Hedging Interest Rate Risk Using a Structural Model of Credit Risk*

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Abstract

Recent evidence has shown that structural models fail to capture interest rate sensitivities of corporate debt. We consider a structural model that incorporates a three-factor dynamic term structure model (DTSM) into the Merton (1974) model. We show that the proposed model largely captures the interest rate exposure of corporate bonds. We also find that for investment-grade bonds, hedging effectiveness substantially improves under the proposed model. Our results indicate that to better capture and hedge the interest rate exposure of corporate bonds, we need to incorporate a more realistic DTSM in the existing structural models.

Keywords: structural models, interest rate risk, hedge ratios
JEL Classifications: G13, G12, G33, G24

1 Introduction

One important and widely used theoretical framework for understanding the credit risk premium is the structural approach of Black and Scholes (1973) and Merton (1974). In addition to the large theoretical literature on various extensions of the original Merton model, there has been a fast growing literature on the empirical performance of structural credit risk models. For instance, recent studies have empirically examined the implications of structural models for corporate bond spreads (Eom, Helwege, and Huang 2004; Ericsson and Reneby 2005; Huang and Huang 2012), spread changes (Collin-Dufresne, Goldstein, and Martin 2001), equity volatility (Huang and Zhou 2008), and corporate bond volatility (Bao and Pan 2013) etc.

In an influential study, Schaefer and Strebulaev (2008) find that while the Merton model substantially underestimates corporate bond yield spreads, the model-implied equity sensitivity of corporate bond returns (hedge ratios) is actually quite consistent with those observed from market data. However, they also document that the Merton model with stochastic interest rates fails to capture the interest rate sensitivity of corporate bond returns and note that this failure "remains an interesting puzzle" (page 3). For convenience we refer to this puzzle as the "interest rate sensitivity puzzle."

In this paper we study hedge ratios implied by structural models with stochastic interest rates. Specifically, we consider the class of so-called Merton-Vasicek (MV) models—namely, the original Merton (1974) model combined with the class of Gaussian dynamic term structure models (GDTSMs). In particular, we implement the four-factor MV model, which incorporates a three-factor GDTSM (into the Merton model). This is motivated by three consideration: First, given that Treasury returns are an important determinant of corporate bond returns (especially for investment-grade bonds), a structural model doing a better job in capturing the behavior of Treasury yields may predict hedge ratios of corporate bonds more accurately. Second, it is known that three factors explain all but a negligible fraction of the variation in the (default-free) term structure. Third, hardly any studies have examined structural models with a multi-factor DTSM. For instance, the term structural model implemented in Schaefer and Strebulaev (2008) is a one-factor Vasicek (1977) model. We estimate the four-factor MV model using data for a sample of US corporate bonds from the TRACE over the period July 2002–December 2012.

Our main findings are as follows: First, we find that indeed the four-factor MV model largely captures the interest rate sensitivity of corporate debt and thus helps mitigate the "interest rate sensitivity puzzle." This result holds for hedge ratios on both corporate bond returns and yield spread changes. Second, we also examine the hedging performance of Treasury bonds when either corporate bond returns or spread changes are to be hedged, and find that hedging effectiveness substantially improves under the four-factor model, at least for investment-grade bonds in our sample. Third, in addition to the interest rate sensitivity of corporate bonds, we also examine their sensitivity to the underlying equity. We find that the four-factor model also provides quite accurate predictions of the equity sensitivities of both corporate bond returns and yield spread changes.

These findings also have implications for structural modeling of credit risk. Although there is an enormous literature on structural models, very few such models go beyond one-factor DTSMs. Perhaps, this is because so far there is no clear evidence that including stochastic interest rates (in a structural model) helps the model better predict bond yield spreads. The empirical results from our analysis indicate that to better capture and hedge the interest rate exposure of corporate bonds, we need to incorporate a more realistic DTSM in the existing structural models. As such, this study helps bring the literature on structural models and the term structure literature closer to each other.

Our paper is closely related to Schaefer and Strebulaev (2008) and extends their article in several ways. First, we document that the two-factor MV model also fails to capture the interest rate sensitivity of yield spread changes. Second, we show that incorporating a three-factor DTSM helps resolve the interest rate sensitivity puzzle. Our paper also draws on insights from Collin-Dufresne, Goldstein, and Martin (2001), who point out the importance of studying changes in the corporate yield spread among other things. As we examine both corporate returns and spread changes using the same class of structural models, our paper to some extent helps bridge these two influential studies.

The literature on empirical tests of structural models goes back to Jones, Mason, and Rosenfeld (1984) and Ogden (1987), both of which focus on callable bonds, however. In addition to those studies mentioned in the beginning of this paper, other studies using individual corporate bond data include Lyden and Saraniti (2000); Bao (2009); Bao and Hou (2013); among others. The two-factor MV model, examined in Schaefer and Strebulaev (2008) and this paper, is introduced by Shimko, Tejima, and Van Deventer (1993). Other two-factor structural models with stochastic interest rates include those of Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Acharya and Carpenter (2002), and Collin-Dufresne and Goldstein (2002) etc.

The rest of the paper is organized as follows. In Section 2, we first review the class of the Merton-Vasicek models and characterize hedge ratios implied by such models. We then present regression models to be used to test whether the models can predict the equity and interest rate sensitivities of corporate debt. Lastly, we define the measure of hedging effectiveness to be used in both simulations and the empirical analysis. Section 3 discusses corporate and Treasury bond data used in our empirical analysis. Section 4 conducts a Monte Carlo simulation to investigate time-series regressions of both corporate bond yield spread changes and returns, using the four-factor MV model as the data-generating process. Section 5 presents and discusses our empirical results. Finally, Section 6 concludes.

2 Model Implications and the Empirical Methodology

In this section we first review the class of structural models with stochastic interest rates to be considered in both simulation and empirical analysis. We then present formulas of hedge ratios implied by these models. Next, we outline the empirical methodology to be used later in the paper. Before proceeding, we note that for comparison and also the analytical tractability, we follow Schaefer and Strebulaev (2008) and focus on zero-coupon bonds in Sections 2.1 and 2.2.

2.1 Structural Models with Stochastic Interest Rates

We focus on the class of the Merton-Vasicek models in this study. The basic structure of these models is the same as that of the Merton (1974) model, except that the risk-free interest rate is assumed to follow a Gaussian dynamic term structure model (GDTSM).

More specifically, the firm is assumed to have a zero-coupon bond outstanding with a maturity of T and face value of K; default can occur at T only. Let $(V_t)_{t\geq 0}$ and $(r_t)_{t\geq 0}$ be the firm asset value process and risk-free interest rate process, respectively. The dynamics of the underlying state vector are given by:

$$dV_t = V_t\,(r_t\,dt + \sigma_v dW^{\mathbb{Q}}_{V,t}), \tag{1}$$

$$r_t = \delta_0 + \delta_1' X_t, \tag{2}$$

$$dX_t = (K^{\mathbb{Q}}_{0,X} + K^{\mathbb{Q}}_{1,X} X_t)dt + \Sigma_X dW^{\mathbb{Q}}_{X,t}, \tag{3}$$

where σ_v is the asset return volatility and $W^{\mathbb{Q}}_A$ a one-dimensional standard Brownian motion under the risk-neutral measure $\mathbb{Q}$; δ_0 is a constant and δ_1 a $k \times 1$ parameter vector; X is a $k \times 1$ vector of state variables that drive the risk-free term structure. Eqs. (2) and (3) are assumed to satisfy the canonical form as specified in Joslin, Singleton, and Zhu (2011; JSZ hereafter); see the appendix.

Note that this structural model is a combination of the Merton model and a k-factor Vesicek term structure model. If the interest rate is constant, then k is zero and we obtain the original Merton model. If $k = 1$, then we have the Shimko, Tejima, and Van Deventer (1993) model, also referred to as the Merton-Vasicek model in Schaefer and Strebulaev (2008). Besides this model, we also consider the special case of $k = 3$ in this study, given the consensus in the DTSM literature that three factors explain all but a negligible fraction of the variation in the risk-free term structure. For convenience, we refer to this model as a four-factor Merton-Vasicek model. One advantage of the class of Merton-Vasicek models specified in Eqs. (1)-(3) is that hedge ratios implied by these models are straightforward to calculate.

2.2 Hedge Ratios in the Class of Merton-Vasicek Models

Let P_t^T denote the time-t price of a default-free zero-coupon bond with maturity T and unity face value. Given the affine structure of the interest rate process specified in Eqs. (2) and (3), P_t^T is exponential-affine in X_t and given by (Duffie and Kan 1996)

$$P_t^T = e^{A_x(T-t)+B_x(T-t)'X_t}. \tag{4}$$

The time-t price of the defaultable zero-coupon bond in this case is given as follows:

$$D_t^T = V_t N(-d_1) + K P_t^T N(d_2), \tag{5}$$

where

$$d_1 = \frac{\ln(\frac{V_t}{K P_t^T}) + S_t^T/2}{\sqrt{S_t^T}}; \quad d_2 = d_1 - \sqrt{S_t^T}.$$

The variable S_t^T denotes the variance of asset returns over the life of corporate debt. If the term structure model is Gaussian, then S_t^T is time-invariant. That is

$$S_t^T \equiv S(T-t) = \int_{T-t}^{0} \left(\sigma_v^2 + B_x'(s)\Sigma_X'\Sigma_X B_x(s) + 2\sigma_v B_x(s)\Sigma_X'\rho\right) ds,$$

where ρ is a k-dimensional vector representing the correlation between $dW_{X,t}$ and $dW_{V,t}$. When $k = 1$, the above integration can be carried out explicitly (see, e.g., Schaefer and Strebulaev 2008). When $k = 3$, numerical integration is needed.

Let TY_t^T and Y_t^T denote respectively the time-t yields of the default-free and defaultable zero-coupon bonds. By definition the time-t yield/credit spread of the defaultable bond equals CS_t^T. Then it follows from Eq. (5) that the model-implied equity sensitivities of the defaultable bond return and yield spread are given as the following:

$$h_E^r \equiv \frac{\partial D/D}{\partial E/E} = \frac{N(-d_1)}{N(d_1)}\frac{E}{D}; \tag{6}$$

$$h_E^{CS} \equiv \frac{\partial (CS)}{\partial E/E} = -\frac{1}{T-t}\frac{N(-d_1)}{N(d_1)}\frac{E}{D}, \tag{7}$$

where the time index in D (the market debt value) and E (the market equity value) is omitted, and $N(d_1)$ is the Black-Scholes-Merton delta of a European call with the strike K. As a special case of the above two formulas, the Merton hedge ratios, denoted $\tilde{h}_E^r$ and $\tilde{h}_E^{CS}$, can be obtained by replacing d_1 and d_2 by $\tilde{d}_1$ and $\tilde{d}_2$, where

$$\tilde{d}_1 = \frac{\ln(V_t/K) + (r + \sigma_v^2/2)(T-t)}{\sigma_v\sqrt{T-t}}; \quad \tilde{d}_2 = \tilde{d}_1 - \sigma_v\sqrt{T-t}.$$

Given the assumption that the interest rate dynamics are driven by a k-factor Gaussian process in the model, we need k Treasury bonds with distinct maturities to hedge the interest rate risk in corporate bond returns or yield spread changes. For illustration, consider the stacked prices on n zero-coupon bonds with maturities $\mathcal{M} = \{m_1, \ldots, m_n\}$ as the following:

$$\mathcal{P}_t = e^{\mathcal{A}_x + \mathcal{B}_x X_t},$$

where $\mathcal{B}_x$ contains rows $\{B_x'(m-t)|m \in \mathcal{M}\}$ for each bond. To make the $n \times k$ matrix $\mathcal{B}_x$ invertible, n needs to match the dimension of the state vector X_t. Suppose k Treasury bonds are used as hedging instruments. It can be shown that the k model-implied interest rate sensitivities of the corporate bond return can be calculated as follows:

$$h_{\mathcal{M}}^r = \frac{KP_t^T N(d_2)}{D_t^T N(d_1)} B_x'(T-t)\mathcal{B}_x^{-1}. \tag{8}$$

Consider the special case where $k = 1$ and a single m-year Treasury security is used for hedging. Both $B'_x(T-t)$ and $\mathcal{B}_x^{-1}$ become scalars. Also, under the JSZ canonical form, we have $r_t = X_t$. It follows from Eq. (8) that

$$h^r_m \equiv \frac{\partial D/D}{\partial P^m/P^m} = \frac{KP^T_t N(d_2)}{D^T_t N(d_1)} \cdot \frac{e^{(T-t)K^{\mathbb{Q}}_{1,X}} - 1}{e^{(m-t)K^{\mathbb{Q}}_{1,X}} - 1}. \tag{9}$$

Note that empirically, the estimation of $K^{\mathbb{Q}}_{1,X}$ may suffer from potential model misspecification and sample uncertainty as the sample may not be long enough to achieve estimation consistency and may be contaminated with measurement errors. In that case, the estimated hedge ratio may not be very close to h^r_m. Nonetheless, Eq. (9) suggests that using a Treasury bond with the same maturity as that of the corporate bond helps reduce the bias. When $m = T$, the second ratio on the right-hand side of the equation equals one and, as a result, h^r_m is much less sensitive to $K^{\mathbb{Q}}_{1,X}$.

It can be shown that the k sensitivities of credit spreads to Treasury bond returns can be written as follows:

$$h^{CS}_{\mathcal{M}} \equiv \frac{\partial(CS)}{\partial P/P} = \left(-\frac{KP^{\tau}_t N(d_2)}{DN(d_1)\tau} + \frac{1}{\tau}\right) B^{\tau\prime}_x \mathcal{B}_x^{-1}. \tag{10}$$

2.3 Regressions Incorporating Model-Implied Hedge Ratios

One objective of this study is to include model-implied hedge ratios in a regression test to see whether the four-factor Merton-Vasicek model provides good predictions of hedge ratios for both corporate bonds and credit spreads, in the spirit of Schaefer and Strebulaev (2008; SS) who focus on the Merton-implied equity sensitivity of corporate bond returns.

We consider hedge ratios implied by the (two-factor) Merton-Vasicek model first. Specifically, we estimate the following set of regression models for each firm i in a given sample:

$$rx^T_{i,t} = \alpha_r + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t} + \beta^r_{i,10} h^r_{i,10} rx^{10}_t + \epsilon_{i,t}; \tag{11}$$

$$\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t} + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t + \epsilon_{i,t}, \tag{12}$$

where $rx^T_{i,t}$ and $CS^T_{i,t}$ are respectively the excess return and yield spread change for bond-i (with maturity T) in month t; $r^E_{i,t}$ and $rx^E_{i,t}$ respectively the month-t return and excess return on the stock of firm i; r^{10}_t and rx^{10}_t respectively the month-t return and excess return of the 10-year Treasury. In addition, $h^r_{i,E}$ and $h^r_{i,10}$ denote the Merton-Vasicek sensitivities of corporate bond returns to equity return and the 10-year Treasury return, respectively; $h^{CS}_{i,E}$ and $h^{CS}_{i,10}$ the model-implied sensitivities of spread changes to equity return and the 10-year Treasury return, respectively.

To test the null that these slope coefficients are equal to one, we follow Collin-Dufresne, Goldstein, and Martin (2001; CDGM) and SS to focus on the means of the slope coefficients estimated from the given sample and examine whether the means are close to one or not. As shown later, results from our empirical analysis indicate that while the null that $\beta^r_{i,E} = 1$ is not rejected, the null that $\beta^r_{i,10} = 1$ is strongly rejected, consistent with SS. Additionally, we find that similar results hold for hedge ratios on yield spread changes that are included in regression (12). That is, the (two-factor) Merton-Vasicek model fails to capture the interest rate sensitivity of corporate debt. As mentioned earlier, the four-factor Merton-Vasicek model with a more realistic DTSM may help mitigate this "interest rate sensitivity puzzle."

