The Market Impact of Options

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Abstract

This article develops a noisy rational expectations model to examine the effect of introducing options on the financial market in an economy with heterogeneous uncertain endowment and information. The model is shown to have a unique equilibrium and demonstrates that adding options cannot always reveal additional directional information that is not contained in the price of an underlying asset. However, the option is not redundant because it is a security for betting volatility. The introduction of options generally changes the stock price and trading of the asset, but the impact on the stock price is small. Furthermore, introducing options is not always Pareto-improving and may cause certain agents to be worse off.

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1 Introduction

The growth of option and other derivatives markets raises a number of concerns regarding the effect of option listings on the underlying assets. One concern is whether the introduction of options destabilizes the underlying market and requires increased regulation. Many studies have empirically examined this issue. This paper explores the effect of option listings on stock prices, market liquidity and welfare from a theoretical perspective.

We develop a static noisy rational expectations model to examine the effect of introducing a series of call options with different strikes on the financial market. The agents in the economy have heterogeneous uncertain endowment and information, so they have different opinions about the future states. As with Huang and Wang (1997), the liquidity trading is derived endogenously from investors' nontraded income, thus allowing us to analyze both the allocational and informational role of the options. We show that in this paper's normal-exponential framework, adding options cannot reveal additional information that is not contained in the underlying asset price. The reason for this result is that the volatility is a publicly known constant in the normal-exponential framework, and the option is more suitable for volatility speculation.[1] In other words, the result suggests that the option does better in revealing volatility information.

While there are no direct informational roles for the options, the new trading opportunities for hedging and volatility speculation provided by the options change investors' demands for securities. Therefore, the introduction of options generally changes the price and volatility of the underlying asset. This is different from the results of Cao (1999) and Vanden (2008). Hence, the option is nonredundant in an economy with information asymmetry.

The driving factor that makes the option nonredundant is the difference in opinions of uncertainty caused by heterogeneous uncertain endowments and information. Once an option is introduced, the investors facing more uncertainty will buy the option. To maintain the optimality of the portfolio, investors will adjust their stock holding to hedge the risk from the option position. Because investors of different types have different views, their adjustments in their stock holdings are generally different, and such

[1] The result of this paper can help explain why option trades empirically contain less information about future stock price changes (Chan, Chung and Johnson, 1993; Chan, Chung and Fong, 2002; Muravyev, Pearson and Broussard, 2013) but implied volatility from option prices is a more efficient forecast for future volatility (Jiang and Tian, 2005).

unbalanced adjustments change the stock price. Numerical analysis also suggests that higher dispersion opinions of the payoffs will cause the option's introduction to have a greater impact on the stock price and volatility.

The welfare effect of introducing options also demonstrates the nonredundancy of options. The introduction of the options always causes someone to benefit, and introducing a complete set of options with all possible strikes is always Pareto-improving. Because more options do not always make everyone better, some regulation or redistribution may be beneficial to society.

In addition to the new trading opportunities, introducing options provides new instruments that informed investors can gain by exploiting uninformed investors. Therefore, options change the investors' incentives to acquire information when the information acquisition decision is endogenized. Differing from Cao (1999) and Massa (2002), our model allows the portion of investors who will acquire information to vary rather than varying information precision based on a fixed ratio of informed investors. The model demonstrates that the introduction of options can increase or reduce the incentives to be informed, thereby changing the underlying asset's price volatility and the degree of informational efficiency of the market.[2]

Our noisy rational expectations model can be regarded as an extension of the works of Detemple and Selden (1991) and Brennan and Cao (1996). Both works explore the effect of option listings on the underlying asset. Detemple and Selden (1991) analyze the issue when the market is incomplete. They demonstrate that the valuation of underlying assets is generically affected when an option is introduced. Brennan and Cao (1996) analyze the effect of introducing a quadratic option. They show that the market is made effectively complete by the quadratic option and that the stock price will stay the same. Our paper analyzes the issue in an economy that is both incomplete and exhibits information asymmetry. Due to the different specifications of the preference, the distribution of the stock's payoff, and the derivative, our results differ. For example, to obtain a clear result, Detemple and Selden (1991) placed strict constraints on investors' probability assessments of the stock's payoff. This means the payoff cannot be a commonly known distribution, such as a normal distribution. Our model uses the convetional normal-exponential framework, facilitating the comparison with the literature.

[2]To some extent, this observation is consistent with the empirical results. Although early studies (Conrad, 1989) show that the introduction of options reduces stock return volatility, more-recent studies (Bollen, 1998) suggest that the introduction of options does not significantly affect the volatility.

In an incomplete market with information asymmetry, Huang and Wang (1997) and Biais and Hillion (1994) also analyze the effects of derivative introduction. However, the derivative Huang and Wang (1997) consider is a collar contract, and Biais and Hillion (1994) analyze the effect in a strategic trading model. Thus, they both show that the introduction of derivative securities can affect the market's informational efficiency, which differs from our results.

Our paper is also related to other studies on the interaction between derivatives and their underlying assets.

In the case of incomplete markets, Ross (1976) argued that the introduction of options can improve allocational efficiency by opening up new spanning opportunities. Bhamra and Uppal (2009) find that the introduction of a nonredundant derivative may increase the stock volatility in a dynamic Lucas endowment economy. An option can also help investors mitigate short sales constraints on stock trading (Diamond and Verrecchia, 1987) and nonnegative wealth constraints (Vanden, 2004).

When there is asymmetric information, Grossman (1988) argues that introducing an option can reveal more information relative to its synthetic counterpart and, hence, reduce the volatility of underlying assets. Assuming that the openness of the option market introduces new noise trading, Back (1993) demonstrates that options trading causes the stock volatility to become stochastic.

If the information acquisition is determined endogenously, the introduction of options will alter investors' incentives to purchase information and, hence, affect the underlying asset indirectly. Cao (1999) shows that introducing a certain type of derivative asset always encourages investors to purchase more-precise information, and it makes the stock price more informative. Massa (2002), using a continuous-time setting, demonstrates that whether the introduction of a derivative increases or decreases the incentive to purchase information depends on the informational structure of the market.

Although these studies provide good reasons for why introducing derivatives affects the underlying assets, few existing models explicitly consider the effect of introducing option contracts for tractability. Many studies assume a specific form of derivative. These studies, include the previously mentioned Brennan and Cao (1996) and Huang and Wang (1997), along with Cao (1999), who only considers generalized straddle[3] and generalized spread, which can only replicate the payoff of a call option at-the-money.

[3] Generalized straddle is a generalization of the quadratic derivative asset proposed by Brennan and Cao (1996).

Back (1993) also only considers the case of introducing a call option at-the-money, while Vanden (2008) considers the log derivative. Because different payoff structures may provide different information and hedging opportunities, introducing different derivatives into the market will have different impacts. [4] Therefore, it is important to understand the impact of introducing a series of derivatives that have an identical payoff as the options in real life.

Our paper is also closely related the recent papers by Chabakauri, Yuan, and Zachariadis (2015), Huang (2014), and Malamud (2015). Chabakauri, Yuan and Zachariadis (2015) derive general conditions for the informational redundancy of assets, but they limit the discussion in a discrete state-space framework. Malamud (2015) studies a noisy REE with derivatives in a continuous-space framework, but he assumes a complete market economy. Our paper differs from those two papers by focusing on the realistic option contract, and it provides a detailed analysis on how options affect the market liquidity, prices and welfare. Huang (2014) also develops a CARA-normal model with options that is close to our paper, but the work assumes that an options market consists of a set of call and put options and does not apply to a single option or a finite number of options case. The proof of uniqueness of the equilibrium is also missing in Huang (2014).

This paper also contributes to the literature on rational expectations equilibrium. We present an example showing that a market that is complete for marketable risks cannot ensure information completeness. This example seems to contradict the conclusion of Grossman (1981), which states that in an asymmetric-information environment with complete markets, the existence of a fully revealing REE is guaranteed under the standard conditions of a competitive equilibrium of the full-information economy. Rather, because the aggregate endowment risk exists in our economy, the markets are essentially incomplete.

The rest of the paper is organized as follows. The model is introduced in Section 2. The equilibrium is solved in Section 3 and analyzed in Section 4. Section 5 considers some possible model extensions, especially the endogenous information acquisition, with an analysis of its effect. Section 6 present the paper's conclusions. The Appendix includes proofs omitted in the main text.

[4] A related problem is how to optimally introduce the derivatives; see Allen and Gale (1994) for optimal security design and optimal financial innovation.

2 The Model

We consider an economy with two dates, 1 and 2, and a single consumption good.

2.1 Securities Market

There is a competitive securities market with three types of traded securities. The first security is a risk-free bond that pays one unit of consumption at the final date 2. The second security is a risky security (stock) with a payoff of:

$$F = V + U \tag{1}$$

where V and U are independent, normally distributed random variables with $V \sim N(\bar{V}, \sigma_v^2)$ and $U \sim N(0, \sigma_u^2)$. Here, $\sim$ denotes the probability distribution of a random variable and $N(\cdot, \cdot)$ denotes the normal distribution with mean and variance. Thus, F gives the terminal payoff of the stock, which we also refer to as its "fundamental" for convenience.

The third type of securities consist of call options on the stock with strike prices k_1, k_2, ..., k_n, and maturity date 2. Without loss of generality, we assume $-\infty < k_1 < k_2 < \ldots < k_n < \infty$. Thus, the final payoff of an option with strike k_j is $(V + U - k_j)^+$. Using the risk-free bond as the numeraire, we denote the prices of the stock and the options as S and C_j, $j = 1, \ldots, n$, respectively. To exclude the arbitrage opportunities, the option prices satisfy $C_1 > C_2 > \cdots > C_n$.

In our analysis, we consider two market structures: market structure I consists of the risk-free bond and the stock; market structure II consists of the the risk-free bond, the stock and the call options on the stock.

2.2 Investors

There are two classes of investors participating in the securities market, denoted by $i = 1, 2$, with relative population weights ω and $1 - \omega$, respectively. Investors are identical in each class, but are potentially different between the two classes with regard to the endowment and the information.

Each investor is initially endowed with one share of the stock and zero units of the bond and options. In addition, investor 1 also receives a non-traded income of XU, where $X \sim N(0, \sigma_x^2)$ and X, U and V are all independent. Investor 2, however, has no non-traded income — all his income comes from his security holdings.[5]

We assume that investor 1 observes X, part of his exposure to the risk in his non-traded income. He also receives private information about V, part of the stock payoff. Investor 2 is assumed to receive no private information. Thus, the information set of investor 1 can be expressed as $\mathcal{F}_1 = \{S, \{C_j, j = 1, 2, ..., n\}, X, V\}$, and investor 2's information set is $\mathcal{F}_2 = \{S, \{C_j, j = 1, 2, ..., n\}\}$[6].

For investor i $(i = 1, 2)$, let $\{\theta_{iS}, \theta_{ij}, j = 1, 2, ..., n\}$ be his holdings of the stock and options and hence investor i's trading of the stock is $\theta_{iS} - 1$. His terminal wealth or consumption (at $t = 2$) will be given by:

$$W_1 = F + (F - S)(\theta_{1S} - 1) + \sum_{j=1}^{n} \left[(F - k_j)^+ - C_j\right] \theta_{1j} + XU \tag{2a}$$

$$W_2 = F + (F - S)(\theta_{2S} - 1) + \sum_{j=1}^{n} \left[(F - k_j)^+ - C_j\right] \theta_{2j} \tag{2b}$$

Finally, investors are assumed to have a utility function over terminal consumption at $t = 2$ of the following form:

$$E\left[- e^{-aW_i} \middle| \mathcal{F}_i\right], \quad i = 1, 2 \tag{3}$$

where $a > 0$ is the absolute risk-aversion coefficient. Clearly in this case, investors have constant absolute risk aversion (CARA).

To ensure the economy is properly defined, we also require[7]

$$a^2 \sigma_x^2 \sigma_u^2 < 1. \tag{4}$$

[5]We can also assume that the income is $X(U + \varepsilon)$ with independent ε. In this case, the non-traded income is not perfectly correlated with the payoff of the stock. Almost all the results remain the same. For simplicity, we consider the simpler case with $\varepsilon = 0$.

[6]In the information sets $\mathcal{F}_1$ and $\mathcal{F}_2$, the information contained in securities prices is essentially the pricing functionals $\{S(V, X), C_j(V, X), j = 1, \cdots, n\}$.

[7]Under normality assumption, the income XU can take large negative values and generate an infinitely negative expected utility for investor 1 before he knows X. To guarantee a finite utility, we assume equation (4) holds. However, this constraint is only needed in section 5.2, as we assume investor 1 knows X in other parts of this paper.

2.3 Equilibrium Notion

The notion of equilibrium we use for the economy defined above is the standard one of rational expectations. It is defined as a set of trades that is contingent on the information that traders have, $\{\theta_{1S}(V, X, S, C_1, \cdots, C_n), \theta_{1j}(V, X, S, C_1, \cdots, C_n), j = 1, ..., n\}$, $\{\theta_{2S}(S, C_1, \cdots, C_n), \theta_{2j}(S, C_1, \cdots, C_n), j = 1, ..., n\}$, and on price functionals $\{S(V, X), C_j(V, X), j = 1, ..., n\}$, such that:

- (1) investors maximize their expected utility:

$$\{\theta_{iS}, \theta_{ij}, j = 1, ..., n\} \in \text{argmax } E\left[-e^{-aW_i}\middle|\mathcal{F}_i\right], \quad i = 1, 2 \tag{5}$$

- (2) markets clear

$$\omega\theta_{1S} + (1 - \omega)\theta_{2S} = 1 \tag{6a}$$

$$\omega\theta_{1j} + (1 - \omega)\theta_{2j} = 0, \quad j = 1, ..., n. \tag{6b}$$

2.4 Discussion

Before proceeding, we discuss in more detail several important aspects of the model. With regard to the market structure, we consider only call options on the stock, not put options. There is, however, no loss of generality in doing so. By put-call parity, we can easily allow put options and then re-express them as portfolios of the stock, bond, and call options (e.g., Merton (1973)). Additionally, we only allow a finite number of options. Thus, options do not complete the market along the dimension of stock payoffs. [8] However, this case of (limited) completeness can be approached by allowing an infinite number of options, as in Cox and Ross (1976) and Breeden and Litzenberger (1979). We do not formally introduce this limiting case as another market structure to consider, but will discuss the situation as a limiting case of market structure II.

