Informative Stock Prices and Optimal Managerial Style*

Fernando Anjos† Chang-Mo Kang‡

Abstract

We model a firm that has an investment opportunity of uncertain quality, and who can hire one of two styles of managers: an operational manager, who has inside private information about the project, or a financial expert, who in addition to inside information can access complementary information embedded in stock prices. Our model shows that firms may prefer operational managers, despite their informational disadvantage. This obtains because although both managerial styles are equally myopic, the knowledge of the specific beliefs held by the market creates a much stronger incentive for financial experts to cater to these beliefs, ignoring valuable inside information. On the other hand, operational managers distort investment policies in an attempt to signal their private information, and this distortion may lead to firms preferring financial experts, even if financial experts discard all inside information and inside information is more important than financial-market information. Our model implies that operational managers are preferred for “hard projects”, characterized by either being long shots (low probability of success) or having low return conditional on success, a result that is consistent with some evidence on firm-executive matching. Finally, our model also delivers the counter-intuitive prediction that under certain conditions operational managers are only preferred for high enough stock-price informativeness.

November 18, 2014

JEL classification: D82, G14, G31.

Keywords: informative stock prices, financial expertise, managerial myopia, asymmetric information.

*The authors thank comments from and discussions with Andres Almazan, Aydoğan Altı, Adolfo de Motta, and John Hatfield. The authors also thank comments from seminar participants at the University of Texas at Austin and conference participants at the 2014 FIRN Annual Conference (Australia).

†University of Texas at Austin, McCombs School of Business, 2110 Speedway Stop B6600, Austin TX 78712. Telephone: (512) 232-6825. E-mail: fernando.anjos@mccombs.utexas.edu

‡University of New South Wales, Australian Business School, Sydney, NSW 2052, Australia, Email: chang.kang@unsw.edu.au
1 Introduction

A strand of research in finance makes the case that managers can create value by using information embedded in stock prices (Dow and Gorton, 1997; Titman and Subrahmanyam, 1999). For example, a firm that is considering to expand into a new geography can attempt to infer the market’s view about this investment opportunity and update its original beliefs accordingly. Since financial markets potentially aggregate the information of many agents in the economy, it seems plausible that such information is relevant, at least at the margin.

If learning about information embedded in financial prices is important for the firm, shareholders should in principle prefer managers who are savvy about financial markets, since extracting the relevant information from stock prices may require this kind of expertise. First, the stock price of a particular firm is driven by a number of factors, and it is probably non-trivial to understand the market’s view about the marginal project. Second, some of the relevant information may also be reflected in the prices of other stocks, namely those of rival, customer, and supplier firms. Simultaneously analyzing multiple assets is both complex and time-consuming; finance-savvy managers may plausibly carry out such an analysis both more effectively and efficiently.

Building on the above motivation, our paper develops a simple model to investigate the desirability of manager financial expertise, taking an information-economics perspective. According to the model, and perhaps counter-intuitively, a firm’s shareholders may actually be better off with a manager who is not savvy about financial markets, even if (i) all managers (savvy about finance or not) demand the same wage and (ii) have the same inside information, i.e., information that is complementary to that of the stock market. This obtains because we assume managers (savvy about finance or not) are myopic — as claimed in much finance research (Stein, 1989; Aghion and Stein, 2008) — and finance-savvy managers are more inclined to decide in a way that caters to stock-market beliefs and ignores inside information. Since non-finance-savvy managers, in the extreme, do not possess any knowledge about stock-market beliefs, they are more shielded from this perverse catering effect (though not entirely,
as we show later). On the other hand, non-finance-savvy managers still have an incentive to signal their own (inside) information about project quality, which distorts their investment policies relative to those that are optimal conditional on their information set. It turns out that under certain conditions this distortion makes finance-savvy managers the better choice for shareholders, even if (i) these managers choose to ignore all inside information and (ii) inside information is a stronger signal about project quality than financial-market information.

Our key contribution is a model that is helpful in framing the information-driven trade-offs associated with executive financial expertise. Beyond the more theoretical contribution, the model may potentially explain some observed empirical patterns. First, the strong perverse catering incentive of finance-savvy managers may justify why a large fraction of firms employs non-finance-savvy CEO’s, even though both finance-savvy and non-finance-savvy executives command a similar pay (Custódio and Metzger, 2014).1 Second, our comparative statics results are consistent with the observed patterns of executive-firm matching documented in Custódio and Metzger (2014), as we discuss after introducing the model in more detail.

Now we turn to the presentation of our theoretical setup. We model a publicly traded firm that has an investment opportunity of uncertain quality. The stock market observes an informative signal about the project, and this signal is observable to the firm’s manager only if she is a financial expert. On the other hand, if the manager is an operational manager, she only observes a private signal about project quality. We interpret this signal as firm-level inside information, and it is also observable by financial-expert managers. We refer to the dichotomy operational manager / financial expert as managerial style and we are interested in understanding which style of manager the firm should hire.2

---

1 In Custódio and Metzger (2014) nearly 60% of non-financial firms hire CEOs who have no work experience in financial sectors.

2 We note that our model is not about some managers—for example, engineers—being able to generate specific types of information about project quality that are not available to other managers. Rather, the model compares managers who are privy to all information inside the firm, but where some managers are
The timeline is as follows. Initially the firm hires a specific style of manager. Then, true project quality is realized, and the inside and stock-market signals are generated; we assume these signals are conditionally independent. Once signals are generated, managers make their investment decision, after which the market updates its beliefs about project quality. These updated beliefs are incorporated into interim stock prices. Finally, at the last stage, the project’s true payoff is realized, with concomitant adjustment to stock prices. We assume the firm’s shareholders have a long-run view and would prefer the manager to maximize expected realized payoffs. In contrast, we assume managers are myopic and wish to maximize interim stock prices instead, in the spirit of Stein (1989).

Given the assumption of myopia, in equilibrium financial experts have no interest in using inside information for making their investment decision. To see this, suppose the manager obtains very positive information about a project, but knows that the market received a bad signal. In the interesting case, where average project quality is not too high, equilibrium low-stock-market-signal prices upon investment are below low-stock-market-signal prices upon no investment. Therefore, the financial-expert manager rationally passes up this project, despite the positive inside information, and, actually, irrespectively of how informative the inside signal is relative to the financial-market signal.\(^3\)

Now we turn to operational managers. These managers are unaware of the specific signal obtained by the stock market but are equally myopic. Specifically, all operational managers care about are interim stock prices, and they use their own information in the way that is most useful for predicting these prices. In particular, when operational managers have positive inside information, this means that it is likely that the stock market received a good signal as well. This creates an incentive for operational managers to invest based on good news. On the other hand, the fact that investment is good news about project quality for the stock market creates an incentive for the manager to over-invest, trying to pass bad projects able to access an additional source of information (the stock market), whereas others are not.\(^3\)

\(^3\)A caveat is in order. This result is somewhat extreme due to the simplifying assumption that managers place zero weight on long-run firm value.
as good, a distortion which is absent in the financial expert’s case.\textsuperscript{4}

Above we outlined each managerial style’s incentives and investment behavior. The key question of our paper is what determines the optimal hiring choice from the perspective of long-run shareholders. We start by noting that the precision of the inside signal needs to be stronger than that of the stock market for operational managers to even have a chance. This is intuitive, since all operational managers are doing is using inside information to forecast the stock-market signal, and for there to be a positive externality for shareholders associated with the manager’s behavior (as compared to financial experts), the information they act on needs to be more correlated with true project quality than the stock-market signal itself. We also believe that this ranking of signal precisions is probably realistic, and we take it as an assumption in the remainder of our analysis. Under such assumption, we obtain two main results: (i) operational managers are preferred for “hard projects”, i.e., projects with low probability of success or low return conditional on success; (ii) operational managers are always preferred for informative-enough stock prices.

To see why operational managers are preferred for hard projects, note that their investment distortion is one-sided: they always invest in projects with good inside information, which is very efficient given the strength of this information; but they sometimes invest in bad projects as well, in an attempt to pass them as good. The incentive to engage in this behavior is however limited, and in particular it is disciplined by interim stock prices. Once the manager has bad news, she knows it is likely that the market has bad news as well. If this is true, then the realized interim price upon investment, in the case of hard projects, is quite low. Therefore, the marginal incentive to invest becomes low as well. In fact, in equilibrium the operational manager ends up being just indifferent between investing or not,\textsuperscript{5} which gives shareholders a baseline level of performance associated with operational managers (the value

\textsuperscript{4}The market is however not fooled in equilibrium and properly discounts the price by incorporating the possibility of inefficient investment. See also Holmström (1999) for details.

\textsuperscript{5}In our setting, full separation does not obtain in equilibrium since, in contrast to the traditional costly signaling model (e.g., Spence, 1973), the cost of signaling (i.e., investment) is not type-specific under the market belief that supports the full separating equilibrium. In that sense, our model is also related to cheap-talk games (e.g., Crawford and Sobel, 1982).
of not investing). Having this baseline level of performance is important for projects that are not too attractive economically, on average. In contrast, financial experts’ only choice for hard projects may be to never invest: Even if they have some information (stock-market signal) about their quality, investing just based on good news may still deliver a negative NPV.\(^6\)

Next we turn to the result that operational managers are always preferred for informative-enough stock prices. We start by noting that both styles’ performance increases in stock-price informativeness: financial experts differentiate better between good and bad projects, and operational managers have fewer incentives to pass bad projects as good, since they assign a higher likelihood that the stock market also has bad news. To understand the result, it is useful to focus on the case where inside signals have full precision and projects are “easy”, i.e. always investing still delivers positive NPV. In this case, both styles perform equally well at the extremes of zero stock-market signal precision and full stock-market signal precision. With zero precision both styles always invest, with full precision both styles act on true project quality. As precision increases from zero, at some threshold financial experts are able to use such information and pick only good projects. From then on, their performance increases linearly with precision. On the other hand, operational managers’ performance initially responds little to increases in the precision of the stock-market signal. This obtains because there is a perverse effect associated with higher stock-market signal precision, namely that stock prices conditional on good stock market news are higher. These higher stock prices, at the margin, create an incentive for mimicking. This effect becomes absent once precision is high enough, since the low-type operational manager assigns a very small chance that the stock market received good news. It turns out that the disproportionately effective role of stock markets in disciplining operational managers for high enough signal precision makes these managers eventually surpass financial experts.\(^7\)

\(^6\)Indeed, we show that for \textit{any} level of stock-price informativeness one can always find hard enough projects that operational managers are preferred.

\(^7\)See also Ferreira, Ferreira, and Raposo (2011) who consider the role of informative stock prices as a substitute for manager monitoring by the board.
We now briefly review the research that our paper most closely relates to. Following the insights of Dow and Gorton (1997) and Titman and Subrahmanyam (1999), much literature ensued, including studies providing empirical evidence for the argument that firms improve their decisions by learning from stock prices (Chen et al., 2007; Bakke and Whited, 2010). Our contribution to this literature is theoretical, and we depart from existing research by developing a model with two added frictions: (i) managers are assumed to be myopic and (ii) there is heterogeneity in managerial ability to extract stock-price information. As argued above, the myopia assumption plays an important role in our results, but it is not by itself a limitation to the beneficial effect of stock-price information. Instead, we simply point out that under myopia it may be better not to learn from stock prices, since such information can potentially crowd out more valuable firm-level information. On the other hand, and even with myopia present, the signaling-driven investment distortions associated with firm-level information may actually make it preferable to rely just on stock-market information.

Our paper also contributes to the literature which analyzes investment distortions induced by the manager’s incentive to influence the opinion of stock-market participants (Jensen, 2004; Aghion and Stein, 2008; Kedia and Phillipon, 2009). In particular, our model is closely related to Kedia and Phillipon (2009), who show that firms engage in significant hiring and investment in order to misrepresent their investment opportunities to the market. To our best knowledge, however, our paper is the first to present a framework that compares the effects of managerial myopia (the disadvantage of financial experts) with the effects of perverse signaling incentives (the disadvantage of operational managers). The determinants of which is the overriding concern are not trivial, and, by shedding light on this economic tension, our model hopefully furthers our understanding of executive-firm matching.

