

Delegated Information Acquisition and Asset Pricing *

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Abstract

I study the joint determination of optimal contracts and equilibrium asset prices in an economy with multiple principal-agent pairs. Principals design contracts that incentivize agents to acquire information. With agency problems, the compensation depends on agents' forecasting accuracy for asset prices and payoffs. Complementarities in information acquisition delegation arise. As more principals hire agents to acquire information, asset prices become less noisy. Consequently, agents are more willing to acquire information because they can forecast asset prices more accurately, thus mitigating agency problems and encouraging other principals to hire agents. This mechanism can explain phenomena, including multiple equilibria and home bias.

KEYWORDS: Information acquisition, Contract, Complementarities

JEL: D62, D83, G14, G31

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1 Introduction

The asset management industry has experienced tremendous growth with current assets under management comparable to global GDP. Not surprisingly, institutional investors now dominate trading activities in all financial markets.¹ While institutions assist their clients in making investment decisions, agency problems may simultaneously arise. In particular, potential moral hazard emerges when institutions' efforts are largely unobservable, raising the issue of optimal contract design. Given institutions' superior capabilities to acquire information, it is commonplace for clients to delegate information acquisition to them and provide incentives for them through optimal contracting. However, the joint determination of optimal contracts, information acquisition delegation and equilibrium asset pricing has not yet been fully explored in the literature.²

This paper contributes to the literature by solving for optimal contracts characterized in a general space and equilibrium asset prices in an economy with multiple principal-agent pairs. I show that the optimal contracts for delegated information acquisition depend on agents' forecasting accuracy for asset prices and payoffs: agents receive high compensation when they produce accurate forecasts. Moreover, I find strategic complementarities in the delegation of information acquisition: the more principals hire agents to acquire information, the more others are willing to do so. As more principals hire agents to acquire information, asset prices become less noisy. As a result, agents are more willing to acquire information because they can forecast asset prices more accurately. Thus, the agency problems are mitigated and other principals are encouraged to hire agents. Such strategic complementarities yield multiple equilibria, and can explain many phenomena, including asset price jumps, herding behaviour, home bias and idiosyncratic volatility comovement.

The model of this paper features delegated information acquisition, optimal contract design, and equilibrium asset pricing, introducing a two-period economy with one risky asset and one risk-free

¹French (2008) documents that financial institutions accounted for more than 80% ownership of equities in the U.S. in 2007, compared to 50% in 1980. TheCityUK (2013) estimates the size of assets under management is around \$87 trillion globally, which is equal to global GDP. Meanwhile, Jones and Lipson (2004) reports that institutional trading volume reached 96% of total equity trading volume in NYSE by 2002.

²Papers studying optimal contracts without any asset pricing implications include Bhattacharya and Pfleiderer (1985) and Dybvig et al. (2010). Papers studying institutions' impacts on asset pricing without asymmetric information or information acquisition include Vayanos and Woolley (2013) and Basak and Pavlova (2013). The most relevant papers are by Kyle, Ou-Yang and Wei (2011) and Malamud and Petrov (2014). However, they only consider restricted contract space. More importantly, my research has new asset pricing implications, such as strategic complementarities.

asset. The risky asset's payoff has two components: the first can be learned by agents and is called fundamental value, while the other cannot be learned and produces residual uncertainty. This economy has a market maker, noisy traders and a mass of principal-agent pairs. The principals are risk neutral while the agents are risk averse. Different principals cannot share agents, and different agents cannot share principals. Before trading, the principals choose whether to hire agents to acquire information regarding fundamental value. When deciding to hire agents, principals design optimal contracts that provide incentives for agents to acquire costly information, after which agents provide forecasts to their corresponding principals. The feasible contracts are general functions of agents' forecasts, the asset price and the payoff. I model agency problems by assuming that agents take hidden actions when acquiring information. When the market opens, the principals submit market orders to the market maker based on agents' forecasts. Having received all orders from the principals and the noisy traders, the market maker then sets the price.

The generality of this model relies on its broad interpretations. The principal-agent pairing can be interpreted as either that between fund managers and in-house analysts, or that between the pension fund trustees/board of directors (within funds) and fund managers. This model can unify both, because the optimal contract problems in the two contexts are essentially equivalent given that agents construct portfolios based on forecasts and principals can directly observe agents' portfolios. Therefore, the assumption regarding who invests is not crucial, and the aforementioned parsimonious model is a natural setting to study information acquisition incentives.

I show that the optimal contracts depend on the agents' forecasting accuracy for the asset price and the payoff. Agents can forecast the asset price and payoff accurately only if they acquire information. Thus, the agents' efforts are related to their forecasting accuracy, which determines their compensation. Specifically, agents receive high compensation when they forecast accurately - in contrast to an economy without agency problems, in which the compensation is constant. As an incentive for accurate forecasting, the bonus decreases with price informativeness and increases with residual uncertainty. When the price becomes more informative or residual uncertainty decreases, it is easier for agents to use information to forecast accurately and then receive high compensation. Consequently, agents are more willing to exert efforts and principals can accordingly provide fewer

incentives. These results predict that the bonus is larger for professionals who trade small/growth stocks featuring greater residual uncertainty.³

Furthermore, I find that the delegation of information acquisition exhibits strategic complementarities. Price informativeness has two counteractive effects: the first is to lower trading profit; and, the second is to mitigate agency problems. Whereas the first effect leads to standard strategic substitutability due to competition in trading, the strategic complementarities in information acquisition delegation originates from the effect of price informativeness on mitigating agency problems. When more principals hire agents to acquire information, the asset price becomes less noisy. As a result, agents are more willing to acquire information because they can forecast the asset price more accurately, and thus agency problems are mitigated. Clearly, strategic complementarities in information acquisition delegation emerge when price informativeness has a larger impact on mitigating agency problems than that on lowering trading profits. This only occurs when the residual uncertainty is large and compensation must consequently rely largely on agents' forecasts for the asset price. This mechanism causes principals to coordinate information acquisition delegation, therefore introducing the possibility of multiple equilibria. The multiplicity of equilibria may lead to the economy switching between low-information and high-information equilibria without any relation to fundamentals, leading to jumps in asset price and price informativeness.

This model, to my knowledge, is new to the literature to combine optimal contracts characterized in a general space, equilibrium asset pricing and delegated information acquisition. Meanwhile, it shows that the agency problem in information acquisition delegation is a new source for strategic complementarities. In particular, my model yields closed-form solutions for both optimal contracts and equilibrium asset pricing. Although this model is intentionally stylized to focus on information acquisition delegation, it captures realistic institutional features. Moreover, it has a number of implications as follows.

The first implication relates to home bias, a long-standing puzzle.⁴ A plausible explanation

³There are two ways to understand this result: (1) The predictable component can be interpreted as regular items in firms' cash flow, while the residual uncertainty can be interpreted as irregular items in firms' cash flows. Normally the small/growth firms have larger irregular items. (2) The small/growth firms, such as tech firms, take a long time to realize their earnings. Thus principals need to rely largely on intermediate stock prices to incentivize agents. The decomposition of predictable component and residual uncertainty is a parsimonious way to capture this feature.

⁴Home bias is well documented by [Fama and Poterba \(1991\)](#), [Coval and Moskowitz \(1999\)](#) and [Grinblatt and Keloharju \(2001\)](#). Despite large benefits from international diversification, [Fama and Poterba \(1991\)](#) find that

is that investors have superior information on home assets. However, [Van Nieuwerburgh and Veldkamp \(2009\)](#) argue that investors can easily acquire information about other assets, which could eliminate the information advantage of home investors and mitigate home bias.⁵ Although investors can freely acquire information, I show that agency problems lead to home bias: investors tend to acquire more information about assets for which they have an information advantage. I extend the model to consider two groups of principals (A and B) and two risky assets (X and Y); group A (B) is endowed with private information only about asset X (Y). I show that group A has higher incentives to acquire information on asset X relative to asset Y , and vice versa. Group A can use the endowed information to monitor agents, and thus group A 's agency problems are less severe when hiring agents to acquire information about asset X relative to asset Y .⁶ Consequently, group A is encouraged to hire agents to acquire information and trade more on asset X . This result is in direct contrast to that of the economy without agency problems, in which the decreasing marginal benefit of information discourages group A from acquiring information about asset X . Interpreting group A as home investors on asset X implies that agency problems can explain home bias.

The mechanism above for home bias can also explain industry bias: investors trade more on the assets within their expertise. This prediction is consistent with [Massa and Simonov \(2006\)](#), who document that Swedish investors buy assets highly correlated with their non-financial income. Moreover, because endowed information is more valuable in monitoring agents when the assets have greater residual uncertainty, the home/industry bias is stronger for these assets. This prediction is consistent with [Kang and Stulz \(1997\)](#) and [Coval and Moskowitz \(1999\)](#), who find that the home bias of U.S. fund managers is stronger when they trade small stocks.

The next implication relates to herding, defined as any behavioral similarity caused by interactions amongst individuals ([Hirshleifer and Teoh, 2003](#)). I extend the model to assume that each principal can choose to hire his agent to acquire either an exclusive signal or a common signal: the former is only accessible to his agent and is conditionally independent of others, while the latter is

households invest nearly all of their wealth in domestic assets. For example, they find that U.S. households invest around 94% of their equity portfolio in the domestic market, while this number is 82% in the UK.

⁵Constraint on international capital flow may explain home bias. However, it is not a major concern currently. In particular, the recent studies ([Seasholes and Zhu, 2010](#) and [Coval and Moskowitz, 1999](#)) find that households/fund managers also have a strong home bias in the U.S. market, which suggests this explanation is not satisfactory.

⁶Normally, the principals can use their private information in the subjective evaluation of agents. Even if the private information is not verifiable, some mechanisms, such as reputation concern, could reveal these information.

accessible to any agent. Under agency problems, I show that principals herd to acquire the common signal when the residual uncertainty is sufficiently large. Herding makes the price sensitive to the common signal itself. Thus, agents are willing to obtain the common signal because this allows them to easily forecast the asset price. In particular, when the residual uncertainty is large, herding emerges because its impact on mitigating agency problems is larger than that on lowering trading profit. This result is in clear contrast to that of the economy without agency problems, in which principals prefer the exclusive signals due to the substitute effect.

Moreover, my model has additional applications. For example, I show that idiosyncratic volatility comovement occurs in a multi-asset extension, in which principals incentivize agents to acquire information on each asset through their forecasting accuracy for the prices of assets with correlated fundamentals. An increase on one asset's idiosyncratic volatility, perhaps due to more noisy traders, discourages information acquisition and consequently leads to higher idiosyncratic volatilities on other correlated assets.

This paper is related to several strands of the literature. First, it is related to literature regarding the optimal contracting in delegated portfolio management, such as [Bhattacharya and Pfleiderer \(1985\)](#), [Stoughton \(1993\)](#), [Dybvig, Farnsworth and Carpenter \(2010\)](#) and [Ou-Yang \(2003\)](#). However, the asset prices play no roles in the aforementioned contracting work. My work on the contracting is most related to [Dybvig et al. \(2010\)](#). They study the optimal contract problem in a complete market, in which the asset price has no informational role; they find that the optimal compensation involves a benchmark. In contrast to their work, I consider the optimal contracts in general equilibrium and the asset prices play informational roles. I find that the compensation depends on agents' forecasting accuracy for the asset prices and the payoffs.

My paper is also related to recent studies on the institutional investors, such as [Basak, Shaprio and Tepla \(2006\)](#), [Basak, Pavlova and Shaprio \(2007, 2008\)](#), [Basak and Makarov \(2014\)](#), [Basak and Pavlova \(2013\)](#), [Dasgupta and Prat \(2006, 2008\)](#), [Dasgupta, Prat and Verardo \(2011\)](#), [Dow and Gorton \(1997\)](#), [He and Krishnamurthy \(2012, 2013\)](#), [Garcia and Vande \(2009\)](#), [Kaniel and Kondor \(2013\)](#), [Buffa, Vayanos and Woolley \(2013\)](#), [Kyle, Ou-Yang and Wei \(2011\)](#) and [Malamud and Petrov \(2014\)](#). In particular, [Buffa, Vayanos and Woolley \(2013\)](#) study the joint equilibrium

determination of optimal contracts and asset prices in a dynamic and multi-asset model. They focus on how the inefficiency of benchmarking arises endogenously and amplifies stock market volatility. However, these authors do not model moral hazard problems in information acquisition. The most relevant works are by [Kyle, Ou-Yang and Wei \(2011\)](#) and [Malamud and Petrov \(2014\)](#). [Kyle, Ou-Yang and Wei \(2011\)](#) consider a moral hazard problem between one principal and one agent in the [Kyle \(1985\)](#) model. They restrict the contract space and solely consider the linear contracts. Furthermore, [Malamud and Petrov \(2014\)](#) also focus on the restricted contract form, which consists of one proportional fee and one option-like incentive fee. My model differs from these papers in the following regard. First, I place no restrictions on the contract space. Second, I find that the agency problems generate strategic complementarities in information acquisition delegation, which is new to this literature.

Moreover, my paper is related to recent studies on the strategic complementarities, including [Dow, Goldstein and Guembel \(2011\)](#), [Froot, Scharfstein and Stein \(1992\)](#), [Garcia and Strobl \(2011\)](#), [Goldstein, Li and Yang \(2013\)](#), [Goldstein and Yang \(2015\)](#) and [Veldkamp \(2006b\)](#). [Froot et al. \(1992\)](#) find that short-term investors herd to acquire similar information. Because they must liquidate assets before payoffs are realized, the short-term investors can profit on their information only if their information is reflected in future prices by the trades of similarly informed investors. [Garcia and Strobl \(2011\)](#) find that relative wealth concern can generate complementarities. Because the investors' utilities are negatively affected by others, they tend to hedge others' impacts by following others' information acquisition decision. [Dow et al. \(2011\)](#) show that information acquisition complementarities emerge when the asset prices affect the firms' investments. [Veldkamp \(2006b\)](#) finds that when the information production has a scale effect, the selling price of information decreases as more investors buy information. In contrast to their work, the strategic complementarities in my model originates from the effect of price informativeness on mitigating agency problems in delegated information acquisition.

Last, my paper is also the literature on relative performance (see [Holmstrom \(1979, 1982\)](#) et al.) or contractual externalities (see [Ozdenoren and Yuan \(2014\)](#), [Gorton, He and Huang \(2010\)](#) et al.). [Gorton, He and Huang \(2010\)](#) also study moral hazard problem in a trading game with

multiple principal-agent pairs and find the optimal contracts for the agents depend on the asset price informativeness. However, the principals' delegation choices in their model are exogenous and they focus on agents' coordination problem induced by mark-to market contracts which take similar spirits of my model. In contrast, the agents in my model have no coordination problems and we focus on strategic complementarities among principals.

The paper is organized as follows. I introduce the model in Section 2 and solve the optimal contracts in Section 3. Section 4 shows the strategic complementarities and multiple equilibria. Section 5 studies three applications. Section 6 discusses the robustness. In particular, I solve a fully-fledged model with non-linear REE to show that the main results are robust in Section 6. Section 7 concludes.

2 Model

2.1 Economy

My model is built on [Kyle \(1985\)](#), in which investors submit market orders and a market maker sets the price according to the total order. My model deviates from [Kyle \(1985\)](#) in the following features: there are a mass of investors and each one has trading constraints.⁷ Investors in my model trade in a competitive market, and no single individual investor has any price impact.

My economy has a mass of principal-agent pairs. The principals trade the risky asset and have incentives to acquire information for profits. However, these principals are unable to acquire information alone, perhaps because of large information acquisition or opportunity costs. Before trading, principals choose whether to hire agents to acquire information. Because agents' efforts are unobservable, a moral hazard problem arises within each pair. When deciding to hire agents, principals design optimal contracts that provide incentives for agents to acquire information. In particular, the population of principals who hire agents is endogenous in my model. My analysis of optimal contracting is similar to that of [Dybvig et al. \(2010\)](#). In particular, I solve optimal contracts without any restriction on the contract space. The optimal contracts will induce agents

⁷The assumptions of a mass of investors in which each one has trading constraints is not new (see [Dow, Goldstein and Guembel, 2011](#), [Goldstein, Ozdenoren and Yuan, 2013](#) and [Malamud and Petrov, 2014](#)).

to make costly efforts and truthfully report signals.

Timeline and Assets. My economy has three periods $t = 0, 1, 2$ and two assets. The first asset is risk-free and the second is risky. The risk-free asset is in zero supply and pays off one unit of consumption good without uncertainty at time $t = 2$. The payoff of the risky asset is denoted by D with two components: V and ϵ . V and ϵ are independent. I call V the fundamental value and ϵ the residual uncertainty. I assume that V depends on equally likely states, h and l , realized at time $t = 2$. V takes V_ω (where $\omega \in \{h, l\}$). Without a loss of generality, I assume that $V_h = \theta$ and $V_l = -\theta$, where $\theta > 0$. The residual uncertainty ϵ is uniformly distributed on $[-M, M]$, where $M > 0$.⁸ At time $t = 0$, principals choose whether to delegate information acquisition to agents. When deciding to hire agents, principals write contracts with their agents. The contract is denoted by π . Otherwise, the principal does nothing at time $t = 0$. At time $t = 1$, the market opens and the principals submit market orders.⁹ After receiving the total orders, a competitive market maker sets the price. I denote the risky asset's price by P .

Players. There are four types of players. The first type is principals, who choose whether to hire agents, design optimal contracts at $t = 0$, and trade the risky asset at $t = 1$. The second type is agents, who decide whether to accept the contracts and exert costly effort to acquire information about the fundamental value V . The third type is noisy traders, and the last type is a risk-neutral competitive market maker.

There are a mass of principal-agent pairs. Each pair is indexed by $i \in [0, \infty)$. Within each pair i , I denote its principal by principal i and denote its agent by agent i . To simplify the analysis, I assume that different principals can not share agents, and vice versa. Each pair can be interpreted as one mutual/hedge fund. There can be many interpretations of principal-agent pairs, such as principals as board directors of funds and agents as fund managers/in-house analysts. Moreover, I assume that the total demand from noisy traders is n , which follows a uniform distribution on $[-N, N]$, where $N > 0$.

Agency Problem. Agent i 's effort is denoted by $e_i \in \{0, 1\}$. When agent i exerts effort, $e_i = 1$;

⁸The assumptions about θ and ϵ are made only to obtain an analytical solution and make the mechanism clear. I will show numerically that the mechanism is robust when θ and ϵ follow more general distributions.

⁹The assumption about market orders is to obtain closed-form solution without losses of any economic insights. In the extension, I allow principals to learn information from the price and then submit limit orders. The numerical results show that the main results are robust.

otherwise, $e_i = 0$. After exerting effort, agent i generates a private signal $s_i \in \{h, l\}$ regarding the risky asset's fundamental value V . I denote the probability with which a signal is correct by

$$\begin{aligned} pe_i + \frac{1}{2}(1 - e_i) &= \text{prob}(s_i = h|V = \theta) \\ &= \text{prob}(s_i = l|V = -\theta), \end{aligned}$$

where s_i is conditionally independent across agents and $p > \frac{1}{2}$. If agent i shirks, his signal is pure noise. If agent i exerts effort, his signal is informative. If I let $\text{prob}(s_i)$ be the unconditional probability of signal s_i , I obtain $\text{prob}(s_i = h) = \text{prob}(s_i = l) = \frac{1}{2}$. Let $\text{prob}^I(V|s_i)$ be the probability of V conditional on signal s_i if agent i exerts effort, and let $\text{prob}^U(V|s_i)$ be the probability of V conditional on signal s_i if he shirks. I then have the following:

$$\text{prob}^I(V = \theta|s_i = h) = \text{prob}^I(V = -\theta|s_i = l) = p, \quad (2.1)$$

$$\text{prob}^U(V = \theta|s_i = h) = \text{prob}^U(V = -\theta|s_i = l) = \frac{1}{2}. \quad (2.2)$$

To acquire information, each agent bears a utility loss. I assume that all agents have the same CARA utility function $-\exp(-\gamma_a \pi + \gamma_a C)$, where π is compensation, C is information acquisition cost and γ_a is risk aversion.¹⁰ All agents have zero initial wealth. Due to hidden actions, there are moral hazard problems followed by truth telling problems between principals and agents.

