# Centralized versus Decentralized Delegated Portfolio Management under Moral Hazard

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**Abstract**

If an investor wants to invest into multiple asset classes, should he delegate to a single portfolio manager to manage all asset classes (centralization)? Or should he delegate to multiple managers, each of whom exclusively manages one asset class (decentralization)? What if within the asset classes, managers could deviate from a high mean return strategy to a low mean return strategy to save private costs? We find: (i) Investors prefer decentralization when asset classes have vastly different mean returns, because investors can use their wealth allocations to temper managers from excessive risk taking; (ii) Investors could prefer centralization even in the absence of skill, because the single manager internalizes correlations across asset classes. Thus, moral hazard is a potential demand-side explanation for empirically why both specialist and generalist funds can simultaneously exist in the market.

**Keywords:** centralized delegated portfolio management, decentralized delegated portfolio management, portfolio choice, optimal contracting

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# 1 Introduction

Delegated portfolio management is a core activity in the modern financial markets — but what is the optimal form of delegation? If an investor wants to access, for example, an Asian macro strategy and an European macro strategy, should the investor delegate the execution of these two strategies to a single global strategy manager (*centralized delegation*)? Or should he delegate separately to an Asian strategy manager and an European strategy manager (*decentralized delegation*)? Moreover, suppose there is a strategy deviation moral hazard risk: instead of delivering the advertised Asian macro strategy, managers could privately deviate to a passive Asian equity index of a lower expected return that the investor could have accessed without delegation. And suppose analogous moral hazard risks could occur in the European asset class. In the presence of such moral hazard problem, which form of delegation is better: centralized delegation or decentralized delegation?

William F. Sharpe coined the term "decentralized investment management" in his Presidential Address to the *American Finance Association* 1981 Annual Meeting. Sharpe (1981, page 233) concludes with:

> There is, of course, much more to this problem [of decentralized investment management]. We have assumed away many important aspects of the principal-agent relationships(s). ... In short, we have clearly provided necessary and sufficient conditions for the traditional final sentence in such a paper: More research on this subject is needed.

Clearly the general literature in principal-agent theory and its specific applications to delegated portfolio management have significantly advanced in the years since 1981. Yet to the best of my knowledge, the problem of understanding the similarities and differences between centralized versus decentralized delegation with the presence of moral hazard remains unexplored, and its solution properties remain elusive. In particular, substantial recent empirical evidence (see literature review in Section 2 below) suggests that moral hazard risks are strongly present in hedge funds, via the forms of fraud, operational risk, misrepresentation of investment strategies and mixed evidence of managerial effort in generating alpha. The key contribution of this paper is an attempt to explore a question opened by Sharpe from decades ago and is made ever more imperative in the modern financial markets.

In this economy, there are two types of individuals: a single *Principal* and multiple *Managers*. The Principal is initially endowed with a single unit of wealth, while Managers have zero initial wealth. All individuals are risk averse with mean-variance preferences over their terminal wealth. The Principal has a strict desire for Managers to be *compliant* and implement a specific pair of investment strategies, and the Principal must delegate to Managers to access these strategies. However, implementing these strategies incurs private costs for the Managers. Moreover, Managers could deviate to alternative *deviant* strategies that are privately costless but have lower mean returns and different correlation structure than the Principal's desired pair of strategies. For simplicity, we take an extreme assumption that the Principal would abandon delegation if it is too costly to implement his desired strategy pair.

In the presence of such moral hazard over investment strategies within each of the two asset classes, the Principal needs to decide which form of delegation is best for him. In the first option, the Principal can choose *centralized delegation*: the Principal delegates all initial wealth to a single Manager $C$. Manager $C$ has two actions: investment strategy choice and portfolio allocation choice. Manager $C$ first needs to select a strategy pair, one strategy from each asset class. Taking any offered contract into account, Manager $C$ will then construct portfolio weights between this strategy pair. In return, the Principal compensates Manager $C$ with a linear contract over the net returns of the resulting portfolio.

Alternatively, in the second option, the Principal can choose *decentralized delegation*: the Principal makes a portfolio choice and decides how much of his initial wealth to delegate to Manager $A$ who will exclusively manage one asset class, and delegate the rest to Manager $B$ who will exclusively manage the other asset class. Both Managers can only pick one strategy from their respective asset classes. Again, the Principal compensates these two Managers with linear contracts over the net returns from their respective asset class.

## 1.1 Results overview

In first best with no moral hazard risk, where the Principal can observe and directly contract on the Managers' strategy choices in each asset class, the comparison of centralized versus decentralized delegation is a simple question of optimal risk sharing.

We next consider the second best case where moral hazard is distinctly present, in that

Managers can privately choose their investment strategies within said asset classes. In centralized delegation, for any given contract, Manager $C$ makes portfolio weight choices and strategy pair choices. A deviant pair of strategies have generically different mean returns with some correlation level than the compliant pair. Given that Manager $C$ has mean-variance preferences, when he deviates, he will naturally put higher portfolio weights to the deviant strategy of one asset class with a higher mean return and a lower portfolio weight to the deviant strategy of another asset class with a lower mean return. This generates a "long-short" trading profit benefit for Manager $C$ out of the deviant strategy pair that is not preferred by the Principal. Again, because the compliant pair has precisely the correct risk-return profile that the Principal strict desires, the Principal has a strict desire to be compliant and will not entertain any other deviant strategy pairs. Thus to ensure compliance, the Principal must compensate Manager $C$ with higher performance fees for the opportunity cost for Manager $C$'s foregone long-short trading profits out of the deviant pair, along with Manager $C$'s private costs for implementing the Principal's desired strategy pair. Furthermore, the Principal must compensate Manager $C$ for differences in contract volatilities that arise out of incentivizing Manager $C$ to implement the compliant pair as opposed to any other deviant pair.

In contrast, under decentralized delegation, even if Manager $A$ or Manager $B$ deviates, he can only deviate in strategies within his own asset class. The aforementioned long-short opportunity cost in centralization simply does not exist for them due to restriction in their respective investment opportunity set. Hence, to ensure compliance from the decentralized Managers, the Principal simply needs to compensate for their private costs, and the mean and volatility differences between the compliant and deviant strategies in their respective asset classes.

With these second best mechanisms of centralization versus decentralization, Table 1 summarizes and highlights the specific components that affect the Principal's decision for the best delegation form.

Of the several implications from Table 1, we highlight the first two. We leave the discussions of the other implications to the main text. As an illustration, the first implication suggests the Principal should not delegate to a single Manager $C$ to manage both equities and treasuries. Historically, equity has earned a higher expected return than treasuries due to the risk premium that equity commands; this is well understood in the asset pricing literature.

| Assets classes with | Cen | Dec | Corollary |
|---|---|---|---|
| Different mean returns | ✗ | ✓ | 6.1 |
| Similar mean returns | ✓ | ✗ | 6.2 |
| Large differences in correlations | ✗ | ✓ | 6.3 |
| Low volatilities | ✗ | — | 6.4 |
| High volatilities without diversification benefits | ✗ | — | 6.4 |
| High volatilities with diversification benefits | ✓ | — | 6.4 |
| Low managerial risk aversion | ✗ | — | 6.5 |

**Table 1:** (Second best) This table summarizes the key asset pricing comparative statics to the second best contracting environment. Here, "Cen" refers to centralized delegation, while "Dec" refers to decentralized delegation. A ✓ signifies it is favorable to the Principal for that form of delegation; a ✗ signifies it is unfavorable; a — signifies it is neither favorable nor unfavorable and we defer the details to the actual result statement.

However, in the context of delegated portfolio management, the concern for the Principal is that the single Manager $C$ could deviate and use excessive leverage (via shorting treasuries) to finance an excessive long position in equities. This "long-short" portfolio could be very contrary to the risk-return preferences of the Principal. Thus, in this illustration, centralized delegation is detrimental for the Principal, while decentralized delegation is beneficial. In decentralized delegation, the Principal directly contracts with, for instance, Manager $A$ who exclusively manages equity and Manager $B$ who exclusively manages treasuries. Since neither Manager $A$ nor Manager $B$ could trade each other's asset class, and the Principal assigns wealth allocations to the Managers, neither Manager $A$ nor Manager $B$ could engage in those detrimental "long-short" strategies. This argument for the Principal to find specialized managers in equities and treasuries is not based on an asset "expertise segmentation" argument as proposed by, for example, He and Xiong (2013). Rather, beyond an "expertise segmentation" supply-side argument, our model provides a potential explanation for why we empirically observe the existence of "specialist" funds that only specialize in one asset class — investors, on the demand-side, want to manage risk-return themselves to temper excessive risk taking by the managers.

The second implication of Table 1 suggests even in the absence of skill, centralized delegation could be better than decentralized delegation. As an illustration and as motivated from the introduction, suppose the Principal wants to access a globally diversified portfolio, specifically an Asian macro portfolio and an European macro portfolio. Suppose managers in this economy have no skill and cannot generate mean returns above their comparable index. That is, the macro strategies have similar expected returns to their comparable index, but it is still privately costly for the managers to execute their macro strategy. Furthermore suppose the two macro strategies have low correlations with each other. In this case, decentralized delegation is not favorable because when the Asian and European managers consider a deviation, they do not take into account the beneficial correlations of their strategies. However, in centralized delegation, as the single manager can construct portfolios across both asset classes, the single manager will internalize correlations when considering any strategy deviations. The single manager is less likely to deviate if the diversification benefits are sufficiently large, even in the absence of skill. Thus, in this case, centralized delegation could still be favorable for the Principal. Hence, our model provides a potential explanation for why investors are willing to invest into some "generalist" funds that have an apparent lack of skill — even in the absence of skill, investors may prefer the diversification benefits of having a single manager managing multiple asset classes.

## 2 Literature Review

The term "decentralized investment management" was first coined by Sharpe in his seminal paper Sharpe (1981) where he argues that an investor would prefer decentralization over centralization for "diversification of style" and "diversification of judgment".[1] Adding to Sharpe's argument, Barry and Starks (1984) demonstrate that risk sharing is another motive for preferring decentralization over centralization. Decentralized delegation is a very real issue faced by practitioners, as recognized in Elton and Gruber (2004), which offers conditions under which "a central decision maker can make optimal decisions without requiring decentralized decision makers to reveal estimates of security returns". More recently, van Binsbergen, Brandt, and Koijen (2008) study the decentralization problem in continuous time and derive the optimal wealth that the investor should allocate among decentralized managers. However, none of the references above have studied a moral hazard problem of any form in comparing centralization versus decentralization, but this is exactly the central research goal of this paper.

[1] While Rosenberg (1977) and Rudd and Rosenberg (1980) predate Sharpe (1981) on discussion of such delegation concepts, Sharpe (1981) consolidated the idea and offers a clearer call for research directions.

There is a small but growing empirical literature on comparing the effectiveness of centralized versus decentralized delegation. Blake, Rossi, Timmermann, Tonks, and Wermers (2013) document that pension fund managers have gravitated from a centralized delegation model to a decentralized delegation model. In the context of mutual funds, Dass, Nanda, and Wang (2013) compare the performance of sole- and team-managed balanced funds. Similarly, Kacperczyk and Seru (2012) ask whether centrally managed or decentrally managed mutual funds perform better.

Although our paper is not specific to the type of funds being delegated to, the prototypical example is hedge funds. Getmansky, Lee, and Lo (2015) and Agarwal, Mullally, and Naik (2015) are recent survey papers of the hedge fund industry. In particular, strong empirical evidence suggests that moral hazard is a substantial concern in hedge funds. Patton (2009) argues that a quarter of the funds that advertise themselves as "market neutral" have significant exposures to the market factor. Brown, Goetzmann, Liang, and Schwarz (2008, 2012) and Brown, Goetzmann, Liang, and Schwarz (2009) argue that proper due diligence to the extent of reducing operational risks of hedge funds is a source of alpha. Bollen and Pool (2012) constructs several performance flags based on hedge fund return patterns as indicators of increased fraud risks. Thus, in light of these empirical evidence, our paper takes strategy deviations as the core source of moral hazard.

Our paper belongs to the vast literature of delegated portfolio management. Stracca (2006) offers a survey on theory findings of delegated portfolio management. The contracting frictions in delegating to a *single* agent has been studied since at least Bhattacharya and Pfleiderer (1985) and Stoughton (1993). These papers are information based, whereby the principal delegates to an agent because the agent can exert private costly effort to acquire a signal of the future value of a security. In our paper, instead of adopting the private costly information acquisition motivation, we assume that the principal delegates to an agent because the principal faces access restrictions to the financial markets. A recent paper by He and Xiong (2013) uses a similar assumption in researching optimal investment mandate delegation. Again, in light of the aforementioned empirical evidence, moral hazard in the form of strategy deviations, and not simply information acquisition, will be the key friction to model in our paper.

Our model fits into the broad literature of optimal delegation forms. The recent work by Gromb and Martimort (2007) discusses the optimal design of contracts for experts who can privately collect a signal. In addition, their paper focuses on risk neutral individuals with limited liability and economies of scale of private costs. Whereas in our paper, we explicitly focus on how risk aversion can play a critical role in portfolio choice and we do not assume economies of scale in private costs. Some key earlier work on delegation to multiple agents are Demski and Sappington (1984), Demski, Sappington, and Spiller (1988), and Holmström and Milgrom (1991). Unlike our paper, these papers also do not explicitly consider the issue of portfolio choice in their moral hazard problems.

# 3 Model Setup

## 3.1 Individuals, Assets and Moral Hazard

There are two time periods $t = 0$ and $t = 1$. There are two classes of individuals: a single *Principal* and three *Managers* $A$, $B$ and $C$. The Principal is initially endowed with \$1 unit of wealth, and the Managers have \$0 initial wealth. Both the Principal and the Managers have mean-variance preferences over their own terminal wealth. The Principal has a risk aversion parameter of $\eta_{\mathrm{P}} > 0$, while the Managers have a risk aversion parameter of $\eta_{\mathrm{M}} > 0$.

There are two risky asset classes, indexed by $\theta$ and $\tau$. Within each asset class, there are two specific investment strategies $\{\mathrm{H}, \mathrm{L}\}$. Thus, for asset class $\theta$, the specific investment strategies are $\{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$, and for asset class $\tau$, they are $\{\tau_{\mathrm{H}}, \tau_{\mathrm{L}}\}$.

The Principal has no access to the financial markets and must delegate to the Managers. We make an assumption on the investment strategy from each asset class that the Principal strictly prefers. See Remark 3.3 for a discussion of the significance and restrictions of this assumption.

**Assumption 3.1.** *The Principal has a strict preference for the strategy pair* $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ *to be implemented over any other strategy pairs.*

Motivated by Assumption 3.1, we consider the "H" investment strategies to be *compliant*[2] and the "L" strategies to be *deviant*[3] . Likewise, we call the strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ the *compliant* strategy pair, and call any strategy pair $(\theta, \tau) \neq (\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ the *deviant* strategy

[2] The term *compliant* refers to the strategies that the Principal strictly prefers the Managers to implement.

[3] The term *deviant* refers to the strategies that the Principal strictly prefers the Managers to *not* implement.

pairs. As a concrete example, we may think of $\theta$ as the Asian equities and $\tau$ as European equities. Accordinginly, $\theta_{\mathrm{H}}$ can represent an active Asian macro equities strategy, while $\theta_{\mathrm{L}}$ can represent a passive Asian market index. Analogously, the $\tau_{\mathrm{H}}$ strategy can represent an active European macro equities, while $\tau_{\mathrm{L}}$ can represent a passive European market index.