As such, we augment each of regressions (11) and (12) with two more Treasury securities along with related model-implied hedge ratios and estimate the following augmented regressions for each firm i:

$$\begin{aligned} rx^T_{i,t} &= \alpha_r + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t} + \beta^r_{i,0.5} h^r_{i,0.5} rx^{0.5}_t + \beta^r_{i,2} h^r_{i,2} rx^2_t + \beta^r_{i,10} h^r_{i,10} rx^{10}_t + \epsilon_{i,t}; \quad (13) \\ \Delta CS^T_{i,t} &= \alpha_{CS} + \beta^{CS}_{i,0.5} h^{CS}_{i,0.5} r^{0.5}_t + \beta^{CS}_{i,2} h^{CS}_{i,2} r^2_t + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t} + \epsilon_{i,t}, \quad (14) \end{aligned}$$

where r^m_t and rx^m_t denote respectively the month-t return and excess return of the m-year Treasury security, with $m = 0.5, 2$; $h^r_{i,m}$ and $h^{CS}_{i,m}$ respectively the sensitivities of excess corporate bond returns and spread changes to returns on the m-year Treasury. Note that all the sensitivities included in the above two regressions are calculated using the four-factor Merton-Vasicek model.

2.4 Hedging Effectiveness

Regression analysis allows us to examine whether a new model provides more accurate predictions of hedge ratios than the existing model. Another useful exercise worth doing is to see whether hedging effectiveness improves under the proposed four-factor Merton-Vasicek model.

As in Green and Figlewski (1999), we assume that the objective of an investor is to minimize the monthly volatility of his hedged bond portfolio position. Suppose that at the end of month j, the investor hedges a portfolio of N_j corporate bonds with Treasury bonds and the host/underlying equity, and makes no additional trade until the end of month $j+1$. At the end of month $j+1$, the position is closed out and the hedging error over the one-month period can be computed as follows:

$$e^r_{j+1} = \frac{\sum_{i=1}^{N_j} \phi_{i,j} D_{i,j} \left(r^D_{i,j+1} - H'_{i,j} r^H_{i,j+1} \right)}{\sum_{i=1}^{N_j} \phi_{i,j} D_{i,j}},$$

where ϕ denotes the number of each bond held in the portfolio, H_j stacked hedge ratios determined at the end of month j, and r^H_{j+1} the returns on hedging instruments over month $j+1$. Hedging errors for credit spreads can be computed in a similar way. In our empirical analysis, we form six portfolios based on bond ratings at the end of each month, with ϕ set equal to the full amount of outstanding principal.

Following Bertsimas, Kogan, and Lo (2000), we use as the summary statistic for hedging errors the terminal root mean squared hedging error (RMSE) at the end of our sample. Note that RMSE is equal to the volatility of hedged position if the mean hedging error is zero. For comparison, we also compute the RMSE of the unhedged bond portfolio ($H_{i,j} = 0$), denoted $RMSE^u$. One measure of hedging effectiveness for strategy l calculates the reduction in the RMSE as a result of hedging as the following:

$$H_{Eff} = 1 - \frac{RMSE^l}{RMSE^u}$$

When the hedge works perfectly, then $H_{Eff} = 1$. If H_{Eff} is negative, it implies that hedging increases volatility relative to the unhedged position. In general, we expect the hedge to be imperfect, and H_{Eff} to be bounded between zero and one.

3 Data

3.1 Corporate Bond Data

We use corporate bond data from Mergent FISD and Enhanced TRACE over the period July 2002–December 2012 in this study. FISD reports details for corporate debt securities, including information about the name of the issuer, seniority, coupon, face value, issuance date, maturity date, credit rating (from the S&P, Moody's, and Fitch), and redemption features etc. Enhanced TRACE provides information on bond transactions, such as the date and time of execution, the transaction price, and the yield to maturity at time of transaction. In particular, Enhanced TRACE includes more high-yield bond transactions and more precise information about transaction volumes than the "standard" TRACE does.

We apply the following standard filters to construct our sample of corporate bonds: (1) The issue's Mergent bond type is in "U.S. corporate debentures" or "U.S. corporate MTN" categories (CDEB or CMTN, respectively); (2) The issuer is an industrial firm; and (3) Bonds are denominated in U.S. dollars, senior unsecured, without embedded options, and with a fixed coupon rate. These filters lead to a sample of 5,305 straight bonds issued by 1,161 companies, which may or may not have relevant trading records complied in TRACE.

We next implement Dick-Nielsen (2013)'s algorithm to identify and correct reporting errors in TRACE data, and more specifically restrict our sample to bond transaction data without duplicates, reversals, and corrections/cancelations. We also exclude trades with commissions (agency transactions), special sale condition, special price, and less than \$100,000 in volume (Bessembinder et al., 2009; Dick-Nielsen, 2009; Dick-Nielsen et al., 2012). Additionally, we apply Rossi (2014)'s price sequence filter to remove transaction prices that appear to be problematic. Finally, following

Collin-Dufresne, Goldstein, and Martin (2001; CDGM), we exclude observations with time-to-maturity less than 4 years. The resulting sample includes a total of 3,800,272 trades on 1,272 bond issues.

To match TRACE with CRSP and COMPUSTAT, we link each bond's cusip to its issuer's permno in CRSP and gvkey in COMPUSTAT. The coverage rate of the matching in our sample is 88%. We assume that financial statements become available 45 days after the last day of each quarter. Following CDGM, we calculate monthly firm leverage ratios by interpolating values of debt level between quarters.[1]

3.2 Treasury Bond Data

We take the underlying quotes on individual Treasury bonds from the CRSP Master file of monthly Treasury bonds. Data on these quotes are used to calculate Treasury bond returns, except for the 10-year maturity for which we use the monthly series of (adjusted) 10-year bond returns from the CRSP US Treasury and Inflation (MCTI) dataset (following Schaefer and Strebulaev 2008). In the calculation of bond excess returns, we use 1-month Treasury bill rates from the CRSP Monthly Risk-Free Rates file as the risk-free rate.

Quotes on individual bonds are also used to construct yields of zero-coupon Treasury bonds, data necessary for estimating a dynamic term structure model (DTSM). The CRSP Fama-Bliss zero yield data set—widely used in the term structure literature—includes bonds with maturities up to five years only. However, a majority of corporate bonds in our sample, especially investment-grade bonds, have more than five years to maturity (the median maturity of all corporate bonds included in our raw sample is 6.78 years). As such, we need to extend the Fama-Bliss data to longer maturities. Following Le and Singleton (2013), we apply similar filters and algorithms (the so-called "bootstrap" method) as described by Fama and Bliss (1987) to data on individual bonds. And we construct a set of monthly "Fama-Bliss" zero yields with maturity up to ten years over

[1] In robustness checks, we reestimate all relevant regressions using leverage ratios that are calculated based on the latest available balance sheet. The results are qualitatively similar to what we obtain using leverages based on linear interpolation.

the period 1990–2012.[2] This allows us to estimate a DTSM with at least 10-year data when we implement and estimate a structural model recursively in our empirical analysis (Section 5).

3.3 Corporate Bond Returns and Credit Spreads

For corporate bond i at month t, we calculate its end-of-month dirty price $B_{i,t}$ as the volume-weighted average of all trades within 7 days of the month-end. Given consecutive monthly prices, the month-t bond log-return is given by $r_{i,t} = \ln(B_{i,t} + C_{i,t}) - \ln(B_{i,t-1})$, where $C_{i,t}$ is the coupon paid in month t. To make sure that our yield-spread and return series are constructed consistently, we do not compute monthly bond yields as the volume-weighted average of yields reported in each transaction and we instead solve for the month-t bond yield $Y_{i,t}$ using the bond price obtained in the first step.

The yield spread, $CS_{i,t}$, is measured against the actual yield of the nearest-maturity Treasury. Note that interpolated yields are not used here in order to be consistent with the analysis of hedging effectiveness discussed in Section 5, where actual Treasury bonds rather hypothetical ones need to be used as a hedging instrument.[3] We remove upper 1% and lower 1% tails of the credit spreads in order to avoid the influence of outliers. Changes in credit spreads at time t, $\Delta CS_{i,t}$ are calculated as $CS_{i,t} - CS_{i,t-1}$.

Lastly, following Schaefer and Strebulaev (2008), we require a corporate bond to have at least 20 consecutive monthly observations to be included in the sample—given that our empirical analysis focuses on time series regressions.[4] This leads to the final sample of 533 corporate bonds from 245 issuers.

[2] Extending the "bootstrap" method to maturities beyond ten years is limited by the fact that there are much fewer observations available for yields on such individual bonds (see, e.g., Gürkaynak, Sack, and Wright 2007). Instead, we use actual yields of Treasury bonds from the CRSP for maturities longer than 10 years.

[3] In an earlier version of the paper we also consider two alternative methods for constructing Treasury yields: One is to linearly interpolate the yields of two nearest-maturity Treasuries (following CDGM). The other is to construct the entire spot curve using a cubic spline curve and then obtain the yield of a Treasury with the same maturity and coupon rate as the corporate bond. Results from yield spread regressions indicate that the main findings are qualitatively the same.

[4] In one earlier version of the paper we also use the Lehman and Datastream bond data, where this filter has little impact, and obtain qualitatively the same results.

Table 1 provides summary statistics on both monthly yield spreads and changes in the spread, for both the full sample and subsamples grouped by subperiod, leverage, rating and maturity. Overall, there are 34,324 bond-months in the whole sample, with the average credit spread of about 2.52%. The median spread is about 1.51% and the standard deviation 4.03%, indicating that credit spreads are widely dispersed and right-skewed. As expected, corporate yield spreads are much higher and more dispersed after July 2007, a few months after the collapse of two Bear Stearns subprime hedge funds. On average both the level and volatility of bond spreads have been generally restored to their pre-crisis numbers since October 2009, when the U.S. unemployment rate peaked at 10.1%.

Table 2 summarizes returns and excess returns on corporate bonds in our sample. Note that corporate bond returns are skewed to the left with fat tails (with a kurtosis of 52.58). In addition, returns on short-term bonds are substantially more skewed than long-term ones. Compared to investment-grade bonds, high-yield bonds tend to have a higher volatility and are more likely to suffer extreme losses.

4 Simulation Studies

This section conducts numerical simulations to examine a given structural model's ability to predict hedge ratios for both corporate bond returns and yield spreads. As true data-generating processes (DGP) are known by construction in such exercises, they allow us to have a better understanding of why hedge ratios implied by a particular structural model may or may not work. More importantly, results from such simulations provide guidance for our empirical analysis of model-implied hedge ratios using actual corporate bond data.

4.1 The Test Procedure

We conduct our simulation analysis in the following steps: First, given time series of V_t and X_t generated by a particular DGP, we compute in each period the prices of corporate bonds and Treasury bonds based on Eq. (4) and (5). Next, we construct time series of independent variables (corporate bond returns and spread changes) and of independent variables, which include equity returns and Treasury bond returns. Then, a hypothetic hedger, who may or may not correctly specify the data-generating process, estimates the model she has in mind using observed bond data. With estimated model parameters, the hedger calculates hedge ratios for both corporate bond returns and spread changes in each period. Lastly, the performance of estimated hedge ratios is recorded at the end of sample. For comparison, results on hedge ratios implied by the Merton model are always reported, regardless of the DGP perceived by the hedger.

In our simulation exercises, given a DGP, we generate 1,000 samples (trials) of 15 years of daily data for each of six rating classes ranging from B to AAA. For a given rating category, the number of bonds simulated in each trial is set equal to that for the same rating group in our empirical sample (for instance, the number of BBB bonds used is 160). Every bond has an initial maturity of 20 years so it has 5 years to maturity at the end of our regression sample period. This range of bond maturities is largely consistent with CDGM's empirical sample in which all bonds matures in at least 4 years and the median maturity is about 10.2 years. We construct times series of bond returns and credit spreads using price observations at the end of each month. For comparison, we specify firms in different rating categories to have different values of initial leverage and asset volatility. In particular, we use the corresponding parameter values of rating-specific D_0/V_0 and σ_v as estimated by Schaefer and Strebulaev (2008; Table 7).

We choose the four-factor Merton-Vasicek model as the DGP in the baseline simulation analysis, motivated by the stylized fact documented in the term structure literature that at least three factors are needed in order for a GDTMS to adequately capture the behavior of the yield curve. For illustration and validation of our simulation procedure, we also consider the (two-factor) Merton-Vasicek model (Shimko, Tejima, and Van Deventer 1993) to be the DGP but present the analysis in the appendix.[5]

[5] In an earlier version of the paper we also consider the following extensions of the Merton model: a jump-diffusion model allowing for jumps in the asset value, a model with stochastic asset volatility, the Black and Cox (1976), and Longstaff and Schwartz (1995) models. Simulation results indicate that even when each of these four models is used as the DGP, Merton equity hedge ratios for corporate bond returns and spread changes are pretty good approximations. This finding provides further evidence on the robustness of the main result of Schaefer and Strebulaev (2008).

4.2 Hedge Ratios Implied by the Two-Factor Merton-Vasicek Model

This subsection investigates the performance of the (two-factor) Merton-Vasicek model, which is close to the original Merton (1974) model albeit a misspecified model in our simulated economy by construction. In other words, the hedger tries to estimate the original Vasicek model using yield data generated from a three-factor GDTSM.[6]

Consider first the results from regressions of corporate bond excess returns against either stock excess returns, or Treasury bond excess returns, or both, reported in columns 2 through 7 of Table 3. When interpreting these results, we need to bear in mind that firms in our simulated economy are much more homogeneous than those in our real sample. Consequently, regression coefficients across firms in each rating group are not as diverse as those in our empirical results, which lead to small standard errors in the computation of t-statistics. Therefore, for a given degree of deviation from unity, the null hypothesis is more likely to be rejected in simulations than in empirical analysis.

Results from univariate regressions show that the estimates of the coefficient $\beta^r_{i,E}$ are insignificantly different from one regardless of rating categories (Panels A1), indicating that on average the Merton-Vasicek model captures the return's sensitivity to equity even though it is a misspecified model. Interestingly, using the Merton hedge ratio ($\tilde{h}^r_{i,E}$) leads to the same conclusion although the estimates of $\tilde{\beta}^r_{i,E}$ are generally a bit lower than those of $\beta^r_{i,E}$.

The Merton-Vasicek model clearly overestimates the corporate bond return's interest rate sensitivity. But using a Treasury security with the same maturity as that of the corporate bond (Panel A4) reduces the bias notably relative to using a fixed 10-year Treasury (Panel A3). The robustness achieved by matching maturities is suggested by Eq. (9).

Results from bivariate regressions reported in Panel A5 show that $\beta^r_{i,E}$ is still insignificantly different from one and that $\beta^r_{i,10}$ is significantly below one (ranging from 0.54 to 0.60), indicating that equity returns and Treasury bond returns affect corporate bond returns separately. It is worth

[6] Eq. (9) shows that in this situation the hedger needs to estimate only two parameters regarding interest rate dynamics, $K^{\mathbb{Q}}_{1,r}$ and σ_r. As far as hedging is concerned, it makes little difference whether a "completely affine" or "essentially affine" model (Duffee, 2002) is estimated. Joslin and Le (2013) find that fitting under the risk-neutral measure is typically given more priority when there is a tension in simultaneous fitting of the physical and risk-neutral dynamics. Therefore, the only damage caused by estimating a "completely affine" model is a highly underestimated $K^{\mathbb{P}}_{1,r}$. Unreported simulation results confirm this conjecture.

noting that the simulation results shown in this panel are quite similar to the empirical results obtained from the same regression with real data (see Panel D of Table 5). Panel A6 conveys the same message as Panel A4 does: replacing the 10-year Treasury in Panel A5 by a Treasury with the same maturity as the corporate bond helps narrow the gap between the slope coefficient and one.

