A Searched-Based Framework of Capital Reallocation*

Feng Dong† Pengfei Wang‡ Yi Wen§

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Abstract

The relative importance of used capital goods to new investment has been increasing significantly over time. To this end, we develop a dynamic general equilibrium model with heterogeneous firms to account for all of the following key empirical regularities on capital reallocation: (i) the amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical, (ii) the probability of selling out used capital is well below 100%, and is procyclical, (iii) both the price of used capital and that of new capital are procyclical, and the former is more volatile than the latter. We show that the interactions between search frictions and financial frictions are essential to explain these facts. Moreover, many other predictions of our framework are also in line with important facts/puzzles on business cycles, such as (i) the dispersion of investment rate is procyclical, (ii) the dispersion of firm productivity is countercyclical. Finally, based on our structural model, we examine the roles of productivity, financial, and search-and-matching shock played in the fluctuations of capital reallocation.

Key Words: Capital Reallocation, Search Frictions, Financial Frictions, Total Factor Productivity, Business Cycles.

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†Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China. Tel: (+86) 21-52301590. Email: fengdong@sjtu.edu.cn

‡Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7612. Email: pfwang@ust.hk

§Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166; and School of Economics and Management, Tsinghua University, Beijing, China. Office: (314) 444-8559. Fax: (314) 444-8731. Email: yi.wen@stls.frb.org

1 Introduction

Resources allocation and reallocation is one of the most central questions in economics. Since capital is one of the key input in production, it is natural for us to investigate its accumulation pattern. As Rüdiger and Bayer (2014) emphasize, the dispersion of firm investment rate is procyclical, which is opposite to the fact by Kehrig (2015) that the dispersion of firm productivity is countercyclical. On the other hand, the importance of investment through the channel of capital reallocation has been increasing over time.[1] Meanwhile, Eisfeldt and Rampini (2006) uses Compustat to show that the proportion of used capital goods in overall investment around 24%. Cui (2014) updates the number as to around 40% in 2012. As revealed by Eisfeldt and Rampini (2006) and confirmed by many other works, the amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical.

The recent empirical findings by Lanteri (2015) suggests another dimension of capital reallocation: the price of used capital is procyclical, just as the price of new capital. More intriguingly, Lanteri (2015) discovers that the price of used capital is more volatile than the price of new capital over the business cycles. Finally, Kurmann and Pestroky-Nadeau (2007) documents that the probability of selling out used capital (the rate of capital reallocation) is well below 100%, and is procyclical. Since a substantial amount of physical capital remains unmatched in each period, then just like labor reallocation, capital reallocation is typically realized in decentralized markets, and is likely to subject to search and matching frictions across firms.

To this end, we develop a dynamic general equilibrium model with heterogeneous firms in a production economy. It turns out that our model is able to generate all of the key empirical regularities on capital reallocation: (i) the amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical, (ii) the probability of selling out used capital is well below 100%, and is procyclical, (iii) both the price of used capital and that of new capital are procyclical, and the former is much more volatile than the latter. We show that the interactions between search frictions and financial frictions are essential to explain these facts. Moreover, many other predictions of our framework are also in line with important facts/puzzles on business cycles, such as (i) the dispersion of investment rate is procyclical, (ii) the dispersion of firm productivity is countercyclical. Finally, based on our structural model, we examine the roles of productivity, financial, and search shock played in the fluctuations of capital reallocation.

Here is the intuition behind our framework. First and foremost, search frictions guarantee equilibrium capital unemployment, *i.e.*, the proportion/probability of selling out capital is below 100%, just emphasized by Kurmann and Petrosky-Nadeau (2007). Given capital unemployment, relaxing borrowing

[1] Captital reallocation means the transfer of ownership through sale, merger, and acquisition, etc. See Eisfeldt and Rampini (2006), among others, for more details.

constraint increases the demand for used capital by relatively productive firms, which in turn alleviates capital misallocation, and decreases the dispersion of firm productivity, which is in line with Kehrig (2015). Meanwhile, the more efficient use of capital from reallocation implies the capital is more concentrated in the hands of relatively productive firms. It then suggests that the dispersion of gross investment rate increases with the amount of capital reallocation, both of which are then procyclical. Both of these predictions are also consistent with the empirical regularities uncovered by Rüdiger and Bayer (2014) and Eisfeldt and Rampini (2006). Moreover, the benefit of capital reallocation, which can be measured by firm dispersion, is predicted to be counter-cyclical accordingly. Equivalently, we can use the distance between the most efficient TFP and the equilibrium TFP to measure the marginal benefit of capital reallocation. Intuitively, when boom arrives, capital reallocation is relatively efficient, and thus the distance shrinks, which in turn implies the benefit of capital reallocation decreases in boom. Just as Eisfeldt and Rampini (2006) shows.

The arbitrage-free condition guarantees that the price of used capital commoves with the price of new capital. Therefore no surprise our model can always generate the procyclicality of the price of used capital. More intriguingly, we show that financial shocks in our framework turns out increasing the relative volatility of the price of used to new capital. Here is the intuition. A positive financial shock alleviates capital misallocation, and thus increases the average price of used capital. In the presence of borrowing constraint, and keeping the price of used capital as given, the increase of the price of used capital continues to relax the borrowing constraint of the relatively productive, which further drives up the average price of used capital. Consequently, the price of used capital tends to be more volatile than that of new capital, just the empirical facts discovered by Lanteri (2015).

As summarized in Table 1, financial shocks turn out be able to explain all the important facts on capital reallocation and many other regularities over business cycles. Meanwhile, no surprisingly, aggregate productivity shock is also able to "replicate" many dimensions of the empirical facts. However, it is worth noting that the predicted benefit of capital reallocation is just opposite to the data. Moreover, the relative volatility of used capital seems failing to match the empirical pattern in Lanteri (2015). Additionally, search (and matching efficiency) shock plays almost the identical role in the fluctuations of capital reallocation over the cycles as does by financial shock. The most salient difference between the financial and search shock is that, the former is able to generate the relatively high volatility of the price of used capital while the latter is not.

Targets	Data	TFP Shock	Financial Shock	Search Shock
the amount of reallocation	+	+	+	+
the benefit of reallocation	−	+	−	−
probability of reallocation	+	+	+	+
price of used and new capital	+	+	+	+
relative volatility of used to new capital	high	almost equal	high	almost equal
dispersion of investment rate	+	+	+	+
TFP dispersion	−	+	−	+

Table 1: Shock Comparison (" + " and " − " denotes procyclical and counter-cyclical respectively)

Main Contribution. To the best of our knowledge, our paper is the first one to model and quantify the implication of search frictions and financial frictions for capital reallocation among heterogeneous firms. We implement our idea by developing a tractable DSGE model with heterogeneous firms and capital accumulation, as well with search and matching frictions in the decentralized markets for capital reallocation. More importantly, our paper is the first one to address all of the following important facts/puzzles on capital reallocation over the business cycles.

- (Rampini and Eisfeldt, 2006 and Cui 2014) The amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical.
- (Kurmann and Pestrosky, 2007) The rate of capital reallocation (probability of selling out used capital) is well below 100%, and is procyclical.
- (Lanteri, 2015) Both the price of used capital and that of new capital is procyclical. The former is more volatile than the latter.
- (Rüdiger and Bayer 2014) The dispersion of firm investment rate is procyclical.
- (Kehrig 2015) The dispersion of firm productivity is countercyclical.

Literature Review. Our paper is closely related three strand of literature. First, our paper belongs to the literature of capital reallocation and misallocation. On the one hand, the seminal work by Eisfeldt and Rampini (2006) establishes the basic pattern of capital reallocation. Cui (2013) use financial frictions to show why capital reallocation can be delayed. Xu (2014) analyzes the effects of merger and acquisition, a form of capital reallocation, on growth. Zhang (2012) and Li and Whited (2014) address the implications of information frictions for the reallocation of financial and physical assets respectively. On the other hand, our work is related to the general discussion on misallocation by Hsieh and Klenow (2009), and the focus on capital allocation across firms with financial frictions by Buera, Kaboski and Shin (2013) and Midrigan and Xu (2014).

Second, since we want to model capital unemployment as suggested by Kurmann and Pestroky-Nadeau (2007), who shows that a substantial amount of physical capital remains unmatched in each period,search and matching frictions, our paper is related to the literature on search and matching frictions. The most classic application is labor search, which is also called the Diamond-Mortensen-Pissarides framework. Moreover, it is related to the recently discussion on credit and capital search, such as Wasmer and Weil (2004), Petrosky-Nadeau and Weil (2013, 2015), Dong, Wang and Wen (2015), Ottonello (2015), and Triper (2015), etc.[2]

Third, the theory of TFP, such as Lagos (2006), Moll (2013), Dong (2014), Petrosky-Nadeau (2014), etc. Complementary to the previous research, our model shows that the equilibrium aggregate TFP is jointly determined by aggregate productivity, financial frictions and search frictions in the decentralized markets for capital reallocation.

Our paper is most related to Kurmann and Petrosky-Nadeau (2007), Lanteri (2015), Cui and Radde (2015), and Ottonello (2015). First, Kurmann and Petrosky-Nadeau (2007) address the implications of search frictions for capital allocation over the business cycles. Second, Lanteri (2015) shows that the price of used capital is procyclical and is more volatile than the price of new capital. Then he builds a model in which new and used capital are imperfect substitutes to explain the empirical results. Third, Ottonello (2015) models and quantifies search frictions in capital allocation for slow recovery in investment in a directed-search framework. Finally, Cui and Radde (2015), who considers the financial interaction between representative entrepreneur and representative household. Their model is used to solve the puzzle by Kiyotaki and Moore (2012). All of the above papers focus on the interaction between household and firms. Complementary to all of the above works, our work sheds light on the implications of search frictions for the reallocation of physical capital across firms.

The rest of the paper proceeds as follows. Section 2 and Section 3 builds and then characterizes the model respectively. Section 4 then launches a series of quantitative analysis after calibation with US economy. Section 5 lists several model extension, and Section 6 concludes. All proofs are documented in the appendix.

[2]Our paper is relevant to monetary search, such as Lagos and Wright (2005), Rocheteau and Wright (2005), Williamson and Wright (2010a, 2010b), Nosal and Rocheteau (2011), etc, and to finance search including Duffie-Garleanu-Pederson (2005), Lagos and Rocheteau (2009), Zhang (2012), Trejos and Wright (2013), Afonso and Lagos (2014), etc., and goods search, such as Bai, Rios-Rull, and Storesletten (2012), Kaplan and Menzio (2015), etc.