Information Acquisition and Trading. At time $t = 0$, some principals hire agents to acquire information. The population of these principals is denoted by λ , where λ is endogenous. I call these principals informed principals; others are referred to as uninformed principals. While deciding to hire agents, informed principal i writes a contract π_i with agent i . At time $t = 1$, all contracts and λ become public information. Upon receiving report s_i from his agent, informed principal i submits a market order X_i conditional on the report to maximize his utility over final wealth $W_{i,1}$, where $W_{i,1} = W_0 + X_i(D - P) - \pi_i$, and $X_i \in [-1, 1]$. This limited position is due to frictions, such as leverage constraint or limited wealth. Then, uninformed principals submit market order

¹⁰ When I model agents' utility nesting cost as $-\exp^{-\gamma_a \pi} - C$, the results do not change. In particular, when I consider general HARA utility function for agents, the main results are robust as shown later.

X_U , where $X_U = 0$ due to symmetric distributions of the asset payoff or price. Given the contracts beforehand, the informed principal i 's optimization problem in trading is the following:

$$\max_{X_i} E(W_0 + X_i(D - P) - \pi_i | s_i). \quad (2.3)$$

The total orders received by the competitive risk-neutral market maker are

$$X = \int_{i=0}^{\lambda} X_i di + n. \quad (2.4)$$

The market maker sets a price equal to the risky asset's expected payoff conditional on X :

$$P = E(D|X). \quad (2.5)$$

Contracting Problem. With agency problems, principals design optimal contracts π that provide incentives for agents to acquire information at time $t = 0$. In accordance with [Dybvig et al. \(2010\)](#), this type of contract induce agents to exert effort and report the true signals. Because [Dybvig et al. \(2010\)](#) assume that the market is complete, there is no informational role of the price. However, the market is not complete in my model. Moreover, the asset price plays an informational role in monitoring agents because it aggregates information from all principals. The contracts in my model are general functions of agents' reports, the asset price and payoff. The agents either accept or reject the contracts. If agents accept the contracts, they exert costly efforts in information acquisition. After acquiring information, they report their signals to the corresponding principals. The specific contract provided by principal i is a general function $\pi_i(s^R(s_i), P, D)$, where $s^R(s_i)$ is agent i 's report conditional on his realized signal s_i .

To formalize my analysis, I consider two cases: without and with the agency problem. In the case without agency problem, I assume that each agent's costly effort and signal can be observed by his principal. This problem may not be realistic, but is useful for further comparison. In the case with agency problem, agents' efforts and signals are unobservable. There is a moral hazard problem followed by a truth telling problem. The revelation principle guarantees that I can focus

solely on the contracts that induce agents to truthfully report signals after exerting efforts. The detailed analysis of the two problems follows:

Without Agency Problem. Principal i chooses $\pi_i(s^R(s_i), P, D)$ at time $t = 0$ and submits demand X_i at time $t = 1$ to maximize his expected utility:

$$\max_{\pi_i(s_i, P, D), X_i(s_i, \pi_i)} \sum_{s_i=\{h, l\}} \text{prob}(s_i) \int \int [W_0 + X_i(D - P) - \pi_i(s_i, P, D)] f^I(P, D|s_i) dP dD, \quad (2.6)$$

where $f^I(P, V|s_i)$ is the conditional joint probability density function when agent i acquires information. Without agency problem, principals design contracts subject to agents' participation constraint,

$$\sum_{s_i=\{h, l\}} \text{prob}(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D) + \gamma_a C}] f^I(P, D|s_i) dP dD = -\exp(-\gamma_a W_a), \quad (2.7)$$

where LHS of Equation (2.7) is agent i 's expected utility given the premise that he exerts costly effort and reports the true signal. Moreover, W_a is the reserve wealth of agents, which can be interpreted as the agents' outside options.

With Agency Problem. With agency problem, the contract satisfies two type of ICs, including the *Ex Ante IC*, which is the incentive-compatibility of effort exerting

$$\begin{aligned} & \sum_{s_i=\{h, l\}} \text{prob}(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D) + \gamma_a C}] f^I(P, D|s_i) dP dD \\ & \geq \sum_{s_i=\{h, l\}} \text{prob}(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s^R(s_i), P, D)}] f^U(P, D|s_i) dP dD, \end{aligned} \quad (2.8)$$

and the *Ex Post IC*, which is the incentive-compatibility of truth reporting ($\forall s_i$ and $s^R(s_i) : s \rightarrow s$)

$$\int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D)}] f^I(P, D|s_i) dP dD \geq \int \int [-\exp^{-\gamma_a \pi_i(s^R(s_i), P, D)}] f^I(P, D|s_i) dP dD, \quad (2.9)$$

where $f^U(P, V|s_i)$ is the conditional joint probability density function when agent i shirks. The RHS of Equation (2.8) is agent i 's expected utility when he shirks. Then, $f^U(P, V|s_i) = f(P, V)$, which is the unconditional joint probability density function. Equation (2.9) induces agents to truthfully report their signals. For any realized signal s_i , the LHS of Equation (2.9) is agent i 's

utility if he reports the truth signal, whereas RHS of Equation (2.9) is the agents i 's utility if he misreports.

Principal i 's choice variables are contingent fees $\pi_i(s_i, P, D)$ and a demand schedule $X_i(s_i)$. Each principal i maximizes his utility through simultaneous decisions over trading and optimal contracting. The trading decisions and optimal contracts depend on the population of informed principals. In the equilibrium, the population of informed principals λ renders the expected utility of informed and uninformed principals equal; the difference in utilities between the two types of principals is the expected net benefit of information. I denote the expected net benefit of information by B , where B is the difference between the maximum value of optimization problem in Equation (2.6) and the initial wealth W_0 . It is clear that B is difference between the trading profit for informed principals and the expected compensation to agents.

2.2 Discussion

Before proceeding, I discuss the assumptions of my model. First, I assume that the principals trade by alone and only agents acquire information. Although this assumption is stylized, my model has broad interpretations. The most direct interpretation is that the principals are fund managers and the agents are in-house analysts. The in-house analysts collect information and report forecasts to fund managers, who trade based on the forecasts. However, the assumption about who invests is not crucial, as is evident if I assume that agents trade instead of principals and that principals can observe or infer agents' contractible portfolios. Because agents construct portfolios based on forecasts, the contracts written upon agents' portfolios, the asset price and the payoff can be transformed into the contracts directly written on agents' forecasts, the asset price and the payoff. In practice, the pension fund trustees/board directors of funds can observe the fund managers' portfolios. Therefore, an alternative interpretation is that the pension fund trustees/board directors of funds, who maximize the households' interests, hire fund managers to simultaneously collect and trade on information. Another interpretation is that the principals are households and the agents are fund managers. Because mutual/hedge funds must disclose their holdings regularly, households could infer the beliefs of fund managers through holding data,

although they are noisy (see [Kacperczyk, Sialm and Zheng, 2007](#), [Cohen, Polk, Silli, 2010](#) and [Shumway, Szeffler and Yuan, 2011](#)). Although households can not choose the management fee, they can use fund flow to provide incentives for fund managers. The fund flow can be viewed as a form of implicit contract.

Furthermore, in accordance with the literature, I assume that the principals are risk-neutral. This assumption simplifies my analysis, while capturing the features of the practice. In practice, principals, such as households or mutual/hedge funds can diversify risks alone. For example, households can allocate money to different assets to diversify risk. In particular, if principals are risk averse, the contracts include a risk-sharing component. However, this risk-sharing component does not overturn my mechanism: an increase in the population of informed principals makes the price more informative and mitigates the agency problems.

The third assumption is that the principals submit market orders and do not learn information from the asset prices. This assumption is not crucial in my model. Introducing learning enables uninformed principals to free ride informed principals by learning information from the price; this affects principals' incentive to acquire information. However, this free-riding problem only affects the strength of the driving force, and will not overturn my mechanism. In particular, this assumption captures my idea in a more complicated dynamic framework, in which there are multiple rounds of trading and principals solely observe current and past prices. It is obvious that such settings will only complicate the model, leading to a loss of tractability, without adding much economic insight. In particular, the numerical results in one extension show that the strategic complementarities are robust when principals can learn information from the asset price.

Last, I assume that the asset payoff has two components: fundamental value and residual uncertainty. Because the agents can exert effort to learn about the fundamental value and the asset price is a noisy signal about this fundamental variable, the presence of residual uncertainty induces principals rely both on the asset payoff and asset price to incentivize agents. Thus the residual uncertainty is important for my results and also show that my results are stronger in small/growth firms featuring large residual uncertainty. There are two interpretations to large residual uncertainty associated with small/growth firms: (1) The fundamental value is learnable,

which can be interpreted as regular items in firms' cash flow, while the residual uncertainty is not learnable, which can be interpreted as irregular items. Normally the small/growth firms have larger irregular items. (2) The small/growth firms, such as tech firms, take a long time to realize their earnings. Thus principals need to rely largely on intermediate stock prices to incentivize agents. The decomposition of predictable component and residual uncertainty is a parsimonious way to capture this feature.

3 Equilibrium

3.1 Equilibrium Definition

I formally introduce the equilibrium concept in this section. I focus on symmetric equilibrium with identical contracts. Before trading, principals choose whether to hire agents to acquire information and the population of these principals is endogenous. These principals design optimal contracts that provide incentives for their agents to acquire information and report truthfully. Given these contracts, all principals submit optimal demands when the market opens and a risk-neutral market maker sets the price after receiving the total orders¹¹.

DEFINITION 3.1. *A symmetric equilibrium is defined as a collection: a price function P set by a risk-neutral competitive market maker, $P(X) : \mathbb{R} \rightarrow \mathbb{R}$; an optimal demand schedule for each principal i , $X_i(s_i) : \mathbb{R} \rightarrow \mathbb{R}$; an optimal contract designed by each principal i , $\pi_i(s_i, P, D) : \mathbb{R}^3 \rightarrow \mathbb{R}$; and an equilibrium population of principals hiring agents to acquire information, λ . This collection satisfies the following:*

(1) *Given the price function solved in Equation (2.5) and the demand schedule solved in Equation (2.3), principal i designs optimal contract $\pi_i(s_i, P, D)$ and the optimal contract problem is equivalent to the problem in Equation (2.6) subject to constraints (2.7), (2.8), and (2.9),*

(2) *Given contract $\pi_i(s_i, P, D)$, agent i decides whether to accept or reject this contract,*

(3) *Given the price function in Equation (2.5) and the optimal contract $\pi_i(s_i, P, D)$, principal i submits demand X_i to solve Equation (2.3),*

¹¹The agents have no coordination problems given the optimal contracts in our equilibrium, which takes similar spirits of Gorton, He and Huang (2010). Gorton, He and Huang (2010) find that the agents will shirk and lie to the principals only if they have exogenous irrelevant noisy signals (rumor).

(4) A risk-neutral competitive market maker sets the price as the risky asset's expected payoff conditional on total orders. The pricing function is solved in Equation (2.5),

(5) If there exists a positive solution to $B(\lambda) = 0$, an equilibrium with information acquisition is obtained. Otherwise, an equilibrium of no information acquisition is obtained ($\lambda = 0$).

(6) All contracts are identical in this economy.

3.2 Equilibrium Characterization

I characterize the equilibrium as one featuring trading strategies and optimal contracting by principals, and a pricing rule by the market maker. I follow a step-by-step approach to illustrate this idea.

Step 1. I first solve for the principals' trading decisions and the market maker's pricing rule given the contracts designed beforehand and the population of informed principals. When the market opens at $t = 1$, the informed principal i submits X_i to maximize $W_0 + X_i(D - P) - \pi_i$, which is his final wealth. Furthermore, uninformed principals submit $X_U = 0$. Because the principals are risk-neutral, there is no hedging demand, and the informed principal i submits $X_i = 1$ after agent i reports $s_i = h$ and submits $X_i = -1$ after agent i reports $s_i = l$. Following the large number theorem, when fundamental value $V = \theta$, the total number of buy orders from informed principals is λp and the total number of sell orders is $\lambda(1 - p)$. Thus the total order received by the market maker is $X = \lambda(2p - 1) + n$. Similarly, the total order received by the market maker is $X = -\lambda(2p - 1) + n$ when $V = -\theta$. Therefore, the total order X is distributed on $[-\lambda(2p - 1) - N, \lambda(2p - 1) + N]$.

Receiving total orders X , the risk-neutral market maker updates his beliefs and sets the price as the risky asset's expected payoff: $P = E(D|X)$. If $-\lambda(2p - 1) + N < \lambda(2p - 1) - N$, the total orders can fully reveal information regarding V and I have $P = V$, which leads to zero trading profits for informed principals. This is impossible because the principals need to pay costs for information. Thus I have the formal lemma regarding the population of informed principals.

LEMMA 3.1. *The population of informed principals satisfies the following:*

$$\lambda < \frac{N}{2p - 1}. \quad (3.1)$$

This lemma is helpful for further analysis. Then, I have the following lemma regarding price:

LEMMA 3.2. *Given λ and contract $\pi(s, P, D)$, the price follows the rule:*

$$P(X) = \begin{cases} \theta & \text{if } N - \lambda(2p - 1) < X \leq N + \lambda(2p - 1), \\ 0 & \text{if } -N + \lambda(2p - 1) \leq X \leq N - \lambda(2p - 1), \\ -\theta & \text{if } -N - \lambda(2p - 1) \leq X < -N + \lambda(2p - 1). \end{cases} \quad (3.2)$$

Lemma 3.2 shows that the price increases with the total orders X due to the correlation between the total orders and the fundamental value V . However, with noisy traders, the total orders do not fully reveal V . In particular, the probability that the price equals V is the following:

$$\text{prob}(P = V|V) = \frac{\lambda(2p - 1)}{N}. \quad (3.3)$$

This probability measures price informativeness. This probability increases with the population of informed principals and the precision of signals, and decreases with the variance of noisy traders' demand.

Step 2. I solve the informed principals' optimal contracts at $t = 0$. As Lemma 3.2 implies, the asset price is informative regarding V . Thus principals will use the price to monitor agents. The contracting problem is reduced to the optimization problem in Equation (2.6) subject to constraints (2.7), (2.8), and (2.9). Due to risk-neutrality, the principals' trading decisions and contracting problems are independent. Then, the contracting problem can be transferred to the following:

$$\max_{\pi_i(s_i, P, D)} \sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int [-\pi_i(s_i, P, D)] f^I(P, D|s_i) dP dD, \quad (3.4)$$

Equation 3.4 shows that principals minimize expected compensation subject to participant constraint and incentive compatibility. However, if the residual uncertainty is sufficiently small, the asset payoff D is perfectly informative about V and thus there is no role of asset price in the contracting, which is not interesting. To avoid this case, I make the following assumption regarding M :

ASSUMPTION **3.1.** M satisfies: $M \geq \theta$.

From Dybvig et al. (2010), the joint conditional pdf or conditional probability of P and D play important roles in optimal contracts. Thus I characterize the joint conditional pdf or the conditional probability of P and D before I solve the optimal contracts. If agent i exerts effort, signal s_i is informative about V and this indicates that $prob^I(V = \theta|s_i = h) = prob^I(V = -\theta|s_i = l) = p$. Then, I have the following lemma:

LEMMA **3.3.** *When s_i is informative about V , the conditional pdf is as follows:*

(1) conditional on $s_i = h$,

$$f^I(P = \theta, D|s_i = h) = \begin{cases} \frac{p}{2M} \frac{\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D \leq M + \theta \\ 0 & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.5)$$

$$f^I(P = 0, D|s_i = h) = \begin{cases} \frac{p}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } M - \theta \leq D \leq M + \theta \\ \frac{1}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D < M - \theta \\ \frac{1-p}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.6)$$

$$f^I(P = -\theta, D|s_i = h) = \begin{cases} 0 & \text{if } M - \theta < D \leq M + \theta \\ \frac{(1-p)}{2M} \frac{\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D \leq M - \theta \end{cases} \quad (3.7)$$

(2) conditional on $s_i = l$,

$$f^I(P = \theta, D|s_i = l) = \begin{cases} \frac{(1-p)}{2M} \frac{\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D \leq M + \theta \\ 0 & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.8)$$

$$f^I(P = 0, D|s_i = l) = \begin{cases} \frac{1-p}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } M - \theta \leq D \leq M + \theta \\ \frac{1}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D < M - \theta \\ \frac{p}{2M} \frac{N-\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.9)$$

$$f^I(P = -\theta, D|s_i = l) = \begin{cases} 0 & \text{if } M - \theta < D \leq M + \theta \\ \frac{p}{2M} \frac{\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D \leq M - \theta \end{cases} \quad (3.10)$$

If agent i shirks, signal s_i is uninformative regarding V and this indicates that $\text{prob}^U(V = \theta | s_i = h) = \text{prob}^U(V = -\theta | s_i = l) = \frac{1}{2}$. Then, I have the following lemma:

LEMMA 3.4. *When s_i is uninformative about V , the conditional pdf is as follows:*

$$f^U(P = \theta, D) = \begin{cases} \frac{1}{4M} \frac{\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D \leq M + \theta \\ 0 & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.11)$$

$$f^U(P = 0, D) = \begin{cases} \frac{1}{4M} \frac{N - \lambda(2p-1)}{N} & \text{if } M - \theta \leq D \leq M + \theta \\ \frac{1}{2M} \frac{N - \lambda(2p-1)}{N} & \text{if } -M + \theta \leq D < M - \theta \\ \frac{1}{4M} \frac{N - \lambda(2p-1)}{N} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.12)$$

$$f^U(P = -\theta, D) = \begin{cases} 0 & \text{if } M - \theta < D \leq M + \theta \\ \frac{1}{4M} \frac{\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D \leq M - \theta \end{cases} \quad (3.13)$$

Lemma 3.3 shows that s_i is correlated with the asset price or the payoff when it is informative. Lemma 3.4 shows that s_i is uncorrelated with the asset price or payoff when it is pure noise. Thus agents' efforts are tied to the accuracy of their forecasts for the asset price and the payoff.

To simplify the optimization problems, in accordance with Grossman and Hart (1983) and Dybvig et al. (2010), I transfer the choice variables. I let:

$$v(s_i, P, D) = \exp[-\gamma_a \pi(s_i, P, D)]. \quad (3.14)$$

I can rewrite the contracting problem in a similar form, in which choice variable becomes $v(s_i, P, D)$.

Then principal i 's contracting problem becomes:

$$\max_{v_i(s_i, P, D)} \sum_{s_i = \{h, l\}} \text{prob}(s_i) \int \int \frac{1}{\gamma_a} \log[v_i(s_i, P, D)] f^I(P, D | s_i) dP dD, \quad (3.15)$$

subject to constraints (2.7), (2.8), and (2.9). Then, I use the first-order approach to solve the optimal contracts in both cases without and with agency problem.

PROPOSITION 3.1. *(Without Agency Problem) The optimal contract without agency problem is:*

$$\pi_i(s_i, P, D) = W_a + C .$$

Proposition 3.1 shows that agents' compensation is constant without agency problem. This is slightly different from the previous literature, which assumes that investors are risk-averse and finds that compensation is a proportional fee for risk-sharing purpose. However, I assume that principals are risk-neutral. Therefore, principals do not care about risk and there is no role for risk-sharing. In fact, the compensation is equal to agents' reserve wealth and information acquisition cost. This case is used later for comparative purposes with the agency problem.

ASSUMPTION 3.2. *C satisfies: $C < -\frac{\log 2 + \log(1-p)}{\gamma_a}$.*

Assumption 3.2 is important because it guarantees that the optimal contract is implementable in the agency problem. I conduct the analysis with agency problems under Assumption 3.2.

