Mind the Gap: An Empirical Foundation for Investment-Based Asset Pricing Models*

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Abstract

Investment-based asset pricing models get traction from a common tenet, namely firms' limited flexibility in adjusting their physical capital. While this inflexibility mechanism is at the core of the investment-based approach, direct empirical evidence regarding this channel is scarce. We provide an empirical foundation for the inflexibility mechanism. We propose a standard modeling framework where firms optimally invest by closing a fraction of the gap between their existing capital stock and a target level of capital. Inflexible firms are sluggish in filling the gap, are more risky, and earn higher expected returns. We use sequential Bayesian techniques to estimate inflexibility and the gap for a large sample of US listed firms, and we test how they affect expected equity returns. Our evidence strongly corroborates the inflexibility mechanism as a first-order driver of cross-sectional differences in returns. Ultimately, our results support the investment-based paradigm as a mainstream avenue to turn anomalies into empirical regularities.

Keywords: Cross-Sectional Returns, Investment-Based Asset Pricing, Inflexibility, Gap, Investment CAPM, Sequential Monte Carlo.

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1. Introduction

The cross-section of expected stock returns is among the most fundamental and debated topics in finance. In the last decades, a colossal anomalies literature erupted and disclosed a zoo of variables associated with cross-sectional spreads in average returns. Since anomalies are not rationalized by traditional risk models, they confound the way practitioners and academics think about risk and returns. In this state of affairs, the recent review article of Zhang (2015) calls attention to investment-based asset pricing models as a mainstream avenue to turn anomalies into empirical regularities without putting market efficiency in peril.

This class of asset-pricing models gets traction from a common tenet, namely firms' limited flexibility in adjusting capital investment. The fundamental economic mechanism at work originates from capital budgeting in the presence of capital adjustment costs. Firms with riskier future investment projects have higher discount rates and choose to be less responsive to shocks to their investment opportunities by sluggishly adjusting their capital stock. In this way, they save capital adjustment costs that would otherwise excessively offset the present discounted value of future investment profits. Other conditions equal, the more inflexible a firm is, the higher must be its discount rates, and the riskier are its shares. In this paper, we refer to this economic mechanism as the *inflexibility mechanism*.

Is the inflexibility mechanism a first-order driver of expected equity returns? The goal of this work is to provide an empirical foundation for it and, ultimately, for the investment-based asset pricing paradigm. While the inflexibility mechanism has been fruitfully exploited to rationalize several asset pricing puzzles in production economies, such as in the seminal papers of Jermann (1998) and Zhang (2005), no existing study directly documented the relationship between firms' inflexibility and returns. We attempt to fill this gap by implementing an empirical test of the inflexibility mechanism.

To do so, we first characterize the optimal investment policy of firms with heterogeneous capital adjustment costs in a simplified investment-based setting along the lines of Sargent (1978) and Caballero and Engel (1999). Firms decide their investment expense by trading off expected profits from risky investment opportunities that arise stochastically and costs of adjusting their capital stock. In the model, firms optimally invest by closing a fraction of the *gap* between their existing capital stock and a target level of capital. The target capital stock is dynamic and it reflects firms' expected investment prospects. The rate at which a certain firm closes the gap captures its flexibility. Firms that face higher adjustment costs are less flexible and exhibit lower speeds of adjustment towards the target. The main prediction of the model unfolds the inflexibility mechanism as an explicit relationship between inflexibility and expected returns. Specifically, discount rates are negatively related to flexibility. When the former are higher the present value of expected investment profits is lower, and firms undertake unresponsive investment policies to save adjustment costs.