Our paper further relates to empirical studies on the work experience of CEOs (Custódio, Ferreira, and Matos, 2013; Custódio and Metzger, 2013, 2014). The study most closely related to our paper is Custódio and Metzger (2014), who investigate the financial policies

---

8See, e.g., Bond, Edmans, and Goldstein (2012) for the recent literature review.
of firms managed by financial-expert CEOs, i.e., executives who have work experience in financial sectors. Custódio and Metzger (2014) find that these CEOs are more financially sophisticated and, as mentioned before, less likely to work in younger companies with higher investment rates and lower profitability. Notwithstanding many explanations being possible for this matching pattern, it is consistent with our result that operational managers are critical for “hard projects”.

Finally, our paper is related to a strand of corporate governance literature. Harris and Raviv (2010) and Chakraborty and Yilmaz (2013) study the optimal governance structure using cheap-talk models. More specifically, Harris and Raviv (2010) ask when corporate shareholders should be given authority over corporate decisions, and Chakraborty and Yilmaz (2013) consider optimal governance in terms of board structure and authority allocation (manager vs. board). Much of these two papers’ effort goes into trying to understand which information structures are desirable, and such spirit is also shared by our research question—operational managers and financial experts are merely different information structures. The similarities of the questions and modeling approach notwithstanding, our model is different in that managers are myopic (and not preference-biased in the sense of cheap-talk games) and the two-sided private information is of a particular kind. Whereas in Harris and Raviv (2010) and Chakraborty and Yilmaz (2013) both players have, in a qualitative sense, equally relevant pieces of information for inferring the truth about project payoffs, in our model operational managers, in the economically interesting case, already know the truth. What these managers do not know is what the other player (stock market) knows about this truth, and interestingly this by itself is relevant.

The remainder of the paper is organized as follows. Section 2 develops the baseline setup, which has a simplified information structure, and section 3 characterizes the equilibrium. Section 4 investigates the conditions under which each style of manager is preferred. Section 5 presents a version of the model for a more general information structure. Section 6 concludes. All proofs and some intermediate results are contained in the appendix.
2 Baseline setup

In this section we analyze our baseline setup, which employs a simplified information structure and focuses on the most interesting results. The more general version of the model is presented in section 5.

2.1 Firm

We consider an all-equity firm that operates in a risk-neutral economy where the market rate of return is normalized to zero. The firm’s assets consist of unit cash and a growth option. The growth option is a project that requires unit investment and yields a random terminal payoff denoted by \( r \). The project is successful with probability \( p \), paying off \( R > 1 \); otherwise the project payoff is zero. To facilitate presentation, we define an indicator \( k \in \{0, 1\} \), where \( k = 1 \) if the firm invests in the project and \( k = 0 \) otherwise. Notice then that the terminal value of the firm can be written as

\[
v := kr + (1 - k).
\]

The firm is run by a manager who makes the investment decision on behalf of shareholders. We denote the manager’s (possibly mixed) strategy by \( \sigma \), and it stands for the likelihood that the firm invests in the project.

2.2 Information structure and managerial style

The firm is publicly traded in a competitive stock market.\(^9\) In the general model, which we defer to section 5, both managers and investors receive informative signals about \( r \). To facilitate presentation and focus on the most interesting features of the model, the baseline setup makes the stark assumption that managers are perfectly informed about \( r \),\(^{10}\) whereas

---

\(^9\)We abstract from asymmetric-information problems between investors in the stock market. For these issues, see Grossman and Stiglitz (1980), Kyle (1985), and Kyle (1989), among others.

\(^{10}\)We show later that our main results are robust to perturbing this stark assumption.
investors receive a signal $s \in \{S_L, S_H\}$ which discloses the actual state of $r$ with probability $q \geq 1/2$. Investors also use the information contained in the managerial investment decision $k$ and update their beliefs about $r$ accordingly.

The distinctive feature of our model is that not all managers possess the same information. Specifically, we consider a binary managerial style, denoted by $\theta \in \{FE, OM\}$. The FE style refers to financial experts, who observe both the project payoff $r$ and the market signal $s$. The OM style refers to operational managers, who only observe $r$. Recovering the signal $s$ from observing stock market activity is plausibly non-trivial, since prices are noisy and affected in complex ways by many different factors. Furthermore, $s$ could be revealed in part from looking at the prices of multiple assets jointly (for example, stocks of competitor firms or suppliers). Financial experts can thus be interpreted as managers who are savvy about stock markets and who are able to correctly back out the market’s belief about the firm’s marginal projects.

2.3 Timeline and objective functions

The sequence of events unfolds as follows:

- $t = 0$: Shareholders appoint $\theta$-style managers.
- $t = 1$: Managers observe $r$ and the stock market receives a signal $s$. The signal $s$ is also observed by managers if $\theta = FE$.
- $t = 2$: Managers choose whether to invest ($k = 1$) or not ($k = 0$). The stock market observes $k$ and forms a competitive interim price $u$ by updating its beliefs about $r$.
- $t = 3$: The terminal value of the firm $v$ is realized.

Notice that in this setting the stock price is formed at $t = 2$ after the market observes the signal $s$ and the firm’s investment $k$, and then updated at $t = 3$ after $r$ is realized.
Shareholders only care about the long run, and they wish to maximize \( v \). On the other hand, managers do not focus on long-term performance \((t = 3)\), but rather maximize the interim stock price \((t = 2)\). This assumption of \textit{managerial myopia} is in line with previous research, e.g., Stein (1989). Myopic managers only care about what the stock market believes about the project,\(^{11}\) and this plays an important role in our results.

3 Managerial style and equilibrium investment strategies

In this section we characterize the investment strategy chosen by each manager style, solving the subgame that starts at period \( t = 1 \). Before proceeding, it is useful to define \( \mu(k, s) \) as the posterior probability that the market assigns to the event \( r = R \) after observing the investment decision \( k \) and the signal \( s \). Then, the interim stock price formed at \( t = 2 \) can be written as follows:

\[
    u(k, s) = 1 + k(\mu(k, s)R - 1) \tag{2}
\]

The interim price is then the baseline value of the firm (unit cash) plus, conditional on investment occurring, the expected net present value of the project. Myopic managers choose the investment strategy \( \sigma \) (probability of investing) that maximizes the expected interim price conditional on their information set, i.e., \( r \) for operational managers and \((r, s)\) for financial experts.

3.1 Operational-manager subgame

Consider the case in which shareholders hire operational managers at \( t = 0 \). Since the stock market forms the interim price \( u \) after observing the firm’s investment \( k \) and the signal \( s \), managers who have profitable projects (i.e., \( r = R \)) would like to convey this information

\(^{11}\)For simplicity our managers are fully myopic. We make this assumption to facilitate exposition; the main results would still hold if managers do not place much weight on long-run outcomes.
to the market via their investment choice. To be a credible signal about $r$, however, the
investment choice of those managers should not be easily mimicked by other managers who
have unprofitable projects (i.e., $r = 0$). As formally shown below, it is indeed the case that
operational managers invest more frequently in profitable projects in equilibrium and, thus,
the market rationally takes the firm’s investment as a positive signal about the project’s
payoff $r$. We first discuss the intuition associated with this signaling mechanism and then
formally solve for the equilibrium investment strategy of operational managers.

3.1.1 Overview of the signaling mechanism

In this section, we provide the preliminary intuition for the signaling mechanism that emerges
as an equilibrium outcome in the operational-manager subgame. We start by noting, as we
demonstrate later, that the stock market views the firm’s investment as economic only after
receiving a good signal $S_H$. Therefore myopic managers would ideally maximize the interim
price by investing only when the market receives $S_H$. However, given their lack of information
about $s$, operational managers cannot condition their investment decision on $s$ but rather
they attempt to infer $s$ from the actual state of $r$. Such an inference is feasible since profitable
projects are more likely to receive a good market signal.\footnote{Formally, $\text{Prob}(s = S_H|r = R) = q > \text{Prob}(s = S_H|r = 0) = 1 - q$.} Therefore, operational managers
who have profitable projects rationally assign a higher probability to the state $s = S_H$ and
have an incentive to invest more frequently (i.e., select a higher $\sigma$). In equilibrium, the market
expects such an investment choice of operational managers and regards the investment itself
as a positive signal about the project’s profitability.

To better illustrate this intuition, consider the interim price $u(1, s)$ formed at $t = 2$.
For notational convenience, we denote by $\sigma_R$ (resp. $\sigma_0$) the investment strategy chosen by
operational managers who have profitable (resp. unprofitable) projects. From Bayes’ rule,
the posterior probabilities $\mu(1, S_H)$ and $\mu(1, S_L)$ can be written as:

\[
\begin{align*}
\mu(1, S_H) &= \frac{pq\sigma_R}{pq\sigma_R + (1 - p)(1 - q)\sigma_0}, \\
\mu(1, S_L) &= \frac{p(1 - q)\sigma_R}{p(1 - q)\sigma_R + (1 - p)q\sigma_0},
\end{align*}
\]

Notice that for any $\sigma_R$ and $\sigma_0$, $\mu(1, S_H) \geq \mu(1, S_L)$, i.e., the stock market rationally assigns a higher probability to the state of $r = R$ (resp. $r = 0$) after receiving the signal $S_H$ (resp. $S_L$). From (2), we also find that this result implies $u(1, S_H) > u(1, S_L)$, i.e., the interim price is higher when the market receives a good signal $S_H$.

Now we turn to the expected interim price, denoted by $\bar{u}_R$ (resp. $\bar{u}_0$), from the standpoint of operational managers who invest in profitable (resp. unprofitable) projects. From (2), the expected interim price can be written as:

\[
\bar{u}_i = E_i[\mu(1, s)]R,
\]

where $E_i[\mu(1, s)] = \text{Prob}(s = S_H|r = i)\mu(1, S_H) + [1 - \text{Prob}(s = S_H|r = i)]\mu(1, S_L)$ for $i \in \{0, R\}$. Notice that $E_R[\mu(1, s)] > E_0[\mu(1, s)]$ since the actual state of $r$ is informative about the market signal $s$, i.e., $\text{Prob}(s = S_H|r = R) > \text{Prob}(s = S_H|r = 0)$. Therefore, $\bar{u}_R > \bar{u}_0$, which implies that operational managers expect a higher interim price when they invest in profitable projects. To illustrate the signaling effect of investment, we can decompose $\bar{u}_R$ as follows:

\[
\bar{u}_R = \bar{u}_0 + R(E_R[\mu(1, s)] - E_0[\mu(1, s)]),
\]

Equation (5) shows that myopic managers with unprofitable projects should invest less frequently since the market is more likely to receive a bad signal about their investment. In this regard, we can interpret the increment in the expected interim price from investment in profitable projects, $R(E_R[\mu(1, s)] - E_0[\mu(1, s)])$, as the signaling benefit.

As formally shown in section 3.1.2, full separation does not occur in our setting since,
if the market regards the investment as a perfect signal about \( r = R \) in equilibrium, managers could fool the market by investing in unprofitable projects.\(^{13}\) The partially revealing equilibrium implies that the signaling mechanism cannot fully address the inefficiency that arises from managerial myopia. Furthermore, partial revelation also implies that the market signal \( s \) is still informative about the project’s payoff \( r \) in equilibrium. If signals were not informative in equilibrium, there would be no wedge between interim prices \( u(1, S_H) \) and \( u(1, S_L) \). Such a wedge is necessary for credible signaling to take place, otherwise both low- and high-type managers would face the same expected interim price, and there would be no scope for credible signaling. This discussion also makes it straightforward that stock-market informativeness \( q \) cannot be too close to 1/2 for separation to be possible, otherwise the wedge between \( u(1, S_H) \) and \( u(1, S_L) \) becomes very small.

### 3.1.2 Characterizing equilibrium

Now we formally solve the operational-manager subgame and characterize the equilibrium, which confirms our intuition discussed in section 3.1.1. As is usual in signaling games, multiple Perfect Bayesian equilibria exist in our setting. In particular, there always exists a pooling equilibrium in which operational managers never invest and the market has off-equilibrium-path beliefs that managers invest only in unprofitable projects. Such an equilibrium is uninteresting and seems unreasonable if there exist more efficient equilibria or if we employ refinement techniques such as those proposed in Kreps and Wilson (1982) or Cho and Kreps (1987). In our analysis, we adopt the equilibrium selection procedure outlined in assumption 1, which we view as reasonable.