Consider next the regression results for corporate bond yield spread changes, reported in columns 9 through 14 of Table 3. Let's focus on bivariate regressions (Panels B5 and B6). Notice that the null that $\beta_{i,E}^{CS}$ equals unity is rejected except for AAA and B bonds, the two smallest rating groups in our sample. The null that $\beta_{i,10}^{CS}$ equals unity is rejected except for B bonds. And the null that $\beta_{i,E}^{CS}$ equals unity is rejected regardless of rating groups. Nonetheless, in those cases where the null is rejected, the magnitude of $\beta_{i,E}^{CS}$ ranges from 1.04 for BB bonds to 1.17 for AA bonds, the magnitude of $\beta_{i,10}^{CS}$ from 1.07 for BB bonds to 1.13 for AAA bonds, and that of $\beta_{i,\tau}^{CS}$ from 1.11 for B bonds to 1.18 for AA bonds. That is, the magnitudes of these three regression coefficients are reasonably close to unity.

To summarize, the above simulation results indicate that although it is misspecified, on average the Merton-Vasicek model captures the equity sensitivity, especially for corporate bond returns. However, the model considerably over-predicts the interest rate sensitivity of corporate bond returns, and it under-predicts that of credit spreads albeit to a less extent.

Panels C1 through C6 report results on the effectiveness of hedging corporate bond returns (columns 1 through 6) or yield spread changes (columns 7 through 12) with different hedging instruments. We make a few observations from these results. First, using equity to hedge spread changes is much more effective than to hedge corporate bond returns (Panels C1 and C2). This implies that to some extent the models are more useful for hedging spreads than for hedging returns. Also, the Merton equity hedge ratio is often more effective than the two-factor MV equity hedge ratio, as it does not depend on term structure parameters. Therefore, it offers some robustness for hedging spreads when the interest rate process is misspecified. This effect is stronger for the spreads of speculative-grade bonds, in which the interest rate risk plays a small role.

Second, using a maturity-matching Treasury bond as a hedging instrument is much more effective than using a 10-year Treasury note. Finally, hedging with both equity and maturity-matching Treasuries is much more effective than using equity along with a fixed-maturity T-note. On the other hand, since the two-factor Merton-Vasicek model misspecifies the equity sensitivity of corporate bond returns, adding equity as an additional hedging instrument could damage the hedging performance. Therefore, we find that values of hedging effectiveness in Panel C6 are lower than corresponding numbers reported in Panel C4.

4.3 Hedge Ratios Implied by the Four-Factor Merton-Vasicek Model

This subsection considers hedge ratios implied by the four-factor Merton-Vasicek model itself. Note that even though this model coincides with the DGP in this exercise, it is assumed that the hedger does not observe the true model parameters. As such, she needs to estimate the three-factor GDSTM first and then calculates hedge ratios using the estimated model. Another implication of this is that three Treasury securities are essential to fully hedge the interest rate risk. Following JSZ, we assume that 6-month, 2-year, and 10-year Treasury yields are observed with measurement error,[7] and that the hedger uses Treasuries with these maturities as hedging instruments. Note that these three yields captures to a large extent the first three principal components of yield curve, as y^{10} roughly corresponds to the "level" factor, $y^{10} - y^{0.5}$ the "slope" factor, and $y^2 - (y^{10} + y^{0.5})/2$ the "curvature" factor.

Table 4 presents results from both regressions of corporate bond returns (Panels A1 through A3) and those of yield spreads (Panels B1 through B3) against either equity return, or returns of the aforementioned three Treasury securities, or both. If we focus on Panels A3 and B3, a few observations can be made from these two panels. First, both $\beta^r_{i,E}$ and $\beta^{CS}_{i,E}$ are insignificantly different from one, regardless of rating groups. Second, coefficients on excess Treasury returns are much closer to one compared to the results in Panel A5 of Table 3. Although the null hypothesis is still rejected in some cases, the absolute deviation of average coefficients from one are all within

[7] The hedger is assumed able to identify these Treasury bonds that are priced exactly. Hence, she uses these three yields as observed factors when estimating the $\mathbb{P}$-measure dynamics. See Section 2 of JSZ for details.

0.08. Third, coefficients on Treasury returns are all insignificantly different from one except for the $\beta^{CS}_{i,0.5}$ in two rating groups; even for these two classes the t-statistics are marginally significant. Overall, we find that a correct identification of DGP significantly improves the model's ability to predict hedge ratios, especially for the interest rate sensitivities.

Columns 1 through 6 in Panels C1-C3 shows effectiveness on hedging bond returns. We interpret these results by comparing them to corresponding panels in Table 3. While a more realistic GDTSM only provides a marginal improvement in hedging the firm-specific risk (Panels C1), it dramatically increases the effectiveness of interest-rate-risk hedging (Panels C2 against Panel C3 of Table 3). Even if we use as benchmark the numbers in Panel C4 of Table 3, which result from a robust hedging scheme, hedging with three Treasuries still offers nontrivial advantage. Results in Panel C3 suggest that, due to the sampling uncertainty and estimation errors, we should not expect to have a perfect hedge even when the DGP is correctly specified. This point of view is relevant when we interpret our empirical results.

When we combine these results with those in columns 7 through 12, we can draw the following conclusion. First, using equity to hedge spread changes is still much more effective than to hedge corporate bond returns (Panel C1). Second, using Treasuries to hedge is much more effective for hedging corporate bonds than for hedging yield spread changes (Panel C2). Finally, if our objective is to minimize RMSEs, hedging with both equity and three Treasuries does not always lead to the best performance; this result holds for both corporate bond returns and spread changes.

We next conduct an empirical analysis to investigate the performance of hedge ratios implied by both the one- and Four-Factor Merton-Vasicek models, based on real data on Treasury securities, individual corporate bonds, and stocks.

5 Empirical Results

This section conducts an empirical analysis of hedge ratios for both corporate bond returns and yield spreads, implied by both the two- and four-factor Merton-Vasicek models. We first consider a regression analysis of hedge ratios implied by the two-factor model. We then redo the regression analysis using hedge ratios implied by the four-factor Merton-Vasicek model. Lastly, we investigate the effectiveness of hedging corporate bond returns or yield spreads based on these two models. Note that for each model we recursively estimate the parameters governing interest rate dynamics, using Treasury yields from January 1990 to the month in which hedge ratios are to be calculated. Estimation of V_t and σ_v is based on daily observations over a 3-month rolling window (Campbell et al., 2008).

5.1 Regressions on Merton-Vasicek Hedge Ratios

We estimate bivariate regressions (11) and (12) along with some special cases of these models, and summarize regression results for corporate bond returns in Table 5 and those for yield spread changes in Table 6, by rating groups. Specifically, for each bond in a given rating category, we estimate time-series regressions of excess returns on corporate bonds (or yield spread changes) against either equity, or Treasury, or both. We then report the means of estimated regression coefficients over individual bonds in a given rating group. Note that the t-statistics shown in the table are no longer based on the flipping Fama-Macbeth approach as adopted by CDGM. Instead, we estimate the standard error using the Schaefer and Strebulaev (2008) method that takes into account the cross-sectional covariations in coefficient estimates.

Consider return regressions first. Results from univariate regressions, reported in Panels A through C, indicate that while the mean estimate of $\beta^r_{i,E}$ is insignificantly different from one, the mean estimate of $\beta^r_{i,10}$ (or $\beta^r_{i,T}$) is significantly different from one, regardless of rating groups. However, the fact that the mean $\beta^r_{i,E} \approx 0.25$ for AAA bonds indicates substantial cross-sectional variation in the $\beta^r_{i,E}$ estimates for the 17 bonds in this rating group. These results for the slope estimates are consistent with evidence from simulations (Panels A1 through A4 of Table 3), indicating that the data-generating process used in simulation captures some important aspects of corporate bond returns. Also, note that as the rating decreases, the R^2 of regressions against equity increases from 1.5% for AAA bonds to 23.6% for B bonds (Panel A) while the R^2 of regressions against the Treasury decreases (Panels B and C). For instance, the R^2 against the 10-year T-note ranges from

1.3% for B bonds to 55.1% for AAA bonds. That is, while equity affects mainly excess returns on HY bonds, Treasuries affect mainly excess returns on IG bonds.

Panel D of Table 5 reports results from bivariate regressions against both the equity and 10-year T-note. Note that the main implications from these results are consistent with those from univariate regression results (Panels A and B). That is, the null that $\beta^r_{i,E} = 1$ is still not rejected and the null that $\beta^r_{i,10} = 1$ is strongly rejected, regardless of rating groups. Specifically, the average $\beta^r_{i,10}$ ranges from 0.52 to 0.71 for IG bonds, is about 0.03 for BB bonds, and 0.22 for B bonds.[8] This finding is consistent with Schaefer and Strebulaev (2008), who note that the Merton-Vasicek model's failure in capturing the interest rate sensitivity of corporate bond returns "remains an interesting puzzle" (page 3).[9] Below we document that there exists a similar puzzle regarding the interest rate sensitivity of corporate yield spread changes (Panel D of Table 6). Section 5.2 shows that both puzzles can be largely resolved by incorporating a three-factor DTSM into the Merton-Vasicek model.

Consider next the results from regressions of yield spread changes, reported in Table 6. As the univariate regression results (Panels A through C) are largely consistent with those from bivariate regressions (Panel D), we focus on the latter in the discussion that follows in this subsection. We make three observations from Panel D. First, the null that $\beta^{CS}_{i,E} = 1$ is not rejected, regardless of bond ratings; that is, the Merton-Vasicek model also predicts the equity sensitivity of corporate yield spread changes. This finding complements Schaefer and Strebulaev (2008)'s for the equity sensitivity of corporate bond returns. Second, the null that $\beta^{CS}_{i,10} = 1$ is strongly rejected except for AA bonds. And the Merton-Vasicek model underestimates the interest rate sensitivity of yield spread changes. Third, if we exclude AAA bonds, the R^2 increases as the rating decreases, going from 11% for AA bonds to 27% for B bonds.

[8] Untabulated results indicate that replacing the 10-year T-note by a T-year Treasury matching the maturity of the corporate bond in the bivariate regressions leads to a larger coefficient on the Treasury return ($\beta^r_{i,T}$) but the null is still rejected.

[9] The average $\beta^r_{i,10}$ ranges from 0.31 to 0.41 for IG bonds, is 0.32 for BB bonds, and -0.19 for B bonds in Schaefer and Strebulaev (2008; Table 11).

To summarize, the regression results based on the Merton-Vasicek model confirm the main finding of Schaefer and Strebulaev (2008) for the sensitivities of corporate bond returns and importantly, show that the similar findings obtain for the sensitivities of yield spread changes.

5.2 Regressions on Four-Factor Merton-Vasicek Hedge Ratios

Given the implications from simulation results represented in Section 4.3 and the insights from the term structure literature, we consider a four-factor model that incorporates a three-factor GDTSM into the Merton model and conduct a regression analysis of hedge ratios implied by this four-factor model.

Consider regression results for corporate bond returns first, reported in Table 7. Notice from Panel A of the table that incorporating a multi-factor term structure model does not affect the Merton-Vasicek model's ability to predict the corporate bond return's equity sensitivity. Comparing Panel B of the table with Panel B of Table 5, we can see that augmenting regressions of corporate bond excess returns on excess returns on 10-year T-notes with excess returns on the 6-month T-bills and 2-year T-notes raises the R^2 considerably except for AA and A bonds. Importantly, $\beta^r_{i,10}$ becomes much higher (and closer to one) under the augmented regressions. In particular, $\beta^r_{i,10}$ is not significantly different from one for BB bonds and about 0.61 for B bonds. Recall from Panel B of Table 5 that the (two-factor) Merton-Vasicek model is unable to capture the interest rate sensitivities of excess returns on corporate bonds in these two rating groups.

Panel C of Table 7 reports the results from regressions of excess corporate bond returns on four hedge ratios implied by the four-factor Merton-Vasicek model, including the equity sensitivity and three interest rate sensitivities. Here are a few observations from the table. First, the null that $\beta^r_{i,E} = 1$ is still not rejected, regardless of rating groups. Second, $\beta^r_{i,10}$ is significantly different from one for BBB and BB bonds only. Even for these two rating groups, the average $\beta^r_{i,10}$ increases substantially when the four-factor model implied hedge ratio, $h^r_{i,10}$, is incorporated into the regressions. More specifically, the average $\beta^r_{i,10}$ increases from 0.52 under the two-factor model to 0.84 for BBB bonds, and from 0.03 under the two-factor model to 0.66. These results indicate that incorporating a multi-factor DTSM allows us to better capture the dynamics of the yield curve and thus the interest rate sensitivities of corporate bond returns.

Consider next regressions of yield spread changes on hedge ratios implied by the four-factor Merton-Vasicek model. Results from univariate regressions against equity returns, reported in Panel A of Table 8, are consistent with those under the two-factor Merton-Vasicek model (Panel A of Table 6). For instance, $\beta_{i,E}^{CS}$ is insignificantly different from one, regardless of rating categories. And equity return is more important to spread changes than to corporate bond returns, as can be seen from the average full sample (adjusted) R^2 being 9.3% for the spread regressions, the same as that for the return regressions (Panel A of Table 7)—as mentioned before, return regressions are usually expected to have much higher R^2 than otherwise identical spread regressions. Panel B of Table 8 shows that the three coefficients on the interest rate sensitivities are insignificantly different from one in most cases. Lastly, note from Panel C of the table that none of the regression coefficients are statistically different from one, except in one case where $\beta_{i,10}^{CS} = 1.3$ (with a t-value of 2.01) for BB bonds.

To summarize, the results from the regression analysis presented in this subsection indicate that incorporating a more realistic DTMS into the Merton-Vasicek model allows us to fully capture the interest rate sensitivities of yield spread changes. In addition, doing so also significantly improves the model's ability to capture the interest rate sensitivities of corporate bond returns and thus mitigates the "interest rate sensitivity puzzle" noted by Schaefer and Strebulaev (2008).

Before proceeding to the analysis of hedging effectiveness, we make one remark about potentials reasons for the inability of the four-factor Merton-Vasicek model to fully capture the interest rate exposure of corporate bond returns. Recent evidence has shown that excess returns on Treasury bonds contain information beyond what is contained in (default-free) yield curve factors (e.g., Cochrane and Piazzesi 2005; Duffee 2011). That is, the three-factor DTSM included in the four-factor MV model is missing some other predictors of excess returns on Treasury bonds. However, this is not necessary a problem for predictions of hedge ratios on spread changes as spread changes are much less affected by Treasury returns than corporate bond returns are.

5.3 Hedging Effectiveness

Regression results presented in Sections 5.1 and 5.2 show that the four-factor Merton-Vasicek model helps mitigate the "interest rate sensitivity puzzle." This subsection examines whether hedging effectiveness improves under this four-factor model.

For comparison, we analyze hedging effectiveness under the two-factor Merton-Vasicek model first, and report results on hedging corporate bond returns in Panel A and those on hedging spread changes in Panel B of Table 9. Note from Panel A that while using equity to hedge corporate bond returns is more effective for HY bonds, using Treasury to hedge is more effective for IG bonds. In addition, using a Treasury with the same maturity T as that of the hedged corporate bond is more effective than using a fixed maturity 10-year Treasury, as implied from Eq. (9). Results shown in Panel B indicate that for HY bonds, model-based hedging of spread changes is often more effective than that of corporate bond returns. Also, it is worth noting from both panels that when equity is used as the hedging instrument, the Merton-based hedge is often more effective than the two-factor model based hedge. This finding provides further evidence on the robustness of the Merton equity hedge ratios.