With some abuse of notation, we use $\theta$ and $\tau$ to index the investment strategies under their respective asset classes $\theta$ and $\tau$. We use $\theta \in \{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$ to denote $\theta$ is an investment strategy from $\{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$ of the asset class $\theta$. Analogous comments for the expression $\tau \in \{\tau_{\mathrm{H}}, \tau_{\mathrm{L}}\}$ will apply. We write $R_\theta$ to denote the net return of a strategy $\theta \in \{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$ in the asset class $\theta$. Again, analogous comments for the notation $R_\tau$ for $\tau \in \{\tau_{\mathrm{H}}, \tau_{\mathrm{L}}\}$ will apply. The set of all possible strategy pair combinations from these two asset classes is then $\mathcal{S} := \{(\theta_{\mathrm{H}}, \tau_{\mathrm{H}}), (\theta_{\mathrm{H}}, \tau_{\mathrm{L}}), (\theta_{\mathrm{L}}, \tau_{\mathrm{H}}), (\theta_{\mathrm{L}}, \tau_{\mathrm{L}})\}$. We denote the set deviant strategy pairs as $\mathcal{S}_{-(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})} := \mathcal{S} \setminus \{(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})\}$.

For each asset class, the Managers can privately choose an investment strategy. However, if they choose to implement the Principal's desired strategies, they will incur a specific private cost. The private cost structure for choosing $(\theta, \tau)$ is,

$$c(\theta) = \begin{cases} c > 0, & \theta = \theta_{\mathrm{H}}, \\ 0, & \theta = \theta_{\mathrm{L}} \end{cases} \quad \text{and} \quad c(\tau) = \begin{cases} c > 0, & \tau = \tau_{\mathrm{H}}, \\ 0, & \tau = \tau_{\mathrm{L}}. \end{cases} \tag{3.1}$$

We may think of the source of this private cost as "effort", in the sense that the Managers need to expend energy to actively manage a more complex investment strategy that the Principal desires for any given asset class.[4]

We denote the means and variances of $\theta \in \{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$ as, $\mu_\theta := \mathbb{E}[R_\theta], \sigma_\theta^2 := \mathrm{Var}(R_\theta)$, respectively. We use analogous notations for $\tau \in \{\tau_{\mathrm{H}}, \tau_{\mathrm{L}}\}$. We denote the correlations of the pairs $(\theta, \tau)$ as $\rho_{\theta\tau} := \mathrm{Corr}(R_\theta, R_\tau)$, for $(\theta, \tau) \in \mathcal{S}$.

We make the following assumptions on the moments of the investment strategies.

**Assumption 3.2.** *Assume that,*

*1. The compliant strategies have identical means*[5]*,* $\mu \equiv \mu_{\theta_{\mathrm{H}}} = \mu_{\tau_{\mathrm{H}}}$*. Moreover, compliant*

[4] Here, we have assumed that both asset classes $\theta$ and $\tau$ have identical private costs but this can be readily relaxed without affecting the qualitative results.

[5] The equivalent means assumption can be easily relaxed at the expense of more complicated expressions of the results.

*strategies have higher means than the deviant ones,*

$$\Delta\mu_\theta := \mu_{\theta_\mathrm{H}} - \mu_{\theta_\mathrm{L}} = \mu - \mu_{\theta_\mathrm{L}} > 0,$$
$$\Delta\mu_\tau := \mu_{\tau_\mathrm{H}} - \mu_{\tau_\mathrm{L}} = \mu - \mu_{\tau_\mathrm{L}} > 0.$$

2. *The volatilities of all investment strategies are identical,*[6]

$$\sigma^2 \equiv \sigma_\theta^2 = \sigma_\tau^2, \quad \textit{for all } \theta, \tau.$$

3. *No perfect correlations between the investment strategies,*

$$|\rho_{\theta\tau}| < 1, \quad (\theta, \tau) \in \mathcal{S}.$$

**Remark 3.3.** *Assumption 3.1 is a critical assumption that has both technical and economic content and is clearly done with some loss of generality. From a technical perspective, as we shall see in the next section, this assumption simplifies the Principal's objective function. Without this assumption, the Principal needs to cycle through all four possible strategy pairs* $(\theta, \tau) \in \mathcal{S}$ *to compute which pair yields the highest value function for himself. While this computation is not difficult from a technical perspective, it is not particularly economically insightful. Furthermore, the sufficient conditions on the parameters that ensure* $(\theta_\mathrm{H}, \tau_\mathrm{H})$ *to be the optimal pair is not economically interesting.*

*Economically, however, this assumption can be motivated in one of the two following ways. Firstly, this assumption may be justified if the Principal has some existing investments that correlate favorably with the compliant pair* $(\theta_\mathrm{H}, \tau_\mathrm{H})$*. Thus, he wants to delegate to Managers who will compliantly implement* $(\theta_\mathrm{H}, \tau_\mathrm{H})$ *for him.*

*Secondly, this motivation can be further economically justified if the Principal has partial access to the financial markets. Suppose the Principal can directly and costlessly access both of the deviant strategies of each asset class,* $\theta_\mathrm{L}$ *and* $\tau_\mathrm{L}$*. Continuing from the opening example with Asian equities and European equities, the deviant strategies would then be a passive Asian index and a passive European index, both of which the Principal could access directly*

[6] The equivalent volatility assumption can be easily relaxed at the expense of more complicated expressions of the results.

*without delegation. If this were the case, the Principal would only want to delegate to have managers implement his preferred strategy pair* $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$*, which are, respectively, the Asian macro and European macro strategies. For instance, if the parameters are such that a deviant strategy pair* $(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})$ *yields higher value for the Principal than* $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$*, then the Principal only needs one outside Manager to manage the asset class* $\theta$ *because the Principal can very well manage* $(\tau_{\mathrm{L}})$ *by himself. Analogous comments apply for other deviant strategy pairs. If this were the case, we would have no meaningful discussion of centralized versus decentralized delegation as in our context.*

*Thus, for the remainder of the paper, Assumption 3.1 is strictly enforced.*

## 3.2 Delegation forms

In the presence of moral hazard over investment strategy choices for each asset class, how should the Principal delegate? For the rest of the paper, we focus on two forms of delegation — *centralized delegation* and *decentralized delegation*. In both forms of delegation, the Principal offers a *linear contract* over the portfolio's net returns.

**Remark 3.4.** *We emphasize that the core idea of the paper is to study the difference between centralized delegation versus decentralized delegation — under the linear contract form. That is, while we do solve for the optimal linear contract, we make no claims that the linear class is optimal. Indeed, while interesting from a contract theory perspective, contract optimality (and its inherent complexity) may detract from the core idea of the paper in understanding the differences between the delegation forms.*

### 3.2.1 Centralized delegation

In *centralized delegation*, the Principal delegates all initial wealth to a single Manager $C$. In the previous example, Manager $C$ can be a global strategy manager who manages both the Asian and European asset classes. Manager $C$ will be responsible for managing both asset classes $\theta$ and $\tau$. Given any contract, Manager $C$ will have both a strategy choice and a portfolio choice. Firstly, from each of the two asset classes, Manager $C$ will pick a strategy $\theta \in \{\theta_{\mathrm{H}}, \theta_{\mathrm{L}}\}$ and a strategy $\tau \in \{\tau_{\mathrm{H}}, \tau_{\mathrm{L}}\}$. Secondly, for each chosen strategy pair $(\theta, \tau)$, Manager $C$ will pick portfolio weight $1 - \psi$ into strategy $\theta$, and portfolio weight $\psi$ into strategy $\tau$. The resulting portfolio $(1 - \hat{\psi}_{(\theta,\tau)}, \hat{\psi}_{(\theta,\tau)})$ will have a net return $\hat{R}_{(\theta,\tau)}$. For

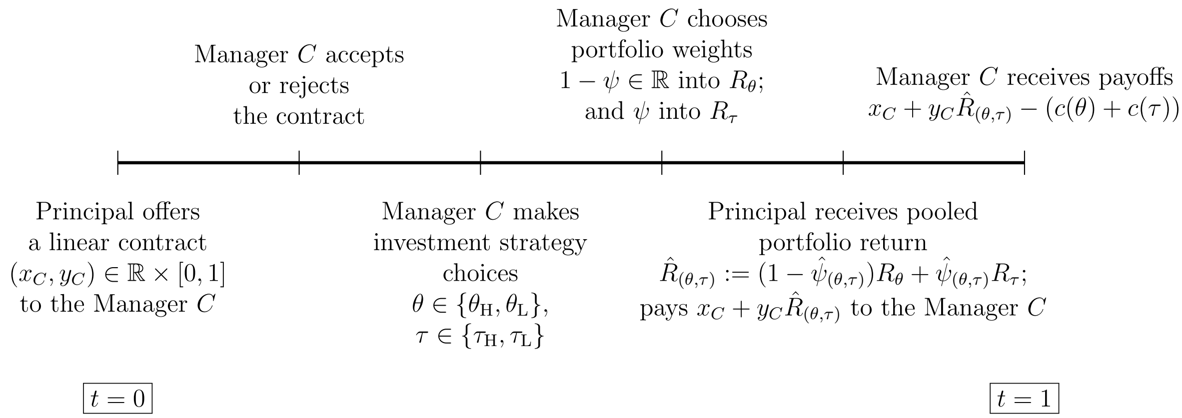


**Figure 1:** Centralized delegation timeline. Manager $C$ has a strategy choice from each asset class and has a portfolio choice between the selected pair of strategies. The Principal only has a contract design choice.

Manager $C$'s service, the Principal offers a linear contract $x_C + y_C \hat{R}_{(\theta,\tau)}$ over the portfolio net return, where $(x_C, y_C) \in \mathbb{R} \times [0,1]$. Thus, $x_C$ is a *fixed (percentage) fee*, while $y_C$ is a *performance (percentage) fee*. See Figure 1 for a timeline.

Thus, the optimization problem under centralized delegation is as follows.

$$\sup_{(x_C, y_C) \in \mathbb{R} \times [0,1]} \mathbb{E}[W_{cP}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}] - \frac{\eta_{\mathrm{P}}}{2} \mathrm{Var}(W_{cP}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}), \tag{Cen}$$

subject to,

$$\begin{aligned}
W_{cP}^{(\theta,\tau)} &:= 1 + \hat{R}_{(\theta,\tau)} - (x_C + y_C \hat{R}_{(\theta,\tau)}), && \text{(3.2a)}\\
W_{C}^{(\theta,\tau)} &:= -(c(\theta) + c(\tau)) + x_C + y_C \hat{R}_{(\theta,\tau)}, && \text{(3.2b)}\\
\tilde{W}_{C}^{(\theta,\tau)} &:= -(c(\theta) + c(\tau)) + x_C + y_C \left((1-\psi)R_\theta + \psi R_\tau\right), && \text{(3.2c)}\\
\hat{\psi}_{(\theta,\tau)} &:= \arg\sup_{\psi \in \mathbb{R}} \ \mathbb{E}[\tilde{W}_{C}^{(\theta,\tau)}] - \frac{\eta_{\mathrm{M}}}{2} \mathrm{Var}(\tilde{W}_{C}^{(\theta,\tau)}), && \text{(3.2d)}\\
\hat{R}_{(\theta,\tau)} &:= (1-\hat{\psi}_{(\theta,\tau)})R_\theta + \hat{\psi}_{(\theta,\tau)} R_\tau, && \text{(3.2e)}\\
0 &\le \mathbb{E}[W_{C}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}] - \frac{\eta_{\mathrm{M}}}{2} \mathrm{Var}(W_{C}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}), && \text{(3.2f)}\\
(\theta_{\mathrm{H}}, \tau_{\mathrm{H}}) &= \arg\max_{(\theta',\tau') \in \mathcal{S}} \ \mathbb{E}[W_{C}^{(\theta',\tau')}] - \frac{\eta_{\mathrm{M}}}{2} \mathrm{Var}(W_{C}^{(\theta',\tau')}). && \text{(3.2g)}
\end{aligned}$$

In (Cen), the Principal maximizes his mean-variance utility over his terminal wealth (3.2a), which is equal to the return from the Manager $C$ managed portfolio, less the fees that the Principal pays. In the optimization, the Principal needs to pick the optimal fixed fees $x_C$ and the optimal performance fees $y_C$ as part of the linear contract. Given the linear contract, Manager $C$ will construct the optimal portfolio, as in (3.2d), out of the two strategies, one each from the two asset classes, to obtain the portfolio returns in (3.2e). For Manager $C$'s service, his terminal wealth is (3.2b). The contract must be such that Manager $C$ is willing to participate and so (3.2f) is Manager $C$'s individual rationality constraint. In the second best case, the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ must also be incentive compatible for Manager $C$, which is (3.2g).

### 3.2.2 Decentralized delegation

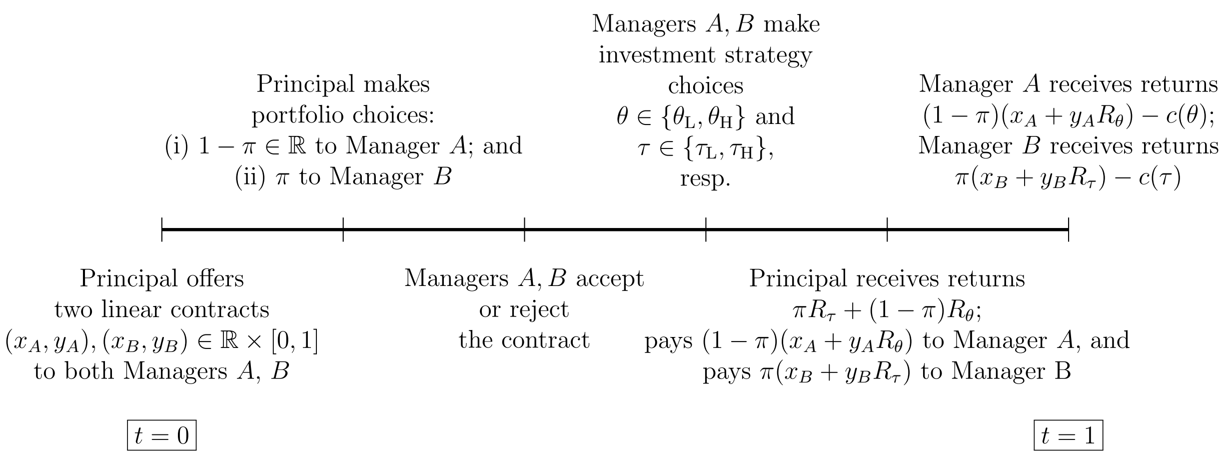


**Figure 2:** Decentralized delegation time line. In contrast to centralized delegation of Figure 1, the Principal now has both contract design choice and portfolio choice. Manager $A$ and Manager $B$ only have strategy choices within their own asset class.

In *decentralized delegation*, the Principal delegates wealth to two different individuals, Manager $A$ and Manager $B$. Manager $A$ is responsible for only managing asset class $\theta$, and Manager $B$ is responsible for only managing asset class $\tau$. Following the earlier example, Manager $A$ is an Asian asset class manager, while Manager $B$ is an European asset class manager. The Principal allocates $1-\pi$ portion of his initial wealth to Manager $A$ and $\pi$ proportion to Manager $B$. In return for the two individuals' services, the Principal offers a linear contract $(1-\pi)(x_A + y_A R_\theta)$ to Manager $A$, and $\pi(x_B + y_B R_\tau)$ to Manager $B$, where $(x_A, y_A), (x_B, y_B) \in \mathbb{R} \times [0,1]$.[7] Thus, $x_A, x_B$ represent the *fixed (percentage) fees* for, respectively, Manager $A$ and Manager $B$, whereas $y_A, y_B$ represent the *performance (percentage) fees*. Please see Figure 2 for the time line.