PROPOSITION 3.2. *(Agency Problem) Given λ , there exists one unique optimal contract in the economy with agency problems. There are two cases regarding optimal contract as follows:*

(1) when $p = 1$, the optimal contract is the following:

$$\pi_i(s_i, P, D) = \begin{cases} -\infty & \text{if } s_i = h \text{ and } P = -\theta \\ -\infty & \text{if } s_i = h \text{ and } D < -M + \theta \\ -\infty & \text{if } s_i = l \text{ and } P = \theta \\ -\infty & \text{if } s_i = l \text{ and } D > M - \theta \\ W_a + C & \text{otherwise} \end{cases} \quad (3.16)$$

(2) when $p < 1$, the optimal contract is the following:

$$\pi_i(s_i = h, P = \theta, D) = \pi_i(s_i = l, P = -\theta, D) = \frac{\log x}{\gamma_a} \quad (3.17)$$

$$\pi_i(s_i = h, P = -\theta, D) = \pi_i(s_i = l, P = \theta, D) = \frac{\log y}{\gamma_a} \quad (3.18)$$

$$\pi_i(s_i = h, P = 0, D) = \begin{cases} \frac{\log x}{\gamma_a} & \text{if } M - \theta < D \leq M + \theta \\ \frac{\log[px+(1-p)y]}{\gamma_a} & \text{if } -M + \theta \leq D \leq M - \theta \\ \frac{\log y}{\gamma_a} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.19)$$

$$\pi_i(s_i = l, P = 0, D) = \begin{cases} \frac{\log y}{\gamma_a} & \text{if } M - \theta < D \leq M + \theta \\ \frac{\log[px+(1-p)y]}{\gamma_a} & \text{if } -M + \theta \leq D \leq M - \theta \\ \frac{\log x}{\gamma_a} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (3.20)$$

where x and y are defined in the Appendix. In particular, $x > y$.

Proposition 3.2 has several interesting features. First, when the signals acquired by agents are perfectly informative ($p = 1$), the agency problem is trivial through an infinite penalty for incorrect forecasts. Given the finite support of the asset price or the asset payoff, if the asset price or payoff deviates to a large extent from the forecasts, the principals know that the agents are shirking. For example, when agent i acquires information and then reports $s_i = h$, it is impossible that the price is $-\theta$. This infinite penalty avoids the agency problem.¹² Second, when the signals acquired by agents are not perfectly informative ($p < 1$), the optimal compensation depends on the agents' forecasting accuracy for the asset price and the payoff. For example, when agents report $s_i = h$, agents receive high compensation when the price or payoff is high and low compensation when the price or payoff is low. Agents can forecast the asset price and the payoff accurately if they acquire information. Thus, the forecasting accuracy is related to agents' efforts. This compensation will encourage agents to exert effort and tell the truth. When $p = 1$, the problem is equivalent to the case without agency problem, which is not analytically interesting. Thus I focus on the case in which $p < 1$ in the following analysis. I formally state the assumption regarding p as follows:

ASSUMPTION 3.3. p satisfies: $p < 1$.

¹²In this basic model, I assume that agents have CARA utilities and do not have limited liability. Thus the infinite penalty can be interpreted as infinite disutilities. For example, if agents have log utilities, $\pi = 0$ provides an infinite penalty for agents. I discuss more general utilities for agents in the following sections.

3.3 Characteristics of Optimal Contract

I show the characteristics of the optimal contract in this section. I focus on how the price informativeness or residual uncertainty affects the compensation.

The bonus, defined by the difference between agents' compensations when they forecast correctly and incorrectly, provides incentives for agents to exert effort. It is given as follows:

DEFINITION 3.2. *The bonus is defined as S_f : $S_f = \frac{\log x - \log y}{\gamma_a}$.*

Because λ measures the price informativeness and M measures the residual uncertainty in the asset's payoff, I show their effects on bonuses as follows:

PROPOSITION 3.3. *Bonus S_f decreases with λ , but increases with M .*

Proposition 3.3 shows that S_f decreases with price informativeness and increases with residual uncertainty. In fact, when the price becomes more informative, agents can forecast the asset price more accurately with information. Agents are therefore more willing to exert efforts. As a result, principals can provide less incentive, which is characterized as a decreased bonus. Similarly, the bonus increases with residual uncertainty. In particular, because both the asset price and the payoff are used in the incentive provision, their effects interact with each other. I then have the following result:

COROLLARY 3.1. *When $\theta = M$, λ has no effect on S_f , that is $\frac{\partial S_f}{\partial \lambda} = 0$ if $\theta = M$.*

When $\theta = M$, the asset payoff is perfectly informative about the fundamental value. Thus, principals solely use the asset payoff in the contracts.

4 Agency Problem and Information Acquisition Complementarity

In this section, I show how agency problems in delegated information acquisition affect the financial market. I show that agency problems generate complementarities and multiple equilibria.

4.1 Case without Agency Problem

Informed principal i 's final wealth $W_{1,i}$ has two components: the first is trading profit, which is $X_i(D - P)$; the second is agents' compensation π_i . Informed principals' expected trading profit

is denoted as E_p , where $E_p = E[X_i(D - P)]$. Thus the expected net benefit from information is $B = E[X_i(D - P) - \pi_i]$. Informed principals' expected trading profit is shown as follows:

LEMMA 4.1. *Informed principals' expected trading profit: $E_p = \theta(2p - 1) \frac{N - \lambda(2p - 1)}{N}$.*

Lemma 4.1 shows that informed principals' trading profits decrease with the population of informed principals because of competition in trading. This effect is called the strategic substitute effect. Because compensation is constant in the first-best problem, the net benefit from information decreases with the population of informed principals. The result is shown as follows:

PROPOSITION 4.1. *(Without Agency Problem) Information acquisition is a strategic substitute without agency problem, that is $\frac{\partial B}{\partial \lambda} < 0$.*

4.2 Agency Problem

With agency problems, the compensation depends on the accuracy of agents' forecasts for the asset price and payoff. In particular, the bonus decreases with the population of informed principals, which leads to decreased compensation, and is the source of the strategic complementarity effect. When the residual uncertainty is large, there is a strategic complementarity effect in the information acquisition delegation; otherwise, there is only a strategic substitute effect. When the residual uncertainty is large, the principals rely largely on agents' forecasts for the asset price to incentive them. Thus, price informativeness has a larger impact on mitigating agency problems than lowering trading profit, which generates strategic complementarities. When the residual uncertainty is small, only the substitute effect exists because price informativeness has little impact on mitigating agency problems. The result of information acquisition delegation is shown as follows:

PROPOSITION 4.2. *(Agency Problem) In an economy with agency problems, I have the following results:*

(1) *for a sufficiently small M , the information acquisition delegation is a strategic substitute. That is, $\frac{\partial B}{\partial \lambda} < 0$.*

(2) *for a sufficiently high M , there exists λ^c satisfying the following: when $\lambda < \lambda^c$, the information acquisition delegation is a strategic complement. That is $\frac{\partial B}{\partial \lambda} > 0$.*

4.3 Multiplicity of Equilibria

As shown in Grossman and Stiglitz (1980) and Hellwig (1980), one unique equilibrium in information acquisition exists with a strategic substitute effect. However, the strategic complementarities may generate multiple equilibria (Dow, Goldstein and Guembel, 2011, Garcia and Strobl, 2011, Goldstein, Li and Yang, 2013 and Veldkamp, 2006a). Proposition 4.2 shows that agency problems produce strategic complementarities when the residual uncertainty is large. Thus, multiple equilibria may emerge in this case. This result is important because it may explain asset price jumps and excess volatilities in the financial market. The equilibrium populations of informed principals in the cases without or with agency problem are denoted by λ_{fb} and λ_{sb} , respectively. Because there is a substitute effect in the cases without or with agency problem with low residual uncertainty, one unique equilibrium exists in both cases, which is shown as follows:

LEMMA 4.2. *There exists one unique equilibrium λ_{fb} regarding information acquisition delegation without agency problem.*

LEMMA 4.3. *When M is sufficiently small, there exists one unique equilibrium λ_{fb} regarding information acquisition delegation in the economy with agency problems.*

Because the contract is very complex, I do not characterize all equilibria in the agency problem with large residual uncertainty. However, my goal is to demonstrate the existence of multiple equilibria. In particular, no information acquisition delegation may emerge as one of the equilibria.

PROPOSITION 4.3. *(Agency Problem) When M is sufficiently large, there are three cases regarding information acquisition delegation in the economy with agency problems,*

(1) when $\theta(2p-1) + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] > 0$, all equilibria are with positive population of informed principals, and at least one equilibrium exists.

(2) when $\theta(2p-1) + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] < 0$ and $\max_{\lambda < \lambda_{fb}} B_{ap}(\lambda) > 0$, there exists at least three equilibria, one of which is $\lambda_{sb} = 0$.

(3) when $\max_{\lambda < \lambda_{fb}} B_{ap}(\lambda) < 0$, the unique equilibrium is no information acquisition delegation.

That is $\lambda_{sb} = 0$.

Proposition 4.3 shows that agency problems may generate multiple equilibria. When the

information acquisition cost is low (first case), agency problems are not severe and principals have incentives to hire agents. In fact, when the information acquisition cost is high (third case), agency problems are severe and thus no principals have incentives to hire agents. In the second interesting case when information acquisition is neither too high nor too low, agency problems produce multiple equilibria, and non-information is one of these equilibria. When residual uncertainty is high, principals must rely heavily on the asset price in the incentive provision. However, when no principals hire agents to acquire information, the price does not incorporate any information, and the incentive provision from asset price fails. Consequently, agency problems are severe, which deters principals from hiring agents. All results are shown in Figure 4.1.¹³

This proposition has implications for asset price jumps or excess volatilities. With multiple equilibria regarding information acquisition, the economy may switch between non-information equilibrium and high-information equilibria without any relation to fundamentals, leading to jumps in the asset price and informativeness. Because a jump is an extreme form of excess volatilities, the same mechanism can also cause excess volatilities in asset price and informativeness. This result implies that the price informativenesses and institutional ownership are more volatile for small/growth stocks or during recessions, which are usually associated with large residual uncertainties. This result also implies that price jumps and excess volatilities are more likely to occur for small/growth stocks or during recessions, which is consistent with [Bennet, Sias and Starks \(2003\)](#), [Campbell, Lettau, Malkiel and Xu \(2001\)](#), [Xu and Malkiel \(2003\)](#), [Ang, Hodrick and Zhang \(2006, 2009\)](#) and [Bekaert, Hodrick and Zhang \(2012\)](#).

I also examine how agency problems affect asset pricing behavior. I focus on the analysis of price informativeness and return volatility. For price informativeness, because the equilibrium is not a linear function of fundamental value or noisy traders' demand, the conditional variance $Var(D|P)$ in the conventional literature is not appropriate for my analysis because this measure depends on the price P . In accordance with [Malamud and Petrov \(2014\)](#), I use the price's expected error as

¹³ I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $C = 0.07$. I also set $M = 5$, $M = 20$ and $M = 200$ for low residual uncertainty, median residual uncertainty and high residual uncertainty cases, respectively.

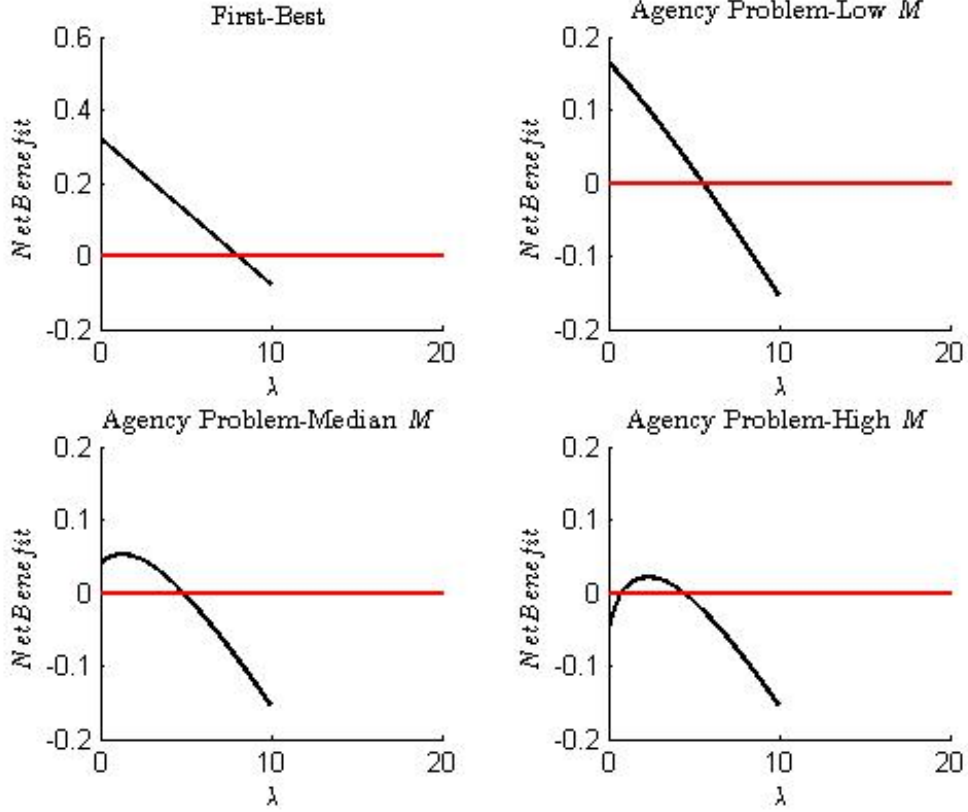


Figure 4.1: Information Acquisition Benefit

price informativeness. When the price is more informative, this expected error is lower:

$$E(|V - P||V) = \frac{\theta[N - \lambda(2p - 1)]}{N}. \quad (4.1)$$

For volatility, I calculate the asset return's volatility $Var(V - P)$ as follows:

$$Var(V - P) = \frac{M^2}{3} + \frac{\theta^2[N - \lambda(2p - 1)]}{N}. \quad (4.2)$$

When the population of informed principals increases, both the expected error of the price and the volatility decrease. Before proceeding, I know that agency problems negatively affect the net benefit from information, which decreases the prices informativeness. Then, price becomes more sensitive to noisy traders' demand, leading to increased volatility. I denote B_{ap} as the net benefit of information in an economy with agency problems, and denote B_{fb} as the net benefit in case without agency problem. I find the following result:

LEMMA 4.4. *Given λ , the net benefit in the agency problem is lower than the case without agency problem. That is $B_{ap} < B_{fb}$.*

I then have the formal result regarding price informativeness and volatility.

PROPOSITION 4.4. *Both price's expected error and volatility are higher in an economy with agency problems than that without agency problem.*

I examine how different parameters affect the population of informed principals. I focus on the case in which M is small because a unique equilibrium exists in this case. When the agents' risk aversion increases, the agency problem becomes more severe and the principals need to provide higher compensation to agents. Thus, I expect that the equilibrium population of informed principals decreases with agents' risk aversion. Furthermore, when M increases, it is more difficult for principals to monitor agents and the agency problem is exacerbated. Thus, the equilibrium population of informed principals decreases with residual uncertainty. These results are shown in the following figures. I note that agents' risk aversion or residual uncertainty does not have any impact on the population of informed principals due to the assumption regarding principals' risk-neutrality. These two figures show that price informativeness is low during recessions, which are associated with large uncertainty.¹⁴

5 Implications

In this section, I extend the basic model in three directions to study its asset pricing implication. First, I show that the agency problems induce principals to herd in terms of acquiring similar information. This may explain investors' herding behavior in trading. Second, I show that the agency problems encourage principals to acquire disproportionately more information on assets about which they already have an information advantage. This may explain the home/industry bias. Moreover, I shows that the agency problems provide a new and rational explanation for the well-known idiosyncratic volatility comovement.

¹⁴ I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $\gamma_a = 1$, $C = 0.05$ and $M = 20$ for analysis of agents' risk aversion. I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $\gamma_a = 1$, and $C = 0.075$ for analysis of residual uncertainty M .

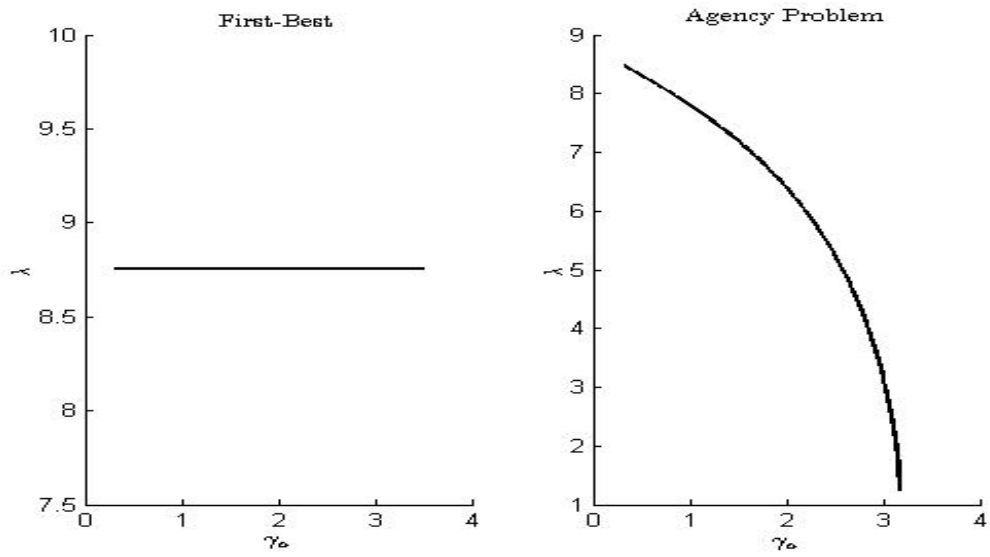


Figure 4.2: Population of Informed Principal and Agents' Risk Aversion

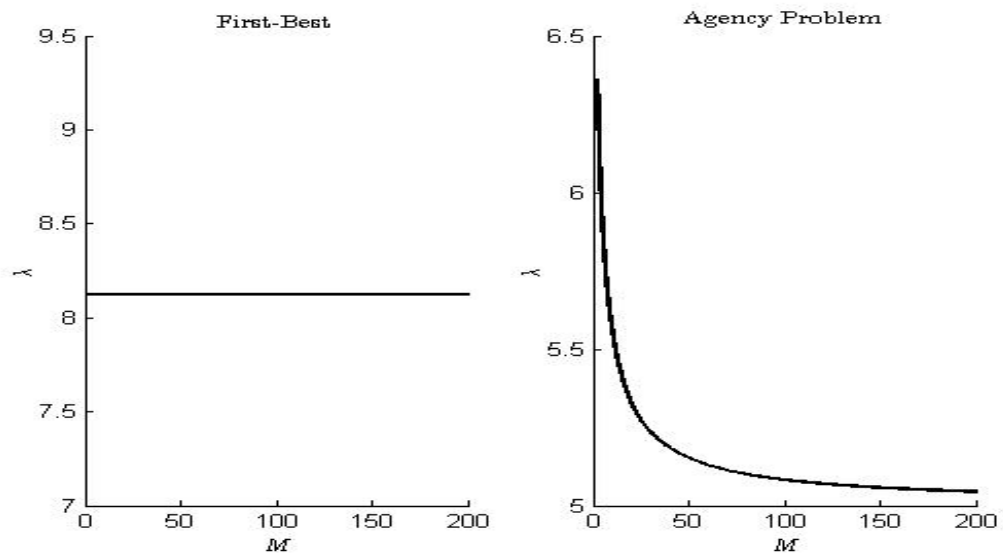


Figure 4.3: Population of Informed Principals and Residual Uncertainty

5.1 Herding

In this section, I show that the agency problems induce principals to herd in terms of acquiring similar information. I assume that each principal can choose to hire his agent to acquire either an exclusive signal, which is conditionally independent and can only be acquire by his agent, or a common signal, which can be acquired by any agent. The exclusive signal is $s_i \in \{h, l\}$. The common signal is $s_c \in \{h, l\}$. I assume the probabilities with which these signals are correct are the same ($p > \frac{1}{2}$):

$$\begin{aligned} p &= \text{prob}(s_i = h|V = \theta) = \text{prob}(s_i = l|V = -\theta) \\ &= \text{prob}(s_c = h|V = \theta) = \text{prob}(s_c = l|V = -\theta). \end{aligned} \tag{5.1}$$

Then, I have the conditional probability of V as follows:

$$\text{prob}^I(V = \theta|s_i = h) = \text{prob}^I(V = -\theta|s_i = l) = p, \tag{5.2}$$

$$\text{prob}^I(V = \theta|s_c = h) = \text{prob}^I(V = -\theta|s_c = l) = p, \tag{5.3}$$

Following the basic model, I assume that if agent i does not exert costly effort, his signal is a pure noise. I denote $\text{prob}^U(V|s_i)$ as probability of V conditional on signal s_i if s_i is a pure noise. Furthermore, the information acquisition costs are the same for all signals, which are denoted by C .