**Assumption 1** If a separating equilibrium exists, we select the most efficient. If not, we select the most efficient pooling equilibrium. \(^{13}\) [For each managerial style subgame, we select the most efficient perfect Bayesian equilibria in which the firm’s expected value at \( t = 0 \) is maximized.]

\(^{13}\)See Kedia and Phillipon (2009) for other models that produce a partially revealing equilibrium.
Assumption 1 is less restrictive than it seems, since separation in the operational-manager subgame, when feasible, is a unique equilibrium. Furthermore, these separating equilibria generate firm values above the baseline threshold of 1 (cash holdings). The implications of assumption 1 for the financial-expert subgame are more subtle and will be discussed later. Proposition 1 below provides an initial equilibrium characterization.

**Proposition 1** In equilibrium, the following is the case:

1. If $R \geq \overline{R}_{OM} := \frac{pq+(1-p)(1-q)[p(1-q)+(1-p)q]}{pq(1-q)}$, the manager invests in all projects with certainty. Otherwise, the manager always invests in profitable projects and randomly invests in unprofitable projects with a non-zero probability. Formally,

   \[
   \begin{cases}
   \sigma_R = \sigma_0 = 1 & \text{if } R \geq \overline{R}_{OM} \\
   \sigma_R = 1, \sigma_0 \in (0, 1) & \text{otherwise.}
   \end{cases}
   \]

   (6)

2. Equilibrium beliefs $\mu(k, s)$ are given by

   \[
   \begin{align*}
   \mu(1, S_H) &= \frac{pq}{pq+(1-p)(1-q)\sigma_0} \\
   \mu(1, S_L) &= \frac{p(1-q)}{p(1-q)+(1-p)\sigma_0},
   \end{align*}
   \]

   which implies $\mu(1, S_H) > \mu(1, S_L)$ (i.e., $S_H$ is strictly good news when investment takes place).

3. If $\sigma_0 \in (0, 1)$ in equilibrium, the market forms higher interim price for investment than for no-investment only when it receives $S_H$, i.e., $u(1, S_H) > 1 > u(1, S_L)$.

Proposition 1 shows that equilibria are one of two (mutually-exclusive) categories, depending on parameter region: (i) pooling, in which the manager always invests in both types of projects; and (ii) separating, in which the manager invests more frequently in profitable projects. As discussed in section 3.1.1, the manager invests in unprofitable projects with a non-zero probability in the separating equilibrium. The equilibrium market belief described
in (7) shows how the market rationally discounts the value of investments for the possibility that the manager invests in unprofitable project (i.e., $\sigma_0 > 0$). As we discuss later, this mechanism is important in deterring the manager from over-investing in unprofitable projects. Given the equilibrium strategies taken by operational managers, the expected value of the firm at $t = 0$ can be written as:

$$v_{OM} := E_{t=0} [v|m=OM] = pR + (1-p)(1-\sigma_0).$$  \hfill (8)

Equation (8) shows that the “cost” associated with appointing operational managers corresponds to the expected loss from investing in unprofitable projects (i.e., $(1-p)\sigma_0$). Notice that while the market is not fooled in equilibrium, the manager is “trapped” in the inefficient action which is to invest in unprofitable projects with some probability. In most of the analysis below, we will focus on the separating equilibrium, since the pooling equilibrium is trivially dominated by the financial-expert subgame equilibrium.

To solve for the equilibrium strategy $\sigma_0$, we use the standard indifference condition that the investment in unprofitable projects does not affect the manager’s expected utility (i.e., the expected interim price). Formally, this condition corresponds to

$$(1-q)u(1,S_H) + q u(1,S_L) = 1 \iff E_{0} [\mu(k=1,s)] R = 1. \hfill (9)$$

Using the results from proposition 1 together with condition (9), we fully characterize the equilibrium as described in proposition 2:

**Proposition 2** If $R < \overline{R}_{OM}$, there always exists a unique separating equilibrium in which $\sigma_0$ is the largest root to the quadratic equation below:

$$\sigma_0^2(1-p)^2q(1-q) + \sigma_0p(1-p) \left[ q^2 + (1-q)^2 - q(1-q)R \right] - p^2q(1-q)(R-1) = 0. \hfill (10)$$
It is also the case that

\[
\lim_{R \to \overline{R}_{OM}} \sigma_0 = 1 \\
\lim_{R \to 1} \sigma_0 = 0.
\]  

Equations (11) and (12) in proposition 2 show that lower \( R \) reduces the managerial incentive to invest in unprofitable projects. This occurs because, as \( R \) approaches 1, the investment in profitable projects gets barely economic and therefore the extent to which managers can attempt to influence the interim price by investing in unprofitable projects becomes very limited. In line with proposition 1, which states that managers always invest in unprofitable projects for \( R \geq \overline{R}_{OM} \), proposition 2 shows that, as \( R \) increases to \( \overline{R}_{OM} \), the equilibrium investment strategy for unprofitable projects also increases to 1.

3.1.3 Project characteristics and the performance of operational managers

In this section we perform some comparative statics exercises and show how the investment behavior and performance of operational managers vary with project characteristics (propositions 3 and 4). We also show that operational managers always create shareholder value relative to the benchmark case of no-investment (proposition 5). This last result plays an important role in the later comparison of operational managers to financial experts.

**Proposition 3** If \( R \geq \overline{R}_{OM} \), i.e., the separating equilibrium exists, the equilibrium investment strategy \( \sigma_0 \) varies locally with each parameter as follows:

\[
\frac{\partial \sigma_0}{\partial R} > 0 \quad (13) \]
\[
\frac{\partial \sigma_0}{\partial q} < 0 \quad (14) \]
\[
\frac{\partial \sigma_0}{\partial p} > 0 \quad (15)
\]
The economic intuition for why $\sigma_0$ increases in $R$ has been discussed in the preceding section: a higher average productivity increases the incentive for the manager to attempt to influence the interim stock price by investing in unprofitable projects. Likewise, proposition 3 shows that higher $p$, which increases the average productivity of projects, makes the manager invest in unprofitable projects more frequently, i.e., higher $\sigma_0$. Finally, the relationship between $\sigma_0$ and $q$ is also straightforward. The more informative the market signal, the more weight the manager with unprofitable projects places on the event that the stock-market received a negative signal, which reduces the incentive to mimic the high type.

**Proposition 4** If $R < R_{OM}$, i.e., the separating equilibrium exists, shareholder ex-ante welfare $\pi_{OM}$ varies locally with each parameter as follows:

\[
\frac{\partial \pi_{OM}}{\partial R} > 0 \quad (16) \\
\frac{\partial \pi_{OM}}{\partial q} > 0 \quad (17) \\
\frac{\partial \pi_{OM}}{\partial p} > 0 \quad (18)
\]

Results in proposition 4 parallel those of the previous proposition. The result is trivial for the case of $q$, since it follows directly from the fact that higher $q$ induces less-frequent investment in low-type projects. However, for $p$ and $R$, two offsetting effects are present: on one hand, the higher $p$ or $R$, the higher the quality of the average project; on the other hand, following the results in proposition 3, an increase in $p$ or $R$ also leads to more-frequent investment in bad projects. In the end, the positive effect dominates.

**Proposition 5** Shareholder ex-ante welfare $\pi_{OM}$ (or average firm value) is always bigger than 1, the benchmark firm value if investment never takes place.

To understand the intuition for the result in proposition 5, note that either the manager does not invest (in which case firm value is 1), or the manager invests and average firm value is a convex combination of $\pi_0$ and $\pi_R$ (the expected interim prices from the perspective of
each type of manager). Since the indifference condition of low-type managers implies $\bar{u}_0 = 1$ and it is also the case that $\bar{u}_R > \bar{u}_0$, average firm value conditional on investment is strictly greater than 1. This implies that unconditional average firm value is also strictly greater than one. Intuitively, it would be impossible in equilibrium to have low-type managers invest so frequently that they destroyed value on average, since this value destruction would show up in rational-expectations interim prices and disincentivize managers from doing so.

### 3.2 Financial-expert subgame

Now consider the subgame in which shareholders hire financial experts at $t = 0$. As with operational managers, financial experts may desire to use investment as a signal for $r = R$. The signaling mechanism however does not work in this case, since financial experts are also informed about $s$. It turns out that for the non-trivial equilibria that we focus on, which generate expected firm value bigger than 1 (cash value), $s$ is a sufficient statistic for determining the strategy of myopic managers. In other words, financial experts ignore true project quality in equilibrium, a result contained in proposition 6 below.\(^{14}\) This result is in line with other literature on myopic managerial behavior, e.g., Stein (1989) or Aghion and Stein (2008).

**Proposition 6** Under the equilibrium-selection assumption 1, all equilibria are dominated (sometimes weakly) by an equilibrium where the investment strategy of financial experts does not depend on the realization of $r$.

Proposition 6 shows that investment does not convey information, and therefore posterior probabilities that $r = R$ are simply

\[
\mu(k, s) = \begin{cases} 
\frac{pq}{pq + (1-p)(1-q)} & \text{if } s = S_H \\
\frac{p(1-q)}{p(1-q) + (1-p)q} & \text{if } s = S_L.
\end{cases}
\]

\(^{14}\)The proof of the proposition contains some additional discussion regarding technical details, that we chose to omit here.
Combining the above with the expression for interim stock price (2), we obtain the optimal strategy for the manager:

\[
\sigma(S_H) = \begin{cases} 
1 & \text{if } R > R_{FE} := \frac{pq + (1-p)(1-q)}{pq} \\
0 & \text{otherwise,}
\end{cases}
\] (20)

and

\[
\sigma(S_L) = \begin{cases} 
1 & \text{if } R > R_{FE} := \frac{p(1-q) + (1-p)q}{p(1-q)} \\
0 & \text{otherwise.}
\end{cases}
\] (21)

Consider the interesting case where \( R_{FE} < R < \overline{R}_{FE} \) and, thus, the manager invests only after observing \( S_H \). Then, the expected value of the firm at \( t = 0 \) when appointing financial experts is given by

\[
\overline{v}_{FE} := E_{t=0}[v|m = FE] = pqR + p(1-q) + (1-p)q,
\] (22)

which naturally increases in \( q \), since higher informativeness of the market signal makes it less likely for the manager to invest in unprofitable projects. The expected value of the firm \( \overline{v}_{FE} \) also increases in \( R \) and \( p \), since the expected value of projects is positively associated with the two parameters.

4 Operational managers vs. financial experts

The previous section characterized the equilibrium investment strategies of each managerial style. In this section, which contains the key analysis of the paper, we compare the performance of financial experts and operational managers, measured by the expected value of the firm at \( t = 0 \), and examine how their performance is affected by the parameters \( R, p \) and \( q \). The comparison of managerial styles is non-trivial, since both financial experts and
operational managers create more value for higher $R$, $p$, and $q$, as shown above.

Before analyzing comparative statics in detail, it is worthwhile to frame the trade-off across managerial styles in general terms. Consider the case of profitable projects, i.e., $r = R$. In this case operational managers always invest, which is in the interest of shareholders. They do so because the stock market positively reacts to investment after receiving a positive signal $S_H$ (i.e., $u(1, S_H) > 1$), and managers who have profitable projects assign a high probability to the event $s = S_H$. That is, the fact that operational managers are imperfectly informed about $s$ makes them not want to pass up any profitable project. This mechanism does not work with financial experts once they know $s = S_L$ to be the case: whenever these managers face a price $u(1, s_L) < 1$, they prefer not to invest even if $r = R$. In short, operational managers are always desirable whenever projects are profitable. The less straightforward part of the trade-off has to do with unprofitable projects, where both managers do not act in the full interest of shareholders: operational managers invest in these projects with unconditional probability $(1 - p)\sigma_0$, and financial experts do so when $s = S_H$, which occurs with unconditional probability $(1 - p)(1 - q)$. For unprofitable projects, it is therefore not clear which managerial style is preferred.

4.1 Variation in average project quality

This section investigates the effects of $R$ (payoff conditional on success) and $p$ (success probability) in the determination of optimal managerial style. Both parameters have similar associated comparative statics, which is intuitive, since both determine overall average project quality. The left (right) panel of figure 1 shows how the performance of each manager style varies with $R$ ($p$).