The optimization problem under decentralized delegation is as follows.

$$\sup_{(x_A,y_A),(x_B,y_B)\in\mathbb{R}\times[0,1]} \; \sup_{\pi\in[0,1]} \; \mathbb{E}[W_P^{(\theta_\mathrm{H},\tau_\mathrm{H})}] - \frac{\eta_\mathrm{P}}{2}\mathrm{Var}(W_P^{(\theta_\mathrm{H},\tau_\mathrm{H})}) \tag{Dec}$$

subject to,

$$W_P^{(\theta,\tau)} := 1 + \pi R_\tau + (1-\pi)R_\theta - \pi(x_B + y_B R_\tau) - (1-\pi)(x_A + y_A R_\theta) \tag{3.3a}$$

$$W_A^{\theta} := (1-\pi)(x_A + y_A R_\theta) - c(\theta) \tag{3.3b}$$

$$W_B^{\tau} := \pi(x_B + y_B R_\tau) - c(\tau) \tag{3.3c}$$

$$0 \le \mathbb{E}[W_A^{\theta_\mathrm{H}}] - \frac{\eta_\mathrm{M}}{2}\mathrm{Var}(W_A^{\theta_\mathrm{H}}), \quad \text{and} \quad 0 \le \mathbb{E}[W_B^{\tau_\mathrm{H}}] - \frac{\eta_\mathrm{M}}{2}\mathrm{Var}(W_B^{\tau_\mathrm{H}}) \tag{3.3d}$$

$$\theta_\mathrm{H} = \operatorname*{arg\,max}_{\theta'\in\{\theta_\mathrm{H},\theta_\mathrm{L}\}} \; \mathbb{E}[W_A^{\theta'}] - \frac{\eta_\mathrm{M}}{2}\mathrm{Var}(W_A^{\theta'}) \tag{3.3e}$$

$$\tau_\mathrm{H} = \operatorname*{arg\,max}_{\tau'\in\{\tau_\mathrm{H},\tau_\mathrm{L}\}} \; \mathbb{E}[W_B^{\tau'}] - \frac{\eta_\mathrm{M}}{2}\mathrm{Var}(W_B^{\tau'}) \tag{3.3f}$$

The Principal's objective (Dec) is to pick the optimal linear contracts to compensate the two Managers, and also to pick the optimal portfolio policy to decide how much wealth to allocate to the Managers' strategies. The Principal's terminal return (3.3a) is equal to the portfolio $(1-\pi, \pi)$ that the Principal decides to allocate to Manager $A$ and $B$'s strategy returns $(R_\theta, R_\tau)$, less the fees owed. Equations (3.3b) and (3.3c) represent, respectively, Manager $A$ and Manager $B$'s terminal wealth. The two Managers' participation constraints are in (3.3d). To induce Manager $A$ and Manager $B$ to pick the Principal's desired strategy pair $(\theta_\mathrm{H}, \tau_\mathrm{H})$, the Managers' incentive compatibility constraints are in (3.3e) and (3.3f).

[7] To actually have a feasible contract, we actually require that $\pi \ge 0$ and $1-\pi \ge 0$. Else, without this requirement, the Principal could demand infinitely large claw back payments from the two Managers. We will see in the subsequent that these conditions do not bind in the presence of the individual rationality constraints.

# 4 First Best

We begin by considering the first best setup, whereby the Principal can directly observe and contract on the private investment strategy choices of the Managers.

## 4.1 Centralized Delegation in First Best

For the first best centralized delegation case, consider problem (Cen) without the incentive compatibility constraint (3.2g).

**Proposition 4.1** (First best centralized delegation)**.** *Consider the first best centralized delegation problem; that is, problem* (Cen) *without the incentive compatibility constraint* (3.2g)*. Fix any strategy pair* $(\theta, \tau) \in \mathcal{S}$*.*

*(a) Given any linear contract* $(x_C, y_C) \in \mathbb{R} \times [0, 1]$*, the optimal portfolio weight to strategy* $\tau$ *of Manager* $C$ *is,* [8]

$$\hat{\psi}^{y_C}_{(\theta,\tau)} = \frac{1}{2}\left(1 + \frac{1}{y_C}\frac{\mu_\tau - \mu_\theta}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})}\right). \tag{4.1}$$

*(b) For any given contract* $(x_C, y_C)$*, the mean and variance of the portfolio return* $\hat{R}^{y_C}_{(\theta,\tau)}$ *are given by,*

$$\begin{aligned}
\mathbb{E}[\hat{R}^{y_C}_{(\theta,\tau)}] &= \frac{1}{y_C}\frac{(\mu_\theta - \mu_\tau)^2}{2\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})} + \frac{\mu_\theta + \mu_\tau}{2}, \\
\mathrm{Var}(\hat{R}^{y_C}_{(\theta,\tau)}) &= \frac{1}{y_C^2}\frac{(\mu_\theta - \mu_\tau)^2}{2\eta_{\mathrm{M}}^2\sigma^2\,(1-\rho_{\theta\tau})} + \sigma^2(1+\rho_{\theta\tau}).
\end{aligned}$$

[8] If one needs the value of $\hat{\psi}^{y_C}_{(\theta,\tau)}$ at $y_C = 0$, we will define it via the limit; that is, $\hat{\psi}^{0}_{(\theta,\tau)} := \lim_{y_C \downarrow 0} \hat{\psi}^{y_C}_{(\theta,\tau)}$. However, as we shall see, the optimal performance fee $y_C$ generically will not be reached at 0 (i.e. due to individual rationality of Manager $C$), and hence the point 0 is not really of concern. For subsequent expressions in this proposition that involves $y_C$ in the denominator, define it through the same limiting argument.

(c) *The optimal fixed and performance fees are, respectively,*

$$\hat{x}_C((\theta,\tau),y_C) = (c(\theta)+c(\tau)) - y_C\mathbb{E}[\hat{R}^{y_C}_{(\theta,\tau)}] + \frac{\eta_{\mathrm{M}}}{2}y_C^2\mathrm{Var}(\hat{R}^{y_C}_{(\theta,\tau)}), \quad \textit{for any } y_C \in [0,1]$$
$$\hat{y}_C^{FB} = \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}, \tag{4.2}$$

*and so under the optimal contract, the optimal portfolio is,*

$$\hat{\psi}_{(\theta,\tau)} = \hat{\psi}^{\hat{y}_C^{FB}}_{(\theta,\tau)} = \frac{1}{2}\left(1+\frac{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}{\eta_{\mathrm{P}}}\frac{\mu_\tau-\mu_\theta}{\eta_M\sigma^2(1-\rho_{\theta\tau})}\right).$$

*In particular, for implementing the Principal's desired investment strategy pair* $(\theta_{\mathrm{H}},\tau_{\mathrm{H}})$*, the optimal portfolio would be,*

$$\hat{\psi}^{y_C}_{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})} = \frac{1}{2}.$$

(d) *The Principal's value function for implementing* $(\theta_{\mathrm{H}},\tau_{\mathrm{H}})$*,*

$$\mathbb{E}[W^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}_{cP})\Big|_{FB} = -2c+1+\mu-\frac{1}{4}\frac{\eta_{\mathrm{P}}\eta_{\mathrm{M}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}\sigma^2(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}).$$

(e) *For any contract* $(x_C,y_C)$*, the Manager* $C$*'s utility for implementing investment strategy pair* $(\theta,\tau)$ *is,*

$$\begin{aligned}&\mathbb{E}[W^{(\theta,\tau)}_C] - \frac{\eta_{\mathrm{M}}}{2}\mathrm{Var}(W^{(\theta,\tau)}_C)\\ &= -(c(\theta)+c(\tau)) + x_C + \frac{(\mu_\theta-\mu_\tau)^2}{4\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})} + \frac{1}{2}(\mu_\theta+\mu_\tau)y_C - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta\tau})y_C^2,\end{aligned}$$

*and in particular for* $(\theta_{\mathrm{H}},\tau_{\mathrm{H}})$*, it is,*

$$\mathbb{E}[W^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}_C] - \frac{\eta_{\mathrm{M}}}{2}\mathrm{Var}(W^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}_C) = -2c + x_C + \mu y_C - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2.$$

In centralized delegation, for any given contract, the portfolio weight $\hat{\psi}^{y_C}_{(\theta,\tau)}$ into strategy $\tau$ made by Manager $C$ is clearly independent of the fixed fees $x_C$ and only dependent on the performance fee $y_C$. Moreover, the core idea in centralized delegation is that Manager $C$ takes any arbitrary contract (in particular, the performance fees) into account in making portfolio choice decisions. This is the fundamental idea in delegated portfolio management, well highlighted by Admati and Pfleiderer (1997) and other: a portfolio manager uses his access to the financial markets to manipulate and partially unwinds the effects of any incentive contracts that a Principal offers to him.

For the sake of the current argument, suppose strategy $\tau$ has a higher mean return than strategy $\theta$, so $\mu_\tau \geq \mu_\theta$ (the converse case is analogous). In this case, naturally Manager $C$ allocates higher portfolio weights $\hat{\psi}^{y_C}_{(\theta,\tau)}$ to strategy $\tau$ and less to strategy $\theta$. If the strategies have high correlations $\rho_{\theta\tau}$, it induces the Manager $C$ to almost take a "long-short" strategy whereby even more weights are allocated to $\tau$ and less are to $\theta$. Holding the performance fees $y_C$ as constant, these portfolio choice implications would be standard to any Markowitz-type mean-variance investor. However, taking the performance fees $y_C$ into account, we see that if the Principal offers very low performance fees, $y_C \downarrow 0$, then Manager $C$ acts almost like a risk-neutral individual and takes only infinitely long-short positions. In contrast, if the Principal offers very high performance fees, $y_C \uparrow 1$, then Manager $C$ makes portfolio choices that are identical to the Markowitz-type mean-variance investor. Thus, to optimally risk share in first best, the optimal performance fees $\hat{y}^{FB}_C$ is equal to the ratio of Principal's risk aversion $\eta_{\mathrm{P}}$ over the sum of both the Principal and Manager $C$'s risk aversions $\eta_{\mathrm{P}}+\eta_{\mathrm{M}}$. As we shall see, this effect of the performance fees $y_C$ on the portfolio choice $\hat{\psi}^{y_C}_{(\theta,\tau)}$ of Manager $C$ will play an important role for centralized delegation in the second best discussion (Section 5).

The optimal fixed fees $\hat{x}_C$ simply makes Manager $C$'s participation constraint (3.2f) bind; that is, the fixed fees are to simply compensate for Manager $C$'s private costs for taking on investment strategy pairs $(\theta, \tau)$, less Manager $C$'s share of the returns, and plus a volatility adjustment.

## 4.2 Decentralized Delegation in First Best

Next, we consider the first best decentralized delegation case. That is, consider the problem (Dec) without the incentive compatibility constraints (3.3e) and (3.3f).

**Proposition 4.2** (First best decentralized delegation). *Consider the first best centralized delegation problem; that is, problem* (Dec) *without the incentive compatibility constraints*

(3.3e) *and* (3.3f). *For any investment strategy* $\theta, \tau$, *define the quantities:*

$$\pi^o_{(\theta,\tau)} := \frac{1}{2}\left[1 + \frac{(\mu_\tau - \mu_\theta)(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1-\rho_{\theta\tau}))}{\eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})}\right],$$
$$y^o_{A,(\theta,\tau)} := \frac{\eta_{\mathrm{P}}\left[(\mu_\theta - \mu_\tau)(1-\rho_{\theta\tau})(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta\tau})) + \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1-\rho^2_{\theta\tau})\right]}{(\mu_\theta - \mu_\tau)\left[(\eta_{\mathrm{M}} + \eta_{\mathrm{P}})^2 - \eta_{\mathrm{P}}^2\rho^2_{\theta\tau}\right] + \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})[\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta\tau})]},$$
$$y^o_{B,(\theta,\tau)} := \frac{\eta_{\mathrm{P}}\left[(\mu_\theta - \mu_\tau)(1-\rho_{\theta\tau})(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta\tau})) - \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1-\rho^2_{\theta\tau})\right]}{(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta\tau}))\left[(\mu_\theta - \mu_\tau)(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1-\rho_{\theta\tau})) - \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta\tau})\right]}.$$

*Then,*

*(a) For any portfolio* $\pi$ *allocated to Manager* $B$ *and performance fees* $(y_A, y_B)$, *the optimal fixed fees are,*

$$\hat{x}_A(\theta, \pi, y_A) = \frac{1}{1-\pi}\left[c(\theta) - (1-\pi)y_A\mu_\theta + \frac{\eta_{\mathrm{M}}}{2}y_A^2(1-\pi)^2\sigma^2\right] \tag{4.3a}$$
$$\hat{x}_B(\tau, \pi, y_B) = \frac{1}{\pi}\left[c(\tau) - \pi y_B\mu_\tau + \frac{\eta_{\mathrm{M}}}{2}y_B^2\pi^2\sigma^2\right] \tag{4.3b}$$

*(b) If* $(\pi^o_{(\theta,\tau)}, y^o_{A,(\theta,\tau)}, y^o_{B,(\theta,\tau)}) \in (0,1)^3$, *then the optimal portfolio policy and optimal performance fee policy of the Principal for implementing strategy* $(\theta, \tau)$ *are* $(\pi^o_{(\theta,\tau)}, y^o_{A,(\theta,\tau)}, y^o_{B,(\theta,\tau)})$.

*(c) In particular, for implementing* $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$, *the optimal portfolio and performance fee polices are,*

$$(\hat{\pi}^{FB}, \hat{y}^{FB}_A, \hat{y}^{FB}_B) = \left(\frac{1}{2}\,,\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}\,,\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}\right),$$

*and the Principal's value function is,*

$$\mathbb{E}[W_P^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})})\Big|_{FB} = -2c + \mu - \frac{1}{4}\frac{\eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}.$$

In decentralization, the Principal allocates equal amount of wealth into Manager $A$ and Manager $B$, and offers the same performance fees to both Managers. This result is immediate since from Assumption 3.2, we had assumed that the compliant strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ have identical means and identical volatilities. Unlike the performance fees of centralization in Proposition 4.1, where the performance fees are simply $\eta_{\mathrm{P}}/(\eta_{\mathrm{P}} + \eta_{\mathrm{M}})$, the performance fees in decentralization must take into account the correlations $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ of the strategies. In centralization, risk management is internalized by the single Manager $C$, and hence the resulting performance fees only need to depend on the risk aversions of the individuals. However, with decentralization, the Principal must handle risk management himself and thus the performance fees must reflect the correlations of strategies, in addition to the individuals' respective risk aversions.

## 4.3 Comparison between Centralized Delegation versus Decentralized Delegation in First Best

Now we can compare centralized delegation versus decentralized delegation under first best.

**Proposition 4.3** (First best centralization versus decentralization)**.** *The difference between the Principal's value function in first best decentralized delegation and first best centralized delegation is,*

$$\left(\mathbb{E}[W_P^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})})\right)\Big|_{FB} - \left(\mathbb{E}[W_{cP}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP}^{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})})\right)\Big|_{FB}$$
$$= \frac{\eta_{\mathrm{M}}\eta_{\mathrm{P}}^2\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\sigma^2}{4(\eta_{\mathrm{P}}+\eta_{\mathrm{M}})(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))}.$$

*Decentralized delegation is better than centralized delegation if and only if the returns of the Principal's desired strategy pair* $(\theta_{\mathrm{H}},\tau_{\mathrm{H}})$ *are strictly positively correlated* $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} > 0$*. Conversely, centralized delegation is better than decentralized delegation if and only if the strategies are strictly negatively correlated* $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < 0$*. The two forms of delegation are equivalent when the strategies are uncorrelated* $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} = 0$*.*

Firstly, by Assumption 3.2, the first best portfolio choices between centralization (Proposition 4.1) and decentralization (Proposition 4.2) are identical: in centralization, Manager $C$ would put equal portfolio weights into strategy $\theta_{\mathrm{H}}$ and $\tau_{\mathrm{H}}$; in decentralization, the Principal would put equal weights into Manager $A$ (who manage strategy $\theta_{\mathrm{H}}$) and Manager $B$ (who manager strategy $\tau_{\mathrm{H}}$). Thus, the essential difference between centralization and decentralization under first best comes down to the performance fee policies, which is then an issue of optimal risk sharing.

When the correlation between the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ is strictly negative, $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < 0$, delegating to a single Manager $C$ is beneficial. Given that Manager $C$ puts equal long positions into both investment strategies $\theta_{\mathrm{H}}$ and $\tau_{\mathrm{H}}$ and is risk averse, a strictly negative correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ lowers the contract volatility for Manager $C$, and thereby it is cheaper for the Principal to risk share with Manager $C$. When the correlations become strictly positive, $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} > 0$, the reverse happens, and decentralized delegation is more appealing to the Principal. When the correlations become positive, delegating to a single Manager $C$ increases Manager $C$'s contract volatility, and thereby making it more expensive for the Principal to risk share. However, with decentralized delegation, neither Manager $A$ nor Manager $B$ directly absorbs the positive correlation effects. Thus, the Principal, via his own portfolio choice, can make it cheaper to risk share with the decentralized Managers. Finally, in the case when the strategies are uncorrelated, $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} = 0$, both centralized and decentralized delegation are identical, since neither the centralized nor decentralized Managers(s) are affected by the correlation structure directly for the purpose of risk sharing.

