Time-Varying Ambiguity and Asset Pricing Puzzles

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Abstract

This paper studies the effects of time-varying Knightian uncertainty (ambiguity) on asset pricing in a Lucas exchange economy. Specifically, it considers a general equilibrium model where an ambiguity-averse agent applies a discount rate that is adjusted not only for the current magnitude of ambiguity but also for the risk associated with its future fluctuations. As such, both the ambiguity level and volatility help raise asset premia and accommodate richer dynamics of asset prices. Employing a novel measure for the ambiguity level, we demonstrate that the estimated model can match the key moments of equity premium and risk-free rate. More importantly, our model explains the credit spread puzzle despite a low default probability. It also accounts for a wide range of dynamic asset pricing phenomena, including the cyclical variations of stock prices and credit spreads, the long-horizon predictability of stock returns and credit spreads, and the low correlation between asset prices and consumption growth. Furthermore, the proposed ambiguity measure is found to exhibit significant predictive power for excess returns on equities and bonds as well as for corporate yield spreads—a finding that justifies uncertainty channels highlighted in the model.

Keywords: Ambiguity, asset pricing, credit spreads, equity premium
JEL Classifications: E43, E44, G12, G13, G33

1 Introduction

In economics, Knightian uncertainty (ambiguity) refers to the situation where the decision maker is uncertain about the probability law governing the state process. Motivated by Ellsberg (1961)'s classic experiments that highlight the distinction between ambiguity aversion and risk aversion, various models of preference have been proposed and subsequently applied to financial economics. A salient feature in many such models is that the degree of Knightian uncertainty is assumed to be constant over time.[1] In this paper, we show that time variations in ambiguity have significant qualitative and quantitative effects on equilibrium asset pricing; they create an additional uncertainty channel that helps to explain the equity premium puzzle, the risk-free rate puzzle, the excess volatility puzzle, the credit spread puzzle, and a wide variety of dynamic asset pricing phenomena.

Building on the continuous-time framework of Chen and Epstein (2002), this paper considers an exchange economy where the representative agent is not endowed with complete knowledge of the aggregate consumption dynamics. Rather, she holds multiple priors (Gilboa and Schmeidler, 1989) that describe the data-generating process. To make decisions robust to model misspecifications, the agent optimally evaluates investments according to the prior that leads to the lowest expected utility. This "worst-case" evaluation brings about a *first-order effect of ambiguity*, wherein the magnitude determined by the difference between the "true" expected growth rate and the worst-case belief used to price assets.

If the set of priors is updated over time, however, the uncertainty about future ambiguity levels also permits a *second-order effect*, in the sense that the magnitude of this effect is bound to the second moment (volatility) of the ambiguity process. In the model proposed in this paper, the agent's preference for early resolution of uncertainty gives rise to positive responses of the marginal rate of substitution to ambiguity shocks. The extent to which these ambiguity shocks are priced depends on their persistence. In studying the model's quantitative predictions, we find both the first-order and second-order effects of Knightian uncertainty are essential in generating high equity premium, low risk-free rates, and excess equity volatility.

To address the credit spread puzzle,[2] we consider an individual firm that chooses the optimal default time to maximize the (levered) equity value (Leland, 1994a,b). As the uncertainty-adjusted

[1] This specification is termed "κ-Ignorance" in Chen and Epstein (2002). Models that correspond to this specification include Miao (2009) and Jeong, Kim, and Park (2015).

[2] The term credit spread puzzle describes a finding by Huang and Huang (2012) that various structural models, once calibrated to match historical default loss experience and leverage ratios, underpredict credit spreads, especially for investment grade bonds. The role of time-varying market price of risk in explaining this puzzle is emphasized in Chen, Collin-Dufresne, and Goldstein (2009).

discount rate increases proportionately with the degree of ambiguity, an ambiguity shock would lower the present value of a given path of future cash flows. It thus leans shareholders more towards exercising their options to default, even if there is no news of the firm's fundamentals. In light of the ambiguity's first-order and second-order effects on marginal utilities, its covariation with the default time implies that default events would become a greater concern for the investor exactly when they are more likely to be incurred. As a result, she demands high risk-neutral default probabilities in valuing defaultable claims. Furthermore, given that corporate defaults cluster when the economic outlook becomes unclear, the liquidation process can be particularly costly during such times, which leads to lower recovery rates. Together, positive correlations of the market price of uncertainty[3] and default timing—and with loss given default—generate higher credit spreads than those implied by models with zero or constant ambiguity.

To assess the adequacy of these two risk channels for capturing the credit spread puzzle, we perform a calibration exercise for each rating category. Based on the estimated dynamics of aggregate variables, we calibrate firm-specific parameters to match historical financial ratios and recovery rates. The calibrated model replicates the average levels of both default rates and the credit component of corporate yield spreads for all investment and speculative grades. The empirical plausibility of the proposed economic mechanism is further tested against data. Based on a sample of defaulted companies, we find the ambiguity level highly significant in explaining the estimated default boundary (with the expected sign). This result verifies the positive correlation between ambiguity and corporate default date as posited by our model, which is also supported by historical data.

The function of time-varying ambiguity is not only to reconcile momentous aspects of asset prices but also to capture their dynamic behaviors. Even with a homoscedastic ambiguity process, the model is able to generate time variations in asset Sharpe ratios. Empirically, the asymmetric conditional heteroscedasticity (the fact that an increase in volatility follows a previous rise in the ambiguity level) magnifies the pricing kernel's covariations with the magnitude of ambiguity. The resulting time-varying market price of uncertainty leads to important time-series predictions, including the procyclical variation of price-dividend ratios, the countercyclical variation of credit spreads, the long-horizon predictability of excess equity returns and credit spreads, and their weak correlations with consumption growth.

There are a few examples of general equilibrium models that succeed in explaining the equity risk

[3]It is a more appropriate term here than market price of risk. For notional convenience, in this paper we refer to "uncertainty" as both risk and ambiguity, unless "Knightian uncertainty" or "subjective uncertainty" is otherwise used in the context.

premium and credit spreads in a unified framework: Chen, Collin-Dufresne, and Goldstein (2009) build on the habit-formation model of Campbell and Cochrane (1999), while Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) use a theoretical framework in the spirit of Bansal and Yaron (2004) that incorporates regime switching.[4] This paper departs from these previous studies in three important aspects.

Firstly, while these studies aim to explain asset pricing puzzles with macroeconomic risk,[5] our explanation is based on time-varying Sharpe ratios that are driven by changes in the ambiguity level. This key difference underlies our model's capacity to account for variations in asset premia above and beyond what are captured by the business cycle. The economic intuition is that an ambiguity shock can lead to a large reaction in the marginal rate of substitution, even without news about the economic fundamentals.

Secondly, unlike these previous studies, which focus exclusively on investment-grade issuers, we examine the credit spreads of speculative-grade bonds, too. As Huang and Huang (2012) emphasize, when explaining the credit spread puzzle it is also important to generate realistic yield spreads on high-yield bonds. Theoretically, we could account for the puzzle by imposing an extremely high risk premium. But if the resulting model were to over-predict speculative-grade yield spreads, then it would merely create a credit spread puzzle in the other direction. With a reasonable firm-level calibration, our model can match the level of credit spreads across all rating classes.

Third, this paper extensively studies the the dynamic properties of equity returns and credit spreads, including their predictability (from the price-dividend ratio) and their (weak) correlations with macroeconomic fundamentals. These properties have not drawn sufficient attention in existing works on the credit spread puzzle. Our model replicates these stylized facts and, more importantly, it provides an accurate account of historical variations in stock prices and yield spreads.

With respect to implementation, an important advantage of our model is that the key driving variable, the level of ambiguity, is measurable. This facilitates our data-driven estimation of model parameters that does not involve market data. Specifically, we construct a novel measure of the economy-wide level of ambiguity by using survey forecasts. Consistent with our model's implication, higher ambiguity levels forecast higher market premia on a broad set of assets, including equities,

[4] Other studies attempting to reconcile the credit spread puzzle include David (2008), Elkamhi et al. (2011), McQuade (2012), Albagli et al. (2013), and Chen et al. (2013). These models either are not set in general equilibrium or do not consider equity premium.

[5] The key ingredient in their models is the market price of risk (Sharpe ratio) that varies with macroeconomic conditions. Huang and Huang (2012) advise caution when linking the credit spread puzzle to the recession risk, since "there is no clear evidence yet that corporate bond defaults, *especially on investment-grade bonds*, are strongly correlated with business cycles."

corporate bonds and Treasury bonds. This predictability is significant in both in-sample inference and out-of-sample analysis. Based on this ambiguity measure, the price-dividend ratio backed out from the model shows a 75% correlation with its historical counterpart. And the correlation between model and actual credit spreads is even higher than 82%. The model-implied time-series capture the cyclical fluctuations in their historical counterparts, and track them well *within* each business cycle, offering strong support for the uncertainty channels highlighted in this paper.

This paper also contributes to a growing body of literature that studies representative-agent asset pricing in the presence of Knightian uncertainty. Our modeling of ambiguity aversion builds on recursive multiple-priors utility introduced and axiomatized by Epstein and Wang (1994), Chen and Epstein (2002), and Epstein and Schneider (2003).[6] In the context of these analyses, our specification of time-varying ambiguity can be interpreted as a reduced form of models of learning under ambiguity, as developed in Epstein and Schneider (2007, 2008), Leippold, Trojani, and Vanini (2008) and Illeditsch (2011), and our results provide further guidance to future works on modeling learning. While these studies focus on model implications for the equity market, the multiple-priors preferences also are applied to other asset classes (Gagliardini et al., 2009; Ilut, 2012). However, the linkage of ambiguity aversion to the credit spread puzzle has not yet been examined in the literature.[7]

Employing a different approach to ambiguity modeling,[8] Drechsler (2013) also studies the implications of time-varying ambiguity for asset pricing.[9] Dreschler's approach and ours differ in three areas. First, Drechsler (2013)'s model also features a predictable (long-run) component and stochastic volatility in consumption growth, and he incorporates large jumps in these two driving state process. Our model, in contrast, shuts down other risk channels to highlight the role of time-varying ambiguity and to keep the model parsimonious. Second, Drechsler (2013) specifies an exogenous dividend process and treats equity as an unlevered claim to the dividend. Our model

[6] Other preference models that describe ambiguity-averse behavior include the "smooth ambiguity" preferences (Klibanoff et al., 2005, 2009) and the multiplier preferences, which is based on the robust control theory (Hansen and Sargent, 2001; Anderson et al., 2003; Strzalecki, 2011). Epstein and Schneider (2010) provides a comprehensive review.

[7] Boyarchenko (2012) uses the robust control model to explain the CDS spreads during the subprime mortgage crisis but not the credit spread puzzle.

[8] Drechsler adopts the framework of robust control and defines the ambiguity level in terms of relative entropy. Epstein and Schneider (2003) draws a head-to-head comparison between the robust control and recursive multiple-priors models. What most distinguishes these modeling approaches is the updating rules imposed for the set of priors. To ensure dynamic consistency, for example, Dreschler's construction of multiplier preferences implies a form of axiom that differs from the rectangularity employed by multiple-priors models.

[9] Ilut and Schneider (2014) incorporate time-varying ambiguity into a business cycle model where agents regard neutral technological changes as ambiguous. Ulrich (2013) considers time-varying inflation ambiguity and its implications for the term structure of Treasury yields.

considers a levered firm in order to explain the credit spread puzzle. Third, while Drechsler (2013) targets properties of option prices and the variance premium, our focus is stylized facts in the equity and corporate bond markets.

The remainder of our paper is organized as follows. Section 2 describes how we measure the economy-level ambiguity, and it illustrates the empirical relevance of the proposed ambiguity measure. In Section 3, we introduce the model and characterize the valuation of different assets in equilibrium. Section 4 outlines the model estimation and discusses quantitative implications on asset pricing puzzles. Section 5 presents our conclusions.

2 Empirical Evidence

2.1 Measuring the Level of Knightian Uncertainty

Before proceeding to our model, we first present empirical evidence of the effect of time-varying ambiguity on asset premia. We accomplish this by creating a proxy for economy-wide Knightian uncertainty. In our model, the level of ambiguity is captured by the "distance" between the most optimistic and the most pessimistic outlooks on economic growth. Accordingly, our empirical proxy is constructed as the cross-sectional range of professional forecasts of next quarter's real output growth. The underlying assumption is that the representative agent aggregates and synthesizes survey forecasts to form her own belief set. Consequently, the more widely dispersed opinions are from survey participants, the lower confidence she has in probability assessments of the future.

The data source used in this study is Blue Chip Financial Forecasts, which conducts monthly surveys that ask approximately 45 financial market professionals for their projections of a set of economic fundamentals covering real, nominal and monetary variables.[10] Within this survey, which is published monthly, forecasts are always made for a specific calendar quarter. To prevent the forecast horizon from varying over time, we sample individual forecasts at a quarterly frequency. For example, forecasts of GDP growth in the second quarter are extracted from the April release, which comes out on April 1. Dictated by the availability of forecasts of real GDP (GNP) growth, the sample period for the rest of this article is from 1985Q1 to 2010Q4.

In practice, the range of survey forecasts, like other non-robust statistic, could be unduly affected by outlier responses. For example, since 2002 Genetski.com has consistently made predictions of GDP growth about 1% higher than any other respondents, until it stopped participating in the

[10] More details on this survey and its comparison to an alternative source, Survey of Professional Forecasters (SPF), are discussed in Appendix A.1. The advantages of Blue Chip over SPF are also discussed in, e.g., Buraschi and Whelan (2012). To the best of our knowledge, the current study is the first to infer the ambiguity level using the Blue Chip data.

survey in October, 2004. Including this single data point would increase our ambiguity measure by at least 1%. To minimize the impact of such outliers, we employ in our analysis the interval between the 90th percentile point and the 10th percentile point of each cross section[11]

$$\widetilde{A}_t = \hat{F}_t^{-1}(0.9) - \hat{F}_t^{-1}(0.1),$$

where $\hat{F}_t(x)$ denotes the time-t empirical distribution of individual forecasts. The use of this "interdecile" range[12] is conceptually consistent with the multiple priors utility; it is unlikely that the agent simply pools experts' opinions without analysis when developing her set of priors. As discussed in Gajdos et al. (2008), the subjective belief set should be distinguished from the set of all logically possible priors, which contains those outliers.

$\widetilde{A}_t$ is plotted as the dashed blue line in Figure 1.[13] We find that it exhibits substantial temporal variation, ranging from 3.8% in the early 1990s recession to about 1.5% during most years under the Clinton Administration. Moreover, taking together with the business cycle, its time-variation seems too systematic to be attributed to pure sampling errors.

Given the dynamics of $\widetilde{A}_t$, it seems that the assumption of a constant ambiguity level stems from model tractability rather than from empirical plausibility. So why does the level of ambiguity vary over time? An obvious answer is learning. With the multiple-prior utility structure, time-invariant ambiguity points to the constancy of the priors' set, which in turn indicates the lack of learning from data (Chen and Epstein, 2002). In contrast, models with time-varying ambiguity admit the possibility that this set actively responds to updates of information. For example, an increase in investors' confidence in their probability assessment could be the product of better information quality.

Figure 1 provides visual evidence for the co-movement of aggregate ambiguity with corporate default frequency and the price-dividend ratio on the CRSP Value-Weighted Market Index. The red line presents annual issuer-weighted default rates as reported in Ou et al. (2011). As noted by Chen (2010) and Bhamra et al. (2010), we can observe significant countercyclical fluctuations in the default rates, which tend to rise before contractions and peak at the troughs of recessions.

[11] Before 1990, each release of Blue Chip survey contains two statistics named "TOP10" and "BOT10", respectively. They represented not the 90th and 10th percentile points but the average values among top-10 and bottom-10 predictions.

[12] A contemporaneous and independent study by Ilut and Schneider (2014) uses the interdecile range of real GDP forecasts, constructed from the SPF data, to infer the confidence about TFP. In an early version of their paper, the *interquartile* range is used as the ambiguity measure. This statistic is defined as the difference between the upper and lower quartiles. Drechsler (2013) proxies the ambiguity level by the standard deviation of SPF forecasts, which is consistent with his entropy-based formulation of the agent's belief set. In Appendix A.2, $\widetilde{A}_t$ is compared with alternative ambiguity measures.

[13] In Figure 1, $\widetilde{A}_t$ is sampled annually to match the frequency of corporate default rates.

However, a closer inspection also reveals that the two lines closely correspond to each other even within business cycles. These remarks apply equally to the price-dividend ratio, represented by the green line with the corresponding y-axis on the right side. While its procyclical variation has been documented extensively, the figure shows that it (negatively) covaries with the ambiguity measure at a frequency higher than the business cycle. Indeed, their correlation is measured at -73.5%, which is much more remarkable than the correlations of the P/D ratio with real output and consumption growth (12% & 21%). Overall, Knightian uncertainty seems to capture both the inter- and intra-cycle variations in default probabilities and in stock prices. This result suggests that it could play an important role in driving the equity premia and credit spreads — a proposition that is substantiated in the following section.

2.2 Predictability of Asset Returns & Credit Spreads

Having demonstrated that there is historical variation in the ambiguity level, our goal in this subsection is to establish whether the proposed measure has explanatory power for the market premia on a number of assets. To this end, we first examine its significance as a predictor of excess stock returns

$$r_{t+1}^{M} = \text{constant} + b\widetilde{A}_t + \epsilon_t,$$

where r_{t+1}^{M} is defined as the difference between the quarterly return on the CRSP value-weighted portfolio and the corresponding three-month T-bill rate. Since $\widetilde{A}_t$ is constructed from three-month-ahead forecasts made at the beginning of each quarter, the economists' forecast horizon matches that of our predictive regression. This specification maps exactly to the concept of *one-period-ahead* conditionals.

The second row of Table 1 reports the regression estimates. Since the quarterly regression involves non-overlapping observations, t-statistics in parentheses are simply based on Newey-West standard errors adjusted for six lags of moving average residuals. We find that the ambiguity measure has significant predictive ability for excess market returns, with adjusted R^2 equal to 4.3%. Higher levels of ambiguity are associated with higher equity returns, implying that Knightian Uncertainty is priced. Moreover, the magnitude of its impact is sizable as well: a one-standard-deviation (0.75%) increase in $\widetilde{A}_t$ raises the expected quarterly return by 2.18%.

To further pin down the role of ambiguity in pricing corporate securities, we estimate the following predictive regressions of credit spreads for various rating classes

$$CS_{t+1} = \text{constant} + b\widetilde{A}_t + \epsilon_t,$$

where CS_{t+1} is the quarter-end yield spreads between Barclays (Lehman) US corporate bond and Treasury bond indices. The next six rows of Table 1 contains the results of forecasting yield spreads of Aaa- to B-rated bonds. All regression coefficients are significant at the 5% level, and $\widetilde{A}_t$ seems to have stronger predictive power for speculative-grade bonds. The latter is not surprising, as ambiguity only affects the "pure" credit component in yield spreads, which accounts for a greater fraction in those lower rating categories (Huang and Huang, 2012; Avramov et al., 2007; Rossi, 2009).

The predictive content of $\widetilde{A}_t$ is not limited to corporate securities. Table 1 also presents the performance of $\widetilde{A}_t$ in forecasting three-month excess returns on Treasury bonds portfolios. Results in the last two rows show that time-varying ambiguity explains a significant fraction of the variations in both intermediate-term and long-term bond returns.Again, the coefficient estimates are positive, indicating that in an ambiguous environment premia tend to be higher.[14]

To reinforce the common predictor conclusion, we perform an out-of-sample analysis based on two metrics. We employ, first, the out-of-sample R-squared proposed by Campbell and Thompson (2008), and, second, the approximately normal test statistics developed in Clark and West (2007). Both statistics are standard in the literature of return predictability (Jones and Tuzel, 2013; Bakshi et al., 2012). In keeping with these studies, we choose a 10-year initialization period to "train" predictive regression models. Then we evaluate the predictive power of $\widetilde{A}_t$ by comparing it to the historical average of the predicted variable. In other words, in both exercises we estimate the benchmark regression using the mean of all return/spread observations up to and including time-t.

Out-of-sample R^2s reported in Table 1 are uniformly positive, suggesting that the use of $\widetilde{A}_t$ as the predictor leads to a lower mean squared forecast error. Among different asset classes, the premia on corporate bonds appear subject to highest predictability, as in-sample results reveal. The p-values from the Clark-West tests are presented in the last column.[15] They confirm the effectiveness of $\widetilde{A}_t$ as a real-time predictor. The only exception is long-term Treasury bonds, for which the ambiguity measure is significant only if the test's size is set at 10%. Otherwise, none of the Clark-West p-values exceeds 0.04. Generally, we find that $\widetilde{A}_t$'s in-sample significance is translated into the out-of-sample context.