With regard to the investors, the key ingredient of our model is the heterogeneity between them, which underlies most of our results. First, the two classes of investors are different in two dimensions: endowment and information. These differences give rise to their desire to trade in the securities market. Second, because investor 1 observes part

[8] The market structure we allow never completes the market in the strict Arrow-Debreu sense since there are no securities whose payoffs span non-traded income shocks X and Z. However, it can fully span the total shock to the stock payoff $F = V + U$.

of his exposure (i.e., X) to the non-traded risk, which is correlated with the risk of the stock, he will trade in the securities market to share this risk with investor 2. Third, investor 1's private information on the stock payoff (i.e., V) gives rise to his additional desire to trade, to speculate. Finally, given that investors within each class are identical and they all have CARA preferences, the simple aggregation results of Wilson (1964) and Rubinstein (1974) hold for each class. It is for this reason, in our discussion throughout the paper, we will treat and refer to investors of each class as a single investor.

In the model described above, we also assume that investors who receive non-traded income also receive private information on the stock's payoff. Moreover, the fraction of these investors is exogenously given. We do so initially to maintain parsimony and to focus on the direct impact of options trading on the market. In Section 5.2, we will relax these restrictions by allowing investors to optimally choose private information acquisition and, hence, endogenize the information structure of the model.

Our model is distinct from the previous studies of the impact of options trading in several important dimensions. First, most of the previous work only allows a specific option. For example, Cao and Brennan (1996) and Cao (1999) only consider a single at-the-money straddle. This is very restrictive. Moreover, it is unrealistic, as the strike prices of options are usually determined ex ante, while the price of the stock is determined in the equilibrium. In our model, we allow any number of options with an arbitrary set of strike prices. Second, previous work introduces "noise" into the market by assuming exogenous shocks to the supply of the stock.[9] By leaving the "noise" outside the model, the resulting conclusions become incomplete as they do not include the potential responses from the components that are not modeled. In our model, we explicitly model the "noise" as arising from the investors' trades for risk sharing. This not only allows us to endogenize the non-informative component in market prices and thus captures the full impact options trading, but it also allows us to examine its welfare implications.

[9]Back (1993) further allows supply shocks to the option.

3 Solution to Equilibrium

We now solve for the equilibrium of the economy as defined in Section 2. For convenience, we defined several coefficients for future use. Let

$$\alpha = \frac{\sigma_v^2}{\sigma_v^2 + a^2\sigma_x^2\sigma_u^4}, \qquad \hat{\sigma}_u^2 = \sigma_u^2 + \alpha a^2 \sigma_x^2 \sigma_u^4. \tag{7}$$

Clearly,

$$0 \leq \alpha \leq 1, \qquad \sigma_u^2 \leq \hat{\sigma}_u^2.$$

We also introduce a variable that is a linear function of the underlying variables X and V:

$$Q = V - \bar{V} - a\sigma_u^2 X. \tag{8}$$

It is worth noting that α gives a measure of information asymmetry in the economy. Because investor 1 privately observes V, he has more private information when σ_v is higher, which corresponds to a higher α. When $\sigma_v = 0$, he receives no private information about the stock's payoff. However, if a higher α is caused by a smaller σ_x, which means there are fewer noises in the economy, the securities prices would be more informative about V, thus indicating less information asymmetry.

3.1 Symmetric Information

Before we consider the equilibrium in the general case when investors are heterogeneous in both endowments and information, we first consider the equilibrium when all investors have symmetric information. This case can be obtained by simply setting $\sigma_v = 0$ in the model. We use this case as a benchmark case to obtain some basic intuition about the model. The results in this case are given in the following theorem.

Theorem 1 *Let $\sigma_v = 0$ in the economy defined in Section 2. A unique equilibrium exists under both market structures I and II. Under market structure I, the equilibrium stock price and stock holdings are given by:*

$$S = V - a\sigma_u^2(\omega X + 1) \tag{9a}$$

$$\theta_{1S} = 1 - (1-\omega)X, \quad \theta_{2S} = 1 + \omega X \tag{9b}$$

where $V = \bar{V}$. Under market structure II, the equilibrium stock price and stock holdings are the same as under market structure I, and the equilibrium options prices and holdings are given by:

$$C_j = \frac{\sigma_u}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{S - k_j}{\sigma_u} \right)^2 \right] + (S - k_j)\, \Phi\left(\frac{S - k_j}{\sigma_u} \right) \tag{10a}$$

$$\theta_{ij} = 0, \quad i = 1, 2;\ j = 1, \ldots, n \tag{10b}$$

where $\Phi(\cdot)$ denotes the cumulative density function of a standard normal distribution.

Under market structure I, we see from the first equality of Equation (9a) that the equilibrium stock price consists of two components. The first component is V, the expected payoff of the stock, given the investors' information. The second component is $a\sigma_u^2(\omega X + 1)$, which represents the risk premium on the stock. Here, σ_u^2 characterizes the risk for one share of the stock for the investors and a represents their risk aversion.

When $X = 0$, the risk premium on the stock is simply $a\sigma_u^2$, as each investor has to hold one share of the stock. When $X \neq 0$, however, the effective stock risk an investor has to bear becomes $\omega X + 1$ because the part of the risk in investor 1's non-traded income, XU, is perfectly correlated with the stock risk, which is also U. Thus, the risk premium becomes $a\sigma_u^2(\omega X + 1)$, where ω is the population weight of investor 1.

Under market structure II, we see from Theorem 1 that investors will hold the same amount of stocks and bonds as under market structure I; the equilibrium stock price also stays the same. Their trading in options is zero. In this case, options are clearly redundant securities. They provide no additional value in facilitating investors' risk sharing, which is the sole reason for trading.

3.2 Asymmetric Information Under Market Structure I

The following theorems describe the equilibrium under the two market structures, respectively.

Theorem 2 *For the economy defined in Section 2, under market structure I, a unique equilibrium exists, in which*

$$S = \bar{V} + \frac{1}{\omega\hat{\sigma}_u^2 + (1-\omega)\sigma_u^2} \left[\omega\hat{\sigma}_u^2 Q + (1-\omega)\sigma_u^2 \alpha Q - a\sigma_u^2 \hat{\sigma}_u^2 \right] \tag{11}$$

and

$$\theta_{1S} = 1 + \frac{1-\omega}{a[\omega\hat{\sigma}_u^2 + (1-\omega)\sigma_u^2]}\left[a(\hat{\sigma}_u^2 - \sigma_u^2) + (1-\alpha)\,Q\right] \tag{12a}$$

$$\theta_{2S} = 1 - \frac{\omega}{1-\omega}(\theta_{1S} - 1). \tag{12b}$$

The economy under market structure I is similar to those considered by Huang and Wang (1997) and Vayanos and Wang (2012).

3.3 Asymmetric Information Under Market Structure II

The more interesting case is under market structure II when options are also traded. We have the following result:

Theorem 3 *For the economy defined in Section 2, under market structure II, a unique equilibrium exists, in which the equilibrium price S, C_j and security holdings θ_{iS}, θ_{ij}, where $i = 1, 2$ and $j = 1, \ldots, n$, are given by the solution to the following equations:*

$$S = \frac{\int_{-\infty}^{\infty} y e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+\theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+\theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy} \tag{13a}$$

$$C_j = \frac{\int_{-\infty}^{\infty} (y-k_j)^+ e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+\theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+\theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}, \quad j = 1, \ldots, n \tag{13b}$$

and

$$\frac{\int_{-\infty}^{\infty} y e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}$$

$$= \frac{\int_{-\infty}^{\infty} y e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{2j}\right]} e^{-\frac{1}{2\hat{\sigma}_u^2}\left(y-\bar{V}-\alpha Q+a\hat{\sigma}_u^2\theta_{2S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{2j}\right]} e^{-\frac{1}{2\hat{\sigma}_u^2}\left(y-\bar{V}-\alpha Q+a\hat{\sigma}_u^2\theta_{2S}\right)^2} dy} \tag{14a}$$

$$\theta_{2S} = 1 - \frac{\omega}{1-\omega}(\theta_{1S} - 1) \tag{14b}$$

$$\frac{\int_{-\infty}^{\infty} (y-k_j)^+ e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{1j}\right]} e^{-\frac{1}{2\sigma_u^2}\left(y-\bar{V}-Q+a\sigma_u^2\theta_{1S}\right)^2} dy}$$

$$= \frac{\int_{-\infty}^{\infty} (y-k_j)^+ e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{2j}\right]} e^{-\frac{1}{2\hat{\sigma}_u^2}\left(y-\bar{V}-\alpha Q+a\hat{\sigma}_u^2\theta_{2S}\right)^2} dy}{\int_{-\infty}^{\infty} e^{-a\left[\sum_{j=1}^{n}(y-k_j)^+ + \theta_{2j}\right]} e^{-\frac{1}{2\hat{\sigma}_u^2}\left(y-\bar{V}-\alpha Q+a\hat{\sigma}_u^2\theta_{2S}\right)^2} dy} \tag{14c}$$

$$\theta_{2j} = -\frac{\omega}{1-\omega}\theta_{1j}, \quad j = 1, \cdots, n. \tag{14d}$$

It should be clear from Theorem 3 that given the state of the economy, which is fully specified by X and V, we need to solve for $n+1$ prices for the traded securities (the stock plus n options) and their holdings by the two investors. The total number of unknowns is $(n+1)\times 3$. We have exactly the same number of equations given by (13) and (14). The solution will give the equilibrium values for S, $\{C_j, j=1,\ldots,n\}$, $\{\theta_{1S}, \theta_{2S}\}$ and $\{\theta_{1j}, \theta_{2j}, j=1,\ldots,n\}$ as functions of X and V through their dependence on Q .

4 Impact of Options Trading under Asymmetric Information

It is obvious from Theorems 2 and 3 that under asymmetric information, the introduction of options will change the market equilibrium. In other words, options are no longer redundant securities. In this section, we examine in detail the impact of options on the market, particularly on investors' risk-sharing, the informational efficiency of the market, the equilibrium price of the stock, and investors' welfare. We do so by comparing these aspects of the market equilibrium under market structures I and II.

4.1 Equilibrium without Options

We first analyze the market equilibrium in the absence of options (i.e., under market structure I). This case allows us to examine in detail investors' trading motives and behavior, which will help us understand the role of options when they are present and the consequent impact on equilibrium.

In our model, investors trade for two reasons: to manage risk and to earn extra returns. For investor 1, the trading motives are direct and clear. First, he trades in the stock to hedge his non-traded risk, to which he has an exposure of X. In addition, when he possesses private information on the stock payoff, from his observation of V, he also trades to speculate on this information for extra returns.

For investor 2, however, the trading motives are more indirect. As investor 1 trades the stock to hedge, its price has to shift, which induces investor 2 to take the other side with an expected extra return. For example, for a positive X, investor 1 has a larger overall risk exposure. He would like to sell the stock to reduce his risk. Consequently, the stock price will decrease, independent of the expected payoff of the stock. This gap between the expected payoff and the market price gives rise to an excess return on the stock, which attracts investor 2 to take a larger stock position. This interaction is clearly captured by the market equilibrium even under symmetric information. Under asymmetric information, however, this trading is tempered by the fact that investor 2 does not observe V and thus perceives a higher risk for the stock than investor 1.

Based on the observations above, we now examine the investors' trading behavior and the resulting equilibrium more formally. Given that investor 1's stock demand is driven by two underlying state variables, his exposure to non-traded risk X and privation information on stock payoff V, the equilibrium stock price will depend on these two variables as well. For example, in the case of symmetric information, the dependence is linear as shown in Theorem 1. More specifically, the price can be expressed a function of $Q = (V - \bar{V}) - a\sigma_u^2 X$. For the moment, let us assume that with asymmetric information, the stock price remains a function of Q only, which will be confirmed as stated in Theorem 2.

Given this form of the price function, the investors' stock demand follows immediately. For investor 1, his expected payoff from the stock is simply V, and the conditional variance is σ_u^2. Given the normality of the underlying shocks and his CARA utility function, his stock demand is

$$\theta_{1S} = \frac{V - S}{a\sigma_u^2} - X.$$

For investor 2, his expected payoffs from the stock and the conditional payoff are

$$\mathrm{E}[V|S] = \mathrm{E}[V|Q] = \bar{V} + \alpha Q \triangleq \hat{V}, \qquad \mathrm{Var}[V|S] = \mathrm{Var}[V|S] = \hat{\sigma}_u^2.$$

His stock demand is

$$\theta_{2S} = \frac{\mathrm{E}[V|S] - S}{a\hat{\sigma}_u^2} = \frac{\bar{V} + \alpha Q - S}{a\hat{\sigma}_u^2} = \frac{\hat{V} - S}{a\hat{\sigma}_u^2}.$$

Market clearing immediately yields the equilibrium stock price (11) in Theorem 2. We can rewrite (11) as follows:

$$S = \left(\frac{\frac{\omega}{\sigma_u^2}}{\frac{\omega}{\sigma_u^2} + \frac{1-\omega}{\hat{\sigma}_u^2}} V + \frac{\frac{1-\omega}{\hat{\sigma}_u^2}}{\frac{\omega}{\sigma_u^2} + \frac{1-\omega}{\hat{\sigma}_u^2}} \hat{V} \right) - a \frac{1}{\frac{\omega}{\sigma_u^2} + \frac{1-\omega}{\hat{\sigma}_u^2}} (1 + \omega X). \tag{15}$$

The first term gives an average of investors' expectation of the stock's payoff. The weight in the average for an investor is proportional to his population weight and inversely proportional to his perception of stock risk. This is intuitive. The higher the population weight of an investor, the greater the impact of his expectation on the price. However, the higher the perceived risk on the stock, the less the impact of his expectation on the price, as he will trade less aggressively in the stock.

The second term gives the risk premium on the stock. As in the case of symmetric information, the risk premium is proportional to the investors risk aversion a, the total exposure to the stock risk $1 + \omega X$ per capita, and an average of the investors' perception of stock risk, given by the harmonic mean of each class of investors' perceived risk, which is $\left(\frac{\omega}{\sigma_u^2} + \frac{1-\omega}{\hat{\sigma}_u^2} \right)^{-1}$.

It is worth noting that under market structure I, the equilibrium price under asymmetric information has a similar form as under symmetric information. The only difference is that the expected stock payoff is replaced by an average of different investors' expected payoff, and the risk is replaced by the harmonic average of different investors' perceived risk. If we set $\sigma_v = 0$, then $V = \bar{V}$, $\hat{\sigma}_u = \sigma_u$, and we obtain the equilibrium price in (9) in Theorem 1.