# 5 Second Best

We come to the core results of the paper. In second best, the Principal cannot directly observe nor contract on the specific investment strategies that the Managers choose within each asset class. In both the second best centralized delegation (Proposition 5.1 below) and the second best decentralized delegation (Proposition 5.2 below), the key emphasis is, respectively, the performance fees and the optimal portfolio policies. In contrast, the fixed fees (i.e. $x_C$ in centralization; and $x_A, x_B$ in decentralization) are relatively straightforward. In both cases, the optimal fixed fees ensure that the Managers will participate and accept the contract. Furthermore, the fixed fees compensate the Managers for their private costs, less the expected performance fee amount payoff, plus a volatility adjustment. This fixed fee form is standard in all linear contracting setups, and hence we focus the remainder of the paper on the performance fees and the portfolios.

## 5.1 Centralized delegation

First state the second best results for centralized delegation.

**Proposition 5.1** (Second best centralized delegation)**.** *Consider the second best centralized delegation problem* (Cen)*. Then:*

*(a) For any performance fee $y_C \in [0,1]$, the optimal fixed fees have the same form as that of first best in* (4.2) *of Proposition 4.1.*

*(b) For any performance fees $y_C \in [0,1]$ and investment strategy pair $(\theta,\tau) \in \mathcal{S}$, the optimal portfolio $\hat{\psi}^{y_C}_{(\theta,\tau)}$ chosen by Manager C is the same as the first best form* (4.1) *of Proposition 4.1.*

*(c) The (three) incentive compatibility constraints on the performance fees $y_C$ for inducing Manager C to implement the strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ are,*

$$\begin{aligned} &-2c + \frac{1}{2}(\mu_{\theta_{\mathrm{H}}} + \mu_{\tau_{\mathrm{H}}})y_C - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2 \\ &\quad \geq -(c(\theta') + c(\tau')) + \frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} + \frac{1}{2}(\mu_{\theta'} + \mu_{\tau'})y_C - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta',\tau'})y_C^2, \end{aligned} \tag{5.1}$$

*for $(\theta',\tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. Moreover, three incentive compatibility constraints* (5.1) *can be equivalently written as a single constraint,*

$$\begin{aligned} 0 \geq \max_{(\theta',\tau')} \Big\{ &-(c(\theta') + c(\tau') - 2c) + \frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} \\ &+ \frac{1}{2}(\mu_{\theta'} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau'} - \mu_{\tau_{\mathrm{H}}})y_C - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2 \Big\}, \end{aligned} \tag{5.2}$$

*where the maximum is taken over the possible deviant strategy pairs $(\theta',\tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. Recall from Assumption 3.2, $\mu \equiv \mu_{\theta_{\mathrm{H}}} = \mu_{\tau_{\mathrm{H}}}$.*

*(d) If the net cost for Manager C to comply and implement the Principal's desired strategy pair $(\tau_{\mathrm{H}}, \tau_{\mathrm{H}})$ is sufficiently low, the Principal will achieve first best. That is, if*

$$\begin{aligned} 0 > \max_{(\theta',\tau')} \Big\{ &-(c(\theta') + c(\tau') - 2c) + \frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} \\ &+ \frac{1}{2}(\mu_{\theta'} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau'} - \mu_{\tau_{\mathrm{H}}})\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}} + \eta_{\mathrm{M}}} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\left(\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}} + \eta_{\mathrm{M}}}\right)^2 \Big\} \end{aligned} \tag{5.3}$$

*then the optimal performance fee is* $\hat{y}_C = \hat{y}_C^{FB}$.

(e) *Suppose the net costs for compliance to Manager* $C$ *is sufficiently high; that is, replace* $>$ *in* (5.3) *with* $\leq$. *If an optimal performance fee* $\hat{y}_C \in [0,1]$ *exists, there necessarily exists some (unique) pair of deviant strategy pair* $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}}) \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$ *that yields the highest net deviation benefit for Manager* $C$. *Consider the following two conditions on* $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$:

(i) *For any strategy pair* $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$, *define (i.e. the quadratic discriminant),*

$$D_{(\theta',\tau')} := \frac{1}{4}\left(\mu_{\theta'} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau'} - \mu_{\tau_{\mathrm{H}}}\right)^2 + \left[-\left(c(\theta') + c(\tau') - 2c\right) + \frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})}\right]\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}). \tag{5.4}$$

*Suppose the strategy pair* $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$ *is such that,*

$$D_{(\theta^b,\tau^b)} \geq 0.$$

(ii) *For the pair* $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$, *define (i.e. the positive quadratic root),*

$$\tilde{y}_{+,(\theta^{\mathrm{b}},\tau^{\mathrm{b}})} := \frac{1}{2}\left(-\frac{\mu_{\theta^{\mathrm{b}}} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau^{\mathrm{b}}} - \mu_{\tau_{\mathrm{H}}}}{2} + \sqrt{D_{(\theta^{\mathrm{b}},\tau^{\mathrm{b}})}}\right) \times \left[-(c(\theta^{\mathrm{b}}) + c(\tau^{\mathrm{b}}) - 2c) + \frac{1}{4}\frac{(\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta^{\mathrm{b}},\tau^{\mathrm{b}}})}\right]^{-1}. \tag{5.5}$$

*Suppose the pair is* $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$ *is such that,*

$$\tilde{y}_{+,(\theta^{\mathrm{b}},\tau^{\mathrm{b}})} \in [0,1].$$

*If both conditions (i) and (ii) hold, then the second best performance fee is* $\hat{y}_C = \tilde{y}_{+,(\theta^b,\tau^b)}$. *If neither condition (i) nor (ii) hold, then no second best contract exists for centralized delegation.*

The right hand side of the incentive compatibility condition (5.2) is the "net cost" for Manager $C$ for being compliant instead of being deviant. We can rewrite and decompose the incentive compatibility condition to the following subparts:

$$0 \geq \max_{(\theta',\tau')} \Big\{ \underbrace{2c - (c(\theta') + c(\tau'))}_{\text{(i) Private costs}} + \underbrace{\frac{1}{2}(\mu_{\theta'} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau'} - \mu_{\tau_{\mathrm{H}}})y_C}_{\text{(ii) Mean performance fees}}$$

$$+ \underbrace{\frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})}}_{\substack{\text{(iii) Trading profits}\\ \text{opportunity costs}}} + \underbrace{\frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} - \rho_{\theta',\tau'})y_C^2}_{\text{(iv) Contract volatility}} \Big\}.$$

(v) Correlation interaction

Let's discuss each of these subparts.

(i) *Private costs*: By being compliant and picking $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$, Manager $C$ needs to incur private costs of $c(\theta_{\mathrm{H}}) + c(\tau_{\mathrm{H}}) = 2c$, but by deviating to $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$, the private costs are strictly lowered to $c(\theta') + c(\tau')$. Hence, $2c - (c(\theta') + c(\tau'))$ represents the net private costs for complying instead of deviating. These effects are common in practically all standard principal-agent models. However, as we will discuss below in (ii) to (v), there are additional effects that arise solely because of Manager $C$'s ability to take an arbitrary contract offered by the Principal, and then subsequently form portfolios upon it.

(ii) *Mean performance fees*: Incentive compatibility for Manager $C$ also comes in the form of the net return differences from implementing the compliant investment strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ versus a deviant pair $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. By implementing the compliant pair and recalling the optimal portfolio choices, Manager $C$ gains an expected performance fee payoff of $(\mu_{\theta_{\mathrm{H}}} + \mu_{\tau_{\mathrm{H}}})y_C/2 = \mu y_C$, whereas by implementing a deviant pair, Manager $C$ has the expected performance fee payoff of $(\mu_{\theta'} + \mu_{\tau'})y_C/2$. However, for the compliant investment strategies, $\mu = \mu_{\theta_{\mathrm{H}}} \geq \mu_{\theta'}$ and $\mu = \mu_{\tau_{\mathrm{H}}} \geq \mu_{\tau'}$ (with one of these weak inequalities being strict). Thus, by being compliant, Manager $C$ enjoys a net gain of $(2\mu - \mu_{\theta'} - \mu_{\tau'})y_C/2 > 0$ in higher performance fee payoffs.

(iii) *Trading profits opportunity costs*: This represents the opportunity cost in trading profits for Manager $C$ by being compliant rather than deviating to an alternative strategy pair

$(\theta', \tau')$. Under the compliant strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$, we had assumed that they have equivalent means $\mu$ and equivalent volatility $\sigma$. Consequently, Manager $C$ would put equal weights into both investment strategies, and hence the optimal portfolio weights would be independent of Manager $C$'s risk aversion, strategies' volatility $\sigma$, and the correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$. In contrast, under the deviant investment strategy pairs $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$, the mean returns of $\theta'$ and $\tau'$ could be different, and the correlation $\rho_{\theta',\tau'}$ is also generically different than the correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ of the compliant pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$. Thus, when Manager $C$ deviates, he is likely to engage into a long-short strategy in the deviant pair. This constitutes a benefit for Manager $C$ that is foregone by being compliant, and hence is an opportunity cost for Manager $C$ that the Principal needs to compensate for in the form of higher performance fees.

(iv) *Contract volatility*: Incentive compatibility for Manager $C$ comes in the form of differences in the contract volatility under the compliant strategy pair and that of deviant strategy pairs. For any investment strategy pair $(\theta, \tau)$, the contract volatility for Manager $C$ is $\sigma^2(1 + \rho_{\theta,\tau})y_C^2$. Adjusting for Manager $C$'s risk aversion, the term $-\frac{1}{4}\eta_{\mathrm{M}}\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2$ is the net change in contract volatility for Manager $C$ from taking the compliant pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ versus a deviant pair $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. The signs of the correlations matter. If $\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} > 0$, that is the deviant strategy $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$ has a strictly higher correlation than the compliant strategy pair, then this represents a net benefit for Manager $C$. Since Manager $C$ is risk averse, picking the compliant strategy pair with a lower correlation is beneficial, and so being compliant reduces the contract volatility. For the reverse case, when $\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < 0$, being compliant increases the contract volatility.

(v) *Correlation interaction term*: Finally, there is an interaction between the (c) trading profits opportunity costs and (d) contract volatility for Manager $C$. On the one hand, a higher correlation $\rho_{\theta',\tau'}$ increases the contract volatility for Manager $C$ when considering a deviation to $(\theta', \tau')$, and is thus detrimental to Manager $C$. On the other hand, a higher $\rho_{\theta',\tau'}$ increases the long-short trading benefit for Manager $C$, and is thus beneficial for Manager $C$.

Finally, the Principal wants to incentivize Manager $C$ as cheaply as possible, which is equivalent to binding the incentive compatibility constraints with the minimal net costs to

Manager $C$ across all possible strategy deviations. See also Corollary B.1 for the explicit conditions on the parameters under which which strategy pair $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$ is the most profitable deviation for Manager $C$.

## 5.2 Decentralized delegation

Next, we state the second best result for decentralized delegation.

**Proposition 5.2** (Second best decentralized delegation)**.** *Consider the second best decentralized delegation case; that is consider, problem* (Dec) *in its entirety.*

*(a) For any portfolio $\pi$ and performance fee policies $(y_A, y_B)$, the optimal fixed fees have the form* (4.3) *of first best decentralization in Proposition 4.2.*

*(b) The incentive compatibility conditions to induce the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ are,*

$$0 \geq c - (1-\pi) y_A \Delta\mu_\theta, \tag{5.6a}$$

$$0 \geq c - \pi y_B \Delta\mu_\tau. \tag{5.6b}$$

*(c) Suppose the private costs $c$ are moderately high*[9] *, then the second best decentralized optimal policies are,*

$$(\hat{\pi}, \hat{y}_A, \hat{y}_B) = \left( \frac{1}{2}\left[1 + \frac{\Delta\mu_\tau - \Delta\mu_\theta}{\Delta\mu_\theta \Delta\mu_\tau} c\right] ,\ \frac{2\Delta\mu_\tau c}{c(\Delta\mu_\tau - \Delta\mu_\theta) + \Delta\mu_\theta\Delta\mu_\tau} ,\ \frac{2\Delta\mu_\theta c}{c(\Delta\mu_\theta - \Delta\mu_\tau) + \Delta\mu_\theta\Delta\mu_\tau} \right).$$

The right hand sides of the two incentive compatibility conditions (5.6) under decentralized delegation represent the "net cost" for Manager $A$ and Manager $B$ for being compliant instead of being deviant. We note the incentive compatibility conditions can be decomposed

[9] The precise conditions for this are in Proposition B.2(aiv). See also Proposition B.2 for further details of the optimal policies of second best decentralized delegation.

as:

$$0 \geq \underbrace{c}_{\substack{\text{(a.A)} \\ \text{Private costs}}} - \underbrace{(1-\pi)y_A\Delta\mu_\theta}_{\text{(b.A) Portfolio mean}}, \quad 0 \geq \underbrace{c}_{\text{(a.B) Private costs}} - \underbrace{\pi y_B \Delta\mu_\tau}_{\text{(b.B) Portfolio mean}}, \tag{5.7}$$

Let's discuss each of these subparts.

(a) *Private costs:* This is the standard private cost effects in most moral hazard models. By being compliant and implementing strategy $\theta_{\text{H}}$, Manager $A$ needs to incur a private cost of $c(\theta_{\text{H}}) = c$. Likewise, when Manager $B$ is compliant and implements strategy $\tau_{\text{H}}$, Manager $B$ incurs a private cost of $c(\tau_{\text{H}}) = c$.

(b) *Portfolio mean:* Manager $A$ is allocated $1 - \pi$ portion of the Principal's wealth and is given a performance fee of $y_A$. When Manager $A$ implements strategy $\theta$, his expected performance fees are $(1-\pi)y_A\mu_\theta$. Thus, when Manager $A$ chooses the compliant strategy, he must forgo the expected performance fees that are generated from the deviant strategy. In all, the expected net benefit for being compliant for Manager $A$ is $(1-\pi)y_A\mu_{\theta_{\text{H}}} - (1-\pi)y_A\mu_{\theta_{\text{L}}} = +(1-\pi)y_A\Delta\mu_\theta$; or equivalently, the expected net cost for being compliant is $-(1-\pi)y_A\Delta\mu_\theta$. The discussion for Manager $B$ is analogous.

Finally, the Principal wants to incentive Manager $A$ and Manager $B$ as cheaply as possible, which is equivalent to binding the incentive compatibility constraints.

**Remark 5.3.** *In the incentive compatibility conditions* (5.6) *of decentralized delegation, while there is a term for portfolio mean returns, but there is no analogous term for portfolio volatility. This is due to Assumption 3.2. If the volatilities of the investment strategies differ, then both Manager A and Manager B will also consider the volatility differences between the compliant versus the deviant strategies.*

# 6 Second Best Centralization versus Decentralization

We compare the similarities and differences in contracting between centralization and decentralization.

## 6.1 Investment opportunity set

We start by considering the effects of the investment opportunity set and their associated asset pricing parameters on the two forms of delegation.

**Corollary 6.1** (Different mean returns)**.** *If the mean return difference between asset classes is* large*, there does not exist an optimal contract in centralized delegation.*

An illustration of Corollary 6.1 could be equity hedge funds and treasuries. Historically, the mean returns of equity hedge funds have been higher than that of treasuries. [10] In particular, this illustration implies that the Principal might prefer decentralized delegation, where he delegates the equity portion of his portfolio to an equity-only manager, and likewise for the treasuries portion. It should be noted this argument is completely independent of a "specialization" or "segmentation" (see He and Xiong (2013)) reasoning, where one might argue that an equity manager becomes an equity manager because he has specialized knowledge in equities. Corollary 6.1 suggests that a Principal might not prefer to delegate equities and treasuries to a single manager because of moral hazard concerns in centralized delegation. In this illustration, the moral hazard concern is simply that the single manager would use high leverage via treasuries to finance excessive positions in equity.