2 Model

2.1 Benchmark

Our benchmark is defined as the scenario in which there are no search and matching frictions in the secondary markets for capital reallocation. However, we proceed with case in which we consider search and matching frictions in the decentralized secondary markets for capital reallocation. It turns out that, the case without search frictions is purely a special case of the model we set up and characterize below. Then we can compare the benchmark with the search-frictions case, which lends a hand for our argument why search and matching frictions are *essential* for our discussion on capital reallocation.

2.2 Environment

Time discrete and proceeds from zero to infinite. The model economy is populated by three kinds of agents, (i) unit measure of firms indexed as $i, j \in [0, 1]$, (ii) unit measure of homogeneous intermediary (dealers) in the secondary market for capital reallocation, (iii) a representative household who owns the intermediary, and makes a decision on consumption, labor supply, and trading firm shares.

Firms receive aggregate and individual productivity shock at the beginning of each period. Then firms decide whether to purchase or sell their capital stock at the secondary capital market. We assume away explicit costs in the decentralized market for capital reallocation.[3] Instead, we only consider search frictions throughout the paper.

More specifically, all capital reallocation has to be realized with bilateral trading in decentralized markets. On the one hand, seller-firms and some dealers randomly search and match with each in the decentralized seller-side markets. On the other hand, buyer-firms and the remaining dealers randomly search and match with each other in the decentralized buyer-side markets. All the trade between firms and dealers are bilateral trade, and the terms of trade is determined by Nash bargaining. The inter-dealer market is centralized, in which supply and demand always equal to each in equilibrium with a common price. The matching technology in the seller-side and buyer-side markets are given by $\mathcal{M}^S\left(x^S, S\right) \leq \min\left\{x^S, S\right\}$, and $\mathcal{M}^B\left(x^B, B\right) \leq \min\left\{x^B, B\right\}$, where x^S and x^B are the measure of dealers in the seller-side and buyer-side decentralized secondary market, and S and B the measure of seller-firms and buyer-firms. Assume both matching functions are assumed to be constant return to scale. Then the matching probability (proportion) for firms (p) and dealers (q) is determined by $p^S \equiv \frac{\mathcal{M}^S\left(x^S,S\right)}{S}$, $q^S \equiv \frac{\mathcal{M}^S\left(x^S,S\right)}{x^S}$, $p^B \equiv \frac{\mathcal{M}^B\left(x^B,B\right)}{B}$, $q^B \equiv \frac{\mathcal{M}^B\left(x^B,B\right)}{x^B}$, and the market tightness is $\theta^S \equiv S/x^S$, and $\theta^B \equiv B/x^B$. To focus on the implication of search frictions for capital reallocation, we do not explicitly consider any information

[3] We address this issue the model extension part.

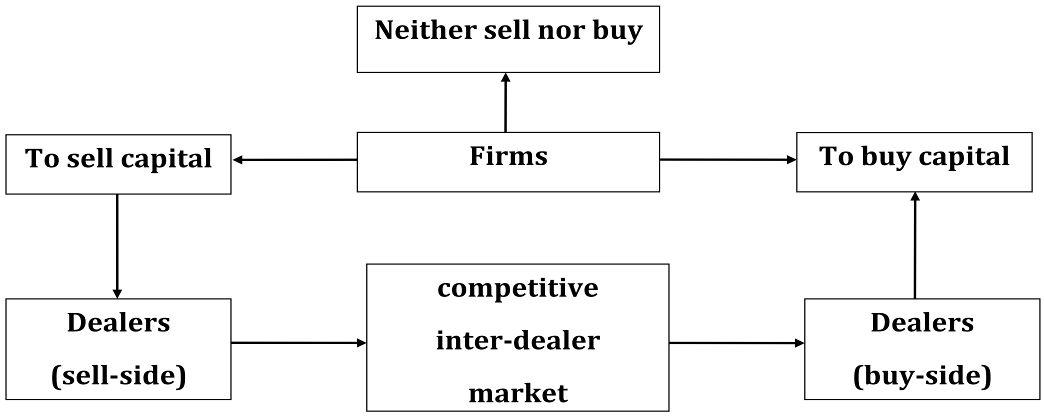

Figure 1: **Capital Reallocation: Firms and Dealers in Decentralized Markets**

frictions.[4] Therefore firm's productivity and its associated outside option is public information, at least to the dealer the firm is matched with. The terms of trade is determined by Nash bargaining. After trading at the decentralized market for capital reallocation, firms make a decision on employment, investment, and dividend distributed to shareholders. See Figure 1 for the illustration of the decision made by firms and dealers in the decentralized market.

2.3 Firms

In each period, firms are heterogeneous two dimension, (i) individual productivity z_t, and (ii) own capital holding k_t. For tractability, we assume these two distributions are orthogonal to each other, and denote them by $F(z)$ and $G(k)$ respectively. The support of z is denoted as $\mathcal{Z} = (z_{\min}, z_{\max})$. The production function for firms with individual productivity z is given by

$$y = \left(Az\widetilde{k}\right)^{\alpha} n^{1-\alpha}, \tag{1}$$

where $\widetilde{k}$ is the total capital used for production by firm-(k, z) *after* capital reallocation, and A the aggregate productivity shock; see next section for more details on $\widetilde{k}$. The specific capital return $R(z)$ is then given by[5]

$$R(z) \equiv \frac{\max_{n\geq 0}\left\{\left(Az\widetilde{k}\right) n^{1-\alpha} - Wn\right\}}{\widetilde{k}} = \alpha\left(\frac{1-\alpha}{W}\right)^{\frac{1-\alpha}{\alpha}} Az, \tag{2}$$

with the associated labor demand as

$$n\left(\widetilde{k}, z\right) = \left(\frac{1-\alpha}{W}\right)^{\frac{1}{\alpha}} Az\widetilde{k}. \tag{3}$$

[4]Complementary to our analysis, Kurlat (2013), Bigio (2014), Fuchs, Green and Papanikolaou (2014), Li and Whited (2014), and Zhang (2012), among others, discuss the reallocation of physical and financial assets with adverse selection.

[5]Rigorously speaking, we should replace $R(z)$ with $R(A, z)$. We use $R(z)$ for the ease of notational simplicity.

The problem of firm-(k,z) at time t is given by

$$V_t(k_t, z_t) = \max_{k_t^S, k_t^B, i_t, d_t} \left\{ d_t + \mathbb{E}_t \left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \int_{z \in \mathcal{Z}} V_{t+1}(k_{t+1}, z) \, dF(z) \right] \right\}, \tag{4}$$

subject to

$$\widetilde{k}_t = k_t - \widetilde{k}_t^S + \widetilde{k}_t^B, \tag{5}$$

$$d_t + i_t = R_t(z_t) \widetilde{k}_t + \widetilde{k}_t^S P_t^S(z_t) - \widetilde{k}_t^B P_t^B(z_t), \tag{6}$$

$$k_{t+1} = (1-\delta) \widetilde{k}_t + \Psi \left(i_t / \widetilde{k}_t \right) \widetilde{k}_t, \tag{7}$$

$$\widetilde{k}_t^S = p_t^S k_t^S, \tag{8}$$

$$\widetilde{k}_t^B = p_t^B k_t^B, \tag{9}$$

$$k_t^S \in [0, k_t], \tag{10}$$

$$k_t^B \in [0, \mu_t P_t k_t]. \tag{11}$$

where $\Psi \left(i_t / \widetilde{k}_t \right) \widetilde{k}_t$ represents the adjustment cost for investment as in Hayashi (1982) and Jermann (1998).[6]. We assume that $\Psi(\iota) = \iota$ and $\Psi'(\iota) = 1$ where ι denotes the steady state investment rate. That is, we assume there exists no adjustment cost for investment in steady state.

Besides, $\left(P_t^S(z_t), P_t^B(z_t) \right)$ denotes the firm-specific capital price in the seller-side and buyer-side dealer markets (more explanation here), and $\mu_t^S \in [0,1]$ the resaleability. for capital sale, and $\mu_t^B > 0$ the collateral constraint for additional capital purchase.[7]

Finally, due to search frictions in the secondary market for capital reallocation, only $\widetilde{k}_t^S \equiv p_t^S k_t^S$ units of capital can be traded between a dealer and a firm-(k_t, z_t) who wants to sell k_t^S units of capital. Meanwhile, only $\widetilde{k}_t^B \equiv p_t^B k_t^B$ units of capital can be traded between a dealer and a firm-(k_t, z_t) who wants to buy k_t^B units of capital.

Remark 1 *It is worth noting that $k_t^S \in \left[0, \mu_t^S k_t\right]$ and $k_t^B \in \left[0, \mu_t^B k_t\right]$ seemingly serve as a more natural setup on the constraints of capital reallocation. We can easily check from Section x.x that, what essentially matters is $\mu_t \equiv \mu_t^B / \mu_t^S$. Therefore we normalize $\mu_t^S = 1$, and focus on the the financial frictions on the demand side.*[8]

[6]Wang and Wen (2013) show under what conditions combining firms with heterogenous investment efficiency and borrowing constraint delivers an aggregate convext adjustment cost for investment.

[7]See Kiyotaki and Gertler (2010), Cui and Radde (2014) for the dicussion on resalelability, and Kiyotaki and Moore (1997), Jermann and Quadrini (2013), Moll (2014), and Wang and Wen (2013).

[8]See Kiyotaki and Moore (2012) for the discussion on the supply side, *i.e.*, resaleability constraint, and see Cui and Radde (2015a,b) who propose a micro foundation for that.

2.4 Household

The objective function of a representative household is given by

$$\max \mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t\left[\log\left(C_t\right)-\psi\frac{N_t^{1+\gamma}}{1+\gamma}\right]\right\}, \tag{12}$$

subject to

$$C_t+\int_{i\in[0,1]}s_{t+1}^i\left(V_t^i-d_t^i\right)di=\int_{i\in[0,1]}s_t^iV_t^idi+\Pi_t^d+W_tN_t, \tag{13}$$

where β is the discount factor of the household, C^h the consumption, N_t the labor supply, $\left(V_t^i, s_t^i\right)$ the price of firm-i and the associated share holdings by the household. The household receive profit from intermediary in the secondary market Π_t^d, and labor income W_tN_t. Denote Λ_t as the Lagrangian multiplier of the household budget constraint in equation (13). Then the first order condition (FOC) on consumption (C_t), labor supply (N_t) and share holding $\left(s_{t+1}^i\right)_{i\in[0,1]}$ is given by

$$\begin{aligned}\Lambda_t &= \frac{1}{C_t}, &(14)\\ \Lambda_tW_t &= \psi N_t^{\gamma}, &(15)\\ V_t^j &= d_t^j+\mathbb{E}_t\left[\left(\frac{\beta\Lambda_{t+1}}{\Lambda_t}\right)V_{t+1}^j\right]. &(16)\end{aligned}$$

where $\beta\Lambda_{t+1}/\Lambda_t$ denotes the pricing kernel.

