Bank Capital and Aggregate Credit

Nataliya Klimenko* Sebastian Pfeil † Jean-Charles Rochet ‡

June 15, 2015

Preliminary and incomplete, please do not cite

Abstract

This paper seeks to explain the role of bank capital in fluctuations of lending and output. We build a continuous time model of an economy in which commercial banks finance their loans by deposits and equity, while facing issuance costs when they raise new equity. The dynamics of the loan rate and the volume of lending in the economy are driven by aggregate bank capitalization. The model has a unique Markovian competitive equilibrium that can be solved in closed form. We use our model to study the effect of minimum capital requirements and find that, *in the short run*, a higher minimum capital ratio simultaneously induces banks to hold more capital and reduces aggregate credit, which creates a trade-off between financial stability and output. However, *in the long run*, this trade-off disappears: there is a range of regulatory capital ratios that yield both financial stability and no reductions in lending. However, excessively high capital requirements lead to serious credit crunches.

Keywords: macro-model with a banking sector, bank capital, capital requirements
JEL: E21, E32, F44, G21, G28

Acknowledgements: we thank Philippe Bacchetta, Bruno Biais, Markus Brunnermeier, Catherine Casamatta, Julien Daubanes, Jean-Paul Décamps, John Geanakoplos, Hans Gersbach, Hendrik Hakenes, Christian Hellwig, Eric Jondeau, Michael Magill, Semyon Malamud, Loriano Mancini, Erwan Morellec, Alistair Milne, Henri Pagès, Bruno Parigi, Greg Phelan, Martine Quinzii, Yuliy Sannikov, Jean Tirole and Stéphane Villeneuve, and seminar participants at ETH Zürich, Princeton, Toulouse School of Economics, EPF Lausanne, the University of Nanterre, Yale and Banque de France.

*University of Zürich. E-mail: nataliya.klimenko@bf.uzh.ch.
†University of Bonn. E-mail: pfeil@uni-bonn.de.
‡University of Zürich, SFI and TSE-IDEI. E-mail: jean-charles.rochet@bf.uzh.ch.

1 Introduction

There is an ongoing debate among scholars and practitioners about the "right" level of bank capital. While proponents of higher capital ratios emphasize the stabilizing effect of bank capital (see e.g. Admati et al. (2010), Admati and Hellwig (2013)), others argue that high leverage of banks is a necessary consequence of their intrinsic role as creators of information insensitive, liquid debt (e.g. deposits).[1] The current paper brings together both aspects in a dynamic general equilibrium model where bank capital plays the role of a loss absorbing buffer that facilitates the creation of liquid, risk-free claims while providing (risky) loans to the real sector.[2]

We consider an economy where firms borrow from banks that are financed by short term deposits and equity. The aggregate supply of bank loans is confronted with the firms' demand for credit, which is decreasing in the nominal loan rate. The firms' default probability depends on undiversifiable aggregate shocks, which ultimately translates into gains or losses for banks. Banks can continuously adjust their volumes of lending to firms. They also decide when to distribute dividends and when to issue new equity. Equity issuance is subject to deadweight costs, which constitutes the main financial friction in our economy.[3]

In a set-up without financial frictions (i.e., no issuance costs for bank equity) the equilibrium volume of lending and the nominal loan rate are constant. Furthermore, dividend payment and equity issuance policies are trivial in this case: Banks immediately distribute all profits as dividends and issue new shares to offset losses and honor obligations to depositors. This implies that, in a frictionless world, there is no need to build up a capital buffer: all loans are entirely financed by deposits.

In the model with financing frictions, banks' dividend and equity issuance strategies become more interesting. In the unique competitive equilibrium, aggregate bank equity, which serves as the single state variable, follows a Markov diffusion process reflected at two boundaries. Banks issue new shares at the lower boundary where book equity is

[1]The perception of financial intermediaries as creators of liquidity ("inside money") has a long tradition within the financial intermediation literature (see, e.g. Diamond and Dybvig (1983), Diamond and Rajan (2001), Gorton (2010), Gorton and Pennachi (1990), Holmström and Tirole (1998, 2011), DeAngelo and Stulz (2014)).

[2]As we abstract from incentive effects of bank capital ("skin in the game"), our model captures best the features of more traditional commercial banks whose business model makes them less prone to risk-shifting than investment banks. The incentive effects of bank capital in a setting allowing for risk-shifting are analyzed for instance in Martinez-Miera and Suarez (2014), De Nicolò et al. (2014), and Van den Heuvel (2008).

[3]Empirical studies report sizable costs of seasoned equity offerings (see e.g. Lee, Lochhead, Ritter, and Zhao (1996), Hennessy and Whited (2007)). Here we follow the literature (see e.g. Décamps et al. (2011) or Bolton et al. (2011)) by assuming that issuing new equity entails a deadweight cost proportional to the size of the issuance.

depleted. When the book value of equity reaches its upper boundary, any further earnings are paid out to shareholders as dividends. Between the two boundaries, the level of equity changes only due to retained earnings or absorbed losses. That is, banks retain earnings in order to increase the loss-absorbing equity buffer and thereby reduce the frequency of costly recapitalizations. The level of capitalization in turns determines the lending and deposit-taking capacity of the banking sector.

Our model generates a strictly positive (except at the dividend payout boundary) risk-adjusted spread for bank loans that is decreasing in the level of aggregate bank equity. To get an intuition for this result, note that loss absorbing equity is most valuable when it is scarce and least valuable when it is abundant. Therefore, the marginal (or market-to-book) value of equity decreases in aggregate bank capitalization. Negative shocks that reduce banks' equity buffers are thus amplified by a simultaneous increase in the market-to-book value, whereas positive shocks are moderated by a simultaneous decrease in the market-to-book value. As a result, banks will lend to firms only if the loan rate incorporates an appropriate premium.

The loan rate dynamics in our model can be obtained in closed form, which enables us to study the long-run behavior of the economy by looking at the properties of the ergodic density function which characterizes the relative time the economy spends in each feasible state in the long run. Our analysis shows that the long-run behavior of the economy is mainly driven by the (endogenous) volatility. In particular, the economy spends most of its time in states with low endogenous volatility. For high recapitalization costs and a low elasticity of demand for bank loans, this can lead to severe credit crunches with persistently high loan rates, low volumes of lending and low levels of bank equity. The occurrence of credit crunches is caused by the following simple mechanism: Assume that a series of adverse shocks has eroded bank capital. Since banks require a larger loan rate when capital is low, this drives down firms' credit demand. As a result, banks' exposure to macro shocks is reduced and thus also the endogenous volatility of aggregate equity, so that the economy may spend a long time in the credit crunch.

We then use our framework to study the impact of minimum capital requirements on lending. Our analysis detects two *short-run* effects: First, imposing a higher capital ratio leads to an increase in the loan rate, thereby, reducing lending. Importantly, this effect is present even when the regulatory constraint is not binding, because banks anticipate that capital requirements might be binding in the future and require a higher lending premium by precautionary motives. Second, bank capitalization increases with the level of minimum capital requirements, i.e., faced with a higher capital ratio, bank recapitalize before their capital gets depleted and build higher capital buffers.

The interplay between these two effects determines the *long-run* effect of minimum capital requirements. For moderate levels of minimum capital ratios such that the regulatory constraint is slack when banks are sufficiently capitalized, the economy spends more time in the states with higher bank capitalization and lower loan rates, so that imposing a higher capital ratio may actually reduce the average loan rate in the long run. This result demonstrates that, contrary to the common intuition, higher capital requirements (provided that their level remains moderate) *do not reduce* but may even expand lending in the long run. By contrast, for high levels of capital ratios, the regulatory constraint is always binding regardless of the level of bank capitalization, and the average loan rate increases with the minimum capital ratio leading to the reductions of lending.

Finally, we consider the long-run impact of minimum capital requirements on social welfare. In our model, the latter is computed as the expected value of the aggregate consumption flows that take the form of dividend payments, recapitalization expenditures and firms' profits.[4] Our numerical analysis shows that increasing the minimum capital ratio above the level that will induce banks to operate with a permanently binding capital requirement is inefficient and there exists an "optimal" level of capital requirement maximizing social welfare.

Related literature. From a technical perspective, our paper is closely related to the recent continuous-time macroeconomic models with financial frictions that study the formation of asset prices in a dynamic endowment economy (see, e.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2013)).[5] Most closely related is the study by Phelan (2014) who explicitly introduces a banking sector in a continuous-time general equilibrium model. In Phelan's paper banks invest in productive capital (land) and depositors extract additional utility from holding bank deposits. While the latter feature is also present in our model, we focus at modelling a lending channel and study the role of bank capital in the loan rate formation.

As in the above-mentioned papers, our model lends itself to studying the full equilibrium dynamics of the economy, which enables us to analyze the long-run impact of capital requirements on lending and welfare. There are a number of recent studies aimed at quantifying the welfare effects of capital regulation. Begenau (2015) analyzes the effect of capital requirements in a dynamic model where banks have excessive risk-taking incentives due to mispriced government guarantees and, as in our model, depositors derive benefits from holding liquid claims. At the heart of her welfare analysis lies the trade-off

[4] As we discuss further, in our model holding deposits in a bank brings additional utility to households, which gives rise to a so-called liquidity premium that ultimately offsets a surplus inherent in deposit holding.

[5] At the same time, the problem of individual banks with respect to dividend distribution and recapitalization in our model shares some similarities with the liquidity management models in Bolton et al. (2011, 2013), Décamps et al. (2011), Hugonnier and Morellec (2015).

between a reduced liquidity provision on the one hand, and lower output volatility and higher credit supply due to lower funding costs on the other. The latter is due to a general equilibrium effect inducing the deposit rates to decline with the supply of deposits. Calibrating the model, she finds an optimal capital requirement of 14% of risky assets. Van den Heuvel (2008) also focuses on the role of capital in mitigating risk-taking incentives in a model where depositors value liquidity. Since in the considered setting without aggregate shocks, capital requirements lead to less liquidity provision, he concludes that the prevailing capital requirements (of 10%) would be too high. Risk-shifting incentives generated by prospective bailouts are also at the heart of the quantitative analysis of Nguyen (2014). Due to equity issuance costs, higher capital requirements cause some banks to exit the market which tends to reduce lending and capital. Yet the remaining banks operate with lower leverage, thus leading to lower default rates and higher consumption. In Martinez-Miera and Suarez (2014), capital requirements serve the purpose of reducing banks' incentives to invest in highly correlated "systemic" assets but come at the expense of reducing credit and output. In contrast to the above-mentioned studies, our focus here is not on the incentive effects of capital but on its role of a loss absorbing buffer - the concept that is often put forward by bank regulators. Moreover, the main contribution of our paper is qualitative: we seek to disentangle the short run and the long run effects of capital regulation rather than provide a quantitative guidance on the optimal level of a minimum capital ratio.

Finally, our paper is also related to the literature on credit cycles that has brought forward a number of alternative explanations for their occurrence. Fisher (1933) identified the famous debt deflation mechanism, that has been further formalized by Bernanke et al. (1996) and Kiyotaki and Moore (1997). It attributes the origin of credit cycles to the fluctuations of the prices of collateral. Several studies also place emphasis on the role of financial intermediaries, by pointing out the fact that credit expansion is often accompanied by a loosening of lending standards and "systemic" risk-taking, whereas materialization of risk accumulated on the balance sheets of financial intermediaries leads to the contraction of credit (see e.g. Aikman et al. (2014), Dell'Ariccia and Marquez (2006), Jimenez and Saurina (2006)). In our model, quasi cyclical lending patterns emerge due to the reflection property of aggregate bank capital.

The rest of the paper is structured as follows. Section 2 presents a static model that illustrates the main mechanisms that will be at work in the continuous-time set up. In Section 3 we solve for the competitive equilibrium in the continuous-time set up and analyze the implications of the equilibrium properties on financial stability. In Section 4 we study the impact of minimum capital requirements on bank policies and welfare. Section 5 concludes. All proofs and computational details are gathered in the Appendix.

2 Static model

Before setting-up the fully fledged dynamic model, it is useful to convey some key intuitions in a static benchmark. The basic reasons why bank capital matters in our framework is that it allows to buffer losses on loans so as to offer perfect safety to depositors.[6]

There are two, essentially equivalent, ways to model why deposits are requested to be completely riskless. The first is to consider, like Gennaioli, Shleifer and Vishny (2013), that depositors are infinitely risk-averse, while other investors (who hold the shares of the bank) are ready to bear some risk if properly remunerated. The second approach, which we follow here, is the one put forward by Stein (2012): there is only one type of (risk-neutral) agents but they derive utility both from consumption and from payment services provided by bank deposits.[7]

2.1 The Model

The static model has only two dates $t = 0$ and $t = 1$.[8] There is one physical good, taken as a numeraire, which can be consumed at $t = 0$ or invested to produce consumption at $t = 1$. There is a continuum of identical risk neutral households with a discount rate ρ. As in Stein (2012), a representative household has utility:[9]

$$U = C_0 + \frac{\mathbb{E}[C_1] + \lambda D}{1+\rho},$$

where C_t denotes consumption at $t = 0, 1$, λ is the liquidity preference and D is aggregate deposits.

Households own the banks and the firms. They also have an endowment w_0 of the good at $t = 0$, which they allocate between consumption and deposits:

$$w_0 = C_0 + D.$$

A typical bank owns e units of the good (E for the whole banking sector), collects deposits d (D on aggregate) and lends $k = d + e$ to firms (K on aggregate). Banks

[6] In this regard, our paper extends the approach arguing that the deposit-taking capacity of banks depends essentially on the banks' ability to diversify risk (see e.g. DeAngelo and Stulz (2014) or Gornall and Strebulaev (2015)).

[7] An implicit assumption is that payment services cannot be provided by risky deposits. Hellwig (2015) proposes a similar model with a "warm glow" theory of deposits.