I assume that the population of principals who hire agents to acquire s_c is λ , and the population of principals who hire agents to acquire s_i is μ . I follow [Garcia and Strobl \(2011\)](#) to define herding equilibrium as follows:

DEFINITION 5.1. *Herding Equilibrium: one equilibrium is herding equilibrium if $\mu = 0$ and $\lambda > 0$*

This definition is following [Hirshleifer and Teoh \(2003\)](#), who define herding as any behavior similarity caused by individuals' interaction. Herding equilibrium occurs only if all informed principals hire agents to acquire the common signal. As argued by [Garcia and Strobl \(2011\)](#), the common signal is less valuable for principals than the exclusive signal because of competition. Thus, without agency problems, herding equilibrium never occur. However, I show that herding equilibrium may emerge in an economy with agency problems through the following mechanism.

There are two groups of informed principals: the first group acquires s_c ; the second group acquires s_i . Each principal in the first group is indexed by principal i , where $i \in [0, \lambda]$. And each principal in the second group is indexed by principal j , where $j \in [0, \mu]$. I denote E_p^c and E_p^I as expected trading profits for principals in the first and second group respectively. I denote B_{fb}^c and B_{fb}^I as net benefits of information for different groups in the economy without agency problem. Moreover, I denote B_{ap}^c and B_{ap}^I as net benefit of information for different groups respectively in the economy with agency problems.

For the first group, principal i submits $X_i = 1$ if $s_c = h$, and submits $X_i = -1$ if $s_c = l$. For the second group, principal j submits $X_j = 1$ if $s_j = h$, and submits $X_j = -1$ if $s_j = l$. To simplify the analysis, I only focus on the herding equilibrium. On the herding equilibrium, $\mu = 0$. In this case, if $s_c = h$, the total orders is $X = \lambda + n$. If $s_c = l$, the total orders is $X = -\lambda + n$. Thus, the total orders X is distributed on $[-\lambda - M, \lambda + M]$. Receiving total orders, the market maker sets the price as follows:

LEMMA 5.1. *Given $\lambda > 0$ and $\mu = 0$, the price follows the rule:*

$$P(X) = \begin{cases} (2p-1)\theta & \text{if } N - \lambda < X \leq N + \lambda \\ 0 & \text{if } -N + \lambda \leq X \leq N - \lambda \\ -(2p-1)\theta & \text{if } -N - \lambda \leq X < -N + \lambda \end{cases} \quad (5.4)$$

To show the existence of a herding equilibrium, I need to calculate expected trading profits for these two groups. Although there is no second group in the herding equilibrium, I also can calculate the expected trading profit for this group assuming one principal j is the marginal principal acquiring an exclusive signal. Then I have the following results:

LEMMA 5.2. *The expected trading profit of principals with the common signal is given by:*

$$E_p^c = (2p-1)\theta \frac{N-\lambda}{N}. \quad (5.5)$$

LEMMA 5.3. *The expected trading profit of principals with an exclusive signal is given by:*

$$E_p^I = (2p - 1)\theta \frac{N - (2p - 1)^2\lambda}{N}. \quad (5.6)$$

Lemma 5.2 and Lemma 5.3 shows that the expected trading profit of principals for the second group is higher than the first group. There is a large price impact when principals trade similarly because of having the same information, which makes the total orders more informative about the common signal and decreases the first group's information advantage. Thus, principals have higher incentives to acquire the exclusive signal than the common signal in the economy without agency problems. However, when the residual uncertainty is sufficiently large, principals herd to the common signals in the economy with agency problems. Herding makes the price sensitive to the common signal. Consequently, agents have strong incentives to acquire the common signal as they can easily forecast the asset price with this signal, which mitigates agency problems in acquiring it. Although the exclusive signals can generate more trading profits, agents could not easily forecast asset price with these signals because of their idiosyncratic noises, which worsens the agency problems in acquiring these signals. This mechanism generates the herding equilibrium. I show the formal result as follows:

PROPOSITION 5.1. *Comparing the economy with and without agency problems, I have*

(1) no herding equilibrium occurs in the economy without agency problem;

when M is small enough, no herding equilibrium occurs in the economy with agency problem; (2)

when $-\frac{\log 4p(1-p)}{\gamma_a} < C < (2p - 1)\theta - W_a$ and M is large enough, the herding equilibrium exists in the economy with agency problems.

Proposition 5.1 shows that the herding equilibrium occurs when the residual uncertainty is large. This result implies that herding is stronger in small/growth stocks, which have considerable uncertainty. It is consistent with [Lakonishok, Shleifer and Vishny \(1992\)](#) and [Wermers \(1999\)](#), who find that institutional investors have stronger herding behavior in small/growth stocks. Although my model is static, it implies that institutional investors tend to follow the lead of others. When more fund managers trade in one specific stock, others observe this and tend to follow their lead

because these followers anticipate that the price will become more informative and the agency problems will be mitigated.

5.2 Home/Industry Bias

In this section, I explore the model's implication for the home/industry bias, which is a long-standing puzzle. As documented by [Fama and Poterba \(1991\)](#), [Coval and Moskowitz \(1999\)](#), [Grinblatt and Keloharju \(2001\)](#), [Huberman \(2001\)](#) and [Seasholes and Zhu \(2010\)](#), both households and institutions prefer to trade the assets which are located around their hometowns or home countries. Though it is possible that some behavior biases drive home bias in households, home bias among institutional investors is still puzzling because they are sophisticated investors. Another plausible explanation is that investors have superior information on home assets, [Van Nieuwerburgh and Veldkamp \(2009\)](#) argue that investors can easily acquire information about other assets, which could eliminate home investors' information advantage and mitigate home bias. Even if investors can freely acquire information, I show that home bias still exists with agency problems in information acquisition.

I extend the basic model to consider two groups of principals: the first group has some opportunity to get free information; the second group has no information. The first group is interpreted as home principals based on the conventional belief that investors have an information advantage on home assets. The population of home principals is denoted by ω . Each principal in this group is indexed by i , where $i \in [0, \omega]$. The second group is called foreign principals. Each principal in this group is indexed by j . Any principals can hire agents to acquire information. Furthermore, I assume that principal i in the first group is endowed by a private signal $s_{h,i}$, which takes the form:

$$s_{h,i} = \{V, \emptyset\}. \quad (5.7)$$

The feature of this signal is that $s_{h,i}$ is a pure noise when $s_{h,i} = \emptyset$, and it is perfectly informative if $s_{h,i} = V$. The possibility that $s_{h,i}$ is perfectly informative is denoted by p_h :

$$prob(s_{h,i} = V) = p_h, \quad (5.8)$$

where $0 < p_h < p$. There are two differences between home and foreign principals: the first is that home principals can use their endowed signals in trading; the second difference is that home principals can use their endowed signals in the contracting.¹⁵ Moreover, I assume that the population of home principals hiring agents is λ , and the population of foreign principals hiring agents is μ . Although home principals may know the fundamental value exactly, they also have incentives to acquire information because they have chances to become uninformed. If the endowed signals are informative, home principals only rely on their endowed signals in trading. Otherwise, they have to rely on signals from agents. Thus, the total orders is $X = p_h\omega + (1 - p_h)(2p - 1)\lambda\omega + (2p - 1)\mu + n$ if $V = \theta$. And the total orders is $X = -p_h\omega - (1 - p_h)(2p - 1)\lambda\omega - (2p - 1)\mu + n$ if $V = -\theta$. To simplify the analysis, I let $\eta = p_h\omega + (1 - p_h)(2p - 1)\lambda\omega + (2p - 1)\mu$. Before proceeding, I define two home bias equilibria as follows:

DEFINITION 5.2. *Weak Home Bias Equilibrium: one equilibrium is weak home bias equilibrium if $\lambda > 0$ and $\mu > 0$.*

DEFINITION 5.3. *Strong Home Bias Equilibrium: one equilibrium is strong home bias equilibrium if $\lambda > 0$ and $\mu = 0$.*

Receiving total orders, the market maker sets the price as follows:

$$P(X) = \begin{cases} \theta & \text{if } N - \eta < X \leq N + \eta \\ 0 & \text{if } -N + \eta \leq X \leq N - \eta \\ -\theta & \text{if } -N - \eta \leq X < -N + \eta \end{cases} \quad (5.9)$$

I calculate the expected trading profits for different groups. I denote $E_{h,p}^1$, $E_{h,p}^2$ and $E_{f,p}$ as expected trading profits for home principals who hire agents, home principals who do not hire, and informed foreign principals respectively. They are shown as follows:

$$E_{f,p} = (2p - 1)\theta \frac{N - \eta}{N}, \quad (5.10)$$

¹⁵If these signals are not verifiable, there exist some mechanisms inducing principals to reveal their private information, such as imposing an infinite penalty when asset payoff deviates considerably from principals' reports. The infinite penalty can be interpreted as reputation concern. One interpretation of these contracts is the subjective evaluation. Or this type of contract can be interpreted as an implicit contract.

$$E_{h,p}^1 = [p_h + (1 - p_h)(2p - 1)]\theta \frac{N - \eta}{N}, \quad (5.11)$$

$$E_{h,p}^2 = p_h \theta \frac{N - \eta}{N}. \quad (5.12)$$

It is clear that the gain from information for home principals is $(1 - p_h)(2p - 1)\theta \frac{N - \eta}{N}$, which is lower than the trading profits of informed foreign principals. This is due to the decreasing marginal benefits of information. Thus, without agency problems, home principals have lower incentive than foreign principals to hire agents to acquire information. However, with agency problems, this is not the case. Home principals can use their endowed information in incentive provision, agency problems are not severe for home principals and home principals may have higher incentive to acquire information than foreign principals. The formal results regarding home bias are as follows:

PROPOSITION 5.2. *Comparing the economy with and without agency problems, I have*

(1) *neither weak home bias equilibrium nor strong home bias equilibrium occurs in the economy without agency problem;*

(2) *when M is small enough, neither weak home bias equilibrium nor strong home bias equilibrium occurs in the economy with agency problem;*

(3) *when both M and N are large enough, a strong herding equilibrium exists in the economy with agency problem when $\theta_1 < \theta < \theta_2$.*

where θ_1 and θ_2 are defined in the Appendix.

Proposition 5.2 shows that home bias occurs when the residual uncertainty is large. This result implies that home bias is stronger when investors trade small/growth stocks. It is consistent with Kang and Stulz (1997) and Coval and Moskowitz (1999). For example, Coval and Moskowitz (1999) find that U.S. fund managers have a stronger home bias when they trade small stocks. It also implies that investors tend to learn more about the assets within their expertise. This prediction is consistent with Massa and Simonov (2006), who find that Swedish investors buy assets highly correlated with their non-financial income.

5.3 Idiosyncratic Volatility Comovement

In this section, I explore the model's implication for idiosyncratic volatility comovement, which is documented by recent studies (see [Bekaert, Hodrick and Zhang, 2012](#), [Kelly, Lustig and Van Nieuwerburgh, 2013](#) and [Herskovic, Kelly, Lustig and Van Nieuwerburgh, 2013](#)). More importantly, because recent studies ([Bansal, Kiku, Shaliastovich and Yaron, 2014](#), [Campbell, Giglio, Polk, Turley, 2014](#) and [Herskovic, Kelly, Lustig and Van Nieuwerburgh, 2013](#)) find that the common factor in idiosyncratic volatilities has significant effects on asset prices, it is important to understand the driving force. In particular, the common factor in idiosyncratic volatilities is not related to the conventional risk factors, and the driving force is still puzzling.

I extend the basic model to consider two risky assets. Each asset is indexed by k , where $k = 1, 2$. Asset k 's payoff is denoted by D_k , which has a fundamental value V_k and a residual uncertainty ϵ_k . I assume ϵ_k is uniformly distributed on $[-M_k, M_k]$, where $M_k > 0$. V_k takes θ_k and $-\theta_k$ with equal probability, where $\theta_k > 0$. In particular, I assume that two assets' residual uncertainties are independent of each other, and also are independent of the two fundamentals. There is a correlation between the two fundamentals as shown:

$$\text{prob}(V_2 = \theta_2 | V_1 = \theta_1) = \text{prob}(V_2 = -\theta_2 | V_1 = -\theta_1) = q, \quad (5.13)$$

$$\text{prob}(V_1 = \theta_1 | V_2 = \theta_2) = \text{prob}(V_1 = -\theta_1 | V_2 = -\theta_2) = q. \quad (5.14)$$

The noisy traders' demand in asset k is denoted by n_k following a uniform distribution on $[-N_k, N_k]$, where $N_k > 0$. Noisy traders' demands are independent of other random variables. I assume that each market has one risk-neutral market maker, who sets the price independently from each other. The price of asset k is denoted by P_k . Furthermore, there are two groups of principals: group k can only trade the risky asset k , perhaps due to market segmentation or trading constraints. The population of informed principals in asset k is λ_k . To simplify the analysis, I assume that λ_1 is exogenous, and λ_2 is endogenous. This assumption is reasonable in many circumstances. For example, there are some insiders or home investors, who are endowed with information. The above assumptions are helpful to make the mechanism in my model clear. If the principals can trade

both assets, it is possible that there exist other possible effects, which may mitigate or exacerbate my mechanism (see [Vayanos and Woolley, 2013](#) and [Cespa and Foucault, 2014](#)). Each informed principal's signal is denoted by $s_{k,i}$. Information structures are the same as the basic model with one risky asset. Then, I have:

$$\text{prob}^I(V_k = \theta_k | s_{k,i} = h) = \text{prob}^I(V_k = -\theta_k | s_{k,i} = l) = p, \quad (5.15)$$

To avoid the price of asset 1 being fully informative about the fundamental, I have $\lambda_1 < \frac{N_1}{2p-1}$. Although the principals in group 2 can not trade the risky asset 1, they still can write contracts on the prices and payoffs of two assets. Specifically, informed principal i in group 2 design contract $\pi_{2,i}(s^R(s_{2,i}), P_1, P_2, D_1, D_2)$, where $s_{2,i}$ is his agent's signal and $s^R(s_{2,i})$ is the report. The reason why principals in asset 2 write this type of contracts is that two assets' fundamentals are correlated and agents' forecasting accuracy for the price of asset 1 is also related to their effort. I follow the procedure in the basic model to solve the equilibrium prices, optimal contracts, and population of informed principals in group 2. I carry out the numerical studies to show how the agency problems generate the idiosyncratic volatility comovement. Figure 5.1 shows that the population of informed principals in asset 2 decreases with the noisy trades' demand in asset market 1 (N_1) in the economy with agency problems. Because P_1 is also informative to V_1 , informed principals in group 2 use the price of asset 1 to monitor their agents. When the noisy traders' demand becomes more volatile in asset 1, P_1 becomes noisier and is more difficult for agents on asset 2 to predict. Consequently, the agents on the asset 2 are less willing to exert effort, which worsen the agency problems and decreases principals' incentives to hire agents on asset 2. This induces the price of asset 2 to become less informative and more sensitive to its noisy traders' demand, leading to increased idiosyncratic volatility (shown in Figure 5.2). Following the same mechanism, when the population of informed principals in group 1 increases, asset 1's price becomes more informative and the principals in group 2 have higher incentives to hire agents (see Figure 5.3). This result is interesting and is related to herding on the industry level([Choi and Sias, 2009](#)).¹⁶

¹⁶In Figure 6.1 and Figure 6.2, I set $\theta_1 = \theta_2 = 2$, $M_1 = M_2 = 5$, $N_2 = 2$, $p = 0.6$, $q = 0.8$, $C = 0.75$, $W_a = 0$, $\lambda_1 = 2$. In Figure 6.3, $\theta_1 = \theta_2 = 2$, $M_1 = M_2 = 5$, $N_1 = N_2 = 2$, $p = 0.6$, $q = 0.8$, $C = 0.75$, $W_a = 0$

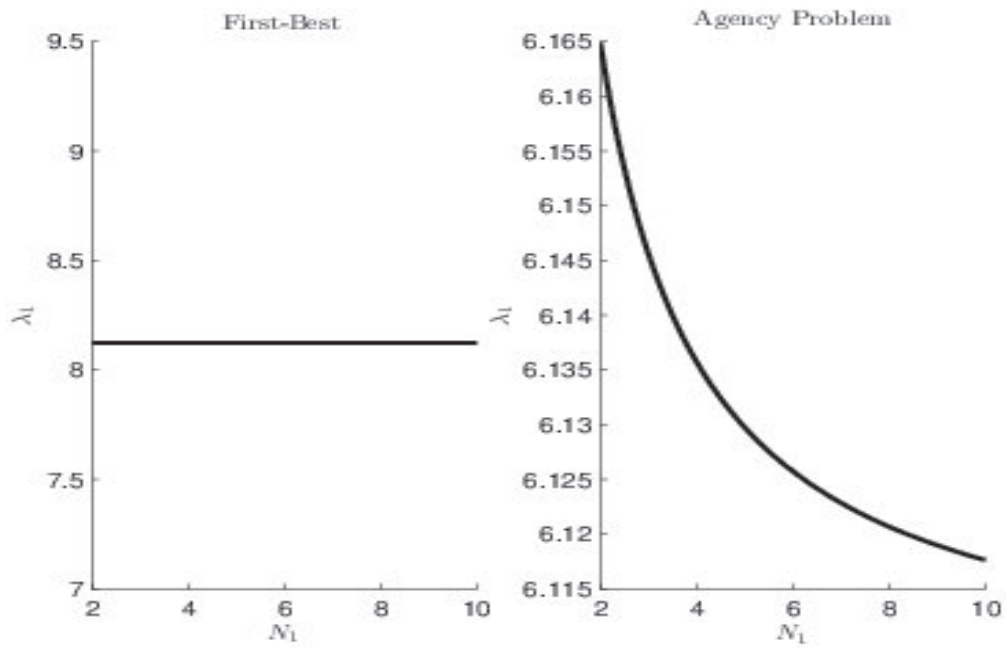


Figure 5.1: Noisy Traders in Asset 1 and Population of Informed Principals in Asset 2