2.5 Equilibrium

An equilibrium consists of a series of prices and quantity such that

1. given the prices, the household, firms and dealers maximize their own objective function;

2. clearing condition in the inter-dealer market, in the labor market (N), in the sharing market (s).

3 Characterization

3.1 Firms

Given any $\widetilde{k}_t$ at hand, FOC on i_t yields

$$1=\Psi'\left(i_t/\widetilde{k}_t\right)\mathbb{E}\left[\left(\frac{\beta\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\partial V_{t+1}\left(k_{t+1},z_{t+1}\right)}{\partial k_{t+1}}\right)\right] \tag{17}$$

We guess and will later verify that firm's value function is linear in capital, *i.e.*,

$$V_t\left(k_t,z_t\right)=\phi_t\left(z_t\right)k_t. \tag{18}$$

Then the FOC on i_t is simplified as

$$1 = \Psi' \left(i_t / \widetilde{k}_t \right) Q_t. \tag{19}$$

where Q_t denotes the Tobin's Q, *i.e.*, the price of new capital goods, such that

$$Q_t \equiv \mathbb{E} \left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \phi_{t+1} \left(z_{t+1} \right) \right]. \tag{20}$$

In turn, the investment is determined by

$$i_t = \omega \left(Q_t \right) \widetilde{k}_t, \tag{21}$$

where $\omega \left(Q_t \right) \equiv \Psi'^{-1} \left(Q_t \right)$. Since $\Psi \left(\iota \right)$ is strictly concave, $\omega \left(Q_t \right)$ strictly increases with Q_t. Substituting equations (18) and (21) into the problem of firm-(k_t, z_t) yields

$$\begin{aligned} \phi_t \left(z_t \right) k_t \quad = \quad & \max_{k_t^S \in [0, k_t],\ k_t^B \in [0, \mu_t P_t k_t]} \{ \left(R_t \left(z_t \right) + \Gamma \left(Q_t \right) \right) k_t \\ & + \left(P_t^S \left(z_t \right) - R_t \left(z_t \right) - \Gamma \left(Q_t \right) \right) \widetilde{k}_t^S \\ & + \left(R_t \left(z_t \right) + \Gamma \left(Q_t \right) - P_t^B \left(z_t \right) \right) \widetilde{k}_t^B \}, \end{aligned} \tag{22}$$

where $\widetilde{k}_t^S = p_t^S k_t^S$, $\widetilde{k}_t^B = p_t^B k_t^B$, and

$$\Gamma \left(Q_t \right) \equiv Q_t \left(1 - \delta \right) + Q_t \Psi \left(\omega \left(Q_t \right) \right) - \omega \left(Q_t \right). \tag{23}$$

Remark 2 *Note that $\Gamma \left(Q_t \right)$ denotes the shadow value of each unit of capital after production. On the one hand, $Q_t \left(1 - \delta \right)$ is the value after depreciation. On the other hand, unit capital implies $\omega \left(Q_t \right)$ unit of new capital, whose return and cost is $Q_t \Psi \left(\omega \left(Q_t \right) \right)$ and $\omega \left(Q_t \right)$ respectively.*

3.2 Bargaining

Seller Side. In the seller-side decentralized market, only p_t^S proportion of capital can be traded between a dealer and a firm-(k_t, z_t) who wants to sell k_t^S units of capital. Denote $P_t^S \left(z_t \right)$ the terms of trade between the firm and the dealer. On the one hand, the profit by the dealer is $\max \left\{ P_t \widetilde{k}_t^S - P_t^S \left(z_t \right) \widetilde{k}_t^S, 0 \right\}$. On the other hand, as indicated by equation (22), the additional benefit received by the firm is

$$\max \left\{ \left(P_t^S \left(z_t \right) - R_t \left(z_t \right) - \Gamma \left(Q_t \right) \right) \widetilde{k}_t^S, 0 \right\}. \tag{24}$$

Therefore the trading surplus is $\max \left\{ P_t - R_t \left(z_t \right) - \Gamma \left(Q_t \right), 0 \right\}$. Since $R_t \left(z_t \right)$ increases with z_t, which is evident from equation (2), the trade on the seller side happens if and only $z_t < z_t^*$, where z_t^* is determined by[9]

$$P_t = R_t \left(z_t^* \right) + \Gamma \left(Q_t \right). \tag{25}$$

[9] Add some discussion on the intuition of equation (25).

Denote 1-η as the bargaining power of firm side.[10] Given P_t and $z_t < z_t^*$, the price agreed between the dealer and seller-firm-z_t, $P_t^S(z_t)$, is determined by Nash bargaining as below,

$$\max_{P_t^S(z_t)} \left(\left(P_t^S(z_t) - R_t(z_t) - \Gamma(Q_t)\right) \widetilde{k}_t^S \right)^{1-\eta} \left(P_t \widetilde{k}_t^S - P_t^S(z_t) \widetilde{k}_t^S \right)^{\eta}, \tag{26}$$

and therefore we have

$$P_t^S(z_t) = (1-\eta) P_t + \eta \left(R_t(z_t) + \Gamma(Q_t)\right). \tag{27}$$

The above equation on $P_t^S(z_t)$ is intuitive. As argued in the previous subsection, $\Gamma(Q_t)$ is the shadow value of each unit of used capital. Therefore $R_t(z_t) + \Gamma(Q_t)$ denotes the expected value of each unit of capital with productivity z_t if this unit of capital is put in production. Therefore the outside option of the dealer and the seller-firm-z_t is P_t and $R_t(z_t) + \Gamma(Q_t)$ respectively. In turn, the Nash bargaining implies the trade price is weighted between these two outside options.

Remark 3 *It is worth noting that, the selling price equation $P_t^S(z_t)$ in (27) has nothing to do with k_t. This is because of the linear structure of value function. See Bianchi and Bigio (2015), among others, for the similar modeling trick. Moreover, $P_t^S(z_t)$ strictly increases with z_t. This is because the individual productivity of seller-firm-z_t is public information, at least to dealers. Since the outside option of seller-firm-z_t strictly increases with z_t, the classic hold-up problem arises from Nash bargaining, which in turn increases $P_t^S(z_t)$. See Zhang (2012), among others, for the interesting implications from the setup in which seller and buyer's outside option (preference) is private information.*

Buyer Side. Similarly, at buyer side, the trading surplus is given by $\max\{R_t(z_t) + \Gamma(Q_t) - P_t, 0\}$. Therefore the trade on the seller side happens if and only $z_t > z_t^*$. Given P_t and $z_t > z_t^*$, $P_t^S(z_t)$ is also determined by a bilateral Nash bargaining such that

$$\max_{P_t^B(z_t)} \left(\left(R_t(z_t) + \Gamma(Q_t) - P_t^B(z_t)\right) \widetilde{k}_t^B \right)^{1-\eta} \left(P_t^B(z_t) \widetilde{k}_t^B - P_t \widetilde{k}_t^B \right)^{\eta}, \tag{28}$$

which suggests that

$$P_t^B(z_t) = (1-\eta) P_t + \eta \left(R_t(z_t) + \Gamma(Q_t)\right). \tag{29}$$

The intuition on $P_t^B(z_t)$ is exactly the same to that on $P_t^S(z_t)$ mentioned in the aforementioned remark.

Remark 4 *For tractability, we have assumed that firms do not directly meet with each other for capital reallocation. Alternatively, we can let firms directly contact each with $i \in \mathcal{B}$ and $j \in \mathcal{S}$. We know that*

[10] The more general setup is to denote $1-\eta^S$ and $1-\eta^B$ as the bargaining power of firms as sellers and buyers respectively. Tractability is well preserved under the general setup. We implicity assume symmetry, *i.e.*, $\eta^S = \eta^B$, for simplicity.

$R(i) \geq R(j)$ holds for $i \in \mathcal{B}$ and $j \in \mathcal{S}$. Since z shock is iid over time and across firm, the price each unit of capital, $x(i,j)$, is determined by Nash bargaining as below,

$$\underset{x\in[R(i),R(j)]}{\arg\max} \left\{ (R(i) - P(i,j))^{\eta} (P(i,j) - R(j))^{1-\eta} \right\}, \tag{30}$$

which delivers

$$P(i,j) = (1-\eta) R(i) + \eta R(j). \tag{31}$$

where $\eta \in (0,1)$ denotes the bargaining power of the firm-buyer side. Then the expected price faced up by $i \in \mathcal{B}$ and $j \in \mathcal{S}$, is given by

$$\overline{P}(i) = \mathbb{E}(P(i,j) | j \in \mathcal{S}), \tag{32}$$

$$\overline{P}(j) = \mathbb{E}(P(i,j) | i \in \mathcal{B}). \tag{33}$$

We can then verify that he linear structure of the model is still well preserved, but the algebra is a little bit messy.

3.3 Firms Revisited

Now we characterize $\phi_t(\cdot)$. Substituting equations (27) and (29) into (22) yields

$$\begin{aligned}\phi_t(z_t) = & \max_{k_t^S\in[0,k_t],k_t^B\in[0,\mu_t P_t k_t]} \{(R_t(z_t) + \Gamma(Q_t)) k_t \\ & + (1-\eta)(R_t(z_t^*) - R_t(z_t)) p_t^S k_t^S + (1-\eta)(R_t(z_t) - R_t(z_t^*)) p_t^B k_t^B\},\end{aligned} \tag{34}$$

which immediately generates the policy function of individual firms on capital reallocation as below.

Lemma 1 *The individual supply and demand of capital reallocation is given by*

$$k_t^S(k_t, z_t) = \begin{cases} k_t, & \textit{if } z_t \leq z_t^* \\ 0, & \textit{otherwise} \end{cases}, \; k_t^B(k_t, z_t) = \begin{cases} 0, & \textit{if } z_t \leq z_t^* \\ \mu_t k_t, & \textit{otherwise} \end{cases}, \tag{35}$$

and thus the amount of firm's own capital after reallocation is given by

$$\widetilde{k}_t(k_t, z_t) = k_t - \widetilde{k}_t^S + \widetilde{k}_t^B = \begin{cases} \left(1 - p_t^S\right) k_t, & \textit{if } z_t \leq z_t^* \\ \left(1 + \mu_t P_t p_t^B\right) k_t, & \textit{otherwise} \end{cases}. \tag{36}$$

As indicated in Lemma 1, the demand and supply for capital reallocation is characterized by a cut-off property. Intuitively, firms whose productivity is low enough choose to sell their capital while firms who are productive enough want to purchase. Due to the linear structure shown in equation (34), both seller- and buyer-firms will choose corner solution, *i.e.*, they would like to sell and buy as much as they can. Due to search frictions in capital reallocation, only p_t^S and p_t^B proportion of firm's plan is realized.