[8] The full model has an infinite horizon and infinitesimal periods (continuous time).

[9] This parsimonious way to model the benefits of liquidity provision is also used in Phelan (2014) and Begenau (2015). As in Brunnermeier and Sannikov (2014), households' consumption can be both positive and negative, which ensures a fully elastic supply of deposits.

compete both on deposit interest rate r and loan rate R.

Lemma 1 *Given the linearity of households' preferences the only positive deposit interest rate in an interior equilibrium is* $r = \rho - \lambda$.

Note that, due to linear preferences, the representative household's utility, which measures social welfare, is independent of deposits volume D and is equal to the initial endowment plus the expected present value of firms' profit $\tilde{\pi}_F$ and banks' profit $\tilde{\pi}_B$ (see details in the Appendix A):

$$U = w_0 + \frac{\mathbb{E}[\tilde{\pi}_F + \tilde{\pi}_B]}{1+\rho}.$$

Although firms are ultimately owned by (dispersed) households, we assume that banks are necessary for collecting deposits and granting loans to the productive sector.[10] The productive sector consists of a continuum of firms endowed with investment projects that are parametrized by a productivity parameter x. The productivity parameter x is distributed according to a continuous distribution with density function $f(x)$ defined on a bounded support $[0, \overline{R}]$.[11]

Firms' projects require each an investment of one unit of good at $t = 0$. A typical project yields x units of good at $t = 1$ with certainty. For the sake of simplicity, we assume that the firm always repays the interest R on the loan. However, it can default on the principal: with some probability, the initial investment is destroyed and the bank only gets R, whereas the firm always gets $(x - R)$. Firms have no own funds and finance themselves via bank loans.[12] Thus, the entire volume of investment in the economy is determined by the volume of bank credit. Firms are protected by limited liability and default when their projects are not successful. Given a nominal loan rate R, only the projects such that $x > R$ will demand financing. Thus, the total demand for bank loans in the economy is:

$$L(R) = \int_R^{\overline{R}} f(x)dx.$$

[10] We assume that firms and banks are run in the interest of their owner, therefore abstracting from any agency issues.

[11] Parameter $\overline{R}$ can be thought of as the maximum productivity among all the firms. This is also the maximum loan rate: when $R > \overline{R}$, the demand for loans is nil.

[12] Firms in our economy should be thought of as small and medium-sized enterprises (SMEs). SMEs represent the major pillar of the real economy and typically highly rely on bank financing. We recognize, however, that the importance of bank financing varies across countries. For example, according to the TheCityUK research report (October 2013), in EU area, bank loans account for 81% of the long term debt in the real sector, whereas in US the same ratio amounts to 19%.

We focus on the simple case where all projects have the same default probability:[13]

$$p + \sigma_0 \varepsilon,$$

where p is the unconditional probability of default, ε represents an aggregate shock faced simultaneously by all firms and σ_0 reflects the change in the default probability caused by the aggregate shock. For simplicity, ε is supposed to take only two values $+1$ (bad state) and -1 (good state) with equal probabilities. The net expected return per loan for a bank after an aggregate shock ε is

$$(R - r - p) - \sigma_0 \varepsilon,$$

where the first term reflects the expected earnings per unit of time and the second term captures the exposure to aggregate shocks.

At the equilibrium of the credit market, the net aggregate output per period in the economy is

$$\Big(F[L(R)] - pL(R)\Big) - \sigma_0 L(R) \varepsilon,$$

where

$$F[L(R)] = \int_R^{\overline{R}} x f(x) dx$$

is the aggregate production function.

Note that $F'[L(R)] \equiv R$,[14] so that the total expected surplus per unit of time, $F[L(R)] - pL(R) - rL(R)$, is maximized for $R_{fb} = r + p$. Thus, in the first best allocation, the loan rate is the sum of two components: the riskless rate and the unconditional probability of default. This implies that banks make zero expected profit, and the total volume of credit in the economy is given by $L(R_{fb})$.

2.2 Competitive equilibrium

We assume for the moment that the banks' equity E is fixed, and characterize the equilibrium loan rate R and the volume of lending $K = L(R)$. Given that $r = \rho - \lambda$, households are indifferent as to the volume of deposits. A firm of productivity x maximizes profit by borrowing if and only if $x > R$ and then obtains profit $(x - R)_+$ with certainty.

[13] The extension to heterogeneous default probability is straightforward and would not change our qualitative results.

[14] Differentiating $F[L(R)] = \int_R^{\overline{R}} x f(x) dx$ with respect to R yields $F'[L(R)]L'(R) = -Rf(R)$. Since $L'(R) = -f(R)$, this implies $F'[L(R)] \equiv R$.

Aggregate profit of the firms is thus:

$$\pi_F = \int_R^{\overline{R}} (x - R) f(x) dx = F[L(R)] - RL(R).$$

Note that default risk is entirely borne by the banks. The profit of a bank that collects deposits d and grants loans k is:

$$\pi_B = k(R + 1 - p - \sigma_0 \varepsilon) - d(1 + r).$$

Since $d = k - e$, this can be written as

$$\pi_B = e(1 + r) + k[R - r - p - \sigma_0 \varepsilon].$$

To make things interesting, assume that $R < r + p + \sigma_0$, which means that the bank makes losses when $\varepsilon = 1$. To guarantee that the bank is able to repay its depositors in full, it must be that

$$e \geq k \frac{[\sigma_0 - (R - r - p)]}{1 + r},$$

which can be viewed as a market-imposed leverage constraint.[15]

The competitive equilibrium is characterized by a loan rate R and a lending volume K that are compatible with expected profit maximization by each individual bank:

$$\max_k \quad e(1 + r) + k(R - r - p) \quad \text{s.t.}$$

$$k[\sigma_0 - (R - r - p)] \leq e(1 + r)$$

and the loan market clearing condition

$$K = L(R).$$

Note that the leverage constraint must be also satisfied at the aggregate level. Then, it is easy to see that, depending on the aggregate level of banks' capitalization E, two cases are possible.

[15] An equivalent interpretation is that banks finance themselves by repos, and the lender applies a hair cut equal to the maximum possible value of the asset (the loan portfolio) that is used as collateral. The justification of the leverage constraint in our framework differs from the other ones that are put forward by academics. One of them relates to the limitation on pledgeable income by bank insiders (Holmstrom and Tirole (1997)) and backs the concept of "inside" equity that plays the role of "the skin in the game" for banks. The other academic justification for leverage constraints stems from the limited resalability of collateral (Kiyotaki and Moore (1997)), thereby, placing emphasis on the asset side of borrowers' balance sheets, whereas we focus here on the banks' liabilities. These three justifications for leverage are different, but not independent.

Case 1: well-capitalized banking sector. When aggregate bank capitalization is sufficiently high, the leverage constraint does not bind and thus the equilibrium loan rate is given by $R^* = r + p \equiv R_{fb}$, which corresponds to the First-Best allocation of credit. This case occurs when

$$E \geq E^* \equiv \frac{L[R_{fb}]\sigma_0}{1+r}.$$

Case 2: undercapitalized banking sector. When aggregate bank capitalization is low, the leverage constraint binds and the equilibrium loan rate is defined implicitly by the unique value $R \equiv R(E)$ such that

$$L(R)[\sigma_0 - (R - r - p)] = E(1+r). \tag{1}$$

Since the left-hand side of Equation (1) is decreasing in R, the loan rate $R(E)$ resulting from (1) is decreasing in E (for $E < E^*$), which implies that $R(E) > r + p$. Thus, this parsimonious model illustrates an important idea: since depositors want riskless deposits, the competitive loan rate $R(E)$ and credit volume $K(E)$ are functions of the aggregate bank capitalization (or loss absorbing capacity) E. An insufficient loss absorbing capacity ($E < E^*$) drives the loan rate away from its First-Best level. Thus, in our model, bank equity plays the role of a loss absorbing buffer, very much in the spirit of the concept that is put forward by regulators.

Note that the shareholder value of any individual bank (i.e., expected present value of its profit) is proportional to its book value e and depends on aggregate capitalization. More specifically, this shareholder value equals

$$v(e,E) = \begin{cases} e\frac{(1+r)}{(1+\rho)}, & E \geq E^* \\ e\frac{(1+r)}{(1+\rho)}\Big[1 + \frac{R(E)-r-p}{\sigma_0-(R(E)-r-p)}\Big], & E < E^*. \end{cases}$$

This implies that the market-to-book value of equity is the same for all banks and is a decreasing function of aggregate bank capitalization:

$$\frac{v(e,E)}{e} \equiv u(E) = \frac{(1+r)}{(1+\rho)}\Big(\frac{\sigma_0}{\sigma_0 - (R(E) - r - p)}\Big).$$

Note that in our model banks are exposed to the same aggregate shocks. Therefore, the fact that an individual bank incurs a loss implies that the entire banking sector incurs losses (symmetrically, collects gains), so that an individual bank's loss is magnified by an increase in $u(E)$ and, vice-versa, each bank's gain is reduced because of the decrease in $u(E)$. As we will see in the continuous-time setting, there exists a particular value of the loan rate $R(E)$ that exactly compensates for the fluctuation of the market-to-book value and allows for an interior choice of lending by banks.

The following proposition summarizes our results for the static benchmark:

Proposition 1 *When $\sigma_0 > R - r - p$, there is a unique competitive equilibrium, in which bank capital matters for the dynamics of lending. The loan rate R is a (weakly) decreasing function of E:*

- *when $E \geq E^*$, $R(E) = R_{fb} \equiv r + p$;*
- *when $E < E^* = \frac{L[R_{fb}]\sigma_0}{1+r}$, $R(E)$ is the unique solution of*

$$L(R)\Big[\sigma_0 - (R - r - p)\Big] = E(1 + r).$$

All the banks have the same leverage $\frac{k}{e} = \frac{L[R(E)]}{E}$ and the same market-to-book value of equity:

$$u(E) = \frac{(1 + r)}{(1 + \rho)}\Big(\frac{\sigma_0}{\sigma_0 - (R(E) - r - p)}\Big),$$

which both are decreasing functions of E.

3 The continuous-time model

We now turn to the dynamic model in continuous time. In the core of the text we assume that $\lambda = \rho$ so that the riskless deposit rate is $r = 0$. The case $r > 0$ is studied in Appendix D, where we also show that $r = \rho - \lambda$ also holds in the continuous time framework.

The main financial friction in our model is that banks face a proportional issuance cost when they want to issue new equity.[16] This cost is supposed to be a decreasing function $\gamma(E)$ of aggregate bank capital. This formulation is aimed at capturing the idea that the more capitalized the banking sector (higher E) is, the less difficult it will be to find investors that are willing to buy new shares of a bank. For simplicity, we neglect other external frictions such as adjustment costs for loans or fixed costs of issuing equity.[17] This implies that our economy exhibits an homotheticity property: all banks' decisions (lending, dividends, recapitalization) are proportional to their equity levels. In other words, all banks make the same decisions at the same moment, up to a scaling factor equal to their equity level. This entails an important simplification: only the aggregate

[16]On top of direct costs of equity issuance, equity issuance costs may also capture inefficiencies caused by asymmetric information, but they are not modeled here.

[17]We also disregard any frictions caused by governance problems inside the banks or government explicit/implicit guarantees.

size of the banking sector, reflected by aggregate bank capitalization, matters for our analysis, whereas the number of banks and their individual sizes do not play any role.

Taking the continuous time limit of the return on assets for banks, we obtain:

$$(R_t - p)dt - \sigma_0 dZ_t, \tag{2}$$

where $\{Z_t, t \geq 0\}$ is a standard Brownian motion.

We investigate the existence of a Markovian competitive equilibrium, where all aggregate variables are deterministic functions of the single state variable, namely, aggregate bank equity, E_t. When $r = 0$, the dynamics of aggregate equity E_t satisfies

$$dE_t = K(E_t)[(R(E_t) - p)dt - \sigma_0 dZ_t] - d\Delta_t + dI_t, \tag{3}$$

where $K(E_t)$ denotes aggregate lending, $d\Delta_t \geq 0$ denotes aggregate dividend payments and $dI_t \geq 0$ are aggregate equity injections. Aggregate deposits are determined by the residual $K(E_t) - E_t$.[18]

Definition 1 *A stationary Markovian competitive equilibrium consists of an aggregate bank capital process E_t, a loan rate $R(E)$ and a credit volume $K(E)$ functions that are compatible with individual banks' profit maximization and the credit market clearing condition $K(E) = L[R(E)]$.*

In the following subsections we show the existence of a unique stationary Markovian equilibrium and study its implications on financial stability.

3.1 The competitive equilibrium

To characterize the competitive equilibrium, we have to determine the optimal recapitalization and financing decisions of individual banks as well as a functional relation between the aggregate level of bank equity E_t and the loan rate R_t. Consider first the optimal decision problem of an individual bank that takes the loan rate function $R_t = R(E_t)$ as given and makes its decisions based on the level of its own equity e_t and aggregate equity E_t. Bank shareholders choose lending $k_t \geq 0$, dividend $d\delta_t \geq 0$ and recapitalization $di_t \geq 0$ policies so as to maximize the market value of equity:[19]

$$v(e, E) = \max_{k_t, d\delta_t, di_t} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\delta_t - (1 + \gamma(E_t))di_t\right)|e_0 = e, E_0 = E\right], \tag{4}$$

[18]Note also that we do not restrict the sign of $K(E) - E$, so that in principle, we also allow for liquid reserves (cash) in the form of negative deposits. However, in the competitive equilibrium, holding liquid reserves turns out to be suboptimal for banks.