[14] The model presented in the next section implies that the risk premia on *real* bonds are low when the ambiguity level is high. If we introduce a positive correlation between innovations in ambiguity and in expected inflation, as exists in the data, the model generates a positive effect of ambiguity on nominal bond risk premia.

[15] As Clark and West (2007) show, both theoretically and numerically, their test statistic is approximately normal after adjusting for the estimation error of the larger model. Thus, we only report the p-values.

3 Model Setup

3.1 Modeling Ambiguity Aversion

Given the empirical evidence on return predictability, this section theorizes about the relationship between time-varying ambiguity and asset premia. Consider a measurable state space $(\Omega, \mathscr{F})$ where each $\mathscr{F}_t \in \mathscr{F}$ can be identified with a partition of Ω. Given a probability measure P, $\mathscr{F}_t$ is the σ-field generated by a d-dimensional Brownian motion B_t defined on $(\Omega, \mathscr{F}, P)$. Suppose that the representative decision-maker does not know the true probability measure P_0 and ranks uncertain consumption streams $C = \{C_t\}_{t=0}^{\infty}$, where C_t: $\Omega \rightarrow \mathscr{R}$ is $\mathscr{F}_t$-measurable. To model preferences in the presence of uncertainty, we adopt the structure of recursive multiple priors

$$V_t^* = \min_{P \in \mathscr{P}^{\Theta}} E_P \left[\int_t^{\infty} f(C_s, V_s^P) ds \mid \mathscr{F}_t \right] \tag{1}$$

where the set $\mathscr{P}$ of priors on $(\Omega, \mathscr{F})$ is constructed by means of Girsanov transformation. In particular, we can define the Radon-Nikodŷm derivative (Z) of P with respect to the reference measure P_0 through a density generator (ϑ_t)

$$dZ_t^{\vartheta} = -Z_t^{\vartheta} \vartheta_t dB_t,^{2} \qquad Z_0^{\vartheta} = 1 \tag{2}$$

$$Z_t^{\vartheta} = \exp\left\{ -\frac{1}{2} \int_0^t |\vartheta_s|^2 ds - \int_0^t \vartheta_s dB_t \right\}, \tag{3}$$

where B_t is a Brownian motion under P_0. It follows that the generated measure $P^{\vartheta}(\omega) = Z(\omega) P_0(\omega)$ is equivalent to P_0.

This continuous-time multiple-priors model (1)–(3) is proposed by Chen and Epstein (2002), who prove the existence and uniqueness of the solution to Eq. (1). The multiple-priors functional form is introduced by Gilboa and Schmeidler (1989) to account for the Ellsberg-type behavior; Epstein and Wang (1994) put forth the recursive version, and Epstein and Schneider (2003) lay its axiomatic foundations. Faced with a multiplicity of approaches to modeling ambiguity, our focus is multiple-priors preferences, which has a well-established framework in continuous time. Strzalecki (2013) shows that the recursive multiple-priors utility, per se, is neutral about the timing of the resolution of uncertainty. This feature allows ambiguity aversion to vary independently without affecting the agent's temporal attitudes, which can be modeled separately using the Kreps-Porteus preferences.

In a pure diffusion environment, ambiguity concerns whether B_t is a Brownian motion.[16] In

[15] We assume that the regularity conditions, as specified in the Appendix D of Duffie (2001), are satisfied so that Z_t^{ϑ} is a martingale under P_0.

[16] Liu et al. (2005) and Drechsler (2013) consider agents who exhibit ambiguity aversion towards jumps in the level or in the expected (long-run) component of consumption growth.

other words, a change of measure from P_0 to $P \in \mathscr{P}$ affects only the drift function of the utility continuation process. To be more precise, the Martingale Representation Theorem implies that the utility process under P_0 can be written as the solution to the backward stochastic differential equation (Duffie and Epstein, 1992)

$$dV_t^{P_0} = -f(C_t, V_t^{P_0})dt + \sigma_{v,t} dB_t. \tag{4}$$

Similarly, for the utility process defined under $P^{\vartheta} \in \mathscr{P}$, we can employ the Girsanov Theorem to express it in terms of B_t

$$dV_t^{P^{\vartheta}} = \left[-f(C_t, V_t^{P^{\vartheta}}) + \vartheta_t \sigma_{v,t}\right] dt + \sigma_{v,t} dB_t. \tag{5}$$

With the multiple-priors utility, the decision-maker's acts reflect her worst-case belief. Therefore, it is natural to guess

$$dV_t^{*} = \left[-f(C_t, V_t^{P^{*}}) + \max_{\vartheta \in \Theta} \vartheta_t \sigma_{v,t}\right] dt + \sigma_{v,t} dB_t. \tag{6}$$

Chen and Epstein (2002) confirm this conjecture and prove the uniqueness of the solution to Eq. (6).

Under Duffie and Epstein (1992)'s parameterizations of recursive preference, the aggregator function f takes the form

$$f(C, V) = \beta\theta V \left[\left(\frac{C}{((1-\gamma)V)^{\frac{1}{1-\gamma}}}\right)^{1-\frac{1}{\psi}} - 1\right] \tag{7}$$

where $\beta > 0$ is the rate of time preference, $\gamma \neq 1$ is the coefficient of relative risk aversion (RRA), and $\psi \neq 1$ is the elasticity of intertemporal substitution (EIS). For notational convenience we set $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. Eq. (7) can be regarded as the continuous-time version of Kreps-Porteus functional form; the normalized utility index V is obtained through an increasing transformation of the initial one parameterized by Epstein and Zin (1989). It follows that the stochastic control problem is to find an optimal consumption strategy c^* to maximize Eq. (1)

$$J_t = \max_{C \in \mathcal{A}} \min_{P \in \mathscr{P}^{\Theta}} E_P \left[\int_t^{\infty} f(C_s, V_s^P) ds \mid \mathscr{F}_t\right] \tag{8}$$

where $C \in \mathcal{A}$ denotes that the control process $\{C_t\}$ is admissible. Market clearing implies that the representative agent takes up the aggregate consumption, i.e. $c_t^* = C_t$.

To illustrate the effect of time-varying ambiguity on asset pricing, we consider a highly stylized model for the driving process. The dynamics of consumption growth is specified as

$$\frac{dC_t}{C_t} = \mu_{c,t} dt + \sigma_c dB_{C,t}, \tag{9}$$

where the sequence $\{\mu_{c,t}\}$ are unknown to the representative agent. Rather, at time t she has a set of beliefs which can be parameterized as

$$\mu_{c,t}^{\mathscr{P}} \in [\mu_c - A_t, \mu_c + A_t], \quad A_t \geq 0$$

where A_t captures the level of ambiguity about the expected growth rates. For the sake of simplicity, we assume that the drift component in the reference model is simply μ_c. Therefore, the less confidence the agent has in her probability assessment of the conditional mean, the wider is the interval $[-A_t, A_t]$. If $A_t \equiv 0$, ambiguity vanishes and we obtain the standard subjective expected utility setting. Note that this specification of the set of priors satisfies the rectangularity and thus ensures dynamic consistency.[17]

To reflect the arrival of news about economic prospects, we allow the ambiguity level A_t to vary over time

$$dA_t = \mu_{A,t}dt + \sigma_{A,t}dB_{A,t}, \tag{10}$$

where $B_{A,t}$ is assumed to be uncorrelated with $B_{C,t}$. For tractability, the drift and diffusion functions are assumed to have an affine structure, which in the next subsection we will parameterize in order to capture key properties of the survey data. As discussed in Section 2.1, evolution of aggregate confidence may reflect information processing and updating by ambiguity-averse investors. Indeed, Epstein and Schneider (2008) formally model the responsiveness of ambiguity to observations and derive important theoretical implications. For the purpose of our study, modeling the origin of variation in ambiguity does not add significant economic insights, while it does complicate the analysis. Hence, the exogenous structure imposed on A_t is to keep the model as simple as possible while retaining the key ingredients needed to reconcile asset pricing puzzles.

3.2 Parameterizations of the Ambiguity Process

As a forcing process, the ambiguity level A_t is assumed to follow an affine process. In this section, we specify the dynamics of A_t (a) to capture key properties of the data and (b) to ensure its non-negativity. Referring to the proxy $\widetilde{A}_t$ for aggregate ambiguity charted in Figure 1, we observe that its level fluctuates around an average value of 2.06 percent. Indeed, Figure 1 reveals that as investors become less confident in probability assessments, the confidence level tends to be subject to progressively larger shocks — a pattern that implies that the ambiguity measure has time-varying volatility and its distribution is right-skewed.

[17] An alternative way to construct restrictions to the prior set is through relative entropy (Sbuelz and Trojani, 2008; Drechsler, 2013). The resulting set of density generators is nonrectangular and thus is not admissible in the recursive multiple-prior setting (Chen and Epstein, 2002; Epstein and Schneider, 2003).

To explore these empirical properties, we firstly perform a variance ratio test on the realized volatility of innovations. Panel A of Table 2 displays the test results. If realized volatility were i.i.d., the term structure of variance ratios would be flat, with magnitudes close to unity. But in Table 2 the empirical variance ratios are all above one, and they increase uniformly with the horizon. The bootstrap analysis indicates that the null of homoscedastic shocks is rejected at the 5% level for all horizons.

Even more prominent is the skewness of the ambiguity level. As shown in Panel B, the number measured for our sample period is 0.82. This positive skewness is significant at the 5% confidence level constructed by 10,000 bootstrap samples. This evidence, together with the heterogeneity of realized volatility that is positively correlated with A_t, prompts a mean-reverting square-root process

$$\begin{aligned} \mu_{A,t} = &\quad \kappa(\bar{A} - A_t), \quad \kappa > 0; \\ \sigma_{A,t} = &\quad \sqrt{a_0 + a_1 A_t}, \quad a_1 > 0. \end{aligned}$$

For the diffusion term, we note that Piazzesi et al. (2007) and Yang (2011) use a similar functional form in discrete time to model other macroeconomic series with similar properties. But because we need to guard against the probability that A_t falls below zero, we restrict a_0 to zero and adopt a Cox-Ingersoll-Ross (CIR) specification

$$\sigma_{A,t} = \sigma_a \sqrt{A_t}.$$

Figure 2 shows that the density function implied by the simulated CIR process fits the empirical one reasonably well, while a simple Ornstein–Uhlenbeck process, with $\sigma_{A,t} = \sigma_A$, is unable to capture the right skewness.

3.3 Equilibrium Prices

In this section, we solve for the value function J by expanding the ϑ^*-expectation of future continuation utility, as presented in Eq. (6). To fully exploit the analytical power afforded by the continuous time, we note that any alternative model can be defined by a probability measure $P^\vartheta \in \mathscr{P}$

$$\frac{dC_t}{C_t} = (\mu_c + a_t)dt + \sigma_c dB^P_{C,t},$$

where $a_t \in [-A_t, A_t]$ and $B^P_{C,t}$ is a Brownian motion under P. In other words, the structure of the density generator is given by $\vartheta_t = (-a_t/\sigma_c, 0)$. This specification describes the component of state dynamics that the agent is uncertain about; there is no ambiguity about the ambiguity itself.[18]

[18] Drechsler (2013) considers an extended specification in which there is Knightian uncertainty about the dynamics of the size of ambiguous beliefs.

These results can be used to confirm the conjecture that the value function is increasing in aggregate consumption. The worst-case belief P^* then corresponds to the density generated by $\vartheta^* = A_t/\sigma_c$, and the stochastic control problem becomes standard.

PROPOSITION 1: *With consumption and ambiguity dynamics as specified above, if $L(A_t)$ solves the following equation*

$$(1-\gamma)(\mu_c - A - \frac{1}{2}\gamma\sigma_c^2) + \frac{\mathfrak{D}^A L^\theta}{L^\theta} + \frac{\theta}{L} - \beta\theta = 0, \qquad \gamma, \psi \neq 1, \tag{11}$$

where $\mathfrak{D}^x$ is the standard Dynkin operator and it satisfies the transversality condition, then the value function is given by

$$J(C_t, A_t) = \frac{C_t^{(1-\gamma)}}{1-\gamma}(\beta L(A_t))^\theta, \tag{12}$$

and L is the price-consumption ratio in equilibrium.

Given that the differential equation (11) does not have a close-form solution, we follow Benzoni et al. (2005) and Chen et al. (2009) by approximating L as an exponential affine function

$$L(A) \approx e^{\eta_0 + \eta_1 A}. \tag{13}$$

As shown in Appendix B, this approach provides an accurate approximation to the problem solution. The price-consumption loading on ambiguity, η_1, depends on the preference configuration. Given a nondegenerate interval for the subjective growth mean ($A_t > 0$), η_1 is negative as long as EIS is larger than one. This corresponds to the scenario wherein the agent recoups her investment in response to shocks that blur economic prospects. Consequently, asset prices tend to drop at times of high subjective uncertainty.

In equilibrium, state prices are shaped by the marginal rates of substitutions of current for future consumption. In particular, given the max-min representation (8), state prices are based on the worst-case density Z^{ϑ^*}. The following proposition sheds light on how ambiguity aversion contributes to the asset premia.

PROPOSITION 2: *The real pricing kernel has dynamics*

$$\begin{aligned} \frac{dM_t}{M_t} = & \; -r_t dt \quad -\Lambda_t' dB_t \\ & \; -r_t dt \quad -\left(\gamma\sigma_c + \frac{A_t}{\sigma_c}, (1-\theta)\eta_1\sigma_a^2 A_t\right)\begin{pmatrix} dB_{C,t} \\ dB_{A,t} \end{pmatrix} \end{aligned} \tag{14}$$

where $r_t = \varrho_0 + \varrho_1 A_t$. The expressions ϱ_0 and ϱ_1 are given in Appendix A.

In the current setup, a nondegenerate set of priors reflects the agent's lack of confidence in her assessment of economic growth. With this interpretation, a wider span of the set demands a proportional increase in the ambiguity premium on the consumption claim, given that the lack of confidence makes the agent evaluate the asset as if the aggregate consumption grows at a rate of $\mu_c - A_t$. This first-order effect is captured by the A_t/σ_c term.

Furthermore, due to the separation between risk aversion and EIS, the risk from fluctuations in the future ambiguity level earns a separate premium. Empirically, this intertemporal substitution effect is enlarged by the inverse-leverage effect in the dynamics of A_t: the higher level of Knightian uncertainty, the greater uncertainty about its future path. If the agent prefers an earlier resolution of this uncertainty ($\gamma > 1/\psi$), shocks to A_t that increase its conditional volatility carry an additional (positive) uncertainty premium. Depending on the persistence of the ambiguity process, these shocks could cause large reactions in the marginal rate of substitution. This second risk channel is characterized by the $(1-\theta)\eta_1\sigma_a^2 A_t$ term in Eq. (14).

To sum up, in our model there are two ways in which time-varying ambiguity influences the market price of uncertainty. In contrast, in models with a constant amount of ambiguity, the first-order effect degenerates into a fixed level and the second-order effect vanishes; such models do not allow for time-varying Sharpe ratios. Also notable, the first-order effect makes the market price of uncertainty fall into the "essentially affine" class introduced by Duffee (2002).

The dual role time-varying ambiguity serves in equilibrium asset pricing is also reflected in its impact on the risk-free rate. As shown in Appendix B, the loading of r_t on A_t has the following expression

$$\varrho_1 = -\frac{1}{\psi} + \frac{1}{2}(\theta - 1)\eta_1^2\sigma_a^2. \tag{15}$$

The first term on the right hand side captures the effect of ambiguity on intertemporal smoothing. Also persisting in the case of constant ambiguity, it underlines the fact that ambiguity about the distribution of future payoffs is a first-order concern. However, the second term is absent in models with either time-invariant ambiguity or constant relative risk aversion ($\theta = 1$). That term lowers the equilibrium interest rate if the agent has a preference for early resolution of uncertainty, as she wants to divest from risky assets when the future level of ambiguity becomes highly uncertain. Therefore, both effects enhance the agent's saving motive and thus help explain the risk-free rate puzzle.

Finally, as the proposed ambiguity measure $\widetilde{A}$ is derived from forecasts of real GDP growth,[19]

[19] The Bluechip survey does not ask participants for the forecast of real consumption expenditures. While the SPF contains this category, the responses are not collected by type of product. Hence, forecasts of nondurables plus service consumption (the variable typically used in model calibration) are unavailable.

we also specify the aggregate output process O_t as

$$\frac{dO_t}{O_t} = \mu_o dt + \sigma_o \left(\sigma_{oc} dB_{C,t} + \sqrt{1-\sigma_{oc}^2} dB_{O,t} \right), \tag{16}$$

where $dB_{O,t}$ is correlated with neither $dB_{C,t}$ nor $dB_{A,t}$. Following Chen et al. (2009), we assume that O_t has the same growth rate as the aggregate consumption C_t, but with different, albeit closely correlated, dynamics. This assumption appears empirically valid: over our sample period, real GDP and real consumption of nondurables plus services have almost identical mean growth rate (1.58% versus 1.54%); the correlation coefficient between their innovations is measured at 63.1%.

We continue to assume that there is ambiguity only about whether $B_{C,t}$ is a Brownian motion. Thus, given the set $\mathscr{P}^\Theta$ of priors as specified above, the expectation of output growth falls within the interval

$$\frac{E^{\mathscr{P}}(dO_t)}{O_t dt} \in \left[\mu_o - \frac{\sigma_o \sigma_{oc} A_t}{\sigma_c}, \mu_o + \frac{\sigma_o \sigma_{oc} A_t}{\sigma_c} \right].$$

It follows that the endogenous ambiguity level A_t is proportional to the empirical proxy defined in the last section,

$$A_t = \frac{\widetilde{A}_t \sigma_c}{2\sigma_o \sigma_{oc}}. \tag{17}$$

Indeed, (17) motivates our formation of the ambiguity measure as the range of survey forecasts because the agent cares about the span of all subjective priors.

Thus far the correlation among innovations in state variables has not been included for parsimony. Empirically, we find that a loss of confidence (in probability assessment) is bad news for future consumption and output growth. For example, the impulse response function shows that a one-standard-deviation shock lowers the consumption growth over the next quarter by roughly 6 basis points. In deriving quantitative implications of the model, we allow for these cross-shock responses by incorporating σ_{ca} and σ_{oa} in Eq. (9) and (16). As shown in section 4.1, estimates of both parameters are significant at the 1% level. Expressions for the pricing kernel and risk-free rate in presence of these correlation terms are given in Appendix B. Intuitively, a negative value of σ_{ca} magnifies the effect of Knightian uncertainty on valuation ratios.

3.4 Firm's Capital Structure

In order to price securities issued by individual firms, we assume that the cash flows to firm j follow the process

$$\frac{d\delta_{j,t}}{\delta_{j,t}} = \frac{dO_t}{O_t} + \sigma_j dB_{j,t}, \tag{18}$$

where $B_{j,t}$ captures the firm-specific risks and σ_j is the idiosyncratic volatility. This CAPM-inspired specification is inspired by the approach employed by Chen et al. (2009), Bhamra et al. (2010) and Chen (2010).

Following the standard capital structure models, we assume that investment decisions are fixed and exogenous for the financing policy. Our benchmark model proposes that bankruptcy costs and differential tax treatment are the major market imperfections that affect corporate decisions. In the future this model can be expanded to include other types of frictions, such as agency costs and transaction (debt retirement and reissuance) costs. For tractability, we assume a stationary debt structure where a firm continuously retires a constant fraction m of existing debt and replaces it with the same amount (of principal) of newly issued debt. In making this assumption, we follow the modeling approach of Leland (1994a, 1998), who shows that although no explicit maturity is stated for the debt the average maturity is equal to $1/m$.

The corporate debt outstanding is composed of coupon bonds with the total coupon payment equal to C. At any time t, therefore, new bonds are issued at a rate $f = mF$, where F is the total face value of all outstanding bonds, with the instantaneous coupon rate $c = mC$ to preserve the debt structure as time elapses. We further assume that the bond indenture provisions prohibit equityholders from selling assets to pay any dividends, and maintain absolute priority for bondholders. Hence, we may think about a situation in which the stockholders will have to make payments to the firm to cover the interest payments. However, shareholders have the contractual right to declare default at any time and turn the firm over to the bondholders. Upon default, the firm incurs a total deadweight cost equal to $\phi(A_t)U^*$, where $0 < \phi < 1$ and U^* is the unlevered value of the equity at the time of default.