**Corollary 6.2** (Similar mean returns)**.** *Suppose the mean returns of strategies within each asset class are* similar*. Then,*

*(a) There does not exist an optimal contract in decentralized delegation.*

*(b) Suppose further the mean returns between the asset classes are similar. Then if the diversification benefits for the compliant investment strategy pair are favorable, an optimal contract will exist in centralized delegation.*

As an illustration, Corollary 6.2(a) implies if portfolio managers lack skill in their own asset class (e.g. it is privately costly to generate alpha, but the alpha is close to zero), decentralized delegation is unfavorable. Corollary 6.2(b) suggests that when there is no skill

[10] Note that the discussion here does not take risk into play. Of course, understanding that hedge funds are riskier than treasuries, it is not surprising that hedge funds have a higher mean return than that of treasuries. And more importantly within the context of our model, we have assumed all strategies have identical volatilities — but it is not difficult to see that a similar statement can be made for risk-adjusted returns for strategies with different volatilities.

within each asset class, and even when the mean returns between the asset classes are similar, centralized delegation could still be favorable as long as there is sufficient diversification benefits by having one single manager managing multiple asset classes.

**Corollary 6.3** (Correlations)**.** *(a) The contracting environment in decentralized delegation is unaffected by correlations of the return strategies.*

*(b) Suppose the return correlation between asset class tends to be* positive and large*. Then, no contract will exist in centralized delegation.*

Corollary 6.3(a) is immediate as each of the decentralized managers is only offered a contract based on their own strategy performance, and hence the correlations of the strategies do not affect the contracting environment. Corollary 6.3(b) is in contrast with the result of Corollary 6.2(b): Corollary 6.2(b) suggests that even in the absent of skills, centralized delegation could be favorable due to possible diversification benefits; however, Corollary 6.3(b) suggests when the diversification benefits are nonexistent, centralized delegation is unfavorable because the single manager has greater incentives to long-short deviant strategy pairs, which are contrary to what the Principal desires. Please see again the discussions after Proposition 5.1, especially the discussions on (iii) trading profits and (v) correlation interaction term.

**Corollary 6.4** (Volatility)**.** *(a) When the volatilities of the investment strategies within an asset class are similar, those volatilities do not affect the contracting environment in decentralized delegation.*

*(b) Investment strategies with extremely* low *volatilities are unfavorable for centralization.*

*(c) Suppose the return correlation between the compliant strategies is* higher *than all of the deviant strategies; that is, suppose $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} \geq \rho_{\theta',\tau'}$ for all $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. Then investment strategies with extremely* high *volatilities are* unfavorable *for centralization.*

*(d) Conversely, suppose the return correlation between the compliant strategies is* lower *than all of the deviant strategies; that is, suppose $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < \rho_{\theta',\tau'}$ for all $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. Then investment strategies with extremely* high *volatilities are* favorable *for centralization, and indeed the first best result can be reached.*

As foreshadowed in the first best discussions of Section 4, it is not surprising to see again that risk (namely volatility and correlations) plays a significant role in delineating whether centralization or decentralization is better in second best. By linear contract assumption and Assumption 3.2 that all strategies have identical volatilities, Corollary 6.4(a) is immediate. [11]

In Corollary 6.4(b), we see that asset classes whose investment strategies have low volatilities are unfavorable for centralization. For investment strategies that have low volatilities, the centralized Manager $C$ would prefer engaging into a "long-short" position, whereby he takes an extreme long position into an asset class whose strategies have high mean returns, and takes an extreme short position into an asset class whose strategies have low mean returns. These extreme long-short positions generate far riskier portfolios than what the Principal would prefer. See also again the discussions of (iii) trading profits after Proposition 5.1.

In Corollary 6.4(c), if the compliant strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ as desired by the Principal has a strictly higher correlation than all of the other deviant pairs, and if the investment strategies have high volatilities, centralization is unfavorable. This is precisely the case when the diversification benefits of centralization are severely mitigated. Observe again the discussions after Proposition 5.1. If Manager $C$ had been compliant and chosen the strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$, for any performance fees $y_C \in [0,1]$, it would result in an trading profit opportunity cost of $\frac{1}{4}\frac{(\mu_{\theta'}-\mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})}$, and a net contract volatility of $\frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} - \rho_{\theta',\tau'})y_C^2$. In the case where $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} \geq \rho_{\theta',\tau'}$, we see that the contract volatility for taking on the compliant strategy pair for Manager $C$ is higher than deviating to other deviant strategy pairs $(\theta', \tau')$. If the volatility $\sigma^2$ of the investment strategies increases, then on the one hand, this magnifies the contract volatility for taking on the compliant pair. On the other hand, the trading profit opportunity costs are minimized because the deviant strategy pairs have inherently higher risk. All together, this implies in a high volatility environment for all asset classes, the excessive contract volatility for being compliant would certainly lead to Manager $C$ to

[11] Even if we were to assume the volatilities of investment strategies are different, it is straightforward to see that the right-hand side of (5.6) would simply have additional terms $+\frac{\eta_{\mathrm{M}}}{2}(1-\pi)^2 y_A^2(\sigma^2_{\theta_{\mathrm{H}}} - \sigma^2_{\theta_{\mathrm{L}}})$ and $+\frac{\eta_{\mathrm{M}}}{2}\pi^2 y_B^2(\sigma^2_{\tau_{\mathrm{H}}} - \sigma^2_{\tau_{\mathrm{L}}})$ for Manager $A$ and Manager $B$, respectively. Depending on the sign of $\sigma^2_{\theta_{\mathrm{H}}} - \sigma^2_{\theta_{\mathrm{L}}}$ and $\sigma^2_{\tau_{\mathrm{H}}} - \sigma^2_{\tau_{\mathrm{L}}}$, the Principal either pays additional fees for increased volatility risk imposed on the Managers, or get savings in fees for decreased volatility risk. Regardless, the key point is that the volatility terms enter linearly into the incentive compatibility conditions for decentralization; this will not be the case for centralization where volatility enters non-linearly into its incentive compatibility condition.

deviate to other deviant strategy pairs.

Corollary 6.4(d) is the converse to (c): when the compliant strategy pair have lower correlations than all of the other deviant strategy pairs, so $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < \rho_{\theta',\tau'}$, then even in a high volatility environment, diversification benefits for centralized delegation are still present. Indeed, in this situation with high volatility, we see that the trading profit opportunity costs become close to nil. Moreover, the deviant strategy pair would always generate a higher contract volatility for Manager $C$ than that of the compliant strategy pair. Within two scenarios, Manager $C$ does not deviate and subsequently, the Principal will reach the first best result.

## 6.2 Managerial risk aversions

**Corollary 6.5** (Risk aversions). *(a) For asset classes whose investment strategies have similar volatilities, managerial risk aversion does not affect the contracting environment in decentralized delegation.*

*(b) When managers have sufficiently low risk aversion, a contract may fail to exist in centralized delegation.*

Corollary 6.5(b) is completely the opposite of standard principal-agent theories.[12] Those theories suggest that it should be cheaper for a principal to compensate a less risk averse agent, because the principal pays the agent a lower risk premium for bearing risk. Corollary 6.5 suggests it is the reverse in this economy — the less risk averse Manager $C$ becomes, the more expensive it is to compensate him. For any given contract, Manager $C$ can simply use the financial markets to modify the intended incentives of the contract.[13]

In centralization, suppose Manager $C$ becomes less risk averse, so $\eta_{\mathrm{M}} \downarrow 0$. Firstly, Manager $C$ becomes less concerned with the volatility difference of the contract, that is $\frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2 \to 0$, for all deviant strategy pairs $(\theta',\tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. Secondly, when Manager $C$ considers a deviation, Manager $C$ cares less about the volatilities of the deviant strategy pairs $(\theta',\tau')$ and less of the correlation of their returns $\rho_{\theta',\tau'}$. Indeed, as Manager $C$ becomes less risk averse, he only cares about the absolute difference $|\mu_{\theta'} - \mu_{\tau'}|$

[12] Say Laffont and Martimort (2001) and Bolton and Dewatripont (2004).

[13] Admati and Pfleiderer (1997, Section V) makes a related point that benchmarked compensations are not relevant to soliciting effort.

between the deviant strategies. In the limit when Manager $C$ becomes risk neutral, he takes an infinitely large long position into strategy with highest mean, and takes an infinitely large short position into the strategy with the lowest mean. Thus, as $\eta_{\mathrm{M}} \downarrow 0$, the long-short trading profit would become infinitely large, $\frac{1}{4}\frac{(\mu_{\theta'}-\mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} \uparrow +\infty$. When this happens, the cost for the Principal to compensate Manager $C$ to ensure his compliance will be excessively high, and thus a contract to implement the Principal's desired investment strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ will fail to exist. [14]

However, in Corollary 6.5(b) for decentralization, because Manager $A$ and Manager $B$ cannot form portfolios across each others' asset classes, they cannot use the financial markets to unwind the effects of a contract as Manager $C$ in centralization. Thus, their risk aversion $\eta_{\mathrm{M}}$ play the standard role as in the usual principal-agent literature.

## 6.3 Managers' private costs

Managers' private costs play a different effect on centralization and decentralization.

In decentralized delegation, private costs $c$ play a critical role to the existence of a contract. Both Manager $A$ and Manager $B$ have completely dedicated themselves to one particular strategy from their respective asset classes, and cannot further form portfolios to maximize risk-return trade-offs. Thus, although Manager $A$ and Manager $B$ are truly risk averse, from the perspective of incentive compatibility, they behave like risk neutral individuals. That is to say, both Manager $A$ and Manager $B$ care only about the private costs $c$ and the mean return differences $\Delta\mu_\theta$ and $\Delta\mu_\tau$ between the compliant strategy and the deviant strategy in their own asset class, and not care about second moment effects of volatility or correlation and even their own risk aversions.[15] Due to this "risk neutrality" in determining incentive compatibility, contracting with decentralized individuals with high private costs could become prohibitively costly, and so much so that a contract to implement the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ could fail to exist.

In contrast, centralized delegation can tolerate a higher level of private costs $c$ before no contract can exist. Given any contract, since Manager $C$ is risk averse, he will pick portfolios

[14] Even if we allow for different volatilities for the different investment strategies (see again Footnote 11), as $\eta_{\mathrm{M}} \downarrow 0$, we collapse back to our current case of (5.6).

[15] As discussed earlier, we had assumed all strategies have equivalent volatilities. But it is not difficult to see that even if strategies in each of the asset classes have different volatilities, the fact that private costs $c$ will still play a first order effect in Manager's consideration for deviation in decentralization.

that generate a high portfolio mean return and a low portfolio volatility. Indeed, save for the differences in risk aversion levels between the Principal and Manager $C$, the portfolio choice behavior of Manager $C$ is analogous to that of the Principal, were the Principal to have direct access to the financial markets. Thus, Manager $C$ behaves like a "quasi-Principal" and private costs $c$ only play a second order effect. This is why for moderately high levels of private costs $c$, the compliant Manager $C$ must pay $2c$ and yet a centralized contract will still exist for Principal to implement his desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$. In sharp contrast, for these same moderately high levels of private costs $c$, decentralized contracts may fail for Manager $A$ and Manager $B$.

# 7 Numerical illustrations

To gain a fuller understanding of the differences and similarities between second best centralized delegation and second best decentralized delegation, we turn to some numerical illustrations of our results. As one can surmise from Proposition 5.1 and Proposition 5.2, it is easiest to display these results in a numerical and graphical fashion. Despite the optimal portfolios and optimal contract are nonlinear in the economic parameters of interest, the results are nonetheless rather straightforward to compute numerically; especially since we do have closed form analytical answers for all of the results. A more explicit analytical solution to the difference in value functions between centralization and decentralization is available under the extreme case when there is only moral hazard over mean returns; see Section B.1.

The base parameters that we use in the numerical illustrations are given in Table 2, unless plotted otherwise. In the figures below, we need to distinguish between two different types of "better". The first type is when contracts for implementing $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ exist for both centralization and decentralization; the *darker* colors indicate which form of delegation is better under this circumstance. The second type is when contracts for implementing $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ do not exist under one form of delegation, but exist for another form of delegation. In the second type, the form of delegation under which contracts exist is better, by default; the *lighter* colors indicate which form of delegation is better under this circumstance.

In Figure 3, we see that high correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ for the compliant strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ favors decentralization, while low correlation favors centralization. This is inherited from the optimal risk sharing result of first best in Proposition 4.3. However, in the presence of

| | | |
|---|---|---|
| Principal's risk aversion parameter | $\eta_{\mathrm{P}}$ | 5 |
| Managers' risk aversion parameter | $\eta_{\mathrm{M}}$ | 3 |
| Compliant investment strategies' mean returns, $\mu \equiv \mu_{\theta_{\mathrm{H}}} = \mu_{\tau_{\mathrm{H}}}$ | $\mu$ | 0.25 |
| Mean return on deviant strategy $\theta_{\mathrm{L}}$ | $\mu_{\theta_{\mathrm{L}}}$ | 0.10 |
| Mean return on deviant strategy $\tau_{\mathrm{L}}$ | $\mu_{\tau_{\mathrm{L}}}$ | 0.08 |
| Volatility of all strategies | $\sigma$ | 0.40 |
| Correlation coefficient of compliant pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ | $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ | 0.20 |
| Correlation coefficient of deviant pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})$ | $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{L}}}$ | 0.30 |
| Correlation coefficient of deviant pair $(\theta_{\mathrm{L}}, \tau_{\mathrm{H}})$ | $\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{H}}}$ | 0.30 |
| Correlation coefficient of deviant pair $(\theta_{\mathrm{L}}, \tau_{\mathrm{L}})$ | $\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{L}}}$ | 0.30 |
| Managers' private costs | $c$ | 0.06 |

**Table 2:** The base parameter assumptions used in Section 7.

moral hazard, when $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ is sufficiently high, a centralized contract to implement $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ for the Principal does not exist. Recalling the discussion on the investment opportunity set in Section 6.1, for any performance fee $y_C \in [0, 1]$, the term $-\frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2$ is the difference between the contract volatility for Manager $C$ implementing a deviant strategy pair $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$ versus that of the compliant strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$. As the correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ of the compliant strategy pair increases, Manager $C$ incurs a high contract volatility for being compliant, whereas a low contract volatility for being deviant. Thus, when the correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ is sufficiently high, Manager $C$ will surely deviate for any performance fee $y_C$ to lower the contract volatility for himself, which then leads to nonexistence of a contract in centralization to implement the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$. For decentralization, high private costs $c$ will lead to contract nonexistence for implementing the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$; this effect was discussed in Section 6.3.