In addition to our main testable hypothesis, the model provides two implications that impose strong restrictions on the data. Indeed, it predicts a sign flip in the relationship between the gap and expected returns. The direction of the return-gap relationship depends on whether a firm is investing or disinvesting. A firm with a large positive gap has a capital stock below the target, and it expands to fill it. The same observed capital adjustment renders a firm with a larger gap less flexible than a firm with a smaller gap because the former effectively closes a smaller fraction of it. The inflexibility channel described above then implies that gap is positively related to expected returns. Symmetrically, a firm with a negative gap disinvests to converge to the target. The more negative the gap, the more costly to fill it. Thus, lower gaps are associated to inflexibility and to higher discount rates whenever firms downsize. In sum, the model predicts that the gap is positively related to returns for expanding firms, and negatively for shrinking firms.

The main challenge for implementing our empirical tests is that both the degree of inflexibility and the gap are inherently unobservable. As a consequence, we need to estimate them using available data. The estimation procedure needs to address three main concerns. First, the estimates must be free of look-ahead bias because they are included in asset pricing tests. Second, the procedure should effectively deal with time series for individual firms that might have a small number of observations. Third, the estimation needs to be computationally efficient because it should be applied to a large cross section of firms. We resort to the sequential Monte Carlo method in the version proposed by Liu and West (2001). This estimation technique well fits the goal of this work. Sequential Monte Carlo takes advantage of its Bayesian nature and specifically of statistical priors for the estimated parameters. For this reason, it deals well with short time series, while the estimates become unstable if the time series are too long (Doucet (1998)). Furthermore, as its name suggests, this method is sequential. The advantages of this feature are twofold. On one hand, it updates the estimates every period simply incorporating the new available information. This on-line updating step ensures computational efficiency because it does not require re-estimating all the parameters every time a new observation becomes available. On the other hand, full-sample information is not needed at any point in the estimation procedure, and this guarantees an output that is free of look-ahead bias.[1]

We use Compustat data on firms' capital stocks from 1961 to 2011 as inputs to estimate flexibility and gaps for a large sample of US listed firms. We set initial priors using the period 1961-1979, and we apply sequential Monte Carlo from 1980 to 2011 to construct our estimates for all firms. We then merge our estimates to monthly stock returns from CRSP for the 1980-2011 period and we implement standard asset pricing tests. The inflexibility mechanism finds strong support in the data. We first sort stocks into deciles based on our measure of inflexibility. Average returns decline monotonically as one moves from low-flexibility to high-flexibility portfolios, with an annualized average spread of 4.88% for the equally-weighted portfolios and of 3.76% for the value-weighted portfolios. Fama-MacBeth monthly cross-sectional regressions reinforce this result. The slope for estimated flexibility is -0.0097, with a t-statistic of 10.31 after controlling for size and book-to-market equity.

Remarkably, the relationship between the gap and average return exhibits the sign flip around zero capital growth that the model predicts. To document this pattern, we first subdivide data in quintiles based on capital growth and, within each quintile, we sort observations in five groups based on the gap. The gap is strongly negatively related to average returns for groups of firms that are disinvesting, and positively related to returns in groups of firms that are investing. Firms with low gaps outperform those with high gaps for firms with negative capital growth, with sizeable annualized spreads ranging from 6.86% to 9.14%. Vice versa, in quintiles of firms with positive capital growth, high-gap firms earn premia from 6.55% to 9.96% per year over low-gap firms. Fama-MacBeth regressions corroborate these results. The estimated slopes for the gap are positive and statistically significant for the top

[1] As we discuss in Section 3, sequential Monte Carlo has other appealing features for this work in comparison to possible alternative estimation methods. In particular, it maps well into the state-space representation suggested by the model, it can handle non-linearities in the state-space unlike Kalman filters, and has computational advantages over standard Markov Chain Monte Carlo (MCMC) techniques.

two quintiles of capital growth, and negative and statistically significant for the bottom two quintiles. All in all, our empirical results support the three testable hypotheses from the model, and strongly indicate the inflexibility mechanism as a key driver of cross-sectional differences in average returns.