Finally, we assume a differential tax system: corporate earnings are taxed instantaneously at a constant rate of τ_c; there is no loss-offset provision of taxes. Individual investors, however, pay taxes on interest income at rate τ_i and on dividend income at rate τ_d, but they are not taxed on their capital gains. It follows that the cash flows to the equity and the debt issued at time t_0, when the firm is solvent, are given by

$$\begin{aligned}
\delta_{e,t} &= (1-\tau_e)(\delta_t - C) - mF + D(\delta_t, A_t), \qquad \text{and} \\
\delta_{d,t} &= e^{-m(t-t_0)}\left((1-\tau_i)C + mF\right),
\end{aligned}$$

respectively, where $\tau_e = 1 - (1-\tau_c)(1-\tau_d)$ is the effective tax rate and D the market price of newly issued debt.

Given that it is in the interest of the equityholders to choose when to default in such a way that the value of equity is maximized, the endogenous default can be formulated as an optimal stopping problem. Fix a domain $\mathcal{S} \subset \mathbb{R} \times (0, \infty)$ for the state vector (δ_t, A_t), and define $\tau_{\mathcal{S}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{S}\}$. Then the optimal default boundary is determined by finding a stopping time $\tau^*(\delta_t, A_t)$ such that

$$E(\delta_t, A_t) = E_t^Q \left[\int_t^{\tau*} e^{-\int_t^s r_u du} \delta_{e,s} ds \right] = \sup_{\tau \in \mathcal{T}} E_t^Q \left[\int_t^{\tau} e^{-\int_t^s r_u du} \delta_{e,s} ds \right]$$

where $\mathcal{T}$ is the set of all stopping times $\tau < \tau_{\mathcal{S}}$. In the language of optimal stopping problems, $\delta_{e,t}$ is the "utility rate" function and the "bequest" function is zero since equityholders receive nothing at default.

In practice, this type of free-boundary problem is usually solved by verifying that a given candidate function e actually coincides with E and that a corresponding stopping time $\tau_{\mathcal{B}}$ is optimal. Derived from the variational inequality verification theorem proved in Øksendal (2003), the following proposition shapes the solution for the maximized equity value.

PROPOSITION 3: *Suppose we can find a function $e : \bar{\mathcal{S}} \to \mathbb{R}$ such that $e \in \mathcal{C}^1(\mathcal{S}) \bigcap \mathcal{C}(\bar{\mathcal{S}})$ and $e \geq 0$ on $\mathcal{S}$. Define*

$$\mathcal{B} = \{(\delta_t, A_t) \in \mathcal{S}; e(\delta_t, A_t) > 0\},$$

as the continuation region.

(a) Suppose $\partial\mathcal{B}$ is a Lipschitz surface. If

(i) (δ_t, A_t) spends zero time at $\partial\mathcal{B}$, i.e. $E_t^Q \left[\int_t^{\tau_{\mathcal{S}}} \mathcal{X}_{\partial D}(\delta_t, A_t) ds \right] = 0$,

(ii) $e \in C^2(\mathcal{S} \backslash \partial\mathcal{B})$ with locally bounded derivatives near $\partial\mathcal{B}$,

(iii) $\mathfrak{D} e(\delta_t, A_t) + \delta_e - r(A_t) e(\delta_t, A_t) \leq 0$ on $\mathcal{S} \backslash \partial\mathcal{B}$,

then $e(\delta_t, A_t) \geq E(\delta_t, A_t)$ for all $(\delta_t, A_t) \in \mathcal{S}$.

(b) Moreover, assume

(iv) $\mathfrak{D} e(\delta_t, A_t) + \delta_e - r(A_t) e(\delta_t, A_t) = 0$ on $\mathcal{B}$,

(v) $\tau_{\mathcal{B}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{D}\} < \infty$ for all $(\delta_t, A_t) \in \mathcal{S}$,

(vi) the family $\{e(\delta_\tau, A_\tau); \tau \in \mathcal{T}, \tau \leq \tau_{\mathcal{B}}\}$ is uniformly integrable, for all $(\delta_t, A_t) \in \mathcal{S}$.

Then $e(\delta_\tau, A_\tau) = E(\delta_\tau, A_\tau)$ and $\tau^* = \tau_{\mathcal{B}}$ is an optimal default time.

(c) Define $\mathcal{U} = \{(\delta_t, A_t) \in \mathcal{S}; \delta_{e,t} > 0\}$. Suppose that for all $(\delta_t, A_t) \in \mathcal{U}$ there exists a neighborhood $\mathcal{W}$ of (δ_t, A_t) such that $\tau_{\mathcal{W}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{W}\} < \infty$. Then $\mathcal{U} \subset \{(\delta_t, A_t) \in \mathcal{S}; e(\delta_t, A_t) > 0\} = \mathcal{B}$. Therefore it is never optimal to default while $(\delta_t, A_t) \in \mathcal{U}$.

These results are deduced from the "high contact principle" (Brekke and Øksendal, 1991), which has been fruitfully applied in optimal stopping and stochastic waves. To find a reasonable guess for the continuation region $\mathcal{B}$, we refer to Proposition 3(c). In view of the expression for $\delta_{e,t}$, which is a non-decreasing function of δ_t, we conjecture that $\mathcal{B}$ has the form $\mathcal{B} = \{(\delta_t, A_t); \delta_t > \delta^*(A_t)\}$ such that $\mathcal{U} \subseteq \mathcal{B}$.[20] In other words, inspired by Goldstein et al. (2001) and Strebulaev (2007), we define the default time in terms of corporate earnings. Therefore, the optimal strategy of shareholders is to find out the critical default barrier δ^* that depends on the current level of ambiguity.

Proposition 3 characterizes the maximized equity value by the partial differential equation (PDE) listed in *(iv)*. The relevant boundary conditions derive from its $\mathcal{C}^1$-property. The continuity condition implies that

$$E(\delta^*(A), A) = 0. \tag{19}$$

Moreover, to ensure differentiability,[21] it is further required that

$$\lim_{\delta \to \delta^*} E_\delta = \lim_{\delta \to \delta^*} E_A = 0. \tag{20}$$

The market value of newly issued debt D, on the other hand, satisfies the PDE

$$\begin{aligned} \mathfrak{D}D(\delta, A) + (1-\tau_i)mC + m^2F - (r(A)+m)D(\delta, A) &= 0, \\ \lim_{\delta \to \delta^*} D(\delta, A) &= (1-\phi)mU(\delta^*(A), A), && (21) \\ \lim_{\delta \to \infty} D(\delta, A) &= ((1-\tau_i)mC + m^2F)\mathscr{PV}^{r+m}(A), && (22) \\ \lim_{\delta \to \infty} D_\delta(\delta, A) &= 0, && (23) \end{aligned}$$

where $\mathscr{PV}$ is the expected present value operator

$$\mathscr{PV}^q(x_t) = E_t^Q \left[\int_t^\infty e^{-\int_t^s (q(x_u)du}) ds \right].$$

The upper boundary conditions (22) and (23) imply that when the firm's payoff grows to infinity, the possibility of default becomes meaningless, so that the debt value tends towards to the price of a default-free claim to the continuous cash flow $(1-\tau_i)mC + m^2F$. $U(\delta_t, A_t)$ in Eq. (21) denotes the value of unlevered assets that can be written as

$$U(\delta_t, A_t) = \delta_t L^o(A_t),$$

where $L^o(A_t)$ is the price-earning ratio. Given the affine model structure, both $\mathscr{PV}^r(A)$ and $L^o(A_t)$ would be solved with a log-linear approximation

$$L^o(A) \approx e^{\xi_0 + \xi_1 A}, \tag{24}$$

$$\mathscr{PV}^r(A) \approx e^{\zeta_0 + \zeta_1 A}. \tag{25}$$

[20]This conjecture is easily verified using the results established in Mordecki (2002) for a general Lévy process.

[21]While the continuous pasting condition (19) is necessary, the smooth pasting condition (20) need not hold in general, although it holds in the optimal default problem. See Alili and Kyprianou (2005) for a detailed discussion.

The details are described in Appendix B.

The free boundary problem as characterized above resembles the one confronted in pricing American options. For our pricing model where the risk-free rate and asset volatility are time-varying and dependent on the ambiguity level, two numerical approaches might be relevant: the short-maturity asymptotic approximation (Lamberton and Villeneuve, 2003; Chevalier, 2005; Levendorskii, 2008), and the simulation-based method proposed in Longstaff and Schwartz (2001). But neither approach is applicable here because there is no such expiration date in the current debt structure. For this reason, we perform a regular perturbation analysis to solve for the prices of levered equity and corporate bond.[22] Specifically, we utilize the persistence in the ambiguity process, which is confirmed by our model estimation in Section 4, by adding a small time-scale parameter ε to its dynamics

$$dA_t = \varepsilon\kappa_A(\bar{A} - A_t)dt + \sigma_a\sqrt{\varepsilon A_t}dB_{A,t}. \tag{26}$$

The small magnitude of ε captures the slow mean-reversion in A_t and makes possible the following expansion with respect to asset prices

$$\begin{aligned} E &= E_0 + \sqrt{\varepsilon}E_1 + \varepsilon E_2 + O(\varepsilon^{\frac{3}{2}}), && (27) \\ D &= D_0 + \sqrt{\varepsilon}D_1 + \varepsilon D_2 + O(\varepsilon^{\frac{3}{2}}), && (28) \end{aligned}$$

where the expansion series solve a series of ordinary differential equations, as presented in Appendix C. These asymptotic approximations can be viewed as an extension of the slow variation asymptotics (Sircar and Papanicolaou, 1999; Lee, 2001) to optimal stopping-time problems (Fouque et al., 2001).

To obtain economic insights from this approximate analytic solution, we focus on the leading-order terms, which essentially solve a reduced, one-dimensional free boundary problem. As might be expected, D_0 and E_0 in (27)-(28) are a variant of Leland (1994a)'s pricing formula with ambiguity

[22] This approach is also used by McQuade (2012), who proposes a resolution of the credit spread puzzle as well, by incorporating stochastic volatility into structural modeling. Yet in contrast to McQuade (2012), who draws a distinction between growth and value firms and aims to provide cross-sectional implications for market and book values of firms' equity, we attempt to derive the effects of time-varying ambiguity on the prices of various asset classes in a *general equilibrium* model.

fixed at the current level

$$
\begin{aligned}
D_0 &= ((1-\tau_i)mC + m^2 F)e^{\zeta_0^m+\zeta_1^m A}\left(1-\left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_{r+m}}\right)+ \\
&\quad m(1-\phi)\delta_0^*(A)e^{\xi_0+\xi_1 A}\left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_{r+m}}, \qquad (29) \\
E_0 &= (1-\tau_e)\delta e^{\xi_0+\xi_1 A} + (\tau_e-\tau_i)Ce^{\zeta_0+\zeta_1 A}\left(1-\left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_r}\right)- \\
&\quad \phi\delta_0^*(A)e^{\xi_0+\xi_1 A}\left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_r} - \frac{D_0(\delta,A)}{m}. \qquad (30)
\end{aligned}
$$

where the expressions for α_{r+m} and α_r are given in Appendix C. δ_0^* is the leading-order term in an according asymptotic expansion of the optimal default boundary

$$
\delta^* = \delta_0^* + \sqrt{\varepsilon}\delta_1^* + \varepsilon\delta_2^* + O(\varepsilon^{\frac{3}{2}}). \qquad (31)
$$

Following standard practices, we price newly issued debt as the present value of coupon payments before default (the first line of Eq. (29)), and recovered firm's value at default (the second line). On the other hand, the value of levered equity is equal to the firm's value minus the total debt value, which can be shown as equal to $1/m$ times the value of new debt. The value of a levered firm in turn consists of the sum of the after-tax value of unlevered assets, the tax benefit and the potential bankruptcy cost, as reflected by the first three terms on the right-hand side of (30) respectively. The first term is usually interpreted as the value of equity before any debt is issued.[23] The second term depends on the level of tax shelter $(\tau_e - \tau_i)C$. However, tax benefits cannot be claimed after default. Therefore, their current value is equal to the product of the tax-sheltering value of coupon payments and the probability of solvency. Finally, the last term in (30) derives from the reorganization costs upon bankruptcy, $\phi U(\delta^*, A)$. The present value of bankruptcy cost is obtained by multiplying this term by the risk-neutral default probability.

As indicated by Eq. (29) and (30), the leading-order prices are equivalent to an application to credit risk modeling of the κ-ignorance specification (Chen and Epstein, 2002), where the ambiguity premium is time-invariant. This is true because of the persistence in the A_t process that makes the agent attach great importance to its current level rather than its long-run mean. This explains why $\bar{A}$ does not appear in the pricing formula for the primary-order terms. For the same reason, the agent would show large reaction to changes in the ambiguity level, which are perceived as being long-lasting. This high ambiguity elasticity of asset prices helps explain the large equity premium.

[23] In the EBIT-based framework there does not exist any all-equity firm. Rather, the value of a debtless firm is divided between equity and government, with the portions determined by the effective tax rate.

To illustrate this point, we present the expression for the uncertainty premium on unlevered equity

$$
\begin{aligned}
& \frac{1}{dt}E_t\left[\frac{dU((1-\tau_e)\delta_t, A_t)}{U((1-\tau_e)\delta_t, A_t)}\right] - r_t + \frac{(1-\tau_e)\delta_t}{U((1-\tau_e)\delta_t, A_t)} \\
= \; & \gamma\sigma_c\sigma_o\sigma_{oc} + (1-\theta)\eta_1\xi_1\sigma_a^2 A_t + \frac{\sigma_o\sigma_{oc}}{\sigma_c}A_t + \qquad (32) \\
& [(\xi_1+\sigma_{oa})\gamma\sigma_{ca} + (1-\theta)\eta_1\sigma_{oa}]\sigma_a^2 A_t. \qquad (33)
\end{aligned}
$$

The first term in (32) is standard in the constant relative risk aversion (CRRA) case. The second term represents risk compensation for the uncertainty in the future level of ambiguity; it is positive if the agent has a preference for early resolution of uncertainty ($\gamma > 1/\psi$) and the EIS is greater than one. As noted above, an increase in the permanence κ_A of the ambiguity process would raise the absolute values of η_1 and ξ_1, and thus would result in greater magnitude of the second term.

While the equity premium in long-run risk models also contains an component akin to the second term in (32), the third term here is unique to ambiguity aversion models. This ambiguity premium component captures the fact that Knightian uncertainty about the distribution of future payoffs is a first-order concern. Therefore, the ambiguity level could have large impact on equity premium even if it is not as volatile as the cash flow process. It is also worth noting that (32) encompasses the κ-ignorance case, wherein the second term vanishes and the ambiguity premium term becomes constant. Finally, (33) reflects the conditional correlation between the aggregate consumption / output and the amount of ambiguity.

As to the uncertainty premium on corporate debt, note that δ_0^* in Eq. (29) and (30) has the following solution

$$
\delta_0^* = \frac{((1-\tau_i)C + mF)\alpha_{r+m}e^{\zeta_0^m+\zeta_1^m A} - (\tau_e-\tau_i)C\alpha_r e^{\zeta_0+\zeta_1 A}}{\phi\alpha_r + (1-\phi)\alpha_{r+m} - 1}e^{-\xi_0-\xi_1 A} \qquad (34)
$$

Again Eq. (34) depends only on the current value of At rather than its historical mean. If the response of price-earning ratio to the amount of ambiguity is negative ($\xi_1 < 0$), δ_0^* is an increasing function of A. The equation indicates that equityholders prefer earlier default when growth prospects are highly uncertain. The resulting positive covariation between the default boundary and the market price of ambiguity makes default events cluster exactly when the marginal rate of substitution is high. Hence, the agent demands a higher premium on corporate bonds than would be the case were there a constant default barrier. As to other determinants, higher marginal benefit of tax $(\tau_e - \tau_i)C$, or lower fractional bankruptcy costs ϕ, lead to an increase in the default boundary.

4 Model Implications

In this section, to illustrate the quantitative impact of time-varying ambiguity on asset prices we assess the model's empirical performance. We start with a formal estimation of the state space. Section 4.2 considers the model's ability to reconcile the credit spread puzzle with a reasonable firm-level calibration. In Section 4.3, we investigate whether the model can match some key moments of levered equity premium and risk-free rate. The last subsection examines the the model's implications for the dynamics of asset prices.

4.1 Data and Estimation

A major obstacle to the estimation of equilibrium asset pricing models is that the key state variable, e.g. the surplus consumption ratio or the long-run component in consumption growth, tends to be unobservable.[24] Advantageously, this model features a fully measurable state vector. Therefore, we obtain model parameters through a maximum likelihood estimation (MLE) of the state process, while leaving the choices of preference parameters similar to the existing studies.Specifically, we consider the state vector $Y_t = \{c_t, o_t, \widetilde{A}_t\}$, where the dynamics of $\widetilde{A}_t$ can be easily derived from Eq. (17) via Itô's lemma.

Following the convention in the literature, C_t is measured as the sum of real personal consumption expenditures (PCE) on nondurable goods and services, and O_t as real GDP per capita. Both data series are retrieved from the U.S. national accounts of Bureau of Economic Analysis and are seasonally adjusted.[25] The estimation frequency is quarterly and the sample period is 1985 to 2010.

Given the specification of Y_t's dynamics, one cannot extract its conditional density in closed form. In principle, the likelihood function can be estimated by Monte Carlo simulation methods. However, this approach is computationally intensive for our model because the simulation has to be performed for every conditioning variable and for every parameter value. Instead, we apply the approximate MLE developed by Aït-Sahalia (1999, 2002, 2008), who constructs explicit expansions for the log-likelihood function of a large class of univariate and multivariate diffusion processes. Given that Y_t is an irreducible process, as defined in Aït-Sahalia (2008), each term in the expansion series needs to be solved from Kolmogorov forward and backward equations. To obtain an analytic expression for these terms, we follow Aït-Sahalia's procedure for approximating the likelihood

[24] In that case, econometricians usually consider a dynamic system with both state equations and observation equations. Under this estimation scheme, asset prices are involved in developing the observation equations, and estimates of preference parameters can also be obtained.

[25] Participants in Blue Chip surveys are asked to provide forecasts of *seasonally adjusted*, and annualized growth rates of real GDP.

function two-dimensionally: in the time interval $\Delta = 1/4$ and in the state variable $Y_{t+\Delta} - Y_t$.

The estimation results are presented in Table 3. A most remarkable aspect of parameter estimates pertains the persistence of the ambiguity level. The mean-reversion rate is estimated at 0.30, corresponding a mean return time of more than 3 years. The unconditional expectation of ambiguity $\bar{A}$ is about 1.11%. This quantity, when combined with estimates of σ_c, σ_o and σ_{oc}, implies that the proposed measure $\widetilde{A}_t$ has a long-run mean of 2.09%, which matches almost exactly the average value observed from the survey data. As expected, innovations in aggregate consumption and in the ambiguity level are negatively correlated.

The preference parameters reported in Table 3 are chosen to take into account the empirical evidence and economic considerations. The time discount factor β in the stochastic differential utility is usually calibrated at somewhere between 0.01 and 0.02.[26] Taking an intermediate value in this range, we fix β at 0.015. We further assume that the RRA coefficient γ is equal to 10, as do Bansal and Yaron (2004). The question of whether the magnitude of the EIS is greater than one is controversial. We set its value to Bansal and Shaliastovich (2013)'s estimate of 1.81, which they derive from bond market data as well as survey forecasts. Jeong et al. (2015), who address the issue of the sensitivity of this estimate to the inclusion of ambiguity aversion, estimate EIS based on the utility structure that we employ in our study. Jeong et al. (2015)'s estimate, too, is generally higher than one, but it is accompanied with large standard errors.

4.2 The Credit Spread Puzzle

In a step towards developing a unified understanding of how time-varying ambiguity affects the pricing of equities and corporate bonds, this section examines the model's implications for the levels and dynamics of credit spreads. As discussed by Chen et al. (2009), this exercise can be viewed as "an out-of-sample test" of potential solutions to the equity premium puzzle. Using calibration methodologies commonly employed in the literature, we choose firm-level parameters to match the average solvency ratios of firms in different rating categories. However, in this calibration experiment, historical default rates are not targets towards which we calibrate the model; instead they serve as criteria used to assess the model performance. This procedure to generate *endogenous* default probabilities is consistent with Bhamra et al. (2010) and Chen (2010).

[26] It is calibrated at 0.01 by Bhamra et al. (2010), at 0.015 by Chen (2010), at 0.0176 by Benzoni et al. (2011) and at 0.02 by Wachter (2013).