Substituting the equilibrium price into the investors' stock demand function, we obtain their equilibrium stock holdings given in (12). In particular, we can rewrite them as follows:

$$\theta_{1S} = 1 + \frac{1-\omega}{\omega\hat{\sigma}_u^2 + (1-\omega)\sigma_u^2}\left[(\hat{\sigma}_u^2 - \sigma_u^2) + (1-\alpha)\left((V-\bar{V})/a - \sigma_u^2 X\right)\right] \tag{16}$$

while θ_{2S} is the same as in (12). Let us first examine the stock holding of investor 1. The first term merely reflects the per capita endowment of the stock share. It is the second term that gives his trading in the stock. We first notice that the second term is proportional to $1-\omega$, the population of investor 2. This is intuitive: with more class-2 investors, there will be more counter-parties in trading for class-1 investors in risk sharing and speculation.

More importantly, there are three components in investor 1's equilibrium trading. The first term is proportional to $\hat{\sigma}_u^2 - \sigma_u^2$, the difference in the perceived risk of the stock by investor 2 and investor 1, respectively. As an uninformed investor, investor 2 perceives a higher risk for the stock than investor 1. Consequently, investor 1 is willing to hold a larger share of the stock on average. This is independent of the speculative bets he makes based on private information or trades to hedge his non-traded risk. As we will see later, it is this component of trading that is closely related to options trading when they are introduced.

The second component is proportional to $V - \hat{V}$, the difference between investor 1's forecasted stock payoff and that of investor 2, who has no private information. This component captures investor 1's speculative bets based on his private information on V. It is inversely proportional to his risk aversion. The third component, which is negatively proportional to X, reflects the trade to hedge investor 1's non-traded risk. The first two components of trading are present only under asymmetric information while the third component is present even under symmetric information, as we see in Theorem 1.

To facilitate comparison, we rewrite the securities holdings and prices as power series in α because it measures information asymmetry which leads to option trading. In particular, we have

$$S = S^{(0)} + b_S^{(1)}\alpha + b_S^{(2)}\alpha^2 + o(\alpha^2) \tag{17a}$$

$$\theta_{1S} = \theta_S^{(0)} + b_{\theta S}^{(1)}\alpha + b_{\theta S}^{(2)}\alpha^2 + o(\alpha^2) \tag{17b}$$

where

$$S^{(0)} = \omega V + (1-\omega)\bar{V} - a\sigma_u^2(\omega X + 1) \tag{18a}$$

$$\theta_S^{(0)} = 1 + \frac{1-\omega}{a\sigma_u^2} Q \tag{18b}$$

and

$$b_S^{(1)} = (1-\omega)\left[Q - a^2\sigma_u^2\sigma_x^2(a\sigma_u^2 - \omega Q)\right] \tag{19a}$$

$$b_S^{(2)} = \omega(1-\omega)a^2\sigma_u^2\sigma_x^2\left[Q - a^2\sigma_u^2\sigma_x^2(a\sigma_u^2 - \omega Q)\right] \tag{19b}$$

$$b_{\theta S}^{(1)} = (1-\omega)\left[-\frac{Q}{a\sigma_u^2} + a\sigma_x^2(a\sigma_u^2 - \omega Q)\right] \tag{19c}$$

$$b_{\theta S}^{(2)} = -\omega(1-\omega)\left[-a\sigma_x^2 Q + a^3\sigma_u^2\sigma_x^4(a\sigma_u^2 - \omega Q)\right]. \tag{19d}$$

Here, we omit the corresponding expressions for θ_2, as it can be easily obtained from (12b) and (17b). Clearly, the zero-th order term gives the stock price and stock-holding of investor 1 under symmetric information. The higher-order terms give the impact of information asymmetry on the stock price and stock holding under market structure I.

4.2 Equilibrium with Options

We now examine how the market equilibrium changes when options are introduced. We first consider the equilibrium under market structure II and then consider how it differs from the equilibrium under market structure I. In this subsection, we state some general results regarding the equilibrium under market structure II. In the following subsections, we will analyze in more detail the impact of options.

In the presence of information asymmetry, the introduction of derivative securities tends to change the informational efficiency of the market. In particular, more private information can be conveyed by the price system as more securities are traded (see, e.g., Grossman (1989)). However, as Huang and Wang (1997) illustrate, when trading for risk sharing is endogenous, the introduction of derivatives can increase the amount of uninformative trading and thus reduce the informational efficiency of the market. In our model, we have the following result:

Proposition 1 *For the economy as defined in Section 2, the introduction of options does not improve the informational efficiency of the market. In particular, for the uninformed investors, their perception of the stock's payoff and its uncertainty is the same under* *market structures I and II.*

From Theorem 3, the above proposition is immediate. As equations (13) and (14) show, under market structure II the equilibrium prices of the stock and options only depend on Q, as under market structure I. Thus, $\mathrm{E}[V|S,\{C\}] = \mathrm{E}[V|Q]$ and $\mathrm{Var}[V|S,\{C\}] = \mathrm{Var}[V|Q]$. This is true regardless of how many options are introduced and what strikes they have. This also implies that the stock price alone is sufficient for the uninformed investor (investor 2) to infer Q.

The intuition behind this result is the following: Investors tend to use the stock itself to bet on the direction of its price movement, while they use options to bet on the volatility of price. In our setting, the price volatility is independent of the mean price, under the normality assumption of all underlying shocks. Thus, trading in the options is also independent of the price forecast, as are their prices. This intuition will become clearer in the following discussions.[10]

Despite their lack of impact on the market's informational efficiency, options are not redundant. They are used by the informed investors to improve risk sharing, as Theorem 3 clearly shows. We will examine the nature of options trading and its market impact next.

To better understand the impact of option trading on the market, we first consider the case when only one call option with strike k is introduced. In this one option case, the equilibrium is characterized in the following theorems.

Theorem 4 characterized the first-order effect of option trading on the market[11].

Theorem 4 *For the economy defined in Section 2, when only one call option with strike k is introduced and α is small*[12]*, the equilibrium price S, C and security holdings θ_{iS},*

[10] Although the introduction of options does not provide additional information in our model, it can if we extend our model to allow richer information structures. For example, suppose there is uncertainty about the volatility of U, i.e., σ_u becomes a random variable and informed investors know its value but uninformed investors do not. Then, trading in options can help the uninformed investors infer the value of σ_u.

[11] Theorem 4 only gives the result when α is small; however, similar result holds when α approaches 1 (see Appendix 7.3).

[12] Here, a small α corresponds to a small σ_v, not a large $a^2\sigma_x^2\sigma_u^2$, because $\alpha \neq 0$ and $\hat{\sigma}_u^2 \neq \sigma_u^2$ for any finite $a^2\sigma_x^2\sigma_u^2$.

θ_{iC}, *where* $i = 1, 2$, *are:*

$$\begin{pmatrix} S \\ C \end{pmatrix} = \begin{pmatrix} S^{(0)} \\ C^{(0)} \end{pmatrix} + \begin{pmatrix} b_S^{(1)} + \delta_S^{(1)} \\ \delta_C^{(1)} \end{pmatrix} \alpha + o(\alpha) \tag{20a}$$

$$\begin{pmatrix} \theta_{1S} \\ \theta_{1C} \end{pmatrix} = \begin{pmatrix} \theta_S^{(0)} \\ 0 \end{pmatrix} + \begin{pmatrix} b_{\theta S}^{(1)} + \delta_{\theta S}^{(1)} \\ \delta_{\theta C}^{(1)} \end{pmatrix} \alpha + o(\alpha) \tag{20b}$$

where $S^{(0)}$, $\theta_S^{(0)}$, $b_S^{(1)}$ *and* $b_{\theta S}^{(1)}$ *are given in (18) and (19),*

$$C^{(0)} = \frac{\sigma_u}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{S^{(0)} - k}{\sigma_u} \right)^2 \right] + \left(S^{(0)} - k \right) \Phi \left(\frac{S^{(0)} - k}{\sigma_u} \right), \tag{21}$$

$$\delta_S^{(1)} = 0 \tag{22a}$$

$$\delta_C^{(1)} = b_S^{(1)} \Phi \left(\frac{S^{(0)} - k}{\sigma_u} \right) + (1 - \omega) \frac{a^2 \sigma_x^2 \sigma_u^3}{2} \phi \tag{22b}$$

$$\delta_{\theta S}^{(1)} = \frac{(1 - \omega)(1 - \Phi) a \sigma_x^2 \sigma_u \phi}{2d} \tag{22c}$$

$$\delta_{\theta C}^{(1)} = -\frac{(1 - \omega) a \sigma_x^2 \sigma_u \phi}{2d} \tag{22d}$$

and

$$x = \frac{k - S^{(0)}}{\sigma_u}, \tag{23a}$$

$$\phi = \phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{k - S^{(0)}}{\sigma_u} \right)^2 \right], \tag{23b}$$

$$\Phi = \Phi(x) = \int_{-\infty}^{x} \phi(t) dt \tag{23c}$$

$$d = [1 - \Phi(x)] \left[(1 + x^2) \Phi(x) - 2x\phi(x) \right] - x\phi(x) - \phi(x)^2. \tag{23d}$$

The first-order impacts of option introduction on the security prices and holdings in the economy with information asymmetry are given by $\delta_{\cdot}^{(1)}$. It shows that the introduction of options has some impacts on the stock trading but little impact on the stock price.

To understand the results of theorem 4, we may assume that the market is already in the equilibrium without options and consider how investors change their securities holdings when a call option is introduced. That is to say, investor 1 already holds $\theta_{10}(\approx \theta_S^{(0)} + b_{\theta S}^{(1)}\alpha + b_{\theta S}^{(2)}\alpha^2)$ shares of stocks. However, the optimal adjustment of the portfolio can be solved in an economy where investors have no endowments and agree upon the expectation of the stock's payoff, i.e., $S_0(\approx S^{(0)} + b_S^{(1)}\alpha + b_S^{(2)}\alpha^2)$, but disagree on the variance of the payoff, i.e., investor 1 thinks of it as σ_u^2 and investor 2 thinks of it as $\hat{\sigma}_u^2$. When α is small, the first order approximation of the investor's preference is a mean-variance preference, i.e., investor 1 will choose $\Delta\theta_{1S}^{(1)}$ and $\Delta\theta_{1C}^{(1)}$ to maximize

$$aE[W_1] - \frac{1}{2}a^2 D[W_1]$$

where $W_1 = \Delta\theta_{1S}^{(1)}(F - S) + \Delta\theta_{1C}^{(1)}((F-k)^+ - C)$, $F \sim N(S_0, \sigma_u^2)$. Investor 2 will choose $\Delta\theta_{2S}^{(1)}$ and $\Delta\theta_{2C}^{(1)}$ to maximize

$$aE[W_2] - \frac{1}{2}a^2 D[W_2]$$

where $W_2 = \Delta\theta_{2S}^{(1)}(F - S) + \Delta\theta_{1C}^{(1)}((F-k)^+ - C)$ and $F \sim N(S_0, \hat{\sigma}_u^2)$. Therefore, given security price S and C, the investors' demands for securities are

$$\begin{pmatrix} \Delta\theta_{1S}^{(1)} \\ \Delta\theta_{1C}^{(1)} \end{pmatrix} = \frac{1}{a}\Sigma^{-1} \begin{pmatrix} S_0 - S \\ C_0^1 - C \end{pmatrix}, \qquad \begin{pmatrix} \Delta\theta_{2S}^{(1)} \\ \Delta\theta_{2C}^{(1)} \end{pmatrix} = \frac{1}{a}\hat{\Sigma}^{-1} \begin{pmatrix} S_0 - S \\ C_0^2 - C \end{pmatrix}.$$

where Σ is the covariance matrix of stock and option payoff from investor 1's perspective and $\hat{\Sigma}$ is the covariance matrix from investor 2's perspective. C_0^i is investor i's shadow pricing of an option before the introduction of the option. When α is sufficiently small, $\|(\hat{\Sigma} - \Sigma)\| = O(\alpha)$ and $\hat{\Sigma}^{-1} = \Sigma^{-1} - \Sigma^{-1}(\hat{\Sigma} - \Sigma)\Sigma^{-1} + o(\alpha)$. By the market clearing condition,

$$\omega \begin{pmatrix} \Delta\theta_{1S}^{(1)} \\ \Delta\theta_{1C}^{(1)} \end{pmatrix} + (1-\omega) \begin{pmatrix} \Delta\theta_{2S}^{(1)} \\ \Delta\theta_{2C}^{(1)} \end{pmatrix} = 0,$$

we have

$$\begin{pmatrix} S \\ C \end{pmatrix} = \begin{pmatrix} S_0 \\ \omega C_0^1 + (1-\omega)C_0^2 \end{pmatrix} + o(\alpha)$$

after some algebra. This is just equation (20a) because

$$C_0^1 = C^{(0)} + \alpha b_S^{(1)} \Phi\left(\frac{S^{(0)} - k}{\sigma_u}\right) + o(\alpha)$$

and

$$C_0^2 = C^{(0)} + \alpha b_S^{(1)} \Phi\left(\frac{S^{(0)} - k}{\sigma_u}\right) + \alpha \frac{a^2\sigma_x^2\sigma_u^3}{2}\phi\left(\frac{S^{(0)} - k}{\sigma_u}\right) + o(\alpha).$$

Therefore, investor 1's demand for securities is

$$\begin{pmatrix} \Delta\theta_{1S}^{(1)} \\ \Delta\theta_{1C}^{(1)} \end{pmatrix} = \frac{1}{a}\Sigma^{-1}\begin{pmatrix} S_0 - S \\ C_0^1 - C \end{pmatrix} = \frac{1}{a}\Sigma^{-1}\begin{pmatrix} 0 \\ (1-\omega)(C_0^1 - C_0^2) \end{pmatrix},$$

i.e., equation (20b).

In this mean-variance economy, the difference in the initial option pricing, i.e., $C_0^2 - C_0^1$, causes the option trading; therefore, it suggests that the disagreement on the volatility of the stock's payoff is the essential factor which causes the option trading. The larger $\hat{\sigma}_u^2 - \sigma_u^2$ is, i.e., more disagreement on the stock volatility, the more option trading there will be. Moreover, the additional securities trading caused by the option introduction only depends on investors' evaluation of the risk in the stock's payoff and is independent of investors' expectations about the payoff. This suggests that the option is more similar to a security for trading volatility and that the investor who views the stock's payoff as more risky should buy the option. It also suggests that options can reduce, though not necessary reconcile, the disagreements about the volatility of the stock's payoff. On the other hand, different expectations about the payoff of the stock affect the stock trading only through θ_{10}, which suggests the stock is sufficient for reconciling the disagreements about the mean of the stock's payoff but is not useful for reducing the disagreements about the volatility of the stock's payoff.