Substituting equation (35) into (34) yields the shadow value of each unit of capital with productivity z_t at the beginning of time t,

$$\begin{aligned}\phi_t(z_t) &= R_t(z_t) + \Gamma(Q_t) \\ &\quad + \underbrace{(1-\eta)\, p_t^S \left(R_t(z_t^*) - R_t(z_t)\right) \mathbf{1}_{\{z_t \le z_t^*\}} + \mu_t P_t (1-\eta)\, p_t^B \left(R_t(z_t) - R_t(z_t^*)\right) \mathbf{1}_{\{z_t > z_t^*\}}}_{\textbf{real-option value from capital reallocation}}.\end{aligned} \tag{37}$$

The first line of the RHS of equation (37) denotes shadow value of each unit of capital in production, where $\Gamma(Q_t)$ is defined in equation (23). The second line denotes the additional benefit from capital reallocation. Note that $\phi_t(z_t)$ is not related to k_t, and therefore the conjecture in equation (18) is verified.

Combining equations (20) and (37) yields Tobin's Q as below,

$$\begin{aligned}Q_t &= \underbrace{\mathbb{E}\left[\left(\frac{\beta\Lambda_{t+1}}{\Lambda_t}\right)\left(\int R_{t+1}(z)\, dF(z) + \Gamma(Q_{t+1})\right)\right]}_{\textbf{classic value}} \\ &\quad \underbrace{+\mathbb{E}\left[\left(\frac{\beta\Lambda_{t+1}}{\Lambda_t}\right)(1-\eta)\, p_{t+1}^S \int_{z_{\min}}^{z_{t+1}^*} \left(R_{t+1}\left(z_{t+1}^*\right) - R_{t+1}(z)\right) dF(z)\right]}_{\textbf{real-option value from selling capital}} \\ &\quad \underbrace{+\mathbb{E}\left[\left(\frac{\beta\Lambda_{t+1}}{\Lambda_t}\right)\mu_{t+1} P_{t+1} (1-\eta)\, p_{t+1}^B \int_{z_{t+1}^*}^{z_{\max}} \left(R_{t+1}(z) - R_{t+1}\left(z_{t+1}^*\right)\right) dF(z)\right]}_{\textbf{real-option value from purchasing capital}},\end{aligned} \tag{38}$$

Then, given Q_t, the *new* investment and *gross* investment is given by

$$i_t(k_t, z_t) = \omega(Q_t)\, \widetilde{k}_t(k_t, z_t), \tag{39}$$

$$i_t^g(k_t, z_t) = \widetilde{k}_t(k_t, z_t) - k_t + i_t(k_t, z_t). \tag{40}$$

3.4 Inter-Dealer Market

The cut-off value property of buying and selling capital yields $\mathcal{S}_t = \{z |\, z \in [z_{\min}, z_t^*], \forall\, k_t\}$ and $\mathcal{B}_t = \{(k_t, z_t)\,|\, z \in [z_t^*, z_{\max}], \forall\, k_t\}$. Then the measure of seller-firm and that of buyer-firm is given by

$$S_t(z_t^*) = F(z_t^*),\ B_t(z_t^*) = 1 - F(z_t^*). \tag{41}$$

In turn, the total supply and demand of capital in the secondary market is

$$K_t^S \equiv \int\int_{\mathcal{S}_t} k_t^S = K_t S_t,\ K_t^B \equiv \int\int_{\mathcal{B}_t} k_t^B = \mu_t P_t K_t B_t. \tag{42}$$

Following Lagos and Rocheteau (2009) and Zhang (2012), we assume there exists a competitive inter-dealer market in which demand equals supply, *i.e.*, $K_t^S p_t^S = K_t^B p_t^B$. Then we have

$$S_t p_t^S = \mu_t P_t B_t p_t^B, \tag{43}$$

or equivalently,

$$\mathcal{M}^S\left(x_t^S, S_t\right) = \mu_t P_t \mathcal{M}_t^B\left(x_t^B, B_t\right), \tag{44}$$

where LHS and RHS of the above equation denotes the total matched amount of capital in the supply-side and demand-side secondary market.

As specified in Section 2, the total measure of dealers is normalized to one, and thus

$$x_t^B + x_t^S = 1. \tag{45}$$

The arbitrage-free condition for dealers at either side of the markets is given by

$$q_t^S\left(P_t - \overline{P}_t^S\right)K_t = q_t^B \mu_t^B\left(\overline{P}_t^B - P_t\right)K_t \equiv \Pi_t^d, \tag{46}$$

which implies

$$P_t = \lambda_t \overline{P}_t^S + (1-\lambda_t)\,\overline{P}_t^B, \tag{47}$$

where

$$\overline{P}_t^S \equiv \mathbb{E}\left(P_t^S(z_t)\,|z_t \in \mathcal{S}_t\right) = (1-\eta)\,P_t + \eta\mathbb{E}\left(R_t(z_t) + \Gamma(Q_t)\,|z_t \le z_t^*\right), \tag{48}$$

$$\overline{P}_t^B \equiv \mathbb{E}\left(P_t^B(z_t)\,|z_t \in \mathcal{B}_t\right) = (1-\eta)\,P_t + \eta\mathbb{E}\left(R_t(z_t) + \Gamma(Q_t)\,|z_t > z_t^*\right), \tag{49}$$

$$\lambda_t \equiv \frac{q_t^S}{q_t^S + q_t^B \mu_t P_t}. \tag{50}$$

Substituting equations (25), (48) and (48) into (47) yields

$$R_t(z_t^*) = \lambda_t \mathbb{E}\left(R_t(z_t)\,|z_t \le z_t^*\right) + (1-\lambda_t)\,\mathbb{E}\left(R_t(z_t)\,|z_t > z_t^*\right). \tag{51}$$

Combining equations (41), (44), (45) suggests that $\left(q_t^S, q_t^B\right)$ is a function of z_t^*. In turn, we can denote λ_t as $\lambda_t(z_t^*)$. Moreover, substituting equation (2) into (25) reveals that

$$z_t^* = \lambda_t \mathbb{E}\left(z_t | z_t \le z_t^*\right) + (1-\lambda_t)\,\mathbb{E}\left(z_t | z_t > z_t^*\right). \tag{52}$$

A Special Case. If $z \sim U(z_{\min}, z_{\max})$, then given λ_t, we can obtain an analytical solution to z_t^* from equation (52) as

$$z_t^* = \lambda_t z_{\min} + (1-\lambda_t)\,z_{\max}. \tag{53}$$

Note that, we treat λ_t as given in equation (52). However, λ_t is related to z_t^* in equilibrium since $\left(q_t^B, q_t^S\right)$ is related to z_t^*. This is because, (B_t, S_t) is determined by z_t^*, and given (B_t, S_t), equations (41) and (44) jointly pin down $\left(x_t^B, x_t^S\right)$, and thus solves $\left(q_t^B, q_t^S\right)$. To sharpen the analysis, we further specify the matching technology as $\mathcal{M}_t^S\left(x_t^S, S_t\right) = \gamma_t\left(x_t^S\right)^\rho \left(S_t\right)^{1-\rho}$ and $\mathcal{M}_t^B\left(x_t^B, B_t\right) = \gamma_t\left(x_t^B\right)^\rho \left(B_t\right)^{1-\rho}$ with $\rho \in (0,1)$. Then equation (44) yields

$$x_t^S = \frac{\left(\mu_t P_t\right)^{\frac{1}{\rho}}\left(\frac{F(z_t^*)}{1-F(z_t^*)}\right)^{\frac{1-\rho}{\rho}}}{1+\mu_t^{\frac{1}{\rho}}\left(\frac{F(z_t^*)}{1-F(z_t^*)}\right)^{\frac{1-\rho}{\rho}}}. \tag{54}$$

In turn, the market tightness is $\theta^S \equiv \frac{S}{x^S} = \frac{F(z^*)}{x^S(z^*)}$, $\theta^B \equiv \frac{B}{x^B} = \frac{1-F(z^*)}{1-x^S(z^*)}$, and thus the matching probability is given by $p^S = \gamma\left(\theta^S\right)^{-\rho}$, $q^S = \gamma\left(\theta^S\right)^{1-\rho}$, $p^B = \gamma\left(\theta^B\right)^{-\rho}$, and $q^B = \gamma\left(\theta^B\right)^{1-\rho}$. Consequently, $\frac{q^B}{q^S} = \left(\frac{\theta^B}{\theta^S}\right)^{1-\rho} = \left(\frac{F(z_t^*)}{1-F(z_t^*)}\right)_t^{\frac{(1-2\rho)(1-\rho)}{\rho}} \left(\mu_t P_t\right)^{\frac{1-\rho}{\rho}}$, and therefore

$$\lambda_t = \frac{1}{1+\left(\frac{q_t^B}{q_t^S}\right)\mu_t} = \frac{1}{1+\left(\frac{F(z_t^*)}{1-F(z_t^*)}\right)^{\frac{(1-2\rho)(1-\rho)}{\rho}}\left(\mu_t P_t\right)^{\frac{1}{\rho}}}. \tag{55}$$

In general, we can obtain z_t^* by combining equation (52) and (55). Moreover, we may obtain multiple equilibria then. Since we want to focus on unique interior solution, we set $\rho = \frac{1}{2}$, which then implies λ_t is independent of z_t^*, and only related to μ_t such that

$$\lambda_t = \frac{1}{1+\left(\mu_t P_t\right)^2}, \tag{56}$$

which then suggests that z_t^* increases with μ_t. That is, relaxing borrowing constraint helps alleviate capital misallocation.