Panel C reports results on the effectiveness of hedging corporate bond returns based on the four-factor Merton-Vasicek model. Comparing this panel with Panel A, we see that the effectiveness of hedging with equity is not affected much under the four-factor model. On the other hand, hedging with Treasury securities becomes much more effective for investment-grade bonds under this model. For instance, the measure of hedging effectiveness H_{Eff} increases from 0.34 to 0.61 for AAA bonds, 0.31 to 0.54 for AA, and 0.24 to 0.38 for A, under the four-factor model. When equity and the three Treasuries are used together to hedge corporate bond returns, H_{Eff} goes from 0.50 to 0.59 (a 18% of increases) for AAA bonds and increases more than 10% for A bonds.

Comparing Panels D and B of Table 9, we see that hedging spread changes with Treasuries is much more effective under the four-factor model, especially for investment-grade bonds. For instance, H_{Eff} is more than doubled for upper tier investment-grade bonds, almost doubled for

BBB bonds (increasing from 3.9% to 7.3%), and increases from 3.0% to 4.8% for BB bonds, and 2.3% to 2.9% for B bonds.

Overall, the above results show that when Treasuries are used to hedge either IG corporate bond returns or spread changes, hedging effectiveness substantially improves under the four-factor model.

6 Conclusion

This paper studies hedge ratios on corporate bond returns and yield spread changes, implied by structural models with stochastic interest rates. Specifically, we consider the class of the models where the constant interest rate in the original Merton (1974) model is replaced by a Gaussian dynamic term structure model (GDTSM). We implement two specifications in this analysis: the two-factor Merton-Vasicek model and the four-factor one that incorporates a three-factor GDTSM (which has not been studied in the literature). For each of these two models, we examine its ability to predict hedge ratios and hedging effectiveness under the model, and compare the ability to hedge interest rate risk to its ability to hedge equity returns.

We find that while it provides quite accurate predictions of the equity sensitivity of corporate bond returns, the two-factor Merton-Vasicek model fails to capture their interest rate sensitivity, consistent with the existing literature. Furthermore, we find the same pattern with the model implied sensitivities of yield spread changes. That is, the structural model allowing for only simple term structure dynamics has difficulty capturing the interest rate sensitivity of corporate debt, regardless of whether corporate bond returns or yield spread changes are considered.

We next examine the four-factor Merton-Vasicek model that includes a three-factor GDTSM and thus allows for much richer term structure dynamics compared to the two-factor Merton-Vasicek. The idea behind this is that as the majority of the price changes for investment-grade bonds are related to Treasury bond price changes, incorporating a more realistic DTSM should help a structural credit risk model better predict the interest rate sensitivity of corporate bonds. Our results show that indeed this four-factor model largely captures the (interest-rate) level sensitivity of corporate bond returns and, in addition, provides quite accurate predictions of the yield spread change's sensitivity to the entire yield curve. Importantly, we also find evidence that for investment-grade bonds in our sample, hedging effectiveness substantially improves under the four-factor model.

Although there is an enormous theoretical literature on structural modeling of credit risk, including a number of studies investigating the role of stochastic interest rates in a structural bond pricing framework, very few such models go beyond one-factor DTSMs. Partly, this reflects the stylized fact documented in the literature that allowing for stochastic interest rates does little to improve the ability of the model to predict bond yield spreads. However, our results indicate that to better capture and hedge the interest rate exposure of corporate bonds, we need to incorporate a more realistic DTSM in the existing structural models. In another words, the empirical results from our analysis provide support for structural models that draw more from the term structure literature.

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A Gaussian Dynamic Term Structure Models (GDTSMs)

We begin with the canonical representation of GDTSMs devoloped by JSZ, which defines the most general admissible GDTSM for a given dimension of the state vector. JSZ show that any canonical GDTSM can be transformed to a unique GDTSM parameterized by $\Theta_X = (\gamma^{\mathbb{Q}}, k_{\infty}^{\mathbb{Q}}, \Sigma_X, K_{0,X}^{\mathbb{P}}, K_{1,X}^{\mathbb{P}})$. It follows that bond pricing is fully determined by $\Theta_X^{\mathbb{Q}} = (\gamma^{\mathbb{Q}}, u_{\infty}^{\mathbb{Q}}, \Sigma_X)$, a subset of Θ_X that governs the risk-neutral dynamics of state variables,

$$
\begin{aligned}
r_t &= \imath \cdot X_t, && (15)\\
dX_t &= (K_{0,X}^{\mathbb{Q}} + K_{1,X}^{\mathbb{Q}} X_t)dt + \Sigma_X dW_{X,t}^{\mathbb{Q}}, && (16)\\
p_t^T &= A_x(\Theta_X^{\mathbb{Q}}, T-t) + B_x(\Theta_X^{\mathbb{Q}}, T-t)' X_t,
\end{aligned}
$$

where $\imath$ is a vector of ones, $K_{0,X}^{\mathbb{Q}} = [k_{\infty}^{\mathbb{Q}} \quad 0_{1\times(k-1)}]'$, $K_{1,X}^{\mathbb{Q}}$ has the real Jordan form determined by the eigenvalue vector $\gamma^{\mathbb{Q}}$, Σ_X is lower triangular and $\epsilon_t^{\mathbb{Q}} \sim N(0, I_N)$. p_t^T denotes the time-t log-price of a zero-coupon bond maturing at T; $\{A_x(T-t), B_x(T-t)\}$ satisfy a form of what is known as a Riccati equation. $\{K_{0,X}^{\mathbb{P}}, K_{1,X}^{\mathbb{P}}\}$ determine the physical measure dynamics,

$$
dX_t = (K_{0,X}^{\mathbb{P}} + K_{1,X}^{\mathbb{P}} X_t)dt + \Sigma_X dW_{X,t}^{\mathbb{P}}. \qquad (17)
$$

If there is no restriction imposed on the market price of risk Λ_t,

$$
\begin{aligned}
\Lambda_t &= \Sigma_X^{-1}\left(K_{0,X}^{\mathbb{P}} - K_{0,X}^{\mathbb{Q}} + (K_{1,X}^{\mathbb{P}} - K_{1,X}^{\mathbb{Q}})X_t\right),\\
&= \Sigma_X^{-1}\left(\lambda_0 + \lambda_1 X_t\right),
\end{aligned}
$$

then $\{K_{0,X}^{\mathbb{P}}, K_{1,X}^{\mathbb{P}}\}$ are free parameters.

In simulation and empirical analysis, we assume that the yields of k zero-coupon bonds, with maturities $\mathcal{M} = \{m_1, \ldots, m_k\}$, are measured without error,

$$\begin{aligned} Y_t^{\mathcal{M}} &= -p_t^{\mathcal{M}} \circ \mathcal{M}^{-1} \\ &= \widetilde{A_x^{\mathcal{M}}} + \widetilde{B_x^{\mathcal{M}}} X_t. \end{aligned}$$

where $\widetilde{B_x^{\mathcal{M}}} = -B_x^{\mathcal{M}} \circ (\mathcal{M}^{-1} \cdot \iota')$ are yield loadings on state factors. According to Theorem 1 in JSZ, the canonical GDTSM (16)–(17) is observationally equivalent to a GDTSM in which the stacked yields $Y_t^{\mathcal{M}}$ can serve as the factors,

$$\begin{aligned} r_t &= \delta_0 + \delta_1 Y_t, \\ dY_t &= (K_{0,Y}^{\mathbb{Q}} + K_{1,Y}^{\mathbb{Q}} Y_t)dt + \Sigma_Y dW_{X,t}^{\mathbb{Q}}, \\ dY_t &= (K_{0,Y}^{\mathbb{P}} + K_{1,Y}^{\mathbb{P}} Y_t)dt + \Sigma_Y dW_{X,t}^{\mathbb{P}}, \end{aligned}$$

where $K_{1,Y} = \widetilde{B_x^{\mathcal{M}}} K_{1,X} \widetilde{B_x^{\mathcal{M}}}^{-1}$ and $K_{0,Y} = \widetilde{B_x^{\mathcal{M}}} K_{0,X} - K_{1,Y} \widetilde{A_x^{\mathcal{M}}}$.

In this study, we estimate GDTSMs using JSZ's maximum likelihood (ML) estimator. Their estimation methods features a complete separation of the $\mathbb{P}$-measure parameters $\{K_{0,Y}^{\mathbb{P}}, K_{1,Y}^{\mathbb{P}}\}$ from those governing risk-neutral bond pricing $\Theta_X^{\mathbb{Q}}$. Absent further restrictions, the ML estimators of $\{K_{0,Y}^{\mathbb{P}}, K_{1,Y}^{\mathbb{P}}\}$ are recovered by standard linear projection. Our full sample for estimation spans from 1990 to 2012. The parameters in our simulation analysis are based on a one-time estimation using the whole sample. The value of estimated parameters are reported in Table 10. In our empirical analysis, GDTSMs are estimated recursively: hedge ratios for August 2002 are computed using model estimates based on observations from January 1990 through July 2002; at the end of August, we repeat this exercise using observations of one additional month, and so on.

B Simulation Results Based on the Shimko et al. (1993) Model

This appendix examines the performance of the Merton sensitivities in the two-factor Merton-Vasicek economy with stochastic interest rates (Shimko et al. 1993). Under this data-generating process (DGP), the short rate r_t is assumed to follow the Vasicek (1977) dynamics:

$$dr_t = \kappa(\bar{r}^{\mathbb{Q}} - r_t)dt + \sigma_r dW_{r,t}^{\mathbb{Q}}. \tag{18}$$

As is the case of multiple-factor GDTSMs, when we estimate the Vasicek (1977) model we pick one particular maturity for which the zero yield is assumed to be uncontaminated by idiosyncratic noise. Specifically, in our empirical and simulation analysis related to the Vasicek model we assume that all yields except for the 10-year bond share an identical standard deviation of measurement error,

$$Y_t^T = \widetilde{A}(T-t) + \widetilde{B}(T-t)r_t + \eta_t, \qquad \eta_t \sim N(0, \sigma_\eta^2).$$

It is well-known that empirical implementation of one-factor DTSMs leads to large estimates of σ_η. For instance, Cheridito, Filipovic, and Kimmel (2007) estimate a variety of one-factor Gaussian and square-root models and their estimates of σ_η ranges from 1.13% to 1.59%. If we use our entire sample to estimate the Vasicek model, $\hat{\sigma}_\eta$ becomes smaller, but still at 0.59%. To assess the impact of measurement error on the hedging performance of structural models, we run our simulation based on three different values of σ_η: 1%, 0.5% and 0.1%. The last value is close to the magnitude of σ_η estimated from a three-factor model.[10]

Tables 11 and 12 illustrates the effect of σ_η's magnitude on the performance of estimated hedge ratios. In this simulation analysis, the data-generating process is the Shimko et al. (1993) model and the hedger uses the correct model to compute optimal hedge ratios. If $\sigma_\eta = 0$, model parameters can be estimated almost perfectly, given a sufficiently long sample. Therefore, the estimated hedge ratios should accurately characterize the sensitivity of corporate bonds to hedging instruments.

[10] With our whole sample, the σ_η estimated from three-factor GDTSM is 7.8 basis points; this value is also used in our simulation study as reported in Tables 3 and 4.

The main message conveyed by Table 11 is that the hedger is able to obtain reasonably accurate hedge ratios only in the case of $\sigma_\eta = 0.1\%$. In particular, results in Panels A3 and A5 indicate that when the measurement error is large, the slope coefficients corresponding to $h^r_{10} rx^{10}_t$ are well below one, which resemble the regression pattern as reported in Schaefer and Strebulaev (2008). Empirically, the hedger implicitly acknowledges large measurement error if she relies on the Shimko et al. (1993) model (or other structural models with one-dimensional interest rate process) to compute hedge ratios. Hence, the empirical result of $\beta^r_{10} \ll 1$ is actually to be expected. With that being said, we also observe from Panels A1 and A5 that h^r_E does not match the true equity sensitivity when σ_η is large. This is inconsistent with the evidence reported in Schaefer and Strebulaev (2008) and in our Section 5, which suggests that measurement error along cannot replicate the empirical results of regression tests. To account for the joint patterns of $\beta^r_E \approx 1$ and $\beta^r_{10} \ll 1$, we need to introduce the channel of model misspecification, as demonstrated in Section 4.

We also find that the hedger can achieve robustness by using the Treasury bond with the same maturity as that of the hedged corporate bond to hedge the latter's interest rate risk. With this hedging scheme, the accuracy of estimated hedge ratios seems almost affected by the magnitude of measurement error, as shown in Panels A4 and A6. For most rating classes, the average regression coefficient β^r_T is almost indistinguishable from one. This superior accuracy is also translated to hedging effectiveness. Table 12 shows that when we use a 10-year Treasury bond alone to hedge corporate bond returns, the hedging performance is acceptable only if σ_η is at the magnitude of 10 basis points. If σ_η is as large as 1%, hedging with a fixed-term bond would lead to even higher RMSEs than no hedging at all. In contrast, hedging with matched maturity would reduce the RMSE by at least 85%. For AAA bonds, the latter hedging scheme results in nearly perfect hedges. The same remarks applies to the hedging of credit spreads as well.

C Estimation of Firm Specific Inputs

To compute the model-implied sensitivities as shown in Eq. (6)–(10), we need to estimate the following two parameters: the market value of firm V and the standard deviation of asset returns

σ_v. At end of each month, we calibrate the Merton (1974) model to real data to obtain the value of these parameters. The essence of our calibration is to match the predicted equity value and equity volatility with their observed values. This is equivalent to solving the following two equations at each point of time:

$$E_t = A_t N(d_1) - Ke^{-r(T-t)} N(d_2); \tag{19}$$

$$\sigma_E = \frac{A_t}{E_t} N(d_1)\sigma_v, \tag{20}$$

where K is the total debt outstanding and σ_E the equity volatility. This approach is used in several studies including Jones, Mason, and Rosenfeld (1984) and Campbell, Hilscher, and Szilagyi (2008).[11]

We also estimate V and σ_v using two alternative methods: the KMV method (Bohn and Crosbie 2003) and the ML estimation method. The former focuses on Eq. (19) and calculates σ_v as the sample standard deviation of the time series of estimated asset returns, $\ln(V_{t+1}) - \ln(V_t)$; the latter is based on a likelihood function of stock price movement that is introduced in Duan (1994), who utilizes Eqs. (1) and (19) but not Eq. (20). We find that empirically these alternative estimators barely help improve the performance of implied sensitivities in our sample. More importantly, both methods require the whole time series to be observed, which prohibits real-time estimation of hedge ratios.

Following Eom, Helwege, and Huang (2004), we estimate K using the firm's total liability (Compustat item LTQ). An alternative measure is the firm's short-term debt (the larger of $DLCQ$ and $LCTQ$) plus one half of its long-term debt ($DLTTQ$), first proposed by Moody's KMV (Bohn and Crosbie 2003). However, we find that although the latter specification has little impact on the model prediction of sensitivities, it lowers the performance of the model spreads in explaining the temporal variations in market spreads. The time to maturity $T-t$ is set equal to the bond-specific maturity instead of the average maturity of all bonds issued by the firm. While this specification

[11]Note that we can obtain A_t and σ_v by solving (19) and (20) either iteratively or simultaneously. In our implementation, we try both ways for each firm and pick those estimates that result in smaller RMSEs.

leads to conceptual inconsistency, as the same firm could have different estimated asset values, it actually improves the model prediction of hedge ratios as well as the hedging performance. The risk-free rate r used is the Treasury yield with time to maturity of $T - t$.