[19]Throughout the paper, we use lower case letters for individual bank variables and upper case letters for aggregate variables.

where aggregate equity E_t evolves according to (3) and

$$de_t = k_t[(R(E_t) - p)dt \quad - \sigma_0 dZ_t] - d\delta_t + di_t. \tag{5}$$

A fundamental property of the individual decision problem of a bank is that the feasible set, in terms of trajectories of $(k_t, d\delta_t, di_t)$, and the objective function are homogeneous of degree one in the individual equity level e_t. Therefore, the value function itself must satisfy:[20]

$$v(e, E) = eu(E),$$

where e reflects the book value of equity and $u(E)$ can be thought of as the market-to-book value of equity for banks.

Using the above property and applying standard dynamic programming methods (see Appendix A), it can be shown that the market-to-book value of equity drives all bank policies in our framework. The optimal dividend and recapitalization policies turn out to be of the "barrier type".[21] In particular, dividends are distributed only when $E_t = E_{max}$, where E_{max} is such that $u(E_{max}) = 1$. In other words, distribution of dividends only takes place when the marginal value of equity capital equals the shareholders' marginal value of consumption. Recapitalizations occur only when $E_t = E_{min}$, where E_{min} satisfies $u(E_{min}) = 1 + \gamma(E_{min})$, i.e., when the marginal value of equity equals the total marginal cost of equity issuance. As long as aggregate bank equity E_t remains in between E_{min} and E_{max}, fluctuations of the individual bank's equity are only caused by retained earnings or absorbed losses. Given that the market-to-book value is the same for all banks, bank recapitalizations and dividend payments in our economy are perfectly synchronized in time.

Maximization with respect to the level of lending k_t shows that the optimal lending policy of the bank is indeterminate, i.e., bank shareholders are indifferent with respect to the volume of lending. Instead, the latter is entirely determined by the firms' demand for credit.[22] We show in Appendix A that the maximization problem (4) has a non-

[20]This useful property of the value function is a natural consequence of the scale invariance property of our model.

[21]The barrier-type recapitalization and payout policies have been extensively studied by the corporate liquidity management literature (see e.g. Jeanblanc and Shiryaev (1996), Milne and Robertson (1996), Décamps et al. (2011), Bolton et al. (2012, 2013), Hugonnier et Morellec (2015) among others) that places emphasis on the loss-absorbing role of corporate liquid reserves in the presence of financial frictions. In our model, the role of book equity is very similar to the role of liquidity buffers in those models. However, we differentiate from this literature by allowing for the feedback loop between the individual decisions and the dynamics of individual book equity via the general equilibrium mechanism that determines the loan rate and thus affects the expected earnings of a bank.

[22]This situation is analogous to the case of an economy with constant returns to scale, in which the equilibrium price of any output is only determined by technology (constant marginal cost), whereas the volume of activity is determined by the demand side.

degenerate solution if and only if the market-to-book ratio simultaneously satisfies two equations:

$$[R(E) - p]\, u(E) = -K(E)\sigma_0^2 u'(E), \tag{6}$$

and

$$\rho u(E) = K(E)[R(E) - p]u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E), \tag{7}$$

where $K(E) = L[R(E)]$.

Combining these two equations, we find that $R(E)$ satisfies a first-order differential equation:

$$R'(E) = -\frac{1}{\sigma_0^2} \frac{2\rho\sigma_0^2 + (R(E) - p)^2}{\Big(L[R(E)] - [R(E) - p]L'[R(E)]\Big)} \equiv -\frac{1}{H[R(E)]}. \tag{8}$$

Given that $L'(R) < 0$, it is easy to see that $R'(E) < 0$: In the states with higher aggregate capital, banks charge lower loan rates, which leads to higher volume of credit and output in the economy. In contrast, when aggregate bank capital gets scarce after a long series of negative aggregate shocks, the loan rate increases, which entails a reduction in credit and output.[23] Thus, in line with empirical evidence (see, e.g., Becker and Ivashina (2014)), our model generates a procyclical pattern of lending.

It is important to emphasize that, for any level of bank capitalization $E \in [E_{min}, E_{max})$, the risk-adjusted credit spread $R(E) - p$ remains *strictly positive*. Indeed, expressing the loan rate $R(E)$ from equation (6) immediately shows that, for any $E > E_{max}$, bank shareholders require a strictly positive premium for accepting to lend:

$$R(E) = p + \underbrace{\sigma_0^2 K(E)\Big[-\frac{u'(E)}{u(E)}\Big]}_{\text{"lending premium"}}. \tag{9}$$

To understand the "raison d'être" for this lending premium, consider the impact of the marginal unit of lending on shareholder value $v(e, E) \equiv eu(E)$. A marginal increase in the volume of lending increases the bank's exposure to aggregate shocks. However, note that the aggregate shock not only affects the individual bank's equity e_t but also aggregate equity E_t and thus the market-to-book ratio $u(E)$ that is decreasing in E.[24] Thus, if there is a negative aggregate shock $dZ_t > 0$ that depletes the individual bank's equity, the effect of this loss on shareholder value gets amplified via the market-to-book

[23] Another remark to be made in light of the negative relation between the loan rate and aggregate equity is that recapitalizations occur when the bank makes a strictly positive profit in expectation, whereas dividends are distributed when the bank makes a zero expected profit.

[24] Intuitively, having an additional unit of equity reduces the probability of facing costly recapitalizations in the short-run, so that the marginal value of equity, $u(E)$, is decreasing with bank capitalization.

ratio. Symmetrically, a positive aggregate shock ($dZ_t < 0$), while increasing book equity, translates into a reduction of $u(E_t)$, which reduces the impact of positive profits on shareholder value. This mechanism gives rise to *effective risk aversion* with respect to variation in aggregate capital, which explains why risk-neutral bankers require a positive spread for accepting to lend.

Note that, given the equilibrium loan rate $R(E)$, the market-to-book value function can be computed by solving the equation (9) under the boundary condition $u(E_{max}) = 1$:

$$u(E) = exp\Big(\int_E^{E_{max}} \frac{R(E) - p}{\sigma_0^2 L[R(E)]} dE \Big) \equiv exp\Big(A(E) \Big). \tag{10}$$

The following proposition summarizes the characterization of the competitive equilibrium.

Proposition 2 *There exists a unique Markovian equilibrium, in which aggregate bank capital evolves according to:*

$$dE_t = L[R(E_t)][(R(E_t) - p)dt - \sigma_0 dZ_t], \qquad E_t \in (0, E_{max}). \tag{11}$$

The loan rate function $R(E)$ is defined in the interval $[R_{min}, R_{max}]$, such that $R_{min} \equiv R(E_{max}) = p$ and $R_{max} \equiv R(0)$, and is implicitly given by the equation

$$E = \int_{R(E)}^{R_{max}} H(s) ds, \qquad where \qquad H(s) = \frac{\sigma_0^2 [L(s) - (s-p)L'(s)]}{2\rho\sigma_0^2 + (s-p)^2}. \tag{12}$$

If the equation[25]

$$A[E(R_{max})] = \log(1 + \gamma(0)) \tag{13}$$

has a solution $R_{max} < \infty$, banks recapitalize at $E_{min} = 0$. Otherwise, recapitalizations never take place and banks default upon reaching $E_{min} = 0$.

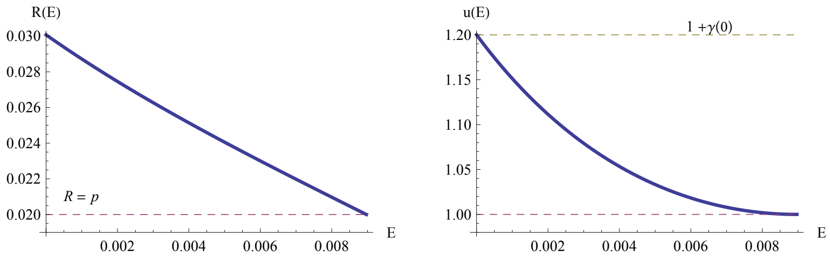
Proposition 2 shows in particular that $E_{min} = 0$, which means that unregulated banks wait until the last moment before recapitalizing. Moreover, when $\lim_{R_{max} \to \infty} A[E(R_{max})] < \log(1 + \gamma(0))$, the recapitalization costs are too high so that banks would never recapitalize,[26] and the banking sector would collapse unless the government bails it out. For the rest of the paper, we will focus on the former case.

The typical patterns of the loan rate $R(E)$ and the market-to-book value $u(E)$ that emerge in the competitive equilibrium are illustrated in Figure 1. Note that the loan rate

[25] The change of the variable of integration transforms Equation (13) into

$$\int_p^{R_{max}} \frac{(R-p)H(R)}{\sigma_0^2 D(R)} dR = \log(1 + \gamma(0)).$$

Figure 1: Loan rate and market-to-book ratio in the competitive equilibrium



Notes: this figure reports the typical patterns of the loan rate $R(E)$ (left panel) and market-to-book ratio $u(E)$ (right panel) in the competitive equilibrium.

function $R(E)$ cannot generally be obtained in closed form. However, it turns out that the dynamics of the loan rate $R_t = R(E_t)$ is explicit. Indeed, applying Itô's lemma to $R_t = R(E_t)$ yields:

$$dR_t = \underbrace{L[R(E_t)]\left((R(E_t)-p)R'(E_t) + \frac{\sigma_0^2 L[R(E_t)]}{2}R''(E_t)\right)}_{\mu(R_t)} dt \underbrace{-\sigma_0 L[R(E_t)]R'(E_t)}_{\sigma(R_t)} dZ_t. \tag{14}$$

After some computations involving the use of (8), one can obtain the drift and the volatility of $R_t = R(E_t)$ in closed form. This yields the following proposition:

Proposition 3 *The loan rate $R_t = R(E_t)$ has explicit dynamics*

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t, \qquad p \leq R_t \leq R_{max}, \tag{15}$$

with reflections at both ends of the support. The volatility function is given by

$$\sigma(R) = \frac{2\rho\sigma_0^2 + (R-p)^2}{\sigma_0\left(1-(R-p)\frac{L'(R)}{L(R)}\right)}. \tag{16}$$

The drift function is

$$\mu(R) = \sigma(R)(R-p)\frac{h(R)}{2}, \tag{17}$$

where

$$h(R) = \frac{\sigma(p)-\sigma(R)}{(R-p)^2} - \frac{1}{\sigma_0} + \frac{\sigma'(R)}{R-p}. \tag{18}$$

[26] It can be shown that the boundary $E_{min} = 0$ is attainable by the process E_t defined in (3). By contrast, the model by Brunnermeier and Sannikov (2014) provides an example of the process that never attains the lowest feasible state, which completely rules out the possibility of defaults even when recapitalizations are not possible.

Overall, the dynamics of E_t and $R_t = R(E_t)$ in the competitive equilibrium depends on the credit demand function $L(R)$ and four parameters: the exposure to aggregate shocks (or fundamental volatility) σ_0, the unconditional probability of default p, the discount factor ρ and the maximum level of the recapitalization cost $\gamma(0)$. In equilibrium with recapitalizations, the loan rate R_t fluctuates between its first-best level p and R_{max}. Moreover, it follows immediately from the expression (13) that the maximum lending premium, $R_{max} - p$, is increasing with the magnitude of financial frictions, $\gamma(0)$. Thus, our model predicts that loan rates, lending and, thereby, output will be more volatile in economies with stronger financial frictions. At the same time, expression (12) shows that the target level of bank capitalization, E_{max}, is increasing with R_{max}. Thus, the loss absorbing capacity of equity becomes more important under stronger financing frictions.[27] By contrast, in the absence of financial frictions, i.e., when $\gamma(E) \equiv 0$, one would have $R_{max} = p$ and $E \equiv 0$, so that there would no role for bank equity and no fluctuations of credit.

3.2 Long run behavior of the economy

We now study the long-run behavior of the economy in the competitive equilibrium. To this end, we look at the long-run behavior of the loan rate, with explicit dynamics given by Equation (15).[28] The behavior of the economy can be described by an ergodic density function which measures the average time spent in the neighborhood of each possible loan rate R: the states with lower R (equivalently, high aggregate capital E) can be interpreted as "boom" states and the states with higher R (equivalently, low aggregate capital E) can be thought of as "bust" states. The ergodic density function can be computed by solving the Kolmogorov forward equation (see details in Appendix A).

Proposition 4 *The competitive loan rate process R_t is ergodic. Its asymptotic distribution is characterized by the probability density function*

$$g(R) = \frac{C_0}{\sigma^2(R)} exp\Big(\int_p^R \frac{2\mu(s)}{\sigma^2(s)} ds \Big), \tag{19}$$

where the constant C_0 is such that $\int_p^{R_{max}} g(R)dR = 1$.

[27] Note that the cost of recapitalization γ only affects R_{max} and E_{max}, without intervening in the expressions of $\mu(R)$, $\sigma(R)$ and $u(E)$.

[28] Working with R_t instead of E_t enables us to provide an analytic characterization of the system's behavior, because the drift and volatility of R_t are closed-form expressions. By contrast, the drift and volatility of the process E_t cannot in general be obtained in closed form, since $R(E)$ has an explicit expression only for particular specifications of the credit demand function.