In Figure 4, we see the effects of the relaxed investment opportunity set of Section 6.1 under centralization. In this example, consider the deviant strategy $\tau_{\mathrm{L}}$ of the asset class $\tau$ (the case for the strategy $\theta_{\mathrm{L}}$ of the asset class $\theta$ is similar). Recall the long-short opportunity cost under centralization is $\frac{1}{4}\frac{(\mu'_\theta - \mu'_\tau)^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})}$ for the deviant strategy pairs $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$. When the deviant strategy is $\tau' = \tau_{\mathrm{L}}$, if its mean return $\mu_{\tau_{\mathrm{L}}}$ is low, Manager $C$ can take small long or even short positions in $\tau_{\mathrm{L}}$ to finance large positions in strategies in the asset class $\theta$. As $\mu_{\tau_{\mathrm{L}}}$ decreases, the long-short opportunity cost for Manager $C$ increases, making centralized delegation unfavorable. In contrast, this opportunity cost does not exist in decentralization. Recall that Manager $B$ is responsible for managing asset class $\tau$. As $\mu_{\tau_{\mathrm{L}}}$ decreases, the

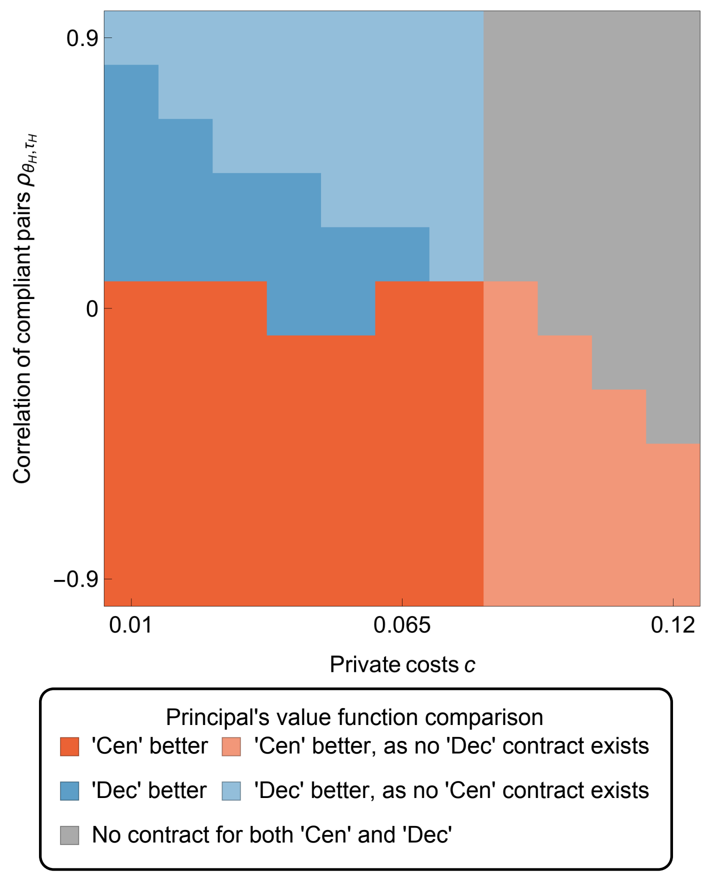


**Figure 3:** Comparing the Principal's value function under centralization versus decentralization: compliant strategy pair correlation $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$ versus private costs $c$.

expected performance fee payoff $y_B\mu_{\tau_{\mathrm{L}}}$ for Manager $B$ when he deviates from the compliant strategy $\tau_{\mathrm{H}}$ to the deviant strategy $\tau_{\mathrm{L}}$ also decreases, and thereby making deviation less profitable for him. Thus, when this happens, the performance fees for Manager $B$ could reach that of the first best result, and thereby making decentralization favorable. However, we note that as $\mu_{\tau_{\mathrm{L}}}$ increases and approaches the mean return $\mu_{\tau_{\mathrm{H}}} = \mu$ of the compliant strategy $\tau_{\mathrm{H}}$, the payoff in performance fees for Manager $B$ to be compliant and deviant become similar. However, Manager $B$ still needs to incur a private cost $c$ to implement the Principal's desired strategy. In such a case when the net benefit for being compliant rather than deviant is small, while Manager $B$ still needs to incur private costs $c$, Manager $B$ will for sure deviate. As a result, a decentralized contract for implementing the Principal's desired strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ could fail to exist, as Manager $B$ will for sure deviate.

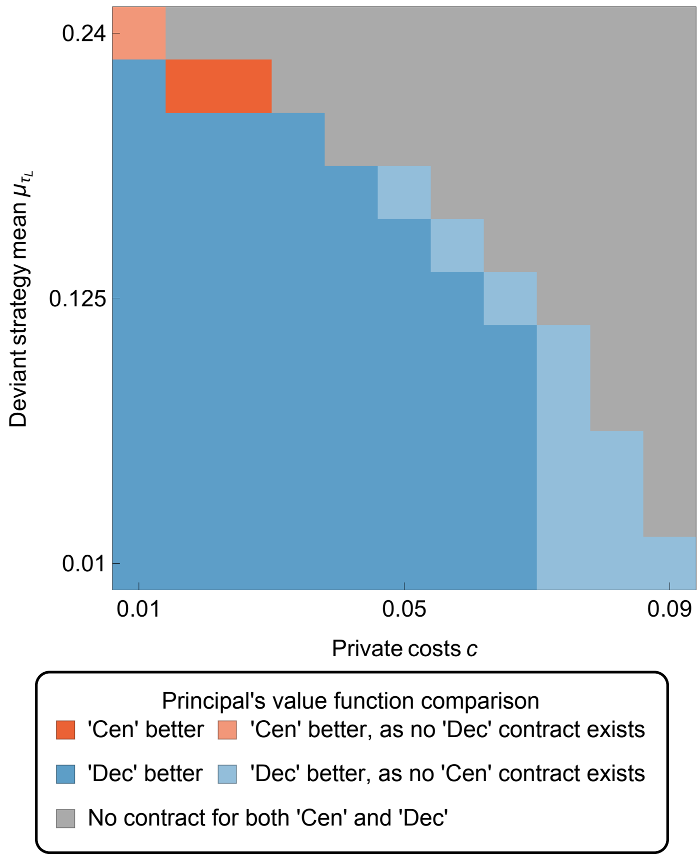


**Figure 4:** Comparing the Principal's value function under centralization versus decentralization: deviant strategy mean return $\mu_{\tau_L}$ versus private costs $c$.

In Figure 5, we see the effects of strategy volatility $\sigma$ and the correlation $\rho_{\theta_H,\tau_H}$ of the compliant pair on the contracting environment. For low correlations, as volatility $\sigma$ increases, it will favor centralization because of the optimal risk sharing effect as discussed even in the first best setup of Proposition 4.3. As already discussed in Figure 3, high correlation $\rho_{\theta_H,\tau_H}$ of the compliant strategy pair will increase the contract volatility for Manager $C$. Here, volatility brings about another perspective on this long-short opportunity cost. As volatility $\sigma$ decreases across all strategies, Manager $C$ will care even more about the mean difference between the deviant strategy pairs, and thus place more extreme long and short positions. This increases the opportunity cost for Manager $C$ to be compliant, and thereby making centralization unfavorable.

In Figure 6, we study the effects of the Principal risk aversion $\eta_P$ and the Managers' risk

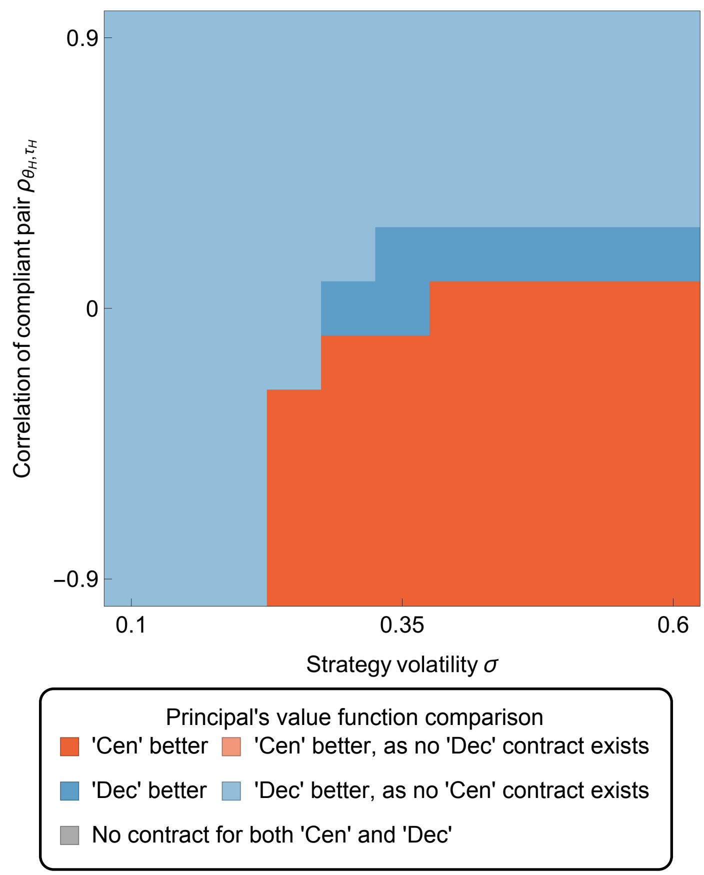


**Figure 5:** Comparing the Principal's value function under centralization versus decentralization: compliant strategy pair correlation $\rho_{\theta_\mathrm{H},\tau_\mathrm{H}}$ versus strategy volatility $\sigma$.

aversion $\eta_\mathrm{M}$ on the contracting environment. As discussed in Section 6.2, in centralization when Manager $C$ becomes less risk averse, he will take even more extreme long-short positions in the deviant strategy pairs, and it will become ever more costly for Principal to induce Manager $C$ to be compliant. In decentralization, by Assumption 3.2 that volatilities are identical across all strategies, Manager $A$ and Manager $B$ will not factor in their risk aversion in a deviation. Note that in one extreme when Manager $C$ is highly risk averse while the Principal is relatively less risk averse, centralization will be favored.

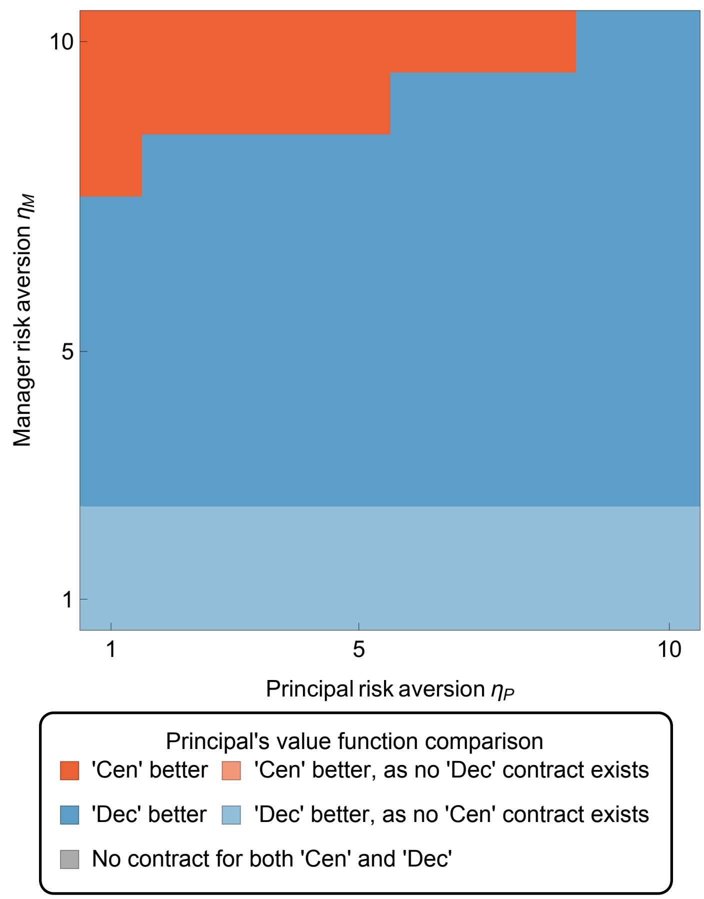


**Figure 6:** Comparing the Principal's value function under centralization versus decentralization: Principal's risk aversion $\eta_{\mathrm{P}}$ versus Managers' risk aversion $\eta_{\mathrm{M}}$.

# 8 Conclusion

From standard portfolio theories, it is potentially difficult to justify why both "generalist" and "specialist" funds can simultaneously exist in the financial markets. On the one hand, standard theories suggest investors should prefer delegating all wealth to one single "generalist" manager because of the diversification benefits realized by having all portfolio choices made under one roof. But empirically, we hardly observe any pension fund or endowment give all their wealth to one single manager. On the other hand, a potential supply-side explanation for why "specialist" funds can exist is that if managers actually have superior skills in their specialized asset classes. But numerous empirical studies have shown that even if skills are indeed present, these managers are hard to identify, or perhaps there are no skills at all. The question is then why do investors demand services of "generalist" managers?

Our contribution is to offer moral hazard in the form of investment strategy deviations as a potential demand-side, and empirically testable, explanation for when an investor would prefer delegating to a "generalist" manager (centralized delegation) and when he would prefer multiple "specialist" managers (decentralized delegation). Using an optimal linear contract, we provide conditions on asset classes' mean returns, volatility and correlations for which delegation form is better for the investor. The key consideration for the investor is the inability to contract on risk management. While centralization allows the single manager to internalize all the risks between asset classes, it also gives the single manager more leeway to deviate. The manager can use his wide access to the financial markets to unwind the effects of any incentive contract, and thereby delivering a portfolio with undesirable risk-return characteristics to the investor. In contrast, in decentralization, the investor is responsible for allocating wealth into different managers, and hence must take into account risk management himself. But since the managers' compensations are not dependent on each others' performance, any incentive compatible contract in decentralization fails to fully account for the investor's risk preferences.

In this paper we have only considered a partial equilibrium model of centralization versus decentralization, and in particular the demand-side effects of a principal investor. It would be fruitful for future research to consider a general equilibrium model of an optimal fund industry organization structure. In particular, if a manager can privately choose investment strategies within each asset class, under what conditions would this manager choose to enter the supply-side of the delegation market? And in equilibrium, what are the optimal contracts and equilibrium delegation market structure? Our demand-side paper is a necessary first step in this research agenda.

# Appendix

## A Proofs for Section 4

*Proof of Proposition 4.1.* (a) Using first order sufficient and necessary conditions, we can see that the value to (3.2d) will be given by (4.1).

(b) Substitute in the optimal portfolio found above into the mean and variance expressions.

(c) Since the fixed fee $x_C \in \mathbb{R}$ is linear respect to the Principal's objective function, it implies that in equilibrium, the individual rationality constraint (3.2f) constraint binds. This implies the Principal's objective function can be rewritten as,

$$\begin{aligned}\mathbb{E}[W_{cP}^{(\theta,\tau)}] - \frac{\eta}{2}\mathrm{Var}(W_{cP}^{(\theta,\tau)}) &= 1 + \mathbb{E}[\hat{R}_{(\theta,\tau)}] - (c(\theta) + c(\tau)) \\ &\quad - \frac{\eta_{\mathrm{P}}}{2} y_C^2 \mathrm{Var}(\hat{R}_{(\theta,\tau)}) - \frac{\eta_{\mathrm{M}}}{2}(1-y_C)^2 \mathrm{Var}(\hat{R}_{(\theta,\tau)}).\end{aligned}$$

Now, by first order conditions on $y_C$, we see that the above becomes a fourth order polynomial (i.e. quartic) equation, and has the following roots,

$$y_C \in \left\{ \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}} \ , \ -\frac{(\mu_\theta - \mu_\tau)^{2/3}}{(\eta_{\mathrm{P}}^2 \sigma^4 (1-\rho_{\theta\tau}^2))^{1/3}} \ , \ \pm \frac{(-1)^{2/3}(\mu_\theta - \mu_\tau)^{2/3}}{(\eta_{\mathrm{P}}^2 \sigma^2 (1-\rho_{\theta\tau}^2))^{1/3}} \right\}.$$

The first root is clearly in $(0,1)$; the second root is negative and hence not in $(0,1)$; the third and fourth roots (with $\pm$) are not in $\mathbb{R}$ since $(-1)^{2/3} \in \mathbb{C}$. Thus, an interior solution exists and is uniquely given by the first root. [16]

(d) Simply substitute in the optimal fixed and optimal fees found earlier.

(e) Analogous to the above.

[16] It should be noted that in general, quartic equations (and naturally arising here because of first order conditions) are notoriously difficult to obtain simple and explicit solutions for. It is conjectured that if one extends to consider more than two risky investment strategies, or that we extend to more general non-linear contracts, it would be difficult to obtain a closed form contract for even first best centralized delegation. Indeed, the most difficult step in the proof of this Proposition 4.1 is this step, as everything else is straightforward. It was actually somewhat surprising to this author that despite a rather complicated first order condition, an economically sensible and intuitive solution for the performance fee arises.