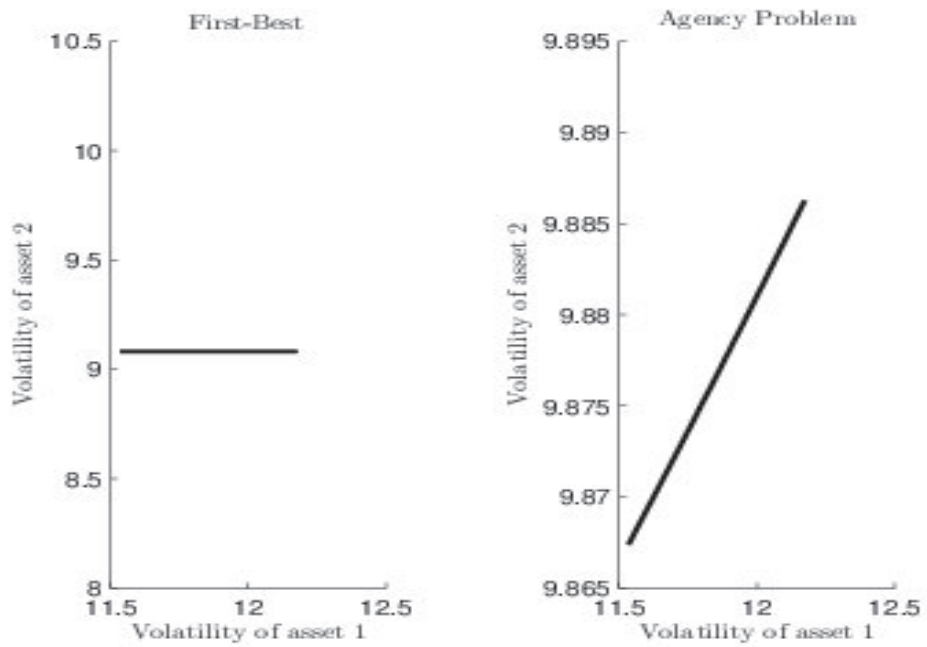


Figure 5.2: Idiosyncratic Volatility Comovement

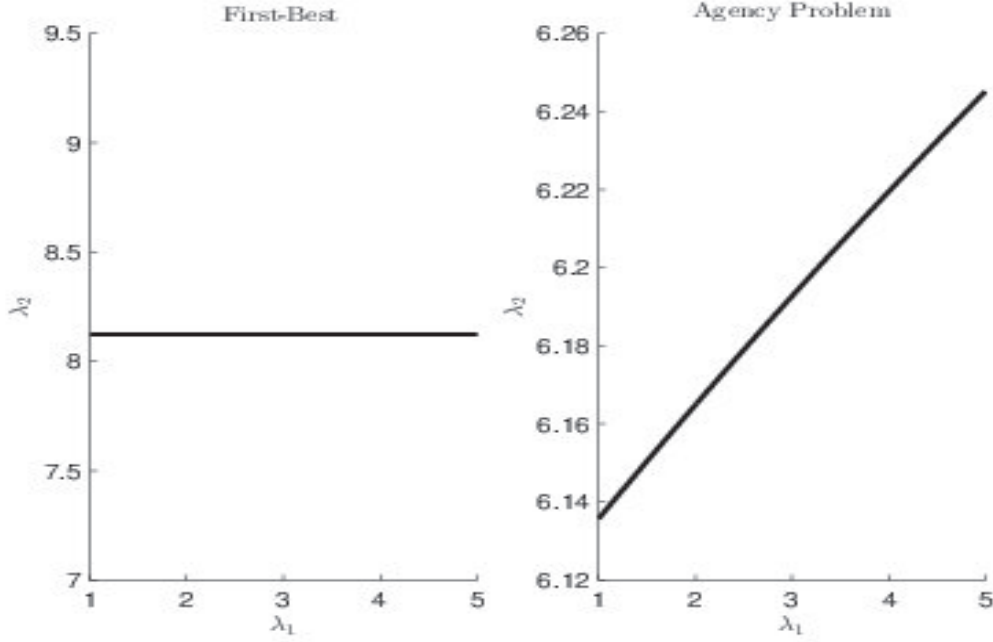


Figure 5.3: Population of Informed Principals both Asset Markets

6 Generalization

My model assumes: (1) agents have CARA utilities; (2) the fundamental value V takes binary values; (3) principals do not learn information from the asset price. This section relaxes these assumptions and shows that the strategic complementarities are robust.

6.1 General Utility Function of Agents

This section shows that my results are robust when agents have a general **hyperbolic absolute risk aversion (HARA)** class of utility functions. The HARA utility function is shown as follows:

$$U(W) = \frac{\gamma}{1-\gamma} \left[\frac{AW}{\gamma} + K \right]^{1-\gamma}, \quad K \geq 0 \quad (6.1)$$

where the utility function is only defined over $\frac{AW}{\gamma} + K > 0$. I know the absolute risk aversion coefficient is given by:

$$-\frac{U''}{U'} = \frac{A\gamma}{AW + K\gamma} \quad (6.2)$$

When $\gamma < 0$, this HARA utility function has an increasing absolute risk aversion, which is implausible. Thus, I only consider the case where $\gamma > 0$. Particularly, this general HARA utility

function has several examples which are largely used in finance or economy, such as power utility, negative exponential utility or logarithmic utility.

ASSUMPTION 6.1. γ satisfies: $\gamma > 0$.

To simplify the analysis, I assume that agents need to incur a utility loss if they exert effort. This utility loss is denoted by C . Thus, the participant constraint and incentive constraints are shown as follows:

$$\sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int \{U[\pi_i(s_i, P, D)] - C\} f^I(P, D|s_i) dP dD = U[W_a], \quad (6.3)$$

where LHS of Equation (6.3) is agent i 's expected utility when he exerts costly effort. Meanwhile, W_a is the reserve wealth of agents.

Ex Ante IC which is the incentive-compatibility of effort constraint

$$\begin{aligned} & \sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int \{U[\pi_i(s_i, P, D)] - C\} f^I(P, D|s_i) dP dD \\ & \geq \sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int [U[\pi_i(s^R(s_i), P, D)]] f^U(P, D|s_i) dP dD \end{aligned} \quad (6.4)$$

Ex Post IC which is incentive-compatibility of truth reporting ($\forall s_i$ and $s^R(s_i) : s \rightarrow s$)

$$\begin{aligned} & \sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int \{U[\pi_i(s_i, P, D)]\} f^I(P, D|s_i) dP dD \\ & \geq \sum_{s_i=\{h,l\}} \text{prob}(s_i) \int \int [U[\pi_i(s^R(s_i), P, D)]] f^I(P, D|s_i) dP dD \end{aligned} \quad (6.5)$$

ASSUMPTION 6.2. I have different cases regarding the information acquisition cost

Case 1: If $\gamma < 1$, $U(W_a) - \frac{C}{2^{p-1}} > 0$;

Case 2: If $\gamma > 1$, $U(W_a) + \frac{C}{2^{p-1}} < 0$;

Case 3: if $\gamma = 1$ and $K = 0$.

This assumption could ensure that the optimal contracts can be implemented and there is interior solution to the contracting for different γ . I show that strategic complementarity effect is robust in the following proposition.

PROPOSITION 6.1. *Under Assumption 6.1 and Assumption 6.2, when the agents have HARA utility in the economy with agency problems, I have the following results:*

(1) *for small enough M , information acquisition delegation is a strategic substitute. That is, $\frac{\partial B}{\partial \lambda} < 0$.*

(2) *for large enough M , there exists λ^{gc} satisfying: when $\lambda < \lambda^{gc}$, information acquisition delegation is a strategic complement. That is $\frac{\partial B}{\partial \lambda} > 0$.*

6.2 More General Distribution of V

In this section, I assume that V takes three values in $\theta, 0$ or $-\theta$. In particular, the distribution of V is symmetric.¹⁷ The probability that $V = 0$ is denoted as p_m . The probability of $V = \theta$ or $V = -\theta$ is given by:

$$\text{prob}(V = \theta) = \text{prob}(V = -\theta) = (1 - p_m)/2, \quad (6.6)$$

After exerting effort, agent i generates a private signal $s_i \in \{h, 0, l\}$ about the risky asset's fundamental value V . The probability with which a signal is correct by

$$\text{prob}(s_i = h|V = \theta) = \text{prob}(s_i = 0|V = 0) = \text{prob}(s_i = l|V = -\theta) = p, \quad (6.7)$$

and

$$\text{prob}(s_i = 0|V = \theta) = \text{prob}(s_i = 0|V = -\theta) = (1 - p)q, \quad (6.8)$$

and

$$\text{prob}(s_i = l|V = \theta) = \text{prob}(s_i = h|V = -\theta) = (1 - p)(1 - q), \quad (6.9)$$

and

$$\text{prob}(s_i = h|V = 0) = \text{prob}(s_i = l|V = -\theta) = (1 - p)/2, \quad (6.10)$$

where s_i is independent across agents and $p \geq \frac{1}{3}$, while $q \geq \frac{1}{2}$. Principal i submits $X_i = 1$ when he

¹⁷ I also can extend my model to consider more general distribution with finite supports, the intuitions are similar. However, the normal distribution is not feasible in my model because the optimal contracts are not implementable as argued by Dybvig et al. (2010). To show our results are robust when the fundamental value and signal have infinite supports, we follow Dybvig et al. (2010) to consider mixture normal distributions to show robustness in the next sections.

receives report $s_i = h$, does nothing when he receives report $s_i = 0$, and submits $X_i = -1$ when he receives report $s_i = l$. Then the market maker sets the price as follows:

$$P(X) = \begin{cases} \theta & \text{if } N < X \leq N + \lambda(p - (1 - p)(1 - q)) \\ \theta \frac{1-p_m}{1+p_m} & \text{if } N - \lambda(p - (1 - p)(1 - q)) < X \leq N \\ 0 & \text{if } -N + \lambda(p - (1 - p)(1 - q)) \leq X \\ & \leq N - \lambda(p - (1 - p)(1 - q)) \\ -\theta \frac{1-p_m}{1+p_m} & \text{if } -N \leq X < -N + \lambda(p - (1 - p)(1 - q)) \\ -\theta & \text{if } -N - \lambda(p - (1 - p)(1 - q)) \leq X < -N \end{cases} \quad (6.11)$$

It is clear that the price increases with the total orders X . It differs from the binary-state case on the feature that the price takes five values. This difference also shows that the problem will become extremely complicate when I consider more a general distribution of V . I carry out the numerical studies to show that information acquisition complementarities is robust in Figure 6.1, as is the relation between residual uncertainty/agents' risk aversion and price informativeness in Figure 6.2 and Figure 6.3.¹⁸

6.3 Learning

This section shows that my results are robust when principals learn information from the asset price. To obtain analytical solution in the non-linear REE, I modify the information structure and the distribution of residual uncertainty. I assume that the residual uncertainty ϵ follows normal distribution $N(0, \sigma_M^2)$. The private signal acquired by agent i is denoted by s_i . Through the costly effort $e_i \in \{0, e\}$, the joint distributions of his signal s_i and the fundamental value V is a mixture distribution as follows:

$$(b + e_i)f^I(s_i, V) + (1 - b - e_i)f^U(s_i, V), \quad (6.12)$$

¹⁸ I set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $C = 0.07$. I also set $M = 5$, $M = 20$ and $M = 200$ for low residual uncertainty, median residual uncertainty and high residual uncertainty cases respectively in Figure 6.1. Then set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $\gamma_a = 1$, $C = 0.05$ and $M = 20$ for analysis of agents' risk aversion in Figure 6.2. I set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $\gamma_a = 1$, and $C = 0.075$ for analysis of residual uncertainty M in Figure 6.3.

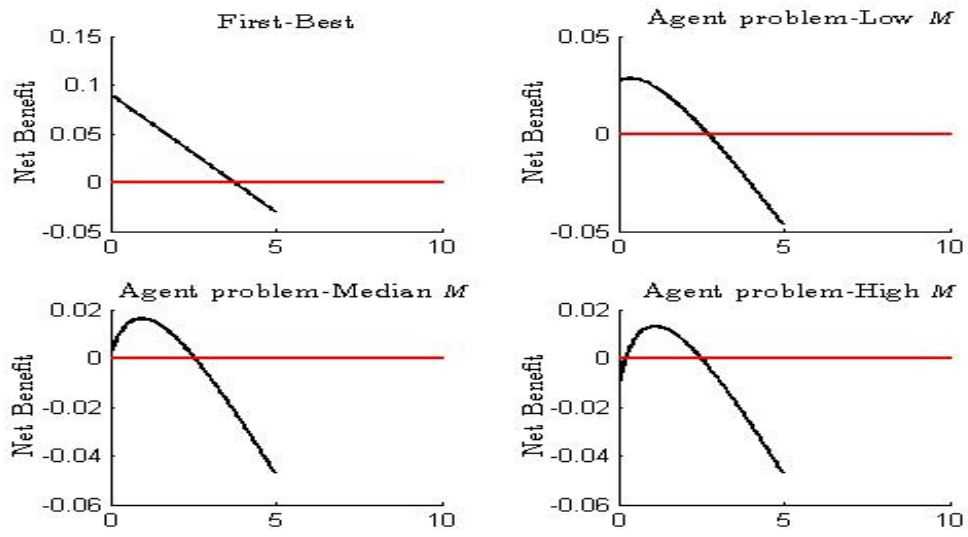


Figure 6.1: Information Acquisition Benefit: Triple-State Case

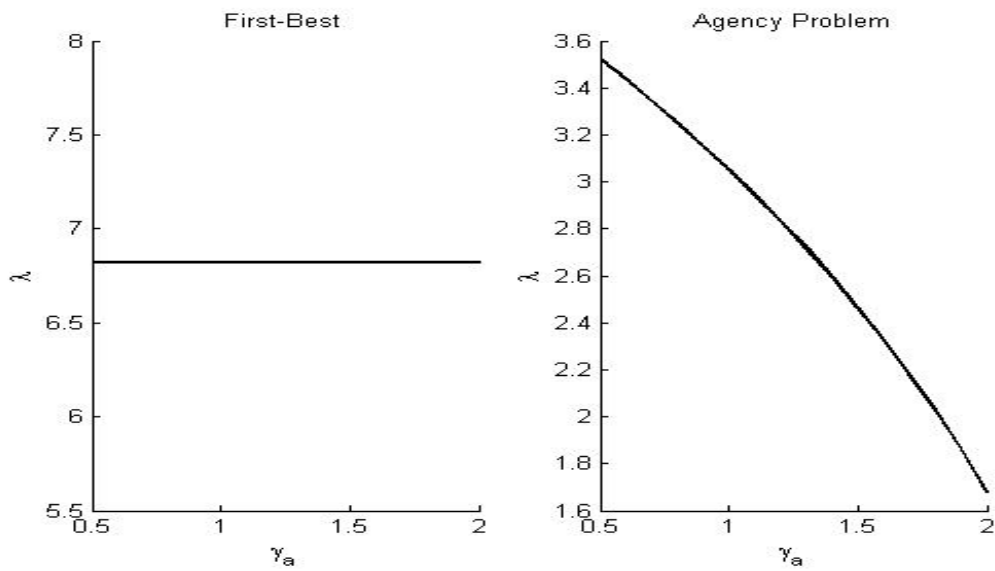


Figure 6.2: Population of Informed Principal and Agents' Risk Aversion: Triple-State Case

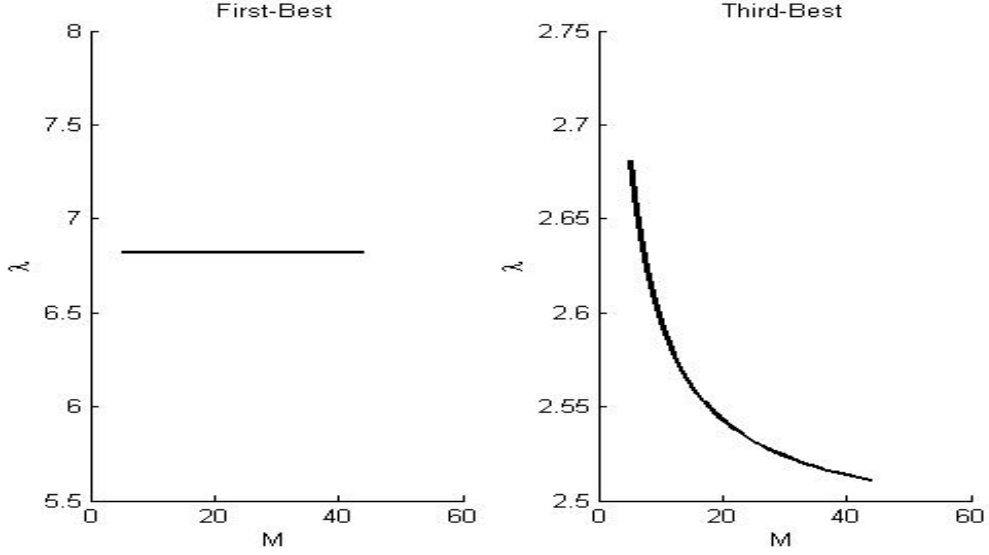


Figure 6.3: Population of Informed Principal and Residual Uncertainty: Triple-State Case

where $e > 0$ and $b + e < 1$. Here, f^I is an "informed" distribution and f^U in an "uninformed" distribution. I assume that s_i and V are independent in the uninformed distribution. Moreover, I assume that the probability density of s_i is: $f(s_i) = \frac{1}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s_i-\theta)^2}{2\sigma^2}} + \frac{1}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s_i+\theta)^2}{2\sigma^2}}$. Meanwhile, "informed" joint distribution $f^I(s_i, V) = \frac{1}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s_i-V)^2}{2\sigma^2}}$, while "uninformed" joint distribution $f^U(s_i, V) = \frac{1}{2} f(s_i)$.

One interpretation of the mixture model is that the signals observed by the agents may be informative or not and the agents cannot tell which occurs. In particular, when agents exert efforts, the probabilities that the signals are informative increase. Meanwhile, the mixture model is a simple sufficient condition when I implement the first-order approach to solve the optimal contracts in a general space. Without a loss of generality, I only consider moral hazard problems in information acquisition. This implies that principals could observe the realized signals acquired by agents, but they could not observe whether the agents exert efforts. Moral hazard problems in information acquisitions can be interpreted in many realistic circumstances, such as data collection. Thus, principal i 's objective function is as follows:

$$\max_{\pi_i(s_i, P, D), X_i(s_i, \pi_i, P)} \int f(s_i) \int \int [W_0 + X_i(D - P) - \pi_i(s_i, P, D)] f^I(P, D | s_i) dP dD ds_i, \quad (6.13)$$

where $X_i(s_i, \pi_i, P)$ is principal i 's demand function conditional on the price P and the signal s_i

reported by his agent. He maximizes his utility function subject to his agent's participant constraint and incentive compatibility as follows:

PC:

$$\int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D) + \gamma_a C}] f^I(P, D | s_i) dP dD ds_i = -\exp(-\gamma_a W_a), \quad (6.14)$$

IC:

$$\begin{aligned} & \int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D) + \gamma_a C}] f^I(P, D | s_i) dP dD ds_i \\ & \geq \int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i, P, D) + \gamma_a C}] f^U(P, D | s_i) dP dD ds_i, \end{aligned} \quad (6.15)$$

where $f^I(P, D | s_i)$ is the conditional probability density given that the agent i exerts effort, and $f^U(P, D | s_i)$ is the conditional probability density given that the agent i shirks.

Now, I assume that there is one continuum of principals and the population of principals hiring agents to acquire information is λ . For the informed principal i , the probability density of s_i conditional on $V = \theta$ is denoted by $\eta_{I,i,h}$, where $\eta_{I,i,h} = \frac{1+b+e}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s-\theta)^2}{2\sigma^2}} + \frac{1-e-b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s+\theta)^2}{2\sigma^2}}$; the probability density of s_i conditional on $V = -\theta$ is denoted by $\eta_{I,i,l}$, where $\eta_{I,i,l} = \frac{1+b+e}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s+\theta)^2}{2\sigma^2}} + \frac{1-e-b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s-\theta)^2}{2\sigma^2}}$. For the uninformed principal i , the probability density of s_i conditional on $V = \theta$ is denoted by $\eta_{U,i,h}$, where $\eta_{U,i,h} = \frac{1+b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s-\theta)^2}{2\sigma^2}} + \frac{1-b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s+\theta)^2}{2\sigma^2}}$; the probability density of s_i conditional on $V = -\theta$ is denoted by $\eta_{U,i,l}$, where $\eta_{U,i,l} = \frac{1+b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s+\theta)^2}{2\sigma^2}} + \frac{1-b}{2} \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s-\theta)^2}{2\sigma^2}}$.