By differentiating the logarithm of the ergodic density defined in (19), we obtain:

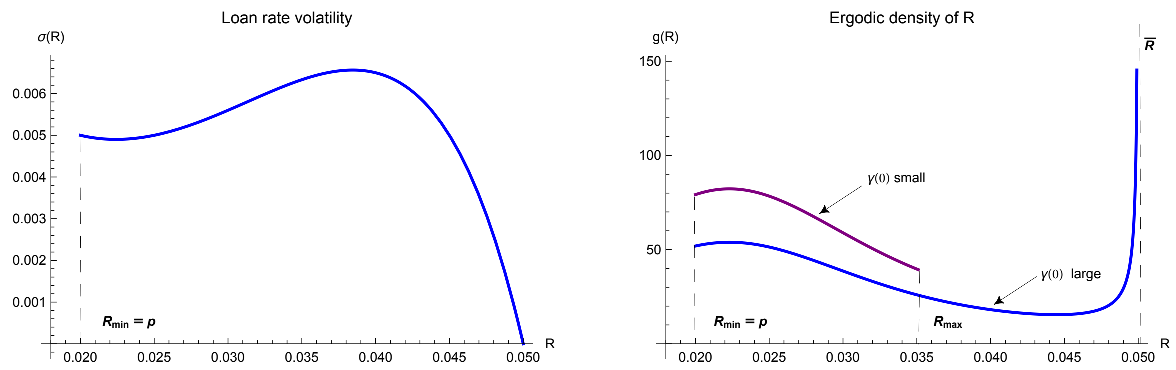
$$\frac{g'(R)}{g(R)} = \frac{2\mu(R)}{\sigma^2(R)} - \frac{2\sigma'(R)}{\sigma(R)}. \tag{20}$$

Using the general formulas for $\sigma(R)$ and $\mu(R)$, it can be shown that $\sigma(p) = 2\rho\sigma_0$, $\sigma'(p) = 2\rho\sigma_0\frac{L'(p)}{L(p)} < 0$ and $\mu(p) = 0$. Hence, $g'(p) > 0$, which means that the state $R = p$ that would correspond to the deterministic steady state is definitely *not* the one at which the economy spends most of the time in the stochastic set up.[29] To get a deeper understanding of the determinants of the system behavior in the long run, we resort to the particular demand specification:

$$L(R) = (\overline{R} - R)^\beta, \tag{21}$$

where $\beta > 0$ and $p < \overline{R}$.

Figure 2: Volatility and ergodic density of R



Notes: this figure reports the typical patterns of the loan rate volatility (left panel) and the ergodic density (right panel).

Figure 2 reports the typical patterns of the endogenous volatility $\sigma(R)$ (the left-hand side panel) and the ergodic density $g(R)$ (the right-hand side panel) for the above loan demand specification. It shows that the extrema of the ergodic density almost coincide with those of the volatility function, i.e., the economy spends most of the time in the states with the lowest loan rate volatility. Intuitively, the economy can get "trapped" in the states with low loan rate volatility because the endogenous drift is generally too small to move it away from these states. In fact, $\sigma(R)$ turns out to be much larger than $\mu(R)$ for any level of R, so that the volatility impact always dominates the drift impact.[30] In this light, relying on the results of the impulse response analysis that is typically used in

[29]See Appendix E for the analysis of the properties of the deterministic steady-state.
[30]The reason is that the factor $h(R)$ in the expression of $\mu(R)$ is very small.

the standard macromodels in order to infer the long-run behavior of the economy would be misleading (see Appendix E).

Note that functions $\sigma(.)$ and $g(.)$ must be truncated (and, in the case of the ergodic density, rescaled) on $[p, R_{max}]$, where R_{max} depends on the magnitude of the maximum level of issuing costs $\gamma(0)$. For the chosen specification of the loan demand function, $L(R) = (\overline{R} - R)^{\beta}$, we always have $R_{max} < \overline{R}$.[31] However, R_{max} can be arbitrary close to $\overline{R}$, which typically happens with very strong financial frictions and low elasticity of credit demand. In that case the economy will spend quite some time in the region where the loan rate is close to R_{max}. We interpret this situation as a persistent "credit crunch": it manifests itself via scarce bank equity capital, high loan rates, low volumes of lending and output.

This "credit crunch" scenario is reminiscent to the "net worth trap" documented by Brunnermeier and Sannikov (2014).[32] In their model, the economy may fall into a recession because of the inefficient allocation of productive capital between more and less productive agents, which they call “experts” and “households” respectively. This allocation is driven by the dynamics of the equilibrium price of capital, which depends on the fraction of the total net worth in the economy that is held by experts. After experiencing a series of negative shocks on their net worth, experts have to sell capital to less productive households, so that the average productivity in the economy declines. Under a reduced scale of operation, experts may struggle for a long time to rebuild net worth, so that the economy may be stuck in a low output region. In our model, the output in the economy is driven by the volume of credit that entrepreneurs can get from banks, whereas the cost of credit depends on the level of aggregate bank capitalization. When the banking sector suffers from a series of adverse aggregate shocks, its loss absorbing capacity deteriorates. As a result, the amplification mechanism working via the market-to-book value becomes more pronounced and bankers thus require a larger lending premium. The productive sector reacts by reducing its demand for credit and the banks have to shrink their scale of operations, which makes it even more difficult to rebuild equity capital.

4 Impact of capital regulation

So far our analysis has been focused on the "laissez-faire" environment in which banks face no regulation. Our objective in this section is to understand the impact of a minimum capital ratio on bank policies and welfare. Let us assume that public authorities enforce a minimum capital requirement, under which each bank must maintain equity capital

[31] To prove this property, it is sufficient to show that the integral $\int_p^{\overline{R}} \frac{(R-p)H(R)}{\sigma_0^2 L(R)} dR$ diverges.

[32] In a partial equilibrium set-up, a similar result is found by Isohätälä, Milne and Robertson (2014).

above a certain fraction of loans, i.e.,

$$e_t \geq \Lambda k_t,$$

where $\Lambda \in (0,1]$ is the minimum capital ratio.[33]

Note that, under such a formulation, banks have two options to comply with minimum capital requirements. The first option is to immediately recapitalize as soon as the regulatory constraint starts binding. The second option consists in cutting on lending and reducing deposit taking. We show below that, in our model, banks use the first option when E_t is small and the latter when E_t is large. In other words, a capital ratio does two things: it forces banks to recapitalize earlier (i.e., $E_{min} > 0$) and to reduce lending as compared to the unregulated case.

To solve for the regulated equilibrium, we again start by looking at the maximization problem of an individual bank. As in the unregulated set-up, bank shareholders maximize the market value of their claim by choosing their lending, recapitalization and dividend policies subject to the regulatory restriction on the volume of lending:

$$v_\Lambda(e,E) \equiv eu_\Lambda(E) = \max_{k_t \leq \frac{e}{\Lambda}, d\delta_t, di_t} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\delta_t - (1+\gamma(E_t))di_t\right)|e_0 = e, E_0 = E\right]. \tag{22}$$

To have the intuition of the solution to the above problem, recall that, in the unregulated case, bank recapitalizations take place only when equity is completely depleted. Thus, it is natural to expect that the regulatory constraint will be binding for relatively low levels of equity. Indeed, in the general case, the bank may find itself in one of two cases: (i) when its level of equity is relatively high, the regulatory constraint is not binding and the volume of lending is still determined by the firms' demand for credit; (ii) in the states with low equity, the regulatory constraint binds and the volume of lending is determined by $k_t = e_t/\Lambda$. Due to the homotheticity property, at each point in time, all banks have the same leverage ratio. Thus, it is legitimate to anticipate the existence of the critical level of bank capital E_c^Λ, such that the regulatory constraint binds (for all banks) for any $E \in [E_{min}^\Lambda, E_c^\Lambda]$ and is slack for any $E \in (E_c^\Lambda, E_{max}^\Lambda]$. This critical threshold E_c^Λ must satisfy

$$\frac{K(E_c^\Lambda)}{E_c^\Lambda} = \frac{1}{\Lambda}.$$

For Λ high enough, E_c^Λ tends to E_{max}^Λ and the unconstrained region disappears entirely. For future reference, we denote the critical leverage ratio above which the unconstrained

[33] Since our model only considers one type of bank assets (loans), we cannot discuss the issue of risk weights or distinguish a leverage ratio from a risk-weighted capital ratio.

region disappears by Λ^*.

Proposition 5 *For all $\Lambda \in (0,1]$, there exists a unique regulated equilibrium, where the support of E_t is $[E^\Lambda_{min}, E^\Lambda_{max}]$. This equilibrium is characterized by one of two regimes:*

a) for $\Lambda \geq \Lambda^*$, the regulatory constraint binds over the entire interval $[E^\Lambda_{min}, E^\Lambda_{max}]$. The loan rate is explicitly given by

$$R(E_t) = L^{-1}[E_t/\Lambda], \tag{23}$$

where L^{-1} is the inverse function of the loan demand. The dynamics of aggregate bank capital is given by:

$$\frac{dE_t}{E_t} = \frac{1}{\Lambda}\Big(L^{-1}[E_t/\Lambda] - p)dt - \sigma_0 dZ_t\Big), \qquad E \in (E^\Lambda_{min}, E^\Lambda_{max}).$$

b) for $\Lambda < \Lambda^*$, the capital constraint only binds for $E \in [E^\Lambda_{min}, E^\Lambda_c]$ and is slack for $E \in (E^\Lambda_c, E^\Lambda_{max}]$, where E^Λ_c is a critical capitalization level. When $E \in (E^\Lambda_{min}, E^\Lambda_c]$, the dynamics of aggregate equity and the loan rate function are defined as in the regime a). When $E \in (E^\Lambda_c, E^\Lambda_{max})$, the loan rate satisfies the first-order differential equation[34] $R'(E) = -1/H(R(E))$ with the boundary condition $R(E^\Lambda_c) = L^{-1}[E^\Lambda_c/\Lambda]$.

In either regime, banks distribute dividends when $E_t = E^\Lambda_{max}$ and recapitalize when $E_t = E^\Lambda_{min}$.

We show in the Appendix C.1 that, in the unconstrained region $(E^\Lambda_c, E^\Lambda_{max})$, the market-to-book value still simultaneously satisfies Equations (6) and (7), whereas in the constrained region $(E^\Lambda_{min}, E^\Lambda_c)$ it satisfies instead

$$\rho = \frac{E(L^{-1}[E/\Lambda] - p)}{\Lambda}\frac{u'_\Lambda(E)}{u_\Lambda(E)} + \frac{\sigma_0^2 E^2}{2\Lambda^2}\frac{u''_\Lambda(E)}{u_\Lambda(E)}, \tag{24}$$

under the condition

$$\frac{u'_\Lambda(E)}{u_\Lambda(E)} \geq -\frac{L^{-1}[E/\Lambda] - p}{\sigma_0^2 E/\Lambda}, \tag{25}$$

with equality at $E = E^\Lambda_c$.

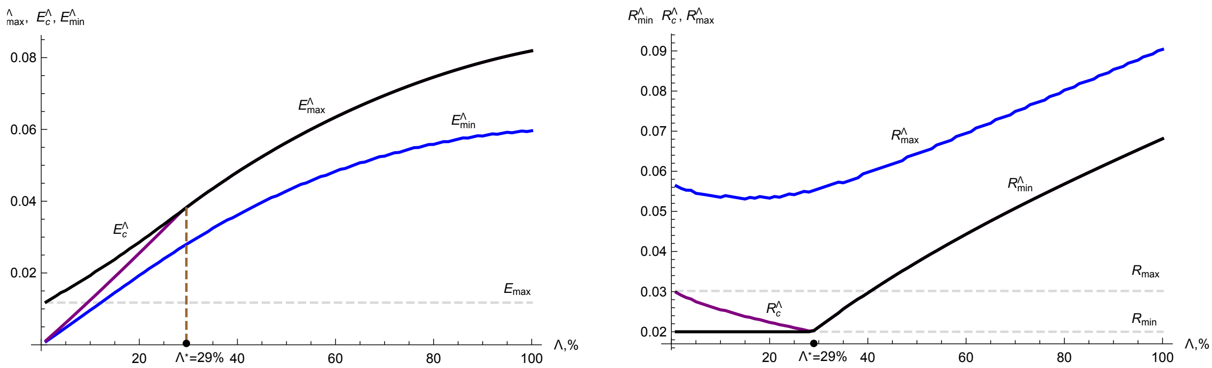
The optimal recapitalization and payout decisions are characterized by two boundaries, E^Λ_{min} (recapitalizations) and E^Λ_{max} (dividend payments), such that $u_\Lambda(E^\Lambda_{min}) = 1 + \gamma(E^\Lambda_{min})$ and $u_\Lambda(E^\Lambda_{max}) = 1$. In Appendix C.1 we provide the detailed description of the computational procedure that enables us to numerically solve for the regulated equilibrium.

[34] The function $H(.)$ is defined in (8). Thus, in the region $E \in (E^\Lambda_c, E^\Lambda_{max})$, the dynamics of aggregate equity is described by the same differential equation as in the unregulated set-up.

4.1 Capital regulation and lending

To get general insight into the impact of capital regulation on the cost of credit and the bank's policies, we perform a comparative static analysis by computing the equilibrium characteristics of bank policies for all values of $\Lambda \in (0,1]$. The left panel of Figure 3 reports the values of E^{Λ}_{min}, E^{Λ}_{max} and E^{Λ}_{c} (solid lines), contrasting them to the values E_{min} and E_{max} computed in the unregulated setting (dashed lines). The minimum and the maximum boundaries of the loan rate are reported in the right panel of Figure 3. This figure shows that, as long as the capital ratio is not too high, the bank may find itself in either constrained or unconstrained region, but above some critical level Λ^* of a capital ratio ($\Lambda^* = 29\%$ in this example), the regulatory constraint is always binding. In contrast with the unregulated set-up, shareholders always recapitalize the bank *before* completely exhausting bank capital.[35] The minimum loan rate is still equal p as long as $\Lambda < \Lambda^*$ and the unconstrained region exists. However, for $\Lambda \geq \Lambda^*$, the increase in Λ entails the upward shift in the entire support of R, as the loan rate becomes entirely determined by the binding regulatory constraint.