The above analysis suggests that investors trade additional stocks mainly for hedging option positions and that the stock demand satisfies $\Delta\theta_{iS}^{(1)} = (\Phi - 1)\Delta\theta_{iC}^{(1)}$, i.e., the ratio between stock trading and the option is just the beta of the option with the stock $1 - \Phi$. Simple algebra shows that the imbalance of investor 1's demand of stock and investor 2's supply of stock is proportional to $\Phi(\frac{k-S_0}{\hat{\sigma}_u}) - \Phi(\frac{k-S_0}{\sigma_u})$. Therefore, there are excess stock demands when $k < S_0$, and there are excess stock supplies when $k > S_0$. However, the imbalance of the stock's demand and supply is of order α^2, indicating that the effect of introducing the option on the stock price is also of order α^2.

To analyze the second-order effect of introducing an option, we approximate the investors' expected utility by incoporating the effect of skewness, as in Kraus and Litzenberge (1976). That is, investor 1 will choose $\Delta\theta_{1S} = \Delta\theta_{1S}^{(1)} + \Delta\theta_{1S}^{(2)}$ and $\Delta\theta_{1C} = \Delta\theta_{1C}^{(1)} + \Delta\theta_{1C}^{(2)}$ to maximize

$$aE[W_1^*] - \frac{1}{2}a^2 D[W_1^*] + \frac{1}{6}a^3\kappa_3[W_1^*]$$

where $W_1^* = \Delta\theta_{1S}(F-S) + \Delta\theta_{1C}((F-k)^+ - C)$, $\kappa_3[\cdot]$ denotes the third central moment and $F \sim N(S_0, \sigma_u^2)$. Investor 2 will choose $\Delta\theta_{2S}$ and $\Delta\theta_{2C}$ to maximize

$$aE[W_2^*] - \frac{1}{2}a^2 D[W_2^*] + \frac{1}{6}a^3\kappa_3[W_2^*]$$

where $W_2^* = \Delta\theta_{2S}(F-S) + \Delta\theta_{1C}((F-k)^+ - C)$ and $F \sim N(S_0, \hat{\sigma}_u^2)$.

The first-order conditions of utility maximization suggest that

$$\frac{\partial}{\partial\Delta\theta_{iS}^{(1)}}(E[W_i] - \frac{1}{2}aD[W_i]) = \frac{\partial}{\partial\Delta\theta_{iC}^{(1)}}(E[W_i] - \frac{1}{2}aD[W_i]) = 0,$$

$$\frac{\partial}{\partial\Delta\theta_{iS}^{(2)}}(E[W_i^*] - \frac{1}{2}aD[W_i^*] + \frac{1}{6}a^2\kappa_3[W_i^*]) = \frac{\partial}{\partial\Delta\theta_{iC}^{(2)}}(E[W_i^*] - \frac{1}{2}aD[W_i^*] + \frac{1}{6}a^2\kappa_3[W_i^*]) = 0.$$

Because investors always agree on the pricing of securities in the equilibrium, we have

$$\begin{aligned}
&\frac{\partial}{\partial\Delta\theta_{1S}^{(1)}}D[W_1] - \frac{\partial}{\partial\Delta\theta_{1S}^{(2)}}(D[W_1^*] - \frac{1}{3}a\kappa_3[W_1^*]) \\
=&\frac{\partial}{\partial\Delta\theta_{2S}^{(1)}}D[W_2] - \frac{\partial}{\partial\Delta\theta_{2S}^{(2)}}(D[W_2^*] - \frac{1}{3}a\kappa_3[W_2^*]) \\
&\frac{\partial}{\partial\Delta\theta_{1C}^{(1)}}D[W_1] - \frac{\partial}{\partial\Delta\theta_{1C}^{(2)}}(D[W_1^*] - \frac{1}{3}a\kappa_3[W_1^*]) \\
=&\frac{\partial}{\partial\Delta\theta_{2C}^{(1)}}D[W_2] - \frac{\partial}{\partial\Delta\theta_{2C}^{(2)}}(D[W_2^*] - \frac{1}{3}a\kappa_3[W_2^*])
\end{aligned}$$

where

$$\frac{\partial}{\partial\Delta\theta_{iS}^{(1)}}D[W_i] = 2\text{Cov}(F, W_i) \quad i = 1, 2,$$

$$\frac{\partial}{\partial\Delta\theta_{iC}^{(1)}}D[W_i] = 2\text{Cov}((F-k)^+, W_i) \quad i = 1, 2,$$

$$\frac{\partial}{\partial\Delta\theta_{iS}^{(2)}}\kappa_3[W_i^*] = 3\text{Cov}(F,(W_i^*)^2) - 6E[W_i^*]\text{Cov}(F,W_i^*) \quad i=1,2,$$

$$\frac{\partial}{\partial\Delta\theta_{iC}^{(2)}}\kappa_3[W_i^*] = 3\text{Cov}((F-k)^+,(W_i^*)^2) - 6E[W_i^*]\text{Cov}((F-k)^+,W_i^*) \quad i=1,2.$$

Therefore,

$$\begin{aligned}
\begin{pmatrix} \Delta\theta_{1S}^{(2)} \\ \Delta\theta_{1C}^{(2)} \end{pmatrix} &= (\Sigma + \frac{\omega}{1-\omega}\hat{\Sigma})^{-1} \\
&\times[\begin{pmatrix} \frac{a}{2}(\text{Cov}(Y,(W_1^*)^2) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}(Y,(W_1^*)^2)) \\ \frac{a}{2}(\text{Cov}((Y-k)^+,(W_1^*)^2) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}((Y-k)^+,(W_1^*)^2)) \end{pmatrix} \\
&+ \begin{pmatrix} aE[W_1^*](\text{Cov}(Y,W_1^*) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}(Y,W_1^*)) \\ aE[W_1^*](\text{Cov}((Y-k)^+,W_1^*) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}((Y-k)^+,W_1^*)) \end{pmatrix}].
\end{aligned} \tag{24}$$

By the property of the mean-variance equilibrium and noticing that $W_1^* - W_1$ is of order α^2, equation (24) can be rewritten as

$$\begin{aligned}
\begin{pmatrix} \Delta\theta_{1S}^{(2)} \\ \Delta\theta_{1C}^{(2)} \end{pmatrix} &= (\Sigma + \frac{\omega}{1-\omega}\hat{\Sigma})^{-1} \\
&\times[\begin{pmatrix} \frac{a}{2}(\text{Cov}(Y,(W_1)^2) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}(Y,(W_1)^2)) \\ \frac{a}{2}(\text{Cov}((Y-k)^+,(W_1)^2) - (\frac{\omega}{1-\omega})^2\hat{\text{Cov}}((Y-k)^+,(W_1)^2)) \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ E[W_1](C_0^1 - C_0^2)(1-2\omega)) \end{pmatrix}] + o(\alpha^2).
\end{aligned} \tag{25}$$

Combining this skewness effect with the second-order effect produced in the mean-variance equilibrium produces the following theorem 5, which characterizes the second-order effect of option trading on the market.

Theorem 5 *For the economy defined in Section 2,when only one call option with strike k is introduced and α is small, the equilibrium price S, C and security holdings θ_{iS}, θ_{iC}, where $i=1,2$, are:*

$$\begin{pmatrix} S \\ C \end{pmatrix} = \begin{pmatrix} S^{(0)} \\ C^{(0)} \end{pmatrix} + \begin{pmatrix} b_S^{(1)} + \delta_S^{(1)} \\ \delta_C^{(1)} \end{pmatrix}\alpha + \begin{pmatrix} b_S^{(2)} + \delta_S^{(2)} \\ \delta_C^{(2)} \end{pmatrix}\alpha^2 + o(\alpha^2) \tag{26a}$$

$$\begin{pmatrix} \theta_{1S} \\ \theta_{1C} \end{pmatrix} = \begin{pmatrix} \theta_S^{(0)} \\ 0 \end{pmatrix} + \begin{pmatrix} b_{\theta S}^{(1)} + \delta_{\theta S}^{(1)} \\ \delta_{\theta C}^{(1)} \end{pmatrix}\alpha + \begin{pmatrix} b_{\theta S}^{(2)} + \delta_{\theta S}^{(2)} \\ \delta_{\theta C}^{(2)} \end{pmatrix}\alpha^2 + o(\alpha^2) \tag{26b}$$

where

$$\delta_S^{(2)} = \frac{a^2\sigma_u^3(\phi+\Phi(x-\Phi x-2\phi))\omega(\delta_{\theta C}^{(1)})^2}{\omega-1} + \frac{1}{2}a^3\phi x\sigma_u^4\sigma_x^2\omega\delta_{\theta C}^{(1)} \tag{27a}$$

$$\begin{aligned}\delta_C^{(2)} =& \frac{1}{2(\omega-1)}[a^2\omega\delta_{\theta C}^{(1)}(aC^{(0)}\phi\sigma_u^3\sigma_x^2(1-\omega)^2-(\Phi-1)\Phi x^3\sigma_u^3\delta_{\theta C}^{(1)}+\sigma_u(\phi(1-2\Phi)x^2\sigma_u^2\\ &+(-\phi^2+\Phi+\Phi^2-2\Phi^3)x\sigma_u^2+\phi(1-3\Phi)\Phi\sigma_u^2)\delta_{\theta C}^{(1)})]\\ &+\frac{1}{8\sigma_u}[4(b_S^{(1)})^2\phi-4a^2b_S^{(1)}\phi x\sigma_u^3\sigma_x^2(\omega-1)+a^3\sigma_u^4\sigma_x^2(a\phi\sigma_u^2(1-x^2)\sigma_x^2(\omega-1)\\ &-8(C^{(0)}\phi+(\phi^2+(\Phi-1)\Phi)\sigma_u)\omega\delta_{\theta C}^{(1)})]\end{aligned} \tag{27b}$$

$$\begin{aligned}\delta_{\theta S}^{(2)} =& \frac{a(1-2\omega)\sigma_u}{2d(1-\omega)}[(\delta_{\theta C}^{(1)})^2((1-\Phi)^2\Phi(1-2\Phi)x^3+\phi(1-\Phi)(1-2\Phi)(1-4\Phi)x^2\\ &-(1-2\Phi)(\phi^2(3-5\Phi)+(1-\Phi)^2\Phi)x+\phi((1-\Phi)^2(3\Phi-2)+\phi^2(2-4\Phi)))\\ &+\delta_{\theta C}^{(1)}aC^{(0)}\phi(\Phi-1)\sigma_x^2(1-\omega)^2]\\ &+\frac{1}{8d\sigma_u^2}[4ab_S^{(1)}\phi(\Phi-1)x\sigma_u^2\sigma_x^2(\omega-1)-a^3\phi(\Phi-1)\sigma_u^5(1-x^2)\sigma_x^4(\omega-1)\\ &+(-8b_S^{(1)}(2C^{(0)}(1-\Phi)^2+\phi^2(1-2\Phi)x\sigma_u-\phi^3\sigma_u+\phi(\Phi-1)\sigma_u(\Phi-1-\Phi x^2\sigma_u))\\ &+4a^2\sigma_u\sigma_x^2((\phi(\Phi-1)\Phi x^3\sigma_u^3+\phi^2(2\Phi-1)x^2\sigma_u^3+\phi(\phi^2+\Phi-1)x\sigma_u^3\\ &+2\phi C^{(0)}(\Phi-1)\sigma_u^2+2(\Phi-1)(\phi^2+(\Phi-1)\Phi)\sigma_u^3)\omega))\delta_{\theta C}^{(1)}]\end{aligned} \tag{27c}$$

$$\begin{aligned}\delta_{\theta C}^{(2)} =& \frac{a(1-2\omega)\sigma_u}{2d(1-\omega)}[(\delta_{\theta C}^{(1)})^2((\Phi-1)\Phi x^3+\phi(2\Phi-1)x^2+(\phi^2+(\Phi-1)\Phi(4\Phi-1))x\\ &+\phi(2+7(\Phi-1)\Phi))+\delta_{\theta C}^{(1)}aC^{(0)}\phi\sigma_x^2(1-\omega)^2]\\ &+\frac{1}{8d\sigma_u^2}[a\phi\sigma_u\sigma_x^2(a^2\sigma_u^4(1-x^2)\sigma_x^2-4b_S^{(1)}x\sigma_u)(1-\omega)+4(b_S^{(1)}(2\phi(1-2\Phi)\sigma_u\\ &-4C^{(0)}(\Phi-1))+a^2\sigma_u^3\sigma_x^2\omega(\phi(2C^{(0)}+x\sigma_u(1-\Phi))+2(\phi^2+\Phi^2-\Phi)\sigma_u))\delta_{\theta C}^{(1)}]\end{aligned} \tag{27d}$$

and $S^{(0)}$, C^0, $\theta_S^{(0)}$, $b_S^{(1)}$, $b_S^{(2)}$, $b_{\theta S}^{(1)}$, $b_{\theta S}^{(2)}$, $\delta_S^{(1)}$, $\delta_C^{(1)}$, $\delta_{\theta S}^{(1)}$, $\delta_{\theta C}^{(1)}$, x, ϕ, Φ *and* d *are given in the previous theorem.*

Theorem 4 shows that the second-order effect of introducing an option on the stock price is given by $\delta_S^{(2)}$, which is not neglectable. Equation (27a) also shows that $\delta_S^{(2)}$ has two terms. The first term is from the skewness effect, which reflects that the option trading can change the portfolio's skewness. The second is caused by the imbalance of the stock's demand and supply in the mean-variance economy and, therefore, is in proportion to $(\Phi(\frac{k-S_0}{\hat{\sigma}_u})-\Phi(\frac{k-S_0}{\sigma_u}))\delta_{\theta C}^{(1)}$, i.e., $\frac{1}{2}a^2\phi x\sigma_u^2\sigma_x^2\delta_{\theta C}^{(1)}$.

In summary, these two theorems demonstrate that the option introduction has a non-neglectable impact on the market and that the impact on the stock trading is more significant than the impact on the stock price. These results seem natural because the market is essentially incomplete, and introducing a new security in an incomplete market has an impact on the existing securities generically. However, as shown in Brennen and Cao (1996), the option introduction has no impact on the stock price. Such a difference arises because the quadratic security introduced in Brennen and Cao (1996) is just the security that can essentially complete the market, while the plain vanilla option can be seen as a linear combination of stock, quadratic security, and some other securities. In other words, trading the quadratic security introduced in Brennen and Cao (1996) can change the investor's opinion about stock volatility without altering his opinion about the stock return's higher moments, but trading the plain vanilla option always changes the investor's opinion about all moments of the stock return. Therefore, the equilibrium stock prices differ in our model from those of Brennen and Cao (1996).