$\blacksquare$

*Proof of Proposition 4.2.* (a) By binding the (IR) constraints (3.3d), we obtain the optimal fixed fee form, and we can rewrite the Principal's objective function as,

$$
\begin{aligned}
&\mathbb{E}[W_P^{(\theta,\tau)}] - \frac{\eta}{2}\mathrm{Var}(W_P^{(\theta,\tau)}) \\
&= -(c(\theta) + c(\tau)) + 1 + \pi(\mu_\tau - \mu_\theta) + \mu_\theta - \frac{\eta_{\mathrm{M}}}{2} y_B^2 \pi^2 \sigma^2 - \frac{\eta_{\mathrm{M}}}{2} y_A^2 (1-\pi)^2 \sigma^2 \\
&\quad - \frac{\eta_{\mathrm{P}}}{2}\Big[\pi^2((1-y_B)^2\sigma^2 + (1-y_A)^2\sigma^2 - 2(1-y_B)(1-y_A)\rho_{\theta\tau}\sigma^2) \\
&\qquad + 2\pi(1-y_A)\big((1-y_B)\rho_{\theta\tau}\sigma^2 - (1-y_A)\sigma^2\big) + (1-y_A)^2\sigma^2\Big]. \qquad \text{(A.1)}
\end{aligned}
$$

(b) By first order conditions applied to (A.1), we arrive at three different stationary points of $(\pi, b, q)$,

$$
\begin{aligned}
(\pi, y_A, y_B) \in \Bigg\{ &\left(0\,,\ \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}}\,,\ 1 + \frac{(\eta_{\mathrm{P}} + \eta_{\mathrm{M}})(\mu_\theta - \mu_\tau) - \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2}{\eta_{\mathrm{P}}\eta_{\mathrm{M}}\rho_{\theta\tau}\sigma^2}\right), \\
&\left(1\,,\ 1 + \frac{(\eta_{\mathrm{M}} + \eta_{\mathrm{P}})(\mu_\tau - \mu_\theta) - \eta_{\mathrm{P}}\eta_{\mathrm{M}}\sigma^2}{\eta_{\mathrm{P}}\eta_{\mathrm{M}}\rho_{\theta\tau}\sigma^2}\,,\ \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{M}} + \eta_{\mathrm{P}}}\right), \\
&(\pi^o, y_A^o, y_B^o)\Bigg\}
\end{aligned}
$$

The first and second stationary points, which would imply zero wealth invested into either of the agents, will violate the individual rationality constraint (3.3d). Thus, only the third stationary point is a candidate for an interior solution.

(c) This is simply applying Assumption 3.2. The value function computation is straightforward.

$\blacksquare$

*Proof of Proposition 4.3.* Use Proposition 4.1 and Proposition 4.2. $\blacksquare$

# B Proofs for Section 5 and Additional Results

*Proof of Proposition 5.1.* (a) This is the same proof as that of Proposition 4.1.

(b) This is evident since the arguments in Proposition 4.1 for deriving Manager $C$'s optimal portfolio choice holds true for any arbitrary contract.

(c) By Assumption 3.2 and Proposition 4.1, if the Principal wants to implement and induce the investment strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$, then the Principal needs to write a contract that prevents Manager $C$ from taking on the deviant strategies $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})}$. These are captured by the incentive compatibility constraints in (5.1). One should note that these three constraints can be collapsed to a single one by equivalently writing,

$$
\begin{aligned}
& -2c + \frac{1}{2}(\mu_{\theta_{\mathrm{H}}} + \mu_{\tau_{\mathrm{H}}})y - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y^2 \\
& \geq \max_{(\theta',\tau')} \left\{ -(c(\theta') + c(\tau')) + \frac{1}{4}\frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} + \frac{1}{2}(\mu_{\theta'} + \mu_{\tau'})y - \frac{1}{4}\eta_{\mathrm{M}}\sigma^2(1+\rho_{\theta',\tau'})y^2 \right\},
\end{aligned} \tag{B.1}
$$

where we take the maximum on the right hand side over $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})}$, which is clearly then equivalent to (5.2).

Note that by Assumption 3.2, we have that $\mu_{\theta'} \leq \mu_{\theta_{\mathrm{H}}}$ and $\mu_{\tau'} \leq \mu_{\tau_{\mathrm{H}}}$, and where at least one of these two inequalities are strict, and hence $\mu_{\theta'} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau'} - \mu_{\tau_{\mathrm{H}}} < 0$. Likewise, $c(\theta') + c(\tau') - 2c < 0$. However, since we only assume that the correlations $\rho_{\theta,\tau}$ for all investment strategy pairs $(\theta, \tau)$ are different, and in particular no special sign and order restrictions, so we have that if $\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} > 0$, then the component is concave in $y$, and if $\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} < 0$, it is convex in $y$. Thus, we have a pointwise maximum of convex and/or concave functions, and in general, one has no particular geometric form of this.

(d) From the condition (5.2), we substitute in the first best solution to check the condition under which none of the incentive compatibility constraints will bind. This is condition (5.3).

(e) Suppose the conditions on the private costs (5.3) are such that a first best solution will not be attained in second best. While we could indeed proceed to use Kuhn-Tucker conditions (with say three Kuhn-Tucker multipliers) to solve for the optimal solution, we can proceed with a much more geometric proof here. Firstly, by (5.1) or equivalently (5.2), it is clear that when a binding solution (that is in $[0, 1]$) exists, only one of the constraints will bind. Suppose that $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}}) \in \mathcal{S}_{-(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})}$ is the pair of deviant investment strategies for which its associated incentive compatibility constraint binds.

Given the quadratic form of constraints, we are motivated to define the discriminant for the binding deviant pairs $(\theta', \tau')$ (5.4). Notice that the sign of the discriminant is heavily dependent on the sign of $\rho_{\theta^{\mathrm{b}}, \tau^{\mathrm{b}}} - \rho_{\theta_{\mathrm{H}}, \tau_{\mathrm{H}}}$. Provided that $D_{(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})} \geq 0$, so that roots will exist for the quadratic associated with the binding incentive compatibility constraint $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$, we compute the roots as,

$$\begin{aligned} \tilde{y}_{\pm,(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})} = & \frac{1}{2}\left(-\frac{\mu_{\theta^{\mathrm{b}}} - \mu_{\theta_{\mathrm{H}}} + \mu_{\tau^{\mathrm{b}}} - \mu_{\tau_{\mathrm{H}}}}{2} \pm \sqrt{D_{(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})}}\right) \\ & \times \left[-(c(\theta^{\mathrm{b}}) + c(\tau^{\mathrm{b}}) - 2c) + \frac{1}{4}\frac{(\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}})^2}{\eta_{\mathrm{M}}\sigma^2(1 - \rho_{\theta_{\mathrm{H}}, \tau_{\mathrm{H}}})}\right]^{-1}. \end{aligned}$$

With our current assumptions, it is not difficult to show that the negative root $\tilde{y}_{-,(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})} < 0$. Thus, let's focus on the positive root $\tilde{y}_{+,(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})}$ of (5.5). We must now recall that our solution must be confined in $[0, 1]$. Hence, a second best solution will exist only if $\tilde{y}_{+,(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})} \in [0, 1]$, and likewise, if $\tilde{y}_{+,(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})} \notin [0, 1]$, then no second best solution will exist.

■

**Corollary B.1.** *Consider the second best centralized delegation setup in Proposition 5.1, and suppose the conditions (i.e. conditions (i) and (ii) of part (d)) for the existence of a second best contract holds. In particular, recall* (5.5). *Then the most profitable deviant investment strategy* $(\theta^b, \tau^b)$ *for Manager C is the following and given under the following conditions, which then leads to the optimal performance fee* $\hat{y}_C = \tilde{y}_{+,(\theta', \tau')}$.

*(a) The optimal performance fee is* $\tilde{y}_{+,(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})}$ *when,*

$$\begin{aligned} 0 \geq \max\Big\{ & c + \frac{1}{4}\frac{(\mu_{\theta_{\mathrm{L}}} - \mu)^2}{\eta_{\mathrm{M}}\sigma^2(1 - \rho_{\theta_{\mathrm{L}}, \tau_{\mathrm{H}}})} - \frac{1}{2}\Delta\mu_{\tau}\tilde{y}_{+,(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{L}}, \tau_{\mathrm{H}}} - \rho_{\theta_{\mathrm{H}}, \tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})}, \\ & 2c + \frac{1}{4}\frac{(\mu_{\theta_{\mathrm{L}}} - \mu_{\tau_{\mathrm{L}}})^2}{\eta_{\mathrm{M}}\sigma^2(1 - \rho_{\theta_{\mathrm{L}}, \tau_{\mathrm{L}}})} - \frac{1}{2}(\Delta\mu_{\theta} + \Delta\mu_{\tau})\tilde{y}_{+,(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{L}}, \tau_{\mathrm{L}}} - \rho_{\theta_{\mathrm{H}}, \tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{H}}, \tau_{\mathrm{L}})}\Big\}. \end{aligned}$$

*(b) The optimal performance fee is* $\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{H}})}$ *when,*

$$0 \geq \max\Big\{c + \frac{1}{4}\frac{(\mu-\mu_{\tau_{\mathrm{L}}})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{L}}})} - \frac{1}{2}\Delta\mu_\theta\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{H}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{L}}}-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{H}})}\ ,$$
$$2c + \frac{1}{4}\frac{(\mu_{\theta_{\mathrm{L}}}-\mu_{\tau_{\mathrm{L}}})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{L}}})} - \frac{1}{2}(\Delta\mu_\theta+\Delta\mu_\tau)\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{H}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{L}}}-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{H}})}\Big\}.$$

*(c) The optimal performance fee is* $\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{L}})}$ *when,*

$$0 \geq \max\Big\{c + \frac{1}{4}\frac{(\mu_{\theta_{\mathrm{L}}}-\mu)^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{H}}})} - \frac{1}{2}\Delta\mu_\tau\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{L}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{L}},\tau_{\mathrm{H}}}-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{L}})}\ ,$$
$$c + \frac{1}{4}\frac{(\mu-\mu_{\tau_{\mathrm{L}}})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{L}}})} - \frac{1}{2}\Delta\mu_\theta\tilde{y}_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{L}})} - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{L}}}-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\tilde{y}^2_{+,(\theta_{\mathrm{L}},\tau_{\mathrm{L}})}\Big\}.$$

*Proof of Corollary B.1.* This is simply rewriting out the condition (5.2) more explicitly. ■

*Proof of Proposition 5.2.* (a) This is simply by binding the (IR) constraints (3.3d).

(b) This is simply rewriting the (IC) constraints (3.3e), (3.3f).

(c) This will be seen as a special case of Proposition B.2(aiv).

■

**Proposition B.2.** *Recall the setup of Proposition 5.2.*

*(a) Consider the following conditions on the private cost c imply the optimal second best decentralized delegation optimal portfolio and performance fee policies* $(\hat{\pi}, \hat{y}_A, \hat{y}_B)$ *have the following form:*

*(i) If c is such that,*

$$0 < c \leq \eta_{\mathrm{P}}\Delta\mu_\theta\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\times$$
$$\min\Big\{\frac{1}{\eta_{\mathrm{P}}\Delta\mu_\theta(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})+\Delta\mu_\tau(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))}\ ,$$
$$\frac{1}{\eta_{\mathrm{P}}\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})+\Delta\mu_\theta(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))}\Big\},$$

*then,*

$$\begin{pmatrix} \hat{\pi} \\ \hat{y}_A \\ \hat{y}_B) \end{pmatrix} = \begin{pmatrix} \hat{\pi}^{FB} \\ \hat{y}_A^{FB} \\ \hat{y}_B^{FB} \end{pmatrix} = \begin{pmatrix} 1/2 \\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})} \\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})} \end{pmatrix}$$

*(ii) If $c$ is such that,*

$$\frac{\Delta\mu_\theta \Delta\mu_\tau \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{P}}\Delta\mu_\theta(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) + \Delta\mu_\tau(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))} < c \leq \min\left\{ \Delta\mu_\theta\,, \frac{\Delta\mu_\theta \Delta\mu_\tau \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{P}}\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) + \Delta\mu_\theta(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))} \right\},$$

*then,*

$$\begin{pmatrix} \hat{\pi} \\ \hat{y}_A \\ \hat{y}_B \end{pmatrix} = \begin{pmatrix} \frac{(\Delta\mu_\theta - c)\left(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right)}{\Delta\mu_\theta\left(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right)} \\ \frac{c\left(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right)}{\eta_{\mathrm{M}}\Delta\mu_\theta + c[\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})]} \\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})} \end{pmatrix}$$

*(iii) If $c$ is such that,*

$$\frac{\Delta\mu_\theta \Delta\mu_\tau \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{P}}\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) + \Delta\mu_\theta(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))} < c \leq \min\left\{ \Delta\mu_\tau\,, \frac{\eta_{\mathrm{P}}\Delta\mu_\theta \Delta\mu_\tau (1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{P}}\Delta\mu_\theta(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) + \Delta\mu_\tau(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))} \right\}$$

*then,*

$$\begin{pmatrix} \hat{\pi} \\ \hat{y}_A \\ \hat{y}_B \end{pmatrix} = \begin{pmatrix} \frac{\eta_{\mathrm{M}}\Delta\mu_\tau + c\left[\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right]}{\Delta\mu_\tau\left[2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right]} \\ \frac{\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})} \\ \frac{c[2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})]}{\eta_{\mathrm{M}}\Delta\mu_\tau + c[\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})]} \end{pmatrix}$$

*(iv) If c is such that,*

$$\eta_{\mathrm{P}}\Delta\mu_\theta\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\max\left\{\frac{1}{\eta_{\mathrm{P}}\Delta\mu_\theta(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})+\Delta\mu_\tau(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))},\right.$$
$$\left.\frac{1}{\eta_{\mathrm{P}}\Delta\mu_\tau(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})+\Delta\mu_\theta(2\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}))}\right\}$$
$$< c < \frac{\Delta\mu_\theta\Delta\mu_\tau}{\Delta\mu_\theta+\Delta\mu_\tau},$$

*then,*

$$\begin{pmatrix}\hat{\pi}\\ \hat{y}_A\\ \hat{y}_B\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\left[1+\frac{\Delta\mu_\tau-\Delta\mu_\theta}{\Delta\mu_\theta\Delta\mu_\tau}c\right]\\ \frac{2\Delta\mu_\tau c}{c(\Delta\mu_\tau-\Delta\mu_\theta)+\Delta\mu_\theta\Delta\mu_\tau}\\ \frac{2\Delta\mu_\theta c}{c(\Delta\mu_\theta-\Delta\mu_\tau)+\Delta\mu_\theta\Delta\mu_\tau}\end{pmatrix}$$

*(v) Else if none of the conditions above are satisfied, then there does not exist an optimal second best decentralized delegation contract.*

*Proof of Proposition B.2.* (a) After binding the (IR) constraints (3.3d) into the Principal's objective function, it remains that the portfolio and performance fee policy $(\pi, y_A, y_B)$ have to respect the (IC) constraints (5.6), and the box constraints $(\pi, y_B, y_A) \in \mathbb{R}\times[0,1]^2$. However, we observe that $(\pi, y_A, y_B)$ being on the boundary of $[0,1]^3$ would immediately violate either the (IR) constraints, the (IC) constraints, or both. Hence, for a solution to exist, $(\pi, y_A, y_B)$ must be in the interior of $[0,1]^3$, that being $(0,1)^3$. Hence, given a feasible solution $(\pi, y_A, y_B) \in (0,1)^3$, we must then cycle through the $2 \times 2 = 4$ cases where either the (IC) constraint (5.6a) of Manager $A$ bind or not, and whether (IC) constraint (5.6b) of Manager $B$ bind or not.

(i) This is the case when we obtain an interior solution and neither (IC) of Manager $A$ nor (IC) of Manager $B$ bind. Substitute in the first best solution from Proposition 4.2, under Assumption 3.2, into (5.6) and replace $\geq$ with $>$ to get the conditions on the private costs $c$.