Figure 3: Minimum capital ratio and bank policies



Notes: this figure illustrates the effect of minimum capital requirements on banks' policies. Solid lines in the left panel depict the optimal recapitalization (E^{Λ}_{min}) and payout (E^{Λ}_{max}) barriers, as well as the critical barrier E^{Λ}_{c} such that the regulatory constraint is binding for $E \leq E^{\Lambda}_{c}$. Solid lines in the right panel depicts the minimum ($R^{\Lambda}_{min} \equiv R(E^{\Lambda}_{max})$) and maximum ($R^{\Lambda}_{max} \equiv R(E^{\Lambda}_{min})$) boundaries for the loan rate, as well as the critical loan rate $R^{\Lambda}_{c} \equiv R(E^{\Lambda}_{c})$ such that the regulatory constraint is binding for any $R \geq R(E^{\Lambda}_{c})$. Dashed lines in both panels illustrate the outcomes of the competitive equilibrium in the unregulated set-up. For $\Lambda > \Lambda^*$, a critical level E^{Λ}_{c} does not exist, i.e., the regulatory constraint is binding for any $E \in [E^{\Lambda}_{min}, E^{\Lambda}_{max}]$. Parameter values used: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\overline{R} = 0.15$, $\gamma(E) = 0.15 \exp\Big(-100E\Big) + 0.05$, $L(R) = \overline{R} - R$.

Figure 4 illustrates the short-run impact of capital regulation on loan rates (left panel) and lending (right panel). In the short run, a higher capital ratio reduces lending. Interestingly, it does so even when the capital constraint is not binding, i.e., when $E > E^{\Lambda}_{c}$. The reason is that banks anticipate that the regulatory constraint will bind in the future

[35] As we discuss in Appendix B.2, recapitalizing at a strictly positive level of capital is (generally) also optimal from the social perspective, as it helps to reduce issuance costs.

and require higher lending premium by precautionary reasons.

Understanding the long run impact of capital requirements on lending, however, requires a more subtle approach than a simple comparison of the lending patterns that arise under different levels of capital ratio. In particular, the probabilistic behavior of the economy should be taken into account. We therefore analyze the long-run impact of capital regulation on lending by looking at the intertemporal average loan rate $\tilde{R}^{\Lambda}$ that is computed according to the following formula:

$$\tilde{R}^{\Lambda} = \int_{R^{\Lambda}_{min}}^{R^{\Lambda}_{max}} R g_{\Lambda}(R) dR,$$

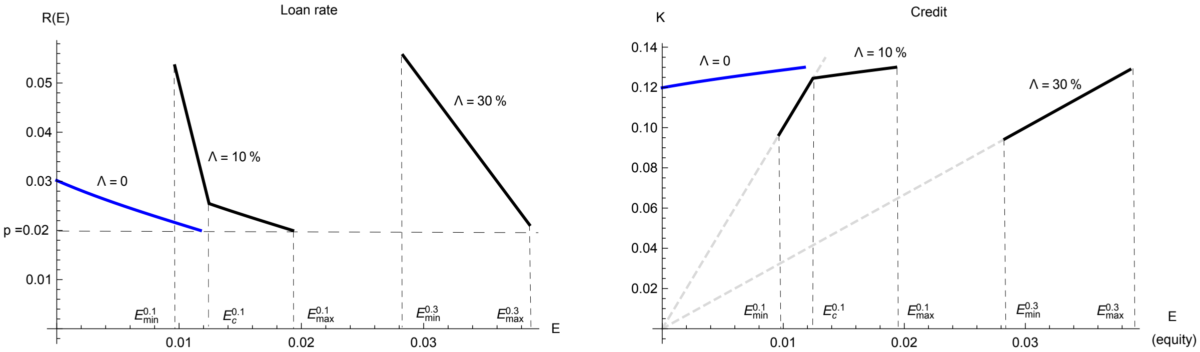
where $g_{\Lambda}(R)$ is the ergodic density function in the regulated equilibrium (see Appendix C.2 for the computational details).

The left panel of Figure 5 reports the typical pattern of the average intertemporal loan rate for all values of $\Lambda \in (0, 1]$. It shows that, when capital ratios remain relatively low, the average loan rate does not actually increase and may even slightly decrease. This result is driven by the behavior of the ergodic density that reflects the probabilistic behavior of the economy in the long run (see Appendix C.2 for details). In fact, the ergodic density is higher in the unconstrained region and is lower in the constrained region. When Λ increases (while remaining relatively low), the constrained region with lower capitalization and higher loan rates expands, but, simultaneously, there is a higher concentration of density in the unconstrained region with higher capitalization and lower loan rates. This implies that, in the long run, banks will spend quite a lot of time in the states with higher capitalization and lower loan rates, so that, for moderate capital ratios, the average loan rate does not increase and may even slightly decrease. As a result, for reasonable levels of capital ratios, there is *no adverse effect on lending* in the long run, as the adverse effect on the loan rate is offset by the positive effect on average capitalization. By contrast, for high levels of capital ratios such that the regulatory constraint becomes always binding, raising a capital ratio would monotonically increase the average loan rate (this is because raising Λ shifts the support of R to the right), thereby, reducing lending.

Overall, our model suggests the existence of a *non-monotonic relationship* between the increase in the minimum capital requirements and lending in the long run, with a certain critical level of capital ratio that maximizes the average lending.[36]

[36] The empirical evidence on the *long-run* impact of minimum capital requirements remains very scarce. The two studies we are aware of that explicitly address this question are Berrospide and Edge (2010) and Bridges et al.(2014). Using the data for U.S. bank holding companies for the period from 1992 to 2009, Berrospide and Edge (2010) find that the long-run impact of exogenous changes in the bank capital ratios on the loan growth is positive but remains very small. Bridges et al.(2014) use a data set for the U.K. banking sector for the period from 1990 to 2011. They find that the increases in the capital requirements induce the banks to cut on lending in the short run, but the loan growth rate is restored

Figure 4: Short run impact of capital regulation on loan rates and lending



Notes: this figure illustrates the short run impacts of capital requirements on loan rates (the left hand side panel) and lending (the right hand side panel). Solid curves $\Lambda = 0$ correspond to the unregulated set-up. Under the lower level of minimum capital ratio ($\approx 10\%$ in our example), the regulatory constraint binds only for the lower levels of bank capitalization. For the higher level of capital ratio (30% in our example), the slope of the lending curve is entirely determined by the binding regulatory constraint. Parameter values used: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\overline{R} = 0.15$, $\gamma(E) = 0.15\exp\Big(-100E\Big) + 0.05$, $L(R) = \overline{R} - R$.

4.2 Capital regulation and social welfare

To complete our analysis, we study the implications of capital requirements on social welfare. In our simple set-up, social welfare equals the aggregate lifetime utility of households and can be computed as the sum of the market value of the firms (i.e., the expected discounted profit of the productive sector) and the aggregate value of banks:[37]

$$W(E) = \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(\pi_F(K_t) + d\Delta_t - (1+\gamma(E_t))\, dI_t\right)|E_t = E\right], \tag{26}$$

where $\pi_F(K) \equiv F(K) - KF'(K)$ denotes the aggregate instantaneous production of firms net of credit costs (recall that the market for bank credit must clear, i.e., $K(E) = L(R(E))$ and $F'(K(E)) = R(E)$).

By using aggregate bank equity E as a state variable, we can apply standard pricing methods to compute the social welfare function. Recall that, in the region $(E^{\Lambda}_{min}, E^{\Lambda}_{max})$, banks neither distribute dividends nor recapitalize, so that the available cash flow consists uniquely of the firms' profit. Therefore, for $E \in (E^{\Lambda}_{min}, E^{\Lambda}_{max})$, the social welfare function,

withing 3 years, which suggests that the negative impact of raising capital requirements in fact vanishes in the long run.

[37]Since recapitalizations are feasible and the deposit rate satisfies $r = \rho - \lambda$ (cf. Appendix D), social welfare in our model is equivalent to the expected intertemporal consumption of households.

$W(E)$, must satisfy the following differential equation:[38]

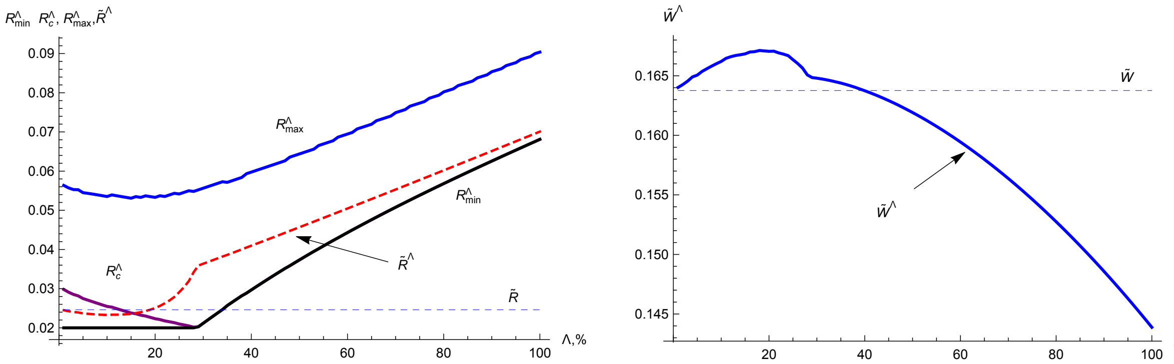
$$\rho W(E) = \pi_F[K(E)] + K(E)\left[F'\left(K\left(E\right)\right) - p\right] W'(E) + \frac{\sigma_0^2}{2} K^2(E) W''(E). \qquad (27)$$

Note that dividend distributions and bank recapitalizations only affect the market value of banks, without producing any immediate impact on the firms' profit. However, bank recapitalizations generate costs that depend on the aggregate bank equity E. These observations yield two boundary conditions, $W'(E_{max}^{\Lambda}) = 1$ and $W'(E_{min}^{\Lambda}) = 1 + \gamma\left(E_{min}^{\Lambda}\right)$, that are needed to numerically compute welfare in the regulated equilibrium.

To keep track of the long-run effect of minimum capital requirements on welfare, we numerically compute the average intertemporal welfare $\tilde{W}^{\Lambda}$ as a function of the regulatory parameter Λ, by using the formula:[39]

$$\tilde{W}^{\Lambda} = \int_{R_{min}^{\Lambda}}^{R_{max}^{\Lambda}} W[E(R)] g_{\Lambda}(R) dR.$$

Figure 5: Long run impact of capital regulation on lending and welfare



Notes: this figure illustrates the long run impacts of capital requirements on the loan rate (the left-hand side panel) and welfare (the right-hand side panel). The blue dashed lines correspond to the levels of the average loan rate and average welfare in the unregulated set-up. Parameter values used: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\overline{R} = 0.15$, $\gamma(E) = 0.15 \exp\Big(-100E\Big) + 0.05$, $L(R) = \overline{R} - R$.

The right panel of Figure 5 illustrates the typical pattern of the intertemporal average welfare $\tilde{W}^{\Lambda}$ as a function of the regulatory parameter Λ (the solid line), contrasting it to the average intertemporal welfare computed in the unregulated set-up (the dashed line). It is easy to see that the pattern of the average welfare almost mirrors the pattern of the

[38]Note that, in the regulated equilibrium, the right-hand side of Equation (27) has different expressions in the regions $[E_{min}^{\Lambda}, E_c^{\Lambda}]$ and $(E_c^{\Lambda}, E_{max}^{\Lambda}]$, and the continuity of $W(E)$ and its first derivative at $E = E_c^{\Lambda}$ is required.

[39]Since the ergodic density is computed as a function of the loan rate R, the social welfare must be rewritten as a function of R too, by using the fact that $E = [R(E)]^{-1}$ and the mapping $E \rightarrow R(E)$ is unique.

average loan rate. The maximum welfare is attained for a larger value of Λ than the one corresponding to the minimum of the average loan rate $\tilde{R}^{\Lambda}$, because of the additional effect generated by lower recapitalization costs. Nevertheless, in all numerical scenarios we tried, the optimal level of minimum capital requirements maximizing social welfare remains below the critical level Λ^* above which the regulatory constraint always binds.[40] This result suggests that very high capital requirements would be harmful for social welfare.

5 Conclusion

This paper develops a general equilibrium model of commercial banking, in which banks satisfy households' needs for safe deposits and channel funds to the productive sector. Bank capital plays the role of a loss-absorbing buffer that insulates banks from the need to undertake costly recapitalizations too often. In our model, the aggregate level of bank capitalization drives the cost and the volume of lending. Specifically, we establish a negative relation between the equilibrium loan rate and the level of aggregate bank capital. The closed-form characterization of the equilibrium dynamics of loan rates enables us to study analytically the long-run behavior of the economy. We show that this behavior is ergodic and is essentially determined by the volatility of the loan rate and the magnitude of financing frictions. The economy never spends a lot of time at the deterministic steady state and, under severe financing frictions, may spend quite a lot of time in a credit crunch regime.

We use our model to study the impact of minimum capital requirements on lending. We find that implementing a higher capital ratio induces two effects: on the one hand, it increases the loan rates for any given level of capital; on the other hand, it induces banks to operate with more capital, so that, for moderate levels of capital ratios, the average intertemporal loan rate does not increase but may even slightly decrease, which implies higher lending. In such a case, the trade-off between stability and growth disappears in the long run. However, for high levels of minimum capital requirements, any increase in a minimum capital ratio entails an increase in the average loan rate leading to the substantial reductions in lending and social welfare.

It should be acknowledged that our model suffers from several limitations. First, it only considers commercial banking activities (deposit taking and lending), while neglecting market activities such as securities and derivatives trading. Second, it only considers diffusion risks that do not lead to actual bank defaults, but merely fluctuations in the size of the banking sector. A consequence of these limitations is that we cannot address

[40] In our numerical simulations, Λ^* ranges in between $22\% - 30\%$ depending on the parameter values.

the important questions of banks' excessive risk-taking and the role of capital regulation in the mitigation of this behavior, which have already been the subject of a large academic literature. Finally, in this paper we have focused on the scenario in which private bank recapitalizations prevent systemic crises from happening. A potential direction of further investigations would be to explore the alternative scenario, allowing to analyze bank bailouts.