4.3 Options Trading and Stock Trading

With the help of theorem 4 and 5, we obtain the following proposition characterizing the impact of option introduction on securities' trading.

Proposition 2 *For the economy defined in Section 2, introducing one call option with strike price k causes investor 1 to sell the option and buy stock. Additionally, investor 1 sells more options when the option is deeper in (out of) the money. Investor 1 sells fewer stocks when the strike price is higher.*

Proposition 2 is intuitive. Once an option is introduced, the investors who perceive a higher risk for the stock will buy the option because they price the option higher, as the option price increases with stock volatility. Due to the positive correlation between the option's payoff and the stock's payoff, as well as the self-financing requirement, the trading in the option leads investors to adjust their stock holdings. Because investor 1 places a lesser likelihood on extreme payoffs of the stock and prefers portfolios with a higher payoff in these states of nature, investor 1 will sell stocks to make the portfolio's payoff a concave function of the stock's payoff. That is, investor 1 profits from investor 2 in states with extreme stock payoff to compensate the loss in other states. With the higher moneyness of the option, investor 1 profits less per option in the states with extreme stock payoff. Therefore, investor 1 will sell more options because the option is farther away from the money. By equation (20b), the change in stock holding is mainly driven by the correlation between the payoffs of option and stock. As the strike price increases, the payoffs of option and stock are less correlated, and hence, investor 1 buy fewer stocks.

Although proposition 2 analyzes the impact of introducing an option assuming V and X, the conclusions hold before the values of V and X are realized as suggested by the following numerical examples.

Figure 1 plots how the expected stock and investor 1's option demands change when the market structure changes from I to II. It clearly shows that investor 1 demands fewer stocks as the strike of the option increases and demands more options when the option is deeper in (out of) the money. In the plots, the strike price is expressed as distances from the corresponding expected stock prices in the economy with the same composition of investors but without the introduction of the call option. More precisely, the corresponding expected stock prices is S_0. The baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\sigma_v = 0.5$, $\sigma_u = 1$, $\sigma_x = 1$, and $\omega = 0.15$.

4.4 Options and Stock Price

Proposition 3 shows the impact of an option on the stock price.

Proposition 3 *For the economy defined in Section 2, when only one call option with strike price k is traded in addition to the stock and the bond,*

1. *the equilibrium stock price S is increasing in k except for an interval around the stock price, i.e., for a given V and X, there exists an interval $[I_0, I_1]$ such that $S \in [I_0, I_1]$ and $\frac{dS}{dk} \leq 0$ when $k \in [I_0, I_1]$, $\frac{dS}{dk} > 0$ when $k \notin [I_0, I_1]$.*

2. *the introduction of an option increases the stock price if the call option is in the money, and the introduction of an option decreases the stock price if the call option is out of the money. That is, for a given V and X, $S > S_0$ if $k < S_0$ and $S < S_0$ if $k > S_0$.*

In other words, the proposition shows that whether an increase in the option strike price results in an increase or a decrease in the stock price depends on the strike price. This result stands in contrast with Detemple and Selden (1991), which states an increase in the option exercise price always results in a decrease in the stock price in a mean-variance economy. The conclusion that the introduction of an option will increase as

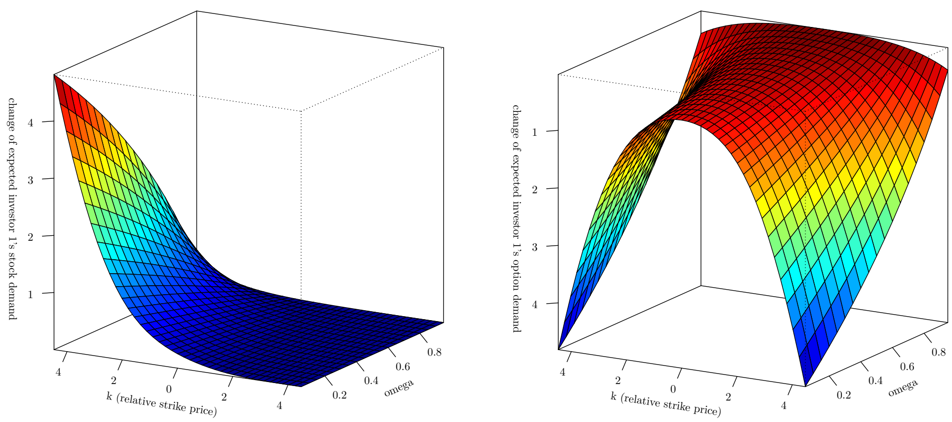

Figure 1: Expected demand changes caused by introducing an option

The graph shows the change in investor 1's expected stock and option demands caused by the introduction of a call option. The strike prices are expressed as the distances from the corresponding expected stock prices in the economy but without the introduction of a call option. The baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\sigma_v = 0.5$, $\sigma_u = 1$, $\sigma_x = 1$, and $\omega = 0.15$.

well as decrease the stock price is also different from the results of Detemple and Selden (1991), which states that introducing an option contract always increases the equilibrium value of the stock.

The intuition regarding why introducing options affects stock price is as follows. Once an option is introduced, investor 1 will sell the option because he gives a lower price for the option. To maintain the optimality of the portfolio, investors need adjust their stock holding to hedge the risk from the option position according to the covariance and higher co-moments between the stock's payoff and the option's payoff. Because investors of different types have different views of these co-moments, their adjustments in their stock holdings are different, and such unbalanced adjustments change the stock price. Because the option demand and the adjustment of the stock holdings are both a non-monotonic function of the option strike price, the relationship between the stock price and the option strike price is non-monotonic.

To see why the introduction of an out-of-the-money option decreases the stock price, we first consider the case of introducing an ATM option, i.e., $k = S_0$. In this case, investor 1's option demand is:

$$\theta_{1C} = \frac{2Z}{a\sigma_u}$$

where Z is the unique real root of the equation

$$\sigma_u(\frac{1}{\sqrt{2\pi}f(Z)} + Z) = \hat{\sigma}_u(\frac{1}{\sqrt{2\pi}f(Z\frac{\omega\hat{\sigma}_u}{(\omega-1)\sigma_u})} + Z\frac{\omega\hat{\sigma}_u}{(\omega-1)\sigma_u})$$

and $f(Z) = e^{0.5Z^2}\Phi(Z)$. Investor 1's stock holding is:

$$\theta_{1S} = \frac{(1-\omega)[V - \hat{V} - a\sigma_u^2(1+X) + a\hat{\sigma}_u^2]}{a[\omega\hat{\sigma}_u^2 + (1-\omega)\sigma_u^2]} - \frac{Z}{a\sigma_u}$$

the option price is:

$$C = \sigma_u(\frac{1}{\sqrt{2\pi}f(Z)} + Z)$$

and the stock price remains S_0.

That is to say, when an ATM option is introduced, investors' adjustments in their stock holdings are always half of their option holding and are hence balanced. Therefore, introducing an ATM option does not change the stock price. However, when the strike price increases from S_0 and the stock price is S_0, the option's payoff is less correlated with the stock's payoff, so less stock holding adjustment is needed per unit option holding. Investor 1 is more sensitive to the change in strike price because he places a lesser likelihood on extreme payoffs for the stock. Consequently, there is greater selling pressure in the stock market, and the equilibrium stock price should decrease from S_0.

Although proposition 3 analyzes the impact of introducing an option assuming V and X, the conclusions hold before the values of V and X are realized, as suggested by the following numerical examples. The varying V and X generate stock price volatility so that we can examine the impact of option introduction on stock price volatility.

Figure 2 shows how the expected stock price and the price volatility change by introducing the option change with ω and the option's strike price. When the strike price is close to the corresponding $E[S_0]$, there are only small changes in the expected price with the introduction of an option, but there are relatively large increases in volatility. When the strike price is far from the corresponding $E[S_0]$, there are small changes in both the expected price and the volatility. Here again, the strike prices are expressed as distances from the corresponding expected stock prices in the economy with the same composition of investors but without the introduction of the call option, i.e., $E[S_0]$. The corresponding baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\sigma_v = 0.5$, $\sigma_u = 1$, $\sigma_x = 1$, and $\omega = 0.15$.

Figure 2 demonstrates that whether adding an option increases or decreases the stock price depends on whether the strike price is lower or higher than $E[S_0]$, confirming the results of Proposition 3. Figure 2 also demonstrates that the introduction of an option that, on average, is ATM always makes the stock more volatile even if, on average, it does not change the stock price. The increased volatility comes from the varying moneyness because the strike is predetermined while the stock price varies with V and X. It also highlights the difference between an ATM option, which is common in the literature such as Brennen and Cao (1996) and Cao (1999), and an option with predetermined strike. Assuming an option is always ATM means that its strike price varies with the stock price, which is not consistent with reality. We can see more clearly the difference between the constant strike and the varying strike when the rational expectations equilibria are derived from the equilibria of the demand schedule games. In the case of a constant strike, traders only submit orders for options with a given constant strike, while in the case of varying strikes, traders must submit orders for options with different strikes.

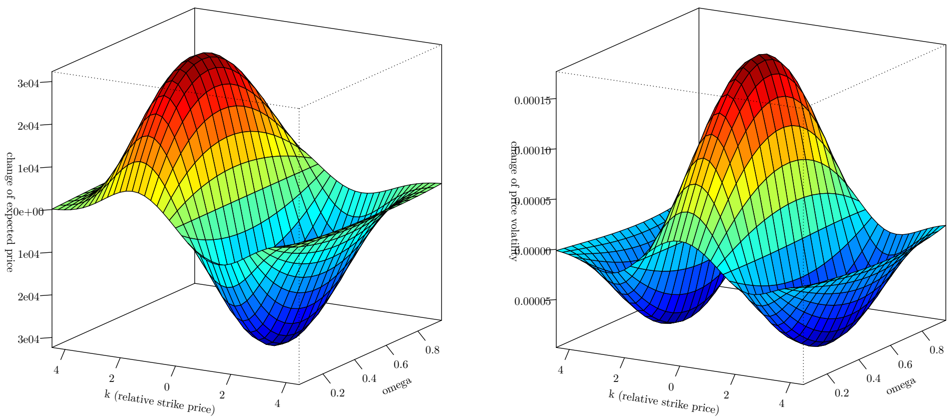

Figure 2: Expected stock price and volatility changes caused by introducing an option

The graph shows the expected stock price and volatility changes caused by the introduction of a call option. The strike prices are expressed as the distances from the corresponding expected stock prices in the economy but without the introduction of a call option. The baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\sigma_v = 0.5$, $\sigma_u = 1$, $\sigma_x = 1$ and $\omega = 0.15$.

Finally, Figure 2 demonstrates that it is the dispersion of the opinions of uncertainty among the investors that determines the impact of option trading on stock price and volatility. It shows that when $\omega = 0.5$ the impact of option introduction on stock price and volatility is the largest. This is because the asymmetric information causes the greatest dispersion of opinions of the uncertainty among the investors when $\omega = 0.5$.

4.5 Options Trading and Investor Welfare

The following proposition demonstrates that the introduction of options will benefit some investors.[13]

Proposition 4 *For the economy defined in Section 2, the introduction of options always benefits someone by increasing the interim utility, that is, the expected utilities of each agent conditional on his private signal. Furthermore, allowing the trading of a new option with a strike price different from the existing options benefits some investors, though not all. Finally, the introduction of options with all possible strikes is interim and ex-ante Pareto improving.*

Such a result is intuitive. When there is background risk, the market is no longer effectively complete. The introduced options provide more instruments for hedging and hence improve the overall allocational efficiency. When a complete set of options with all strikes is introduced, the market is effectively complete and thus it is always Pareto-improving. Figure 3 demonstrates that the introduction of a call option in the economy may reduce an investor's expected utility before he receives his private signals. The corresponding baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\omega = 0.15$, $\sigma_v = 0.5$, $\sigma_u = 1$, and $\sigma_x = 1$. Again, the strike prices are expressed as distances from the corresponding expected stock prices in the economy without the introduction of a call option.

4.6 Impact of More Options

Theorems 4 and 5 can be extended to a multiple options case so we can analyze the impact of option introduction on existing options if there already exist some options. Intuitively, investors can better share risks using available securities when a new option is introduced so that investors' evaluation of the risk in the payoff of the stock will be less

[13] Hakansson (1982) also analyzes the impact of opening option markets on welfare and concludes that the introduction of option trading leads either to Pareto equivalence, to a Pareto redistribution, or to a Pareto improvement. However, his conclusion comes from the strong endowment neutrality of opening option markets, a condition that is not satisfied in the economy considered in this paper.

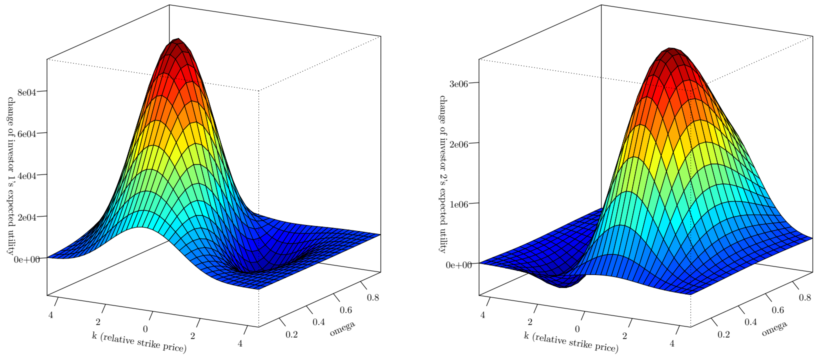

Figure 3: Investors' expected utility changes caused by option introduction

The plot shows the changes in expected utilities of investors caused by introducing an option with strike k. The strike prices are expressed as distances from the corresponding expected stock prices in the economy with the same composition of investors but without option introduction. The baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\omega = 0.15$, $\sigma_v = 0.5$, $\sigma_u = 1$, and $\sigma_x = 1$.

risky, which will lead to lower implied volatility[14]. We examine the impact of introducing a new call option with a different strike on existing options based on numerous numerical exercises.

Figure 4 plots the implied volatilities of options when different numbers of options are introduced. Each line in the figure corresponds to the implied volatilities for the options that are traded in the economy. The lower line corresponds to the economy with more options introduced. It clearly shows that additional options always lower the existing options' implied volatilities. Here, the strike prices are expressed as distances from the corresponding stock prices in the economy with the same composition of investors but without the introduction of the call option, i.e., S_0. The corresponding baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\sigma_v = 0.5$, $\sigma_u = 1$, $\sigma_x = 1$, and $\omega = 0.15$.