4.2.1 Firm-Level Calibration

A key variable in the valuation of default claims is the cost of default ϕ. Given that the default timing and default boundary are endogenous in our model, the default loss, too, should be modeled to allow for covariation with the state of the economy. As would be illustrated in Section 4.2.3, our model indicates that the default timing tends to be positively correlated with the degree of ambiguity. It follows that financial distress would be enormously costly when prospects for economic growth are highly unclear (Shleifer and Vishny, 1992). Given this insight, we parameterize the default cost as a linear function of the ambiguity level

$$\phi_t = b_0 + b_1 A_t.$$

To obtain empirical estimates of b_0 and b_1, we follow Davydenko, Strebulaev, and Zhao (2012), who estimate ϕ from the market values of defaulted firms. To simplify their estimation procedure, we slightly modify two of its steps. Firstly, the market prices of bank loans are estimated as a linear-quadratic function of bond prices, as described in their appendix. Davydenko et al. (2012, DSW hereafter) find that this specification can explain more than 75% of variations in bank debt prices available in their dataset. Secondly, we assume that the jump-to-default risk premium, defined as the ratio of risk-neutral intensity to actual default intensity, is constant over time. Its value is set at 2.31, which is the empirical estimate reported by Driessen (2005).[27] DSW find that their estimates are fairly insensitive to the assumption of this risk premium factor, whether it is specified as time-varying or fixed. Our estimates indicate that the mean cost for all corporate defaults in our sample is 24.7%, consistent with DSW's result. By regressing these estimates on the ambiguity measure, we obtain the following results

$$\begin{array}{ccccc} \phi_{j,t} = 0.113 & + & 6.294 & \widetilde{A}_t. \\ (6.61) & & (5.28) & \end{array}$$

The effect of ambiguity on the default cost is statistically significant and economically substantial. The slope coefficient implies that a one-percent increase in A_t pushes up the proportional cost by 11.8%, which constitutes our estimate of b_1.

Table 4 summarizes several calibrated firm-level parameters in addition to b_0 and b_1. Parameters on tax rates are set to be consistent with Chen (2010)'s calibration, as reported in Panel A. Panel B shows calibration targets that enable the model to generate differential default rates across credit

[27] Similar results are presented in Berndt et al. (2005), who estimate the jump-risk premium from the data on credit default swaps.

ratings. Like Huang and Huang (2012), we study the following ratings by Moody's: Aaa, Aa, A, Baa, Ba, and B. In most studies in the literature, the leverage ratio is used as the proxy for firms' financial health. In a departure from these studies, our model defines default time in terms of a firms cash flow rather than its value. For this reason, we directly calibrate the debt structure to earning-based financial metrics; that is, we match the initial proportion of F to δ_0 to the historical debt-earning ratio for each rating category — a practice advocated by Chen (2010), who argues for the use of non-market-based financial ratios in testing the implications of structural models. Employing the same principle, we calibrate the parameter σ_j such that the model matches firms earning volatilities rather than their asset volatilities.

The distribution of solvency ratios across rating groups is retrieved from the special report of Moody's Financial Metrics. It indicates that these ratios, besides the conventional leverage ratio, do receive serious consideration in rating agencies' analytical process. Indeed, it is emphasized in the report that "when Moody's does analyze financial ratios, it uses a multivariate approach." The quantities in the second row of Panel B are the median debt-EBITDA ratios, and are strictly monotonic with ratings. In the third row, the volatility of net revenue is used as a proxy for the earning volatility. An implicit assumption underlying this practice is that the EBITDA margin is constant and thus the revenue and EBITDA have the same growth rate. The level of risk shows a weak relationship to ratings. For instance, the idiosyncratic volatility for Aaa issuers is 9.7%, higher than the 8.5% for Aa issuers.

With values of F and σ_j determined for each credit rating, we solve for the total coupon payment C by following Leland (1994a)'s procedure. That is, the coupon is set so that the market price of newly issued debt D equals its principal value mF. As an illustration, consider Aaa-rated firms whose historical debt-to-EBITDA ratio is 0.90. When the value of corporate earning starts at $\delta_0 = 100$, the face value of debt equals 90. Depending on the current level of ambiguity, the coupon payment $\bar{C}$ making the debt priced at par varies. For example, $\bar{C} = 5.61$ if A_0 is at its historical mean. To determine the unconditional expectation of model-generated credit spreads, we follow Chen et al. (2009) by calculating the following conditional spreads for selected grids on A_0

$$\bar{C}(A_0)\left(\frac{1}{F} - \frac{1}{(\bar{C}(A_0) + mF)\mathscr{P}\mathscr{V}^{r+m}(A_0)}\right).$$

Thus, credit spreads are computed based on a nondefaultable bond with the same coupon rate and interest frequency. The reported model spread for each rating group is the population average over the steady-state distribution of A_t.

4.2.2 Default Rates and Credit Spreads

One important empirical consideration in studying the credit spread puzzle is what kind of measure for history spread levels should be used for comparison. The standard practice in the literature is to set the firm's leverage so that it is equal to the average leverage of firms in a particular rating class, and then compare model-implied spreads with (weighted) average actual spread over a period. However, David (2008) and Bhamra et al. (2010) are critical of this "averages-to-averages" comparison, as it is subject to the convexity bias in credit spreads.[28] In this paper, we adopt a "medians-to-medians" strategy: first, the model is calibrated to the median financial ratios for a given rating group, as listed in Table 4; second, the model's prediction is compared with the median spread across all bonds in that rating category. Besides avoiding the convexity issue, the use of medians makes our calibration robust to the presence of potential outliers.

To obtain a long-span bond sample covering the entire 1985-2010 period, we consolidate corporate bond data from Lehman Brothers and Merrill Lynch databases: for the 1985-1997 period, we extract all quotes (matrix prices are discarded) on noncallable U.S. bonds from Lehman Brothers Fixed Income Database, whose availability is restricted after 1998; thenceforth we switch to Merrill Lynch as the data source for bid quotes.[29] To be included into the sample, a bond must belong to a Lehman (now Barclays) or Merrill Lynch (now BofA ML) index and have at least 24 *actual* monthly quotes during a 7-15 year maturity period. To construct credit spreads, we follow the market convention to use the nearest-maturity Treasury bond as the benchmark. Monthly yields of individual Treasuries are obtained from CRSP Monthly Bonds Master file.

Table 5 reports the median of the expected 10-year credit spread for individual bonds. We find that for each rating group the average spread for a typical bond is smaller than the average spread of the corresponding Barclays corporate bond index. Note that the latter is used by Huang and Huang (2012) as a reference for comparison with model-predicted 10-year spreads. Three factors might account for this spread discrepancy. First, the distribution of credit spread is known to be right-skewed, and so the mean is likely to be higher than the median. Second, the bond pool underlying these indices includes callable bonds, which are known to have generally higher credit spreads than noncallable ones (Caouette et al., 1998). Third, our bond sample contains only bonds

[28] Huang and Huang (2012) and Chen et al. (2009) find that if structure models are calibrated to the historical default loss experience, the convexity effect is small and makes the historical spread even more of a puzzle.

[29] The reason for retrieving bid prices instead of mid price is that the Warga Database contains only bid prices. Both data sources are widely used in the corporate bond literature (Collin-Dufresne et al., 2001; Elton et al., 2001; Eom et al., 2004; Chen et al., 2007; Schaefer and Strebulaev, 2008). We examine their comparability in term of data quality and find a close match in their prices between December 1996 and December 1997, the period when these two data sets overlap.

close to 10-year maturity, but the indices are based on a wide array of bonds with diverse maturities (their average time to maturity is 10.45 years). Table 5 indicates that this discrepancy is not due to the switch to Merrill Lynch as the data source; it has the same magnitude in the subsample before 1998. The last panel shows that over the second subsample period the average spread for a typical noncallable bond is close to the average level of Merrill Lynch U.S. Corporate/High Yield 7-10Y Option-Adjusted Spreads.

Table 6 compares model-generated default probabilities and credit spreads to their empirical counterparts. In contrast to many existing calibration studies, wherein attention is confined to investment-grade bonds, our goal is to match the targets for both investment-grade and speculative-grade debt. As Huang and Huang (2012) observe, when explaining the credit spread puzzle, it is important to ensure that the proposed models do not over-predict credit spreads on high-yield bonds. McQuade (2012), who considers model implications for all rating categories, makes the same point.

The second column in Table 6 reports the average cumulative default probability over the ten-year horizon for issuers of each initial credit ratings. These statistics are based on historical default experiences from 1983 to 2010 (Ou et al., 2011), which closely match our sample period. As shown in the third column, the model captures well the historical default rates for most rating categories, although it underpredicts the default probability for Aaa-rated issuers. Given that the latter rarely default, this shortcoming might best be attributed to a potentially large measurement error. In untabulated results, we find significant variations in the model-implied default rates, which increase uniformly with the initial level of ambiguity. Taking Baa bonds as an example, the ten-year forward probability of default is 3.7% if $A_0 = 0.5\%$, while it rises to 6.6% if $A_0 = 1.8\%$.

While matching the historical default rates indicates how well the model quantifies the credit risk under the physical measure, the model-generated credit spreads assess the model performance under the risk-neutral measure. The results are reported in the right panel of Table 6. It is well-known that non-credit factors, such as bond liquidity, account for a sizeable portion of yield spreads, especially for investment-grade bonds. Thus, in testing the model's predictions, we correct the target spreads for the fractional liquidity components estimated by Longstaff, Mithal, and Neis (2005), and by Chen, Cui, He, and Milbradt (2013). For example, the total yield for a typical Baa bond is about 182 basis points, as reported in Table 6; the estimate by Longstaff et al. (2005) of Baa bonds' liquidity fraction is 29%. The credit component should account for $182 \times 0.71 \approx 129$ basis points, as shown in the column labeled "Target_1". As expected, the liquidity fraction of total spread becomes smaller as the rating quality of the bond decreases, while the absolute magnitude of the liquidity component decreases with the credit rating.

We find that the model successfully captures the level of spreads for both investment-grade bonds and high-yield bonds. Our result indicates that we do not force the model to match the historical spreads on high-quality bonds by overstating the amount of compensation demanded for per unit of default risk. In other words, this model improves the pricing of corporate debt by producing a higher level of uncertainty premium and by proposing a better structure for the market price of uncertainty. To underline the effect of time-varying ambiguity, we also tabulate the credit spreads generated by a simplified model with $A_t \equiv \bar{A}$. The second column from the right indicates that the simplified model substantially underperforms the model with time-varying ambiguity, especially for investment-grade bonds. Indeed, the simplified model shows fairly limited improvement over the baseline model where the agent is ambiguity neutral. This finding is consistent with Chen et al. (2009), who conclude that time-varying asset Sharpe ratios are essential for explaining the credit spread puzzle.

4.2.3 Theoretical Insights and Empirical Verification

However, time-varying Sharpe ratios alone do not suffice to account for the high level of historical spreads — a point illustrated by the baseline model of Chen et al. (2009), where the default boundary is constant. To explain the credit spread puzzle, Sharpe ratios need to show a strong correlation with the default time. In our model, it is accomplished by the optimal default boundary δ^* that increases with the level of ambiguity. This positive covariation in turn drives up the risk-neutral default probability far beyond its counterpart under the physical measure.

To demonstrate the effect of time-varying ambiguity on the firm's default decision, in Figure 3 we plot δ^* as a function of A_t. We focus on a typical Baa-rated firm with the current earning level normalized to 100. The figure shows that the default boundary is higher when the outlook for economic growth is more unclear than usual. In other words, the management is more likely to exercise their default option earlier in the presence of a high degree of ambiguity. While Knightian uncertainty has little impact on total cash flows to the firm, it generates the co-movement between the default probability and market prices of uncertainty.

Note that the ambiguity level also determines the asset valuation ratios. Numerically, its negative effect on the price-earning ratio L^o dominates the positive effect on the earning-based default boundary. Thus, the asset value at the time of default U^* is negatively related to A_t, as indicated by the red dashed line in Figure 3.[30] The direction in which the default boundary moves

[30]This countermovement of default boundaries based on the cash flows and on the asset value is also documented

actually constitutes an important empirical basis used in model validation and verification. For instance, Davydenko (2012) finds that the value-based boundary is procyclical; this result supports the prediction of Chen (2010) and Bhamra et al. (2010).

With the same methodology, we test the model implications by examining the relationship between observed boundary levels and the degree of ambiguity in a cross-sectional regression setting. Asset values at default are estimated based on observed market prices of debt and equity, as well as empirical estimates of the risk-neutral default probability. In this study, default boundaries are obtained as a byproduct of the estimation of the default cost. Following the convention in structural modeling, we express default boundary as a fraction of the face value of outstanding debt. We find that on average the location of the default boundary is measured at 68% of the book value of debt, a value very close to the 66% reported by Reisz and Perlich (2007) and by Davydenko (2012).

Table 7 shows the results of regressions on the ambiguity measure and other theoretical determinants. To correct for selection bias, we use Heckman (1976, 1979)'s procedure of two-stage regressions. In the table, the inverse Mills ratio term turns out statistically insignificant in all nine regression models listed. Therefore, the null of no selection bias cannot be rejected, and t-statistics are computed based on unadjusted standard errors. The first set of regression indicates that a high degree of ambiguity lowers the level of value-based boundary, which is consistent with the model's implication. The ambiguity factor alone explains 4.9% of variations in observed boundaries, which is substantial compared to the R^2 values reported by Davydenko. This effect is significant even after controlling for other determinants suggested by structural models. Moreover, the ambiguity measure seems to "crowd out" the significance of GDP growth rate, which is used to proxy for macroeconomic conditions. This result implies that the ambiguity level contains relevant information on corporate default behaviors above and beyond that contained in the business cycle.

4.3 The Levered Equity Premium and Risk-Free Rate

Corporate bonds and equities issued by the same firm are different contingent claims on the same cash flows. Given its implications for bond pricing, this section examines our model's ability to match some key statistics of historical equity returns and moments of the risk-free rate. In explaining these empirical regularities, we focus on the levered equity as a residual claim to earnings. Our approach departs from most studies on equilibrium equity premium by introducing the default risk dimension. As emphasized by Toft and Prucyk (1997) and Bhamra et al. (2010), this approach preserves the direct link between the pricing of corporate securities and the firm-level decisions. In

by Hackbarth et al. (2006), Chen (2010) and Bhamra et al. (2010).

contrast to exogenously specified dividend dynamics, it can generate a set of consistent processes for the levered equity and derivatives that are based on it.

Table 8 compares the model-implied moments of equity returns to the data. The "Portfolio" column reports real equity returns constructed using the CRSP value-weighted index. While the statistics are computed from the data for the 1985-2010 period, they appear fully consistent with the numbers for the entire post-war sample.[31] Accordingly, model-implied moments are computed based on the unlevered and levered claims to the aggregate output O_t.[32] For the levered equity, the initial ratio of debt to cash flow is calibrated at 2.61, which is the median value for the Baa credit rating. This calibration responds to the finding of Avramov et al. (2007) that the average rating of 3,578 public firms rated by Standard & Poor's is approximately BBB.

As documented in Bhamra et al. (2010), the uncertainty premium on the levered equity (5.9%) is higher than that of the unlevered equity (4.1%), because the former is on a residual claim of corporate cash flows after the interest expenses and taxes are deducted. The levered equity premium matches the historical mean of excess stock returns.

Table 8 also presents the implications of a κ-ignorance model. Columns under the name "Constant Ambiguity" show that the time-invariant ambiguity results in significantly lower uncertainty premia on both unlevered and levered equities. To understand the contribution of temporal variations in A_t, recall the unlevered equity premium discussed in Section 3.4. Leaving all correlation terms aside, Eq. (32) indicates that it is the sum of three conceptually different components. In the κ-ignorance case, the first and third components—the standard risk exposure and the pure ambiguity premium—are retained. Numerically, however, they are subordinate to the second component, which is caused by the uncertainty associated with time-variations in the ambiguity level, or more fundamentally, by the interaction of learning and ambiguity aversion. This is reflected in the 1.1% unlevered equity premium with time-invariant ambiguity, against the 4.1% with time-varying ambiguity. This result is consistent with the findings of Leippold et al. (2008) that the equity premium without learning "is too small for practical purposes".

The model can also generate excess equity volatility. The model-implied volatility for levered equity returns is 12.4% per annum. While this is a bit below that found in the data, it falls within two standard errors of the empirically estimated volatility.[33] As A_t follows a CIR process,

[31] For example, the excess returns are averaged out at 6.1 percent per annum, very close to the 6.2 percent estimated over the sample from 1947 to 2001 (Piazzesi et al., 2007).

[32] To obtain the model's implications for the levered equity, we run 1,000 simulations based on the Euler discretization. Each simulation generates 26 years' daily observations. Simulating the model at lower frequencies makes the chance of getting negative values for A_t higher than 0.001.

[33] The standard error of equity volatility is estimated at 2.78%. It is Newey-West adjusted with the lag length equal

its conditional standard deviation rises with the square root of its level. Consequently, equity volatility is a non-decreasing and concave function of the ambiguity level. Given that equity price is non-increasing in A_t, the model produces "the leverage effect" (Black, 1976).[34] In contrast, the model with invariant ambiguity predicts that the equity volatility is constant and equal to the volatility of dividends.

The exercise just described could raises questions about the extent to which the aggregate stock index is representative of a hypothetic Baa-level firm that is only exposed to systematic shocks. In response to this concern, we calibrate the model using the method described in Section 4.2.1—the firm's earnings follow Eq. (18) with the total volatility calibrated to 12.16%, which is the median volatility for Baa firms. These calibration results are then compared with the statistics for a typical Baa-rated firm (the median values among all Baa firms).[35] As indicated by the third column in Panel A, a typical Baa firm has both a higher mean excess return and a higher return volatility than the market index.

The last column shows the model's implications for an individual Baa firm. Intuitively, the inclusion of idiosyncratic cash flow risk should increase the default risk embedded in levered equity, which is confirmed by the increased equity premium and volatility implied by the model. These two quantities are largely consistent with their data counterparts listed in the third column. Note that in the presence of idiosyncratic risk, the ambiguity level plays a limited role in determining the condition volatility of equity returns — a pattern that is consistent with empirical evidence that the volatility asymmetry is generally stronger for aggregate market index returns than for individual stocks (Kim and Kon, 1994; Tauchen et al., 1996; Andersen et al., 2001). Moreover, the model reproduces the stylized fact that individual firm volatility is approximately twice the level of market volatility.[36]

As demonstrated in Chen et al. (2009), a key statistics to be matched in explaining the credit spread puzzle is the average Sharpe ratio for a typical firm. In a departure from their procedures, where the firm-level Sharpe ratio is an explicit calibration target, we calibrate our model to cor-

to 6 quarters.

[34]Here we slightly abuse this terminology. The term leverage effect reflects the fact that Black (1976) attributes the asymmetric return-volatility relationship to changes in financial leverage. In our model this negative correlation applies to unlevered equity, too, although we find that levered equity volatility increases with leverage (i.e., it decreases with credit rating). In other words, the volatility asymmetry implied by our model captures the volatility feedback effect rather than the leverage effect. Numerically we can confirm that the sensitivity of price-dividend ratio to ambiguity, ξ_1, increases with volatility parameter σ_a.

[35]The median is obtained from a July 1985 to December 2010 sample of BBB-rated stocks listed in the CRSP. The sample's beginning is determined by the first date for which credit ratings by S&P are available on the COMPUSTAT database. To be included into the sample, the stock must have at least twelve consecutive monthly return observations.

[36]See, for example, Section 1 in Chen et al. (2009).

porate earning volatility, which endogenously produces Sharpe ratios for individual firms. Based on equity returns, the median Sharpe ratio of Baa-rated firms is about 0.28, which is remarkably consistent with the 0.29 predicted by the model. If we consider only the systematic component in a Baa firm's cash flow, the model's prediction is roughly comparable to the Sharpe ratio for the market portfolio. However, when the ambiguity level is further restricted to a constant the model-implied Sharpe ratio jumps to about 0.79. This reveals that the restricted model delivers unrealistically high returns (on levered equity) per unit of standard deviation.

The model-generated mean of the risk-free rate, 1.18%, is almost identical to the historical counterpart but a degenerate set of priors implies a significantly higher level. As shown in Eq. (15), the restriction $A_t \equiv \bar{A}$ lessens the impact of Knightian uncertainty, which therefore works only through the misspecification doubts about the expected consumption growth. In other words, the κ-ignorance model preserves the first term in ϱ_1. In equilirium, this drift perturbation term decreases r_t by merely $\bar{A}/\psi = 0.61\%$, or not enough to account for the risk-free rate puzzle. Therefore, the second term $(\theta - 1)\eta_1^2\sigma_a^2/2$, which is missing in the constant-ambiguity case, is necessary to capture the low mean of historical interest rates. That term symbolizes the precautionary saving motive caused by unknown future levels of ambiguity, and it leads to another 0.48% reduction in the unconditional expectation of r_t.[37] Hence, time-varying ambiguity is crucial for the model to match the empirical moments of risk-free rates.