Remark 5 *Equation (52) reveals that the cut-off value z_t^* is not related to the aggregate state variables (A_t, K_t), but is only determined by μ_t, financial frictions in capital demand. This is at least partially because of our removing away the price effect when specifying the borrowing constraint $k_t^B \leq \mu_t k_t$. If we further introduce the price effect as in Kiyotaki and Moore (1997), then the borrowing constraint is modified as $k_t^B \in [0, \mu_t P_t k_t]$. Under such alternative setup, z_t^* in equation (52) would be be affected by P_t, which in turn is a forward looking variable.*

Remark 6 *(Self-fulfilling Corner Solution) As shown above, although $\rho = \frac{1}{2}$ can guarantee the interior solution to z_t^* is unique, we may still have other kind of solution. Note that we always have a self-confirming equilibrium in which the secondary market for capital reallocation is completely collapsed. On the one hand, if no one is willing to sell capital, then no one is able to buy, and vice versa.*

3.5 Aggregation

First, using equation (39), the aggregate investment, and the law of motion of the aggregate capital stock is given by

$$I_t \equiv \int\int i_t dk_t dz_t = \omega\left(Q_t\right) K_t, \tag{57}$$

and

$$K_{t+1} \equiv \int \int \left[(1-\delta) \widetilde{k}_t + \Psi \left(i_t / \widetilde{k}_t \right) \widetilde{k}_t \right] dk_t dz_t = (1-\delta) K_t + \Psi \left(I_t / K_t \right) K_t. \tag{58}$$

Besides, using equation (44) and (40), we get that

$$I_t^{gross} \equiv \int \int i_t^{gross} dk_t dz_t = I_t. \tag{59}$$

Meanwhile, the aggregate resource constraint is

$$Y_t = C_t + I_t. \tag{60}$$

Following Jermann (1998), we assume $\Psi(i/k) = \frac{\iota_{SS}^{1/\sigma}}{1-1/\sigma} (i/k)^{1-1/\sigma} - \frac{\iota_{SS}/\sigma}{1-1/\sigma}$ where ι_{SS} denotes the investment-capital ratio in steady state, and $\sigma \in (0,1)$ a parameter for adjustment cost. Since $\Psi(\iota_{SS}) = \iota_{SS}$, equation (58) immediately implies $\iota_{SS} = \delta$. Then equation (19) which implies $\omega(Q_t) = \delta Q_t^{\sigma}$. In turn, equation (57) and (58) can be rewritten as

$$I_t = \delta Q_t^{\sigma} K_t, \tag{61}$$

and

$$K_{t+1} = \left(1 + \frac{\delta \sigma}{1-\sigma} \left(1 - Q_t^{\sigma-1} \right) \right) K_t. \tag{62}$$

Second, given the aggregate productivity A_t, the aggregate capital K_t and labor supply N_t, and the cut-off value z_t^*, we characterize the aggregate output and the associated TFP as below.

Proposition 1 *The aggregate output is given by*

$$Y_t = (TFP_t \cdot K_t)^{\alpha} N_t^{1-\alpha}, \tag{63}$$

where

$$TFP_t = A_t \cdot \left\{ \mathbb{E}(z) + p_t^S S_t \left[\mathbb{E}(z | z \geq z_t^*) - \mathbb{E}(z | z \leq z_t^*) \right] \right\}, \tag{64}$$

which strictly increases with z_t^* (cut-off value), A_t (aggregate productivity) and γ_t (matching efficiency). Moreover, the wage rate is

$$W_t = (1-\alpha) \left(\frac{Y_t}{N_t} \right). \tag{65}$$

The benefit and amount of capital reallocation. On the one hand, if there is no capital reallocation, *i.e.*, $z_t^* = z_{\min}$, then equation (64) implies $TFP_t = \underline{TFP}_t \equiv A_t \mathbb{E}(z)$. Intuitively, if there exists no capital reallocation, then our model framework is reduced to the classical investment theory with adjustment cost. On the other hand, the best allocation implies $z_t^* = z_{\max}$, with $TFP_t = \overline{TFP}_t \equiv A_t z_{\max}$. Since

TFP_t increases with z_t^*, in general $TFP_t \in \left(\underline{TFP}_t, \overline{TFP}_t\right)$, and thus the benefit of capital reallocation (CR_t^B) is then given by

$$CR_t^B \equiv \overline{TFP}_t - TFP_t. \tag{66}$$

Alternatively, as suggested by Eisfeldt and Rampini (2006), we can also use productivity dispersion to measure CR_t^B. The above two measurement is in line with each other. This is because, as argued above, if there were no frictions, then $z_t^* = z_{\max}$, *i.e.*, all capital should be distributed to the most productive firms, in which case the dispersion is zero. In the end, the amount and benefit of capital reallocation (CR_t^A) is given by

$$CR_t^A = p_t^S S_t K_t = \mu_t p_t^B B_t K_t. \tag{67}$$

Decomposition of TFP. Since search frictions involve in extensive margin, we can then decompose the TFP into extensive and intensive margin. Denote

$$TFP_t^{EXT} \equiv \left(\frac{K_t^S}{K_t}\right) \cdot \left(\frac{\mathcal{M}^S\left(x_t^S, S_t\right)}{S_t}\right) = S_t p_t^S = \mu_t B_t p_t^B, \tag{68}$$

$$TFP_t^{INT} \equiv A_t \left[\mathbb{E}\left(z \middle| z \geq z_t^*\right) - \mathbb{E}\left(z \middle| z \leq z_t^*\right)\right]. \tag{69}$$

Note that TFP_t^{EXT} is the successfully allocated mount for each unit of capital, TFP_t^{INT} the average gain. Then CR_t^B can be rewritten as

$$TFP_t^{GAIN} = TFP_t - \underline{TFP}_t = TFP_t^{EXT} \cdot TFP_t^{INT}, \tag{70}$$

and thus the decomposition of the benefit of capital reallocation is given by

$$\frac{\Delta TFP_t^{GAIN}}{TFP_t^{GAIN}} = \frac{\Delta TFP_t^{EXT}}{TFP_t^{EXT}} + \frac{\Delta TFP_t^{INT}}{TFP_t^{INT}}. \tag{71}$$

We finish this part by characterizing the dispersion of investment rate. As argued by Bachmann and Bayer (2014), the investment dispersion is procyclical, which is just opposite to the dispersion of productivity.

Although at the aggregate level $\frac{I_t^{gross}}{K_t} = \omega\left(Q_t\right)$, as implied in equation (59) and (62), there is heterogeneity at the firm level on gross investment rate as below.

$$\frac{i_t^{gross}\left(k_t, z_t\right)}{k_t} \equiv \frac{\widetilde{k}_t\left(k_t, z_t\right) - k_t + i_t\left(k_t, z_t\right)}{k_t} = \left(1 + \omega\left(Q_t\right)\right)\left(\frac{\widetilde{k}_t\left(k_t, z_t\right)}{k_t}\right) - 1. \tag{72}$$

where $\widetilde{k}_t\left(k_t, z_t\right)$ is given by equation (36).

Corollary 1 ***(Dispersion of Investment Rate)*** *The standard deviation of the gross investment rate and that of firm growth rate is given by*

$$std\left(\frac{i_t^{gross}(k_t,z_t)}{k_t}\right)=(1+\omega(Q_t))\frac{\mathcal{M}_t^S\left(x_t^S,S_t\right)}{\sqrt{S_tB_t}}, \tag{73}$$

which increases with Q_t *and* z_t^*.

Since Q_t and z_t^* tends to increase with μ_t, the above corollary implies that the dispersion of investment rate is procyclical.

4 Quantitative Analysis

4.1 Calibration

Standard Parameters: First, as standard in the literature, we set the quarterly discount factor as $\beta=0.985$, capital share $\alpha=0.33$, depreciation rate $\delta=0.025$, and normalize the aggregate productivity $A=1$, the inverse Frisch elasticity of labor supply $\gamma=1$, the coefficient of labor disutility $\psi=1.75$. Following Miao and Wang (2010), we set the parameter for investment adjustment cost $\sigma=0.25$.

Matching Technology: We assume Cobb-Douglas matching technology such that $\mathcal{M}_t^S\left(x_t^S,S_t\right)=\xi_t\left(x_t^S\right)^{\rho}(S_t)^{1-\rho}$ and $\mathcal{M}_t^B\left(x_t^B,B_t\right)=\xi_t\left(x_t^B\right)^{\rho}(B_t)^{1-\rho}$ with $\rho\in(0,1)$. In turn, the market tightness is $\theta^S\equiv\frac{S}{x^S}=\frac{F(z^*)}{x^S(z^*)},\theta^B\equiv\frac{B}{x^B}=\frac{1-F(z^*)}{1-x^S(z^*)}$. Consequently, the matching probability is given by

$$p^S=\gamma\left(\theta^S\right)^{-\rho},\ q^S=\gamma\left(\theta^S\right)^{1-\rho},\ p^B=\gamma\left(\theta^B\right)^{-\rho},\ q^B=\gamma\left(\theta^B\right)^{1-\rho}. \tag{74}$$

We assume symmetry between firms and dealers by setting $\eta=0.5$. Moreover, following the literature on labor search, we let the matching elasticity equal to the bargaining power, *i.e.*, $\rho=1-\eta=0.5$.

Productivity Distribution: We assume individual productivity conforms to a Power distribution *i.e.*, $F(z)=z^{\epsilon}$ with $z\in(0,1)$. According to Kurmann and Petrosky (2007). We may take the average such that $p^S=0.86$. Moreover, the proportion of used capital that is successfully purchased over total investment is $\zeta_t=\frac{S_tp_t^SK_t}{S_tp_t^SK_t+I_t}=\frac{S_tp_t^S}{S_tp_t^S+\omega(Q_t)}$.In steady, $Q_t=1$, and thus $\omega(Q_t)=\delta$. Then above measurement is simplified as $\zeta=\frac{p^SF(z^*)}{p^SF(z^*)+\delta}$. As shown by Eisfeldt and Rampini (2006), $\zeta=24\%$. Then we know that $F(z^*)=(z^*)^{\epsilon}=\left(\frac{\zeta}{1-\zeta}\right)\left(\frac{\delta}{p^S}\right)$. Additionally, equation (52) can be rewritten as $z^*=\lambda\cdot\left(\frac{\epsilon}{1+\epsilon}\right)z^*+(1-\lambda)\cdot\left(\frac{\epsilon}{1+\epsilon}\right)\left(\frac{1-(z^*)^{1+\epsilon}}{1-(z^*)^{\epsilon}}\right)$, where $\lambda\approx\frac{1}{1+\left(\frac{(z^*)^{\epsilon}}{1-(z^*)^{\epsilon}}\right)^{\frac{(1-2\rho)(1-\rho)}{\rho}}\mu^{\frac{1}{\rho}}}$. Therefore we have two equations on two unknowns (z^*,ϵ). In turn, we obtain $\epsilon=0.75$.

Other Parameterization: Credit Market instruments to non-financial assets is 0.7. Then we have $\mu=0.711$. Moreover, we use p^S to back out coefficient of the matching efficiency as $\xi=0.88$. Finally, we summarize the parameterization Table 2.