[14] Here, the implied volatility for the option with strike k_j is defined to be the volatility σ solving the following equation:

$$C_j = \frac{\sigma}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{S - k_j}{\sigma} \right)^2 \right] + (S - k_j)\, \Phi \left(\frac{S - k_j}{\sigma} \right)$$

where S and C_j are the stock price and the option price, respectively.

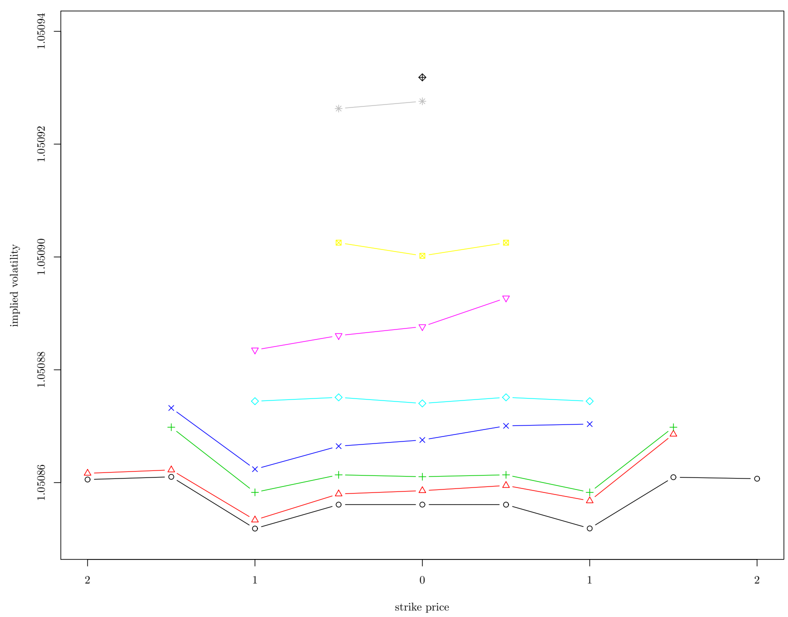

Figure 4: implied volatilities when different numbers of options are introduced

The plot shows the implied volatilities of options when different numbers of options are introduced. Each line in the figure corresponds to the implied volatilities for the options that are traded in the economy. For example, in the economy corresponding to the lowest line there are nine options introduced with strikes (-2,-1.5,-1,-0.5,0,0.5,1,1.5,2). The strike prices are expressed as distances from the corresponding stock prices in the economy with the same composition of investors but without option introduction. The baseline parameters are $\bar{V} = 0$, $a = 0.5$, $\omega = 0.15$, $\sigma_v = 0.5$, $\sigma_u = 1$, and $\sigma_x = 1$.

5 Extensions

5.1 Model Robustness

In the paper, we have shown that options do not reveal directional information that is not already revealed by the underlying asset's price. Although this result is derived from some specific assumptions mentioned in section 2, we show here that the result is robust to more general assumptions.

The first possible extension is introducing infinite numbers of call options. Heuristically, the model can be solved as we have done in Theorem 3 except that there are infinite numbers of equations for options similar to equations (13b). Thus, the securities' prices remain functions of $Q = (V - \bar{V}) - a\sigma_u^2 X$, suggesting that options do not reveal directional information that is not already revealed by the stock price.

A special case is introducing a complete set of call options with all possible strike prices. In this case, a complete set of Arrow-Debreu securities on the payoff of the stock can be replicated by portfolios of call options, so it is identical to assume that there are zero-supplied Arrow-Debreu securities in the market. Note that the distribution of strike prices is always symmetric about the stock price when a complete set of call options is introduced, the stock price S remains unchanged and is given by equation (11), and the state price density for state $F = k$ is given by

$$q_k = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left(-\frac{(k-S)^2}{2\sigma_d^2}\right)$$

where $\frac{1}{\sigma_d^2} = \frac{\omega}{\sigma_u^2} + \frac{1-\omega}{\hat{\sigma}_u^2}$. Thus, the price of the call option with strike price K is

$$C_K = \frac{\sigma_d}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{S-K}{\sigma_d}\right)^2\right] + (S-K)\,\Phi\left(\frac{S-K}{\sigma_d}\right).$$

Another possible extension is to introduce a derivative with a more general payoff $G(F)$. Replacing $(F-k)^+$ by $G(F)$ in the proof of Theorem 3, we can see that such a derivative cannot reveal additional directional information.

5.2 Endogenous Information Acquisition

In this section, we discuss briefly how the introduction of options changes investors' incentives to acquire information in an extended model. We assume a new type of investors who are identical to investor 1 except that they have no nontraded income.

They are called type 3 investors. The population weights for investors 1, 2, and 3 are ω, $(1-\omega)(1-\lambda)$, and $(1-\omega)\lambda$, respectively. For simplicity, we only consider the case of adding a complete set of Arrow-Debreu securities.

We assume investor 3 must pay a constant cost of δ to acquire the information about V and X, while investor 1 always acquires information about V at no cost because he knows X and can infer V from the stock price. Such an assumption is similar to that of Grossman and Stglitz (1980) but is different from Cao (1999), where informed investors choose the precision of the signals.

As a benchmark, we first consider how many investors choose to acquire costly information in the economy without an option. To determine the proportion λ of investors who want to become informed, we only need to compare the ex-ante expected utility of being informed with the utility of being uninformed. The following theorem illustrates the comparative advantage of being informed. The theorem is almost identical to the result of Grossman and Stglitz (1980).

Theorem 6 *In the economy without options, the expected utility of the type 3 investor conditional on the public (the price) information and taking into account the cost C of obtaining the signal is given by*

$$E[U_{30}|S] = e^{a\delta}\frac{\sigma_u}{\sigma_2}E[U_{20}|S]$$

where $\sigma_2^2 = \sigma_u^2 + \frac{\sigma_u^4 a_1^2 \sigma_x^2}{\sigma_v^2+\sigma_u^4 a_1^2\sigma_x^2}\sigma_v^2$, $a_1 = a\frac{\omega}{\omega+\lambda(1-\omega)}$, U_{20}, *and* U_{30} *are the expected utilities of investors 2 and 3 conditional on their own private information.*

Obviously, the theorem means that as more uninformed investors choose to be informed, the expected utility of the informed investor decreases relative to the expected utility of the uninformed investor. Indeed, as more investors are informed, the informativeness of the stock price increases, and the informational advantage of the informed decreases. Therefore, there is a strategic substitutability in information acquisition.

When the options are introduced, although options do not provide additional information, informed investors can use the option market to exploit uninformed investors. On the other hand, the expanded trading opportunities provided by options change the investors' optimal portfolio because the gains of the informed investors from exploiting the uninformed investors in the stock market may be reduced. Therefore, the introduction of options changes the incentives of the investor to acquire information. The following theorem characterizes the comparative advantage of being informed in the economy with a complete set of Arrow-Debreu securities.

Theorem 7 *In the economy with a complete set of Arrow-Debreu securities, the expected utility of the type 3 investor conditional on the public (the price) information, and taking into account the cost δ of obtaining the signal, is given by*

$$E[U_3|S] = \exp\{aC - \frac{1}{2}(\frac{1}{\sigma_u^2} - \frac{1}{\sigma_2^2})\sigma^2\}E[U_2|S]$$

where U_2 and U_3 are the expected utilities of investors 2 and 3 conditional on their own private information, $\frac{1}{\sigma^2} = \frac{\omega_1}{\sigma_u^2} + \frac{1-\omega_1}{\sigma_2^2}$.

Obviously, if $\frac{\sigma_u}{\sigma_2} > \exp\{-\frac{1}{2}(\frac{1}{\sigma_u^2} - \frac{1}{\sigma_2^2})\sigma^2\}$, the informed investors have more informational advantages in the economy with a complete set of Arrow-Debreu securities. Hence, adding a complete set of options will induce more uninformed investors to acquire information. On the other hand, if $\frac{\sigma_u}{\sigma_2} < \exp\{-\frac{1}{2}(\frac{1}{\sigma_u^2} - \frac{1}{\sigma_2^2})\sigma^2\}$, adding a complete set of options will induce fewer uninformed investors to acquire information.

Because $\frac{\sigma_u}{\sigma_2}\exp\{\frac{1}{2}(\frac{1}{\sigma_u^2} - \frac{1}{\sigma_2^2})\sigma^2\}$ is an increasing function of ω if ω is small but is a decreasing function of ω if ω is large, when a small proportion of investors are of type 1, adding a complete set of options will induce more uninformed investors to acquire information. However, when a large proportion of investors are of type 1 and price is informative, adding a complete set of options will induce fewer uninformed investors to acquire information. Thus, information structure, i.e., how many investors are of type 1, is an important factor in determining the uninformed investors' incentives to acquire information. Intuitively, there is more information in the market when there are more type 1 investors and the stock price is more informative; hence, the purchase of information is less advantageous. Thus, the informed investor's gain from exploiting uninformed investors with the introduction of options is limited, while the expanded trading opportunities provided by options always improve the uninformed investors' portfolio allocation. Consequently, fewer uninformed investors prefer to acquire information.

As a simple illustration, figure 5 shows how many uninformed investors will choose to acquire information in two different economies. The dashed line represents the proportion λ of investors who want to become informed in the economy without options, and the solid line represents the proportion λ of investors who want to become informed in the economy with a complete set of options. The corresponding parameters are $\bar{V} = 0$,

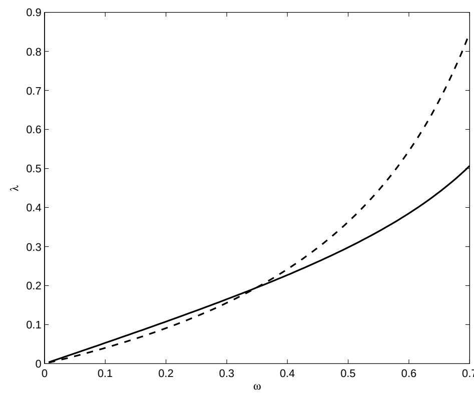

Figure 5: Information structure and the incentives to be informed

The dashed line represents the proportion λ of investors who want to become informed in the economy without options. The solid line represents the proportion λ of investors who want to become informed in the economy with a complete set of options. The parameters are $\bar{V} = 0$, $a = 1$, $\sigma_v = 1$, $\sigma_u = 1$, $\sigma_x = 1$, and $\delta = 0.15$.

$a = 1$, $\sigma_v = 1$, $\sigma_u = 1$, $\sigma_x = 1$, and $\delta = 0.15$. When there are many investors of type 1, i.e., ω is large, the proportion λ of investors who want to become informed in the economy with options is lower than the proportion λ of investors who want to become informed in the economy without options.

6 Conclusions

This study presents a rational expectations equilibrium model in an economy with information asymmetry to analyze the effect of the introduction of options on the underlying asset. Different from previous studies, the paper allows the introduction of arbitrary numbers of call options with predetermined strike price. We show that the introduction of call options cannot reveal additional information to the investors. Thus, given the information structure of the market, the addition of the option does not affect the informational efficiency of the market but does affect the allocational efficiency of the market. In terms of welfare, the introduction of the options always causes someone to benefit, and introducing a sufficient number of options is always Pareto-improving.

The model shows that the introduction of options will generally change the stock price. Once only one call option is introduced, the stock price increases if the call option is in the money, and the stock price decreases if the call option is out of the money. The equilibrium option trading suggests that the option is a security for betting volatility, and the disagreement about the payoff's uncertainty, not the disagreement about the expected payoff, is the only factor that leads to option trading and makes the option non-redundant.

When the information acquisition decision is endogenized, we show that the introduction of options does not always increase investors' incentives to acquire information. Thus, the introduction of options does not necessarily reduce the underlying asset's price volatility. Further analysis shows that the information structure, i.e., how many investors are of type 1, is an important factor in determining the uninformed investors' incentives to acquire information. When there are already many informed investors, adding options will reduce the incentives of the uninformed investors to acquire information.

We conclude with remarks on possible future research. One direction is to explore the impacts of several options in more detail. We have provided a preliminary numerical analysis in section 4.6, and more-rigorous analyses are left for future research. Another direction is to allow for a richer information structure, such as the diverse information structure in Diamond and Verrecchia (1981). A third direction is to extend the analysis to a multi-period and multi-asset model.

7 Appendix

7.1 Proof of Theorems 1, 2, and 3

We only give the proof of Theorem 3 because and Theorem 2 can be considered as a special case by setting the option trading to zero. Setting $\omega = 1$ in Theorems 2 and 3 we can obtain Theorem 1.

We first prove that Equations (13) and (14) do characterize an equilibrium. We then show the uniqueness of the equilibrium.

For investor 1, no matter what the form of the securities pricing functionals is, his expected utility is always

$$-\int_{-\infty}^{\infty} e^{-a(Xy+y+(y-S)(\theta_{1S}-1)+\sum_{j=1}^{n}((y-k_j)^+-C_j)\theta_{1j})} g(y)dy$$

where $g(y) = \frac{1}{\sqrt{2\pi}\sigma_u} \exp\{-\frac{1}{2\sigma_u^2}(y-V)^2\}$. The expected utility can be rewritten as

$$-e^{aS(\theta_{1S}-1)+\sum_{j=1}^{n} C_j\theta_{1j}+0.5a^2\sigma_u^2(X+\theta_{1S})^2-aV(X+\theta_{1S})} \int_{-\infty}^{\infty} e^{-a(\sum_{j=1}^{n}(y-k_j)^+\theta_{1j})} f(y)dy$$

where $f(y) = \frac{1}{\sqrt{2\pi}\sigma_u} \exp\{-\frac{1}{2\sigma_u^2}(y - V + a\sigma_u^2(X + \theta_{1S}))^2\}$. Thus, given the equilibrium securities prices, the first-order condition of investor 1's utility maximization problem leads to Equations (13).

Noticing Equations (13) always hold no matter how much information is contained in securities prices and investor 2 always knows investor 1's demands for securities, Equations (13) allow investor 2 to extract information about V and X from S and C_j ($j = 1, .., n$). Because demands $(\theta_{iS}, \theta_{ij})$ are essentially observable, S and C_j depend on V and X only through Q and because S is a monotonic function of Q[15], investor 2 actually observes Q indirectly. Therefore, conditional on the securities prices, the final payoff of stock $V + U$ maintains a normal distribution with mean $\bar{V} + \alpha Q$ and variance $\hat{\sigma}_u^2$ from the perspective of investor 2.