4.4 Dynamics of Asset Prices

While several extant preference-based models capture levels of equity premium and credit spreads, their ability to match the historical behavior of asset prices has not been fully demonstrated. In this section, we review some stylized facts about the dynamics of equity returns and extend them to the corporate debt market. Section 4.4.1 investigates the forecasting power of the price-dividend ratio: using this ratio equity returns and credit spreads are predictable but consumption and dividend growth are not. The weak correlation between asset markets and macroeconomics, as discussed in Section 4.4.2, poses another serious challenge. We find that our model can reproduce these interesting findings and, more importantly, it accounts of historical fluctuations in the price-dividend ratios and credit spreads over the last 26 years.

[37] Note that allowing for serial covariations between c_t and A_t results in another 0.05% decrease in ϱ_1, which partially accounts for the gap between risk-free rates generated by the models with and without time-varying ambiguity. This cross-shock effect is reflected by the last term in Eq. (36b).

4.4.1 Predictive Regressions

Panel A in Table 9 presents long-horizon regressions of excess stock returns on the log price-dividend ratio in simulated and historical data. Our sample spans only a 26-year period, and consequently our predictive regressions are run at a quarterly frequency. Mindful of issues arising from the use of overlapping observations, we compute Hodrick (1992) standard errors to remove the moving average structure in the error terms.[38] Like Campbell and Shiller (1988) and Fama and French (1988), we find that high equity prices imply low expected returns; both the absolute values of slope coefficients and the R^2 increase with the return horizon.

The last four columns show that the model captures the negative relationship between equity premium and price-dividend ratio: high equity prices relative to dividends imply low ambiguity levels, and therefore predict low future expected returns on stocks in excess of the risk-free rate. For unlevered equity, the model-implied predictability appears too strong compared to that suggested by the data; bear in mind that the log P/D ratio and equity premium are perfectly correlated. Introducing default risk into the equity produces more realistic regression statistics. At the 5-year horizon, the model-implied coefficient and R^2 are almost identical to those in the data.

The price-dividend ratio's link with the ambiguity level also connects it to credit spreads. Given that ambiguity follows a mean-reverting process, we may expect some long-horizon predictability of credit spreads. This is confirmed by results in Panel B: over the long run, high price-dividend ratios forecast high credit spreads for the Baa bond index; at the one-year horizon, the P/D ratio is not a powerful predictor.[39] Regressions with the simulated data display a similar pattern. For an individual Baa firm, the model-generated R^2s are substantially lower than their empirical counterparts because of the presence of idiosyncratic volatility. If we consider only systematic variations in the corporate cash flow, the model-implied predictability is aligned with that observed from market data, as shown in the last two columns.

The historical variation in price-dividend ratios could also be driven by changes in expected dividends growth, in addition to movements in equity premium. However, previous studies (Campbell, 1999; Lettau and Ludvigson, 2001) find that the P/D ratio is not useful for forecasting either

[38]Based on Hodrick (1992)'s t-statistics, the predictive ability of price-dividend ratio is marginally significant at a short horizon of one year, but it disappears for longer horizons. We emphasize that our focus is not on testing the predictability of stock returns. Even if we ignore the issues associated with statistical inference for the long-horizon predictive regressions, existing evidence for return predictability is sensitive to changing samples (Goyal and Welch, 2003; Ang and Bekaert, 2007). Instead, the question examined here is whether the model can reproduce the seeming forecasting power of price-dividend ratio.

[39]Shorter-horizon results, not reported in the table, suggest a negative relationship, which is consistent with the persistence in the ambiguity evaluation. But the predictability is not significant either.

consumption growth or dividend growth. In Panel C, we report the model-implied predictability in dividend growth to assess its consistency with the empirical results. The estimated slope coefficients for our sample are positive and somewhat higher than those reported by Beeler and Campbell (2012), who use quarterly data from 1947Q2 to 2008Q4, but they are still insignificant at any horizon. For both unlevered and levered equity the average coefficients implied from the simulated samples are negative[40]; however, they are all within one standard error of corresponding empirical estimates. At the same time, the model-implied 90% confidence bands for regression coefficients and R^2 contain the data counterparts.

According to (18), there is zero predictability in the firm's cash flow growth. In our simulation exercise, the P/D ratio exhibits a weak negative correlation with the future growth rate in the long run due to the correlation term $\sigma_{oa}\sigma_a\sqrt{A_t}dB_{A,t}$. For the unlevered equity, its P/D ratio is linear in the ambiguity level. So when it is currently high, the ambiguity level is likely to bounce back to its long-run mean in the future. This tendency will negatively affect output/dividend growth, especially in the long run. For the levered equity, this negative correlation is further weakened by the nonlinearity in the relationship between the P/D ratio and ambiguity level.

For comparative purposes, Panel D provides regression results where consumption growth is used as the dependent variable, as this regression specification is also estimated in Bansal and Yaron (2004) and Wachter (2013). Again, both the data and the model suggest that price-dividend ratios do not have any statistical or economic significance in predicting growth rates. The model-implied average slope coefficients match the data well. While the empirical R^2s are slightly lower than the model's predictions, they still fall into the 90% confidence band. Overall, the model accounts for the absence of predictability in dividend & consumption growth.

4.4.2 Correlations

Cochrane and Hansen (1992) and Campbell and Cochrane (1999) point out that the weak correlation between consumption growth and stock returns in the U.S. data is inconsistent with the implication of many consumption-based models. For models solely driven by shocks to consumption growth, these two series should be highly, if not perfectly, correlated. Similarly, if we simply embed a Merton model inside these equilibrium models, we would expect changes in credit spreads to have a strong but negative correlation with consumption growth. But empirically we find that this correlation is as low as it is in the case of equity returns. These low correlations tend to

[40]Note that positive slope coefficient is not a permanent feature of predictive regressions of dividend growth. For example, negative but insignificant coefficients are documented by Campbell (1999), whose sample covers the 1947Q2 - 1996Q3 period.

foil economists' attempt to link the stock and credit markets with the macroeconomics. However, adding dynamics to the agent's ambiguity aversion allows for new insights to understand the lack of co-movement in these time series.

Panel A in Table 10 shows the correlation of consumption growth with market returns in actual and simulated data. At a quarterly horizon, the contemporaneous correlation is measured at 0.10, close to the 0.12 reported in Campbell and Cochrane (1999). Equity returns are negatively correlated with previous consumption growth and positively correlated with subsequent consumption growth. To account for the weak correlation, exogenous shocks to the economy, besides consumption shocks, are necessary. With innovations in the ambiguity level, the model implies a mean correlation of 0.22. The simulated data suggests little correlation of returns with previous and future consumption, but its 90% confidence intervals embrace the empirical correlation coefficients for all leads and lags.

When studying consumption's correlation with credit spreads, we examine the spreads for both investment-grade and high-yield bonds. To disentangle variations in the credit components of yield spreads, Panel B focuses on the results of the lowest investment-grade rating, Baa, and the highest speculative-grade rating, Ba. In the data, there is a negative but weak correlation between spread changes and economic fundamentals. In our model, changes in credit spreads result only from uncertainty in ambiguity innovations, and as such they should be orthogonal to consumption growth without the correlation term. Incorporating the empirical co-movement between the consumption and ambiguity processes leads to correlation coefficients of -0.20 and -0.25. Consistent with the data, the model implies a greater magnitude of correlation for high-yield bonds.

4.4.3 Historical Variations

An important advantage of our model is that it enables us to identify the time series of financial variables based on the empirical measure for ambiguity and the calibrated model. A comparison of these time series with their historical counterparts provides an intuitive way to evaluate the model's performance. Figure 4 presents the model's prediction for the price-dividend ratio of levered equity, and the actual price-dividend ratio on the CRSP value-weighted index. It is unclear how well the index represents a levered claim to aggregate output; thus attention must be paid to the historical fluctuations in (rather than the level of) the model-implied ratios. We conclude that the model provides an accurate account of serial variation in the P/D ratio: it can reproduce the countercyclical behavior of stock prices, as documented in Campbell and Shiller (1988) and Fama and French (1989), and it captures shorter-term fluctuations within each business cycle. This high-frequency covariation incarnates an important model implication: changes in the ambiguity level can lead to significant responses of asset prices, even without shocks to the economic fundamentals. Indeed, the correlation between model-predicted and historical P/D ratios is 75%, which is much higher than the number reported by Chen et al. (2009) (21%), who studied a different and shorter sample period.

As Figure 5 suggests, the model is particularly useful for fitting the dynamics of historical credit spreads. The ambiguity-implied Baa spread is depicted as the red dotted line. The blue and green lines correspond to Barclays aggregate Baa bond spread and Merrill Lynch BBB 7-10Y Option-Adjusted Spread, respectively. Again, the model's prediction captures the countercyclical pattern of historical spreads, and it exhibits similar dynamics *within* each business cycle. Indeed, its correlations with both yield spread indices are higher than 82%. As a benchmark, the corresponding correlation coefficients generated by Chen et al. (2009)'s and Buraschi et al. (2013)'s models are 72% and 41%, respectively.[41] In Figure 5, the largest discrepancies between the observed and model spreads occur around financial crises. This result is to be expected given that the liquidity shortage tends to play an important role in these episodes, which are not modeled in this paper. In summary, whereas the evidence presented in Figure 1 is merely suggestive of the role of ambiguity in explaining the credit spread puzzle, that provided in Figure 5 is considerably more conclusive.

5 Conclusion

How do changes in aggregate uncertainty affect asset prices? In many established preference-based models, the representative agent knows the objective probability law precisely. Under this assumption, a change in uncertainty is typically described as an anticipated change in the quantity of consumption risk or in the degree of risk aversion. In this paper, we depart from this assumption by considering a continuous-time Lucas economy where the agent cannot estimate probabilities reliably and has an aversion to the resulting ambiguity. In our model, an increase in uncertainty causes the agent to feel even less confident in assigning probabilities.

This paper proposes a tractable way to incorporate these changes in Knightian uncertainty into an equilibrium asset pricing model. With the max-min expected utility, the agent cares about the span of all subjective priors (probability measures), and sets much higher prices for state-contingent claims that pay off in concurrence with a great width of the span. In this setting, time-varying

[41] The Baa-Aaa yield spread predicted by David (2008)'s model achieves a 80% correlation with its historical counterpart. However, this result is based on a unique calibration method: the model is fed with the time series of a market-based leverage measure — the market value of assets divided by the book value of debt.

ambiguity stands for a nondegenerate set of priors that capture the difficulty experienced by the agent in assessing economic growth. With this interpretation, a negative shock to her confidence in probability assessment leads to a large reaction in the marginal rate of substitution, even without news about the fundamentals.

Methodologically, time-varying ambiguity is important in that it exerts both the first-order and second-order effects on asset premia. The first-order effect is directly bound to the difference between the true consumption growth and the current worst-case belief used by the agent to evaluate assets; the second-order effect reflects the agent's desire to hedge against permanent shocks to the degree of ambiguity. Both channels are essential to generate the large premia embedded in equity and corporate bond prices and to imply reasonable values for the mean and volatility of the government bill rate. These theoretical implications are supported by the empirical evidence for the predictability of realized asset premia produced by our novel measure of the ambiguity level. Also consistent with the models predictions is the fact that the historical corporate default barrier varies in response to changes in the amount of ambiguity.

Our ambiguity- based model provides a better fit with historical variations in stock prices and credit spreads than do other existing frameworks that price equity and corporate debt jointly. Our results suggest that economists should look beyond the business cycle when characterizing the dynamics of asset prices. In the models of Chen (2010) and Bhamra et al. (2010), the fit of the . With this interpretation, a negative shock to her confidence in probability assessment leads to a large reaction in the marginal rate of substitution, even in the absence of news about the fundamentals. Methodologically, time-varying ambiguity is important in that it exerts both first-order and second-order effects on asset premia. The first-order effect is directly bound to the difference between the true consumption growth and the current worst-case belief used by the agent to evaluate assets. The second-order effect reflects the agents desire to hedge against permanent shocks to the degree of ambiguity. Both channels are needed to generate the large premia embedded in equity and corporate bond prices and to imply reasonable values for the mean and volatility of the government bill rate. These theoretical implications are supported by the empirical evidence for the predictability of realized asset premia produced by our novel measure of the ambiguity level. Also consistent with the models predictions is the fact that the historical corporate default barrier varies in response to changes in the amount of ambiguity. Our ambiguity- based model provides a better fit with historical variations in stock prices and credit spreads than do other existing frameworks that price equity and corporate debt jointly. Our results suggest that economists should look beyond the business cycle when characterizing the dynamics of asset prices. In the framework of

Chen (2010) and Bhamra et al. (2010), the model's fit to the historical yield spreads is sacrificed through a discretization of the expected consumption growth and consumption volatility. In this manner they are able to deliver closed-form solutions to equity and debt prices with *dynamic* capital structure. Given this insight, an extension of our model to allow for endogenous capital structure presents an interesting topic for future research. It would be also useful to study how change in the ambiguity level affects firms investment decisions.

A Ambiguity about the Real GDP Growth

A.1 Details of the Data Set

Blue Chip Financial Forecasts provide an extensive panel of data on expectations by professional economists from leading financial institutions and service companies. Each economist is asked to forecast a large set of macroeconomic and financial variables. Our ambiguity measure draws on the individual-level forecasts of real Gross National Product (GNP). The name of this variable was changed to real Gross Domestic Product (GDP) after January, 1992.

Besides the Blue Chip survey, the Survey of Professional Forecasters (SPF) is another widely used source of survey data.[42] For the objective of this study, the Blue Chip's data appears to bear greater relevance, for two reasons.

First, as shown in Table A1, the Blue Chip keeps the number of respondents fairly stable over the last thirty years, which is always within the interval between 44 and 50. By contrast, the number of forecasters participating in the SPF varies enormously, from 9 to 53. This feature is undesirable at least in the multiple-priors framework, because changes in the subjective set of priors should not be a mere reflection of rapid turnover of survey participants. Instead, they are supposed to mirror updates of beliefs.

Second, Table A1 indicates that the composition of the Blue Chip's forecaster panel is maintained almost constant over time. On average, about 94% of analysts appearing in the current period's publication also participate in the survey of the preceding period. Instead, it is not unusual that a batch of respondents suddenly drops out of the SPF for a large number of periods, and suddenly re-enters.[43] If the forecast is more closely associated with the individual participant, the highly fluctuant constitution makes the panel unsuitable for our extraction of the set of priors.

A.2 Alternative Measures of Ambiguity

In providing empirical support for a link between ambiguity and variance premium, Drechsler (2013) proxies the size of ambiguity by the cross-sectional standard deviation in growth forecasts. By the same spirit, Ulrich (2013) uses the variance of inflation forecasts to quantify the ambiguity about the inflation dynamics. These empirical measures are conceptually consistent with the way they construct the set of alternative beliefs. Specifically, they set a upper bound for the (growth rate of) relative entropy[44] which, in this paper's setting, is equivalent to a upper bound for the norm

[42] Livingston Survey also provides economists' forecasts of real GDP growth rate, but at a semiannual frequency.

[43] The problems associated with changing composition of the SPF panel are discussed exhaustively in Engelberg et al. (2011).

[44] It is the Kullback-Leibler distance between an alternative probability measure and the reference measure.

of the density generator (ϑ_t). This approach to restricting the amount of ambiguity is adopted in Hansen and Sargent (2008)'s robust control framework.

With multiple-priors preferences the belief set is restricted by a *rectangular* set of density generators. This rectangularity inspires our use of cross-sectional range, rather than standard deviation, as the ambiguity measure. As discussed in Epstein and Schneider (2003), which type of restriction to be imposed in ambiguity modeling "will typically depend on the application". Empirically, nevertheless, we can still draw a comparison between these two cross-sectional measures of growth ambiguity to see which one possesses greater explanatory power for asset premia.

Panel A of Table A2 assesses the predictive performance of standard-deviation-based measure. The in-sample results indicate that it has significant forecasting power for credit spreads, to a degree similar to that of our measure $\widetilde{A}_t$. However, it loses the statistical significance when used to predict the excess returns on equity and Treasury bonds. In contrast, $\widetilde{A}_t$ is significant in predicting the market premia on all these three asset classes, as we have seen from Table 1. The out-of-sample analysis conveys a similar message: the standard deviation of forecasts can predict credit spreads but not equity and bond returns. When used to forecast excess equity returns, it cannot beat historical average, as evidenced by the negative out-of-sample R^2s.[45] Its insignificance is surprising given Drechsler (2013)'s finding that it has nontrivial correlation with the variance premium, which is shown to have significant predictive power for equity returns. The information content of our measure $\widetilde{A}_t$ is not surpassed by the variance premium. Untabulated result shows that in conjunction with the variance premium $\widetilde{A}_t$ remains significant in the predictive regression and leads to an adjusted R^2 of 7.8%.[46] This result highlights the distinction between ambiguity measures based on interdecile range and standard deviation of survey forecasts.

In an early version of Ilut and Schneider (2014), their measure for the ambiguity level is the difference between the 75th percentile and the 25th percentile in each cross-section of forecasts. The resulting interquartile range is actually Philadelphia Fed's definition of "forecast dispersion", and is included in its SPF database.[47] We construct this measure using the Blue Chip data. Panel B presents its association with realized asset premia. Comparing the results with Table 1, we find that it possesses the same degree of predictive power as the interdecile range. The only noticeable difference is that the interquartile range is significant only at the 10% level in the out-of-sample test for the predictability of equity returns.

[45] Under the null of no predictability, the out-of-sample R^2 is expected to be negative. See Campbell and Thompson (2008) for a detailed discussion.

[46] We thank Ivan Shaliastovich for suggesting this exercise.

[47] In statistics, the interquartile range is often used to find outliers in data instead of removing them.

Table A1: Survey Data Summary and Description

Yr-Qtr	No_BC	Pct_BC	No_SPF	Pct_SPF	Yr-Qtr	No_BC	Pct_BC	No_SPF	Pct_SPF	Yr-Qtr	No_BC	Pct_BC	No_SPF	Pct_SPF
1985Q1	45		24	75%	1993Q4	47	94%	34	79%	2002Q3	49	96%	35	91%
1985Q2	48	94%	29	69%	1994Q1	50	94%	30	83%	2002Q4	48	98%	35	83%
1985Q3	47	100%	31	71%	1994Q2	47	96%	35	80%	2003Q1	47	96%	37	78%
1985Q4	48	98%	25	84%	1994Q3	47	96%	30	87%	2003Q2	46	96%	35	86%
1986Q1	46	98%	26	77%	1994Q4	50	92%	30	73%	2003Q3	47	96%	30	80%
1986Q2	46	98%	28	75%	1995Q1	49	98%	29	83%	2003Q4	47	94%	34	71%
1986Q3	43	100%	24	71%	1995Q2	48	98%	52	52%	2004Q1	46	100%	32	81%
1986Q4	45	91%	24	71%	1995Q3	48	96%	52	85%	2004Q2	46	96%	32	78%
1987Q1	46	94%	22	86%	1995Q4	49	88%	47	87%	2004Q3	46	98%	30	87%
1987Q2	50	92%	28	71%	1996Q1	49	92%	38	92%	2004Q4	44	95%	34	79%
1987Q3	50	100%	22	91%	1996Q2	47	94%	42	74%	2005Q1	47	91%	36	86%
1987Q4	44	100%	20	80%	1996Q3	49	92%	42	79%	2005Q2	48	94%	46	59%
1988Q1	48	96%	18	72%	1996Q4	49	98%	37	84%	2005Q3	50	94%	53	79%
1988Q2	47	96%	19	74%	1997Q1	49	98%	38	82%	2005Q4	50	98%	51	94%
1988Q3	49	94%	18	78%	1997Q2	47	96%	40	83%	2006Q1	50	96%	53	87%
1988Q4	48	98%	15	80%	1997Q3	47	96%	37	86%	2006Q2	49	92%	53	92%
1989Q1	49	96%	17	71%	1997Q4	48	92%	38	82%	2006Q3	50	94%	51	86%
1989Q2	49	98%	17	71%	1998Q1	45	98%	32	94%	2006Q4	50	94%	51	86%
1989Q3	47	100%	15	73%	1998Q2	48	91%	29	76%	2007Q1	51	98%	49	86%
1989Q4	50	94%	17	71%	1998Q3	48	88%	32	69%	2007Q2	51	100%	53	81%
1990Q1	48	100%	14	64%	1998Q4	45	96%	33	73%	2007Q3	50	100%	49	88%
1990Q2	48	98%	9	78%	1999Q1	47	91%	33	76%	2007Q4	48	100%	48	81%
1990Q3	47	98%	13	54%	1999Q2	46	91%	37	65%	2008Q1	47	96%	50	84%
1990Q4	47	96%	30	33%	1999Q3	48	90%	37	81%	2008Q2	46	96%	50	84%
1991Q1	48	94%	35	63%	1999Q4	46	89%	43	70%	2008Q3	46	98%	47	83%
1991Q2	48	96%	42	79%	2000Q1	47	91%	36	83%	2008Q4	45	98%	51	76%
1991Q3	50	96%	33	91%	2000Q2	44	91%	34	76%	2009Q1	46	87%	43	91%
1991Q4	48	100%	39	79%	2000Q3	46	93%	32	84%	2009Q2	46	89%	51	78%
1992Q1	49	96%	40	90%	2000Q4	46	98%	34	82%	2009Q3	48	90%	34	94%
1992Q2	48	98%	37	95%	2001Q1	43	98%	35	83%	2009Q4	48	94%	41	66%
1992Q3	48	96%	36	83%	2001Q2	46	89%	33	94%	2010Q1	49	96%	42	90%
1992Q4	48	98%	40	83%	2001Q3	47	100%	33	82%	2010Q2	48	100%	44	89%
1993Q1	49	−−*	34	94%	2001Q4	42	95%	29	79%	2010Q3	46	100%	36	92%
1993Q2	50	94%	36	78%	2002Q1	49	86%	35	63%	2010Q4	48	94%	43	74%
1993Q3	49	98%	35	83%	2002Q2	47	96%	37	84%					

* *In January 1993, the Blue Chips changed the coding system used to identify individual forecasters.*

This table provides descriptive results on the panelists participating in the Blue Chip Financial Forecasts and the Survey of Professional Forecasters. The columns named "No_BC" and "No_SPF" report the number of respondents contained in the survey of each period. The columns named "Pct_BC" and "Pct_SPF" show the percentage of current panel members that also respond to the last period's survey. The sample period spans from 1985Q1 to 2010Q4.

