N_1}{N_1+N_2}\Phi\left[\frac{k-N_1}{\sqrt{N_1}}\bar{z}\right]+\frac{N_2}{N_1+N_2}\Phi\left[\frac{-\hat{k}-N_2}{\sqrt{N_2}}\bar{z}\right]\geq\Phi\left[\frac{N_1\sqrt{N_1}\left(-1+\frac{k}{N_1}\right)\bar{z}}{N_1+N_2}+\frac{N_2\sqrt{N_2}\left(-1-\frac{\hat{k}}{N_2}\right)\bar{z}}{N_1+N_2}\right].$$

Under (75), It follows

$$\left(N_2+\hat{k}\right)\left(\sqrt{N_1+N_2}-\sqrt{N_2}\right)>(k-N_1)\left(\sqrt{N_1+N_2}-\sqrt{N_1}\right)\iff$$

$$\left(-N_1\sqrt{N_1}+\sqrt{N_1}k\right)+\left(-\sqrt{N_2}N_2-\sqrt{N_2}\hat{k}\right)>\sqrt{N_1+N_2}k-\sqrt{N_1+N_2}\hat{k}-\sqrt{N_1+N_2}\left(N_1+N_2\right)\iff$$

$$\frac{N_1\sqrt{N_1}\left(-1+\frac{k}{N_1}\right)\bar{z}}{N_1+N_2}+\frac{N_2\sqrt{N_2}\left(-1-\frac{\hat{k}}{N_2}\right)\bar{z}}{N_1+N_2}>\frac{k-\hat{k}-(N_1+N_2)}{\sqrt{N_1+N_2}}\bar{z}. \tag{78}$$

Put together, we establish (77).

Next I show that under (75), the bilateral acquisition surplus is positive when $N_2\geq N_1$. Since prices are set between other firms, only bilateral prices between firms N and $N+1$ are to be set. The value of firm N_1 without acquisition is

$$V_{N_1}^{12\ldots N_1}=\left(1-\frac{N_1-1}{2}\delta-\frac{k}{N_1}\right)\bar{z}\sigma-\Phi\left[\frac{k-N_1}{\sqrt{N_1}}\bar{z}\right]c-\Phi\left(\frac{N_1-1}{2}\delta-\bar{z}\right)c+\Phi\left(-\sqrt{N_1}\bar{z}\right)c.$$

The value of firm N_1+1 without the acquisition link is

$$V_{N_1+1}^{(N_1+1)(N_1+2)\ldots(N_1+N_2)}=\left(1+\frac{\hat{k}}{N_2}\right)\bar{z}\sigma-\Phi\left[\frac{-N_2-\hat{k}}{\sqrt{N_2}}\bar{z}\right]c.$$

With the acquisition link, the value of firm N_1, and firm N_1+1 are respectively

$$V_{N_1}^{12\ldots(N_1+N_2)}=\left(1-\frac{N_1-1}{2}\delta-\frac{k-\hat{k}}{N_1+N_2}\right)\bar{z}\sigma-\Phi\left[\frac{k-\hat{k}-(N_1+N_2)}{\sqrt{N_1+N_2}}\bar{z}\right]c-\Phi\left(-\bar{z}+\frac{N_1-1}{2}\delta\right)c+\Phi\left(-\sqrt{N_1}\bar{z}\right)c;$$

$$V_{N_1+1}^{12\ldots(N_1+N_2)}=\left(1-\frac{k-\hat{k}}{N_1+N_2}\right)\bar{z}\sigma-\Phi\left[\frac{k-\hat{k}-(N_1+N_2)}{\sqrt{N_1+N_2}}\bar{z}\right]c.$$