The first-order conditions of the utility maximization problem of both investors and the market clearing condition lead to Equations (14). Thus Equations (13) and (14) characterize an equilibrium.

Now, we show this is the only equilibrium, which we prove in two steps.

The first step is to show that uninformed investors cannot learn more from securities prices than from Q. We have shown that uninformed investors can always infer Q from securities prices. Because securities prices are uniquely determined by Q and $(\theta_{1S}, \theta_{11}, \cdots, \theta_{1n})$, according to Equations (13), knowing Q is sufficient for an uninformed investor to recover securities prices because he knows $(\theta_{1S}, \theta_{11}, \cdots, \theta_{1n})$. Therefore, securities prices cannot provide more information than Q[16].

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[15]From Equations (13), we have

$$\frac{\partial S}{\partial Q} = \frac{\int_{-\infty}^{\infty} ye^A B dy \int_{-\infty}^{\infty} e^A dy - \int_{-\infty}^{\infty} ye^A dy \int_{-\infty}^{\infty} e^A B dy}{(\int_{-\infty}^{\infty} e^A dy)^2}$$

where $A = -a\left[\sum_{j=1}^{n}(y - k_j)^+ \theta_{1j}\right] - \frac{1}{2\sigma_u^2}\left(y - \bar{V} - Q + a\sigma_u^2\theta_{1S}\right)^2$, $B = \frac{1}{\sigma_u^2}\left(y - \bar{V} - Q + a\sigma_u^2\theta_{1S}\right)$. Simple algebra leads to

$$\frac{\partial S}{\partial Q} = \int_{-\infty}^{\infty} \left(\frac{e^A}{\int_{-\infty}^{\infty} e^A dy}\right) y^2 dy - \left[\int_{-\infty}^{\infty} \left(\frac{e^A}{\int_{-\infty}^{\infty} e^A dy}\right) y dy\right]^2.$$

So $\frac{\partial S}{\partial Q} > 0$, S is monotone in Q.

[16]The equilibrium securities must be of the form $\{S(Q(V, X), C_j(Q(V, X))\}$ can also be seen from the fact that S and C_j solve the following equations in the equilibrium:

$$\begin{aligned} \omega\theta_{1S}(Q, S, C_1, \cdots, C_n) + (1 - \omega)\theta_{2S}(S, C_1, \cdots, C_n) &= 1 \\ \omega\theta_{1j}(Q, S, C_1, \cdots, C_n) + (1 - \omega)\theta_{2j}(S, C_1, \cdots, C_n) &= 0 \end{aligned}$$

where investor 1's demands depend on V and X through Q given the securities prices because of Equations (13).

The second step is to prove that the solution of Equations (14) is unique. These equations are the first-order conditions of the minimization problem:

$$\min_{\{\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n}\}} F(\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n}) \tag{28}$$

where

$$F = [\int_{-\infty}^{\infty} e^{-a(y(X+\tilde{\theta}_{1S})+\sum_{j=1}^{n}(y-k_j)^{+}\tilde{\theta}_{1j})-\frac{1}{2\sigma_u^2}(y-V)^2} dy]^{\omega}$$
$$\times [\int_{-\infty}^{\infty} e^{-a(y(1+\frac{\omega(\tilde{\theta}_{1S}-1)}{\omega-1})+\sum_{j=1}^{n}(y-k_j)^{+}\frac{\omega\tilde{\theta}_{1j}}{\omega-1})-\frac{(y-\bar{V}-\alpha Q)^2}{2\hat{\sigma}_u^2}} dy]^{(1-\omega)}$$

By Artin's theorem (Marshall, Olkin and Arnold, 2009, P649), both

$$\ln[\int_{-\infty}^{\infty} e^{-a(y(X+\tilde{\theta}_{1S})+\sum_{j=1}^{n}(y-k_j)^{+}\tilde{\theta}_{1j})-\frac{1}{2\sigma_u^2}(y-V)^2} dy]$$

and

$$\ln[\int_{-\infty}^{\infty} e^{-a(y(1+\frac{\omega(\tilde{\theta}_{1S}-1)}{\omega-1})+\sum_{j=1}^{n}(y-k_j)^{+}\frac{\omega\tilde{\theta}_{1j}}{\omega-1})-\frac{(y-\bar{V}-\alpha Q)^2}{2\hat{\sigma}_u^2}} dy]$$

are strictly convex functions of $(\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n})$, so $\ln F(\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n})$ is also a convex function. This means $F(\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n})$ is strictly convex. The optimization problem and the F.O.C equations have the same solution.

Given V and X, $F(\tilde{\theta}_{1S},\tilde{\theta}_{11},...,\tilde{\theta}_{1n})$ will approach infinity and increase with $|\tilde{\theta}_{1S}|$ and any $|\tilde{\theta}_{1j}|$ when $|\tilde{\theta}_{1s}|$ or any $|\tilde{\theta}_{1j}|$ is sufficiently large. Hence the minimum of F is in the interior of $(n+1)$-dimensional cube $[-K,K]^{n+1}$, where K is a sufficiently large positive number. The compact $[-K,K]^{n+1}$ and strictly convex F ensure the existence of a unique solution of the optimization problem. Thus, the solution of Equations (14) is unique, and the rational expectations equilibrium in the economy is unique. Thus, we prove the theorem.

7.2 Proof of Theorems 4 and 5

Obviously, we only need to prove Theorem 5. If only one call option is introduced, Equations (14) can be rewritten as

$$\frac{\int_{-\infty}^{\infty} y e^{aB(y)} e^{-\frac{1}{2\sigma_u^2}(y-S_0)^2} dy}{\int_{-\infty}^{\infty} e^{aB(y)} e^{-\frac{1}{2\sigma_u^2}(y-S_0)^2} dy} = \frac{\int_{-\infty}^{\infty} y e^{a\frac{\omega}{\omega-1}B(y)} e^{-\frac{1}{2\hat{\sigma}_u^2}(y-S_0)^2} dy}{\int_{-\infty}^{\infty} e^{a\frac{\omega}{\omega-1}B(y)} e^{-\frac{1}{2\hat{\sigma}_u^2}(y-S_0)^2} dy} \tag{29a}$$

$$\frac{\int_{-\infty}^{\infty} (y-k)^+ e^{aB(y)} e^{-\frac{1}{2\sigma_u^2}(y-S_0)^2} dy}{\int_{-\infty}^{\infty} e^{aB(y)} e^{-\frac{1}{2\sigma_u^2}(y-S_0)^2} dy} = \frac{\int_{-\infty}^{\infty} (y-k)^+ e^{a\frac{\omega}{\omega-1}B(y)} e^{-\frac{1}{2\hat{\sigma}_u^2}(y-S_0)^2} dy}{\int_{-\infty}^{\infty} e^{a\frac{\omega}{\omega-1}B(y)} e^{-\frac{1}{2\hat{\sigma}_u^2}(y-S_0)^2} dy} \tag{29b}$$

where

$$B(y) = -y\Delta\theta_{1S} - (y-k)^+\Delta\theta_{1C},$$

$$\Delta\theta_{1S} = \theta_{1S} - 1 - \frac{(1-\omega)[V - \hat{V} - a\sigma_u^2(1+X) + a\hat{\sigma}_u^2]}{a[\omega\hat{\sigma}_u^2 + (1-\omega)\sigma_u^2]}.$$

Note that when $\alpha = 0$, the solution is $\Delta\theta_{1S} = 0$ and $\Delta\theta_{1C} = 0$, so we can assume

$$\Delta\theta_{1S} = \alpha\Delta\theta_{1S}^{(1)} + \alpha^2\Delta\theta_{1S}^{(2)} + o(\alpha^2),$$

$$\Delta\theta_{1C} = \alpha\Delta\theta_{1C}^{(1)} + \alpha^2\Delta\theta_{1C}^{(2)} + o(\alpha^2),$$

when α is sufficiently small. Thus,

$$B(y) = -\alpha(y\Delta\theta_{1S}^{(1)} + (y-k)^+\Delta\theta_{1C}^{(1)}) - \alpha^2(\Delta\theta_{1S}^{(2)} + (y-k)^+\Delta\theta_{1C}^{(2)}) + o(\alpha^2).$$

$$e^{aB(y)} = 1 + aB(y) + \frac{1}{2}a^2(B(y))^2 + o(\alpha^2).$$

In addition, we have

$$C_0^2 - C_0^1 = \alpha\frac{a^2\sigma_x^2\sigma_u^3}{2}\phi\left(\frac{S_0-k}{\sigma_u}\right) + \alpha^2\frac{a^4\sigma_x^4\sigma_u^3(\sigma_u^2 - (S_0-k)^2)}{8}\phi\left(\frac{S_0-k}{\sigma_u}\right) + o(\alpha^2)$$

$$S_0 = S^{(0)} + \alpha b_S^{(1)} + \alpha^2 b_S^{(2)} + o(\alpha^2)$$

Substitute these equations and Equations (26) into Equations (29) and rearrange them. It is now easy to check that Equations (29) do hold asymptotically. The expression of S and C can be obtained by substituting Equations (26) into Equations (13).

7.3 Equilibrium Analysis when α approaches 1

In the main body of this paper, we provide an approximated solution to the equilibrium when α is small (σ_v^2 is small), that is,when there is little information asymmetry. Noting when σ_x^2 is small, which indicates that α approaches 1[17], and noting that there is little information asymmetry, the equilibrium approximates the symmetric information equilibrium.

To solve the equilibrium, we define $\hat{\alpha} = 1 - \alpha$, and thus, $\sigma_u^2 = \sigma_u^2 + \hat{\alpha}\sigma_v^2$. As in section 4.1, we rewrite the securities holdings and stock price as power series in $\hat{\alpha}$ under market structure I. In particular, we have

$$\begin{aligned} S &= S^{(0)} + \hat{b}_S^{(1)}\hat{\alpha} + \hat{b}_S^{(2)}\hat{\alpha}^2 + o(\hat{\alpha}^2) \\ \theta_{1S} &= \hat{\theta}_S^{(0)} + \hat{b}_{\theta S}^{(1)}\hat{\alpha} + \hat{b}_{\theta S}^{(2)}\hat{\alpha}^2 + o(\hat{\alpha}^2) \end{aligned}$$

where

$$\begin{aligned} S^{(0)} &= \omega V + (1-\omega)\bar{V} - a\sigma_u^2(\omega X + 1) + (1-\omega)Q \\ \hat{\theta}_S^{(0)} &= 1 \end{aligned}$$

and

$$\begin{aligned} \hat{b}_S^{(1)} &= (\omega - 1)(Q + a\sigma_v^2) \\ \hat{b}_S^{(2)} &= \omega(1-\omega)\sigma_v^2\frac{Q + \sigma_v^2}{\sigma_u^2} \\ \hat{b}_{\theta S}^{(1)} &= (1-\omega)\frac{Q + a\sigma_v^2}{a\sigma_u^2} \\ \hat{b}_{\theta S}^{(2)} &= -\omega(1-\omega)\frac{\sigma_v^2(Q + a\sigma_v^2)}{a\sigma_u^4}. \end{aligned}$$

Based on the same argument as in Theorem 4, when one call option with strike k is introduced and $\hat{\alpha}$ is small,

$$C_0^2 - C_0^1 = \hat{\alpha}\frac{\sigma_v^2}{2\sigma_u}\phi\left(\frac{S_0 - k}{\sigma_u}\right) + \hat{\alpha}^2\frac{\sigma_v^4(\sigma_u^2 - (S_0 - k)^2)}{8\sigma_u^5}\phi\left(\frac{S_0 - k}{\sigma_u}\right) + o(\hat{\alpha}^2)$$

the equilibrium price S, C and security holdings θ_{iS}, θ_{iC}, where $i = 1, 2$, can be written

[17]We do not consider the case where α approaches 1 due to small a or σ_u^2, because the economy is not properly defined when $a = 0$ or $\sigma_u = 0$.

as:

$$\begin{pmatrix} S \\ C \end{pmatrix} = \begin{pmatrix} S^{(0)} \\ C^{(0)} \end{pmatrix} + \begin{pmatrix} \hat{b}_S^{(1)} + \hat{\delta}_S^{(1)} \\ \hat{\delta}_C^{(1)} \end{pmatrix} \hat{\alpha} + o(\hat{\alpha}) \tag{33a}$$

$$\begin{pmatrix} \theta_{1S} \\ \theta_{1C} \end{pmatrix} = \begin{pmatrix} \hat{\theta}_S^{(0)} \\ 0 \end{pmatrix} + \begin{pmatrix} \hat{b}_{\theta S}^{(1)} + \hat{\delta}_{\theta S}^{(1)} \\ \hat{\delta}_{\theta C}^{(1)} \end{pmatrix} \hat{\alpha} + o(\hat{\alpha}) \tag{33b}$$

where

$$\hat{\delta}_S^{(1)} = 0 \tag{34a}$$

$$\hat{\delta}_C^{(1)} = \hat{b}_S^{(1)} \Phi \left(\frac{S^{(0)} - k}{\sigma_u} \right) + (1 - \omega) \frac{\sigma_v^2}{2\sigma_u} \phi \tag{34b}$$

$$\hat{\delta}_{\theta S}^{(1)} = \frac{(1 - \omega)(1 - \Phi)\sigma_v^2 \phi}{2da\sigma_u^2} \tag{34c}$$

$$\hat{\delta}_{\theta C}^{(1)} = -\frac{(1 - \omega)\sigma_v^2 \phi}{2da\sigma_u^2} \tag{34d}$$

and x, ϕ, Φ, and d are given by Equations (23).

7.4 Proof of Propositions 2 and 3

Propositions 2 and 3 can be easily obtained based on Theorems 4 and 5.

Here, we provide a proof that does not rely on the assumption that α is small.