Parameter	Value	Description
β	0.99	discount factor
α	0.33	capital income share
δ	0.025	depreciation rate
A	1	aggregate productivity
γ	1	inverse Frisch elasticity of labor supply
ψ	1.75	coefficient of labor disutility
σ	0.25	parameter of investment adjustment cost
η	0.5	bargaining power of dealers
ρ	0.5	matching elasticity
ξ	0.88	matching efficiency
μ	0.711	parameter of borrowing constraint
ϵ	0.75	parameter of individual productivity distribution

Table 2: Parameterization

4.2 Transition Dynamics

The dynamic system on $(Y_t, TFP_t, I_t, C_t, K_t, N_t, W_t, P_t)$ is given by

$$
\begin{aligned}
Y_t &= (TFP_t K_t)^{\alpha} N_t^{1-\alpha}, \\
TFP_t &= A_t \left\{ \frac{\epsilon}{1+\epsilon} + (z_t^*)^{\epsilon} p_t^S \left[\mathbb{E}(z|\, z \geq z_t^*) - \mathbb{E}(z|\, z \leq z_t^*) \right] \right\}, \\
\frac{W_t}{C_t} &= \psi N_t^{\gamma}, \\
K_{t+1} &= \left(1 + \frac{\delta\sigma}{1-\sigma} \left(1 - Q_t^{\sigma-1}\right) \right) K_t, \\
P_t &= \alpha \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}} A_t z_t^* + (1-\delta) Q_t + \left(\frac{\delta}{1-\sigma} \right) (Q_t - Q_t^{\sigma}), \\
Q_t &= \mathbb{E}\left[\left(\frac{\beta C_t}{C_{t+1}} \right) \left(\alpha \left(\frac{1-\alpha}{W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} \mathbb{E}(z) + \Gamma(Q_{t+1}) \right) \right] \\
&\quad + \mathbb{E}\left[\left(\frac{\beta C_t}{C_{t+1}} \right) \alpha \left(\frac{1-\alpha}{W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} p_{t+1}^S \int_{z_{\min}}^{z_{t+1}^*} \left(z_{t+1}^* - z \right) dF(z) \right] \\
&\quad + \mathbb{E}\left[\left(\frac{\beta C_t}{C_{t+1}} \right) \alpha \left(\frac{1-\alpha}{W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} p_{t+1}^B \mu_{t+1} \int_{z_{t+1}^*}^{z_{\max}} \left(z - z_{t+1}^* \right) dF(z) \right], \\
z_t^* &= \lambda_t \mathbb{E}(z_t | z_t \leq z_t^*) + (1-\lambda_t) \mathbb{E}(z_t | z_t > z_t^*), \\
Y_t &= C_t + I_t, \\
I_t &= \delta Q_t^{\sigma} K_t, \\
W_t &= (1-\alpha) \left(\frac{Y_t}{N_t} \right),
\end{aligned}
$$

Moreover, $\left(x_t^S, \theta_t^S, \theta_t^B, p_t^S, q_t^S, p_t^B, q_t^B, \lambda_t, \chi_t\right)$ emerges in the presence of search frictions:

$$
\begin{aligned}
\mathcal{M}^S\left(x_t^S, F\left(z_t^*\right)\right) &= \mu_t \mathcal{M}_t^B\left(1-x_t^S, \left(1-F\left(z_t^*\right)\right)\right), \\
\theta_t^S &\equiv \frac{F\left(z_t^*\right)}{x_t^S}, \\
\theta_t^B &\equiv \frac{1-F\left(z_t^*\right)}{x_t^B}, \\
p_t^S &= \mathcal{M}_t^S\left(\frac{1}{\theta_t^S}, 1\right), \\
q_t^S &= \mathcal{M}^S\left(1, \theta_t^S\right), \\
p_t^B &= \mathcal{M}_t^B\left(\frac{1}{\theta_t^B}, 1\right), \\
q_t^B &= \frac{\mathcal{M}^B\left(1-x_t^S, 1-F\left(z_t^*\right)\right)}{1-x_t^S}, \\
\lambda_t &= \frac{q_t^S}{q_t^S+q_t^B \mu_t P_t} = \frac{1}{1+\left(q_t^B/q_t^S\right)\mu_t P_t}
\end{aligned}
$$

Finally, the amount and the benefit of capital reallocation, and the average bid-ask spread in the decentralized markets for used capital is given by

$$
\begin{aligned}
CR_t^B &= \overline{TFP}_t - TFP_t, \\
CR_t^A &= p_t^S S_t K_t.
\end{aligned}
$$

Based on the calibration, we take three pieces of impulse response exercise in this part: (i) A_t, aggregate productivity shock, (ii) μ_t, financial shocks, (iii) ξ_t, matching efficiency shock. All those three shocks are assumed to follow an AR(1) process with persistence coefficient 0.95. We summarize the results in Figures 2, 3 and 4 respectively.

Before checking out the transition dynamics driven by those shocks, we first summarize all the key empirical facts related to our framework:

1. (Rampini and Eisfeldt, 2006 and Cui 2014) The amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical.

2. (Kurmann and Pestrosky, 2007) The rate of capital reallocation (probability of selling out used capital) is well below 100%, and is procyclical.

3. (Lanteri, 2015) Both the price of used capital and that of new capital is procyclical. The former is more volatile than the latter.

4. (Rüdiger and Bayer 2014) The dispersion of firm investment rate is procyclical.

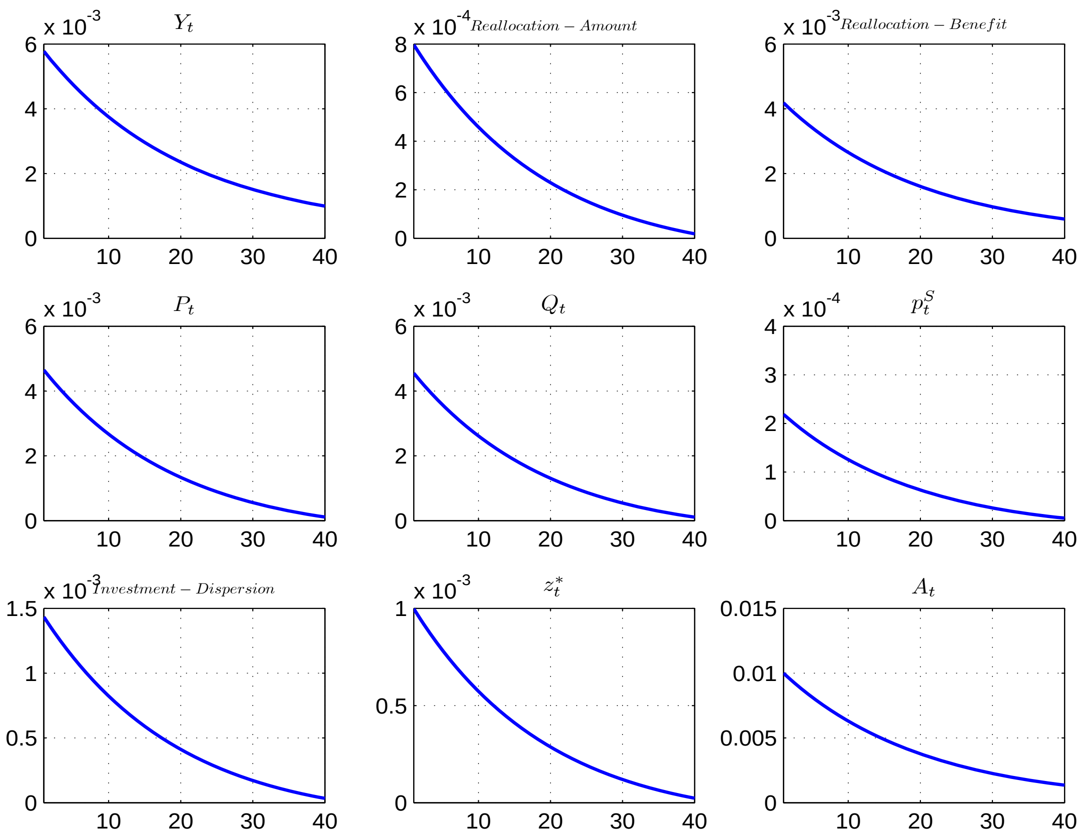

Figure 2: **IRF of Aggregate Productivity Shock (A_t)**

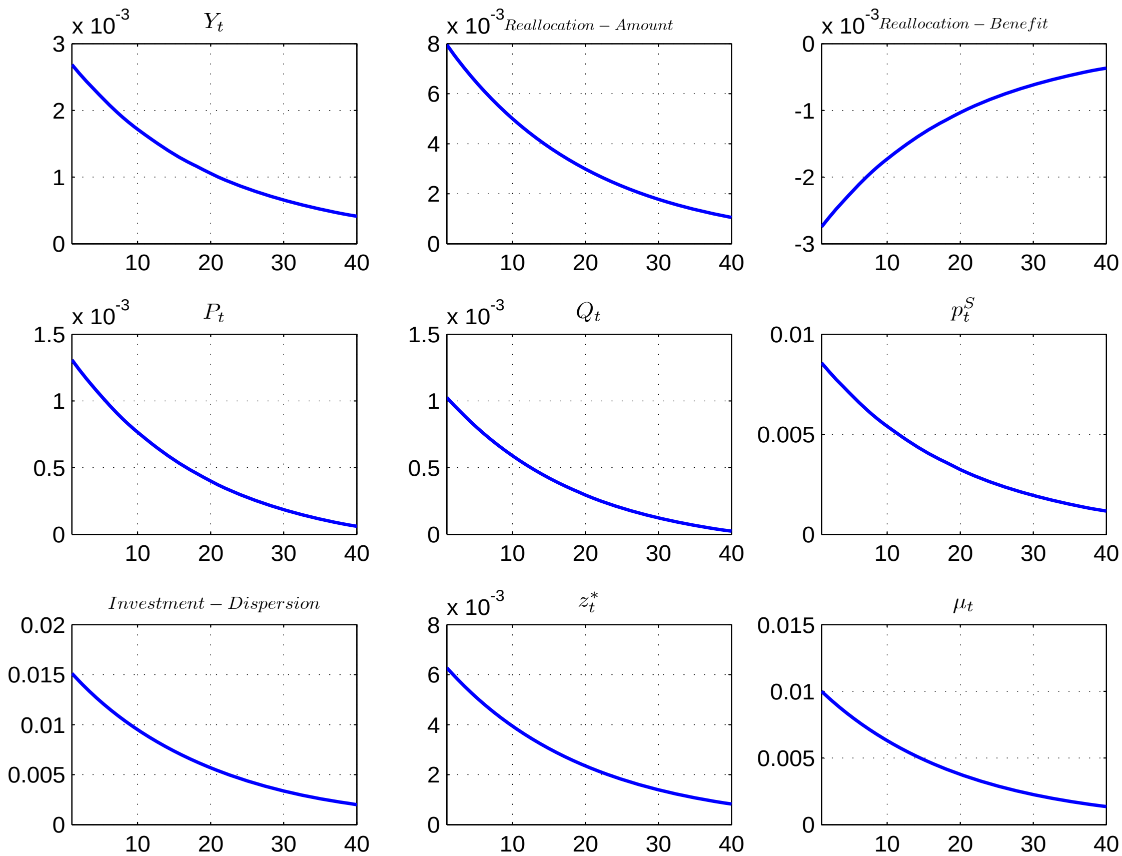

Figure 3: **IRF of Financial Shock** (μ_t)

5. (Kehrig 2015) The dispersion of firm productivity is countercyclical.