(ii) This is the case when only (IC) of Manager $A$ binds and when that of Manager $B$ does not bind. This happens when, after substituting the first best solution into

(5.6), and we obtain,

$$\Delta\mu_\theta \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) > 2c(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})),$$
$$\Delta\mu_\tau \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}) \leq 2c(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})).$$

The binding condition also allows for us to get the portfolio policy $\pi$ as a function of $y_A$. Via first order conditions on the objective function, substitute back and then we solve for $(\pi, y_A, y_B)$. However, we still need to satisfy the interior box constraints $(\pi, y_A, y_B) \in (0,1)^3$. We have $y_A \in (0,1)$ holding. Here, $y_B > 0$ and to have $y_B < 1$, we need,

$$c < \Delta\mu_\tau.$$

Under such condition, we would also have $\pi \in (0,1)$. Putting those three conditions on the private cost $c$ together yields the displayed condition.

(iii) This is the case when (IC) of Manager $A$ does not bind, but that of Manager $B$ does bind. The argument is completely analogous to the previous one.

(iv) This is the case when both (IC)'s of Manager $A$ and Manager $B$ bind. Here, we need to differentiate between two sub-cases — when $\Delta\mu_\theta = \Delta\mu_\tau$ and when $\Delta\mu_\theta \neq \Delta\mu_\tau$. If $\Delta\mu_\theta = \Delta\mu_\tau \equiv \Delta\mu$, then we immediately have that

$$(\pi, y_A, y_B) = (1/2, 2c/\Delta\mu, 2c/\Delta\mu).$$

So, the condition to ensure that $(\pi, y_A, y_B) \in (0,1)^3$ is clearly when,

$$c < \frac{\Delta\mu}{2}.$$

Suppose $\Delta\mu_\theta \neq \Delta\mu_\tau$, and without loss of generality, suppose $\Delta\mu_\theta > \Delta\mu_\tau$. To have $y_A > 0$, we would need,

$$\frac{\Delta\mu_\theta \Delta\mu_\tau}{\Delta\mu_\theta - \Delta\mu_\tau} > c,$$

and to have $y_A < 1$, one would need,

$$c < \frac{\Delta\mu_\theta \Delta\mu_\tau}{\Delta\mu_\theta + \Delta\mu_\tau}.$$

Finally, to have $\pi > 0$, we would need,

$$c < \frac{\Delta\mu_\theta \Delta\mu_\tau}{\Delta\mu_\theta - \Delta\mu_\tau}.$$

Putting these conditions together implies we need,

$$\frac{\eta(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})}{2\left(\eta_{\mathrm{M}} + \eta_{\mathrm{P}}(1+\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})\right)}\Delta\mu_\theta \leq c < \min\left\{\frac{\Delta\mu_\theta\Delta\mu_\tau}{\Delta\mu_\theta - \Delta\mu_\tau}, \frac{\Delta\mu_\theta\Delta\mu_\tau}{\Delta\mu_\theta + \Delta\mu_\tau}\right\}.$$

Simplifying and generalizing to the case when $\Delta\mu_\theta < \Delta\mu_\tau$, we have the displayed condition.

■

*Proof of Corollary 6.1.* Let $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$ be the argmax of (5.1). We consider the limit $|\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}}| \uparrow \infty$. If $\mu_{\theta^{\mathrm{b}}} \equiv \mu_{\tau^{\mathrm{b}}}$, the statement is vacuous and uninteresting. Without loss of generality, assume $\mu_{\theta^{\mathrm{b}}} > \mu_{\tau_{\mathrm{L}}}$. We can rewrite (5.2) as,

$$\begin{aligned} 0 \geq &-(c(\theta^{\mathrm{b}}) + c(\tau^{\mathrm{b}}) - 2c) + \frac{1}{4}\frac{(\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta^{\mathrm{b}},\tau^{\mathrm{b}}})} \\ &+ \frac{1}{2}((\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}}) - \mu_{\theta_{\mathrm{H}}} + 2\mu_{\tau^{\mathrm{b}}} - \mu_{\tau_{\mathrm{H}}})y_C - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta^{\mathrm{b}},\tau^{\mathrm{b}}} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2 \end{aligned}$$

Thus, as $(\mu_{\theta^{\mathrm{b}}} - \mu_{\tau^{\mathrm{b}}}) \uparrow \infty$, we have the statement $0 \geq \infty$, and hence, no real root $y_C$ can exist. ■

*Proof of Corollary 6.2.* Suppose that $\mu_{\theta_{\mathrm{H}}} = \mu_{\theta_{\mathrm{L}}}$ and $\mu_{\tau_{\mathrm{H}}} = \mu_{\tau_{\mathrm{L}}}$.

(a) From (5.6), we would have the conditions,

$$0 \geq c \quad \text{and} \quad 0 \geq c,$$

and clearly, this is impossible. Thus, no optimal contract will exist for Manager $A$ and Manager $B$, and hence no contract can exist in decentralized delegation.

(b) Suppose further that $\mu \equiv \mu_\theta = \mu_\tau$ for all $(\theta, \tau)$. Then (5.1) simplifies to,

$$0 \geq \max_{\theta',\tau'}\left\{-(c(\theta') + c(\tau') - 2c) - \frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'} - \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2\right\}.$$

If there exists a pair of investment strategies $(\theta^{\mathrm{b}}, \tau^{\mathrm{b}})$ that satisfy this condition, then the optimal performance fee would follow form (5.5). Note that this condition is satisfied when the private costs are not too high, and that $\rho_{\theta',\tau'}$ is sufficiently larger than $\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}}$; that is, the complaint strategy pair $(\theta_{\mathrm{H}}, \tau_{\mathrm{H}})$ has sufficiently low correlations relative to the possible deviant strategy pairs.

■

*Proof of Corollary 6.3.* (a) The incentive compatibility conditions (5.6) clearly do not depend on the return correlations $\rho_{\theta,\tau}$ for any $(\theta, \tau) \in \mathcal{S}$.

(b) Let's consider the triple limit $\rho_{\theta',\tau'} \uparrow +1$ for each $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$ on the incentive compatibility condition (5.1) for centralized delegation. Observing that,

$$\lim_{\substack{\rho_{\theta',\tau'}\uparrow+1,\\ (\theta',\tau')\in\mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}}} \frac{1}{4}\frac{(\mu_{\theta'}-\mu_{\tau'})^2}{\eta_{\mathrm{M}}\sigma^2(1-\rho_{\theta',\tau'})} \uparrow +\infty,$$

$$\lim_{\substack{\rho_{\theta',\tau'}\uparrow+1,\\ (\theta',\tau')\in\mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}}} -\frac{1}{4}\eta_{\mathrm{M}}^2\sigma^2(\rho_{\theta',\tau'}-\rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}})y_C^2 < \infty, \quad \text{for any } y_C \in [0,1],$$

which leads to the incentive compatibility condition $0 \geq \infty$. Thus, no centralized delegation contract can exist.

■

*Proof of Corollary 6.4.* (a) This statement is immediate by Assumption 3.2.

(b) From the incentive compatibility condition (5.2), as $\sigma^2 \downarrow 0$, we have the statement (ignoring the terms independent of $\sigma^2$) that $0 \geq +\infty + 0 = +\infty$, and thus no real root for $y_C$ can exist.

(c) From the incentive compatibility condition (5.2), when $\rho_{(\theta_{\mathrm{H}},\tau_{\mathrm{H}})\geq\rho_{(\theta',\tau')}}$ for all $(\theta', \tau') \in \mathcal{S}_{-(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}$, as $\sigma^2 \uparrow +\infty$, we have the statement (ignoring the terms independent of $\sigma^2$) that $0 \geq 0 + \infty = +\infty$, and thus no real root for $y_C$ can exist.

(d) The proof is symmetric to that of (c), but we lead to the statement that $0 \geq 0 - \infty = -\infty$, which is always true.

■

*Proof of Corollary 6.5.* (a) This is immediate from Assumption 3.2.

(b) From the incentive compatibility condition (5.2), as $\eta_M \downarrow 0$, we obtain the statement (ignoring the terms that are independent of $\eta_M$) that $0 \geq +\infty + 0 = +\infty$. This implies no real root for $y_C$ can exist.

■

## B.1 When there is only moral hazard over mean returns

An interesting special case that neither neither potentially favors nor biases centralized delegation is when there is no moral hazard over correlations, $\rho \equiv \rho_{\theta\tau}$ for all $(\theta, \tau)$, and the potential mean return losses between the two investment strategies are identical, $\Delta\mu \equiv \Delta\mu_\theta = \Delta\mu_\tau$. In this case, the incentive compatibility constraints (5.6) of decentralized delegation have the form,

$$0 \geq c - (1-\pi) y_A \Delta\mu, \tag{B.2a}$$

$$0 \geq c - \pi y_B \Delta\mu, \tag{B.2b}$$

which is effectively the same form as before, but the incentive compatibility constraint (5.1) for centralized delegation reduces to,

$$0 \geq \max_{(\theta', \tau')} \{2c - (c(\theta') + c(\tau')) + \Delta\mu y_C\} = 2c + \Delta\mu y_C \tag{B.3}$$

Thus, in this special case for centralized delegation, the centralized Manager $C$ has incentives that are very much aligned with the Principal, as the alternative investment strategies $(\theta_{\rm L}, \tau_{\rm L})$ have the same mean $\mu_{\theta_{\rm L}} = \mu_{\tau_{\rm L}} = \mu - \Delta\mu$, same volatility and same correlations, this implies that a long-short strategy is not profitable.

**Corollary B.3.** *Assume that there is no moral hazard over correlations $\rho \equiv \rho_{\theta\tau}$ for all strategy pairs $(\theta, \tau) \in \mathcal{S}$, and the mean return differences between the two strategies are identical, $\Delta\mu \equiv \Delta\mu_\theta = \Delta\mu_\tau > 0$.*

*(a) Consider the second best centralized delegation problem.*

(i) *The optimal performance fee is,*

$$\hat{y}_C = \begin{cases} \hat{y}_C^{FB}, & 0 < c < \frac{1}{2}\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}\Delta\mu \\ \frac{2c}{\Delta\mu}, & \frac{1}{2}\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}\Delta\mu \leq c < \frac{\Delta\mu}{2} \\ \varnothing, & \textit{otherwise.} \end{cases}$$

(ii) *The associated Principal's value function in second best centralized delegation is,*

$$\begin{aligned} &\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})} \\ &= \begin{cases} -2c + \mu - \frac{\eta_{\mathrm{P}}\eta_{\mathrm{M}}}{4(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})}\sigma^2(1+\rho), & 0 < c < \frac{1}{2}\frac{\eta}{\eta+\eta_{\mathrm{M}}}\Delta\mu, \\ -2c + \mu - \frac{\left[4\eta_{\mathrm{M}}c^2 + \eta_{\mathrm{P}}(\Delta\mu - 2c)^2\right](1+\rho)\sigma^2}{4(\Delta\mu)^2}, & \frac{1}{2}\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}\Delta\mu \leq c < \frac{\Delta\mu}{2}, \\ -\infty, & \textit{otherwise.} \end{cases} \end{aligned}$$

(b) *Consider the second best decentralized delegation problem.*

(i) *The optimal portfolio and performance fee policies are,*

$$(\hat{\pi}, \hat{y}_A, \hat{y}_B) = \begin{cases} \left(\frac{1}{2}, \frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}, \frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}\right), & 0 < c < \frac{1}{2}\frac{\eta(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}\Delta\mu, \\ \left(\frac{1}{2}, \frac{2c}{\Delta\mu}, \frac{2c}{\Delta\mu}\right), & \frac{1}{2}\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)} \leq c < \frac{\Delta\mu}{2}, \\ \varnothing, & \textit{otherwise.} \end{cases}$$

(ii) *The associated Principal's value function in second best decentralized delegation is,*

$$\begin{aligned} &\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})} \\ &= \begin{cases} -2c + \mu - \frac{\eta_{\mathrm{P}}\eta_{\mathrm{M}}(1+\rho)\sigma^2}{4(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))}, & 0 < c < \frac{1}{2}\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}\Delta\mu, \\ -2c + \mu - \frac{\sigma^2\left[4(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))c^2 - \eta_{\mathrm{P}}(1+\rho)\Delta\mu(\Delta\mu - 4c)\right]}{4(\Delta\mu)^2}, & \frac{1}{2}\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)} \leq c < \frac{\Delta\mu}{2}, \\ -\infty, & \textit{otherwise.} \end{cases} \end{aligned}$$

(c) *Let's compute the difference between the Principal's value function under second best decentralized delegation and that of second best decentralized delegation.*

*(i) Suppose* $\rho \in (-1,0)$*. Then* $\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}} > \frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}$*, and,*

$$\left(\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) - \left(\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right)$$
$$= \begin{cases} \frac{\eta_{\mathrm{M}}\eta_{\mathrm{P}}^2\rho(1+\rho)\sigma^2}{4(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))}, & 0 < c < \frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}, \\ \frac{\sigma^2\left[4c\Delta\mu\eta_{\mathrm{P}}(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})-4c^2(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))-(\Delta\mu)^2\eta_{\mathrm{P}}^2(1+\rho)\right]}{4(\eta_{\mathrm{P}}+\eta_{\mathrm{M}})(\Delta\mu)^2}, & \frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)} \le c < \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}, \\ \frac{\eta_{\mathrm{M}}c^2\sigma^2}{(\Delta\mu)^2}\rho, & \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}} \le c < \frac{\Delta\mu}{2}, \\ \textit{undefined}, & \textit{otherwise.} \end{cases}$$

*In particular, for all* $c \in (0, \Delta\mu/2)$*,*

$$\left(\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) - \left(\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) < 0.$$

*(ii) Suppose* $\rho \in (0,1)$*. Then* $\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)} > \frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}$*, and,*

$$\left(\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) - \left(\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right)$$
$$= \begin{cases} \frac{\eta_{\mathrm{M}}\eta^2\rho(1+\rho)\sigma^2}{4(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})(\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))}, & 0 < c < \frac{1}{2}\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{P}}+\eta_{\mathrm{M}}}, \\ \frac{\sigma^2(1+\rho)}{4}\left(\eta_{\mathrm{P}} + \frac{4c[c(\eta_{\mathrm{M}}+\eta_{\mathrm{P}})-\eta_{\mathrm{P}}\Delta\mu]}{(\Delta\mu)^2} - \frac{\eta_{\mathrm{P}}\eta_{\mathrm{M}}}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho))}\right), & \frac{1}{2}\frac{\eta_{\mathrm{P}}}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}} \le c < \frac{1}{2}\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)}, \\ \frac{\eta_{\mathrm{M}}c^2\sigma^2}{(\Delta\mu)^2}\rho, & \frac{1}{2}\frac{\eta_{\mathrm{P}}(1+\rho)}{\eta_{\mathrm{M}}+\eta_{\mathrm{P}}(1+\rho)} \le c < \frac{\Delta\mu}{2}, \\ -\infty, & \textit{otherwise.} \end{cases}$$

*In particular, for all* $c \in (0, \Delta\mu/2)$*,*

$$\left(\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) - \left(\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) > 0.$$

*(iii) If* $\rho = 0$*, then for all* $c \in (0, \Delta\mu/2)$*,*

$$\left(\mathbb{E}[W_P] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_P)\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) - \left(\mathbb{E}[W_{cP}] - \frac{\eta_{\mathrm{P}}}{2}\mathrm{Var}(W_{cP})\Big|_{SB,(\theta_{\mathrm{H}},\tau_{\mathrm{H}})}\right) = 0.$$

*Proof of Corollary B.3.* This is a special case of Proposition 5.1 and Proposition 5.2. ■

Corollary B.3 illustrates that when there is no moral hazard over correlations, $\rho \equiv \rho_{\theta_{\mathrm{H}},\tau_{\mathrm{H}}} = \rho_{\theta_{\mathrm{L}},\tau_{\mathrm{L}}}$, and that the mean return losses due to moral hazard are equal, $\Delta\mu \equiv \Delta\mu_{\theta} = \Delta\mu_{\tau}$, then this substantially aligns the interests of the centrally delegated single Manager $C$. And as a result, in this special case, our results are essentially identical to the first best case of Proposition 4.3 that we had studied earlier. In particular, centralized delegation is favored when the correlations are negative $\rho < 0$, decentralized delegation is favored when the correlations are positive $\rho > 0$, and both forms of delegation are equal when the investment strategies are uncorrelated $\rho = 0$.

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