Appendix A. Proofs

Proof of Lemma 1. The utility of the representative consumer is

$$U = C_0 + \frac{\mathbb{E}[\tilde{C}_1] + \lambda D}{1+\rho},$$

where the consumptions at $t = 0$ and $t = 1$ are given by aggregate budget constraints:

$$C_0 = w_0 - D,$$

$$\tilde{C}_1 = (1+r)D + \tilde{\pi}_B + \tilde{\pi}_F,$$

where $\tilde{\pi}_B$ and $\tilde{\pi}_F$ denote respectively the aggregate profits of the banks and of the firms. Inserting the expressions of C_0 and $\tilde{C}_1$ expressions into U yields

$$U = w_0 + \frac{\mathbb{E}[\tilde{\pi}_B + \tilde{\pi}_F]}{1+\rho} + D\Big[-1 + \frac{1+r+\lambda}{1+\rho}\Big].$$

Maximization with respect to D at an interior solution $D > 0$ is only possible when the term between brackets is zero, which is equivalent to $r = \rho - \lambda$.[41]

Proof of Proposition 1. Omitted.

Proof of Proposition 2. By the standard dynamic programming arguments, shareholder value $v(e, E)$ must satisfy the Bellman equation:[42]

$$\begin{aligned} \rho v = \max_{k\geq 0, d\delta \geq 0, di \geq 0} \quad & \Big\{ d\delta(1 - v_e) - di(1 + \gamma(E) - v_e) + \\ & + k[(R(E) - p)v_e + \sigma_0^2 K(E) v_{eE}] + \frac{k^2\sigma_0^2}{2} v_{ee} \\ & + K(E)(R(E) - p)v_E + \frac{\sigma_0^2 K^2(E)}{2} v_{EE} \Big\}. \end{aligned} \tag{A1}$$

Using the fact that $v(e, E) = eu(E)$, one can rewrite the Bellman equation (A1) as follows:

$$\begin{aligned} \rho u(E) = \max_{k\geq 0, d\delta \geq 0, di \geq 0} \quad & \Big\{ \frac{d\delta}{e}[1 - u(E)] - \frac{di}{e}[1 + \gamma(E) - u(E)] + \\ & + \frac{k}{e}[(R(E) - p)u(E) + \sigma_0^2 K(E) u'(E)] + \\ & + K(E)(R(E) - p)u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E) \Big\} \end{aligned} \tag{A2}$$

A solution to the maximization problem in k only exists when

$$\frac{u'(E)}{u(E)} \leq -\frac{R(E) - p}{\sigma_0^2 K(E)}, \tag{A3}$$

[41] As we show in Appendix D, the same equality holds in the continuous-time framework.
[42] For the sake of space, we omit the arguments of function $v(e, E)$.

with equality when $k > 0$.

Under conjecture that $R(E) \geq p$ (which will be verified ex-post), it follows from the above expression that $u(E)$ is a decreasing function of E. Then, the optimal payout policy maximizing the right-hand side of (A2) is characterized by a critical barrier E_{max} satisfying

$$u(E_{max}) = 1, \tag{A4}$$

and the optimal recapitalization policy is characterized by a barrier E_{min} such that

$$u(E_{min}) = 1 + \gamma(E_{min}). \tag{A5}$$

In other words, dividends are only distributed when E_t reaches E_{max}, whereas recapitalization occurs only when E_t reaches E_{min}. Given (A3), (A4), (A5) and $k > 0$, it it easy to see that, in the region $E \in (E_{min}, E_{max})$, market-to-book value $u(E)$ satisfies:

$$\rho u(E) = K(E)(R(E) - p)u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E). \tag{A6}$$

Note that, at equilibrium, $K(E) = L[R(E)]$. Taking the first derivative of (A3), we can compute $u''(E)$. Inserting $u''(E)$ and $u'(E)$ into (A6) and rearranging terms yields:

$$R'(E) = -\frac{1}{\sigma_0^2} \frac{2\rho\sigma_0^2 + (R(E) - p)^2}{\Big(L[R(E)] - [R(E) - p]L'[R(E)]\Big)}. \tag{A7}$$

Since $L'(R(E)) < 0$, it is clear that $R'(E) < 0$ if $R(E) > p$. To verify that $R(E) > p$ for any $E \in [E_{min}, E_{max})$, it is sufficient to show that $R_{min} \equiv R(E_{max}) \geq p$.

To obtain R_{min}, let $V(E) \equiv Eu(E)$ denote the market value of the entire banking sector. At equilibrium, dividends are distributed when the marginal value of bank capital equals the marginal value of dividends, which implies $V'(E_{max}) = 1$. Similarly, recapitalizations take place when the marginal value of bank capital equals the marginal costs of recapitalizing the banks, which implies $V'(E_{min}) = 1 + \gamma(E_{min})$. Given that $V'(E) = u(E) + Eu'(E)$, it must hold that $Eu'(E) = 0$. Hence, $u'(E_{max}) = 0$ and $E_{min} = 0$. Inserting $u'(E_{max}) = 0$ into the binding condition (A3) immediately shows that $R_{min} = p$, so that $R(E) > p$ for any $E \in [E_{min}, E_{max})$.

Hence, the loan rate $R(E)$ can be computed as a solution to the differential equation (A7), which yields:

$$\int_0^E R'(s)ds = R(E) - R_{max}, \tag{A8}$$

where $R_{max} \equiv R(0)$.

To obtain E_{max}, we use the fact that individual banks' optimization with respect to the recapitalization policy implies $u(E_{min}) = 1 + \gamma(E_{min})$. Integrating equation (A3) in between $E_{min} = 0$ and E_{max}, while taking into account the condition $u(E_{max}) = 1$, yields an equation that implicitly determines E_{max}:

$$\underbrace{u(E_{max})exp\Big(\int_0^{E_{max}} \frac{R(E) - p}{\sigma_0^2 L[R(E)]} dE\Big)}_{u(0)} = 1 + \gamma(E_{max}). \tag{A9}$$

The system of equations (12) and (13) immediately follows from the change of variable of integration in equations (A8) and (A9), i.e., $dR = R'(E)dE$.

Proof of Proposition 3. Omitted.

Proof of Proposition 4. Consider the process R_t that evolves according to

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t, \qquad p \leq R_t \leq R_{max}, \tag{A10}$$

with reflections at both ends of the support.

Let $g(t, R)$ denote the probability density function of R_t. It must satisfy the forward Kolmogorov equation:

$$\frac{\partial g(t,R)}{\partial t} = -\frac{\partial}{\partial R}\Big\{\mu(R)g(t,R) - \frac{1}{2}\frac{\partial}{\partial R}\Big[\sigma^2(R)g(t,R)\Big]\Big\}. \tag{A11}$$

Since the process R_t is stationary, we have $\frac{\partial g(t,R)}{\partial t} = 0$ and thus $g(t, R) \equiv g(R)$. Integrating Equation (A11) over R yields:

$$\mu(R)g(R) = \frac{1}{2}\frac{\partial}{\partial R}\Big[\sigma^2(R)g(R)\Big],$$

where the constant of integration is set to zero because of reflection properties of the process. Solving the above equation by using the change of variable $\hat{g}(R) = \sigma^2(R)g(R)$ ultimately yields:

$$g(R) = \frac{C_0}{\sigma^2(R)} exp\Big(\int_p^{R_{max}} \frac{2\mu(s)}{\sigma^2(s)} ds\Big), \tag{A12}$$

where the constant C_0 is chosen so as to normalize the solution to 1 over the region $[p, R_{max}]$, i.e., $\int_p^{R_{max}} g(R)dR = 1$.

To ensure that the distribution of R is non-degenerate, it is sufficient to check that $\sigma(R) > 0$ for any $R \in [p, R_{max}]$. From the expression of $\sigma(R)$, it is easy to see that this condition holds for any loan demand specifications such that $L'(R) < 0$ and $L(R) > 0$.

Appendix B. Computing social welfare

B.1. Social welfare in the competitive equilibrium

In this appendix we provide an illustrative example of how the welfare function corresponding to the competitive allocation of credit can be computed.[43] For this we use the results derived in Section 3.1 and consider the simple case where the credit demand is linear, i.e., $L(R) = \overline{R} - R$. Given the linear specification, the loan rate $R(E)$ can be computed in closed form:

$$R(E) = p + \sqrt{2\rho}\sigma_0 \tan\left(\frac{\sqrt{2\rho}}{\sigma_0(\overline{R} - p)}(E_{max} - E)\right), \tag{B1}$$

[43] The computation of welfare in the regulated equilibrium is similar.

and thus

$$K(E) = \overline{R} - p - \sqrt{2\rho}\sigma_0 \tan\left(\frac{\sqrt{2\rho}}{\sigma_0(\overline{R}-p)}(E_{max}-E)\right). \tag{B2}$$

To recover the production function, $F[L(R)]$, recall that $F'(L(R)) = R$. Using the fact that $R = \overline{R} - L$, we obtain $F'(L) = (\overline{R} - L)$ and, thereby,

$$F[L(R)] = \overline{R}L(R) - \frac{[L(R)]^2}{2}. \tag{B3}$$

At equilibrium, we have $L[R(E)] = K(E)$, so the that firm' expected profit is given by

$$\pi_F(K(E)) = F(K(E)) - K(E)F'(K(E)) = \frac{[K(E)]^2}{2}.$$

Then, the social welfare function follows the ODE:

$$\rho W(E) = \frac{[K(E)]^2}{2} + K(E)(\overline{R} - K(E) - p)W'(E) + \frac{\sigma_0^2}{2}[K(E)]^2 W''(E), \tag{B4}$$

with the boundary conditions $W'(0) = 1 + \gamma(0)$ and $W'(E_{max}) = 1$.

B.2. Social welfare in the Second Best

We now solve for the benchmark case of a benevolent social planner choosing dividend distribution, recapitalization and lending policy such as to maximize social welfare (26). The solution to the social planner's maximization problem is of the "barrier type" as well. In the region $(E^{sb}_{min}, E^{sb}_{max})$, where banks neither distribute dividends nor recapitalize, the available cash flow consists uniquely of the firms' profit. Therefore, the social welfare function must satisfy differential equation (27) for $E \in (E^{sb}_{min}, E^{sb}_{max})$ and bank lending $K^{sb}(E)$ is determined by the following first order condition:[44]

$$F''(K^{sb})K^{sb}\left[W'(E) - 1\right] - \left[F'(K^{sb}) - p\right]W'(E) + \sigma_0^2 K^{sb} W''(E) = 0.$$

Optimality of E^{sb}_{max} is ensured by the super contact condition $W''\left(E^{sb}_{max}\right) = 0$. For the recapitalization boundary, E^{sb}_{min}, two cases can arise. First, if $W''(0) \leq \gamma'(0)$, then $E^{sb}_{min} = 0$, while for $W''(0) > \gamma'(0)$, also the case $E^{sb}_{min} > 0$ can arise. In the latter case, E^{sb}_{min} is determined by the super contact condition $W''\left(E^{sb}_{min}\right) = \gamma'\left(E^{sb}_{min}\right)$.[45]

For the numerical implementation we again resort to the linear specification with $L(R) = \overline{R} - R$. Note that searching for the socially optimal $K^{sb}(E)$ is equivalent to searching for the socially optimal $R(E)$. Inserting $K^{sb}(E) = \overline{R} - R^{sb}(E)$ into (B4) yields:

$$\rho W(E) = \frac{[\overline{R} - R^{sb}(E)]^2}{2}\left(1 + \sigma_0^2 W''(E)\right) + (\overline{R} - R^{sb}(E))(R^{sb}(E) - p)W'(E). \tag{B5}$$

[44] For the sake of space, we abstain from writing the argument of $K^{sb}(E)$.

[45] These conditions can be formally derived by solving the constrained optimization problem of maximizing the welfare function with respect to the recapitalization and dividend barriers, by using the method of Lagrange multipliers.

The first-order condition with respect to $R^{sb}(E)$ implies:

$$R^{sb}(E) = \overline{R} - \frac{(\overline{R} - p)W'(E)}{2W'(E) + \sigma_0^2 W''(E) - 1}. \tag{B6}$$

Substituting (B6) in (B5) yields a simple second-order differential equation:

$$W''(E) = \frac{2\rho W(E)[2W'(E) - 1] - (\overline{R} - p)^2[W'(E)]^2}{2\rho\sigma_0^2 W(E)}, \tag{B7}$$

that can be solved numerically under the boundary conditions $W'(E^{sb}_{max}) = 1$ and $W'(E_{min}) = 1 + \gamma(E_{min})$. The free boundaries E^{sb}_{min} and E^{sb}_{max} satisfy the system of equations:[46]

$$W''(E^{sb}_{max}) = 0 \tag{B8}$$
$$W''(E^{sb}_{min}) = \gamma'(E_{min}). \tag{B9}$$

It is easy to see from (B6), at the target level of aggregate bank equity, we have $R^{sb}(E^{sb}_{max}) = p$.

Appendix C. Solving for the regulated equilibrium

C.1. Optimal bank policies

Consider the shareholders' maximization problem stated in (22). By the standard dynamic programming arguments and the fact that $v_\Lambda(e, E) = eu_\Lambda(E)$, where $u_\Lambda(E)$ satisfies the following Bellman equation:

$$\begin{aligned}\rho u_\Lambda(E) = \max_{d\delta\geq 0, di\geq 0} \quad & \left\{\frac{d\delta}{e}[1 - u_\Lambda(E)] - \frac{di}{e}[1 + \gamma(E) - u_\Lambda(E)]\right\} + \\ + \max_{0<k\leq e/\Lambda} \quad & \left\{\frac{k}{e}[(R(E) - p)u_\Lambda(E) + \sigma_0^2 K(E)u'_\Lambda(E)]\right\} + \\ & + K(E)[R(E) - p]u'_\Lambda(E) + \frac{\sigma_0^2 K^2(E)}{2}u''_\Lambda(E).\end{aligned} \tag{C1}$$

A solution to (C1) exists only if $K(E) \leq E/\Lambda$, and

$$B(E) := \frac{R(E) - p}{\sigma_0^2 K(E)} \geq -\frac{u'_\Lambda(E)}{u_\Lambda(E)} \equiv \alpha(E), \tag{C2}$$

with equality when $K(E) < E/\Lambda$.