The average bilateral surplus minus the average total surplus is

$$\frac{\hat{V}_{N_1}^A+\hat{V}_{N_1+1}^A-\hat{V}_{N_1}-\hat{V}_{N_1+1}}{2}-\frac{C_{g1}+C_{g2}-C_{\text{total}}}{N_1+N_2}$$
$$=\frac{N_2-N_1}{2(N_1+N_2)}\left[\frac{N_2k+N_1\hat{k}}{N_1N_2}\bar{z}\sigma+\left(\Phi\left[\frac{k-N_1}{\sqrt{N_1}}\bar{z}\right]-\Phi\left[\frac{-\hat{k}-N_2}{\sqrt{N_2}}\bar{z}\right]\right)c\right].$$

which is non-negative when $N_2\geq N_1$ and $C_{g1}+C_{g2}-C_{\text{total}}>0$. Whereas if $N_1>N_2$, the average bilateral surplus is smaller than the average social surplus. Under condition (76), the bilateral surplus is negative. Q.E.D.

As a sufficient condition for a positive social surplus, (75) sets a lower bound for the positive liquidity shock $\hat{k}$. The relative cardinality of the two groups is key in determining the sign of the bilateral surplus. When $N_2>N_1$, the pair $\{N_1,N_1+1\}$ on average gets bigger surplus than an average firm. When $N_1=N_2$, we recover the case in subsection A.6.2 when the sign of bilateral surplus matches that of the social surplus. When $N_1>N_2$, under condition (76), bilateral surplus can be negative even if social surplus is positive. (76) implies an upper bound for $\hat{k}$, thus is especially relevant when the potential acquirer does not have an abundant supply of liquidity.

Table A.I. Distressed Acquisition Likelihood and Log Z-score

	Pr(Completing an Acquisition of a Distressed Firm)				
	(1)	(2)	(3)	(4)	(5)
Log Z-score	0.153*	0.145*	0.142*	0.284**	0.317***
	[0.070]	[0.070]	[0.067]	[0.094]	[0.094]
Firm Controls		yes	yes	yes	yes
Year Fixed-Effects			yes		yes
2006-2013				yes	yes
Observations	57,035	57,035	57,035	14,490	14,490
Firm Fixed-Effects	yes	yes	yes	yes	yes

Notes: This table reports the results from a fixed-effects logit regression. The sample includes commercial banks, savings institutions and bank holding companies. The dependent variable Pr(Completing an Acquisition of a Distressed Firm) takes the value of one if institution i completes an acquisition of a distressed firm at time $t+4$, and zero otherwise. Firm controls include quarterly CAR, ROA, and asset size. Regression coefficients are reported with standard errors in the square bracket. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

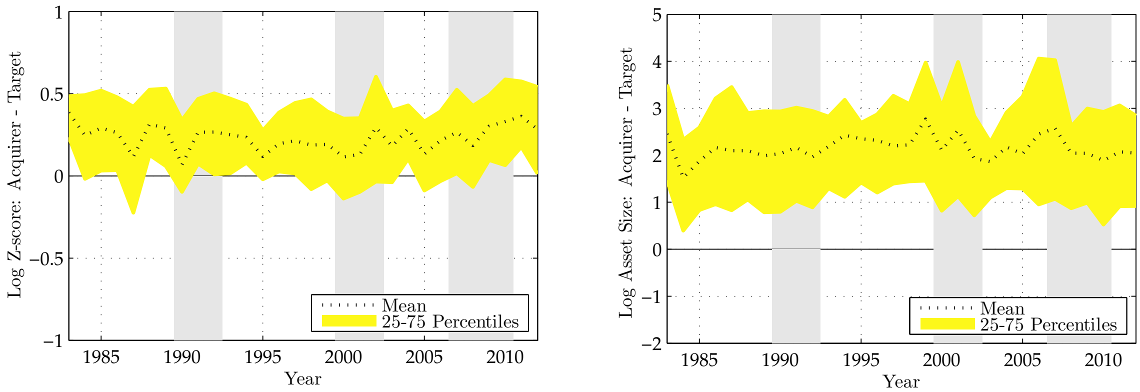
B Additional Empirical Results

Here I provide additional empirical evidence in support of the assumptions of the model.