Figure 1 shows that operational managers are preferred by shareholders for lower $R$ and lower $p$. Propositions 7 and 8 show that this observation is the case in general.

**Proposition 7** Suppose that $R < R_{OM}$, i.e., there exists a separating equilibrium in the operational-manager subgame. Then there always exists a unique threshold $R^* \in (R_{FE}, R_{OM})$
Figure 1: Manager Performance and Variation in Average Project Quality. The figure plots the ex-ante expected value of firms at $t = 0$ in two cases: (i) operational managers (solid black line) and (ii) financial experts (dashed blue line). The left panel varies success payoff $R$ (choice of remaining parameters: $p = 0.5$, $q = 0.6$). The right panel varies success probability $p$ (choice of remaining parameters: $R = 3$, $q = 0.6$).

such that $\bar{v}_{OM} > \bar{v}_{FE}$ if and only if $R < \bar{R}$.

**Proposition 8** There always exists a unique threshold $p^*$ such that $\bar{v}_{OM} > \bar{v}_{FE}$ if and only if $p < p^*$.

The intuition for the existence of thresholds for $R$ and $p$ that make a particular managerial style optimal is related to each style’s “disadvantage”. In the choice of investment strategies, financial experts care only about the market signal $s$. For any level of stock-market signal precision $q$, there is always a sufficiently low average project quality (i.e., low $R$ and/or low $p$), such that these managers never invest. Lower average project quality, on the other hand, leads operational managers to invest more efficiently since, as shown in propositions 3 and 4, the lower payoff of profitable projects reduces the managerial latitude to influence the interim price by investing in unprofitable projects. Therefore, for lower average quality projects, shareholders prefer operational managers who do not pass up profitable projects. As average project quality increases, however, operational managers invest in unprofitable projects more frequently and at some point, their investment distortion becomes severe.
enough to make shareholders prefer financial experts who follow only stock-market signals.

4.2 Variation in stock market informativeness

Next we turn to the effect of stock-market signal precision $q$ on the performance of each manager style. Figure 2 shows how the expected firm values at $t = 0$, i.e., $v_{OM}$ and $v_{FE}$, vary with $q$. Specifically, the left (resp. right) panel represents the case in which the unconditional expected return from investing is negative (resp. positive), i.e., $pR < 1$ (resp. $pR > 1$). As mentioned before, both managers improve with $q$: financial experts act on more accurate signals about project quality, and operational managers have less leeway to pretend that bad projects are good (see proposition 3 for the effect of $q$ on $\sigma_0$). In the extreme case in which the market signal is perfectly informative, i.e., $q = 1$, the performance of both styles of managers converges to first-best.

![Figure 2: Manager Performance and Variation in Stock Market Informativeness.](image)

The left panel of figure 2 shows that operational managers perform better for low $q$. This result obtains because $pR < 1$: when $q$ is small it is better for the financial expert
to pass up all projects, since on average they are low-quality; the operational manager, on the other hand, acts on true project quality (although over-investing) and always delivers some shareholder value, as discussed in the previous section. As $q$ increases beyond a certain threshold, the left panel of figure 2 shows that financial experts start investing and temporarily outperform operational managers.\textsuperscript{15} However, once $q$ is large enough, operational managers dominate once more. This result obtains in general as stated in proposition 9.

**Proposition 9** If $R < \overline{R}_{OM}$, there always exists $q^* < 1$ such that

$$\forall q \in [q^*, 1) : \overline{v}_{OM} > \overline{v}_{FE}. \quad (23)$$

To understand the result in proposition 9, it is useful to focus on the right panel of figure 2, where the average project is valuable. Almost by construction, both managers perform the same at the extremes of $q = 1/2$ and $q = 1$. With $q = 1/2$, both types of managers always invest, the financial expert because there is no information to differentiate projects, the operational manager because the absence of a wedge between the high interim price $u(1, S_H)$ and the low interim price $u(1, S_L)$ eliminates the possibility of credibly signaling project quality via investment behavior. With $q = 1$, on the other hand, the stock market knows the true quality of every project, and both managerial styles have a clear incentive to only invest in good projects.

The interesting question, then, is “why do financial experts dominate for relatively low levels of stock-market informativeness (say $q = 0.7$ in the right panel of figure 2), but the situation reverses for relatively high $q$ (say 0.9)?” To answer this question, we first comment on how the performance of financial experts responds to $q$. As $q$ increases from 1/2, at some point financial experts start investing only after a high signal $S_H$ is generated. Once these managers act in this fashion, their performance improves linearly with $q$, reflecting the ever greater accuracy of the signal in terms of picking the right projects.

\textsuperscript{15}This not need obtain in general. For low enough $pR$, operational managers will dominate financial experts for any $q$. 

23
Operational managers’ performance responds in a more complex way to $q$. To understand how this works, recall the indifference condition of the low-type operational manager:

$$(1 - q) u(1, S_H) + q u(1, S_L) = 1,$$

i.e., the expected interim price, from the perspective of a manager who observes $r = 0$, needs to be 1 (the payoff of not investing) in equilibrium. Suppose we set $q$ at moderate levels and determine the equilibrium rate of investment in low projects ($\sigma_0$) induced by the expression above. What happens if we slightly increase $q$, and how does this relate to the notion that the informed market is disciplining the manager? There are three effects associated with increasing $q$ (assume for a moment that we hold $\sigma_0$ fixed). First, the LHS of the indifference condition places more weight on the low price. Second, the low price $u(1, s_L)$ becomes lower, since the more-informative signal makes it more likely that the true state is $r = 0$. Finally, the high price $u(1, S_H)$ becomes higher, since the more-informative signal makes it more likely that the true state is $r = R$. Whereas the first two effects give the manager an incentive to reduce $\sigma_0$ to re-equilibrate after $q$ increases (making both prices higher), the third effect creates a (partial) perverse incentive to mimic, since the payoff from successfully pretending, the high price, is now higher.

The problem for relatively low $q$ is that the perverse incentive associated with a higher $u(1, S_H)$ is relatively important. This makes the frequency of investment in bad projects $\sigma_0$ relatively unresponsive to increases in $q$. On the other hand, financial experts’ performance responds linearly to increases $q$, as we explained above. Therefore, as we move away from $q = 1/2$, the financial expert initially distances herself from the operational manager. As $q$ further increases, however, the perverse incentive is gradually shut down, since the term $(1 - q)u(1, S_H)$ is converging to zero, and $\sigma_0$ becomes very responsive to $q$. Indeed, this increased responsiveness is enough that it allows operational managers to “catch up” with the linear increases in financial-expert performance. For large values of $q$, operational managers end
up outperforming financial experts. This occurs because high stock-marker informativeness is disproportionately effective at disciplining operational managers.

5 Generalized model

In this section we generalize the baseline model by relaxing the assumption that managers observe the project’s payoff \( r \). Specifically, we consider a setting in which managers receive an informative signal \( m \in \{M_L, M_H\} \) which discloses the actual state of \( r \) with probability \( q_m \geq 1/2 \) and which is conditionally independent of the market signal \( s \) given \( r \): Formally,

\[
Prob(m = M_L | r = 0) = Prob(m = M_H | r = R) = q_m
\]

\[
Prob(s \cap m | r) = Prob(s | r) \times Prob(m | r).
\]

Notice that the baseline model in which managers observe \( r \) corresponds to the case in which \( q_m = 1 \). To focus on the case of interest, we assume that \( E(v|m = M_L, k = 1) = Prob(r = R|m = M_L)R < 1 \), i.e., it is not profitable to invest only after receiving a negative signal \( M_L \). This assumption is natural and crucial in most analysis results.

To facilitate the presentation, we denote the market signal precision (which was denoted as \( q \) in the baseline model) as \( q_s \), i.e.,

\[
Prob(s = S_L | r = 0) = Prob(s = S_H | r = R) = q_s.
\]

As shown in proposition 6, financial experts rely only upon the market signal \( s \) in the choice of investment strategies and thus the equilibrium investment strategy is determined as described in (20) and (21). In the remainder of this section, we solve the model by taking the following steps: we first solve for the equilibrium of operational-manager subgame; and then we find the optimal managerial style by comparing the expected value of \( v \) achieved by each managerial style.
5.1 Operational-manager subgame

Consider a subgame in which operational managers were appointed at \( t = 0 \). Within this setting, as opposed to the baseline model, managers do not observe \( r \) but only receive a signal \( m \) before taking investment strategies. For a notational convenience, we denote as \( \sigma_H \) the investment strategy chosen that operational managers choose after receiving \( m = M_H \) (resp. \( m = M_L \)). As in the analysis of the baseline model, we focus our analysis on the most efficient Perfect Bayesian Nash Equilibrium. While, in the baseline model, \( R > 1 \) is a sufficient condition for the presence of an equilibrium in which the expected value of the firm at \( t = 0 \) is greater than one (i.e., the firm’s value without investment), it is not in this general setting since, even after receiving \( M_H \), the managers face uncertainty about the project’s payoff \( r \). More specifically, as formally shown in lemma 1 in appendix, there exists an equilibrium in which the expected value of the firm at \( t = 0 \) is greater than 1 (i.e., cash holdings) if and only if

\[
R \geq R_{OM}^H := \frac{\Prob(S_H | M_H) \prob(m = H) \prob(s_H, m = H)}{\Prob(S_H | M_H) \prob(m = H) \prob(s_H, m = L) + \Prob(S_L | M_H) \prob(m = L) \prob(s_L, m = L) + \Prob(S_L | M_H) \prob(m = L) \prob(s_L, m = H)} \geq 1
\]

which is the case of interest in our analysis below.

Before proceeding, we redefine \( \mu(k, s) \) which is the posterior probability that the market assigns to the event \( r = R \) after observing the investment decision \( k \) and the signal \( s \): By Bayes’ rule,

\[
\mu(1, S_H) = \frac{p(q_m \sigma_H + (1 - q_m) \sigma_L) q_s}{p(q_m \sigma_H + (1 - q_m) \sigma_L) q_s + (1 - p)(1 - q_m + q_m \sigma_L)(1 - q_s)}
\]

(24)

\[
\mu(1, S_L) = \frac{p(q_m \sigma_H + (1 - q_m) \sigma_L)(1 - q_s)}{p(q_m \sigma_H + (1 - q_m) \sigma_L)(1 - q_s) + (1 - p)(1 - q_m + q_m \sigma_L)q_s}
\]

(25)

Notice that for any \( \sigma_H \) and \( \sigma_L \), \( \mu(1, S_H) \geq \mu(1, S_L) \), i.e., the stock market rationally assigns higher probability to the state of \( r = R \) (resp. \( r = 0 \)) after receiving the signal \( S_H \) (resp. \( S_L \)).

Now we turn to the investment strategy chosen by operational managers. After receiving \( M_i \) (\( i \in \{L, H\} \)), the managers rationally expect that their investment leads to the interim
price as follows:

\[ \overline{u_i} = E_i[\mu(1, s)]R, \]

where \( E_i[\mu(1, s)] = \text{Prob}(s = S_H|m = M_i)\mu(1, S_H) + [1 - \text{Prob}(s = S_H|m = M_i)]\mu(1, S_L) \)

and

\begin{align*}
\text{Prob}(s = S_H|m = M_H) &= \frac{pq_m q_s + (1 - p)(1 - q_m)(1 - q_s)}{pq_m + (1 - p)(1 - q_m)} \quad (26) \\
\text{Prob}(s = S_H|m = M_L) &= \frac{p(1 - q_m)q_s + (1 - p)q_m(1 - q_s)}{p(1 - q_m) + (1 - p)q_m}. \quad (27)
\end{align*}

The equation (27) shows the manager’s belief on \( s = S_H \) given the inside signal \( m = M_H \). Specifically, the numerator can be decomposed as \( pq_m q_s \) and \( (1 - p)(1 - q_m)(1 - q_s) \) which correspond to the probability with which both the market and the manager receive correct signals (i.e., \( r = R \)) and incorrect signals (i.e., \( r = 0 \)), respectively. On the other hand, (27) shows the manager’s belief on \( s = S_H \) given the inside signal \( m = M_L \). Intuitively, the two components in the numerator, \( p(1 - q_m)q_s \) and \( (1 - p)q_m(1 - q_s) \), correspond to the probability with which only the market signal is correct (i.e., \( r = R \)) and only the managerial signal is correct (i.e., \( r = 0 \)), respectively.