The equilibrium securities prices can always be written as the solution of the following equations:

$$S = \frac{\int_{-\infty}^{\infty} y e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}{\int_{-\infty}^{\infty} e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy} = \frac{\int_{-\infty}^{\infty} y e^{a\frac{\omega}{1-\omega}B'(y)} e^{-\frac{(y-S_0)^2}{2\hat{\sigma}_u^2}} dy}{\int_{-\infty}^{\infty} e^{a\frac{\omega}{1-\omega}B'(y)} e^{-\frac{(y-S_0)^2}{2\hat{\sigma}_u^2}} dy}$$

$$C = \frac{\int_{-\infty}^{\infty} (y-k)^+ e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}{\int_{-\infty}^{\infty} e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy} = \frac{\int_{-\infty}^{\infty} (y-k)^+ e^{a\frac{\omega}{1-\omega}B'(y)} e^{-\frac{(y-S_0)^2}{2\hat{\sigma}_u^2}} dy}{\int_{-\infty}^{\infty} e^{a\frac{\omega}{1-\omega}B'(y)} e^{-\frac{(y-S_0)^2}{2\hat{\sigma}_u^2}} dy}$$

where $B'(y) = y\Delta\theta_{1S} + (y-k)^+\theta_{1C}$. Because $(\Delta\theta_{1S}, \theta_{1C})$ minimizes the convex function $F_1 + F_2$, where

$$F_1 = \ln \int_{-\infty}^{\infty} e^{-aB'(y) - \frac{(y-S_0)^2}{2\sigma_u^2}} dy$$

and

$$F_2 = \frac{\omega - 1}{\omega} \ln \int_{-\infty}^{\infty} e^{a \frac{\omega}{1-\omega} B'(y) - \frac{(y-S_0)^2}{2\hat{\sigma}_u^2}} dy$$

are both convex,

$$(\nabla(F_1 + F_2)(\Delta\theta_{1S}, \theta_{1C}) - \nabla(F_1 + F_2)(0,0))^T (\Delta\theta_{1S}, \theta_{1C}) > 0,$$

i.e., $\theta_{1C} < 0$. Now, assume that $k > S_0$ and $\Delta\theta_{1S}^*$ solves

$$S = \frac{\int_{-\infty}^{\infty} y e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}{\int_{-\infty}^{\infty} e^{-aB'(y)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}$$

given $\theta_{1C} = \theta_{1C}^* < 0$, where θ_{1C}^* is investor 1's equilibrium option holding. Then,

$$\frac{d\Delta\theta_{1S}^*}{d\theta_{1C}^*} < 0$$

and

$$\frac{\Delta\theta_{1S}^*}{\theta_{1C}^*} = -\frac{\int_{k}^{\infty} e^{-a(y\Delta\theta_{1S}^* + (y-k)^+ \theta_{1C}^*)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}{\int_{-\infty}^{\infty} e^{-a(y\Delta\theta_{1S}^* + (y-k)^+ \theta_{1C}^*)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}.$$

Let

$$h(t) = \frac{\int_{k}^{\infty} e^{-ta(y\Delta\theta_{1S}^* + (y-k)^+ \theta_{1C}^*)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy}{\int_{-\infty}^{\infty} e^{-ta(y\Delta\theta_{1S}^* + (y-k)^+ \theta_{1C}^*)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy},$$

simple algebra shows

$$h(0) = -\frac{d\Delta\theta_{1S}^*}{d\theta_{1C}^*}|_{\theta_{1C}^*=0}$$

and $\frac{dh(t)}{dt}|_{t=0} > 0$. This implies that when $t < 0$ and $|t|$ is sufficiently small, $S_0 < h(t)$. Let

$$H(t) = \int_{-\infty}^{\infty} (y - S_0) e^{-ta(y\Delta\theta_{1S}^* + (y-k)^+ \theta_{1C}^*)} e^{-\frac{(y-S_0)^2}{2\sigma_u^2}} dy.$$

Then,

$$\frac{dH(t)}{dt} = \frac{H(t)}{t} + ta^2 \int_{-\infty}^{\infty} (y\Delta\theta_{1S}^* + (y-k)^+\theta_{1C}^*)^2 e^{-at(y\Delta\theta_{1S}^* + (y-k)^+\theta_{1C}^*) - \frac{(y-S_0)^2}{2\sigma_u^2}} dy < 0$$

when $t < 0$ because

$$\int_{-\infty}^{\infty} (y-m) e^{-aB'(y) - \frac{(y-S_0)^2}{2\sigma_u^2}} dy = -a \int_{-\infty}^{\infty} B'(y) e^{-aB'(y) - \frac{(y-S_0)^2}{2\sigma_u^2}} dy.$$

Therefore, for any $t < 0$, $S_0 < h(t)$.

$$(\nabla(F_1 + F_2)(\Delta\theta_{1S}, \theta_{1C}) - \nabla(F_1 + F_2)(\Delta\theta_{1S}^*, \theta_{1C}^*))^T (\Delta\theta_{1S} - \Delta\theta_{1S}^*, 0) > 0$$

and

$$(\nabla(F_1)(\Delta\theta_{1S}, \theta_{1C}) - \nabla(F_1)(\Delta\theta_{1S}^*, \theta_{1C}^*))^T (\Delta\theta_{1S} - \Delta\theta_{1S}^*, 0) > 0$$

imply that

$$(-\frac{\partial(F_1 + F_2)}{\partial\Delta\theta_{1S}} (\Delta\theta_{1S}^*, \theta_{1C}^*))(S - S_0) > 0$$

and hence, $S < S_0$ when $k > S_0$. In the case of $k < S_0$, because the equilibrium demands $(\Delta\theta_{1S}, \theta_{1C}) = (-\Delta\theta_{1S}^1 - \theta_{1C}^1, -\Delta\theta_{1S}^1)$, where $(\Delta\theta_{1S}^1, \theta_{1C}^1)$ are the equilibrium demands for the economy with strike price $2S_0 - k$, it is easy to see that $S > S_0$ when $k < S_0$.

It is obvious that $\frac{dS}{dk} < 0$ when $|k - S_0|$ is small based on the first conclusion of Proposition 3. We now show that when $|k - S_0|$ is sufficiently large, $\frac{dS}{dk} > 0$. Noting that

$$\frac{dS}{dk} = \frac{\partial S}{\partial k} + \frac{\partial S}{\partial\Delta\theta_{1S}} \frac{\partial\Delta\theta_{1S}}{\partial k} + \frac{\partial S}{\partial\theta_{1C}} \frac{\partial\theta_{1C}}{\partial k},$$

we have

$$\frac{dS}{dk} = (I + \frac{\omega}{1-\omega} M^* M^{-1})^{-1} \frac{\partial S^*}{\partial k} + (I - (I + \frac{\omega}{1-\omega} M^* M^{-1})^{-1}) \frac{\partial S}{\partial k},$$

where

$$M = \text{Var}\begin{pmatrix} Y \\ Y-k \end{pmatrix}, \qquad M^* = \text{Var}\begin{pmatrix} Y^* \\ (Y-k)^+ \end{pmatrix},$$

$$\frac{\partial S}{\partial k} = \theta_{1C}\mathrm{Cov}(Y, 1_{\{Y>k\}}), \quad \frac{\partial S^*}{\partial k} = -\frac{\omega}{1-\omega}\theta_{1C}\mathrm{Cov}(Y^*, 1_{\{Y^*>k\}}),$$

the density of Y is

$$\frac{e^{-aB'(y)-\frac{(y-S_0)^2}{2\sigma_u^2}}}{e^{-aB'(y)-\frac{(y-S_0)^2}{2\sigma_u^2}}dy},$$

the density of Y^* is

$$\frac{e^{\frac{\omega}{1-\omega}aB'(y)-\frac{(y-S_0)^2}{2\sigma_u^2}}}{\int_{-\infty}^{\infty} e^{\frac{\omega}{1-\omega}aB'(y)-\frac{(y-S_0)^2}{2\sigma_u^2}}}.$$

When $|k - S_0|$ is sufficiently large, both M and M^* are nearly diagonal and can be approximated well by their diagonals, Therefore, some algebra leads to

$$sign(\frac{dS}{dk}) = sign(\frac{\mathrm{Cov}(Y, 1_{\{Y>k\}})}{\mathrm{Var}(Y)} - \frac{\mathrm{Cov}(Y^*, 1_{\{Y^*>k\}})}{\mathrm{Var}(Y^*)}) > 0.$$

7.5 Proof of Proposition 4

Recall that the expected utilities of investors 1 and 2 after observing the signal and prices are

$$U_1 = -\frac{e^{a(XV+S(\theta_{1S}-1)+\sum_{j=1}^n C_j\theta_{1j})}}{\sqrt{2\pi}\sigma_u} \times \int_{-\infty}^{\infty} e^{-a(y(X+\theta_{1S})+\sum_{j=1}^n (y-k_j)^+\theta_{1j})-\frac{(y-V)^2}{2\sigma_u^2}} dy$$

$$U_2 = -\frac{e^{a(S(\theta_{2S}-1)+\sum_{j=1}^n C_j\theta_{2j})}}{\sqrt{2\pi}\hat{\sigma}_u} \int_{-\infty}^{\infty} e^{-a(y\theta_{2S}+\sum_{j=1}^n (y-k_j)^+\theta_{2j})-\frac{(y-\bar{V}-\alpha Q)^2}{2\hat{\sigma}_u^2}} dy$$

Thus, simple algebra leads to

$$(-U_1)^{\omega}(-U_2)^{1-\omega} = \frac{e^{a\omega XV}}{(2\pi)\sigma_u\hat{\sigma}_u}F(\theta_{1S}, \theta_{11}, ..., \theta_{1n})$$

Thus, F is proportional to the weighted geometric average of the investor's utility, indicating that we can solve the equilibrium by solving the social planner's problem where the social planner's utility function is a weighted geometric average of the individual investor's utility functions. The result follows directly from the fact that adding more securities always expands the feasible set of the social planner's utility maximization problem and, hence, increases the social planner's utility.

7.6 Proof of Theorems 6 and 7

When there are three types of investors, the equilibrium can be analyzed in a similar manner, so we omitted the details.

Let U_{i0} be the interim utility of investor i in the economy without options, and let U_i be the interim utility of investor i in the economy with a complete set of Arrow-Debreu securities. Because closed-form equilibrium prices and demands exist in both cases, we can obtain the utilities by definition:

$$U_{10} = -e^{aXV} e^{-aS(X+1) - \frac{(V-S)^2}{2\sigma_u^2}}$$

$$U_{20} = -e^{-aS} e^{-\frac{(\mu_2 - S)^2}{2\sigma_2^2}}$$

$$U_{30} = -e^{-aS} e^{-\frac{(V-S)^2}{2\sigma_u^2}}$$

$$U_1 = -e^{aXV} \frac{\sigma}{\sigma_u} e^{-(\frac{V^2}{2\sigma_u^2} - \frac{S^2}{2\sigma^2}) + (\frac{\omega_1}{2\sigma_u^2} + \frac{1-\omega_1}{2\sigma_2^2})(S^2+\sigma^2) + (\frac{\omega_1 V}{\sigma_u^2} + \frac{(1-\omega_1)\mu_2}{\sigma_2^2})S + a(\omega-1)XS}$$

$$U_2 = -\frac{s}{\sigma_2} e^{-(\frac{V^2}{2\sigma_u^2} - \frac{S^2}{2\sigma^2}) + (\frac{\omega}{2\sigma_u^2} - \frac{1-(1-\omega)\lambda}{2\sigma_u^2} + \frac{1-\omega_1}{2\sigma_2^2})(S^2+\sigma^2) + (\frac{\omega_1 V}{\sigma_u^2} + \frac{(1-(1-\omega)\lambda)V}{\sigma_u^2} + \frac{(1-\omega_1)\mu_2}{\sigma_2^2})S + a\omega XS}$$

$$U_3 = -\frac{s}{\sigma_u} e^{-(\frac{V^2}{2\sigma_u^2} - \frac{S^2}{2\sigma^2}) + (\frac{\omega_1}{2\sigma_u^2} - \frac{\omega_1}{2\sigma_2^2})(S^2+\sigma^2) + (\frac{\omega_1 V}{\sigma_u^2} + \frac{(1-\omega)\lambda V}{\sigma_u^2} - \frac{\omega_1 \mu_2}{\sigma_2^2})S + a\omega XS}$$

where $a_1 = a\frac{\omega}{\omega+\lambda(1-\omega)}$, $\omega_1 = \omega + (1-\omega)\lambda$, $\mu_2 = \frac{\sigma_v^2}{\sigma_v^2+\sigma_u^4 a_1^2 \sigma_x^2}(V - \sigma_u^2 a_1 X)$, $\sigma_2^2 = \sigma_u^2 + \frac{\sigma_u^4 a_2^2 \sigma_x^2}{\sigma_v^2+\sigma_u^4 a_2^2 \sigma_x^2}\sigma_v^2$, $\sigma = \sqrt{\frac{1}{\frac{\omega_1}{\sigma_u^2} + \frac{1-\omega_1}{\sigma_2^2}}}$, $S = \frac{\frac{\omega_1}{\sigma_u^2}V + \frac{1-\omega_1}{\sigma_2^2}\mu_3}{\frac{\omega_1}{\sigma_u^2} + \frac{1-\omega_1}{\sigma_2^2}} - \frac{a(\omega x+1)}{\frac{\omega_1}{\sigma_u^2} + \frac{1-\omega_1}{\sigma_2^2}}$.

Thus

$$U_1 = U_{10} \frac{\sigma}{\sigma_u} e^{\frac{1}{2}(1 - \frac{\sigma^2}{\sigma_u^2})}$$

$$U_2 = U_{20} \frac{\sigma}{\sigma_2} e^{\frac{1}{2}(1 - \frac{\sigma^2}{\sigma_2^2})}$$

$$U_3 = U_{30} \frac{\sigma}{\sigma_u} e^{\frac{1}{2}(1 - \frac{\sigma^2}{\sigma_u^2})}$$

Because $0 < Ye^{0.5(1-Y^2)} \leq 1$ for any positive Y and the utility of any investor is always negative, we always have $U_i > U_{i0}$ and the conclusion of the theorem follows. Theorem 6 is a direct application of the claim in section 4.6 of Vives (2008). Simple algebra leads to

$$\begin{aligned}
E[U_3|S] =& \frac{\sigma}{\sigma_u} e^{\frac{1}{2}(1-\frac{\sigma^2}{\sigma_u^2})} E[U_{30}|S] \\
=& \frac{\sigma}{\sigma_u} e^{\frac{1}{2}(1-\frac{\sigma^2}{\sigma_u^2})} e^{aC} \frac{\sigma_u}{\sigma_2} E[U_{20}|S] \\
=& \frac{\sigma}{\sigma_u} e^{\frac{1}{2}(1-\frac{\sigma^2}{\sigma_u^2})} e^{aC} \frac{\sigma_u}{\sigma_2} \frac{\sigma_2}{\sigma} e^{-\frac{1}{2}(1-\frac{\sigma^2}{\sigma_2^2})} E[U_2|S]
\end{aligned}$$

It follows that

$$E[U_3|S] = e^{a\delta - \frac{1}{2}(\frac{1}{\sigma_u^2} - \frac{1}{\sigma_2^2})\sigma^2} E[U_2|S]$$

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