Table 1: Summary Statistics on Corporate Bond Yield Spreads

	# of obs.	Yield Spreads (%)					Changes in the Yield Spread (%)				
		Mean	Median	Stdev	5%	95%	Mean	Median	Stdev	5%	95%
Full sample	34324	2.525	1.512	4.026	0.395	7.102	-0.002	-0.013	1.632	-0.884	0.874
Panel A: Subperiod groups											
2002.07-2007.06	21453	1.877	1.176	2.512	0.361	5.516	-0.038	-0.017	1.156	-0.738	0.594
2007.07-2009.10	6095	4.818	2.730	7.561	0.861	14.094	0.142	0.041	3.110	-1.757	2.233
2009.11-2012.12	6776	2.513	1.901	2.160	0.420	6.562	-0.019	-0.028	0.675	-0.716	0.669
Panel B: Leverage groups											
L	6865	1.031	0.772	1.459	0.249	2.483	0.004	-0.002	0.527	-0.457	0.514
2	6868	1.344	1.088	1.281	0.347	3.037	-0.004	-0.008	0.514	-0.533	0.553
3	6862	2.105	1.715	1.833	0.604	4.736	-0.031	-0.029	0.876	-0.806	0.670
4	6881	2.421	1.776	2.029	0.570	6.112	-0.023	-0.017	0.842	-0.931	0.900
H	6848	5.731	4.007	7.475	0.740	15.240	0.045	-0.023	3.376	-1.896	2.233
Panel C: Rating groups											
AAA	1129	8.740	5.861	9.210	1.166	23.203	-0.135	-0.056	5.250	-4.286	4.101
AA	994	0.615	0.550	0.358	0.157	1.198	0.003	-0.003	0.231	-0.332	0.323
A	2744	0.827	0.649	0.627	0.201	2.053	0.000	-0.006	0.344	-0.456	0.436
BBB	11381	1.184	0.987	0.763	0.378	2.640	-0.003	-0.006	0.420	-0.525	0.517
BB	10164	2.207	1.677	1.892	0.561	5.637	-0.020	-0.022	0.760	-0.867	0.764
B	4928	3.985	3.079	5.414	1.119	8.091	0.057	-0.029	2.196	-1.179	1.379
C & NR	2984	6.154	4.655	6.988	1.568	15.375	0.013	-0.026	3.082	-1.848	1.995
Panel D: Maturity groups											
Short (4-8yr)	12691	2.099	1.095	3.731	0.313	6.485	-0.008	-0.020	1.496	-0.938	0.915
Median (9-15yr)	5684	2.983	1.967	4.069	0.630	7.863	0.011	-0.013	1.561	-0.899	0.928
Long (>15yr)	10949	2.975	1.875	4.374	0.722	7.563	0.001	-0.006	1.865	-0.798	0.776

This table reports summary statistics for both levels of the monthly corporate bond yield spread and changes in the bond yield spread by sample period (Panel A), leverage (Panel B), rating category (Panel C), and time-to-maturity (Panel D), respectively. Five leverage groups are formed from low (L) to high (H) according to the average leverage for firms in the entire sample period. Rating groups are formed based on the time-series average of each bond's S&P ratings over the sample period. Three maturity groups are formed according to time-to-maturity: between 4 and 8 years (Short), between 9 and 15 years (Median), and greater than 15 years (Long). The sample period is from July 2002 to December 2012 and divided into two subperiods: 2002:Q3-2007:Q2, 2007:Q3-2009:Q4 and 2010:Q1-2012:Q4. N is the number of bonds in each category; the "5%" and "95%" show the 5th and 95th percentile.

Table 2: Summary Statistics on Corporate Bond Returns

	# of obs.	Corporate Bond Returns					Excess Corp. Bond Returns				
		Mean	Median	Stdev	5%	95%	Mean	Median	Stdev	5%	95%
full sample	34324	0.817	0.547	4.888	-3.793	5.940	0.534	0.310	4.424	-3.519	5.109
Panel A: Subperiod groups											
2002.07-2007.06	21453	0.799	0.494	3.956	-3.134	5.242	0.486	0.216	3.605	-3.023	4.451
2007.07-2009.10	6095	0.740	0.686	8.557	-9.380	10.762	0.415	0.407	7.776	-8.698	8.631
2009.11-2012.12	6776	0.938	0.691	3.182	-2.877	5.482	0.790	0.624	2.693	-2.683	4.641
Panel B: Leverage groups											
L	6865	0.560	0.388	2.344	-2.521	4.028	0.303	0.159	2.085	-2.390	3.350
2	6868	0.652	0.438	2.823	-2.528	4.339	0.337	0.162	2.274	-2.403	3.493
3	6862	0.950	0.685	3.280	-3.149	5.764	0.680	0.466	2.946	-2.882	4.690
4	6881	0.894	0.652	4.266	-4.010	6.165	0.607	0.432	3.735	-3.678	5.230
H	6848	1.028	0.785	8.765	-8.667	10.609	0.741	0.594	8.096	-8.272	9.186
Panel C: Rating groups											
AAA	1129	0.580	0.352	2.365	-2.821	4.580	0.415	0.232	2.203	-2.793	4.012
AA	994	0.461	0.339	1.990	-2.230	3.326	0.270	0.149	1.852	-2.141	2.996
A	2744	0.675	0.470	2.982	-2.746	4.548	0.390	0.226	2.594	-2.553	3.667
BBB	11381	0.876	0.615	3.627	-3.315	5.875	0.606	0.377	3.087	-3.052	4.910
BB	10164	0.850	0.796	5.367	-5.435	7.062	0.545	0.545	5.032	-5.251	6.316
B	4928	0.972	0.884	8.637	-8.833	10.700	0.676	0.626	8.224	-8.678	9.520
C & NR	2984	2.453	1.272	13.525	-12.504	18.789	1.817	0.927	11.892	-11.232	15.269
Panel D: Maturity groups											
Short (4-8yr)	12691	0.654	0.444	3.030	-1.945	3.829	0.405	0.214	2.763	-1.920	3.305
Median (9-15yr)	5684	0.972	1.018	6.315	-5.561	7.590	0.678	0.784	5.837	-5.236	6.431
Long (>15yr)	10949	1.116	1.052	7.048	-6.785	8.657	0.758	0.746	6.300	-6.093	7.360

This table reports summary statistics for both corporate bond returns and excess returns relative to the one-month T-bills by sample period (Panel A), leverage (Panel B), rating category (Panel C), and time-to-maturity (Panel D), respectively. Five leverage groups are formed from low (L) to high (H) according to the average leverage for firms in the entire sample period. Rating groups are formed based on S&P ratings of bonds. Three maturity groups are formed according to time-to-maturity: between 4 and 8 years (Short), between 9 and 15 years (Median), and greater than 15 years (Long). The sample period is from July 2002 to December 2012 and divided into two subperiods: 2002:Q3-2007:Q2, 2007:Q3-2009:Q4 and 2010:Q1-2012:Q4. N is the number of bonds in each category; the "5%" and "95%" show the 5th and 95th percentile.

Table 3: Simulation Analysis of Hedge Ratios Implied from the Two-Factor Merton-Vasicek Model

	AAA	AA	A	BBB	BB	B		AAA	AA	A	BBB	BB	B
	Panel A1: $rx^T_{i,t} = \alpha_r + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$							Panel B1: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$					
$\beta^r_{i,E}$	0.35	0.82	0.84	0.89	0.93	0.97	$\beta^{CS}_{i,E}$	1.38	1.18	1.14	1.10	1.06	1.02
	(−0.10)	(−0.30)	(−0.99)	(−1.06)	(−0.95)	(−0.83)		(1.36)	(2.40)	(4.98)	(4.77)	(3.13)	(1.91)
	Panel A2: $rx^T_{i,t} = \alpha_r + \tilde{\beta}^r_{i,E} \tilde{h}^r_{i,E} rx^E_{i,t}$							Panel B2: $\Delta CS^T_{i,t} = \alpha_{CS} + \tilde{\beta}^{CS}_{i,E} \tilde{h}^{CS}_{i,E} r^E_{i,t}$					
$\tilde{\beta}^r_{i,E}$	−0.05	0.78	0.82	0.88	0.92	0.97	$\tilde{\beta}^{CS}_{i,E}$	1.42	1.14	1.10	1.07	1.04	1.01
	(−0.12)	(−0.33)	(−1.04)	(−1.18)	(−1.10)	(−0.92)		(1.08)	(1.59)	(3.25)	(2.92)	(1.90)	(1.06)
	Panel A3: $rx^T_{i,t} = \alpha_r + \beta^r_{i,10} h^r_{i,10} rx^{10}_t$							Panel B3: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t$					
$\beta^r_{i,10}$	0.65	0.65	0.64	0.64	0.63	0.61	$\beta^{CS}_{i,10}$	1.28	1.16	1.10	0.97	0.94	0.89
	(−7.50)	(−13.78)	(−29.31)	(−29.09)	(−20.54)	(−16.25)		(0.75)	(0.40)	(0.12)	(−0.81)	(−1.37)	(−2.08)
	Panel A4: $rx^T_{i,t} = \alpha_r + \beta^r_{i,T} h^r_{i,T} rx^T_t$							Panel B4: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,T} h^{CS}_{i,T} r^T_t$					
$\beta^r_{i,T}$	0.88	0.84	0.84	0.84	0.85	0.86	$\beta^{CS}_{i,T}$	1.51	1.35	1.30	1.26	1.20	1.12
	(−4.33)	(−6.21)	(−10.44)	(−10.81)	(−8.53)	(−5.85)		(3.33)	(5.20)	(10.45)	(9.25)	(5.46)	(2.64)
	Panel A5: $rx^T_{i,t} = \alpha_r + \beta^r_{i,10} h^r_{i,10} rx^{10}_t + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$							Panel B5: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$					
$\beta^r_{i,E}$	0.80	0.99	0.97	0.99	0.99	0.99	$\beta^{CS}_{i,E}$	1.37	1.17	1.12	1.08	1.05	1.01
	(−0.06)	(−0.04)	(−0.29)	(−0.24)	(−0.29)	(−0.43)		(1.37)	(2.36)	(4.72)	(4.43)	(2.75)	(1.44)
$\beta^r_{i,10}$	0.60	0.57	0.57	0.56	0.56	0.54	$\beta^{CS}_{i,10}$	1.13	1.11	1.10	1.09	1.07	1.05
	(−4.97)	(−12.38)	(−26.43)	(−27.78)	(−18.53)	(−15.46)		(2.04)	(3.17)	(5.31)	(5.02)	(2.66)	(1.27)
	Panel A6: $rx^T_{i,t} = \alpha_r + \beta^r_{i,T} h^r_{i,T} rx^T_t + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$							Panel B6: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,T} h^{CS}_{i,T} r^T_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$					
$\beta^r_{i,E}$	1.25	1.11	1.08	1.06	1.03	1.01	$\beta^{CS}_{i,E}$	1.36	1.16	1.11	1.08	1.04	1.01
	(0.94)	(1.60)	(3.11)	(2.88)	(1.73)	(0.72)		(1.38)	(2.35)	(4.62)	(4.28)	(2.58)	(1.23)
$\beta^r_{i,T}$	0.71	0.69	0.68	0.68	0.67	0.65	$\beta^{CS}_{i,T}$	1.24	1.18	1.16	1.15	1.13	1.11
	(−5.23)	(−7.75)	(−11.11)	(−12.24)	(−11.13)	(−11.20)		(2.38)	(3.96)	(7.90)	(7.81)	(5.76)	(5.06)

Continued on next page

Table 3 – Continued

Effectiveness of											
Hedging Corporate Bond Returns						Hedging Corporate Bond Spreads					
AAA	AA	A	BBB	BB	B	AAA	AA	A	BBB	BB	B
Panel C1: Hedging with Equity											
-0.04	0.01	0.07	0.10	0.08	0.07	0.23	0.52	0.57	0.61	0.64	0.65
(0.09)	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)	(0.36)	(0.11)	(0.04)	(0.03)	(0.03)	(0.02)
Panel C2: Hedging with Equity (Merton)											
-0.01	0.01	0.08	0.10	0.08	0.04	0.25	0.58	0.65	0.69	0.72	0.74
(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	(0.04)	(0.33)	(0.11)	(0.04)	(0.03)	(0.03)	(0.02)
Panel C3: Hedging with 10-Year Treasury Bonds											
0.02	0.04	0.10	0.12	0.11	0.04	-0.36	-0.03	0.00	0.02	0.04	0.03
(0.22)	(0.12)	(0.05)	(0.05)	(0.07)	(0.09)	(4.85)	(0.85)	(0.35)	(0.23)	(0.18)	(0.11)
Panel C4: Hedging with T-Year Treasury Bonds											
0.82	0.77	0.77	0.74	0.65	0.48	-0.16	0.04	0.00	0.03	0.07	0.08
(0.03)	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)	(1.12)	(0.09)	(0.04)	(0.04)	(0.04)	(0.03)
Panel C5: Hedging with Equity & 10-Year Treasury Bonds											
-0.02	-0.02	-0.01	-0.02	-0.02	-0.05	-0.21	-0.05	0.03	0.11	0.34	0.48
(0.13)	(0.06)	(0.03)	(0.03)	(0.04)	(0.07)	(3.44)	(0.86)	(0.51)	(0.40)	(0.39)	(0.25)
Panel C6: Hedging with Equity & T-Year Treasury Bonds											
0.66	0.65	0.72	0.68	0.61	0.36	0.09	0.53	0.52	0.58	0.69	0.74
(0.19)	(0.05)	(0.01)	(0.01)	(0.02)	(0.03)	(1.11)	(0.16)	(0.06)	(0.04)	(0.03)	(0.02)

Panels A1-A6 and B1-B6 report results, by rating groups, from regressions of excess corporate bond returns and spread changes against either equity, or Treasury (10- or $(T-t)$-year), or both, using simulated 15 years of monthly data from the four-factor Merton-Vasicek model. 1,000 samples are generated for each rating class, and each bond has an initial maturity of 20 years. Parameters governing the interest-rate dynamics are listed in Table 10; the rating-dependent initial leverage (D_0/V_0) and asset volatility (σ_v) are both from Schaefer and Strebulaev (2008). The hedger is assumed to mis-specify the data-generating process: she estimates model parameters and computes hedge ratios as if observed data is generated from a two-factor (the Shimko et al. (1993)) model. The reported coefficient values are averages of the resulting 1,000 regression estimates for the corresponding slope coefficient. Associated t-statistics in parentheses are calculated based on the standard error estimator outlined in Collin-Dufresne, Goldstein, and Martin (2001). The t-statistics for coefficients related to the Merton (1974) sensitivities, $(\tilde{h}^{CS}_{i,E}, \tilde{h}^{r}_{i,E})$, and the Merton-Vasicek sensitivities, $(h^r_{i,E}, h^r_{i,10}, h^r_{i,T}, h^{CS}_{i,E}, h^{CS}_{i,10}, h^{CS}_{i,T})$, are computed against unity. Panels C1-C6 reports simulation results on the effectiveness of hedging corporate bond returns (columns 2 through 6) or yield spreads (columns 7 through 13) with either equity, or Treasury, or both. Monthly rebalancing is assumed. Measure of hedging effectiveness used is 1-$RMSE_h/RMSE_u$, where $RMSE_h$ ($RMSE_u$) is the root mean square error of the hedged (unhedged) position.