Liquid firms acquire the distressed firms To confirm the assumption made in the model that more liquid firms acquire the distressed firms, I match the quarterly firm-level data with the acquisition dataset using the acquirer entities and acquisition completion dates, and perform fixed-effects logit regressions. The dependent variable is a dummy indicating whether a firm conducts a distressed acquisition at a certain quarter. I assume that an acquisition takes on average four quarters to complete, so it starts four quarters prior to the merger completion date recorded in the Chicago Fed dataset. The independent variable of interest is the firm's estimated log Z-score. Results reported in Table A.I confirm that a firm with higher log Z-score has a higher likelihood of acquiring a distressed firm. For a one-standard-deviation increase in log Z-score (.58), the log odds ratio of a distressed acquisition increases by 0.09 ($=0.153 \times 0.58$). The economic and statistical significance of the coefficient is robust to including firm-level controls, year fixed effects, and only considering the post-2006 period.

Among the identified 3,153 distressed acquisitions, a clear pattern emerges among the acquirer-target pairs: the acquirer has higher Z-score and bigger asset size relative to the target. The results are depicted in Figure A.I. The plots show the distributions of the acquirer-minus-target log Z-score (Panel 1(a)) and log asset size (Panel 1(b)). Both distributions are significantly above zero, implying that more stable firms acquire smaller and distressed targets.

Risk exposure to counterparties through links In the theoretical analysis, a link with the distressed firm is modeled as a bilateral forward swap contract, which increases the financial



(a) Log Z-score: Acquirer - Target

(b) Log Asset Size: Acquirer-Target

Figure A.I. Log Z-score and Asset Size: Acquirer - Target. This figure plots the distribution of log Z-score and log asset size of the acquirer-target wedge for the identified 3,153 distressed acquisitions in 1983-2013. Shaded bars indicate NBER recessions.

Table A.II. Effect of Target log Z-score on Acquirers' Future Z-score

	$\log z_{i,t+1} - \log z_{i,t}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Target Log Z-score	0.248***	0.310***			0.326*	0.291**	0.250***
	[0.060]	[0.064]			[0.130]	[0.095]	[0.060]
Acquisition Dummy			0.624***	0.884***			
			[0.186]	[0.206]			
Distressed Acquisition		-1.268**		-1.373**			
Dummy		[0.447]		[0.464]			
Observations	1,326,071	1,326,071	1,326,071	1,326,071	435,635	98,737	1,326,071
Firm Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed-Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
NBER Recessions					Yes		
Top Firms (A>$1B)						Yes	
Year-quarter Dummy							Yes

Notes: This table reports the coefficients from a fixed-effects regression. The sample includes commercial banks, savings institutions and bank holding companies. The dependent variable $\log z_{i,t+1} - \log z_{i,t}$ is the growth rate of log Z-score for firm i at quarter t. The target log Z-score is the level of log Z-score of the target firm at the acquisition completion date if firm i has an acquisition at quarter t. The dummy variables take 1 (and 0 otherwise) if firm i has an acquisition or a distressed acquisition at quarter t. Firm controls include total assets, total equity, net income, and current level log Z-score. Regression coefficients are reported with standard errors in the square bracket. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

distress of the acquirer and thus negatively affects its Z-score. To confirm this assumption, I perform fixed-effects regressions of growth rate in log Z-score on target log Z-score, and the dummy variables representing acquisition and distressed acquisition, controlling for firm-level characteristics. The regression results summarized in Table A.II show strong support for the model assumption. The estimates suggest that the effect of the log Z-score of the targets on the growth rate of Z-score of the acquirers is positive and significant. The economic magnitude of the effect is sizable: a one-standard-deviation decrease in target log Z-score decreases future log Z-score of the acquirer by 0.16, more than four times the magnitude of its average level. Results in columns (3) - (4) show that, while in general completing an acquisition has a positive impact on the future Z-score of the acquirer, completing an acquisition of a distressed target has a significantly negative impact on the future Z-score of the acquirer. These findings are robust to controlling for recession periods, restricting to only top firms with asset size larger than $1 billion, and including year-quarterly dummy.