Notice that \( \text{Prob}(s = S_H|m = M_H) > \text{Prob}(s = S_H|m = M_L) \), i.e., the manager believes that the market is more likely to receive a good signal when the manager himself receives a positive signal. This result is intuitive since both signals \( s \) and \( m \) are informative about \( r \). It also implies that \( \overline{u}_H > \overline{u}_L \), i.e., after receiving a positive signal \( M_H \), managers assign higher probability to the state that the market also receives a good news \( S_H \) about their project and, thus, forms a higher interim price for their investment. As discussed in section 3.1.1, the higher expected interim price makes the managers invest more frequently after receiving \( M_H \) and in turn the market rationally takes the investment itself as a positive signal about \( m = M_H \) which is also a good signal for the project’s payoff. Proposition formally characterizes the operational-manager subgame equilibrium in which the managers use investment as a signal about their own information:
Proposition 10 In equilibrium, the following is the case:

1. The equilibrium investment strategies are determined as:

\[
\begin{cases}
\sigma_H = \sigma_L = 1 & \text{if } R \geq R_{OM}^e := \frac{\text{Prob}(S_H|M_H)p_{q_S}}{p_{q_S} + (1-p)(1-q_S)} + \frac{1}{p(1-q_S) + (1-p)q_S} \\
\sigma_H = 1, \sigma_L = 0 & \text{if } R \leq R_{OM}^e := \frac{\text{Prob}(S_H|M_H)p_{q_M}q_S}{p_{q_M}q_S + (1-p)(1-q_M)(1-q_S)} + \frac{1}{p_{q_M}q_S + (1-p)(1-q_M)(1-q_S)} \\
\sigma_H = 1, \sigma_L \in (0, 1) & \text{otherwise}
\end{cases}
\]

2. Equilibrium beliefs \(\mu(1,s)\) are given by

\[
\begin{cases}
\mu(1,S_H) = \frac{p_{q_S}(q_M + \sigma_L(1-q_M))}{p_{q_S}(q_M + \sigma_L(1-q_M)) + (1-p)(1-q_S)(1-q_M + \sigma_L q_M)} \\
\mu(1,S_L) = \frac{p(1-q_S)(q_M + \sigma_L(1-q_M))}{p(1-q_S)(q_M + \sigma_L(1-q_M)) + (1-p)q_S(1-q_M + \sigma_L q_M)}
\end{cases}
\] (28)

which implies \(\mu(1,S_H) > \mu(1,S_L)\).

3. If \(R \in (R_{OM}^e, R_{OM})\), \(\sigma_L\) solves

\[R \left[ \text{Prob}(S_H|M_L)\mu(1,S_H) + \text{Prob}(S_L|M_L)\mu(1,S_H) \right] = 1.\] (29)

4. If \(\sigma_L \in [0, 1)\) in equilibrium, the market forms higher interim price for investment than for no-investment only when it receives \(S_H\), i.e., \(u(1,S_H) > 1 > u(1,S_L)\).

Proposition 10 confirms that most characteristics of the operational-manager subgame equilibrium in the baseline model, described in proposition 1, is robust to the possibility that the managers can receive incorrect signal about \(r\). Specifically, for sufficiently high \(R\) (i.e., \(R < R_{OM}^e\)), there exists a pooling equilibrium in which the managers always in the project while, for lower \(R\), there emerges a separating equilibrium in which the managers always invest after receiving \(M_H\) but not after acquiring \(M_L\).

Notably, proposition 10 also shows that, as opposed to the baseline model, a full separating equilibrium can exist if \(R\) is sufficiently low. The absence of a full separating equilibrium
in the baseline model is due to the fact that, when the market believes that managers do not invest in unprofitable projects, it takes the investment as a perfect signal about \( r = R \) and therefore the manager can fool the market by investing in unprofitable projects. In the general setting, the investment has only limited effect on the interim price for two reasons: (i) The managers may receive incorrect signal about \( r \); (ii) The two signals \( s \) and \( m \) are complementary to each other and, thus, the inside signal is more likely to be incorrect when the two signals are inconsistent with each other. To better illustrate the intuition, suppose that the managers do not invest after receiving \( M_L \), i.e., \( \sigma_L = 0 \) in equilibrium. Then, the market beliefs in (28) can be written as:

\[
\begin{align*}
\mu(1, S_H | \sigma_L = 0) &= \frac{pq_s q_m}{pq_s q_m + (1-p)(1-q_s)(1-q_m)} \\
\mu(1, S_L | \sigma_L = 0) &= \frac{p(1-q_s)q_m}{p(1-q_s)q_m + (1-p)q_s(1-q_m)}
\end{align*}
\]

Notice that for \( q_m < 1 \), \( \mu(1, S_L | \sigma_L = 0) < \mu(1, S_H | \sigma_L = 0) < 1 \), i.e., when the market believes that managers do not invest after receiving a negative signal \( M_L \), the market regards the investment itself as a perfect signal about \( m = M_H \) but not as a perfect signal of \( r = R \) since the managerial signal \( m \) is incorrect with probability \( 1 - q_m \). Furthermore, the market rationally expects that the managerial signal \( M_H \) is more likely to be correct when the market also receives a good signal \( S_H \). This implies that, even in the case in which the market takes investment as a perfect signal about \( m = M_H \), the market forms higher price for investment after receiving \( S_H \), i.e., \( \pi_H > \pi_L \) while in the baseline model \( \pi_H = \pi_L \). For sufficiently low \( R \), the managers better off by investing in projects only after receiving \( M_H \), i.e., \( \pi_H > 1 > \pi_L \).

We conclude the analysis of the operational-manager subgame by investigating the partial effect of key parameters upon the equilibrium investment strategy \( \sigma_L \). The following proposition confirms that all the results presented in proposition 3 also hold in the general setting:

**Proposition 11** If \( R \in \left[ R_{OM}^e, R_{OM}^c \right] \), i.e., the partially separating equilibrium exists, the
equilibrium investment strategy $\sigma_L$ varies locally with each parameter as follows:

$$\frac{\partial \sigma_L}{\partial R} > 0 \quad (30)$$

$$\frac{\partial \sigma_L}{\partial q_s} < 0 \quad (31)$$

$$\frac{\partial \sigma_L}{\partial p} > 0 \quad (32)$$

Proposition 11 shows that operational managers invest more frequently after receiving a negative signal $S_L$ as the project’s payoff $R$ and the probability of success $p$ increases or as the market signal precision $q_s$ decreases. These results are consistent with the comparative static results of the baseline model in which the increases in $R$ and $p$ or the decrease in $q_s$ leads operational managers to more investment in unprofitable projects.

5.2 Operational managers vs. financial experts

Now we turn to the optimal managerial style appointed by shareholders at $t = 0$. First, consider the expected firm value at $t = 0$ when shareholders choose financial experts. As in the baseline model analysis, we focus on the case in which $R \in [R_{FE}^e, R_{FE}^e]$, i.e., financial experts invest in the project only after observing a positive market signal $S_H$. Since the investment decisions of financial experts are the same as in the baseline model, the expected value of the firm at $t = 0$ is formed as expressed in (22).

Next, consider the expected firm value at $t = 0$ in the case in which operational managers are appointed. From the equilibrium investment strategies of operational managers, illustrated in proposition 10, we can write the expected firm value as:

$$v_{OM}^e := E_{t=0}[v|m = OM]$$

$$= p[q_m + (1-q_m)\sigma_L]R + [p(1-q_m) + (1-p)q_m](1-\sigma_L), \quad (33)$$

which shows that the firm value decreases as operational managers invest more frequently
after receiving a negative signal $S_L$. Furthermore, (22) and (33) present the importance of each signal precision in the choice of managerial style. Specifically, operational managers outperform only when the inside signal $m$ is more informative than the market signal $s$ (i.e., $q_m > q_s$).\footnote{Notice that this result is immediate from the assumption $E(v|m = M_L, k = 1) = \text{Prob}(r = R|m = M_L)R < 1$, i.e., the investment is unprofitable given $M_L$.} This result is intuitive since, if the market has better information about the project, the ability of financial experts to collect such information improves the efficiency of investments. In what follows, we examine how the key parameters affects the performance of each style of managers.

5.2.1 Variation in average project quality

Consider first the effect of $R$ on the performance of managers. The following proposition shows that, as in the baseline model, higher payoff $R$ makes financial experts outperform operational managers:

Proposition 12 Suppose that $q_s < q_m$ and $R < \bar{R}_{OM}$, i.e., there exists a separating equilibrium in the operational-manager subgame. Then there always exists a unique threshold $\bar{R}_e \in (\bar{R}_{FE}, \bar{R}_{OM})$ such that $\bar{v}_{OM} > \bar{v}_{FE}$ if and only if $R < \bar{R}_e$.

The intuition about the effect of $R$ on the performance of each style of managers is already discussed in the baseline model. Proposition 12 shows that the result holds as far as, relative to the market, the managers have better information about the project.

5.2.2 Variation in stock market informativeness

Now we turn to the effect of $q_s$ on the performance of each style of managers. Not surprisingly, the results presented in proposition 9 may not hold in this general setting since, as shown in (22) and (33), financial experts outperform if $q_s > q_m$ which is not the case in the baseline model. By focusing on the case in which $q_s \leq q_m$, we examine the extent to which the baseline model results can be generalized. This case is also consistent with the corporate
governance literature that considers corporations in which, relative to shareholders, managers have private information.

In proposition 10, we define a threshold \( R_{OM} \) which is a function of \((p, q_m, q_s)\). To facilitate the presentation in the comparative statics with respect to \( q_s \), we fix the values of \( p \) and \( q_m \) and define \( R_{OM}^e(q_s) \). The following proposition confirms that for sufficiently low payoff \( R \), the baseline model results can be generalized:

**Proposition 13** If \( R < R_{OM}^e(q_m) \), there exists a threshold \( q_s^e \in [1/2, q_m) \) such that

\[
q_s \in (q_s^e, q_m) \iff v_{OM} > v_{FE}.
\]

### 5.2.3 Variation in the informativeness of managerial signal

By considering the possibility that the manager receives an incorrect signal about the project, we can also examine how the precision of managerial signal affects the performance of each style of managers. While more precise managerial signal improves the performance of operational managers by reducing the erroneous investment decision (i.e., investment in unprofitable projects after receiving a positive signal \( M_H \)), its effect on \( \sigma_L \) is ambiguous. More specifically, higher \( q_m \) decreases \( Prob(S_H|M_L) \) while reducing \( \mu(1, S_H) \) and \( \mu(1, S_L) \). Intuitively, as the managerial signal becomes more precise about the project, it get also more informative about the market signal and, thus, the managers assign higher probability to the state \( s = S_H \) after receiving a negative signal \( M_L \) and take a lower investment strategy. This disciplinary mechanism however may be offset by the market belief update which also affect the managerial choice of investment strategies. Specifically, as the manager becomes more likely to receive a positive signal for profitable projects, the market rationally takes the investment as more informative signal about the project. Such a market belief in turn increases the managerial latitude to manipulate the stock price by investing in the projects after acquiring a negative signal. The following proposition shows that, despite the ambiguous effect of \( q_m \) on \( \sigma_L \), it improves the performance of operational managers relative to
Proposition 14 If $R \in [R_{OM}^e, \overline{R}_{OM}^e]$, i.e., the partially separating equilibrium exists, more precise managerial signal makes operational managers outperform, or formally,

$$\frac{\partial v_{OM} - v_{FE}}{\partial q_m} \geq 0.$$  \hspace{1cm} (34)

6 Conclusion

We propose a model where firms can be run by financial experts or operational managers. Whereas both styles of managers have inside information about the quality of projects, financial experts can additionally retrieve informative signals about project quality from stock prices. Our analysis shows that firms may prefer operational managers, despite their informational disadvantage. Specifically, when the managers are myopic, the knowledge of the specific beliefs held by the market creates a much stronger incentive for financial experts to cater to these beliefs, ignoring valuable inside information. On the other hand, operational managers distort investment policies in an attempt to signal their private information. Our model implies that operational managers are preferred for “hard projects”, characterized by either being long shots (low probability of success) or having low return conditional on success, a result that is consistent with some evidence on firm-executive matching. Finally, our analysis also delivers the counter-intuitive prediction that under certain conditions operational managers are only preferred for high enough stock-price informativeness.