Table 4: Simulation Analysis of Hedge Ratios Implied from the Four-Factor Merton-Vasicek Model

	AAA	AA	A	BBB	BB	B
	Panel A1: $rx_{i,t}^T = \alpha_r + \beta_{i,E}^r h_{i,E}^r rx_{i,t}^E$					
$\beta_{i,E}^r$	0.67	0.83	0.84	0.88	0.92	0.97
	(−0.13)	(−0.46)	(−1.53)	(−1.65)	(−1.38)	(−0.96)
	Panel A2: $rx_{i,t}^T = \alpha_r + \beta_{i,0.5}^r h_{i,0.5}^r rx_t^{0.5} + \beta_{i,2}^r h_{i,2}^r rx_t^2 + \beta_{i,10}^r h_{i,10}^r rx_t^{10}$					
$\beta_{i,0.5}^r$	0.91	0.92	0.92	0.92	0.92	0.92
	(−0.47)	(−0.64)	(−1.33)	(−1.28)	(−0.84)	(−0.53)
$\beta_{i,2}^r$	0.92	0.93	0.93	0.93	0.93	0.93
	(−1.27)	(−2.17)	(−4.46)	(−4.22)	(−2.64)	(−1.48)
$\beta_{i,10}^r$	0.98	0.98	0.98	0.98	0.98	0.98
	(−1.64)	(−2.76)	(−5.88)	(−5.55)	(−3.41)	(−1.97)
	Panel A3: $rx_{i,t}^T = \alpha_r + \beta_{i,E}^r h_{i,E}^r rx_{i,t}^E + \beta_{i,0.5}^r h_{i,0.5}^r rx_t^{0.5} + \beta_{i,2}^r h_{i,2}^r rx_t^2 + \beta_{i,10}^r h_{i,10}^r rx_t^{10}$					
$\beta_{i,E}^r$	0.85	0.96	0.96	0.97	0.98	0.99
	(−0.12)	(−0.20)	(−0.82)	(−0.92)	(−0.76)	(−0.46)
$\beta_{i,0.5}^r$	0.92	0.92	0.92	0.92	0.92	0.92
	(−0.48)	(−0.65)	(−1.35)	(−1.34)	(−0.93)	(−0.79)
$\beta_{i,2}^r$	0.92	0.93	0.93	0.93	0.93	0.93
	(−1.27)	(−2.21)	(−4.61)	(−4.43)	(−3.11)	(−2.39)
$\beta_{i,10}^r$	0.98	0.98	0.98	0.98	0.98	0.98
	(−1.63)	(−2.82)	(−6.16)	(−5.97)	(−4.12)	(−3.14)

	AAA	AA	A	BBB	BB	B
	Panel B1: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,E}^{CS} h_{i,E}^{CS} r_{i,t}^E$					
$\beta_{i,E}^{CS}$	1.02	1.02	1.02	1.02	1.02	1.01
	(0.30)	(0.84)	(2.82)	(3.01)	(2.45)	(1.75)
	Panel B2: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,0.5}^{CS} h_{i,0.5}^{CS} r_t^{0.5} + \beta_{i,2}^{CS} h_{i,2}^{CS} r_t^2 + \beta_{i,10}^{CS} h_{i,10}^{CS} r_t^{10}$					
$\beta_{i,0.5}^{CS}$	0.81	0.83	0.86	0.87	0.89	0.90
	(−0.32)	(−0.50)	(−0.87)	(−0.82)	(−0.47)	(−0.27)
$\beta_{i,2}^{CS}$	0.94	0.94	0.95	0.95	0.95	0.96
	(−0.19)	(−0.36)	(−0.75)	(−0.72)	(−0.44)	(−0.27)
$\beta_{i,10}^{CS}$	0.98	0.99	0.99	0.99	0.99	0.99
	(−0.18)	(−0.32)	(−0.70)	(−0.68)	(−0.46)	(−0.33)
	Panel B3: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,E}^{CS} h_{i,E}^{CS} r_{i,t}^E + \beta_{i,0.5}^{CS} h_{i,0.5}^{CS} r_t^{0.5} + \beta_{i,2}^{CS} h_{i,2}^{CS} r_t^2 + \beta_{i,10}^{CS} h_{i,10}^{CS} r_t^{10}$					
$\beta_{i,E}^{CS}$	1.01	1.01	1.01	1.01	1.00	1.00
	(0.24)	(0.40)	(1.33)	(1.37)	(1.01)	(0.54)
$\beta_{i,0.5}^{CS}$	0.87	0.90	0.90	0.91	0.91	0.91
	(−0.61)	(−0.94)	(−1.97)	(−2.02)	(−1.34)	(−0.83)
$\beta_{i,2}^{CS}$	0.94	0.95	0.95	0.95	0.96	0.96
	(−0.47)	(−0.88)	(−1.95)	(−1.95)	(−1.33)	(−1.00)
$\beta_{i,10}^{CS}$	0.99	0.99	0.99	0.99	0.99	0.99
	(−0.38)	(−0.77)	(−1.71)	(−1.69)	(−1.21)	(−0.97)

Continued on next page

Table 4 – Continued

Effectiveness of											
Hedging Corporate Bond Returns						Hedging Corporate Bond Spreads					
AAA	AA	A	BBB	BB	B	AAA	AA	A	BBB	BB	B
Panel C1: Hedging with Equity											
-0.01	0.03	0.14	0.17	0.11	0.06	0.52	0.68	0.69	0.71	0.74	0.74
(0.03)	(0.04)	(0.02)	(0.02)	(0.04)	(0.08)	(0.13)	(0.09)	(0.03)	(0.03)	(0.02)	(0.02)
Panel C2: Hedging with Three Treasury Securities											
0.88	0.82	0.76	0.77	0.80	0.65	0.11	0.06	-0.01	0.03	0.07	0.14
(0.15)	(0.12)	(0.10)	(0.10)	(0.09)	(0.15)	(0.14)	(0.09)	(0.07)	(0.04)	(0.06)	(0.09)
Panel C3: Hedging with Equity & Three Treasury Securities											
0.79	0.62	0.52	0.49	0.56	0.44	0.20	0.62	0.55	0.61	0.67	0.72
(0.11)	(0.08)	(0.07)	(0.06)	(0.07)	(0.10)	(0.26)	(0.18)	(0.13)	(0.10)	(0.08)	(0.05)

Panels A1-A3 and B1-B3 report results, by rating groups, from regressions of excess corporate bond returns and spread changes against either equity, or Treasury (10- or $(T-t)$-year), or both, using simulated 15 years of monthly data from the four-factor Merton-Vasicek model. 1,000 samples are generated for each rating class, and each bond has an initial maturity of 20 years. Parameters governing the interest-rate dynamics are listed in Table 10; the rating-dependent initial leverage (D_0/V_0) and asset volatility (σ_v) are both from Schaefer and Strebulaev (2008). The hedger is assumed to correctly specify the data-generating process but she needs to estimate model parameters using observed equity prices and Treasury yields, with the latter contaminated with measure error. With estimated parameters, hedge ratios are computed based on Eq. (6)–(8) and Eq. (10). The reported coefficient values are averages of the resulting 1,000 regression estimates for the corresponding slope coefficient. Associated t-statistics in parentheses are calculated based on the standard error estimator outlined in Collin-Dufresne, Goldstein, and Martin (2001). The t-statistics for coefficients related to model-implied sensitivities, $(h^r_{i,E}, h^r_{i,0.5}, h^r_{i,2}, h^r_{i,10}, h^{CS}_{i,E}, h^{CS}_{i,0.5}, h^{CS}_{i,2}, h^{CS}_{i,10})$, are computed against unity. Panels C1-C3 reports simulation results on the effectiveness of hedging corporate bond returns (columns 2 through 6) or yield spreads (columns 7 through 13) with either equity, or Treasury, or both. Monthly rebalancing is assumed. Measure of hedging effectiveness used is 1-$RMSE_h/RMSE_u$, where $RMSE_h$ ($RMSE_u$) is the root mean square error of the hedged (unhedged) position.

Table 5: Regressions of Corporate Bond Returns on Merton-Vasicek Hedge Ratios

This table reports results from time-series regressions of monthly corporate bond excess returns against either Treasury, or equity, or both that incorporate hedge ratios implied by the Merton-Vasicek model (Shimko, Tejima, and Van Deventer 1993). $rx_{i,t}^{T}$ and $rx_{i,t}^{E}$ are respectively the month-t excess returns on bond-i with T years to maturity and on firm-i's equity. rx_{t}^{10} and rx_{t}^{T} denote the month-t excess returns of the 10- and T-year Treasuries, respectively. $h_{i,E}^{r}, h_{i,10}^{r}$, and $h_{i,T}^{r}$ are the sensitivities of corporate bond returns to equity return, the 10-year Treasury return, and the T-year Treasury return, respectively, under the Merton-Vasicek model. The reported coefficient values are averaged estimates across bonds. Associated t-statistics in parentheses are computed against unity and calculated based on the standard error estimator outlined in Schaefer and Strebulaev (2008). N is the number of bonds in each rating category. The sample period is from July 2002 to December 2012.

	All	AAA	AA	A	BBB	BB	B
Panel A: $rx_{i,t}^{T} = \alpha_r + \beta_{i,E}^{r} h_{i,E}^{r} rx_{i,t}^{E}$							
Intercept	0.008	0.007	0.006	0.006	0.009	0.010	0.009
	(18.00)	(8.59)	(10.63)	(11.45)	(13.52)	(7.45)	(5.62)
$\beta_{i,E}^{r}$	0.987	0.246	1.128	0.920	0.980	0.964	1.257
	(-0.06)	(-0.30)	(0.16)	(-0.23)	(-0.07)	(-0.12)	(0.38)
$\bar{R}^2$	0.088	0.015	0.023	0.029	0.081	0.131	0.236
N	533	17	43	176	160	78	41
Panel B: $rx_{i,t}^{T} = \alpha_r + \beta_{i,10}^{r} h_{i,10}^{r} rx_{t}^{10}$							
Intercept	0.006	0.002	0.003	0.003	0.007	0.010	0.010
	(11.90)	(5.37)	(4.87)	(5.91)	(9.65)	(5.33)	(5.44)
$\beta_{i,10}^{r}$	0.379	0.656	0.667	0.561	0.509	-0.065	-0.071
	(-13.74)	(-7.95)	(-5.53)	(-7.92)	(-8.12)	(-7.15)	(-12.23)
$\bar{R}^2$	0.252	0.551	0.520	0.386	0.187	0.029	0.013
Panel C: $rx_{i,t}^{T} = \alpha_r + \beta_{i,T}^{r} h_{i,T}^{r} rx_{t}^{T}$							
Intercept	0.006	0.002	0.003	0.003	0.007	0.008	0.010
	(12.30)	(4.66)	(4.94)	(5.85)	(9.26)	(8.06)	(5.29)
$\beta_{i,T}^{r}$	0.496	0.715	0.744	0.656	0.557	0.137	-0.024
	(-16.29)	(-6.53)	(-3.93)	(-8.87)	(-8.14)	(-11.80)	(-13.06)
$\bar{R}^2$	0.263	0.604	0.539	0.382	0.213	0.010	0.029
Panel D: $rx_{i,t}^{T} = \alpha_r + \beta_{i,10}^{r} h_{i,10}^{r} rx_{t}^{10} + \beta_{i,E}^{r} h_{i,E}^{r} rx_{i,t}^{E}$							
Intercept	0.006	0.002	0.003	0.003	0.007	0.010	0.008
	(11.36)	(5.09)	(5.71)	(5.27)	(10.08)	(4.97)	(4.21)
$\beta_{i,E}^{r}$	1.010	0.758	0.989	1.014	0.942	0.966	1.256
	(0.05)	(-0.27)	(-0.01)	(0.04)	(-0.14)	(-0.11)	(0.41)
$\beta_{i,10}^{r}$	0.431	0.708	0.668	0.623	0.518	0.028	0.222
	(-12.82)	(-9.16)	(-5.70)	(-8.47)	(-9.17)	(-6.82)	(-8.28)
$\bar{R}^2$	0.345	0.542	0.533	0.441	0.281	0.164	0.294

Table 6: Regressions of Corporate Yield Spread Changes on Merton-Vasicek Hedge Ratios

This table reports results from time-series regressions of monthly changes in the corporate bond yield spread against either Treasury, or equity, or both that incorporate the Shimko, Tejima, and Van Deventer (1993; STV) sensitivities. $\Delta CS_{i,t}^{T}$ is the month-t change in the yield spread of bond-i with T years to maturity. $r_{i,t}^{E}$, r_t^{10}, and r_t^{T} denote the month-t returns on firm-i's equity, the 10-year Treasury, and the T-year Treasury, respectively. $h_{i,E}^{CS}, h_{i,10}^{CS}$, and $h_{i,T}^{CS}$ are the sensitivities of corporate bond yield spread changes to equity return, the 10-year Treasury return, and the T-year Treasury return, respectively, under the STV model (a combination of the Merton and Vasicek models). The reported coefficient values are averaged estimates across bonds. Associated t-statistics in parentheses are computed against unity and calculated based on the standard error estimator outlined in Schaefer and Strebulaev (2008). N is the number of bonds in each rating category. The sample period is from July 2002 to December 2012.

	All	AAA	AA	A	BBB	BB	B
Panel A: $\Delta CS_{i,t}^{T} = \alpha_{CS} + \beta_{i,E}^{CS} h_{i,E}^{CS} r_{i,t}^{E}$							
Intercept	-0.016	-0.001	-0.007	-0.015	-0.022	0.002	0.029
	(-2.31)	(-0.47)	(-2.49)	(-3.05)	(-3.53)	(0.17)	(0.62)
$\beta_{i,E}^{CS}$	1.097	0.990	0.836	1.143	1.060	1.108	1.143
	(0.38)	(-0.00)	(-0.19)	(0.33)	(0.24)	(0.13)	(0.14)
$\bar{R}^2$	0.094	0.056	0.037	0.064	0.116	0.126	0.190
N	533	17	43	176	160	78	41
Panel B: $\Delta CS_{i,t}^{T} = \alpha_{CS} + \beta_{i,10}^{CS} h_{i,10}^{CS} r_t^{10}$							
Intercept	-0.061	-0.011	-0.015	-0.028	-0.051	-0.074	-0.054
	(-5.23)	(-3.14)	(-2.89)	(-5.26)	(-7.18)	(-2.94)	(-1.48)
$\beta_{i,10}^{CS}$	3.639	0.981	3.056	3.054	3.603	5.144	3.996
	(7.62)	(-0.01)	(1.91)	(3.46)	(4.46)	(3.95)	(2.82)
$\bar{R}^2$	0.116	0.135	0.096	0.108	0.139	0.157	0.121
Panel C: $\Delta CS_{i,t}^{T} = \alpha_{CS} + \beta_{i,T}^{CS} h_{i,T}^{CS} r_t^{T}$							
Intercept	-0.038	-0.013	-0.018	-0.023	-0.048	-0.058	-0.026
	(-9.19)	(-3.86)	(-3.17)	(-5.10)	(-7.35)	(-4.23)	(-1.03)
$\beta_{i,T}^{CS}$	3.180	2.666	3.083	2.659	3.253	4.735	3.059
	(10.90)	(1.87)	(3.20)	(5.62)	(6.29)	(6.21)	(2.49)
$\bar{R}^2$	0.135	0.193	0.136	0.123	0.148	0.147	0.133
Panel D: $\Delta CS_{i,t}^{T} = \alpha_{CS} + \beta_{i,10}^{CS} h_{i,E10}^{CS} r_t^{10} + \beta_{i,E}^{CS} h_{i,E}^{CS} r_{i,t}^{E}$							
Intercept	-0.030	-0.010	-0.017	-0.023	-0.046	-0.035	0.014
	(-6.00)	(-3.34)	(-2.64)	(-5.02)	(-6.79)	(-2.64)	(0.29)
$\beta_{i,E}^{CS}$	1.053	1.404	0.961	0.999	1.028	1.150	1.093
	(0.31)	(0.34)	(-0.07)	(-0.00)	(0.10)	(0.30)	(0.09)
$\beta_{i,10}^{CS}$	3.722	2.475	2.664	3.108	3.175	6.880	5.267
	(6.84)	(2.11)	(1.53)	(3.56)	(2.80)	(5.72)	(2.70)
$\bar{R}^2$	0.182	0.180	0.105	0.132	0.202	0.215	0.274

Table 7: Regressions of Corporate Bond Returns on Four-Factor Merton-Vasicek Hedge Ratios

This table reports results from regressions of corporate bond excess returns, that incorporate hedge ratios, against either the excess equity return (Panel A), or excess returns on 0.5-, 2-, and 10-year Treasuries (Panel B), or both (Panel C). $rx^T_{i,t}$, $rx^E_{i,t}$, and rx^m_t denote respectively the month-t excess returns on the corporate bond, equity, and the m-year Treasury security; $h^r_{i,E}$ and $h^r_{i,m}$ are respectively the equity and interest rate sensitivities of the corporate bond return implied by the four-factor Merton-Vasicek model, where $m = 0.5, 2, 10$. The regressions are estimated for each bond, and the reported coefficient values are averaged estimates across bonds. Associated t-statistics in parentheses are computed against unity and calculated based on the standard error estimator outlined in Schaefer and Strebulaev (2008). N is the number of bonds in each rating category. The sample period is from July 2002 to December 2012.