Table A2: Predictive Power of Alternative Ambiguity Measures

	Coefficient	t-stat	Adj R^2	Out-of-Sample R^2	Clark-West p-value
Panel A: Forecasting with the standard deviation of survey forecasts					
r^M_{t+1}	2.871	(1.388)	0.015	-0.031	0.149
CS^{Aaa}_{t+1}	0.374	(1.976)	0.057	0.035	0.079
CS^{Aa}_{t+1}	0.700	(3.589)	0.140	0.115	0.018
CS^{A}_{t+1}	0.797	(2.933)	0.106	0.100	0.035
CS^{Baa}_{t+1}	0.930	(2.604)	0.095	0.088	0.024
CS^{Ba}_{t+1}	3.665	(3.476)	0.286	0.256	0.079
CS^{B}_{t+1}	5.070	(4.197)	0.308	0.318	0.018
rx^{Int}_{t+1}	0.931	(1.398)	0.004	0.009	0.098
rx^{Long}_{t+1}	2.393	(1.361)	0.011	0.017	0.070
Panel B: Forecasting with the interquartile range of survey forecasts					
r^M_{t+1}	3.422	(2.150)	0.022	0.017	0.078
CS^{Aaa}_{t+1}	0.444	(1.811)	0.106	0.083	0.055
CS^{Aa}_{t+1}	0.668	(2.434)	0.157	0.127	0.044
CS^{A}_{t+1}	0.843	(2.383)	0.148	0.131	0.031
CS^{Baa}_{t+1}	0.984	(2.652)	0.133	0.123	0.015
CS^{Ba}_{t+1}	2.421	(2.605)	0.221	0.216	0.001
CS^{B}_{t+1}	3.395	(2.935)	0.240	0.274	0.000
r^{Int}_{t+1}	2.136	(2.259)	0.023	0.021	0.063
rx^{Long}_{t+1}	4.402	(2.065)	0.017	0.025	0.049

This table contains results of predictive regressions on two alternative measures of the ambiguity level. The first one (in Panel A) is the cross-sectional standard deviation of economist forecasts of real GDP growth; the second one (in Panel B) is the interquartile range (the difference between the 75th percentile and the 25th percentile) of these forecasts. r^M_t is defined as the difference between the continuously compounded return on the CRSP value-weighted index and the contemporaneous return on a 3-month Treasury bill. CS^{Aaa}_t — CS^B_t denote the credit spreads of Barclays (Lehman Brothers) bond indices. r^{Int}_{t+1} and r^{Long}_{t+1} are quarterly returns on the Barclays long-term and intermediate-term Treasury indices in excess of the contemporaneous return on a 3-month Treasury bill. For regressions of excess returns, t-statistics computed following the procedures in Hodrick (1992); for regressions of credit spreads, the method of Newey and West (1987) is applied with a lag truncation parameter of 6. The last two columns report Campbell and Thompson (2008)'s out-of-sample R^2s as well as p-values of the Clark-West test. The sample period spans from 1985Q1 to 2010Q4.

B Equilibrium Prices and Affine Approximation

Given the value function of the representative agent as defined in Eq. (8), to make $J(C_t, A_t) + \int_0^t f(C_s, J(C_s, A_s))ds$ a martingale under the most pessimistic probability measure we have

$$\mathfrak{D}^{C,A} J + f(C, J) - \theta^*(A) J_C C \sigma_c = 0.$$

Conjecture that J has the functional form (12). Substituting it into the differential equation above, we obtain

$$(1-\gamma)\left(\mu_c - A - \frac{1}{2}\gamma\sigma_c^2 + (\frac{L_A}{L}\theta - \frac{1}{2}\gamma\sigma_{ca})\sigma_{ca}\sigma_a^2 A\right) + \frac{\mathfrak{D}^A L^\theta}{L^\theta} + \frac{\theta}{L} - \beta\theta = 0, \qquad \gamma, \psi \neq 1. \quad (35)$$

Eq. (11) in Proposition 1 is a special case with the correlation parameter σ_{ca} equal to 0.

To solve the model, we employ Collin-Dufresne and Goldstein (2005)'s log-linear approximation that essentially minimizes the expected squared approximation error. Specifically, we seek an exponential affine expression for the price-consumption ratio as presented in Eq. (13), such that the left hand side of Eq. (35) is linear in A_t except for the θ/L term. In that case, the parameters η_0 and η_1 can be obtained by solving the following optimization problem

$$\min_{\eta_0, \eta_1} E\left[(n_0 + n_1 A_t - \theta e^{-\eta_0 - \eta_1 A_t})^2\right],$$

where

$$\begin{aligned} -n_0 &= (1-\gamma)(\mu_c - \frac{1}{2}\gamma\sigma_c^2) + \theta(\eta_1 \kappa_A \bar{A} - \beta), \\ -n_1 &= (1-\gamma)\left((\eta_1\theta - \frac{1}{2}\gamma\sigma_{ca})\sigma_{ca}\sigma_a^2 - 1\right) - \theta(\eta_1\kappa_A - \frac{1}{2}\theta\eta_1^2\sigma_a^2). \end{aligned}$$

Figure A1 makes a comparison of this approximate price-consumption ratio with the one numerically solved from Eq. (11). These two solutions turn out fairly close to each other. Indeed, when the ambiguity level is around its long-run mean, they are almost indistinguishable from each other. This result demonstrates the accuracy of the exponential affine approximation.

In the same manner, we can also derive an approximate solution for the price-output ratio L_t^o. By Itô's product rule, the expected market return on a claim to aggregate output can be written as

$$r - \frac{1}{L^o} = \frac{1}{dt} E^Q\left[\frac{dL^o}{L^o} + \frac{dO}{O} + \frac{dL^o}{L^o}\frac{dO}{O}\right].$$

Again, we approximate the $L^o(A)$ as a simple exponential formula (24), with ξ_0 and ξ_1 minimizing the mean squared error

$$\min_{\zeta_0, \zeta_1} E\left[(m_0 + m_1 A_t - e^{-\xi_0 - \xi_1 A_t})^2\right],$$

where

$$
\begin{aligned}
-m_0 &= -\varrho_0 + \mu_c - \gamma\sigma_c\sigma_o\sigma_{oc} + \xi_1\kappa_A\bar{A}, \\
-m_1 &= -\varrho_1 - \sigma_o\sigma_{oc}/\sigma_c - (\gamma\sigma_{ca} + (1-\theta)\eta_1)\sigma_{oa}\sigma_a^2 - \xi_1\kappa_A \\
&\quad + \left(\frac{1}{2}\xi_1^2 - \xi_1\sigma_{oa} - \xi_1(\gamma\sigma_{ca} + (1-\theta)\eta_1)\right)\sigma_a^2.
\end{aligned}
$$

Similarly, parameters ζ_0 and ζ_1 in Eq. (25) can be identified by solving the following optimization problem

$$
\min_{\zeta_0,\zeta_1} E\left[(l_0 + l_1 A_t - e^{-\zeta_0-\zeta_1 A_t})^2\right],
$$

where

$$
\begin{aligned}
-l_0 &= -\varrho_0 + \zeta_1\kappa_A\bar{A}, \\
-l_1 &= -\varrho_1 - \zeta_1\kappa_A + \left(\frac{1}{2}\zeta_1^2 - \gamma\sigma_{ca}\zeta_1 - (1-\theta)\eta_1\zeta_1\right)\sigma_a^2.
\end{aligned}
$$

With first-order conditions for optimal consumption choice under ambiguity, Chen and Epstein (2002) drive the following stochastic discount factor (SDF)

$$
M_t = e^{\int_0^t f_J(C_s,J_s)ds} f_C(C_t, J_t) Z_t^{\theta*},
$$

where $Z^{\theta*}$ is the Radon-Nikodŷm derivative corresponding to the worse-case prior. The multiple prior preference is reflected in the presence of $Z^{\theta*}$: if P_0 is the only belief in the prior set $\mathscr{P}$, it degenerates to one, and SDF becomes the standard one in Duffie and Epstein (1992) and Duffie and Skiadas (1994).

To prove Proposition 2, we apply Itô's lemma to the pricing kernel. It follows that

$$
r_t = -E\left(\frac{dM_t}{M_t dt}\right) = \gamma\left[\mu_c - A - \frac{1}{2}(\gamma+1)\sigma_c^2 + \frac{L_A}{L}(\theta-1)\sigma_{ca}\sigma_a^2\right] - \frac{\mathfrak{D}^A L^{\theta-1}}{L^{\theta-1}} - \frac{\theta-1}{L} + \beta\theta.
$$

Connecting it with Eq. (11), we obtain the risk-free rate as a linear function of the ambiguity level

$$
r_t = \varrho_0 + \varrho_1 A_t
$$

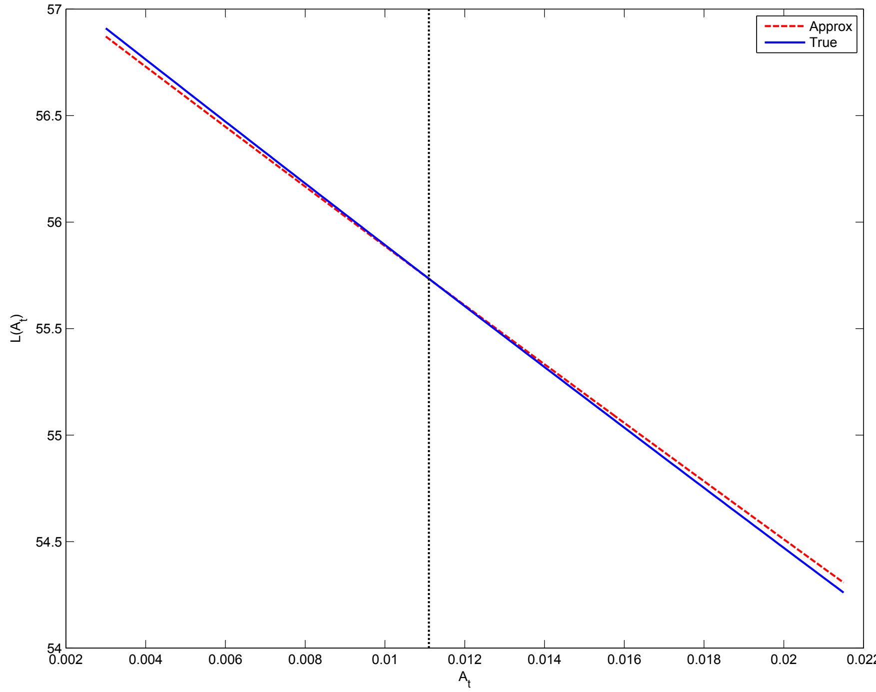
where

$$
\varrho_0 = \rho\mu_c + \beta - \frac{1}{2}\gamma(1+\rho)\sigma_c^2, \tag{36a}
$$

$$
\varrho_1 = -\rho + \frac{1}{2}(\theta-1)\eta_1^2\sigma_a^2 + \sigma_{ca}[(\theta-1)\eta_1 - \frac{1}{2}\gamma(1+\rho)\sigma_{ca}]\sigma_a^2. \tag{36b}
$$

If the consumption growth is a random walk independent of innovations in the ambiguity level, the last term in Eq. (36b) vanishes and the expression for ϱ_1 coincides with Eq. (15).

Figure B1: Approximate Solution for the Price-Consumption Ratio



This figure shows the accuracy of our approximation of the price-consumption ratio L_t as an exponential affine function of the ambiguity level A_t. The red dashed line displays the approximate solution for $L(A)$, and the blue solid line the solution literally evaluated from the differential equation (11). The ODE is solved using the MATLAB function "ode45", with the initial condition

$$L(0) = \frac{\theta}{(1-\gamma)(\mu_c - \frac{1}{2}\gamma\sigma_c^2) - \beta\theta}.$$

The grid line to the X-axis represents the historical mean of A_t.

C Pricing of Corporate Securities under Time-Varying Ambiguity

As a standard approach to endogenous default problems, we look for a solution to a free boundary problem analogous to that in Leland (1994a,b), with the additional spatial variable A_t. The free boundary δ^* has to be determined as part of the problem

$$\mathscr{L}^r_{\delta,A}E + (1-\tau_e)(\delta - C) - mF + D(\delta, A) = 0, \quad (37)$$

$$\mathscr{L}^{r+m}_{\delta,A}D + (1-\tau_i)mC + m^2F = 0, \quad (38)$$

with boundary conditions given in Section 3.4.

Given the scale parameter ε incorporated in (27), it seems convenient to write the operator as a sum of components that are separated by the different powers of ε

$$\mathscr{L}^r_{\delta,A} = \mathscr{L}^r_0 + \sqrt{\varepsilon}\mathscr{L}_1 + \varepsilon\mathscr{L}_2$$

where

$$
\begin{aligned}
\mathscr{L}_0^r &= \mu_\delta^Q \delta \frac{\partial}{\partial \delta} + \frac{1}{2}\bar{\sigma}^2 \delta^2 \frac{\partial^2}{\partial \delta^2} - r\cdot, \\
\mathscr{L}_1 &= \sigma_{oa}\sigma_a^2 A \frac{\partial^2}{\partial \delta \partial A} - [\gamma \sigma_{ca} + (1-\theta)\eta_1]\sigma_a^2 A \frac{\partial}{\partial A}, \\
\mathscr{L}_2 &= \kappa_A(\bar{A} - A)\frac{\partial}{\partial A} + \frac{1}{2}(\sigma_a^2 A)\frac{\partial^2}{\partial A^2}
\end{aligned}
$$

We note that $\mathscr{L}_0$ is simply the Black-Scholes operator with the risk-neutral drift

$$
\mu_\delta^Q = \mu_c - \sigma_o \sigma_c \sigma_{oc}\gamma - \sigma_o \sigma_{oc} A/\sigma_c - [\gamma\sigma_{ca} + (1-\theta)\eta_1]\sigma_{oa}\sigma_a^2,
$$

and conditional variance

$$
\bar{\sigma}^2 = \sigma_o^2 + \sigma_{oa}^2 \sigma_a^2 A + \sigma_j^2.
$$

$\mathscr{L}_2$ is the Dynkin operator of the A_t process under the physical measure. $\mathscr{L}_1$ contains the mixed derivative, due to the covariation between aggregate output and the ambiguity, and the first-order derivative due to the uncertainty premium imposed on the A_t process.

As discussed in Section 3.4, we look for an asymptotic solution for E and D as an expansion in powers of $\sqrt{\varepsilon}$. Inserting the expansion series in (27)-(28) into the PDE (37) and (38), we obtain

$$
\begin{aligned}
(1-\tau_i)mC + m^2F + \quad & \mathscr{L}_0^{r+m} D_0 + \sqrt{\varepsilon}(\mathscr{L}_1 D_0 + \mathscr{L}_0^{r+m} D_1) \\
+ \quad & \varepsilon(\mathscr{L}_2 D_0 + \mathscr{L}_1 D_1 + \mathscr{L}_0^{r+m} D_2) + O(\varepsilon^{\frac{3}{2}}) = 0, \qquad (39) \\
(1-\tau_e)(\delta - C) - mF + \quad & D(\delta, A) + \mathscr{L}_0^r E_0 + \sqrt{\varepsilon}(\mathscr{L}_1 E_0 + \mathscr{L}_0^r E_1) \\
+ \quad & \varepsilon(\mathscr{L}_2 E_0 + \mathscr{L}_1 E_1 + \mathscr{L}_0^r E_2) + O(\varepsilon^{\frac{3}{2}}) = 0. \qquad (40)
\end{aligned}
$$

Equating to zero the first few terms independent of ε leads us to the solution for the lead-order prices as shown in Eq. (29) and (30), where

$$
\begin{aligned}
\alpha_r &= -\frac{\mu_\delta^Q - \bar{\sigma}^2/2 + \sqrt{(\mu_\delta^Q - (\sigma_o^2 + \sigma_{oa}^2\sigma_a^2 A + \sigma_j^2)/2)^2 + 2r(\sigma_o^2 + \sigma_{oa}^2\sigma_a^2 A + \sigma_j^2)}}{\sigma_o^2 + \sigma_{oa}^2\sigma_a^2 A + \sigma_j^2}, \\
\alpha_{r+m} &= -\frac{\mu_\delta^Q - (\sigma_o^2 + \sigma_j^2)/2 + \sqrt{(\mu_\delta^Q - (\sigma_o^2 + \sigma_j^2)/2)^2 + 2(r+m)(\sigma_o^2 + \sigma_j^2)}}{\sigma_o^2 + \sigma_j^2}.
\end{aligned}
$$

Next, the order $\sqrt{\varepsilon}$ terms in the expansion (39) and (40) give the following equations for the first-order corrections

$$
\begin{aligned}
\mathscr{L}_1 D_0 + \mathscr{L}_0^{r+m} D_1 &= 0, \\
\mathscr{L}_1 E_0 + \mathscr{L}_0^r E_1 &= 0.
\end{aligned}
$$

Boundary conditions are constructed by substituting the expansion series into original conditions (19) and (23)

$$\begin{aligned}
D_1(\delta_0^*) + \delta_1^* \frac{\partial D_0}{\partial \delta}|_{\delta=\delta_0^*} &= (1-\phi) m \delta_1^* e^{\xi_0+\xi_1 A_t}, \\
\lim_{\delta\to\infty} D_1(\delta) &= 0, \\
E_1(\delta_0^*) &= 0,
\end{aligned}$$

where we have used the fact that $\lim_{\delta\to\infty} D_0(\delta)$ does not depend on δ. Finally, we can use (20) to find the order $\sqrt{\varepsilon}$ term in the expansion (31) for the optimal default barrier

$$\frac{\partial E_1}{\partial \delta}|_{\delta=\delta_0^*} + \delta_1^* \frac{\partial^2 E_0}{\partial \delta^2}|_{\delta=\delta_0^*} = 0.$$

Since δ_0^* has been determined in Eq. (34), the first-order correction terms E_1 and D_1 actually solve a one-dimensional fixed boundary problem.

Likewise, the second-order price corrections are the solution to the following problem

$$\begin{aligned}
\mathscr{L}_2 D_0 + \mathscr{L}_1 D_1 + \mathscr{L}_0^{r+m} D_2 &= 0, \\
\mathscr{L}_2 E_0 + \mathscr{L}_1 E_1 + \mathscr{L}_0^{r} E_2 &= 0, \\
D_2(\delta_0^*) + \delta_1^* \frac{\partial D_1}{\partial \delta}|_{\delta=\delta_0^*} + \delta_2^* \frac{\partial D_0}{\partial \delta}|_{\delta=\delta_0^*} + \delta_1^{*2} \frac{\partial^2 D_0}{\partial \delta^2}|_{\delta=\delta_0^*} &= (1-\phi) m \delta_2^* e^{\xi_0+\xi_1 A_t}, \\
\lim_{\delta\to\infty} D_2(\delta) &= 0, \\
E_2(\delta_0^*) &= 0, \\
\frac{\partial E_2}{\partial \delta}|_{\delta=\delta_0^*} + \delta_1^* \frac{\partial^2 E_1}{\partial \delta^2}|_{\delta=\delta_0^*} + \delta_2^* \frac{\partial^2 E_0}{\partial \delta^2}|_{\delta=\delta_0^*} + \delta_1^{*2} \frac{\partial^3 E_0}{\partial \delta^3}|_{\delta=\delta_0^*} &= 0.
\end{aligned}$$

References

Aït-Sahalia, Y. (1999). Transition densities for interest rate and other nonlinear diffusions. *Journal of Finance 54*(4), 1361–1395.