Our work contributes to the literature that studies which information structures are more efficient for running a corporation, and it delivers novel testable implications for the matching of firms and executives.
Appendix – Proofs

A.1 Additional lemmas

Lemma 1 In the general setting in which \( q_m \in (1/2, 1] \), the investment equilibrium of operational manager subgame exists if and only if

\[
R \geq R_{OM}^H := \frac{1}{\Prob(S_H|M_H)pqmqs + \Prob(S_L|M_H)pqm(1-q_s)}.
\]  

(A.1)

For all \( p \in (0, 1) \) and \( q_m, q_s \in (1/2, 1] \), \( R_{OM}^H \geq 1 \).

A.2 Proofs

Proof of lemma 1. For \( R < R_{OM}^H := \frac{1}{\Prob(S_H|M_H)pqmqs + \Prob(S_L|M_H)pqm(1-q_s)} \), \( u_H < 1 \) for all \( \sigma_H, \sigma_L \in [0, 1] \). Therefore, the best response investment strategy of operational managers correspond to \( \sigma_H = \sigma_L = 0 \). ■

Proof of proposition 1.

We will first show that \( \sigma_R > 0 \). Suppose to the contrary that \( \sigma_R = 0 \). Then, for any \( \sigma_0 > 0 \), \( \bar{u}_R = \bar{u}_0 = 0 \), which implies that the manager who observes \( r = 0 \) strictly prefers no-investment, i.e., \( \sigma_0 = 0 \). Thus, there is no separating equilibrium such that \( \sigma_R = 0 \). Now we turn to \( \sigma_0 > 0 \). Suppose to the contrary that \( \sigma_0 = 0 \) in equilibrium. Then, since \( R > 1 \), \( \bar{u}_R = \bar{u}_0 > 1 \), which implies that the manager who observes \( r = 0 \) strictly prefers investment, i.e., \( \sigma_0 = 1 \), which contradicts the assumption \( \sigma_0 = 0 \). Thus, \( \sigma_0 > 0 \). Next, note that in equilibrium, the low-type manager either is indifferent between investing or not, in which case the expected interim price \( \bar{u}_0 = 1 \); or, she strictly prefers to invest, which requires \( \bar{u}_0 > 1 \). But since \( \bar{u}_R \geq \bar{u}_0 \) (see section 3.1.1), the high-type manager must at least be indifferent, i.e. \( \sigma_R \geq \sigma_0 \). Furthermore, \( \sigma_R \geq \sigma_0 > 0 \) implies \( \bar{u}_R > \bar{u}_0 \), so it needs to be the case that \( \sigma_R = 1 \) in equilibrium.
So far we have shown that in separating equilibrium \( \sigma_R = 1 \) and \( \sigma_0 > 0 \). The corresponding equilibrium beliefs in (7) follow immediately from (3) and (4). Now we will show that \( \sigma_0 = 1 \) if and only if \( R \geq R_{OM} = \frac{pq + (1-p)(1-q)[p(1-q) + (1-p)q]}{pq(1-q)} \). The expected interim price \( \bar{u}_0 \) can be written as

\[
\bar{u}_0 = \left[ \frac{pq(1-q)}{pq + (1-p)(1-q)\sigma_0} + \frac{pq(1-q)}{p(1-q) + (1-p)q\sigma_0} \right] R. \tag{A.2}
\]

Notice that \( \bar{u}_0 \) decreases in \( \sigma_0 \) and increases in \( R \). After a few steps of simple algebra, we can show that \( \bar{u}_0 = 1 \) at \((\sigma_0, R) = (1, R_{OM})\). This implies that (i) if \( R \geq R_{OM} \), \( \bar{u}_0 \geq 1 \) for all \( \sigma_0 \leq 1 \) and (ii) otherwise, there always exists \( \sigma_0 \in (0, 1) \) such that \( \bar{u}_0 = 1 \). Finally, in the separating equilibrium in which \( \sigma_0 < 1 \), the manager who observes \( r = 0 \) should be indifferent about investment, i.e., \( \bar{u}_0 = 1 \). Since \( \bar{u}_0 \) is the convex combination of two interim prices \( u(1,S_H) \) and \( u(1,S_L) \), and \( u(1,S_H) > u(1,S_L) \), the interim prices necessarily satisfy \( u(1,S_H) > 1 > u(1,S_L) \).

**Proof of proposition 2.**

By plugging (A.2) into the equilibrium condition \( \bar{u}_0 = 1 \) and taking a few steps of simple algebra, we can show that the equilibrium investment strategy \( \sigma_0 \) solves the quadratic equation (10). Next, since the coefficient on the quadratic term of (10) is positive and the constant term is negative, the smallest root of (10) is negative and, thus, cannot be the equilibrium. Finally, in the proof of proposition 1, we show that \( \bar{u}_0 = 1 \) at \((\sigma_0, R) = (1, R_{OM})\). By continuity, this implies that (11) holds. To prove (12), we can show that, at \( R = 1 \), equation (10) can be rewritten as

\[
\sigma_0^2(1-p)^2q(1-q) + \sigma_0p(1-p) \left[ q^2 + (1-q)^2 - q(1-q) \right] = 0
\]

of which the largest root is \( \sigma_0 = 0 \). By continuity, this proves (12).

**Proof of proposition 3.**

Let us denote the left-hand-side of the quadratic (10) by \( H(\sigma_0) \). Since the quadratic term
of $H(\sigma_0)$ is positive and the equilibrium investment strategy corresponds to the largest root, it is immediate that $\frac{\partial H}{\partial \sigma_0} > 0$ in equilibrium. Thus, by the implicit function theorem, $\text{Sign}(\frac{\partial \sigma_0}{\partial X}) = -\text{Sign}(\frac{\partial H}{\partial X})$ for each parameter $X \in \{R, q, p\}$. First, consider the derivative of $H$ with respect to $R$:

$$\frac{\partial H}{\partial R} = -p(1 - p)q(1 - q)\sigma_0 - p^2 q(1 - q) \leq 0.$$ 

Therefore, $\frac{\partial \sigma_0}{\partial R} \geq 0$. Next, the derivative of $H$ with respect to $q$ can be written, after some manipulation, as

$$\frac{\partial H}{\partial q} = (2q - 1)[2p(1 - p)\sigma_0 - (1 - p)^2 \sigma_0^2 + \sigma_0 p(1 - p)R + p^2 (R - 1)].$$ \hspace{1cm} (A.3)

Since $H(\sigma_0) = 0$ in equilibrium, (A.3) can be further rewritten as:

$$\frac{\partial H}{\partial q} = (2q - 1) \left[ 2p(1 - p)\sigma_0 + \frac{p(1 - p)(q^2 + (1 - q)^2)}{q(1 - q)} \right],$$

which implies that $\frac{\partial H}{\partial q} > 0$ and therefore $\frac{\partial \sigma_0}{\partial q} < 0$. Finally consider the derivative of $H$ with respect to $p$:

$$\frac{\partial H}{\partial p} = -q(1 - q)[2(1 - p)\sigma_0^2 + (2p + \sigma_0(1 - 2p))R - 2p] + (1 - 2p)(q^2 + (1 - q)^2)\sigma_0.$$

By multiplying $(1 - p)$ with $\frac{\partial H}{\partial p}$,

$$(1 - p)\frac{\partial H}{\partial p} = -2(1 - p)^2 q(1 - q)\sigma_0^2 - (1 - p)q(1 - q)(2p + \sigma_0(1 - 2p))R \nonumber$$

$$+ 2p(1 - p)q(1 - q) + (1 - p)(1 - 2p)(q^2 + (1 - q)^2)\sigma_0.$$

Since $H(\sigma_0) = 0$ in equilibrium, $(1 - p)\frac{\partial H}{\partial p}$ can be rewritten, after simplification, as:

$$(1 - p)\frac{\partial H}{\partial p} = -\frac{(1 - p)^2}{p} q(1 - q)\sigma_0^2 + pq(1 - q) - pq(1 - q)R.$$
Since $R > 1$ and $p < 1$, $\frac{\partial H}{\partial p} < 0$ and hence $\frac{\partial \sigma_0}{\partial p} > 0$. □

**Proof of proposition 4.**

First we show that $\frac{\partial \pi_{OM}}{\partial R} > 0$. From (8), we can derive

$$\frac{\partial \pi_{OM}}{\partial R} = p - (1 - p) \frac{\partial \sigma_0}{\partial R}.$$ 

Using the derivatives of $H$ (see proof of proposition 3) with respect to $R$ and $\sigma_0$, we can rewrite $\frac{\partial \pi_{OM}}{\partial R}$ as

$$\frac{\partial \pi_{OM}}{\partial R} = \frac{X}{Y},$$

where

$$X := p(1 - p)[(1 - p)q(1 - q)\sigma_0 + p(q^2 + (1 - q)^2 - q(1 - q)(1 + R))],$$

$$Y := 2(1 - p)^2q(1 - q)\sigma_0 + p(1 - p)(q^2 + (1 - q)^2) - p(1 - p)q(1 - q)R.$$

Notice that $Y = \frac{\partial H}{\partial \sigma_0}$, which is positive in equilibrium (see the proof of proposition 3) and, thus, $\frac{\partial \pi_{OM}}{\partial R} > 0$ if and only if $X > 0$. Since $H(\sigma_0) = 0$ in equilibrium, $X$ can be written as

$$X = \frac{p^2q(1 - q)[p(R - 1) - (1 - p)\sigma_0]}{\sigma_0},$$

which is positive since $\pi_{OM} - 1 = p(R - 1) - (1 - p)\sigma_0 > 0$ in equilibrium as proved in proposition 5.

The result that $\frac{\partial \pi_{OM}}{\partial q} > 0$ follows directly from the fact that $\frac{\partial \sigma_0}{\partial q} < 0$.

Finally we show that $\frac{\partial \pi_{OM}}{\partial p} > 0$, which differentiating expression (8) is equivalent to

$$R - (1 - \sigma_0) - (1 - p)\frac{\partial \sigma_0}{\partial p} > 0 \iff \frac{\partial \sigma_0}{\partial p} < \frac{R - 1 + \sigma_0}{1 - p}. \quad (A.4)$$
Using $H$ together with the implicit function theorem it is straightforward to obtain

\[
\frac{\partial \sigma_0}{\partial p} = \frac{2\sigma_0^2(1-p)q(1-q) + \sigma(2p-1)[(1-q)^2 + q^2 - q(1-q)R] - 2pq(1-q)(1-R)}{2\sigma_0(1-p)^2q(1-q) + p(1-p)[(1-q)^2 + q^2 - q(1-q)R]} \tag{A.5}
\]

Inserting (A.5) into expression (A.4), and after some tedious but simple algebra, one can write

\[
\frac{\partial \bar{v}_{OM}}{\partial q} > 0 \iff \sigma_0 > p \left(1 - p\right) \frac{(1 - 2q^2 - q(1-q)R)}{(1 - 2q^2 + q(1-q)R)} \tag{A.6}
\]

After some simplification, one can write the explicit expression for $\sigma_0$ as

\[
\sigma_0 = p \left[\sqrt{(1-2q)^2 - 2q(1-q)(1-2q)^2R + q^2(1-q)^2R^2 - (1-q)^2 - q^2 + q(1-q)R}\right] / 2(1-p)q(1-q) \tag{A.7}
\]

Inserting equation (A.7) into expression (A.6), and after much simplification, one can write

\[
\frac{\partial \bar{v}_{OM}}{\partial q} > 0 \iff (1-q)^2 + 4q(1-q)R > 1.
\]

Since $R > 1$, to show that the above expression holds it is enough to show

\[(1-2q)^2 + 4q(1-q) \geq 1 \iff 1 \geq 1,
\]

which concludes the proof. ■

**Proof of proposition 5.**

In equilibrium, $\bar{v}_{OM} = p\bar{u}_R + (1-p)\sigma_0\bar{u}_0 + (1-p)(1-\sigma_0)$. From the separating equilibrium condition that $\bar{u}_R > \bar{u}_0 = 1$, it is immediate that $\bar{v}_{OM} > 1$. ■

**Proof of proposition 6.**

Suppose that the investment decision of financial experts is contingent on $r$. For $s = S_i$ ($i = H, L$), if $\mu(1,S_i)R > 1$, financial experts who observe $S_i$ would choose $k = 1$ regardless of $r$; similarly, if $\mu(1,S_i)R < 1$, the manager would choose $k = 0$ regardless of $r$. Thus, the

38
equilibrium condition would have to be $\mu(1, S_i)R = 1$ for all $S_i$, where both interim prices are 1. This in turn implies that ex-ante average firm value is 1, which is the benchmark value of not investing. On the other hand, managers using strategies that are contingent just on $s$ is straightforward to rationalize in equilibrium, with off-equilibrium-path beliefs that assign high enough likelihood that managers deviating from these strategies have bad projects. If such equilibria are more efficient (which is the case for many parameter regions), then under assumption 1 we should select these equilibria.