Due to the assumption about risk-neutral principals, the optimal contracts and asset pricing can be solved separately as our basic model. I solve the model following step-by-step: (1) in the first step, I solve the asset pricing; (2) in the second step, I solve the optimal contract given the population of informed principals; (3) in the third step, I calculate the net benefit of information acquisition to show the strategic complementarities.

Asset Pricing In order to maximize the final wealth, it is necessary to compute the conditional expectation of V for different groups. According to the Bayes's rule, the posterior probability $p_K(s_i, P)$ of state h for principal i of type K after observing s_i and P is given by:

$$p_K(s_i, P) = \frac{f_h(P)\eta_{K,i,h}}{f_h(P)\eta_{K,i,h} + f_l(P)\eta_{K,i,l}} \quad (6.16)$$

where $f_\omega(P)$, $\omega = h, l$ is the probability density of the equilibrium price conditional on the corresponding state of the world. Given the posterior probabilities, principals' demand schedules are shown in the following lemma.

LEMMA 6.1. *For any $K = I, U$, there exists a threshold $X_K(P)$ such that the demand schedule for principal i of type K is given by:*

$$X_{K,i} = \begin{cases} 1 & \text{if } s_i \geq X_K(P), \\ -1 & \text{if } s_i < X_K(P), \end{cases} \quad (6.17)$$

where the threshold $X_K(P)$ is uniquely determined by the condition:

$$p_K(X_K(P), P) = \frac{P + \theta}{2\theta}. \quad (6.18)$$

Having showing the demand schedules, the aggregate demand can be calculated as follows. Conditional on $V = \theta$, the aggregate demand is given by

$$D(P, \theta) = \lambda[1 - (1 + b + e)\Phi(X_I(P) - \theta) - (1 - b - e)\Phi(X_I(P) + \theta)] \quad (6.19)$$

$$+ (1 - \lambda)[1 - (1 + b)\Phi(X_U(P) - \theta) - (1 - b)\Phi(X_U(P) + \theta)]; \quad (6.20)$$

conditional on $V = -\theta$, the aggregate demand is given by

$$D(P, -\theta) = \lambda[1 - (1 + b + e)\Phi(X_I(P) + \theta) - (1 - b - e)\Phi(X_I(P) - \theta)] \quad (6.21)$$

$$+ (1 - \lambda)[1 - (1 + b)\Phi(X_U(P) + \theta) - (1 - b)\Phi(X_U(P) - \theta)], \quad (6.22)$$

where Φ is the cumulative distribution function for normal distribution $N(0, \sigma^2)$. Consequently,

given realized demand from noisy traders, the market clearing condition takes the form as follows:

$$D(P, V) = n \quad (6.23)$$

Thus, the probability density of price P conditional on the value V is denoted as $f_i(P)$. They are calculated as follows:

$$\begin{aligned} f_h(P) &= \frac{\lambda}{2N} [(1+b+e)\phi(X_I(P) - \theta) + (1-b-e)\phi(X_I(P) + \theta)]X_I'(P) \\ &\quad + \frac{(1-\lambda)}{2N} [(1+b)\phi(X_U(P) - \theta) + (1-b)\phi(X_U(P) + \theta)]X_U'(P) \end{aligned} \quad (6.24)$$

$$\begin{aligned} f_l(P) &= \frac{\lambda}{2N} [(1+b+e)\phi(X_I(P) + \theta) + (1-b-e)\phi(X_I(P) - \theta)]X_I'(P) \\ &\quad + \frac{(1-\lambda)}{2N} [(1+b)\phi(X_U(P) + \theta) + (1-b)\phi(X_U(P) - \theta)]X_U'(P) \end{aligned} \quad (6.25)$$

where ϕ is the probability density function for the normal distribution $N(0, \sigma^2)$. The key to solve the asset pricing is to solve the thresholds X_I and X_U . I follow [Malamud and Petrov \(2014\)](#) to solve both. I denote the likelihood ratio by:

$$L_K(X) = \eta_{K,i,h} / \eta_{K,i,l}. \quad (6.26)$$

From the Lemma 6.1, I have the following condition:

$$L_I(X_I(P)) = L_U(X_U(P)). \quad (6.27)$$

Thus, the relation between $X_U(P)$ and $X_I(P)$ is: $X_U(P) = L_U^{-1}(L_I(X_I(P)))$. It indicates that the solution of X_I can characterize the asset pricing. I have the following result regarding X_I , X_U and the probability density of price P :

PROPOSITION 6.2. *There exists a monotone increasing, absolutely continuous solution $X_I(P)$, $P \in (-\theta, \theta)$ to*

$$2 \log L_K(X_I(P)) = \log \frac{P + \theta}{\theta - P}.$$

Meanwhile, $X_I(P)$, $X_U(P) = L_U^{-1}(L_K(X_I(P)))$ and

$$f_h(P) = \frac{\lambda}{2N}[(1+b+e)\phi(X_I(P)-\theta) + (1-b-e)\phi(X_I(P)+\theta)]X_I'(P) + \frac{(1-\lambda)}{2N}[(1+b)\phi(X_U(P)-\theta) + (1-b)\phi(X_U(P)+\theta)]X_U'(P), \quad (6.28)$$

and

$$f_l(P) = \frac{\lambda}{2N}[(1+b+e)\phi(X_I(P)+\theta) + (1-b-e)\phi(X_I(P)-\theta)]X_I'(P) + \frac{(1-\lambda)}{2N}[(1+b)\phi(X_U(P)+\theta) + (1-b)\phi(X_U(P)-\theta)]X_U'(P) \quad (6.29)$$

form a rational expectations equilibrium.

Contracting I use the first-order approach to solve the optimal contracts. First, I let l_1 and l_2 be the Lagrange multipliers on the PC and IC. I can get the expression for optimal compensation as follows:

$$\pi(s_i, P, D) = \frac{\log(l_1 + l_2 - l_2 \frac{\exp^{-\gamma_a C} f^U(P, D|s_i)}{f^I(P, D|s_i)}) + \log(\gamma_a)}{\gamma_a} \quad (6.30)$$

Net Benefit of Information Now I calculate the net benefit of information. Conditional on the fundamental value V and asset price P , the expected trading profit of principal i of type $K = I, U$ is calculated by:

$$\begin{aligned} E_{K,h,P} &= \text{prob}(s_i < X_K(P)|V = \theta, P)(P - \theta) + \text{prob}(s_i \geq X_K(P)|V = \theta, P)(\theta - P) \\ &= \frac{1+b+e}{2}\Phi(X_K - \theta)(P - \theta) + \frac{1-b-e}{2}\Phi(X_K + \theta)(P - \theta) \\ &\quad + \frac{1+b+e}{2}[1 - \Phi(X_K - \theta)](\theta - P) + \frac{1-b-e}{2}[1 - \Phi(X_K + \theta)](\theta - P), \end{aligned} \quad (6.31)$$

$$\begin{aligned} E_{K,l,P} &= \text{prob}(s_i < X_K(P)|V = -\theta, P)(P + \theta) + \text{prob}(s_i \geq X_K(P)|V = -\theta, P)(-\theta - P) \\ &= \frac{1+b+e}{2}\Phi(X_K + \theta)(P + \theta) + \frac{1-b-e}{2}\Phi(X_K - \theta)(P + \theta) \\ &\quad + \frac{1+b+e}{2}[1 - \Phi(X_K + \theta)](-\theta - P) + \frac{1-b-e}{2}[1 - \Phi(X_K - \theta)](-\theta - P). \end{aligned} \quad (6.32)$$

Then, the expected trading profit for different groups of principals is as follows:

$$E_K = \frac{1}{2} \int E_{K,h,P} f_h(P) dP + \frac{1}{2} \int E_{K,l,P} f_l(P) dP. \quad (6.33)$$

Consequently, the net benefit from information is: $B = E_I - E_U - E(\pi)$. Now, I numerically show

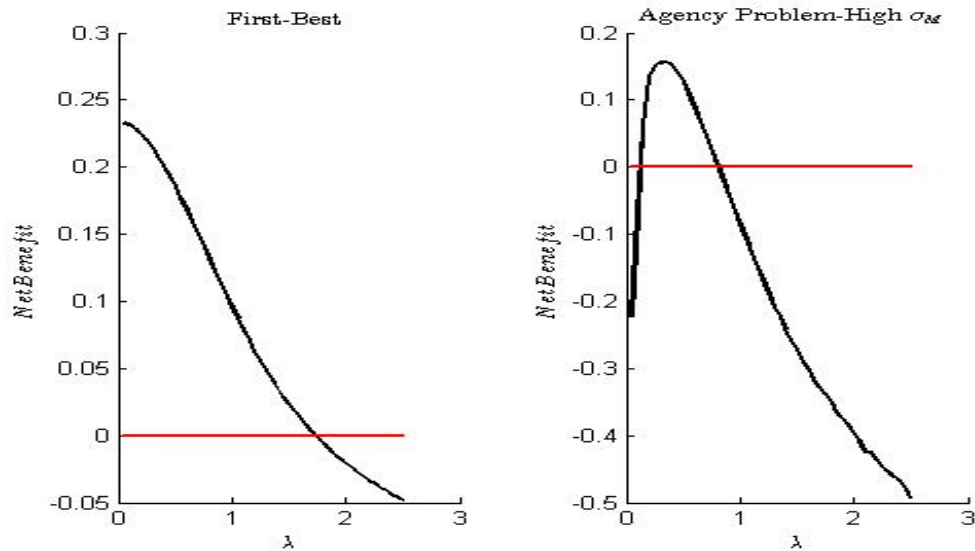


Figure 6.4: Information Acquisition Benefit: Learning Case

that the strategic complementarities are robust when the residual uncertainty has large variance in the following figure.¹⁹

7 Conclusion

I show that optimal contracts depend on the accuracy of agents' forecasts for the asset prices and payoffs. Agents receive high compensation when they produce an accurate forecast. The bonus, as a reward for an accurate forecast, decreases with price informativeness and increases with residual uncertainty of the asset payoffs. When the price becomes more informative or the residual uncertainty decreases, agents can forecast the asset prices or payoffs more accurately with information. Consequently, agents are more willing to exert efforts in acquiring information. Thus, the principals can decrease the bonus. These results predict that the bonus is larger for professionals, who trade or cover small/growth stocks with larger residual uncertainty or assets with lower institutional ownership.

More importantly, I show that agency problems in delegated information acquisition play important roles in shaping institutional investors' behavior and asset pricing. The novelty of my model is that agency problems generate a strategic complementarities in information acquisition delegation.

¹⁹ I set $\theta = 0.5$, $\sigma = 1$, $W_a = 0$, $C = 0.1$, $N = 10$ $\sigma_M = 50$

When more principals hire agents to acquire information, the price becomes less noisy, which make it easier for agents to forecast. Therefore, agents are more willing to exert effort, thereby mitigating agency problems. In turn, other principals are more willing to hire agents. These strategic complementarities lead to multiple equilibria, which have implications for jumps and excess volatilities in asset prices or price informativeness. In particular, multiple equilibria occur when the asset payoff's residual uncertainty is large. This can provide a potential explanation for observed excess volatilities in small/growth stocks or during recessions. My results also predict that price informativeness or institutional ownership tend to have jumps for small/growth stocks. The extensions of this model demonstrate that the agency problems could provide explanations for some phenomena, including idiosyncratic volatility comovement, herding behavior and home/industry bias. Moreover, my model predicts that the herding or home/industry bias is stronger for small/growth stocks.

The driving force for my results is as follows: the price becomes more informative when more principals hire agents to acquire information, which mitigates agency problems. Thus, it is clear that the assumptions about risk-averse principals or non-learning will not overturn the main mechanisms. However, relaxing these assumptions is interesting. If principals are risk-averse, I expect that the optimal contract will consist of two components: the first is agents' forecasting accuracy, and the second is proportional fee attributable to risk sharing. Meanwhile, to simplify the analysis, I assume that the principals behave competitively and do not have price impact. It is also interesting to study principals' strategic trading in the presence of the agency problem. The challenge to solve a model with strategic trading relies on the solutions with optimal contracts and asset prices. If I follow [Kyle \(1985\)](#) to assume that the asset payoff follows normal distribution, the asset pricing is easily solved, while the optimal contract is not implementable as argued by [Dybvig et al. \(2010\)](#). I leave this extension for further study.

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8 Appendix

8.1 Proofs

This appendix provides all proofs omitted above.

Proof of Lemma 3.1. If $\lambda \geq \frac{N}{2p-1}$, market maker will know that $V = \theta$ if $X > -N + \lambda(2p - 1)$ and $V = -\theta$ if $X < N - \lambda(2p - 1)$. Then market makers will always set $P = V$. If this is the case, informed investors' trading will always equals to zero because $X_i(V - P) = 0$. If the trading profit is zero, investors have no incentive to acquire information. Thus I can conclude that the population of informed investors can not be larger than $\frac{N}{2p-1}$.

□

Proof of Lemma 3.2. On the support $[-\lambda(2p - 1) - N, \lambda(2p - 1) + N]$, the conditional pdf of X follows

$$f(X|V = \theta) = \begin{cases} 1 & \text{if } -N + \lambda(2p - 1) \leq X \leq N + \lambda(2p - 1) \\ 0 & \text{if } X < -N + \lambda(2p - 1) \end{cases} \quad (8.1)$$

$$f(X|V = -\theta) = \begin{cases} 0 & \text{if } X > N - \lambda(2p - 1) \\ 1 & \text{if } -N - \lambda(2p - 1) \leq X \leq N - \lambda(2p - 1) \end{cases} \quad (8.2)$$

Using Bayesian updating, $prob(V = \theta|X) = \frac{\frac{1}{2}f(X|V=\theta)}{\frac{1}{2}f(X|V=\theta) + \frac{1}{2}f(X|V=-\theta)}$. Thus conditional on X , market maker's belief about probability of $V = \theta$ follows:

$$prob(V = \theta|X) = \begin{cases} 1 & \text{if } N - \lambda(2p - 1) < X \leq N + \lambda(2p - 1) \\ \frac{1}{2} & \text{if } -N + \lambda(2p - 1) \leq X \leq N - \lambda(2p - 1) \\ 1 & \text{if } -N - \lambda(2p - 1) \leq X \leq -N + \lambda(2p - 1) \end{cases} \quad (8.3)$$

Then because $P = prob(V = \theta|X)\theta - [1 - prob(V = \theta|X)]\theta$, I can get the price function.

□

Proof of Lemma 3.3 and Lemma 3.4. Step 1 $f^I(P = \theta, D, s_i = h) = f^I(P = \theta, D|s_i = h) \times \text{prob}(s_i = h)$. Then

$$\begin{aligned}
f^I(P = \theta, D, s_i = h) &= \text{prob}(P = \theta, s_i = h) \times f^I(D|P = \theta, s_i = h) \\
&= \text{prob}(V = \theta, s_i = h) \times \text{prob}(N - \lambda(2p - 1) \leq n \leq N + \lambda(2p - 1)) \times f^I(D|P = \theta, s_i = h) \\
&= \begin{cases} \frac{1}{2M} \frac{p}{2} \frac{\lambda(2p-1)}{N} & \text{if } -M + \theta \leq D \leq M + \theta \\ 0 & \text{if } -M - \theta \leq D < -M + \theta \end{cases}
\end{aligned} \tag{8.4}$$

Since $\text{prob}(s_i = h) = \frac{1}{2}$, I can get $f^I(P = \theta, D|s_i = h)$ in the Lemma.

Step 2 $f^I(P = -\theta, D, s_i = h) = f^I(P = -\theta, D|s_i = h) \times \text{prob}(s_i = h)$. Then

$$\begin{aligned}
f^I(P = -\theta, D, s_i = h) &= \text{prob}(P = -\theta, s_i = h) \times f^I(D|P = -\theta, s_i = h) \\
&= \text{prob}(V = -\theta, s_i = h) \times \text{prob}(N - \lambda(2p - 1) \leq n \leq -N + \lambda(2p - 1)) \times f^I(D|P = -\theta, s_i = h) \\
&= \begin{cases} 0 & \text{if } M - \theta < D \leq M + \theta \\ \frac{1}{2M} \frac{1-p}{2} \frac{\lambda(2p-1)}{N} & \text{if } -M - \theta \leq D \leq M - \theta \end{cases}
\end{aligned} \tag{8.5}$$

Since $\text{prob}(s_i = h) = \frac{1}{2}$, I can get $f^I(P = -\theta, D|s_i = h)$ in the Lemma.

Step 3 $f^I(P = 0, D, s_i = h) = f^I(P = 0, D|s_i = h) \times \text{prob}(s_i = h)$. Then

$$\begin{aligned}
f^I(P = 0, D, s_i = h) &= \text{prob}(P = 0, s_i = h) \times f^I(D|P = 0, s_i = h) \\
&= \text{prob}(V = \theta, s_i = h) \times \text{prob}(-N + \lambda(2p - 1) \leq n \leq N - \lambda(2p - 1)) \times f^I(D|P = \theta, s_i = h) \\
&+ \text{prob}(V = -\theta, s_i = h) \times \text{prob}(-N + \lambda(2p - 1) \leq n \leq N - \lambda(2p - 1)) \times f^I(D|P = -\theta, s_i = h) \\
&= \begin{cases} \frac{p}{4M} \frac{N - \lambda(2p-1)}{N} & \text{if } M - \theta \leq D \leq M + \theta \\ \frac{1}{4M} \frac{N - \lambda(2p-1)}{N} & \text{if } -M + \theta \leq D < M - \theta \\ \frac{1-p}{4M} \frac{N - \lambda(2p-1)}{N} & \text{if } -M - \theta \leq D < -M + \theta \end{cases}
\end{aligned} \tag{8.6}$$

Step 4: Then Lemma 3.3 and Lemma 3.4 can be derived following the same process above. \square

Proof of Proposition 3.1. I prove this proposition in two steps.

Step 1 (proof of existence and uniqueness)

$$\max_{v_i(s_i, P, D)} \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int \int \frac{1}{\gamma_a} \log[v_i(s_i, P, D)] f^I(P, D|s_i) dD, \quad (8.7)$$

subject to participation constraint:

$$\sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i, P, D) f^I(P, D|s_i) dD = \exp^{-\gamma_a W_a - \gamma_a C} \quad (8.8)$$

Then I let $f = \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int \int \log[v_i(s_i, P, D)] f^I(P, D|s_i) dD$ and then

$$\mathcal{D}_1 = \left\{ - \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i, P, D) f^I(P, D|s_i) dD \geq - \exp^{-\gamma_a W_a - \gamma_a C} \right\} \quad (8.9)$$

It is obvious that f is a strictly concave function and \mathcal{D}_1 is convex. Then I can conclude that the local maximum of f over \mathcal{D}_1 is a global solution to this optimization. This implies that the solution in the first-order approach is the global solution to this problem.