First, as shown in Figure 2, the aggregate productivity shock implies the amount of capital reallocation is procyclical, which fits the empirical regularity. However, the generated benefit of capita reallocation is also procyclical, which is at odds with the data pattern. Although both the price of the used and new capital (P_t and Q_t) is shown to be procyclical, the volatility of the former is not significantly larger than the latter.

Second, Figure 3 suggests that, the time series generated by the financial shock is in line with all the aforementioned empirical facts. In particular, the financial shock implies that the volatility of P_t is evidently larger than that of Q_t. Here is the intuition. According to equation (25), we have $P_t = R_t(z_t^*) + \Gamma(Q_t)$, which suggests that P_t increases with z_t^* and Q_t. Given any Q_t, equation (52) suggests that z_t^* increases with μ_t and P_t. Therefore the financial shock (μ_t) amplifies the interactions between P_t and z_t^*, and thus increases the relative volatility of P_t to Q_t.

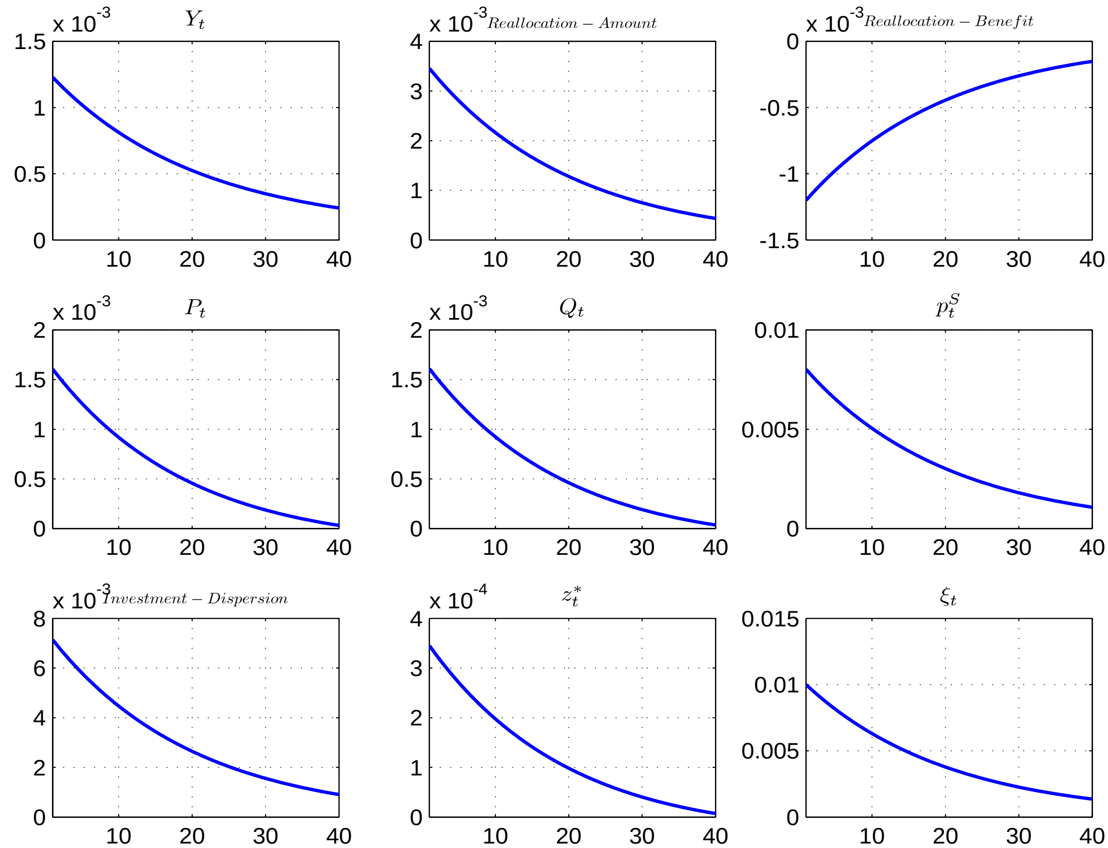

Figure 4: **IRF of Matching-Efficiency Shock (ξ_t)**

Third, Figure 4 implies that, the matching-efficiency shock can almost "replicate" all the qualitative results driven by financial shocks. However, the volatility of P_t is almost the same to that of Q_t under the matching-efficiency shock, just as the case with aggregate productivity shock. Combining all the findings from Figures 2 to 4 yields Table 3.

Targets	Data	TFP Shock	Financial Shock	Search Shock
the amount of reallocation	$+$	$+$	$+$	$+$
the benefit of reallocation	$-$	$+$	$-$	$-$
probability of reallocation	$+$	$+$	$+$	$+$
price of used and new capital	$+$	$+$	$+$	$+$
relative volatility of used to new capital	high	almost equal	high	almost equal
dispersion of investment rate	$+$	$+$	$+$	$+$
TFP dispersion	$-$	$+$	$-$	$+$

Table 3: Shock Comparison (" $+$ " and " $-$ " denotes procyclical and counter-cyclical respectively)

4.3 Capital Reallocation: No Search Frictions

If there were no search and matching frictions in the secondary capital markets, then all firms are confronted with the same centralized price P_t for used capital. Meanwhile, $p_t^S = p_t^B = 1$, *i.e.*, no search frictions in selling or purchasing used capital. Therefore, the problem of firm-(k_t, z_t) is modified as

$$V_t(k_t, z_t) = \max_{k_t^S, k_t^B, i_t, d_t} \left\{ d_t + \mathbb{E}_t \left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \int_{z \in \mathcal{Z}} V_{t+1}(k_{t+1}, z) \, dF(z) \right] \right\}, \tag{75}$$

subject to

$$\begin{aligned} \widetilde{k}_t &= k_t - k_t^S + k_t^B, \\ d_t + i_t &= R_t(z_t) \widetilde{k}_t + \left(k_t^S - k_t^B\right) P_t, \\ k_{t+1} &= (1-\delta) \widetilde{k}_t + \Psi\left(i_t / \widetilde{k}_t\right) \widetilde{k}_t, \\ k_t^S &\in [0, k_t], \; k_t^B \in [0, \mu_t k_t]. \end{aligned}$$

Then we can always guess and verify that $V_t(k_t, z_t) = \phi_t(z_t) k_t$, where $\phi_t(z_t)$ is characterized by

$$\phi_t(z_t) k_t = \max_{k_t^S \in [0, k_t], \; k_t^B \in [0, \mu_t k_t]} \{ (R_t(z_t) + \Gamma(Q_t)) k_t + (P_t - R_t(z_t) - \Gamma(Q_t)) \left(k_t^S - k_t^B\right) \}. \tag{76}$$

where $\Gamma(Q_t)$ is defined in equation (23). Then we know that

$$\begin{aligned} k_t^S(k_t, z_t) &= \begin{cases} \mu_t^S k_t, & \text{if } z_t \le z_t^* \\ 0, & \text{otherwise} \end{cases}, \\ k_t^B(k_t, z_t) &= \begin{cases} 0, & \text{if } z_t \le z_t^* \\ \mu_t k_t, & \text{otherwise} \end{cases}, \\ \widetilde{k}_t(k_t, z_t) &= \begin{cases} 0, & \text{if } z_t \le z_t^* \\ (1+\mu_t) k_t, & \text{otherwise} \end{cases}, \end{aligned}$$

where the cut-off value z_t^* satisfies equation (25). In turn,

$$\phi_t(z_t) = R_t(z_t) + \Gamma(Q_t) + (R_t(z_t^*) - R_t(z_t)) \mathbf{1}_{\{z_t \le z_t^*\}} + \mu_t (R_t(z_t) - R_t(z_t^*)) \mathbf{1}_{\{z_t > z_t^*\}}.$$

Consequently, the Tobin's Q is characterized by

$$\begin{aligned} Q_t = \; & \mathbb{E}\left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \left(\int R_{t+1}(z) \, dF(z) + \Gamma(Q_{t+1}) \right) \right] \\ & + \mathbb{E}\left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \int_{z_{\min}}^{z_{t+1}^*} \left(R_{t+1}\left(z_{t+1}^*\right) - R_{t+1}(z) \right) dF(z) \right] \\ & + \mathbb{E}\left[\left(\frac{\beta \Lambda_{t+1}}{\Lambda_t} \right) \mu_{t+1} \int_{z_{t+1}^*}^{z_{\max}} \left(R_{t+1}(z) - R_{t+1}\left(z_{t+1}^*\right) \right) dF(z) \right] \end{aligned} \tag{77}$$

Since we now remove search frictions in the decentralized markets for used capital, then the clearing condition implies $K_t^S = K_t^B$. Equivalently, when there are no search frictions, $\mathcal{M}^S\left(x_t^S, S_t\right) = S_t$, and $\mathcal{M}_t^B\left(x_t^B, B_t\right) = B_t$, and thus equation (44) can be simplified as $S_t = \mu_t B_t$, where the aggregate supply and demand, K_t^S and K_t^B, is given by equation (42), and the measure of sellers and buyers, S_t, and B_t, is given by $S_t = F\left(z_t^*\right)$, and $B_t = 1 - F\left(z_t^*\right)$. Consequently we obtain $F\left(z_t^*\right) = \frac{\mu_t}{1+\mu_t}$. The TFP is modified as

$$TFP_t = A_t\left\{\mathbb{E}\left(z\right) + F\left(z_t^*\right)\left[\mathbb{E}\left(z\right| z \geq z_t^*\right) - \mathbb{E}\left(z\right| z \leq z_t^*\right)\right]\right\}. \tag{78}$$

5 Extension

5.1 Explicit Reallocation Cost

For simplicity, we have assumed away any explicit cost when firms search for capital reallocation. We can relax this problem by introducing either re-installment cost or search cost associated with reallocation. Accordingly, the trading surplus for each unit matched capital is $\max\left\{P_t - R_t\left(z_t\right) - \Gamma\left(Q_t\right) - \varphi^S, 0\right\}$ and $\max\left\{R_t\left(z_t\right) + \Gamma\left(Q_t\right) - P_t - \varphi^B, 0\right\}$ respectively in the seller-side and buyer-side decentralized markets.

Define $\left(z_t^S, z_t^B\right)$ as

$$\begin{aligned} R_t\left(z_t^S\right) &= P_t - \Gamma\left(Q_t\right) - \varphi^S. && (79) \\ R_t\left(z_t^B\right) &= P_t - \Gamma\left(Q_t\right) + \varphi^B. && (80) \end{aligned}$$

Then we can easily prove that a *wedge* emerges such that $z_t^S < z_t^B$, which is due to the introduction of explicit cost. Moreover, firms' strategy in the secondary market is given by

1. If $z_t < z_t^S$, then the firm wants to sell capital as much as she can.
2. If $z_t > z_t^B$, then the firm wants to buy used capital as much as she can from the secondary market.
3. If $z_t^S < z_t < z_t^B$, then the firm neither sells nor buys capital from the secondary market.