	All	AAA	AA	A	BBB	BB	B
Panel A: $rx^T_{i,t} = \alpha_r + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$							
Intercept	0.008	0.008	0.006	0.006	0.009	0.010	0.009
	(18.04)	(7.59)	(10.60)	(11.44)	(13.53)	(7.73)	(5.60)
$\beta^r_{i,E}$	1.010	0.685	1.133	0.921	0.981	0.966	1.262
	(0.05)	(-0.18)	(0.16)	(-0.22)	(-0.07)	(-0.11)	(0.38)
$\bar{R}^2$	0.093	0.027	0.018	0.030	0.086	0.117	0.219
N	533	17	43	176	160	78	41
Panel B: $rx^T_{i,t} = \alpha_r + \beta^r_{i,0.5} h^r_{i,0.5} rx^{0.5}_t + \beta^r_{i,2} h^r_{i,2} rx^2_t + \beta^r_{i,10} h^r_{i,10} rx^{10}_t$							
Intercept	0.006	0.002	0.004	0.003	0.007	0.009	0.010
	(12.84)	(4.71)	(5.74)	(7.09)	(9.78)	(6.54)	(7.01)
$\beta^r_{i,0.5}$	0.788	0.828	0.843	0.829	0.795	0.834	0.518
	(-7.71)	(-4.73)	(-2.48)	(-3.92)	(-4.20)	(-1.99)	(-4.81)
$\beta^r_{i,2}$	0.815	0.850	0.916	0.836	0.815	0.845	0.554
	(-6.80)	(-3.68)	(-1.21)	(-3.94)	(-3.87)	(-1.89)	(-4.51)
$\beta^r_{i,10}$	0.836	0.856	0.903	0.855	0.835	0.888	0.607
	(-6.11)	(-3.73)	(-1.48)	(-3.60)	(-3.49)	(-1.39)	(-3.86)
$\bar{R}^2$	0.272	0.623	0.524	0.372	0.237	0.088	0.130
N	533	17	43	176	160	78	41
Panel C: $rx^T_{i,t} = \alpha_r + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t} + \beta^r_{i,0.5} h^r_{i,0.5} rx^{0.5}_t + \beta^r_{i,2} h^r_{i,2} rx^2_t + \beta^r_{i,10} h^r_{i,10} rx^{10}_t$							
Intercept	0.006	0.002	0.003	0.003	0.007	0.010	0.009
	(16.85)	(6.79)	(7.94)	(8.69)	(13.66)	(7.77)	(7.64)
$\beta^r_{i,0.5}$	0.794	0.826	0.834	0.824	0.784	0.857	0.559
	(-11.26)	(-7.15)	(-3.59)	(-6.16)	(-6.88)	(-2.40)	(-7.27)
$\beta^r_{i,2}$	0.814	0.853	0.870	0.825	0.805	0.874	0.606
	(-10.26)	(-5.12)	(-2.69)	(-6.17)	(-6.30)	(-2.17)	(-6.20)
$\beta^r_{i,10}$	0.840	0.536	0.935	0.945	0.836	0.661	0.775
	(-3.00)	(-1.79)	(-0.30)	(-0.49)	(-2.15)	(-3.00)	(-1.23)
$\beta^r_{i,E}$	0.997	1.037	0.834	0.916	0.914	1.040	1.203
	(-0.01)	(0.04)	(-0.17)	(-0.13)	(-0.17)	(0.03)	(0.35)
$\bar{R}^2$	0.359	0.619	0.533	0.438	0.297	0.181	0.296
N	533	17	43	176	160	78	41

Table 8: Regressions of Corporate Yield Spread Changes on Four-Factor Merton-Vasicek Hedge Ratios

This table reports results from regressions of yield spread changes, that incorporate hedge ratios, against either the equity return $r^E_{i,t}$ (Panel A), or returns on 0.5-, 2-, and 10-year Treasuries (Panel B), or both (Panel C). $\Delta CS^T_{i,t}$ is the month-t change in the yield spread for bond-i. rx^m_t denotes the month-t return on the m-year Treasury security; $h^{CS}_{i,E}$ and $h^{CS}_{i,m}$ are respectively the equity and interest rate sensitivities of the spread change implied by the four-factor Merton-Vasicek model, where $m = 0.5, 2, 10$. The regressions are estimated for each bond, and the reported coefficient values are averaged estimates across bonds. Associated t-statistics in parentheses are computed against unity and calculated based on the standard error estimator outlined in Schaefer and Strebulaev (2008). N is the number of bonds in each rating category. The sample period is from July 2002 to December 2012.

	All	AAA	AA	A	BBB	BB	B
Panel A: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$							
Intercept	-0.017	-0.002	-0.007	-0.015	-0.022	0.003	0.021
	(-2.43)	(-0.63)	(-2.40)	(-3.05)	(-3.54)	(0.24)	(0.42)
$\beta^{CS}_{i,E}$	1.046	1.177	0.863	1.042	1.046	1.109	1.042
	(0.25)	(0.10)	(-0.16)	(0.15)	(0.18)	(0.17)	(0.05)
$\bar{R}^2$	0.093	0.058	0.038	0.064	0.106	0.117	0.196
N	533	17	43	176	160	78	41
Panel B: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,0.5} h^{CS}_{i,0.5} r^{0.5}_t + \beta^{CS}_{i,2} h^{CS}_{i,2} r^2_t + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t$							
Intercept	-0.027	-0.012	-0.012	-0.020	-0.025	-0.001	-0.030
	(-3.97)	(-3.75)	(-3.37)	(-3.90)	(-3.90)	(-0.11)	(-1.02)
$\beta^{CS}_{i,0.5}$	1.346	2.246	1.396	1.343	1.410	1.138	0.920
	(2.70)	(1.48)	(0.87)	(1.56)	(2.03)	(0.34)	(-0.16)
$\beta^{CS}_{i,2}$	1.308	1.905	1.415	1.348	1.356	1.135	0.918
	(2.48)	(1.06)	(1.00)	(1.55)	(1.85)	(0.34)	(-0.18)
$\beta^{CS}_{i,10}$	1.241	1.847	1.395	1.258	1.304	0.953	0.920
	(1.99)	(1.01)	(0.99)	(1.29)	(1.62)	(-0.11)	(-0.18)
$\bar{R}^2$	0.142	0.199	0.143	0.127	0.145	0.151	0.145
N	533	17	43	176	160	78	41
Panel C: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,0.5} h^{CS}_{i,0.5} r^{0.5}_t + \beta^{CS}_{i,2} h^{CS}_{i,2} r^2_t + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$							
Intercept	-0.037	-0.012	-0.013	-0.020	-0.050	-0.039	0.011
	(-4.78)	(-2.51)	(-3.45)	(-3.66)	(-6.23)	(-2.76)	(0.21)
$\beta^{CS}_{i,0.5}$	0.969	1.266	0.920	0.886	1.016	0.955	0.955
	(-0.62)	(0.64)	(-0.34)	(-1.27)	(0.18)	(-0.43)	(-0.23)
$\beta^{CS}_{i,2}$	0.958	1.288	0.964	0.894	0.962	0.972	0.938
	(-0.89)	(0.89)	(-0.16)	(-1.21)	(-0.48)	(-0.29)	(-0.43)
$\beta^{CS}_{i,10}$	1.177	1.156	1.420	1.114	1.127	1.329	1.257
	(1.89)	(0.91)	(0.55)	(1.64)	(1.07)	(2.01)	(1.26)
$\beta^{CS}_{i,E}$	1.071	0.838	1.036	1.097	0.884	1.165	1.202
	(0.06)	(0.82)	(0.07)	(0.14)	(-0.04)	(0.05)	(0.07)
$\bar{R}^2$	0.191	0.204	0.149	0.131	0.198	0.216	0.271
N	533	17	43	176	160	78	41

Table 9: Effectiveness of Hedging Corporate Bond Returns and Yield Spreads

This table reports empirical results on the effectiveness of hedging corporate bond returns (Panels A and C) or yield spreads (Panels B and D) with either equity, or Treasuries, or both. Treasuries used for hedging include the 6-month T-bill, 2- and 10-year T-notes, and T-year Treasuries with T being the years to maturity of the hedged corporate bond. Hedge ratios used include those based on the Merton (1974), two-factor Merton-Vasicek (Shimko, Tejima, and Van Deventer 1993) (denoted MV), and four-factor Merton-Vasicek (4FMV) models. Hedged portfolios are rebalanced monthly. Measure of hedging effectiveness is 1-$RMSE_h/RMSE_u$, where $RMSE_h$ ($RMSE_u$) is the root mean square error of the hedged (unhedged) position. The sample period is from July 2002 to December 2012.

Hedging Instruments	Hedging Effectiveness by Rating Groups						
	All	AAA	AA	A	BBB	BB	B
Panel A: Hedging Corporate Bond Returns (Merton-Vasicek)							
Equity-M	0.101	-0.013	0.004	0.024	0.043	0.108	0.091
Equity-MV	0.091	-0.002	0.014	0.022	0.056	0.102	0.120
10y Treasury	0.085	0.344	0.314	0.236	0.041	-0.108	-0.045
T-yr Treasury	0.165	0.519	0.460	0.314	0.067	-0.059	-0.025
Equity-MV, 10y Trea.	0.172	0.390	0.374	0.304	0.106	0.016	0.099
Equity-MV, T-yr Trea.	0.252	0.501	0.457	0.342	0.195	0.094	0.181
Panel B: Hedging Corporate Yield Spreads (Merton-Vasicek)							
Equity-M	0.192	-0.047	-0.091	0.125	0.154	0.194	0.112
Equity-MV	0.140	0.076	0.016	0.099	0.131	0.181	0.094
10y Treasury	0.022	-0.063	0.019	0.035	0.039	0.030	0.023
T-yr Treasury	0.033	-0.024	0.034	0.053	0.056	0.039	0.028
Equity-MV, 10y Trea.	0.119	-0.017	0.025	0.103	0.128	0.158	0.074
Equity-MV, T-yr Trea.	0.150	0.014	0.051	0.134	0.160	0.193	0.104
Panel C: Hedging Corporate Bond Returns (4F Merton-Vasicek)							
Equity-4FMV	0.119	0.003	0.002	0.027	0.046	0.111	0.125
6m, 2y, 10y Trea.	0.177	0.612	0.538	0.375	0.032	-0.062	-0.048
E-4FMV, 6m, 2y, 10y Trea.	0.278	0.587	0.469	0.385	0.154	0.116	0.176
Panel D: Hedging Corporate Yield Spreads (4F Merton-Vasicek)							
Equity-4FMV	0.203	0.021	0.040	0.115	0.179	0.220	0.094
6m, 2y, 10y Trea.	0.038	0.021	0.042	0.072	0.073	0.048	0.029
E-4FMV, 6m, 2y, 10y Trea.	0.165	0.027	0.042	0.139	0.189	0.222	0.086

Table 10: Maximum Likelihood Estimates of a Three-Factor GDTSM

$\text{diag}(K^{\mathbb{Q}}_{1,X})$	−0.101	−0.212	−0.730
	(0.024)	(0.038)	(0.118)
$r^{\mathbb{Q}}_{\infty}(\times 10^2)$		7.006	
		(0.687)	
$\Sigma_X(\times 10^2)$	4.730	0	0
	(0.355)		
	−5.528	2.097	0
	(0.289)	(0.194)	
	0.818	−2.232	0.900
	(0.064)	(0.257)	(0.043)
$K^{\mathbb{P}}_{1,X}$	−0.146	0.274	−0.157
	(0.052)	(0.143)	(0.073)
	0.047	−0.410	0.409
	(0.026)	(0.182)	(0.241)
	−0.166	−0.140	−0.997
	(0.085)	(0.092)	(0.239)
$X^{\mathbb{P}}_{\infty}(\times 10^2)$	0.031	−0.023	0.005
	(0.006)	(0.007)	(0.002)
$\sigma_{\eta}(\times 10^2)$		0.078	
		(0.003)	

The three-factor Gaussian dynamic term structure model (GDTSM), specified in Eqs. (2) and (3) under $\mathbb{Q}$, is estimated with maximum likelihood using month-end Treasury yields with maturities of six months, and one, two, three, five, seven and ten years, over the period 1990–2012. The model is normalized to the canonical form proposed by Joslin, Singleton, and Zhu (2011). Reported parameter values are annualized, in the sense that persistence parameters, $I + K_{1,X}$, are raised to the power of 12 and volatility parameters, Σ_X, are multiplied by $\sqrt{12}$. Quantities in parentheses are standard errors from 1,000 Monte Carlo simulations, under the null hypothesis that the estimated model is true; each sample of simulated data consists of 276 monthly observations of seven bond yields with the same maturities as those used in model estimation.