On a more technical note, there does exist a continuum of equilibria satisfying the condition $\mu(1, S_i)R = 1$, i.e., where posteriors are the same and equal to $1/R$. For the sake of completeness, we next characterize these equilibria. Denote by $\sigma(r, s)$ the strategy of financial experts, i.e., the likelihood that they choose $k = 1$ with information set $(r, s)$. Then stock-market posteriors (after observing signal and investment) are now written as

$$
\begin{align*}
\mu(1, S_H) &= \frac{pq\sigma(R, S_H)}{pq\sigma(R, S_H) + (1-p)(1-q)\sigma(0, S_H)} \\
\mu(1, S_L) &= \frac{p(1-q)\sigma(R, S_L)}{p(1-q)\sigma(R, S_L) + (1-p)q\sigma(0, S_L)}.
\end{align*}
$$

To illustrate that this type of equilibrium can exist, suppose we set $\sigma(R, .) = 1$ (although there are other equilibria where this is not the case). Furthermore, to assure existence, let us choose a small $R$:

$$R < \frac{pq}{pq + (1-p)(1-q)}.$$ 

Then we can always find $\sigma(0, S_H) \in (0, 1)$ such that $\mu(1, S_H)R = 1$, since setting $\sigma(0, S_H) = 1$ yields

$$\mu(1, S_H)R < 1,$$

and setting $\sigma(0, S_H) = 0$ yields

$$\mu(1, S_H)R = R > 1.$$ 

Similarly we can find the appropriate $\sigma(0, S_L)$.
Proof of proposition 7.

To prove the proposition, we take the following three steps: we first show that $\bar{v}_{OM} > \bar{v}_{FE}$ for $R \leq R_{FE}$; then show that $\bar{v}_{OM} < \bar{v}_{FE}$ at $R = \bar{R}_{OM}$; and finally prove that there exists a unique value of $\bar{R}^* \in [R_{FE}, \bar{R}_{OM}]$ such that $\bar{v}_{OM} = \bar{v}_{FE}$ if and only if $R = \bar{R}^*$. By continuity of $\bar{v}_{OM} - \bar{v}_{FE}$ in $R$, these three properties are sufficient to prove the proposition. First, for $R \leq R_{FE}$, financial experts never invest and therefore $\bar{v}_{FE} = 1$. By proposition 5, $\bar{v}_{OM} > 1$ for all $R > 1$ and thus $\bar{v}_{OM} > \bar{v}_{FE}$ for all $R \in (1, R_{FE}]$. Now we turn to the case in which $R = \bar{R}_{OM}$. In this case, operational managers take $\sigma_0 = 1$ as shown in proposition 1 while financial experts invest only when $s = S_H$, since $\bar{R}_{OM} < \bar{R}_{FE}$. By (8) and (22), the investment decisions of each style of managers imply that

$$
\bar{v}_{OM} - \bar{v}_{FE} = p(1 - q)\bar{R}_{OM} - p(1 - q) - (1 - p)q = \frac{1}{q}[-p(1 - p)(1 - q)(2q - 1) - (1 - p)^2q(2q - 1)]
$$

Therefore, $\bar{v}_{OM} - \bar{v}_{FE} < 0$ at $R = \bar{R}_{OM}$. Now we will show that there exists a unique value of $\bar{R}^* \in [R_{FE}, \bar{R}_{OM}]$ such that $\bar{v}_{OM} = \bar{v}_{FE}$ if and only if $R = \bar{R}^*$. From (8) and (22), it is immediate that

$$
\bar{v}_{OM} > \bar{v}_{FE} \iff \sigma_0 < \bar{\sigma} := 1 - q + \frac{p(1 - q)(R - 1)}{1 - p}.
$$

Since the equilibrium strategy $\sigma_0$ is the largest root of the quadratic equation $H(\sigma_0) = 0$ and the coefficient on the quadratic term of $H(\sigma_0)$ is positive, the above condition can be rewritten as

$$
\bar{v}_{OM} > \bar{v}_{FE} \iff H(\bar{\sigma}) > 0. \quad (A.8)
$$

By inserting $\sigma_0 = \bar{\sigma}$ into (10) and taking a few steps of algebra, we obtain a quadratic equation in $R$:

$$
F(R) := -p^2 q^2 (1 - q)(R - 1)^2 + p(1 - q)[(1 - p)q(1 - 2q) + p(1 - 3q)](R - 1)
$$
\[(1 - p)[p(3q^2 - 3q + 1) + (1 - p)q(1 - q)^2]\]
\[= 0. \tag{A.9}\]

Notice that \( F(R) = \frac{H(\sigma)}{1-q} \) and thus \( F(R) > 0 \iff H(\sigma) > 0. \) By (A.8), the previous two findings (i.e., \( \bar{v}_{OM} > \bar{v}_{FE} \) at \( R = R_{FE} \) and \( \bar{v}_{OM} < \bar{v}_{FE} \) at \( R = R_{OM} \)) imply that \( F(R_{FE}) > 0 \) and \( F(R_{OM}) < 0, \) which in turn implies that the quadratic equation \( F(R) = 0 \) must have a solution \( R^* \in (R_{FE}, R_{OM}) \) and such a solution is unique. By construction, \( \bar{v}_{OM} > \bar{v}_{FE} \) at \( R = R^*. \)

\textbf{Proof of proposition 8.}

First, \( F(R) \) in (A.9) is a quadratic function of \( p \) and we denote it as \( I(p). \) As shown in the proof of proposition 7,

\[ \bar{v}_{OM} > \bar{v}_{FE} \iff I(p) > 0. \]

For any \( q, \)

\[ \frac{\partial R_{OM}}{\partial p}, \frac{\partial R_{FE}}{\partial p} < 0 \]

and therefore there always exist \( p_{OM}, p_{FE} \in (0, 1) \) such that

\[ p = p_{OM} \iff R = R_{OM} \]
\[ p = p_{FE} \iff R = R_{FE}. \]

Furthermore, since \( R_{OM} > R_{FE}, p_{OM} > p_{FE}. \) In the proof of proposition 7, we show that \( F(R_{OM}) < 0 \) and \( F(R_{FE}) > 0 \) which implies that \( I(p_{OM}) < 0 \) and \( I(p_{FE}) > 0, \) respectively. Therefore, there always exists a unique value \( p^* \in (p_{FE}, p_{OM}) \) such that

\[ p > p^* \iff I(p) < 0, \]

which concludes the proof (see the proof of proposition 7 for more details).
Proof of proposition 9.

Proposition 7 implies that for any \((R, p)\) there always exists a set of \(q\) such that \(v_{FE} > v_{OM}\). Hence, to prove the claim in the proposition, it is enough to show that

\[
\begin{align*}
\left. v_{OM} \right|_{q=1} &= \left. v_{FE} \right|_{q=1} \\
\left. \frac{\partial v_{OM}}{\partial q} \right|_{q=1} &< \left. \frac{\partial v_{FE}}{\partial q} \right|_{q=1},
\end{align*}
\]

which implies the existence of a neighborhood around \(q = 1\) where the operational manager is strictly preferred. The equilibrium condition (10) implies that \(\sigma_0\) approaches to 0 as \(q\) increases to 1. From equations (22) and (8), it is then straightforward to show that the equality (A.10) holds. Next we turn to showing (A.11), by computing the relevant derivatives. By the implicit function theorem, we have

\[
\left. \frac{\partial \sigma_0}{\partial q} \right|_{q=1} = \left. \frac{\partial H/\partial q}{\partial H/\partial \sigma} \right|_{q=1} = \frac{p(1-R)}{1-p},
\]

and so

\[
\left. \frac{\partial v_{OM}}{\partial q} \right|_{q=1} = -(1-p) \left. \frac{\partial \sigma}{\partial q} \right|_{q=1} = p(R-1).
\]

Turning to financial experts, we have

\[
\left. \frac{\partial v_{FE}}{\partial q} \right|_{q=1} = p(R-1) + (1-p),
\]

and thus (A.11) holds. □

Proof of proposition 10. The proof for the equilibrium strategy \(\sigma_H = 1\) is very similar with the proof of \(\sigma_R = 1\) in proposition 1 and thus omitted here. The equilibrium beliefs in (28) are immediately obtained by plugging the equilibrium strategy \(\sigma_H = 1\) in (24) and (25). Now we turn to the equilibrium strategy \(\sigma_L\). When \(m = M_L\), managers take the investment
strategy as:

$$\sigma_L = \begin{cases} 
0 & \text{if } \pi_L < 1 \\
1 & \text{if } \pi_L > 1 \\
\in (0,1) & \text{otherwise.}
\end{cases}$$  \hspace{1cm} (A.14)

If \( R \leq R_{OM}^e \), \( \pi_L \leq 1 \) for all \( \sigma_L \in [0,1] \). This implies that for \( R < R_{OM}^e \), \( \sigma_L = 0 \) is the best investment strategy for managers since \( \pi_L \) decreases in \( \sigma_L \). Likewise, if \( R \geq R_{OM}^e \), \( \pi_L \geq 1 \) for all \( \sigma_L \in [0,1] \) and, thus, the equilibrium investment strategy corresponds to \( \sigma_L = 1 \). Finally, if \( R \in (R_{OM}^e, R_{OM}) \), the equilibrium strategy \( \sigma_L \) should satisfy the equilibrium condition \( \pi_L = 1 \) which corresponds to (29) in proposition 10. We concludes the proof by showing that \( u(1, S_H) > 1 > u(1, S_L) \) for the equilibrium strategy \( \sigma_L \in [0,1] \). Since \( \pi_L \) is a convex combination of \( u(1, S_H) \) and \( u(1, S_L) \), the equilibrium condition \( \pi_L = 1 \) implies that \( u(1, S_H) > 1 > u(1, S_L) \).

\[ \text{Proof of proposition 11.} \] For \( R \in [R_{OM}^e, R_{OM}] \), the equilibrium condition (29) should hold. Notice that the LHS of (29) is \( \pi_L \). From (29), it is obvious that \( \frac{\partial \mu}{\partial \pi_L} < 0 \). By the implicit function theorem, this implies that \( \text{Sign}(\frac{\partial \mu}{\partial X}) = \text{Sign}(\frac{\partial \pi_L}{\partial X}) \) for all parameters \( X \in \{R, q_s, p\} \). First, consider the derivative of \( \pi_L \) with respect to \( R \):

$$\frac{\partial \pi_L}{\partial R} = \text{Prob}(S_H | M_L)\mu(1, S_H) + \text{Prob}(S_L | M_L)\mu(1, S_H) > 0.$$  \hspace{1cm} (A.15)