Step 2: (Solution). I denote Lagrange multiplier of by λ_1 . Then I can get $v_i(s_i, P, D) = \frac{1}{\gamma_a} \frac{1}{\lambda_1}$ and $\frac{1}{\gamma_a \lambda_1} = \exp^{-\gamma_a W_a - \gamma_a C}$. Then I can conclude that $\pi_i(s_i, P, D) = W_a + C$. \square

Proof of Proposition 3.2. Step 1 (proof of existence and uniqueness in the second-best) The second-best case is proposed by [Dybvig et al. \(2010\)](#) where the principals are able to observe agents' signals, but are not able to observe agents' hidden actions. Thus, there is not misreporting problem. Then I will show that the agency problem in my study is equivalent to this second-best case since the signals or fundamental value V take binary states. Particularly, the IC in the second-best case is:

$$\sum_{s_i=\{h,l\}} \frac{1}{2} \int \int v_i(s_i, P, D) [f^I(P, D|s_i) - \exp^{-\gamma_a C} f^U(P, D|s_i)] dP dD \leq 0 \quad (8.10)$$

Then I let $f = \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int \int \log[v_i(s_i, P, D)] f^I(P, D|s_i) dD$ and then

$$\begin{aligned} \mathcal{D}_2 = \{ & - \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i, P, D) f^I(P, D|s_i) dD \geq - \exp^{-\gamma_a W_a - \gamma_a C}; \\ & \sum_{s_i=\{h,l\}} \frac{1}{2} \int \int v_i(s_i, P, D) [f^I(P, D|s_i) - \exp^{-\gamma_a C} f^U(P, D|s_i)] dP dD \leq 0\} \end{aligned} \quad (8.11)$$

$$v_i(s_i, P, D) \geq 0$$

It is obvious that f is a strictly concave function over \mathcal{D}_2 , while \mathcal{D}_2 is convex. Then I can conclude that the local maximum of f over \mathcal{D}_2 is a global solution to this optimization. This implies that the solution in the first-order approach is the global solution to this problem.

Step 2 (case when $p = 1$). The first order condition should be:

$$1 = [\lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)}] \gamma_a v_i(s_i, P, D) \quad (8.12)$$

When $p = 1$, if $\lambda_2 > 0$ I have following cases:

$$\text{when } s_i = h \text{ and } P = -\theta: \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)} = -\infty \quad (8.13)$$

$$\text{when } s_i = h, P = 0 \text{ and } -M - \theta \leq D < -M + \theta: \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)} = -\infty \quad (8.14)$$

$$\text{when } s_i = l \text{ and } P = \theta: \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)} = -\infty \quad (8.15)$$

$$\text{when } s_i = l, P = 0 \text{ and } M - \theta \leq D \leq M + \theta: \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)} = -\infty \quad (8.16)$$

First-order approach will fail here and this indicates that $\lambda_2 = 0$. When $\lambda_2 = 0$, I can conclude that IC will not be binding. I substitute $1 = \lambda_1 \gamma_a v_i(s_i, P, D)$ into PC and get $\frac{1}{\lambda_1 \gamma_a} = \exp^{-\gamma_a W_a - \gamma_a C}$. Then I can get result shown in the proposition.

Step 3: (case when $p < 1$). I denote Lagrange multiplier of PC by λ'_1 and Lagrange multiplier

of IC by λ'_2 . Then I can get

$$1 = [\lambda'_1 + \lambda'_2 - \lambda'_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)}] \gamma_a v_i(s_i, P, D) \quad (8.17)$$

Then I let: $\lambda_1 = \lambda'_1 \gamma_a$, $\lambda_2 = \lambda'_2 \gamma_a$ and $q = \exp^{-\gamma_a C}$ (where $q < 1$)

From Lemma 3.3 and Lemma 3.4, I know that:

(1) When $s_i = h$,

$$\frac{1}{v_i(s_i = h, P = \theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} \quad (8.18)$$

$$\frac{1}{v_i(s_i = h, P = -\theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} \quad (8.19)$$

$$\frac{1}{v_i(s_i = h, P = 0, D)} = \begin{cases} \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} & \text{if } M - \theta \leq D \leq M + \theta \\ \lambda_1 + \lambda_2 - \lambda_2 q & \text{if } -M + \theta \leq D < M - \theta \\ \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (8.20)$$

(2) When $s_i = l$,

$$\frac{1}{v_i(s_i = l, P = \theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} \quad (8.21)$$

$$\frac{1}{v_i(s_i = l, P = -\theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} \quad (8.22)$$

$$\frac{1}{v_i(s_i = l, P = 0, D)} = \begin{cases} \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} & \text{if } M - \theta \leq D \leq M + \theta \\ \lambda_1 + \lambda_2 - \lambda_2 q & \text{if } -M + \theta \leq D < M - \theta \\ \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (8.23)$$

To simplify the analysis, I let $x = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p}$ and $y = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)}$. Then it is clear that I have $\lambda_1 + \lambda_2 - \lambda_2 q = px + (1-p)y$. I substitute $v_i(s_i = l, P = 0, D)$ into PC and IC.

Step 4 (case when $p < 1$). After rearrangement, I have:

$$\begin{aligned} & \frac{1}{x} p \frac{\lambda(2p-1)}{N} + \frac{1}{x} \frac{p\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{px+(1-p)y} \frac{M-\theta}{M} \frac{N-\lambda(2p-1)}{N} \\ & + \frac{1}{y} \frac{(1-p)\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{y} (1-p) \frac{\lambda(2p-1)}{N} = \exp^{-\gamma_a W_a - \gamma_a C} \end{aligned} \quad (8.24)$$

$$\frac{1}{y} = \frac{1}{x} + \frac{\exp^{-\gamma_a W_a} (1 - \exp^{-\gamma_a C})}{(p - \frac{1}{2}) (\frac{\lambda(2p-1)}{N} + \frac{\theta}{M} \frac{N-\lambda(2p-1)}{N})} \quad (8.25)$$

I let $a_1 = \frac{\lambda(2p-1)}{N} + \frac{\theta}{M} \frac{N-\lambda(2p-1)}{N}$, $a_2 = \frac{\exp^{-\gamma_a W_a} (1 - \exp^{-\gamma_a C})}{p-0.5}$, then I have $y = \frac{a_1 x}{a_1 + a_2 x}$. From the above two equations, I have:

$$\frac{a_1}{x} + \frac{1 - a_1}{px + (1-p) \frac{a_1 x}{a_1 + a_2 x}} = \exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a} (1 - \exp^{-\gamma_a C})}{2p - 1} \quad (8.26)$$

I let $g(x) = \frac{a_1}{x} + \frac{1 - a_1}{px + (1-p) \frac{a_1 x}{a_1 + a_2 x}}$. It is obvious that $g(x)$ is a decreasing function of x when $x > 0$. This concludes that there exists unique solution. Let $b = \exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a} (1 - \exp^{-\gamma_a C})}{2p - 1}$.

It is obvious that there is one unique positive solution when $b > 0$. I have:

$$x = \frac{-[ba_1 + (1-p)a_1 a_2 - a_2] + \sqrt{[ba_1 + (1-p)a_1 a_2 - a_2]^2 + 4bpa_2 a_1}}{2bpa_2} \quad \text{and} \quad y = \frac{a_1 x}{a_1 + a_2 x}$$

Step 5: Now I prove that this second-best is equivalent to the agency problem in my model. I need to prove that agents' utility in truth telling is higher than that when they misreport after receiving informative signals, while agents' utility in truth telling in information acquisition is higher than that when they randomly reports without any information. This is to prove that :

$$\begin{aligned} & \frac{1}{x} p \frac{\lambda(2p-1)}{N} + \frac{1}{x} \frac{p\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{px+(1-p)y} \frac{M-\theta}{M} \frac{N-\lambda(2p-1)}{N} \\ & + \frac{1}{y} \frac{(1-p)\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{y} (1-p) \frac{\lambda(2p-1)}{N} \\ & \leq \frac{1}{y} p \frac{\lambda(2p-1)}{N} + \frac{1}{y} \frac{p\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{px+(1-p)y} \frac{M-\theta}{M} \frac{N-\lambda(2p-1)}{N} \\ & + \frac{1}{x} \frac{(1-p)\theta}{M} \frac{N-\lambda(2p-1)}{N} + \frac{1}{x} (1-p) \frac{\lambda(2p-1)}{N} \end{aligned} \quad (8.27)$$

Since $\frac{1}{y} > \frac{1}{x}$ and $p > \frac{1}{2}$, it is easy to show the above inequality always holds.

□

Proof of Corollary 3.1 and Proof of Proposition 3.3. First, because $y = \frac{a_1 x}{a_1 + a_2 x}$, it is obvious that $S_f = \frac{1}{\gamma_a} \log\left(1 + \frac{\exp^{-\gamma_a W_a} (1 - \exp^{-\gamma_a C})}{p - 0.5} \frac{x}{\frac{\lambda(2p-1)}{N} + \frac{\theta}{M} \frac{N - \lambda(2p-1)}{N}}\right)$. I let $z = \frac{x}{\frac{\lambda(2p-1)}{N} + \frac{\theta}{M} \frac{N - \lambda(2p-1)}{N}}$.

For S_f , the signs of $\frac{\partial S_f}{\partial \lambda}$ and $\frac{\partial S_f}{\partial M}$ depend on $\frac{\partial z}{\partial \lambda}$ and $\frac{\partial z}{\partial M}$ respectively. For the equation $\frac{a_1}{x} + \frac{1 - a_1}{px + (1-p)\frac{a_1 x}{a_1 + a_2 x}} = b$, the LHS is decreasing with z and decrease with a_1 . Because RHS is constant with a_1 , then I know that $\frac{\partial z}{\partial a_1} < 0$. Then I have

$$\frac{\partial z}{\partial \lambda} = \frac{\partial z}{\partial a_1} \left(1 - \frac{\theta}{M}\right) \frac{(2p-1)}{N} < 0 \quad (8.28)$$

$$\frac{\partial z}{\partial M} = -\frac{\partial z}{\partial a_1} \frac{\theta}{M^2} \frac{N - \lambda(2p-1)}{N} > 0 \quad (8.29)$$

From equation, it is clear that when $\theta = M$, I have $\frac{\partial z}{\partial \lambda} = 0$.

□

Proof of Lemma 4.1 and Proposition 4.1. When $s_i = h$, I know that $X_i = 1$; When $s_i = l$, $X_i = -1$. So I can calculate expected trading profit as follows:

$$E_p = \text{prob}(s_i = h)E(D - P|s_i = h) + \text{prob}(s_i = l)E(P - P|s_i = l) \quad (8.30)$$

$$= \theta(2p-1) \frac{N - \lambda(2p-1)}{N} \quad (8.31)$$

Then it is obvious that $\frac{\partial B}{\partial \lambda} < 0$.

□

Proof of Proposition 4.2. Let $K = \frac{1}{2}a_1 \log(x) + \frac{1}{2}a_1 \log(y) + (1 - a_1) \log[px + (1 - p)y]$, I can get $B = \theta(2p-1) \frac{N - \lambda(2p-1)}{N} - K$. Then $\frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \frac{\partial K}{\partial a_1} \frac{\partial a_1}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \frac{\partial K}{\partial a_1} \left(1 - \frac{\theta}{M}\right) \frac{\theta(2p-1)}{N}$.

Step 1 (M is small enough) I know that for $M \geq \theta$, $\lim_{M \rightarrow \theta} \frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} < 0$. Because $\frac{\partial B}{\partial \lambda}$ is a continuous function, this implies that there exists a cutoff M^c satisfying $M < M^c$, $\frac{\partial B}{\partial \lambda} < 0$.

Step 2 (M is large enough) I know that for $M = +\infty$, $\lim_{\lambda \rightarrow 0} \frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \lim_{a_1 \rightarrow 0} \frac{\partial K}{\partial a_1} \frac{\theta(2p-1)}{N}$

$$\begin{aligned} \lim_{a_1 \rightarrow 0} \frac{\partial K}{\partial a_1} &= \lim_{a_1 \rightarrow 0} \left\{ \frac{1}{2} \log(x) + \frac{1}{2} \log(y) - \log[px + (1-p)y] \right. \\ &\quad \left. + \frac{1}{2} a_1 \frac{1}{x} \frac{\partial x}{\partial a_1} + \frac{1}{2} a_1 \frac{1}{y} \frac{\partial y}{\partial a_1} + (1-a_1) \frac{1}{px+(1-p)y} \left[p \frac{\partial x}{\partial a_1} + (1-p) \frac{\partial y}{\partial a_1} \right] \right\} \end{aligned} \quad (8.32)$$

Because $y = \frac{a_1 x}{a_1 + a_2 x}$, I know that $\lim_{a_1 \rightarrow 0} x = \frac{1}{bp}$, $\lim_{a_1 \rightarrow 0} y = 0$, $\lim_{a_1 \rightarrow 0} \frac{\partial x}{\partial a_1} = \text{finite}$

$$\lim_{a_1 \rightarrow 0} \frac{\partial y}{\partial a_1} = \lim_{a_1 \rightarrow 0} \left(\frac{x}{a_1 + a_2 x} - \frac{a_1 x}{(a_1 + a_2 x)^2} + \frac{a_1}{a_1 + a_2 x} \frac{\partial x}{\partial a_1} - \frac{a_1 x}{(a_1 + a_2 x)^2} \frac{\partial x}{\partial a_1} \right) = \text{finite}.$$

Thus I can obtain $\lim_{a_1 \rightarrow 0} \frac{\partial K}{\partial a_1} = -\infty$. Then I can conclude that when M is large enough and λ small enough, $\frac{\partial B}{\partial \lambda} > 0$. This concludes my proof. □

Proof of Proposition 4.3. Step 1. When $\lambda = 0$, I know that $B_{ap}(0) = \theta(2p - 1) + \log b$.

Step 2. If $B_{ap}(0) < 0$ and $\max_{\lambda < \lambda_{fb}} B(\lambda) < 0$, the unique equilibrium is no information acquisition equilibrium.

Step 3. If $B_{ap}(0) < 0$ and $\max_{\lambda < \lambda_{fb}} B_{ap}(\lambda) > 0$, I prove that there exist three equilibria. The first one is non-information acquisition equilibrium because $B_{ap}(0) < 0$ and $\frac{\partial B_{ap}(0)}{\partial \lambda} > 0$. I let λ^* be the solution to $\max_{\lambda < \lambda_{fb}} B(\lambda)$. Then there exists one solution in $(0, \lambda^*)$. Moreover, because $B_{ap} < B_{fb}$, I know that $B(\lambda_{fb}) < 0$, thus exists one solution in $(\lambda^*, \lambda_{fb})$. Step 4. If $B_{ap}(0) > 0$, because $B_{ap}(\lambda_{fb}) < 0$, then there exists at least one positive solution in $(0, \lambda_{fb})$ □

Proof of Proposition 4.4. This result is direct because I know that $B_{ap} < B_{fb}$. □

Proof of Lemma 5.1. I know that $E(V|X) = \theta p(V = \theta|X) - \theta p(V = -\theta|X)$. Then because $p(V = \theta|X) = \frac{p(V=\theta, X)}{P(X)}$, then I have:

$$p(V = \theta, X) = \begin{cases} \frac{p}{2} & \text{if } N - \lambda < X \leq N + \lambda \\ \frac{1}{2} & \text{if } -N + \lambda \leq X \leq N - \lambda \\ \frac{1-p}{2} & \text{if } -N - \lambda \leq X < -N + \lambda \end{cases} \quad (8.33)$$

$$p(V = -\theta, X) = \begin{cases} \frac{1-p}{2} & \text{if } N - \lambda < X \leq N + \lambda \\ \frac{1}{2} & \text{if } -N + \lambda \leq X \leq N - \lambda \\ \frac{p}{2} & \text{if } -N - \lambda \leq X < -N + \lambda \end{cases} \quad (8.34)$$

Then I have:

$$p(V = \theta|X) = \begin{cases} p & \text{if } N - \lambda < X \leq N + \lambda \\ \frac{1}{2} & \text{if } -N + \lambda \leq X \leq N - \lambda \\ 1 - p & \text{if } -N - \lambda \leq X < -N + \lambda \end{cases} \quad (8.35)$$

Thus, I conclude the proof. □

Proof of Lemma 5.2. The proof is shown as follows:

$$\begin{aligned} E_p^c &= p(s_c = h)E(V - P|s_c = h) + p(s_c = l)E(P - V|s_c = l) \\ &= (2p - 1)\theta\frac{N-\lambda}{N} \end{aligned} \quad (8.36)$$

□

Proof of Lemma 5.3. Because

$$\begin{aligned} E(P|s_j = h) &= (2p - 1)\theta * \text{prob}(s_c = h|s_j = h)\frac{\lambda}{N} \\ &\quad - (2p - 1)\theta * \text{prob}(s_c = l|s_j = h)\frac{\lambda}{N} \\ &= \theta\frac{\lambda}{N}(2p - 1)^3 \end{aligned} \quad (8.37)$$

Following the same logic, I can get $E(P|s_j = l) = -\theta\frac{\lambda}{N}(2p - 1)^3$. Then I calculate expected trading profit of investors who acquire private signal as:

$$E_p^I = (2p - 1)\theta\frac{N-(2p-1)^2\lambda}{N} \quad (8.38)$$

□

Proof of Proposition 5.1. Step 1. I prove that no herding equilibrium occurs in the economy without agency problem. Following the analysis of optimal contract, I know that the payments $\pi = W_a + C$. Then when $\lambda > 0$ and $\mu = 0$ in the herding equilibrium, I will have $E_p^c - W_a - C = 0 > E_p^I - W_a - C$. Because this is impossible, I conclude that herding equilibrium will not occur.

Step 2. I calculate the optimal payment scheme provided by principals who acquire s_c in the

herding equilibrium. Because $f^I(P = -(2p - 1)\theta, D|s_c = h) = f^I(P = (2p - 1)\theta, D|s_c = l) = 0$, the optimal scheme following the proof of Proposition 2.2, I know that

$$\pi(P = -(2p - 1)\theta, s_c = h, D) = \pi(P = (2p - 1)\theta, s_c = l, D) = -\infty \quad (8.39)$$

Otherwise, $\pi = W_a + C$.

Step 3. I calculate the optimal payment scheme provided by principals who acquire s_i in the herding equilibrium in this step. Before calculation of optimal payment scheme, I calculate pdf of P and D conditional on s_i . To simply the analysis, I only consider the case when M goes to infinity. When M goes to infinity, I know that pdf of P and D conditional on s_i is equivalent to pdf of P conditional on s_i . Then I have the following cases if agents acquire information:

$$prob^I(P|s_i = h) = \begin{cases} \frac{\lambda[p^2+(1-p)^2]}{N} & \text{if } P = (2p - 1)\theta \\ \frac{N-\lambda}{N} & \text{if } P = 0 \\ \frac{2\lambda p(1-p)}{N} & \text{if } P = -(2p - 1)\theta \end{cases} \quad (8.40)$$

$$prob^I(P|s_i = l) = \begin{cases} \frac{2\lambda p(1-p)}{N} & \text{if } P = (2p - 1)\theta \\ \frac{N-\lambda}{N} & \text{if } P = 0 \\ \frac{\lambda[p^2+(1-p)^2]}{N} & \text{if } P = -(2p - 1)\theta \end{cases} \quad (8.41)$$

Then I have the following cases if agents do not acquire information:

$$prob^U(P) = \begin{cases} \frac{\lambda}{2N} & \text{if } P = (2p - 1)\theta \\ \frac{N-\lambda}{N} & \text{if } P = 0 \\ \frac{\lambda}{2N} & \text{if } P = -(2p - 1)\theta \end{cases} \quad (8.42)$$

Following the first-order approach in above proof, I know that

$$\frac{1}{v(s_i = h, P = (2p - 1)\theta)} = \frac{1}{v(s_i = l, P = -(2p - 1)\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2[p^2 + (1 - p)^2]} \quad (8.43)$$

$$\frac{1}{v(s_i = h, P = -(2p - 1)\theta)} = \frac{1}{v(s_i = l, P = (2p - 1)\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{4p(1 - p)} \quad (8.44)$$

$$\frac{1}{v(s_i = h, P = 0)} = \frac{1}{v(s_i = l, P = 0)} = \lambda_1 + \lambda_2 - \lambda_2 \exp(-\gamma_a C) \quad (8.45)$$

I let $p_1 = p^2 + (1 - p)^2$, $x_1 = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p_1}$, $y_1 = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1 - p_1)}$, $a_{11} = \frac{\lambda}{N}$, $a_{21} = \frac{\exp(-\gamma_a W_a)(1 - \exp^{-\gamma_a C})}{p_1 - 0.5}$ and $b_1 = \exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1 - \exp^{-\gamma_a C})}{2p_1 - 1}$. When $b_1 < 0$, the solution to solve the optimal contract does not exist.