Table 11: Regression Results Based on Simulated Data Using the Two-Factor Merton-Vasicek Model

σ_η		AAA	AA	A	BBB	BB	B		AAA	AA	A	BBB	BB	B
		Panel A1: $rx_{i,t}^T = \alpha_r + \beta_{i,E}^r h_{i,E}^r rx_{i,t}^E$							Panel B1: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,E}^{CS} h_{i,E}^{CS} r_{i,t}^E$					
0.010		−60.43	−30.40	−27.21	−19.57	−12.59	−4.29		1.45	1.80	2.14	2.09	2.09	1.73
		(−1.66)	(−6.68)	(−21.56)	(−23.06)	(−18.52)	(−14.34)		(1.54)	(6.52)	(21.86)	(23.70)	(19.51)	(14.58)
0.005	$\beta_{i,E}^r$	1.52	−4.60	−6.62	−4.74	−3.18	−0.48	$\beta_{i,E}^{CS}$	1.03	1.16	1.32	1.32	1.34	1.21
		(0.03)	(−1.95)	(−9.76)	(−10.87)	(−10.14)	(−7.48)		(0.11)	(2.18)	(9.87)	(10.94)	(10.04)	(7.48)
0.001		24.28	4.82	1.60	1.22	0.77	1.01		0.86	0.92	0.99	1.00	1.03	1.00
		(1.76)	(2.17)	(1.23)	(0.65)	(−0.88)	(0.04)		(−1.65)	(−1.96)	(−0.61)	(−0.04)	(1.31)	(0.25)
		Panel A2: $rx_{i,t}^T = \alpha_r + \tilde{\beta}_{i,E}^r \tilde{h}_{i,E}^r rx_{i,t}^E$							Panel B2: $\Delta CS_{i,t}^T = \alpha_{CS} + \tilde{\beta}_{i,E}^{CS} \tilde{h}_{i,E}^{CS} r_{i,t}^E$					
0.010		−425.46	−91.99	−72.20	−39.56	−19.49	−4.78		10.30	5.26	5.31	4.00	3.08	1.88
		(−0.91)	(−4.01)	(−12.50)	(−14.14)	(−12.42)	(−12.16)		(2.56)	(6.49)	(16.29)	(17.54)	(13.97)	(12.08)
0.005	$\tilde{\beta}_{i,E}^r$	10.77	−12.78	−16.91	−9.29	−4.78	−0.51	$\tilde{\beta}_{i,E}^{CS}$	7.22	3.33	3.17	2.44	1.92	1.30
		(0.09)	(−1.08)	(−5.90)	(−6.99)	(−7.25)	(−6.32)		(2.82)	(5.95)	(13.92)	(14.32)	(10.86)	(7.59)
0.001		152.77	13.16	3.05	1.92	0.99	1.11		6.27	2.73	2.42	1.88	1.48	1.09
		(0.93)	(1.46)	(1.06)	(1.00)	(−0.04)	(0.61)		(3.31)	(6.06)	(12.81)	(12.45)	(8.24)	(3.28)
		Panel A3: $rx_{i,t}^T = \alpha_r + \beta_{i,10}^r h_{i,10}^r rx_t^{10}$							Panel B3: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,10}^{CS} h_{i,10}^{CS} r_t^{10}$					
0.010		0.27	0.27	0.27	0.27	0.27	0.28		0.21	0.22	0.22	0.22	0.22	0.22
		(−22.34)	(−37.72)	(−79.31)	(−77.46)	(−52.55)	(−41.12)		(−7.54)	(−15.32)	(−34.24)	(−35.74)	(−27.63)	(−26.06)
0.005	$\beta_{i,10}^r$	0.60	0.60	0.60	0.60	0.60	0.61	$\beta_{i,10}^{CS}$	0.50	0.51	0.51	0.51	0.51	0.49
		(−15.26)	(−26.50)	(−55.38)	(−53.97)	(−36.57)	(−27.85)		(−6.15)	(−12.15)	(−26.83)	(−27.82)	(−21.08)	(−19.41)
0.001		0.97	0.98	0.98	0.98	0.98	0.99		0.84	0.84	0.85	0.84	0.84	0.81
		(−3.61)	(−6.04)	(−11.78)	(−10.68)	(−5.99)	(−1.17)		(−3.47)	(−7.10)	(−14.97)	(−15.26)	(−11.21)	(−9.69)
		Panel A4: $rx_{i,t}^T = \alpha_r + \beta_{i,T}^r h_{i,T}^r rx_t^T$							Panel B4: $\Delta CS_{i,t}^T = \alpha_{CS} + \beta_{i,T}^{CS} h_{i,T}^{CS} r_t^T$					
0.010		1.00	1.00	1.00	1.00	1.01	1.02		0.82	0.87	0.89	0.90	0.93	0.95
		(1.91)	(2.79)	(5.33)	(4.92)	(2.89)	(2.27)		(−5.09)	(−7.45)	(−14.54)	(−13.12)	(−7.82)	(−4.84)
0.005	$\beta_{i,T}^r$	1.00	1.00	1.00	1.00	1.01	1.01	$\beta_{i,T}^{CS}$	0.91	0.92	0.93	0.93	0.94	0.93
		(1.79)	(3.05)	(6.47)	(6.65)	(4.78)	(4.53)		(−2.58)	(−4.24)	(−8.70)	(−8.48)	(−5.86)	(−5.00)
0.001		1.00	1.00	1.00	1.00	1.01	1.02		0.87	0.88	0.88	0.88	0.87	0.84
		(3.13)	(5.67)	(11.73)	(11.64)	(8.41)	(6.99)		(−3.26)	(−6.19)	(−13.17)	(−13.30)	(−9.73)	(−8.21)

σ_η		AAA	AA	A	BBB	BB	B
		Panel A5: $rx^T_{i,t} = \alpha_r + \beta^r_{i,10} h^r_{i,10} rx^{10}_t + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$					
0.010		0.32	0.32	0.31	0.31	0.31	0.30
		(−19.55)	(−33.76)	(−72.90)	(−72.01)	(−50.10)	(−39.63)
0.005	$\beta^r_{i,T}$	0.69	0.69	0.69	0.69	0.68	0.67
		(−10.80)	(−18.89)	(−40.30)	(−39.79)	(−27.82)	(−22.34)
0.001		1.13	1.12	1.12	1.12	1.11	1.11
		(11.00)	(18.42)	(37.73)	(35.95)	(23.13)	(16.11)
0.010		−64.59	−31.09	−27.36	−19.64	−12.57	−4.31
		(−1.78)	(−6.91)	(−21.87)	(−23.26)	(−18.58)	(−14.39)
0.005	$\beta^r_{i,E}$	−9.18	−6.44	−7.05	−4.96	−3.15	−0.52
		(−0.49)	(−2.88)	(−11.39)	(−12.38)	(−11.01)	(−8.36)
0.001		6.19	1.67	0.87	0.84	0.81	0.95
		(0.94)	(0.94)	(−0.68)	(−1.20)	(−1.87)	(−1.12)
		Panel A6: $rx^T_{i,t} = \alpha_r + \beta^r_{i,T} h^r_{i,T} rx^T_t + \beta^r_{i,E} h^r_{i,E} rx^E_{i,t}$					
0.010		1.00	1.00	1.00	1.00	1.00	1.00
		(1.33)	(1.12)	(0.49)	(−0.04)	(−0.90)	(−0.75)
0.005	$\beta^r_{i,T}$	1.00	1.00	1.00	1.00	1.00	1.00
		(0.99)	(1.13)	(1.68)	(1.39)	(0.45)	(0.05)
0.001		1.00	1.00	1.00	1.00	1.00	1.00
		(1.68)	(2.53)	(4.55)	(4.21)	(2.49)	(1.41)
0.010		0.66	0.74	0.74	0.79	0.85	0.94
		(−2.01)	(−4.45)	(−10.29)	(−10.12)	(−6.80)	(−4.45)
0.005	$\beta^r_{i,E}$	0.92	0.94	0.94	0.95	0.97	0.98
		(−1.14)	(−2.26)	(−4.83)	(−4.90)	(−3.41)	(−2.83)
0.001		0.96	0.97	0.98	0.98	0.98	0.99
		(−1.53)	(−2.75)	(−5.45)	(−5.50)	(−3.82)	(−3.32)

σ_η		AAA	AA	A	BBB	BB	B
		Panel B5: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,10} h^{CS}_{i,10} r^{10}_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$					
0.010		0.25	0.25	0.25	0.25	0.24	0.26
		(−7.77)	(−16.22)	(−38.58)	(−40.89)	(−32.31)	(−29.59)
0.005	$\beta^{CS}_{i,T}$	0.56	0.57	0.57	0.57	0.57	0.58
		(−5.91)	(−11.62)	(−26.39)	(−27.76)	(−21.31)	(−20.06)
0.001		0.93	0.94	0.95	0.95	0.96	0.96
		(−2.76)	(−4.83)	(−9.80)	(−9.77)	(−6.80)	(−5.62)
0.010		1.47	1.81	2.13	2.09	2.08	1.72
		(1.63)	(6.66)	(21.96)	(23.79)	(19.46)	(14.66)
0.005	$\beta^{CS}_{i,E}$	1.10	1.20	1.33	1.32	1.33	1.21
		(0.58)	(2.98)	(11.06)	(12.15)	(10.66)	(8.28)
0.001		0.98	0.99	1.00	1.00	1.01	1.00
		(−0.66)	(−0.61)	(0.26)	(0.55)	(0.93)	(0.67)
		Panel B6: $\Delta CS^T_{i,t} = \alpha_{CS} + \beta^{CS}_{i,T} h^{CS}_{i,T} r^T_t + \beta^{CS}_{i,E} h^{CS}_{i,E} r^E_{i,t}$					
0.010		0.83	0.89	0.91	0.92	0.95	0.98
		(−4.88)	(−6.74)	(−12.05)	(−10.36)	(−5.38)	(−2.58)
0.005	$\beta^{CS}_{i,T}$	0.93	0.95	0.96	0.97	0.98	0.99
		(−2.38)	(−3.32)	(−5.96)	(−5.21)	(−2.68)	(−1.28)
0.001		0.95	0.97	0.97	0.98	0.98	0.99
		(−2.63)	(−4.24)	(−8.09)	(−7.52)	(−4.57)	(−2.64)
0.010		0.77	0.84	0.86	0.89	0.92	0.96
		(−2.15)	(−3.94)	(−7.84)	(−7.77)	(−5.25)	(−3.58)
0.005	$\beta^{CS}_{i,E}$	0.92	0.95	0.95	0.96	0.97	0.99
		(−1.32)	(−2.43)	(−4.91)	(−5.02)	(−3.57)	(−2.85)
0.001		0.96	0.97	0.98	0.98	0.99	0.99
		(−1.58)	(−2.84)	(−5.48)	(−5.62)	(−3.89)	(−3.10)

This table reports results, by rating groups, from regressions of excess corporate bond returns (Panels A1-A6) and spread changes (Panels B1-B6) against either equity, or Treasury (10- or T-year), or both, using simulated 15 years of data from the two-factor Merton-Vasicek model (Shimko, Tejima, and Van Deventer 1993). The rating dependent initial leverage (D_0/V_0) and asset volatility (σ_v), and the correlation coefficient ρ of -0.15 are all from Schaefer and Strebulaev (2008). The parameters used for the interest rate process are estimated from the 1990-2012 sample. They include $\kappa^{\mathbb{Q}} = 0.053$, $\bar{r}^{\mathbb{Q}} = 0.130$, $\sigma_r = 0.012$, $\kappa^{\mathbb{P}} = 0.116$, and $\bar{r}^{\mathbb{P}} = 0.059$. 1,000 samples are generated for each rating class, and each bond has an initial maturity of 20 years. In each trial, parameters governing the interest-rate dynamics are assumed to be unknown; to determine hedge ratios, investors need to estimate the model using observed bond yields, which are contaminated with measure error. σ_η denotes the standard deviation of measure error. The reported coefficient values are averages of the resulting 1,000 regression estimates for the corresponding slope coefficient. Associated t-statistics in parentheses are calculated based on the standard error estimator outlined in Collin-Dufresne, Goldstein, and Martin (2001). The t-statistics for coefficients related to the Merton (1974) sensitivities, $(\tilde{h}^{CS}_{i,E}, \tilde{h}^r_{i,E})$, and the Merton-Vasicek sensitivities, $(h^r_{i,E}, h^r_{i,10}, h^r_{i,T}, h^{CS}_{i,E}, h^{CS}_{i,10}, h^{CS}_{i,T})$, are computed against unity.

Table 12: Simulation Analysis of Hedging Effectiveness: Based on the Two-Factor Merton-Vasicek Model

This table reports simulation results on the effectiveness of hedging corporate bond returns (columns 2 through 6) or yield spreads (columns 7 through 13) with either equity, or Treasury, or both. Parameter T refers to the maturity of the corporate bond to be hedged. The data-generating process used is the two-factor Merton-Vasicek model (Shimko, Tejima, and Van Deventer 1993). Model parameter values used are described in Table 11. Hedge ratios considered include both the Merton (1974) and Merton-Vasicek sensitivities. Monthly rebalancing is assumed. Measure of hedging effectiveness used is 1-$RMSE_h/RMSE_u$, where $RMSE_h$ ($RMSE_u$) is the root mean square error of the hedged (unhedged) position.

	Hedging Corporate Bond Returns						Hedging Corporate Bond Spreads					
σ_η	AAA	AA	A	BBB	BB	B	AAA	AA	A	BBB	BB	B
	Panel A: Hedging with Equity											
1.0	-0.00	-0.01	0.00	0.00	-0.02	-0.05	0.08	0.15	0.23	0.26	0.28	0.34
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)
0.5	-0.00	-0.00	0.02	0.03	0.01	-0.02	0.11	0.20	0.30	0.34	0.35	0.42
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.07)	(0.08)	(0.06)	(0.06)	(0.06)	(0.06)
0.1	0.00	0.01	0.05	0.06	0.04	0.03	0.18	0.29	0.41	0.44	0.45	0.53
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.09)	(0.08)	(0.06)	(0.06)	(0.06)	(0.05)
	Panel B: Hedging with Equity (Merton-Based)											
1.0	-0.00	-0.00	-0.00	-0.00	-0.02	-0.05	0.05	0.14	0.23	0.25	0.27	0.33
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.14)	(0.08)	(0.05)	(0.05)	(0.06)	(0.06)
0.5	-0.00	-0.00	0.01	0.02	0.00	-0.01	0.08	0.19	0.30	0.33	0.34	0.42
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.02)	(0.14)	(0.09)	(0.06)	(0.06)	(0.06)	(0.06)
0.1	0.00	0.01	0.02	0.04	0.04	0.04	0.15	0.28	0.41	0.44	0.45	0.53
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.03)	(0.15)	(0.10)	(0.06)	(0.06)	(0.06)	(0.06)
	Panel C: Hedging with 10-Year Treasury Bonds											
1.0	-0.22	-0.24	-0.29	-0.29	-0.25	-0.26	-0.14	-0.13	-0.15	-0.15	-0.14	-0.13
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.11)	(0.08)	(0.06)	(0.06)	(0.06)	(0.05)
0.5	-0.04	-0.03	0.01	0.01	0.00	-0.02	-0.01	-0.03	-0.09	-0.09	-0.05	-0.04
	(0.06)	(0.06)	(0.05)	(0.05)	(0.06)	(0.07)	(0.11)	(0.08)	(0.06)	(0.06)	(0.05)	(0.05)
0.1	0.49	0.44	0.36	0.37	0.42	0.42	0.26	0.17	-0.01	-0.01	0.08	0.07
	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.09)	(0.07)	(0.06)	(0.06)	(0.05)	(0.04)
	Panel D: Hedging with T-Year Treasury Bonds											
1.0	0.99	0.98	0.94	0.93	0.91	0.85	0.30	0.17	-0.35	-0.18	0.25	0.44
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.11)	(0.10)	(0.13)	(0.11)	(0.06)	(0.04)
0.5	1.00	0.98	0.97	0.96	0.94	0.86	0.53	0.52	0.51	0.49	0.46	0.33
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.08)	(0.07)	(0.06)	(0.07)	(0.06)	(0.05)
0.1	1.00	0.98	0.97	0.96	0.93	0.83	0.35	0.29	0.22	0.19	0.20	0.13
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.09)	(0.07)	(0.05)	(0.04)	(0.05)	(0.04)

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Table 12 – Continued

σ_η	Hedging Corporate Bond Returns						Hedging Corporate Bond Spreads					
	AAA	AA	A	BBB	BB	B	AAA	AA	A	BBB	BB	B
	Panel E: Hedging with Equity & 10-Year Treasury Bonds											
1.0	-0.23	-0.27	-0.38	-0.40	-0.35	-0.40	-0.04	0.05	0.14	0.17	0.20	0.27
	(0.07)	(0.07)	(0.07)	(0.08)	(0.07)	(0.09)	(0.10)	(0.10)	(0.07)	(0.07)	(0.07)	(0.07)
0.5	-0.05	-0.06	-0.09	-0.11	-0.10	-0.19	0.13	0.22	0.30	0.33	0.37	0.45
	(0.06)	(0.07)	(0.05)	(0.06)	(0.06)	(0.08)	(0.11)	(0.09)	(0.07)	(0.07)	(0.07)	(0.06)
0.1	0.47	0.37	0.23	0.20	0.25	0.16	0.66	0.65	0.56	0.59	0.69	0.79
	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.03)
	Panel F: Hedging with Equity & T-Year Treasury Bonds											
1.0	0.96	0.91	0.82	0.78	0.79	0.70	0.30	0.17	-0.40	-0.21	0.29	0.63
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.13)	(0.14)	(0.15)	(0.13)	(0.09)	(0.05)
0.5	0.95	0.89	0.83	0.80	0.79	0.64	0.68	0.68	0.54	0.61	0.77	0.87
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)	(0.01)
0.1	0.92	0.85	0.83	0.79	0.76	0.56	0.83	0.85	0.82	0.84	0.88	0.92
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)