Therefore, \( \frac{\partial \pi_L}{\partial R} > 0 \). Next, to show that \( \frac{\partial \pi_L}{\partial p} > 0 \), we rewrite \( \mu(1, S_H) \) and \( \mu(1, S_L) \) in (28) as:

$$\left\{ \begin{array}{l} 
\mu(1, S_H) = \frac{1}{1 + \frac{(1-p)(1-q_s)(1-q_m + \sigma_L q_m)}{pq_s (q_n + \sigma_L q_m)}} \\
\mu(1, S_L) = \frac{1}{1 + \frac{(1-p)q_s (1-q_m + \sigma_L q_m)}{p(1-q_s)q_m + \sigma_L (1-q_m)}}
\end{array} \right.$$ 

which shows that both \( \mu(1, S_H) \) and \( \mu(1, S_L) \) increase in \( p \). The derivative of \( \pi_L \) with respect
to $p$ can be written as:

$$\frac{\partial \overline{u}_L}{\partial p} = \text{Prob}(S_H|M_L)\frac{\partial \mu(1,S_H)}{\partial p} + \text{Prob}(S_L|M_L)\frac{\partial \mu(1,S_L)}{\partial p} + \mu(1,S_H)\frac{\partial \text{Prob}(S_H|M_L)}{\partial p} + \mu(1,S_L)\frac{\partial \text{Prob}(S_L|M_L)}{\partial p},$$

and thus we can prove $\frac{\partial \overline{u}_L}{\partial p} > 0$ by showing that $\mu(1,S_H)\frac{\partial \text{Prob}(S_H|M_L)}{\partial p} + \mu(1,S_L)\frac{\partial \text{Prob}(S_L|M_L)}{\partial p} > 0$. The derivative of $\text{Prob}(S_H|M_L)$ and $\text{Prob}(S_L|M_L)$ with respect to $p$ is can be written as:

$$\frac{\partial \text{Prob}(S_H|M_L)}{\partial p} = \frac{q_m(1 - q_m)(2q_s - 1)}{(p(1 - q_m) + (1 - p)q_m)^2},$$

$$\frac{\partial \text{Prob}(S_L|M_L)}{\partial p} = -\frac{q_m(1 - q_m)(2q_s - 1)}{(p(1 - q_m) + (1 - p)q_m)^2},$$

which implies that

$$\mu(1,S_H)\frac{\partial \text{Prob}(S_H|M_L)}{\partial p} + \mu(1,S_L)\frac{\partial \text{Prob}(S_L|M_L)}{\partial p} = \frac{q_m(1 - q_m)(2q_s - 1)}{(p(1 - q_m) + (1 - p)q_m)^2}(\mu(1,S_H) - \mu(1,S_L)) \geq 0.$$

Thus, $\frac{\partial \overline{u}_L}{\partial p} > 0$. Finally, to obtain $\frac{\partial \sigma_L}{\partial q_s}$, we rewrite the equilibrium condition in (29) as:

$$J(\sigma_L) = RJ_1(\sigma_L) - [p(1 - q_m) + (1 - p)q_m]J_2(\sigma_L) = 0, \quad (A.16)$$

where

$$J_1(\sigma_L) = p^2q_s(1 - q_s)(p(1 - q_m) + (1 - p)q_m)(q_m + \sigma_L(1 - q_m))^2 + p(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_Lq_m) \{p(1 - q_m)(q_s^3 + (1 - q_s)^3) + (1 - p)q_mq_s(1 - q_s)\}$$
and

\[ J_2(\sigma_L) = p^2 q_s (1 - q_s) (q_m + \sigma_L (1 - q_m))^2 + (1 - p)^2 q_s (1 - q_s) (1 - q_m + \sigma_L q_m)^2 \]

\[ + p(1 - p) (q_s^2 + (1 - q_s)^2) (q_m + \sigma_L (1 - q_m)) (1 - q_m + \sigma_L q_m). \]

Notice that (A.16) is derived by multiplying \([pq_s(q_m + \sigma_L(1 - q_m)) + (1 - p)(1 - q_s)(1 - q_m + \sigma_L q_m)][p(1 - q_s)(q_m + \sigma_L(1 - q_m)) + (1 - p)q_s(1 - q_m + \sigma_L q_m)][p(1 - q_m) + (1 - p)q_m]\) on both sides of (29). Now, using implicit function theorem, we can obtain \(\frac{\partial \sigma_L}{\partial q_s}\) by differentiating \(J\) with respect to \(\sigma_L\) and \(q_s\). In this proof above, we already show that \(\frac{\partial \sigma_L}{\partial R} > 0.\) From (A.16), it is straightforward to show that \(\frac{\partial J}{\partial R} > 0\) since \(J_1(\sigma_L) > 0.\) Therefore, by implicit function theorem, \(\frac{\partial J}{\partial \sigma_L} < 0\) which implies that it is sufficient to show that \(\frac{\partial J}{\partial q_s} < 0\) if and only if \(R < \frac{(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_L q_m)}{1 - q_m}\). By differentiating \(J\) with respect to \(q_s\) and solving a few steps of algebra, we obtain:

\[
\frac{\partial J}{\partial q_s} = R(2q_s - 1)[-p^2(p(1 - q_m) + (1 - p)q_m)(q_m + \sigma_L(1 - q_m))^2 \\
+ p(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_L q_m)(3p(1 - q_m - (1 - p)q_m))]
- (p(1 - q_m) + (1 - p)q_m)(2q_s - 1)[-p^2(q_m + \sigma_L(1 - q_m))^2 \\
+ 2p(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_L q_m) - (1 - p)^2(1 - q_m + \sigma_L q_m)^2]
\]

By the equilibrium condition in (A.16), \(\frac{\partial J}{\partial q_s}\) can be rewritten as:

\[
\frac{\partial J}{\partial q_s} = \frac{2q_s - 1}{q_s(1 - q_s)} p(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_L q_m)[Rp(1 - q_m) - p(1 - q_m) - (1 - p)q_m].
\]

Since \(\frac{2q_s - 1}{q_s(1 - q_s)} > 0\) for all \(q_s > 1/2\) and \(p(1 - p)(q_m + \sigma_L(1 - q_m))(1 - q_m + \sigma_L q_m) > 0, \frac{\partial J}{\partial q_s} < 0\) if and only if \(Rp(1 - q_m) - p(1 - q_m) - (1 - p)q_m < 0\) which is the case by assumption. ■

**Proof of proposition 12.** We take similar steps with the proof of proposition 7. First, for \(R \leq R_{FE}\), financial experts do not invest and thus \(\nu_{FE}^c = 1.\) Since \(q_s < q_m, \frac{R_{OM}}{R_{FE}}\) defined
in lemma 1 is less than \( R_{FE} \) and, thus operational managers take \( \sigma_H = 1 \) and \( \overline{\nu}_{OM} > 1 \) for \( R \in [R_{OM}^H, R_{FE}] \). Now we turn to the case in which \( R = R_{OM} \) and thus operational managers take \( \sigma_L = 1 \). In this case, \( \overline{\nu}_{OM} - \overline{\nu}_{FE} = p(1 - q_s)\overline{R}_{OM} - p(1 - q_s) - (1 - p)q_s \). Since \( \frac{\partial \text{Prob}(S_H|M_L)}{\partial q_m} < 0 \), \( \overline{R}_{OM} > \overline{R}_{OM}^e \) for all \( q_m \in (1/2, 1) \). In the proof of proposition 7, we show that \( p(1 - q_s)\overline{R}_{OM} - p(1 - q_s) - (1 - p)q_s < 0 \) and thus \( \overline{\nu}_{OM} - \overline{\nu}_{FE} < 0 \). Finally, we prove the existence of a unique value \( \overline{R}_e \in (\overline{R}_{FE}, \overline{R}_{OM}) \) such that \( \overline{\nu}_{OM} > \overline{\nu}_{FE} \) if and only if \( R < \overline{R}_e \). By (22) and (33), we can show that

\[
\overline{\nu}_{OM} > \overline{\nu}_{FE} \Leftrightarrow \sigma_L < \overline{\sigma} := \frac{(q_m - q_s)(p(R - 1) + 1 - p)}{(1 - p)q_m - p(1 - q_m)(R - 1)} \tag{A.17}
\]

By (A.16), \( \frac{\partial J}{\partial \sigma_L} < 0 \) in equilibrium and therefore it is the case that

\[
\overline{\nu}_{OM} > \overline{\nu}_{FE} \Leftrightarrow J(\overline{\sigma}) > 0. \tag{A.18}
\]

Notice that \( J(\sigma_L) \) is a quadratic function of \( \sigma_L \). After a few steps of algebra, we can show that the coefficient of the quadratic term is \( [p(1 - q_m)q_s + (1 - p)q_m(1 - q_s)](1 - q_s) + (1 - p)q_mq_s]p(1 - q_m)(R - 1) - (1 - p)q_m \) and therefore the quadratic can be written as \( [p(1 - q_m)q_s + (1 - p)q_m(1 - q_s)](1 - q_s) + (1 - p)q_mq_s]p(1 - q_m)(R - 1) - (1 - p)q_m \). Since the coefficient of linear term and the constant term are also linear in \( R \), \( J(\overline{\sigma}) \) is a quadratic function of \( R \) which is denoted as \( F^e(R) \). As shown in the proof of proposition 7, this is sufficient to prove the unique existence of \( \overline{R}_e \in (\overline{R}_{FE}, \overline{R}_{OM}) \). See the proof of proposition 7 for the details. ■

**Proof of proposition 13.** To prove the proposition, it is sufficient to show that \( \overline{\nu}_{OM} > \overline{\nu}_{FE} \) in the neighborhood of \( q_s = q_m \). For \( R < \overline{R}_{OM}^e(q_m) \), there exists \( \overline{q}_s < q_m \) such that

\[
\forall q_s \geq \overline{q}_s, \sigma_L = 0
\]

since \( \sigma_L = 0 \) at \( q_s = q_m \) and \( \frac{\partial \sigma_L}{\partial q_s} < 0 \). Thus, for \( q_s \in [\overline{q}_s, q_m] \), \( \overline{\nu}_{OM} > \overline{\nu}_{FE} \). ■
Proof of proposition 14. First, consider \( \frac{\partial \text{Prob}(S_H|M_L)}{\partial q_m} \). By differentiating (27) with respect to \( q_m \), we obtain

\[
\frac{\partial \text{Prob}(S_H|M_L)}{\partial q_m} = \frac{p(1-p)(1-2q_s)}{(p(1-q_m)+(1-p)q_m)^2} < 0
\]

Now consider the values of \( \sigma_L \) and \( q_m \) that make \( \mu(1,S_H) \) and \( \mu(1,S_L) \) fixed. From (28), we find that both \( \mu(1,S_H) \) and \( \mu(1,S_L) \) are fixed as far as \( \frac{1-q_m+\sigma_Lq_m}{q_m+\sigma_L(1-q_m)} \) are fixed. Now suppose that \( \sigma_L \) varies with \( q_m \) such that \( \mu(1,S_H) \) and \( \mu(1,S_L) \) are fixed. Then, since \( \frac{\partial \text{Prob}(S_H|M_L)}{\partial q_m} < 0 \) and \( \mu(1,S_H) > \mu(1,S_L) \), \( u_L \) decreases as \( q_m \) increases. Therefore, to satisfy the equilibrium condition (29), the equilibrium strategy \( \sigma_L \) should increase \( \mu(1,S_H) \) and \( \mu(1,S_L) \) as \( q_m \) decreases.

Now consider \( u_H \). By differentiating (27) with respect to \( q_m \), we obtain

\[
\frac{\partial \text{Prob}(S_H|M_H)}{\partial q_m} = \frac{p(1-p)(2q_s-1)}{(p(1-q_m)+(1-p)q_m)^2} > 0.
\]

Thus, as \( q_m \) increases, \( \text{Prob}(S_H|M_H) \) increases and the equilibrium market beliefs \( \mu(1,S_H) \) and \( \mu(1,S_L) \) also increase. Since \( \mu(1,S_H) > \mu(1,S_L) \), these facts jointly imply that \( u_H \) also increase in \( q_m \). Now consider the performance of each style of managers. First, \( \bar{v}_{FE} \) does not vary over \( q_m \). To examine the effect on \( \bar{v}_{OM} \), we rewrite it as:

\[
\bar{v}_{OM} = q_m \bar{u}_H + (1-q_m)\sigma_L \bar{u}_L + (1-q_m)(1-\sigma_L) = q_m(\bar{u}_H - 1) + (1-q_m)\sigma_L(\bar{u}_L - 1) + 1 = q_m(\bar{u}_H - 1) + 1 \quad \text{by (29).} 
\]

(A.19)

Therefore, \( \bar{v}_{OM} \) increases in \( q_m \) which proves the proposition.\( \blacksquare \)
References