Aït-Sahalia, Y. (2002). Maximum likelihood estimation of discretely sampled diffusions: A closed-form approximation approach. *Econometrica 70*(1), 223–262.

Aït-Sahalia, Y. (2008). Closed-form likelihood expansions for multivariate diffusions. *Annals of Statistics*, 906–937.

Aït-Sahalia, Y. (2012). Users guide for the matlab library implementing closed form mle for diffusions. Department of Economics and Bendheim Center for Finance, Princeton University.

Albagli, E., C. Hellwig, and A. Tsyvinski (2013). Dynamic dispersed information and the credit spread puzzle. Working paper, Yale University.

Alili, L. and A. E. Kyprianou (2005). Some remarks on first passage of lévy processes, the american put and pasting principles. *Annals of Applied Probability 15*(3), 2062–2080.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens (2001). The distribution of realized stock return volatility. *Journal of Financial Economics 61*(1), 43–76.

Anderson, E. W., L. P. Hansen, and T. J. Sargent (2003). A quartet of semigroups for model specification, robustness, prices of risk, and model detection. *Journal of the European Economic Association 1*(1), 68–123.

Ang, A. and G. Bekaert (2007). Stock return predictability: Is it there? *Review of Financial Studies 20*(3), 651–707.

Avramov, D., T. Chordia, G. Jostova, and A. Philipov (2007). Momentum and credit rating. *Journal of Finance 62*(5), 2503–2520.

Avramov, D., G. Jostova, and A. Philipov (2007). Understanding changes in corporate credit spreads. *Financial Analysts Journal 63*(2), 90–105.

Bakshi, G., G. Panayotov, and G. Skoulakis (2012). The baltic dry index as a predictor of global stock returns, commodity returns, and global economic activity. Working Paper, University of Maryland.

Bansal, R. and I. Shaliastovich (2013). A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies 26*(1), 1–33.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance 59*(4), 1481–1509.

Beeler, J. and J. Y. Campbell (2012). The long-run risks model and aggregate asset prices: An empirical assessment. *Critical Finance Review 1*(1), 141–182.

Benzoni, L., P. Collin-Dufresne, and R. S. Goldstein (2005). Can standard preferences explain the prices of out-of-the-money S&P 500 put options. NBER Working Paper.

Benzoni, L., P. Collin-Dufresne, and R. S. Goldstein (2011). Explaining asset pricing puzzles associated with the 1987 market crash. *Journal of Financial Economics 101*(3), 552–573.

Berndt, A., R. Douglas, J. Duffie, M. Ferguson, and D. Schranz (2005). Measuring default risk premia from default swap rates and edfs. Working Paper, Carnegie Mellon University.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010). The levered equity risk premium and credit spreads: A unified framework. *Review of Financial Studies 23*(2), 645–703.

Black, F. (1976). Stuedies of stock price volatility changes. In *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics*, pp. 177–181.

Boyarchenko, N. (2012). Ambiguity shifts and the 2007–2008 financial crisis. *Journal of Monetary Economics 59*(5), 493–507.

Brekke, K. A. and B. Øksendal (1991). The high contact principle as a sufficiency condition for optimal stopping. In *Stochastic Models and Option Values: Applications to Resources, Environment, and Investment Problems*, pp. 187–208.

Buraschi, A., F. Trojani, and A. Vedolin (2013). Economic uncertainty, disagreement, and credit markets. *Management Science 60*(5), 1281–1296.

Buraschi, A. and P. Whelan (2012). Term structure models with differences in beliefs. Working Paper, Imperial College Business School.

Campbell, J. and J. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(2), 205–251.

Campbell, J. Y. (1999). Asset prices, consumption, and the business cycle. *Handbook of macroeconomics 1*, 1231–1303.

Campbell, J. Y. and R. J. Shiller (1988). Stock prices, earnings, and expected dividends. *The Journal of Finance 43*(3), 661–676.

Campbell, J. Y. and S. B. Thompson (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies 21*(4), 1509–1531.

Caouette, J. B., E. I. Altman, and P. Narayanan (1998). *Managing credit risk: the next great financial challenge*, Volume 2. John Wiley & Sons.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, H., R. Cui, Z. He, and K. Milbradt (2013). Quantifying liquidity and default risks of corporate bonds over the business cycle. Working paper, Booth School of Business, University of Chicago.

Chen, L., P. Collin-Dufresne, and R. Goldstein (2009). On the relation between the credit spread puzzle and the equity premium puzzle. *Review of Financial Studies 22*(9), 3367–3409.

Chen, L., D. Lesmond, and J. Wei (2007). Corporate yield spreads and bond liquidity. *Journal of Finance 62*(1), 119–149.

Chen, Z. and L. Epstein (2002). Ambiguity, risk, and asset returns in continuous time. *Econometrica 70*(4), 1403–1443.

Chevalier, E. (2005). Critical price near maturity for an american option on a dividend-paying stock in a local volatility model. *Mathematical Finance 15*(3), 439–463.

Clark, T. E. and K. D. West (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics 138*(1), 291–311.

Cochrane, J. H. and L. P. Hansen (1992). Asset pricing explorations for macroeconomics. In *NBER Macroeconomics Annual 1992, Volume 7*, pp. 115–182. MIT Press.

Collin-Dufresne, P. and R. Goldstein (2005). Improving on the Campbell-Shiller Log-Linearization Approximation. Working Paper, UC Berkeley and University of Minnesota.

Collin-Dufresne, P., R. Goldstein, and J. Martin (2001). The determinants of credit spread changes. *Journal of Finance 56*(6), 2177–2207.

David, A. (2008). Inflation uncertainty, asset valuations, and the credit spreads puzzle. *Review of Financial Studies 21*(6), 2487–2534.

Davydenko, S. A. (2012). When do firms default? a study of the default boundary. Working Paper, University of Toronto.

Davydenko, S. A., I. A. Strebulaev, and X. Zhao (2012). A market-based study of the cost of default. *Review of Financial Studies 25*(10), 2959–2999.

Drechsler, I. (2013). Uncertainty, time-varying fear, and asset prices. *Journal of Finance 68*(5), 1837–1883.

Driessen, J. (2005). Is default event risk priced in corporate bonds? *Review of Financial Studies 18*(1), 165–195.

Duffee, G. (2002). Term premia and interest rate forecasts in affine models. *Journal of Finance 57*(1), 405–443.

Duffie, D. (2001). Dynamic asset pricing theory. *Princeton University Press*.

Duffie, D. and L. Epstein (1992). Stochastic differential utility. *Econometrica 60*(2), 353–394.

Duffie, D. and C. Skiadas (1994). Continuous-time security pricing: A utility gradient approach. *Journal of Mathematical Economics 23*(2), 107–132.

Elkamhi, R., J. Ericsson, and M. Jiang (2011). Time-varying asset volatility and the credit spread puzzle. Working Paper, University of Iowa.

Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. *Quarterly Journal of Economics*, 643–669.

Elton, E., M. Gruber, D. Agrawal, and C. Mann (2001). Explaining the rate spread on corporate bonds. *Journal of Finance 56*(1), 247–277.

Engelberg, J., C. F. Manski, and J. Williams (2011). Assessing the temporal variation of macroeconomic forecasts by a panel of changing composition. *Journal of Applied Econometrics 26*(7), 1059–1078.

Eom, Y., J. Helwege, and J. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies 17*(2), 499–544.

Epstein, L. G. and M. Schneider (2003). Recursive multiple-priors. *Journal of Economic Theory 113*(1), 1–31.

Epstein, L. G. and M. Schneider (2007). Learning under ambiguity. *Review of Economic Studies 74*(4), 1275–1303.

Epstein, L. G. and M. Schneider (2008). Ambiguity, information quality, and asset pricing. *Journal of Finance 63*(1), 197–228.

Epstein, L. G. and M. Schneider (2010). Ambiguity and asset markets. *Annual Review of Financial Economics 2*(1), 315–346.

Epstein, L. G. and T. Wang (1994). Intertemporal asset pricing under knightian uncertainty. *Econometrica 62*(2), 283–322.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of Financial Economics 22*(1), 3–25.

Fama, E. F. and K. R. French (1989). Business conditions and expected returns on stocks and bonds. *Journal of financial economics 25*(1), 23–49.

Fouque, J.-P., G. Papanicolaou, and K. R. Sircar (2001). From the implied volatility skew to a robust correction to black-scholes american option prices. *International Journal of Theoretical and Applied Finance 4*(4), 651–675.

Gagliardini, P., P. Porchia, and F. Trojani (2009). Ambiguity aversion and the term structure of interest rates. *Review of Financial Studies 22*(10), 4157–4188.

Gajdos, T., T. Hayashi, J.-M. Tallon, and J.-C. Vergnaud (2008). Attitude toward imprecise information. *Journal of Economic Theory 140*(1), 27–65.

Gilboa, I. and D. Schmeidler (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics 18*(2), 141–153.

Goldstein, R., N. Ju, and H. Leland (2001). An ebit-based model of dynamic capital structure. *Journal of Business 74*(4), 483–512.

Goyal, A. and I. Welch (2003). Predicting the equity premium with dividend ratios. *Management Science 49*(5), 639–654.

Hackbarth, D., J. Miao, and E. Morellec (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics 82*(3), 519–550.

Hansen, L. P. and T. J. Sargent (2001). Robust control and model uncertainty. *American Economic Review 91*(2), 60–66.

Hansen, L. P. and T. J. Sargent (2008). *Robustness*. Princeton university press.

Heckman, J. J. (1976). The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models. *Annals of Economic and Social Measurement 5*(4), 475–492.

Heckman, J. J. (1979). Sample selection bias as 3 specification error. *Econometrica 47*(1), 153–162.

Hodrick, R. J. (1992). Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *Review of Financial studies 5*(3), 357–386.

Huang, J.-Z. and M. Huang (2012). How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies 2*(2), 153–202.

Illeditsch, P. K. (2011). Ambiguous information, portfolio inertia, and excess volatility. *Journal of Finance 66*(6), 2213–2247.

Ilut, C. (2012). Ambiguity aversion: Implications for the uncovered interest rate parity puzzle. *American Economic Journal: Macroeconomics 4*(3), 33–65.

Ilut, C. L. and M. Schneider (2014). Ambiguous business cycles. *American Economic Review 104*(8), 2368–99.

Jeong, D., H. Kim, and J. Y. Park (2015). Does ambiguity matter? estimating asset pricing models with a multiple-priors recursive utility. *Journal of Financial Economics 115*(2), 361–382.

Jones, C. S. and S. Tuzel (2013). New orders and asset prices. *Review of Financial Studies 26*(1), 115–157.

Kim, D. and S. J. Kon (1994). Alternative models for the conditional heteroscedasticity of stock returns. *Journal of Business*, 563–598.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005). A smooth model of decision making under ambiguity. *Econometrica 73*(6), 1849–1892.

Klibanoff, P., M. Marinacci, and S. Mukerji (2009). Recursive smooth ambiguity preferences. *Journal of Economic Theory 144*(3), 930–976.

Lamberton, D. and S. Villeneuve (2003). Critical price near maturity for an american option on a dividend-paying stock. *Annals of Applied Probability 13*(2), 800–815.

Lee, R. W. (2001). Implied and local volatilities under stochastic volatility. *International Journal of Theoretical and Applied Finance 4*(01), 45–89.

Leippold, M., F. Trojani, and P. Vanini (2008). Learning and asset prices under ambiguous information. *Review of Financial Studies 21*(6), 2565–2597.

Leland, H. E. (1994a). Bond prices, yield spreads, and optimal capital structure with default risk. Working Paper, University of California at Berkeley.

Leland, H. E. (1994b). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance 49*(4), 1213–1252.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. *Journal of Finance 53*(4), 1213–1243.

Lettau, M. and S. Ludvigson (2001). Consumption, aggregate wealth, and expected stock returns. *Journal of Finance 56*(3), 815–849.

Levendorskii, S. (2008). American and european options in multi-factor jump-diffusion models, near expiry. *Finance and Stochastics 12*(4), 541–560.

Liu, J., J. Pan, and T. Wang (2005). An equilibrium model of rare-event premia and its implication for option smirks. *Review of Financial Studies 18*(1), 131.

Longstaff, F. A., S. Mithal, and E. Neis (2005). Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market. *Journal of Finance 60*(5), 2213–2253.

Longstaff, F. A. and E. S. Schwartz (2001). Valuing american options by simulation: A simple least-squares approach. *Review of Financial studies 14*(1), 113–147.

McQuade, T. J. (2012). Stochastic Volatility and Asset Pricing Puzzles. Working paper, Harvard University.

Miao, J. (2009). Ambiguity, risk and portfolio choice under incomplete information. *Annals of Economics and Finance 10*(2), 257–279.

Mordecki, E. (2002). Optimal stopping and perpetual options for lévy processes. *Finance and Stochastics 6*(4), 473–493.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Øksendal, B. (2003). Stochastic differential equations: An introduction with applications. *Springer, Berlin*.

Ou, S., D. Chlu, and A. Metz (2011). Corporate default and recovery rates, 1920–2010. Special Comment, Moodys Investors Service.

Piazzesi, M., M. Schneider, and S. Tuzel (2007). Housing, consumption and asset pricing. *Journal of Financial Economics 83*(3), 531–569.

Reisz, A. S. and C. Perlich (2007). A market-based framework for bankruptcy prediction. *Journal of Financial Stability 3*(2), 85–131.

Rossi, M. (2009). Realized Volatility, Liquidity, and Corporate Yield Spreads. Working paper, University of Notre Dame.

Sbuelz, A. and F. Trojani (2008). Asset prices with locally constrained-entropy recursive multiple-priors utility. *Journal of Economic Dynamics and Control 32*(11), 3695–3717.

Schaefer, S. and I. Strebulaev (2008). Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *Journal of Financial Economics 90*(1), 1–19.

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–1366.

Sircar, R. K. and G. C. Papanicolaou (1999). Stochastic volatility, smile & asymptotics. *Applied Mathematical Finance 6*(2), 107–145.

Strebulaev, I. A. (2007). Do tests of capital structure theory mean what they say? *Journal of Finance 62*(4), 1747–1787.

Strzalecki, T. (2011). Axiomatic foundations of multiplier preferences. *Econometrica 79*(1), 47–73.

Strzalecki, T. (2013). Temporal resolution of uncertainty and recursive models of ambiguity aversion. *Econometrica 81*(3), 1039–1074.

Tauchen, G., H. Zhang, and M. Liu (1996). Volume, volatility, and leverage: A dynamic analysis. *Journal of Econometrics 74*(1), 177–208.

Toft, K. B. and B. Prucyk (1997). Options on leveraged equity: Theory and empirical tests. *Journal of Finance 52*(3), 1151–1180.

Ulrich, M. (2013). Inflation ambiguity and the term structure of us government bonds. *Journal of Monetary Economics 60*(2), 295–309.

Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance 68*(3), 987–1035.

Yang, W. (2011). Long-run risk in durable consumption. *Journal of Financial Economics 102*(1), 45–61.

Table 1: Asset Return and Credit Spread Predictability by the Ambiguity Level

	Coefficient	t-stat	Adj R^2	Out-of-Sample R^2	Clark-West p-value
r^M_{t+1}	2.923	(2.736)	0.043	0.038	0.035
CS^{Aaa}_{t+1}	0.239	(2.140)	0.093	0.072	0.007
CS^{Aa}_{t+1}	0.384	(3.026)	0.159	0.122	0.022
CS^{A}_{t+1}	0.473	(2.764)	0.144	0.130	0.006
CS^{Baa}_{t+1}	0.566	(2.842)	0.137	0.126	0.002
CS^{Ba}_{t+1}	1.438	(3.014)	0.260	0.236	0.000
CS^{B}_{t+1}	1.939	(3.536)	0.262	0.255	0.004
r^{Int}_{t+1}	1.112	(3.091)	0.032	0.025	0.030
r^{long}_{t+1}	2.774	(2.416)	0.024	0.019	0.064

This table contains results from regressing (one-quarter-ahead) excess stock returns, credit spreads and excess bond returns on the proposed measure of ambiguity level. r^M_t is defined as the difference between the continuously compounded return on the CRSP value-weighted index and the contemporaneous return on a 3-month Treasury bill. CS^{Aaa}_t — CS^B_t denote the credit spreads of Barclays (Lehman Brothers) bond indices. r^{Int}_{t+1} and r^{Long}_{t+1} are quarterly returns on the Barclays long-term and intermediate-term Treasury indices in excess of the contemporaneous return on a 3-month Treasury bill. t-statistics are computed following the procedures of Newey and West (1987) with a lag truncation parameter of 6. The last two columns report Campbell and Thompson (2008)'s out-of-sample R^2s as well as p-values of the Clark-West test. The sample period spans from 1985Q1 to 2010Q4.

Table 2: Moments of the Ambiguity Level

Panel A: Variance Ratios Tests				Panel B: Properties of Skewness	
Horizon(yr)	VR(i)	90%	95%		
2	1.232	1.108	1.142	Sample	0.820
5	1.841	1.191	1.278	90%	0.504
10	2.363	1.241	1.398	95%	0.358

Panel A shows the variance ratios (VR) for the absolute value of the residuals from a AR(5) model for the ambiguity measure,

$$\widetilde{A}_t = \sum_{j=1}^{5} B_j L^j \widetilde{A}_t + \epsilon_t,$$

where L is a lag operator. Bootstrap percentiles are computed by resampling the residual series for 10,000 times, under the null hypothesis that realized volatility were i.i.d.. Panel B presents skewness estimates and corresponding bootstrap confidence percentiles. The sample period spans from 1985Q1 to 2010Q4.

Table 3: Estimation of Model Parameters

Parameter	Estimate	s.e.	Parameter	Estimate	s.e.
Consumption & Output			Ambiguity		
$\mu_c \times 10^2$	1.54	(0.32)	κ_A	0.30	(0.10)
$\sigma_c \times 10^2$	0.82	(0.19)	$\bar{A} \times 10^2$	1.11	(0.12)
σ_{ca}	−0.11	(0.04)	$\sigma_a \times 10^2$	8.13	(2.05)
$\sigma_o \times 10^2$	1.21	(0.22)			
σ_{oc}	0.64	(0.11)			
σ_{oa}	−0.39	(0.08)			

This table reports estimated model parameter values based on the closed-form maximum likelihood estimation of the model using U.S. data at a quarterly frequency. The sample period spans from 1985Q1 to 2010Q4. Quantities shown in parentheses are the misspecification-robust standard error obtained using the Huber sandwich estimator (Aït-Sahalia, 2012). They are reported as the significand sharing the same scientific notation with the corresponding point estimate.

Table 4: Calibration of Corporate-Level Parameters

Panel A: Parameters on Tax Rates and Default Losses

τ_c	τ_d	τ_i	b_0	b_1
0.35	0.12	0.296	0.113	11.846

Panel B: Calibration Target by Credit Rating

Rating Groups	Aaa	Aa	A	Baa	Ba	B
Debt/EBIT	0.90	1.43	1.85	2.61	3.25	5.03
EBIT Volatility	9.82	8.60	10.52	12.16	16.58	13.64

Panel A reports the calibration values of parameters that do not vary among different credit ratings. τ_c denotes the corporate tax rate, τ_d the personal tax rate on dividends and τ_i the tax rate on coupon income. Panel B shows some target parameters for individual firms' calibration by credit rating group. Debt-EBIT ratio is measured as (Short-Term Debt + Long-Term Debt) / EBITDA, corresponding to F/δ in the model. EBIT volatility is measured as standard deviation of trailing five years of net revenue growth. Aggregate metrics reported in the table are the median values by credit rating. The underlying data are taken from Moodys Financial Metrics, a data and analytics platform that provides as-reported and adjusted financial data, ratios, models and interactive rating methodologies.

Table 5: Average Credit Spreads of Ten-Year Bonds

	Aaa	Aa	A	Baa	Ba	B
1985-2010:						
Individual Bonds	77	91	117	182	298	488
Barclays Idx.	95	112	147	210	390	580
1985-1997:						
Individual Bonds	70	87	106	153	276	459
Barclays Idx.	84	109	125	179	340	532
1998-2010:						
Individual Bonds	80	95	131	198	334	506
Barclays Idx.	106	115	169	242	420	609
BofA ML Idx.	74	100	132	194	372	

This table shows the levels of historical averaged 10-year credit spreads across ratings categories. For rows named "Individual Bonds", average yield spread is firstly calculated for each bond with maturity between 7 and 15 years, and the median value is then taken across bonds. Individual bond's quotes are obtained from Lehman Brothers fixed-income data set for the 1985-1997 period, and from Merrill Lynch for the 1998-2010 period. The "Barclays Idx." rows report the average credit spreads of Barclays U.S. Aggregate Corporate/High Yield Bond Indices, and the "BofA ML Idx." rows show the means of Bank of America Merrill Lynch U.S. Corporate/High Yield 7-10Y Option-Adjusted Spreads.