In turn, equation (34) can be generalized as

$$\begin{aligned} \phi_t\left(z_t\right) = & \max_{k_t^S \in [0,k_t], k_t^B \in [0,\mu_t k_t]} \{\left(R_t\left(z_t\right) + \Gamma\left(Q_t\right)\right) k_t \\ & + (1-\eta)\left(R_t\left(z_t^S\right) - R_t\left(z_t\right)\right) p_t^S k_t^S + (1-\eta)\left(R_t\left(z_t\right) - R_t\left(z_t^B\right)\right) p_t^B k_t^B\}, \end{aligned} \tag{81}$$

Note that $z_t^S = z_t^B = z_t^*$ if and only if $\varphi^S = \varphi^B = 0$. It is also worth noting that, a new sub-group of firms emerges such that all firms stick to *voluntary* not-reallocation while the other subgroups are subject to *involuntary* (partial) not-reallocation due to search frictions.

5.2 Endogenous Market-Making in Secondary Markets

We assume there exists unit measure of dealers in the secondary markets. We can endogenize the market-making activity by introducing *free entry conditions*. More specifically, x_t^S and x_t^B measure of dealers enter the seller-side and buyer-side secondary markets respectively after paying a fixed cost, like Dong, Wang and Wen (2015) did. We need to modify equations (46) accordingly. We may obtain an endogenous increasing-returns-to-scale aggregation.

6 Conclusion

The relative importance of used capital goods to new investment has been increasing significantly over time. To this end, we develop a dynamic general equilibrium model with heterogeneous firms to account for all of the following key empirical regularities on capital reallocation: (i) the amount of capital reallocation is procyclical while the benefit of capital reallocation is counter-cyclical, (ii) the probability of selling out used capital is well below 100%, and is procyclical, (iii) both the price of used capital and that of new capital are procyclical, and the former is more volatile than the latter. We show that both search frictions and financial frictions are essential to explain these facts. Moreover, many other predictions of our framework are also in line with important facts/puzzles on business cycles, such as (i) the dispersion of investment rate is procyclical, (ii) the dispersion of firm productivity is countercyclical. Finally, based on our structural model, we examine the roles of productivity, financial, and search shock played in the fluctuations of capital reallocation.

Appendix

A Proofs

Proof of Lemma 1: We obtain from equation (22) the demand and supply in the secondary market for capital reallocation as below.

$$k_t^B(k_t, z_t) = \begin{cases} 0, & \text{if } R_t(z_t) + \Gamma(Q_t) - P_t^B(z_t) < 0 \\ \mu_t P_t k_t, & \text{otherwise} \end{cases}, \tag{A.1}$$

$$k_t^S(k_t, z_t) = \begin{cases} k_t, & \text{if } R_t(z_t) + \Gamma(Q_t) - P_t^S(z_t) < 0 \\ 0, & \text{otherwise} \end{cases}. \tag{A.2}$$

Substituting equation (25) into the above equations yields desired results.

Proof of Proposition 1: We break the proof into two parts. To start with, the clearing condition in the labor market is given by

$$\int\int n_t dG(k_t) dF(z_t) = N_t, \tag{A.3}$$

where

$$n_t\left(\widetilde{k}_t, z_t\right) = \left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}} A_t z_t \widetilde{k}_t, \tag{A.4}$$

and

$$\widetilde{k}_t(k_t, z_t) = k_t - \widetilde{k}_t^S + \widetilde{k}_t^B = \begin{cases} \left(1 - p_t^S\right) k_t, & \text{if } z_t \le z_t^* \\ \left(1 + \mu_t p_t^B\right) k_t, & \text{otherwise} \end{cases}. \tag{A.5}$$

Substituting equation (A.4) and (A.5) into (A.3) yields

$$\left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}} A_t K_t \left(\int_{z_{\min}}^{z_t^*} z_t\left(1 - p_t^S\right) dF(z_t) + \int_{z_t^*}^{z_{\max}} z_t\left(1 + \mu_t p_t^B\right) dF(z_t)\right) = N_t. \tag{A.6}$$

Note that

$$\begin{aligned}
&\int_{z_{\min}}^{z_t^*} z_t\left(1 - p_t^S\right) dF(z_t) + \int_{z_t^*}^{z_{\max}} z_t\left(1 + \mu_t p_t^B\right) dF(z_t) \\
=\ & \int_{z_{\min}}^{z_t^*} z_t dF(z_t) + \int_{z_t^*}^{z_{\max}} z_t dF(z_t) + \mu_t p_t^B \int_{z_t^*}^{z_{\max}} z_t dF(z_t) - p_t^S \int_{z_{\min}}^{z_t^*} z_t dF(z_t) \\
=\ & \mathbb{E}(z) + \mu_t p_t^B \left(1 - F(z_t^*)\right) \mathbb{E}(z|\, z \ge z_t^*) - p_t^S F(z_t^*) \mathbb{E}(z|\, z \le z_t^*) \\
=\ & \mathbb{E}(z) + p_t^S S_t \left[\mathbb{E}(z|\, z \ge z_t^*) - \mathbb{E}(z|\, z \le z_t^*)\right],
\end{aligned} \tag{A.7}$$

where the last equality is held because of equation (43), the clearing condition in the inter-dealer market. Combining equation (A.6) and (A.7) yields

$$\frac{1-\alpha}{W_t} = \left(\frac{N_t}{A_t\left(\mathbb{E}(z) + p_t^S S_t \left[\mathbb{E}(z|\, z \ge z_t^*) - \mathbb{E}(z|\, z \le z_t^*)\right]\right) K_t}\right)^{\alpha}. \tag{A.8}$$

Consequently, we have

$$\begin{aligned}
Y_t &\equiv \int\int y_t\left(\widetilde{k}_t, n_t\right) dG\left(k_t\right) dF\left(z_t\right) &\text{(A.9)}\\
&= \int\int \frac{W_t n_t\left(\widetilde{k}_t, z_t\right)}{1-\alpha} dG\left(k_t\right) dF\left(z_t\right) \\
&= \left(\frac{W_t}{1-\alpha}\right) N_t &\text{(A.10)}\\
&= \left(A_t\left(\mathbb{E}\left(z\right) + p_t^S S_t\left[\mathbb{E}\left(z\right| z \geq z_t^*\right) - \mathbb{E}\left(z\right| z \leq z_t^*\right)\right]\right) K_t\right)^{\alpha} N_t^{1-\alpha}, &\text{(A.11)}
\end{aligned}$$

where the last equality holds because of equation (A.8). In the end, as a by-product, equation (A.10) implies

$$W_t = \left(1-\alpha\right) \frac{Y_t}{N_t}. \tag{A.12}$$

It remains for us to prove that TFP_t strictly increases with z_t^*. On the one hand, since $S_t = F\left(z_t^*\right)$, and $B_t = 1 - F\left(z_t^*\right)$, using Implicit Function Theorem on equation (44) yields that $\partial x_t^S / \partial z_t^* > 0$. Since $S_t p_t^S = \mathcal{M}_t^S\left(x_t^S, S_t\right)$, then we know that $S_t p_t^S$ strictly increases with z_t^*. On the other hand, $\mathbb{E}\left(z\right| z \geq z_t^*\right) - \mathbb{E}\left(z\right| z \leq z_t^*\right)$ also strictly increases with z_t^* when z conforms to the Power distribution, *i.e.*, $F\left(z\right) = z^{\epsilon}$ with $\epsilon < 1$.[11] Here are the details:

$$\begin{aligned}
\mathbb{E}\left(z\right) &= \frac{\epsilon}{1+\epsilon}, &\text{(A.13)}\\
\mathbb{VAR}\left(z\right) &= \frac{\epsilon}{\left(\epsilon+2\right)\left(\epsilon+1\right)^2}, &\text{(A.14)}\\
\mathbb{E}\left(z|z \leq z^*\right) &= \left(\frac{\epsilon}{1+\epsilon}\right) z^*, &\text{(A.15)}\\
\mathbb{E}\left(z|z > z^*\right) &= \left(\frac{\epsilon}{1+\epsilon}\right)\left(\frac{1-\left(z^*\right)^{1+\epsilon}}{1-\left(z^*\right)^{\epsilon}}\right), &\text{(A.16)}\\
\int_{z_{\min}}^{z^*} \left(z^* - z\right) dF\left(z\right) &= \frac{\left(z^*\right)^{1+\epsilon}}{1+\epsilon}, &\text{(A.17)}\\
\int_{z^*}^{z_{\max}} \left(z - z^*\right) dF\left(z\right) &= \frac{\epsilon}{1+\epsilon} + \frac{\left(z^*\right)^{1+\epsilon}}{1+\epsilon} - z^*, &\text{(A.18)}\\
\frac{d}{dz^*} TFP_t^{INT}\left(z_t^*\right) &\equiv \frac{d\left(\mathbb{E}\left(z\right| z \geq z^*\right) - \mathbb{E}\left(z\right| z \leq z^*\right))}{dz^*} \begin{cases} > 0, & \text{if } \epsilon < 1 \\ = 0, & \text{if } \epsilon = 1 \\ < 0, & \text{if } \epsilon > 1 \end{cases}. &\text{(A.19)}
\end{aligned}$$

[11] It is also true if z conforms to a Pareto distribution, *i.e.*, $F\left(z\right) = 1 - z^{-\frac{1}{\epsilon}}$ with $\epsilon > 0$.

Proof of Corollary 1: Equation (36) implies

$$\mathbb{E}\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right) = 1, \tag{A.20}$$

$$\mathbb{E}\left(\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right)^2\right) = \left(1-p_t^S\right)^2 F\left(z_t^*\right) + \left(1+\mu_t p_t^B\right)\left(1-F\left(z_t^*\right)\right). \tag{A.21}$$

Therefore

$$\begin{aligned} std\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right) &= \sqrt{\mathbb{V}\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right)} && \text{(A.22)}\\ &= \sqrt{\mathbb{E}\left(\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right)^2\right)-\left(\mathbb{E}\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right)\right)^2} && \text{(A.23)}\\ &= \frac{\mathcal{M}_t^S\left(x_t^S,S_t\right)}{\sqrt{S_t B_t}}. && \text{(A.24)}\end{aligned}$$

In turn,

$$std\left(\frac{i_t^{gross}\left(k_t,z_t\right)}{k_t}\right) = \left(1+\omega\left(Q_t\right)\right) std\left(\frac{\widetilde{k}_t\left(k_t,z_t\right)}{k_t}\right) = \left(1+\omega\left(Q_t\right)\right)\frac{\mathcal{M}_t^S\left(x_t^S,S_t\right)}{\sqrt{S_t B_t}}.$$

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