The optimal dividend and recapitalization policies are characterized by barriers E^Λ_{max} and E^Λ_{min} such that $u_\Lambda(E^\Lambda_{max}) = 1$ and $u_\Lambda(E^\Lambda_{min}) = 1 + \gamma(E^\Lambda_{min})$. Moreover, by the same reason than in the unregulated equilibrium, it must hold that $E^\Lambda_{max}u'_\Lambda(E^\Lambda_{max}) = 0$ and $E^\Lambda_{min}u'_\Lambda(E^\Lambda_{min}) = 0$. This implies $u'_\Lambda(E^\Lambda_{max}) = 0$ and $u'_\Lambda(E^\Lambda_{min}) = 0$.[47]

[46] These equations immediately follow from the constrained optimization of the shareholder value. When $W''(0) < \gamma'(0)$, $E^{sb}_{min} = 0$.

[47] Note that in the unregulated equilibrium, we had $E_{min} = 0$. Under capital regulation, this is no

With the conjecture that there exists some critical threshold E_c^Λ such that the regulatory constraint is binding for $E \in [E_{min}^\Lambda, E_c^\Lambda]$ and is slack for $E \in [E_c^\Lambda, E_{max}^\Lambda]$, equation (C1) can be rewritten as follows:

$$\rho = -\pi_B(E)\alpha(E) + \frac{\sigma_0^2 K^2(E)}{2}[\alpha^2(E) - \alpha'(E)] + \mathbb{1}_{E\in[E_{min}^\Lambda, E_c^\Lambda]}\frac{[R(E) - p - \sigma_0^2 K(E)\alpha(E)]}{\Lambda} \tag{C3}$$

where $\mathbb{1}_{(.)}$ is the indicator function and $\pi_B(E)$ denotes the aggregate expected profit of banks:

$$\pi_B(E) = K(E)[R(E) - p],$$

the volume of credit $K(E)$ satisfies

$$K(E) = \begin{cases} E/\Lambda, & E \in [E_{min}^\Lambda, E_c^\Lambda] \\ L[R(E)], & E \in (E_c^\Lambda, E_{max}^\Lambda], \end{cases}$$

and the loan rate $R(E)$ is given by

$$R(E) = \begin{cases} L^{-1}[E/\Lambda], & E \in [E_{min}^\Lambda, E_c^\Lambda] \\ R'(E) = -1/H(R(E)), \quad R(E_c^\Lambda) = L^{-1}[E_c^\Lambda/\Lambda], & E \in (E_c^\Lambda, E_{max}^\Lambda], \end{cases}$$

where L^{-1} is the inverse function of the demand for loans and function $H(.)$ is defined in (12). The critical threshold E_c^Λ must satisfy equation

$$\alpha(E_c^\Lambda) = B(E_c^\Lambda).$$

If $\alpha(E) < B(E)$ for any $E \in [E_{min}^\Lambda, E_{max}^\Lambda]$, then the regulatory constraint is always binding and $\alpha(E)$ satisfies equation (C3) with $E_c^\Lambda = E_{max}^\Lambda$.

The condition $u'_\Lambda(E_{min}^\Lambda) = 0$ yields the boundary condition $\alpha(E_{min}) = 0$. Similarly, the condition $u'_\Lambda(E_{max}^\Lambda) = 0$ translates into the boundary condition $\alpha(E_{max}) = 0$.

Numerical procedure to solve for the regulated equilibrium. This numerical algorithm solving for the regulated equilibrium can easily be implemented with the *Mathematica* software. Λ is taken as a parameter.

- Pick a candidate value $\hat{E}_{min}^\Lambda$.
- Assume that the regulatory constraint always binds. Solve ODE (C3) for $\alpha(E)$ under the boundary condition $\alpha(\hat{E}_{min}) = 0$.
- Compute a candidate value $\hat{E}_{max}^\Lambda$ such that satisfies equation $\alpha(\hat{E}_{max}^\Lambda) = 0$.
- Check whether $\alpha(\hat{E}_{max}^\Lambda) \le B(\hat{E}_{max}^\Lambda)$.
- Conditional on the results of the previous step, one of the two scenarios is possible:

longer possible, since $E_{min}^\Lambda = \Lambda K(E_{min}^\Lambda)$.

a) if $\alpha(\hat{E}^{\Lambda}_{max}) \leq B(\hat{E}^{\Lambda}_{max})$, then the regulatory constraint is always binding for a given Λ and the market-to-book value $u_{\Lambda}(\hat{E}^{\Lambda}_{min})$ can be computed according to

$$u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = u_{\Lambda}(\hat{E}^{\Lambda}_{max}) exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{max}} \alpha(E)dE\Big) = exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{max}} \alpha(E)dE\Big).$$

b) if $\alpha(\hat{E}^{\Lambda}_{max}) > B(\hat{E}^{\Lambda}_{max})$, solve equation

$$\alpha(\hat{E}^{\Lambda}_{c}) = B(\hat{E}^{\Lambda}_{c})$$

to find the critical level of equity $\hat{E}^{\Lambda}_{c}$ above which the regulatory constraint is slack. Compute the market-to-book value $u_{\Lambda}(\hat{E}^{\Lambda}_{min})$ according to:

$$u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = A_0 exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{c}} \alpha(E)dE\Big),$$

where

$$A_0 = exp\Big(\int_{\hat{E}^{\Lambda}_{c}}^{\hat{E}^{\Lambda}_{max}} \frac{R(E) - p}{\sigma_0^2 L[R(E)]} dE\Big).$$

- If $u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = 1 + \gamma(\hat{E}^{\Lambda}_{min})$, then $E^{\Lambda}_{min} = \hat{E}^{\Lambda}_{min}$, $E^{\Lambda}_{max} = \hat{E}^{\Lambda}_{max}$ and $E^{\Lambda}_{c} = \hat{E}^{\Lambda}_{c}$ (if exists). Otherwise, pick a different $\hat{E}^{\Lambda}_{min}$, repeat the procedure from the beginning.

C.2. Ergodic density function in the regulated equilibrium

Consider first the case when Λ is sufficiently high, so that the regulatory constraint is binding for any level of E. It is easy to show that, in this case, the dynamics of the loan rate is characterized by the process[48]

$$dR_t = \mu_{\Lambda}(R_t)dt + \sigma_{\Lambda}(R_t)dZ_t, \qquad R^{\Lambda}_{min} \leq R_t \leq R^{\Lambda}_{max}, \tag{C4}$$

where

$$\sigma_{\Lambda}(R) = -\frac{\sigma_0}{\Lambda}\frac{L(R)}{L'(R)}, \tag{C5}$$

$$\mu_{\Lambda}(R) = -\sigma_{\Lambda}(R)\left(\frac{R-p}{\sigma_0} + \frac{\sigma_{\Lambda}(R)}{2}\frac{L''(R)}{L'(R)}\right). \tag{C6}$$

In our numerical simulations, we stick to the following specification of the credit demand function: $L(R) = (\overline{R} - R)^{\beta}$, where $\beta > 0$ and $\overline{R} > p$. Under this specification,

[48] Note that, one can alternatively use R as a state variable, while looking for the unique mapping $E(R)$. In a Markov Equilibrium, one must have $-\sigma_0 K(R) = \sigma_{\Lambda}(R)E'(R)$ and $(R-p)K(R) = \mu_{\Lambda}(R)E'(R) + \sigma^2_{\Lambda}(R)/2E''(R)$. Using the fact that, under the binding regulatory constraint, $E(R) = \Lambda K(R)$ and, at equilibrium, $K(R) = L(R)$, we immediately obtain Expressions (C5) and (C6).

we obtain

$$\sigma_\Lambda(R) = \frac{(\overline{R} - R)\sigma_0}{\beta\Lambda}, \tag{C7}$$

$$\mu_\Lambda(R) = -\sigma_\Lambda(R)\left(\frac{R-p}{\sigma_0} + \frac{(1-\beta)\sigma_0}{2\beta\Lambda}\right). \tag{C8}$$

Let

$$\phi_0 := \frac{\beta^2\Lambda^2}{\sigma_0^2}, \qquad \phi_1 := \frac{1}{\sigma_0^2}\Big[(1-\beta)\sigma_0^2 + 2\beta\Lambda(\overline{R}-p)\Big] - 2, \qquad \phi_2 := \frac{2\beta\Lambda}{\sigma_0^2}.$$

Then, for relatively high levels of Λ such that the regulatory constraint always binds, the ergodic density function of R is given by[49]

$$g_\Lambda(R) = C_1\phi_0(\overline{R} - R)^{\phi_1} exp\Big(\phi_2 R\Big), \tag{C9}$$

where the constant C_1 is such that $\int_{R^\Lambda_{min}}^{R^\Lambda_{max}} g_\Lambda(R)dR = 1$.

For the relatively low levels of Λ, the unconstrained and the constrained regions coexists. The ergodic density function is discontinuous at R_c^Λ[50] and can be computed according to the following formula:

$$g_\Lambda(R) = C_1 \begin{cases} \frac{1}{\sigma^2(R)} exp\Big(\int_p^R \frac{2\mu(s)}{\sigma^2(s)}ds\Big), & R \in [R^\Lambda_{min}, R^\Lambda_c), \\ \frac{1}{\sigma^2_\Lambda(R)} exp\Big(\int_{R^\Lambda_c}^R \frac{2\mu_\Lambda(s)}{\sigma^2_\Lambda(s)}ds\Big) exp\Big(\int_p^{R^\Lambda_c} \frac{2\mu(s)}{\sigma^2(s)}ds\Big), & R \in [R^\Lambda_{max}, R^\Lambda_c], \end{cases}$$

where the constant C_1 is such that $\int_{R^\Lambda_{min}}^{R^\Lambda_{max}} g_\Lambda(R)dR = 1$, and $\mu(R)$ and $\sigma(R)$ are given in (16) and (17), respectively.

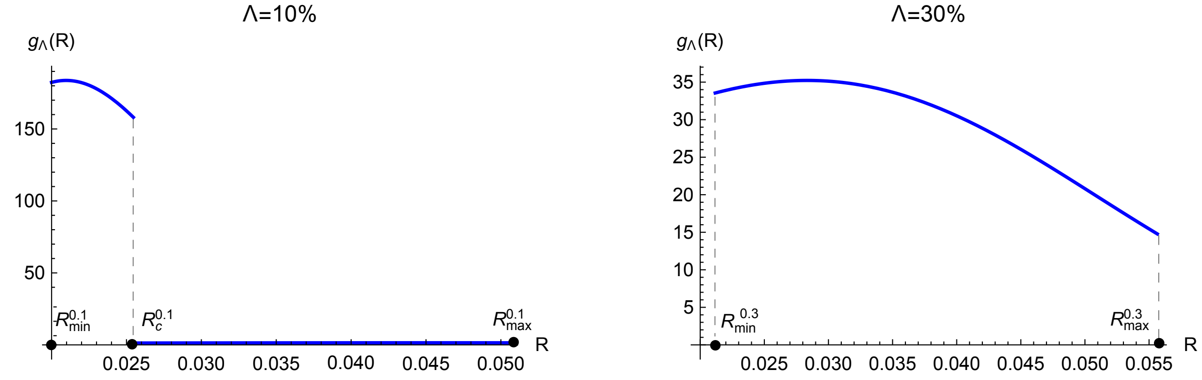
In the left-hand side panel of Figure 6 we report the typical pattern of the ergodic density function of R that emerges when Λ is relatively low (in our example, $\Lambda = 0.10$). In this case, the ergodic density function is defined in two intervals: $[p, R^\Lambda_c]$ and $(R^\Lambda_c, R^\Lambda_{max}]$, and exhibits discontinuity at R^Λ_c. As long as $\Lambda < \Lambda^*$ and Λ increases, the interval $[p, R^\Lambda_c]$ shrinks, so that the density becomes concentrated around p (equivalently, around E^Λ_{max}). At the same time, the constrained region (with a lower density) expands. The overall impact of tightening a minimum capital ratio on the average loan rate will be driven by the trade-off resulting from having a higher density on the shorter interval of R and lower density on the larger interval of R.

The right-hand side panel of Figure 6 illustrates the typical pattern of the ergodic density function for $\Lambda > \Lambda^*$ ($\Lambda = 30\%$ in this example). In this case, the ergodic density function is continuous in the whole interval $[R^\Lambda_{min}, R^\Lambda_{max}]$.

[49] The general formula for the ergodic density function in the regulated set-up is similar to the one stated in Proposition 4. After simplification, it can be expressed as in (C9).

[50] This discontinuity is caused by the discontinuity in the endogenous volatility function $\sigma(R)$, which in turns is caused by the discontinuity of $R'(E)$ at E^Λ_c.

Figure 6: The ergodic density of R under the mild and tight minimum capital ratios



Notes: this figure illustrates the typical patterns of the ergodic density function of R under a low capital ratio (the left-hand side panel, $\Lambda = 10\%$) and a high capital ratio (the right-hand side panel, $\Lambda = 30\%$). In the former case, the regulatory constraint is binding for $R \in [R_c^{\Lambda}, R_{max}^{\Lambda}]$. In the latter case, the regulatory constraint binds for any $R \in [R_{min}^{\Lambda}, R_{max}^{\Lambda}]$.

Appendix D. Competitive equilibrium with $r > 0$

In this appendix, we show how the deposit rate r is determined and solve for the competitive equilibrium in the set up where $r > 0$.