Step 4. If $\exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1 - \exp^{-\gamma_a C})}{2p_1 - 1} < 0$ and $E_p^c = (2p - 1)\theta \frac{N - \lambda}{N} - W_a - C > 0$ for some positive λ , I can get the results in the proposition. □

Proof of Proposition 5.2. Step 1. In the first-best case, it is clear that the optimal payment scheme is constant. That is $\pi = W_a + C$. Because the net benefit of information acquisition for home investors is $(1 - p_h)(2p - 1)\theta \frac{N - \eta}{N} - W_a - C$ and the net benefit of information acquisition for foreign investors is $(2p - 1)\theta \frac{N - \eta}{N} - W_a - C$. If $\lambda > 0$, this indicates that

$$(1 - p_h)(2p - 1)\theta \frac{N - \eta}{N} - W_a - C = 0 \quad (8.46)$$

Moreover, this indicates that $(2p - 1)\theta \frac{N - \eta}{N} - W_a - C > 0$. Thus, it implies that μ should be infinity. This is impossible because price will be fully revealing when μ goes to infinity. Then trading profit will become zero and this violate the assumption that $(1 - p_h)(2p - 1)\theta \frac{N - \eta}{N} - W_a - C = 0$. Therefore, I can conclude that $\lambda = 0$. This implies that neither weak herding equilibrium nor strong herding equilibrium occur in the first-best case.

Step 2. I only prove that strong herding equilibrium occurs under some condition in the economy

with agency problem. Particularly, I try to find the condition under which $\lambda = 1$ and $\mu = 0$. For the foreign investors, the approach to solve the optimal contract is similar to the proof of Proposition 3.2. I only replace $\lambda(2p - 1)$ with η in the proof. When both of M and N go to infinity, I know that net benefit of information acquisition for foreign investors is $B_{f,ap}(0) = (2p - 1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1 - \exp^{-\gamma_a C})}{2p - 1}]$.

Step 3. I take the following steps to solve the optimal contract for the home investors. The conditional pdf of $s_{h,i}$ when s_i is informative is shown as follows:

$$\text{prob}^I(s_{h,i}|s_i = h) = \begin{cases} p_h p & \text{if } s_{h,i} = \theta \\ 1 - p_h & \text{if } s_{h,i} = \emptyset \\ p_h(1 - p) & \text{if } s_{h,i} = -\theta \end{cases} \quad (8.47)$$

$$\text{prob}^I(s_{h,i}|s_i = l) = \begin{cases} p_h(1 - p) & \text{if } s_{h,i} = \theta \\ 1 - p_h & \text{if } s_{h,i} = \emptyset \\ p_h p & \text{if } s_{h,i} = -\theta \end{cases} \quad (8.48)$$

The conditional pdf of $s_{h,i}$ when s_i is uninformative is shown as follows:

$$\text{prob}^U(s_{h,i}) = \begin{cases} \frac{p_h}{2} & \text{if } s_{h,i} = \theta \\ \frac{1}{2} & \text{if } s_{h,i} = \emptyset \\ \frac{p_h}{2} & \text{if } s_{h,i} = -\theta \end{cases} \quad (8.49)$$

Following the first-order approach in proof of Proposition, I know that

$$\frac{1}{v(s_i = h, s_{h,i} = \theta)} = \frac{1}{v(s_i = l, s_{h,i} = -\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p} \quad (8.50)$$

$$\frac{1}{v(s_i = h, s_{h,i} = -\theta)} = \frac{1}{v(s_i = l, s_{h,i} = \theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1 - p)} \quad (8.51)$$

$$\frac{1}{v(s_i = h, s_{h,i} = \emptyset)} = \frac{1}{v(s_i = l, s_{h,i} = \emptyset)} = \lambda_1 + \lambda_2 - \lambda_2 \exp(-\gamma_a C) \quad (8.52)$$

This is similar to the proof of Proposition 3.2, I let $x = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p}$, $y = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1-p)}$, $a_{12} = p_h$, $a_{22} = \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{p-0.5}$ and $b_2 = \exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{2p-1}$.

Following proof of Proposition 4.2, I know that the net benefit of informaiton acquisition for home investors is $B_{h,ap}(p_h) = (1 - p_h)(2p - 1)\theta - K$ (where $K = \frac{1}{2}p_h \log(x) + \frac{1}{2}p_h \log(y) + (1 - p_h) \log[px + (1 - p)y]$).

When $p_h = 0$, I know that $B_{h,ap}(0) = (2p - 1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}]$. Then I denote the derivative of $B_{h,ap}(p_h)$ with p_h by $\frac{\partial B_{h,ap}}{\partial p_h}$. Then I know that for very small positive ϵ , I know that $B_{h,ap}(\epsilon) = (2p - 1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] + \epsilon * \frac{\partial B_{h,ap}}{\partial p_h}$. Because $\frac{\partial B_{h,ap}(0)}{\partial p_h}$ is infinity and θ is not in the function $B_{h,ap}$, there exists small enough θ satisfying $(2p - 1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] = -\epsilon$. In this case, I know that $B_{h,ap}(\epsilon) > 0$ and $B_{h,ap}(0) < 0$. I let $\theta_1 = \frac{-\log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] - \epsilon}{2p-1}$ and $\theta_2 = \frac{-\log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}]}{2p-1}$. This implies foreign investors never have incentive to acquire information, but home investors have incentive to acquire informtion under the condition: $\theta_1 < \theta < \theta_2$.

□

Proof of Proposition 6.1. Following the similar process in the proof of Proposition 3.2 and Proposition 4.2, I know that

$$\pi_i(s_i = h, P = \theta, D) = \pi_i(s_i = l, P = -\theta, D) = \pi_1 \quad (8.53)$$

$$\pi_i(s_i = h, P = -\theta, D) = \pi_i(s_i = l, P = \theta, D) = \pi_3 \quad (8.54)$$

$$\pi_i(s_i = h, P = 0, D) = \begin{cases} \pi_1 & \text{if } M - \theta \leq D \leq M + \theta \\ \pi_2 & \text{if } -M + \theta \leq D < M - \theta \\ \pi_3 & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (8.55)$$

$$\pi_i(s_i = l, P = 0, D) = \begin{cases} \pi_3 & \text{if } M - \theta \leq D \leq M + \theta \\ \pi_2 & \text{if } -M + \theta \leq D < M - \theta \\ \pi_1 & \text{if } -M - \theta \leq D < -M + \theta \end{cases} \quad (8.56)$$

As I know that $\pi_1 > \pi_2 > \pi_3$ if all PC and IC in the second-best are binding as shown below. Then it is similar as proof of Proposition 3.2, I know that this contract can satisfies ex ante IC and ex post IC. Then I let $U_1 = U(\pi_1), U_2 = U(\pi_2)$ and $U_3 = U(\pi_3)$. Particularly, the PC and IC follows:

$$a_1 p U_1 + (1 - a_1) U_2 + a_1 (1 - p) U_3 = U(W_a) + C \quad (8.57)$$

$$U_1 = U_3 + \frac{C}{a_1(p - 0.5)} \quad (8.58)$$

It is clear that $U_2 = U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1} U_3$.

Now I prove information acquisition complementarity is robust when M is infinite and λ is small enough for different γ as follows.

Now the principals' optimization problem becomes to minimize

$$\min_{\pi_1, \pi_2, \pi_3} a_1 p \pi_1 + (1 - a_1) \pi_2 + a_1 (1 - p) \pi_3 \quad (8.59)$$

This problem can be transferred to:

$$\min_{U_3} G(U_3) \quad (8.60)$$

where $G(U_3) = a_1 p U^{-1}(U_3 + \frac{C}{a_1(p-0.5)}) + (1 - a_1) U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1} U_3) + a_1 (1 - p) U^{-1}(U_3)$.

Since a_1 is a linear function of λ . The first-order condition with λ is equivalent to the first-order condition with a_1 , I have $\frac{\partial G(U_3)}{\partial U_3} = 0$. It is easy to check that Assumption *ass:crra* can ensure there exists interior solution to the contracting problem. Particularly, I have the following three cases:

Case 1: If $0 < \gamma < 1$, I know that U_3 should satisfy: $U_3 > 0$ and $U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3 > 0$.

Case 2: If $\gamma > 1$, I know that U_3 should satisfy: $U_3 + \frac{C}{a_1(p-0.5)} < 0, U_3 < 0$ and $\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3 < 0$.

Case 3: If $\gamma = 1$ and $K = 0$, then I know that $U(W) \cong \ln(W)$ (where \cong represents liner transformation).

It is obvious that $U^-(x) = [(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}} - K] \frac{\gamma}{A}$. I let the solution to this problem to be U_3^* . Then the minimum value of $G(U_3)$ is $G(a_1, U_3^*)$. Now I have effect of a_1 on $G(a_1, U_3^*)$ is:

$$\begin{aligned} \frac{\partial G(a_1, U_3^*)}{\partial a_1} &= pU^{-1}(U_3 + \frac{C}{a_1(p-0.5)}) - U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3) + (1-p)U^{-1}(U_3) \\ &+ \frac{\partial G(a_1, U_3^*)}{\partial U_3^*} \frac{\partial G(U_3^*)}{\partial a_1} - a_1 p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1^2(p-0.5)} \\ &+ (1-a_1) \frac{\partial [U^{-1}(\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)]}{\partial (\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)} (-\frac{1}{(1-a_1)^2}U_3) \end{aligned} \quad (8.61)$$

For any cases, I know that $\frac{\partial G(a_1, U_3^*)}{\partial U_3^*} \frac{\partial G(U_3^*)}{\partial a_1} = 0$. I show information acquisition is complementary case by case.

Case 1 ($\gamma < 1$): I know that: $(1-a_1) \frac{\partial [U^{-1}(\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)]}{\partial (\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)} (-\frac{1}{(1-a_1)^2}U_3) < 0$ because $U_3 < 0$ and $-U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3) + (1-p)U^{-1}(U_3) < 0$ and then $pU^{-1}(U_3 + \frac{C}{a_1(p-0.5)}) - p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1(p-0.5)} = p \frac{\gamma}{A} (\frac{1-\gamma}{\gamma})^{\frac{1}{1-\gamma}} (U_3 + \frac{C}{a_1(p-0.5)})^{\frac{1}{1-\gamma}-1} [U_3 - \frac{\gamma}{1-\gamma} \frac{C}{a_1(p-0.5)}] - p \frac{\gamma}{A} K < p \frac{\gamma}{A} (\frac{1-\gamma}{\gamma})^{\frac{1}{1-\gamma}} (\frac{C}{a_1(p-0.5)})^{\frac{1}{1-\gamma}-1} [U_3 - \frac{\gamma}{1-\gamma} \frac{C}{a_1(p-0.5)}] - p \frac{\gamma}{A} K$

Because $U_3 > 0$ and $U_3 < U(W_a) + C$, I know that $\lim_{a_1 \rightarrow 0} \frac{\partial G(a_1, U_3^*)}{\partial a_1} = -\infty$. For the net benefit of information acquisition B , I have $\lim_{a_1 \rightarrow 0} \frac{\partial B}{\partial a_1} = \infty$ for large enough M and small enough λ .

Case 2 ($\gamma > 1$) From the proof of , I know that $p(\frac{A\pi_1}{\gamma} + K)^\gamma + (1-p)(\frac{A\pi_3}{\gamma} + K)^\gamma = (\frac{A\pi_2}{\gamma} + K)^\gamma$.

Let $\pi'_i = (\frac{A\pi_i}{\gamma} + K)^\gamma$, I know that $\pi_i = [(\pi'_i)^{\frac{1}{\gamma}} - K] \frac{\gamma}{A}$, which is a concave function of π'_i . Then

I can have

$$pU^{-1}(U_3 + \frac{C}{a_1(p-0.5)}) - U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3) + (1-p)U^{-1}(U_3) = p \pi_1 + (1-p)\pi_3 - \pi_2 < 0$$

Because $U^-(x) = [(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}} - K] \frac{\gamma}{A}$, I know that $\frac{\partial [U^{-1}(x)]}{\partial x} = \frac{1}{A} (\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}-1}$ and $\frac{\partial^2 [U^{-1}(x)]}{\partial x^2} > 0$

Then from FOC $\frac{\partial G(U_3)}{\partial a_1} = 0$, I know that

$$a_1 p \frac{\partial[U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial(U_3 + \frac{C}{a_1(p-0.5)})} + (1-a_1) \frac{\partial[U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)]}{\partial(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)} + a_1(1-p) \frac{\partial[U^{-1}(U_3)]}{\partial(U_3)} = 0 \quad (8.62)$$

Thus, I have

$$p \frac{\partial[U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial(U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1(p-0.5)} + \frac{\partial[U^{-1}(\frac{U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)]}{\partial(\frac{U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)} \frac{1}{1-a_1} U_3 = p \frac{\partial[U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial(U_3 + \frac{C}{a_1(p-0.5)})} [\frac{C}{a_1(p-0.5)} - \frac{a_1 U_3}{(1-a_1)^2}] - \frac{a_1 U_3}{(1-a_1)^2} (1-p) \frac{\partial[U^{-1}(U_3)]}{\partial(U_3)}$$

Because $[U(W_a) - \frac{C}{2p-1}] \frac{1-a_1}{a_1} < U_3 < -\frac{C}{a_1(p-0.5)}$, I know that $\lim_{a_1 \rightarrow 0} \frac{C}{a_1(p-0.5)} - \frac{a_1 U_3}{(1-a_1)^2} = \infty$.

For $\frac{\partial[U^{-1}(x)]}{\partial x} = \frac{1}{A} (\frac{1-\gamma}{\gamma} x)^{\frac{1}{1-\gamma}-1}$, I know that $\lim_{a_1 \rightarrow 0} \frac{\partial[U^{-1}(U_3)]}{\partial(U_3)} = 0$ because $U_3 \rightarrow -\infty$.

Because $U_3 + \frac{C}{a_1(p-0.5)} > U(W_a) + C$, I have $\frac{\partial[U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial(U_3 + \frac{C}{a_1(p-0.5)})} > \frac{\partial[U^{-1}(x)]}{\partial x} |_{x=U(W_a)+C}$

Thus, I can conclude that

$$\lim_{a_1 \rightarrow 0} p \frac{\partial[U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial(U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1(p-0.5)} + \frac{\partial[U^{-1}(\frac{U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)]}{\partial(\frac{U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)} \frac{1}{1-a_1} U_3 = \infty$$

Therefore, it is easy to show that $\lim_{a_1 \rightarrow 0} \frac{\partial G(a_1, U_3^*)}{\partial a_1} = -\infty$ and I conclude that $\lim_{a_1 \rightarrow 0} \frac{\partial B}{\partial a_1} = \infty$

for large enough M and small enough λ .

Case 3 for $U(W) = \ln(W)$, I directly calculate

$$\frac{\partial G(U_3)}{\partial a_1} = a_1 p \exp(U_3 + \frac{C}{a_1(p-0.5)}) + a_1(1-p) \exp(U_3) - a_1 \exp[\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3] = 0$$

$$\text{Thus } \exp(\frac{1}{1-a_1}U_3) [p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)] = \exp[\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1}]$$

$$\text{I have } U_3 = U(W_a) - \frac{C}{2p-1} - (1-a_1) \log[p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)]$$

$$\text{Then } \frac{G(a_1, U_3^*)}{\partial a_1} = \exp\{U(W_a) - \frac{C}{2p-1} + a_1 \log[p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)]\}$$

$$\text{Then I let } g(a_1) = a_1 \log[p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)]$$

$$\text{I know that } \lim_{a_1 \rightarrow 0} g(a_1) = \frac{a_1^2}{1} \frac{p \exp(\frac{C}{a_1(p-0.5)})}{p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)} \frac{C}{a_1^2(p-0.5)} = \frac{C}{(p-0.5)}$$

$$\text{Then I know that } \frac{\partial g}{\partial a_1} = \frac{1}{a_1} [a_1 \log[p \exp(\frac{C}{a_1(p-0.5)}) + (1-p)] - \frac{C}{(p-0.5)}]$$

$$\text{Thus I have } \lim_{a_1 \rightarrow 0} \frac{\partial g}{\partial a_1} = -\infty$$

Then I can conclude that $\lim_{a_1 \rightarrow 0} \frac{\partial G(a_1, U_3^*)}{\partial a_1} = -\infty$. For the net benefit of information acquisition

B , I have $\lim_{a_1 \rightarrow 0} \frac{\partial B}{\partial a_1} = \infty$ for large enough M and small enough λ .

□

Proof of Lemma 6.1. For principal i , his expected trading profit when he submits 1 is

$(2p_K(s_i, P) - 1)\theta - P$, while his expected trading profit when he submits -1 is $P - (2p_K(s_i, P) - 1)\theta$. Because $p_K(s_i, P)$ is increasing with s_i , principal i is indifferent between submitting 1 and -1 when $(2p_K(s_i, P) - 1)\theta - P = 0$. This concludes the proof. \square

Proof of Proposition 6.2. First, we have the condition as follows:

$$p_I(X_I, P) = p_U(X_U, P) = \frac{P + \theta}{2\theta} = \frac{1}{1 + \frac{f_i}{f_h} \frac{1}{L_I}}. \quad (8.63)$$

Then we have $\log\left(\frac{P+\theta}{\theta-P}\right) = \log\left(\frac{f_h}{f_i}\right) + \log(L_I)$. Denote $B(P) = \frac{\partial L_U^{-1}(L_I(X_I))}{\partial P}$. From the expressions of f_h and f_i , we have

$$\begin{aligned} \log\left(\frac{f_h}{f_i}\right) &= \log\left(\frac{\lambda(1+b+e)\phi(X_I-\theta)X'_I + \lambda(1-b-e)\phi(X_I+\theta)X'_I + (1-\lambda)(1+b)\phi(X_U-\theta)BX'_I + (1-\lambda)(1-b)\phi(X_U+\theta)BX'_I}{\lambda(1+b+e)\phi(X_I+\theta)X'_I + \lambda(1-b-e)\phi(X_I-\theta)X'_I + (1-\lambda)(1+b)\phi(X_U+\theta)BX'_I + (1-\lambda)(1-b)\phi(X_U-\theta)BX'_I}\right) \\ &= \log\left(\frac{[(1+b+e)\phi(X_I-\theta) + (1-b-e)\phi(X_I+\theta)][\lambda X'_I + \frac{(1-\lambda)(1+b)\phi(X_U-\theta)B + (1-\lambda)(1-b)\phi(X_U+\theta)BX'_I}{(1+b+e)\phi(X_I-\theta) + (1-b-e)\phi(X_I+\theta)}]}{[(1+b+e)\phi(X_I+\theta) + (1-b-e)\phi(X_I-\theta)][\lambda X'_I + \frac{(1-\lambda)(1+b)\phi(X_U+\theta)B + (1-\lambda)(1-b)\phi(X_U-\theta)BX'_I}{(1+b+e)\phi(X_I+\theta) + (1-b-e)\phi(X_I-\theta)}]}\right) \end{aligned} \quad (8.64)$$

Because $L_I(X_I) = L_U(X_U)$, we have

$$\begin{aligned} &\frac{(1-\lambda)(1+b)\phi(X_U-\theta)B + (1-\lambda)(1-b)\phi(X_U+\theta)}{(1+b+e)\phi(X_I-\theta) + (1-b-e)\phi(X_I+\theta)} \\ &= \frac{(1-\lambda)(1+b)\phi(X_U+\theta)B + (1-\lambda)(1-b)\phi(X_U-\theta)}{(1+b+e)\phi(X_I+\theta) + (1-b-e)\phi(X_I-\theta)} \end{aligned} \quad (8.65)$$

Thus, $\log\left(\frac{f_h}{f_i}\right) = \log(L_I)$ \square