Table 6: Model Implications for Default Rates and Credit Spreads

	10-yr Default Prob (%)		10-yr Credit Spread (bp)			
	Data	Model	Target_1	Target_2	Constant Ambiguity	Time-Varying Ambiguity
Aaa	0.19	0.04	39	33	8	32
Aa	0.50	0.42	46	39	18	49
A	2.22	1.18	66	67	53	78
Baa	4.89	4.24	129	131	85	153
Ba	21.34	14.76	247	241	164	225
B	45.19	40.13	−−	−−	312	407

This table shows historical and model-implied 10-year default probabilities and 10-year credit spreads by credit rating group. Estimates of historical default frequancies for senior unsecured bonds are from Ou et al. (2011). Columns named "Target_1" and "Target_2" report median corporate yield spreads, as listed in Table 5, corrected for the non-default component. They are calculated based on the fractional sizes of the liquidity component estimated by Longstaff et al. (2005) and Chen et al. (2013), respectively. The column named "Constant Ambiguity" reports model-generated credit spreads for a spacial case with $A_t \equiv \bar{A}$.

Table 7: The Effect of Ambiguity on the Default Boundary

$\widetilde{A}_t$	−0.198	−0.146	−0.193	−0.175	−0.150	−0.192	−0.203	−0.194	−0.184
	(−3.450)	(−2.053)	(−2.796)	(−3.648)	(−2.606)	(−3.305)	(−3.551)	(−3.446)	(−2.998)
$Maturity_{j,t}$		−0.091							
		(−6.723)							
rf_t			−0.452						
			(−0.211)						
$Vol_{j,t}$				−0.286					
				(−2.781)					
$Coupon_{j,t}$					−2.845				
					(−3.232)				
$Cost_{j,t}$						0.322			
						(3.040)			
$Payout_{j,t}$							0.395		
							(0.553)		
GDP_t								0.038	
								(0.837)	
$Tax_{j,t}$									0.245
									(1.088)
$1/Mills$	−0.082	−0.105	−0.075	−0.047	−0.061	−0.119	−0.084	−0.086	−0.132
	(−0.793)	(−1.172)	(−0.682)	(−0.679)	(−0.727)	(−1.073)	(−0.834)	(−0.845)	(−1.562)
Const.	0.966	0.965	0.979	1.099	1.095	0.713	0.927	0.936	0.706
	(4.622)	(4.164)	(4.769)	(4.230)	(4.558)	(2.635)	(4.582)	(4.393)	(4.589)
Adj R^2	0.049	0.208	0.045	0.113	0.086	0.165	0.124	0.048	0.038
N	230	230	230	230	225	114	227	230	144

This table reports cross-sectional regressions of the empirically observed boundary on the ambiguity level and other theoretical determinants. The default boundary is defined as the value of firm's asset at default, scaled by the face value of debt. The regression coefficients are estimated with the Heckit model (Heckman, 1976, 1979). $1/Mills$ denotes the inverse Mills ratio obtained from the first-stage regression, which includes all independent variables at the second stage and the quick ratio. The values of t-statistics are reported in parentheses. The sample period spans from 1983 to 2010.

Table 8: Simulated and Sample Moments of Equity Return and Risk-Free Rate

	Data		Constant Ambiguity		Time-Varying Ambiguity		
	Portfolio	Individual	Unlevered	Levered	Unlevered	Levered	Individual
$E(r_{e,t} - r_t)$	6.13	8.98	1.11	2.09	4.13	5.94	7.81
$\sigma(r_{e,t})$	16.58	32.97	1.18	2.67	6.72	12.39	26.63
$E(r_{e,t} - r_t)/\sigma(r_{e,t} - r_t)$	41.49	28.33	93.97	78.62	73.75	48.60	29.24
$E(r_t)$	1.21		1.70			1.18	
$\sigma(r_t)$	1.18		0			0.92	

This table shows historical and model-implied unconditional moments of the equity returns and the risk-free rates. In the "Data" section, the "Portfolio" column reports the return on the NYSE/AMEX/NASDAQ value-weighted index, and the "Individual" column contains the median descriptive statistics of monthly returns on all Baa rated stocks listed on CRSP. The "Time-Varying Ambiguity" section reports the implications of our benchmark model, while the "Constant Ambiguity" section shows a special case with $A_t \equiv \bar{A}$. Columns labeled "unlevered" and "levered" present the results on unlevered and levered perpetual claims to the aggregate output. The "Individual" column corresponds to a typical Baa firm whose earnings growth rate is also subject to idiosyncratic shocks. The sample period spans from 1985 to 2010.

Table 9: Long-Horizon Predictive Regressions

Panel A: Predictability of Excess Returns

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	−0.15	(0.08)	0.10	−0.35	0.19	−0.18	0.13
3	−0.39	(0.23)	0.16	−0.73	0.33	−0.49	0.21
5	−0.78	(0.44)	0.29	−1.08	0.43	−0.78	0.30

Panel B: Predictability of Credit Spreads

Horizon	Data			Individual		Portfolio	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.45	(0.24)	0.04	0.39	0.03	0.42	0.06
3	0.81	(0.26)	0.14	0.56	0.06	0.69	0.21
5	0.84	(0.38)	0.15	0.58	0.07	0.64	0.17

Panel C: Predictability of Dividend Growth

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.0172	(0.0477)	0.0036	-0.0105	0.03	-0.0068	0.01
				[-0.07, 0.05]	[0.00, 0.09]	[-0.04, 0.04]	[0.00, 0.05]
3	0.0237	(0.1261)	0.0027	-0.0258	0.07	-0.0092	0.02
				[-0.17, 0.12]	[0.00, 0.20]	[-0.14, 0.13]	[0.00, 0.11]
5	0.1751	(0.2487)	0.1244	-0.0378	0.10	-0.0151	0.04
				[-0.25, 0.17]	[0.00, 0.26]	[-0.20, 0.18]	[0.00, 0.14]

Panel D: Predictability of Consumption Growth

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.0025	(0.0068)	0.0052	-0.0033	0.02	-0.0021	0.02
				[-0.04, 0.03]	[0.00, 0.08]	[-0.03, 0.03]	[0.00, 0.06]
3	−0.0045	(0.0166)	0.0035	-0.0094	0.05	-0.0026	0.02
				[-0.11, 0.09]	[0.00, 0.20]	[-0.04, 0.04]	[0.00, 0.09]
5	−0.0070	(0.0326)	0.0075	-0.0135	0.07	-0.0074	0.04
				[-0.17, 0.13]	[0.00, 0.27]	[-0.09, 0.08]	[0.00, 0.17]

This table shows regression results of excess stock returns $\sum_{j=1}^{4h}(r_{e,t+j/4} - r_{t+j/4})$, credit spreads $CS_{Baa,t+h}$, consumption growth $\sum_{j=1}^{4h} \Delta c_{t+j/4}$, and dividend growth $\sum_{j=1}^{4h} \Delta \delta_{e,t+j/4}$ on the log price-dividend ratios for $h = 1$, 3, and 5 years. In Panel A, C and D, "s.e." denotes Hodrick (1992) standard errors; in Panel B it corresponds to Newey and West (1987) standard errors. The entries for the model are the mean, 5% and 95% percentiles (in brackets) based on 1,000 simulated samples with 26×252 daily observations that are aggregated to a quarterly frequency. The sample period spans from 1985 to 2010.

Table 10: Correlations of Consumption Growth with Equity Returns and Credit Spreads

Panel A: Consumption Growth w/ Stock Returns

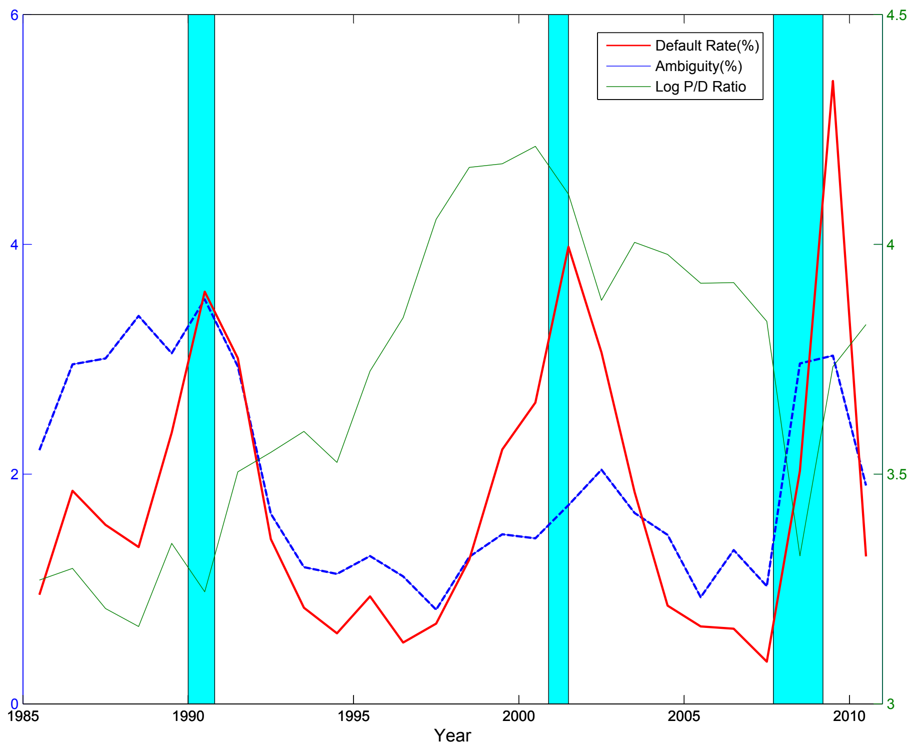
	Data	Model		
		Unlevered	Levered	90% Intvl.
$r_{e,t}, \Delta c_{t-2}$	-0.05	-0.01	-0.01	[-0.21, 0.19]
$r_{e,t}, \Delta c_{t-1}$	-0.01	-0.01	-0.02	[-0.22, 0.18]
$r_{e,t}, \Delta c_{t}$	0.10	0.26	0.22	[0.03, 0.34]
$r_{e,t}, \Delta c_{t+1}$	0.15	0.00	0.00	[-0.19, 0.17]
$r_{e,t}, \Delta c_{t+2}$	0.13	0.00	0.01	[-0.18, 0.19]

Panel B: Consumption Growth w/ Changes in Credit Spreads

	Baa			Ba		
	Data	Model	90% Intvl.	Data	Model	90% Intvl.
$\Delta CS_t, \Delta c_{t-2}$	0.17	0.02	[-0.16, 0.22]	0.19	0.01	[-0.17, 0.19]
$\Delta CS_t, \Delta c_{t-1}$	0.15	0.02	[-0.17, 0.21]	0.12	0.00	[-0.18, 0.17]
$\Delta CS_t, \Delta c_{t}$	-0.12	-0.20	[-0.33, -0.03]	-0.17	-0.25	[-0.42, -0.08]
$\Delta CS_t, \Delta c_{t+1}$	-0.16	0.00	[-0.18, 0.18]	-0.18	-0.01	[-0.18, 0.17]
$\Delta CS_t, \Delta c_{t+2}$	-0.21	-0.01	[-0.20, 0.18]	-0.14	-0.02	[-0.20, 0.16]

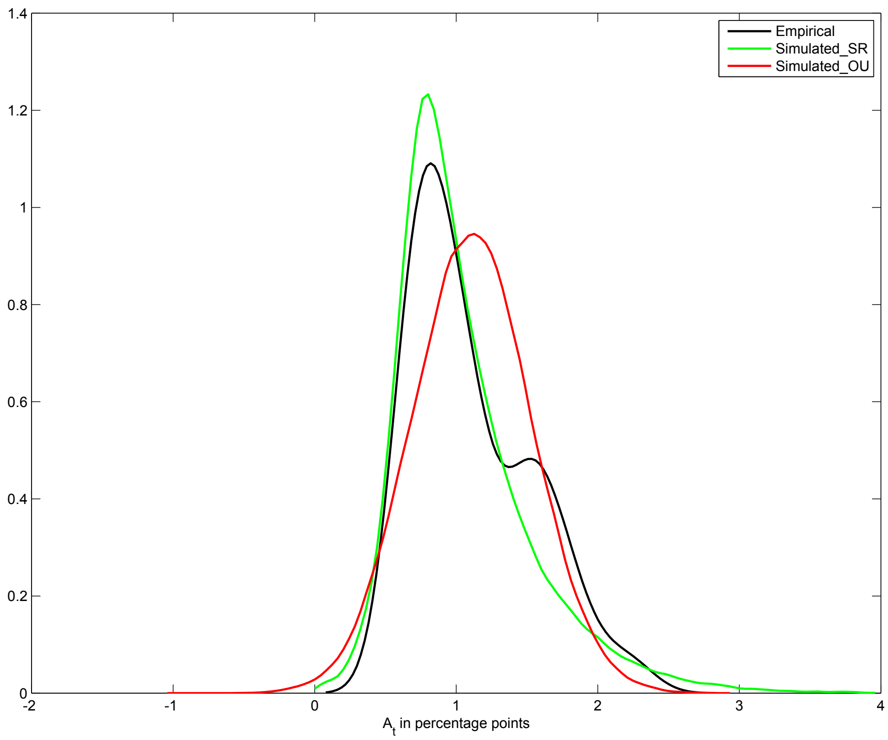
This table shows historical and model-implied correlation of consumption growth with stock returns, and with changes in credit spreads, for different leads and lags. The column labeled "Data" in Panel A reports the correlation between growth rate of aggregate consumption and returns on the NYSE/AMEX/NASDAQ value-weighted index. The "Data" columns in Panel B display the correlations of consumption growth with changes in yield spreads of Barclays U.S. Corporate Baa Bond Index and Barclays U.S. High Yield Ba Bond Index over Barclays U.S. Treasury Bond Index. The entries for the model are the means, 90% confidence intervals based on 1,000 simulated samples with 26×252 daily observations that are aggregated to a quarterly frequency. All empirical data involved cover the period from 1985 to 2010, and are sampled at a quarterly frequency.

Figure 1: Historical Default Rates, Levels of Ambiguity, and Price-Dividend Ratios



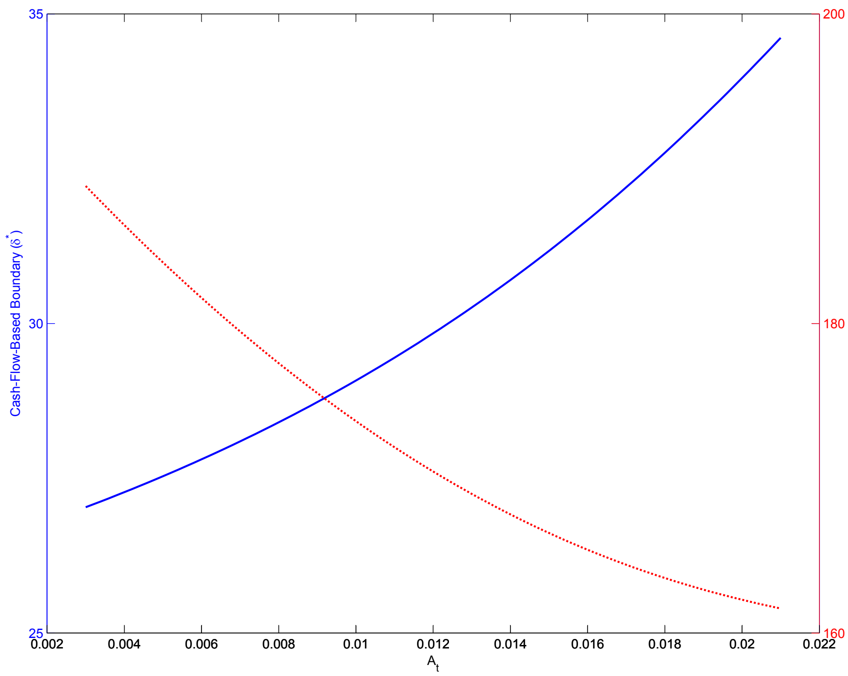
This figure shows the Moody's corporate default rates (red solid line), the annualized measure of ambiguity level constructed from professional forecasts (blue dashed line), and the logarithm price-dividend ratio on the NYSE/AMEX/NASDAQ value-weighted index (green dash-dot line). The y-axis on the left side applies to the first two time series, and the y-axis on the right side the last. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010.

Figure 2: Density Functions of the Amount of Ambiguity A_t



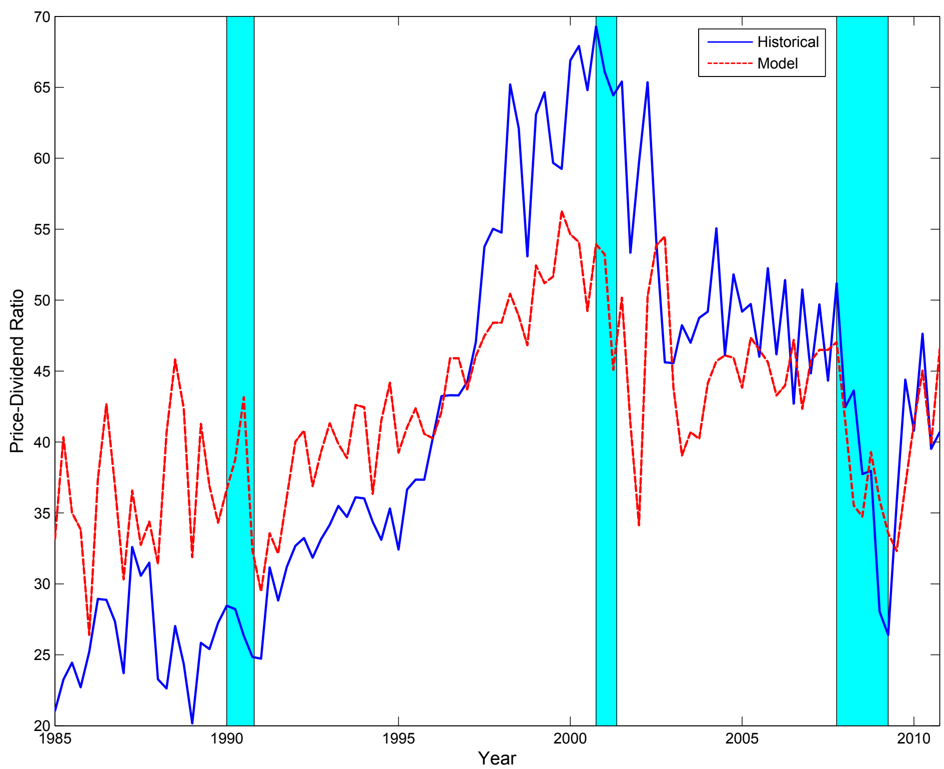
This figure shows empirical densities of A_t (black lines) and simulated densities (green and red lines). "Simulated_SR" denotes the simulated density of the estimated mean-reverting square-root process, with $\mu_{A,t} = \kappa_A(\overline{A} - A_t)$ and $\sigma_{A,t} = \sigma_a\sqrt{A_t}$ in Eq. (10), and "Simulated_OU" the corresponding Ornstein-Uhlenbeck process with $\sigma_{A,t} = \sigma_A$.

Figure 3: Optimal Default Boundary as a Function of the Ambiguity Level



This figure plots the optimal default barriers for a typical Baa-rated issuer, against the level of ambiguity. The solid blue line shows the default boundary in terms of the corporate earnings $\delta^*(A)$. The dashed red line shows the unlevered asset value at default $U^*(A)$. The optimal boundary is computed assuming the firm's cash flow starts at $\delta_0 = 100$.

Figure 4: Historical and Modeled Price-Dividend Ratios



This figure displays the realized time variations in price-dividend ratio and the model's prediction. The solid blue line shows the historical price-dividend ratio on the NYSE/AMEX/NASDAQ value-weighted index. The dashed red line shows the model-generated P/D ratio computed based on historical values of the ambiguity measure $\widetilde{A}_t$. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010.

Figure 5: Historical and Modeled Spreads in Yields between Baa and Default-Free Bonds

This figure displays the realized time variations in 10-year credit spreads for Baa bonds. The solid blue line shows the historical credit spread of Barclays U.S. Aggregate Corporate Baa Bond Index, and the dashed green line corresponds to Bank of America Merrill Lynch U.S. Corporate BBB Option-Adjusted Spread. The dotted red line shows the model-generated credit spread computed based on historical values of the ambiguity measure $\widetilde{A}_t$. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010.