Lifetime utility (welfare) of a given household in the infinite horizon, continuous-time version of the model is equal to

$$W^h(E) = \max_{C_t^h, D_t^h} \quad \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(C_t^h + \lambda D_t^h\right) dt | E_0 = E\right]. \tag{D1}$$

where C^h and D^h denotes an individual household's consumption and deposits (the aggregate values are denoted by C and D). A household's deposits evolve according to

$$dD_t^h = \left[rD_t^h - C_t^h + \pi_F^h\left(E_t\right)\right] dt + d\Delta_t^h - \left(1 + \gamma\left(E_t\right)\right) dI_t^h, \tag{D2}$$

where π_F^h denotes a household's share in aggregate firm profits and likewise, $d\Delta_t^h$ and dI_t^h denote an individual household's share of dividend payments and recapitalizations by banks. We can now use (D2) to eliminate C^h from (D1):

$$\begin{aligned} W^h(E) = & \max_{D_t^h} \quad \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(\left(r+\lambda\right) D_t^h dt - dD_t^h dt\right) | E_0 = E\right] \\ & + \mathbb{E}\left[e^{-\rho t}\pi_F^h\left(E_t\right) dt | E_0 = E\right] + E_t^h u\left(E_t\right) \\ = & \max_{D_t^h} \quad \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(r+\lambda-\rho\right) D_t^h dt | E_0 = E\right] \\ & + \mathbb{E}\left[e^{-\rho t}\pi_F^h\left(E_t\right) dt | E_0 = E\right] + E_t^h u\left(E_t\right). \end{aligned}$$

Note that E^h denotes an individual household's share in aggregate bank equity and the last equality follows from integration by parts. It is then immediate, that for all interior values of D_t^h, it has to hold that

$$r = \rho - \lambda.$$

Now we consider the case that $\rho > \lambda$, implying that $r > 0$ and, thus, the dynamics of equity value of an individual bank are given by:

$$de_t = re_t dt + k_t[(R(E_t) - p - r)dt \quad - \sigma_0 dZ_t] - d\delta_t + di_t. \tag{D3}$$

The aggregate equity of the banking sector evolves according to:

$$dE_t = [K(E_t)(R(E_t) - p - r) + rE_t]dt - \sigma_0 K(E_t)dZ_t - d\Delta_t + dI_t. \tag{D4}$$

Solving the shareholders' maximization problem in the same way as we did in the proof of Proposition 2 and allowing for $k > 0$ yields us two equations:

$$\frac{u'(E)}{u(E)} = -\frac{R(E) - p - r}{\sigma_0^2 K(E)}, \tag{D5}$$

$$(\rho - r)u(E) = [rE + K(E)(R(E) - p - r)]u'(E) + \frac{\sigma_0^2 K^2(E)}{2}u''(E). \tag{D6}$$

Substituting $u'(E)$ and $u''(E)$ into (D6), while using the equilibrium condition $K(E) = L[R(E)]$, enables us to express $R'(E)$:

$$R'(E) = -\frac{1}{\sigma_0^2}\frac{2(\rho - r)\sigma_0^2 + (R(E) - p - r)^2 + 2(R(E) - p - r)rE/L[R(E)]}{\Big(L[R(E)] - [R(E) - p - r]L'[R(E)]\Big)}. \tag{D7}$$

Applying the same arguments as in the setting with $r = 0$, we can show that $E_{min} = 0$ and $R_{min} = r + p$. The boundary R_{max} can be computed numerically by solving equation

$$\int_p^{R_{max}} E'(R)\frac{(R - p - r)}{\sigma_0^2 L(R)}dR = \log(1 + \gamma(0)), \tag{D8}$$

where $E'(R) = 1/R'(E)$.

Note that the left-hand side of the above expression is increasing in R_{max}. Hence, there exists a unique solution to (D8), which guarantees the uniqueness of the equilibrium.

Appendix E. Impulse response analysis

In this Appendix we apply the impulse response methodology to study the stability of the deterministic steady state. Such an exercise enables us to illustrate that the system behavior in a stochastic environment can be in sharp contrast to the behavior predicted by the impulse response analysis.

The usual methodology to analyze the long-term behavior of macro-variables in a DSGE model is to linearize it around the deterministic steady-state and perturb the system by a single unanticipated shock. The equivalent here would be to look at the case where $dZ_t = 0$ for $t > 0$. The dynamics of the system then becomes deterministic and can be described by the ordinary differential equation (linearization is not needed here):

$$dR_t = \mu(R_t)dt,$$

where the initial shock determines $R_0 > p$.

It is easy to see from expression (17) that $\mu(p) = 0$. Hence, the frictionless loan rate ($R_t = p$) is an equilibrium of the deterministic system that is further referred to as the deterministic steady-state (DSS). It is *locally* stable when $\mu'(p) < 0$ and is *globally* stable when $\mu(R) < 0$ for all R. After some computations, it can be shown that

$$\mu'(p) = 2\rho^2\sigma_0^2 \frac{L''(p)}{L(p)}.$$

Hence, the DSS is locally stable when $L''(p) < 0$. Moreover, it also follows from (18), that condition $L''(R) < 0$ ensures global stability.

Illustrative example. Under our usial specifications for the demand for loans, $L(R) = (\overline{R} - R)^\beta$, the volatility of the loan rate is

$$\sigma(R) = \frac{[2\rho\sigma_0^2 + (R-p)^2]\,(\overline{R} - R)}{\sigma_0[\overline{R} + (\beta - 1)R - \beta p]}. \tag{E1}$$

The drift of the loan rate is given by

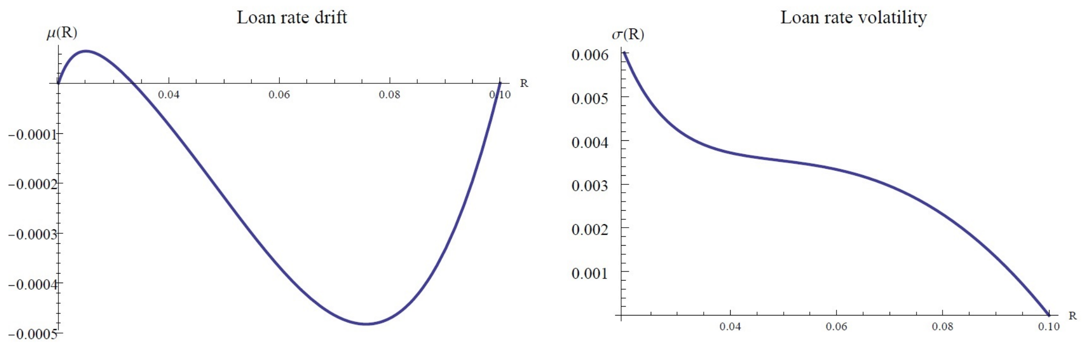
$$\mu(R) = \sigma(R)\frac{\beta(R-p)Q(R)}{2\sigma_0[\overline{R} + (\beta-1)R - \beta p]^2}, \tag{E2}$$

where $Q(R)$ is a quadratic polynomial:

$$Q(R) = (1-\beta)((R-p)^2 - 2\rho\sigma_0^2) - 2(R-p)(\overline{R} - p). \tag{E3}$$

Given the above specification, it can be easily shown that, when $\beta < 1$ (which is equivalent to $L''(R) < 0$), $\mu'(p) < 0$ and $\mu(R) < 0$ in the entire interval $[p, \overline{R}]$. Thus, the DSS is locally and globally stable. By contrast, when $\beta > 1$ (which is equivalent to $L''(R) > 0$), the DSS is locally unstable, i.e., $\mu'(p) > 0$, and there exists a unique $R^* \in (p, \overline{R})$ such that $\mu(R)$ is positive in the region $(0, R^*)$ and negative in the region $(R^*, \overline{R})$ (see Figure 7).

Figure 7: The loan rate drift and volatility, $\beta > 1$



References

Admati, A., DeMarzo, P., Hellwig, M., and P. Pfleiderer, 2011. Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive. Mimeo.

Admati, A., and M. Hellwig, 2013. The Bankers' New Clothes: What's Wrong with Banking and What to Do about It. Princeton University Press.

Aikman, D., Haldane, A. G. and B. D. Nelson, 2014. Curbing the Credit Cycle. The Economic Journal. doi: 10.1111/ecoj.12113.

Becker, B., and V. Ivashina, 2011. Cyclicality of credit supply: Firm level evidence. Journal of Monetary Economics 62, 76-93.

Begenau, J., 2015. Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model. Working Paper.

Berrospide, J. M. and R. M. Edge, 2010. The Effects of Bank Capital on Lending: What Do We Know, and What Does it Mean? International Journal of Central Banking 6, 5-54.

Bernanke, B., Gertler, M., and S. Gilchrist, 1996. The Financial accelerator and the flight to quality. The Review of Economics and Statistics 78, 1 - 15.

Bolton, P., H. Chen, and N. Wang, 2011. A Unifed Theory of Tobin's q, Corporate Investment, Financing, and Risk Management. Journal of Finance, 66, 1545 - 1578.

Bolton, P., H. Chen, and N. Wang, 2013. Market Timing, Investment, and Risk Management. Journal of Financial Economics 109, 40 - 62.

Bridges, J., Gregory, D., Nielsen, M., Pezzini, S., Radia, A. and M. Spaltro, 2014. The impact of capital requirements on bank lending. Bank of England Working Paper n°486.

Brunnermeier, M. K. and Y. Sannikov, 2014. A Macroeconomic Model with a Financial Sector. American Economic Review 104, 379 - 421.

DeAngelo, H., and R. M. Stulz, 2014. Why High Leverage is Optimal for Banks. Fisher College of Business WP 2013-03-08.

Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve, 2011. Free Cash Flow, Issuance Costs, and Stock Prices. Journal of Finance, 66, 1501-1544.

Dell'Ariccia, G., and R. Marquez, 2006. Lending Booms and Lending Standards. Journal of Finance 61, 2511 - 2546.

De Nicolò, G., A. Gamba, and M. Lucchetta, 2014. Microprudential Regulation in a Dynamic Model of Banking. Review of Financial Studies 27, 2097 - 2138.

Diamond, D., and P. Dybvig, 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 99, 689-721.

Diamond, D., and R. Rajan, 2000. A theory of bank capital. Journal of Finance 55, 2431-2465.

Gennaioli, N., Schleifer, A., and R.W. Vishny, 2013. A Model of Shadow Banking. Journal of Finance 68, 1331 - 1363.

Gornall, W., and I. A., Strebulaev, 2015. Financing as a Supply Chain: The Capital Structure of Banks and Borrowers. Working Paper.

Gorton, G., 2010. Slapped by the Invisible Hand: The Panic of 2007. Oxford University Press, Oxford, UK.

Gorton, G., Pennacchi, G., 1990. Financial intermediaries and liquidity creation. Journal of Finance 45, 49-72.

He, Z. and A. Krishnamurthy, 2012. A Model of Capital and Crises. Review of Economic Studies 79, 735-77.

He, Z. and A. Krishnamurthy, 2013. Intermediary Asset Pricing. American Economic Review 103, 732 - 770.

Hellwig, M., 2015. Liquidity provision and Equity Funding of Banks. Discussion Paper, Max Planck Institute.

Hennessy, C. A. and T. M. Whited, 2007. How costly is external financing: Evidence from a structural estimation. Journal of Finance 62, 1705 - 1745.

Holmström, B., Tirole, J., 1997. Financial intermediation, loanable funds, and the real sector. Quarterly Journal of Economics 112, 663-691.

Holmström, B., Tirole, J., 1998. Private and public supply of liquidity. Journal of Political Economy 196, 1-40.

Hugonnier, J. N., and E. Morellec, 2015. Bank Capital, Liquid Reserves, and Insolvency Risk. Working Paper, Swiss Finance Institute and EPFL.

Institute for International Finance, 2010. Interim Report on the Cumulative Impact on the Global Economy of Proposed Changes in Banking Regulatory Framework, Washington, DC: Institute for International Finance.

Isohätälä, J., Milne A., and D. Roberston, 2014. The net worth trap: investment and output dynamics in the presence of financing constraints. Bank of Finland Research Discussion Paper No. 26/2014.

Jeanblanc, M., and A. N. Shiryaev, 1995. Optimization of the flow of dividends. Russian Mathematical Surveys 50, 257 - 277.

Jiménez, G., and J. Saurina, 2006. Credit Cycles, Credit Risk, and Prudential Regulation. International Journal of Central Banking 2, 65-98.

Kiyotaki, N. and J. Moore, 1997. Credit Cycles. Journal of Political Economy 105, 211 - 48.

Lee, I., S. Lochhead, J. Ritter, and Q. Zhao, 1996. The Costs of Raising Capital. Journal of Financial Research 19 , 59-74.

Lucas, R., 1978. Asset Prices in an Exchange Economy. Econometrica 46, 1429 - 1445.

Martinez-Miera, D., and R. Repullo, 2010. Does Competition Reduce the Risk of Bank Failure? Review of Financial Studies 23, 3638 -3664.

Martinez-Miera, D., and J. Suarez, 2014. A Macroeconomic Model of Endogenous Systemic Risk Taking. Mimeo, CEMFI.

Meh, C., and K. Moran, 2010. The role of bank capital in the propagation of shocks. Journal of Economic Dynamics and Control 34, 555 - 576.

Nguyên, T., 2014. Bank Capital Requirements: A quantitative Analysis. Working Paper.

Pandit, V., 2010. We must rethink Basel, or growth will suffer. Financial Times, 10 November.

Phelan, G., 2014. Financial Intermediation, Leverage, and Macroeconomic Instability. Working Paper.

Stein, J., 2012. Monetary Policy as Financial Stability Regulation. Quarterly Journal of Economics 127, 57-95.

TheCityUK report, October 2013. Alternative finance for SMEs and mid-market companies.

Van den Heuvel, S. J., 2008. The welfare cost of bank capital requirements. Journal of Monetary Economics 55, 298 - 320.