Our paper mainly adds to the literature that interprets cross-sectional differences in returns through the lens of investment-based and the q-theory models of investment. A non-exhaustive list includes Berk, Green, and Naik (1999); Zhang (2005); Liu, Whited, and Zhang (2009); Belo, Xue, and Zhang (2013); Belo, Lin, and Bazdresch (2014). Our main contribution to this line of literature is that we provide direct empirical evidence that validates one tenet of these models. In fact, while the inflexibility mechanism is at the core of the investment-based approach, empirical evidence regarding this channel is still limited. Our evidence is also consistent with the empirical factor model of Hou, Xue, and Zhang (2014), which is motivated by the investment-based approach. From a methodological point of view, our paper can be viewed as a finance application of Bayesian estimation techniques. A partial list of other applications includes Korteweg and Polson (2009); Korteweg (2010); and Korteweg and Sorensen (2010).

The rest of the paper is outlined as follows. Section 2 presents the theoretical framework and derives the testable hypotheses related to the inflexibility mechanism. Section 3 describes the sequential Monte Carlo approach to estimation. Section 4 discusses our asset pricing tests and our empirical results. Section 5 concludes.

2. Hypotheses Development

In this section we propose a simple model to develop testable predictions that guide our empirical asset pricing tests of the inflexibility mechanism in Section 4.

Consider an economy with two periods, t and $t+1$, populated by heterogeneous firms, indexed by i. Firms produce a homogeneous non-durable good using capital as an input. Firm i chooses the level of capital to maximize its market value, defined as the discounted value at time t of its operating profits. Firm i starts with an initial capital stock $k_{i,t}$, and invests the amount $i_{i,t} = k_{i,t+1} - (1-\delta_i)k_{i,t}$ such that productive capital at the beginning of period $t+1$ is $k_{i,t+1}$, where $0 \leq \delta_i \leq 1$ is the depreciation rate. Firms produce in both periods, and generate operating revenues $\Pi(k_{i,\tau}, z_{i,\tau}) \equiv z_{i,\tau} f_i(k_{i,\tau})$, where $z_{i,\tau}$ is an exogenous Markovian productivity shock with support $\mathcal{Z}$, $f_i : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ such that $f_i(0) = 0$ is an increasing, concave, and twice continuously differentiable production function, where $\tau \equiv \{t, t+1\}$ and $\mathcal{R}_+$ denotes the set of non-negative real numbers.

Firms are subject to investment frictions in that they incur deadweight costs $\Psi_i(k_{i,t}, k_{i,t+1})$ of adjusting their capital stock from $k_{i,t}$ to $k_{i,t+1}$. Adjustment costs are deducted directly from operating revenues at time t, and are quadratic in the investment, that is

$$\Psi_i(k_{i,t}, k_{i,t+1}) \equiv \frac{\theta_i}{2} i_{i,t}^2$$

with $\theta_i \geq 0$. θ_i is a firm-specific cost parameter that captures the magnitude of investment frictions that firms are exposed to. As standard, the functional form of $\Psi_i(k_{i,t}, k_{i,t+1})$ implies that firms' marginal adjustment costs rise with investment.

Following Li and Zhang (2010) and Wu, Zhang, and Zhang (2010), firm i has a gross discount rate denoted as R_i, which differs across firms because of, for instance, different loadings on macroeconomic priced risk factors[2]. Firm i chooses optimal investment $i_{i,t}$ to maximize its market value $V_{i,t}$ at the beginning of period t:

$$V_{i,t} \equiv \max_{i_{i,t}} \Pi(k_{i,t}, z_{i,t}) - i_{i,t} - \Psi_i(k_{i,t}, k_{i,t+1}) + \frac{1}{R_i} E\left[\Pi(k_{i,t+1}, z_{i,t+1}) + (1-\delta_i)k_{i,t+1}\right]$$

where $E[\cdot]$ denotes the expectation operator with respect to the information set at time t. $V_{i,t}$ is the cum-dividend value of firm i's equity, that is the sum of the free cash flow $d_{i,t} \equiv \Pi(k_{i,t}, z_{i,t}) - i_{i,t} - \Psi_i(k_{i,t}, k_{i,t+1})$ at date t and of the discounted value of the free cash flow $d_{i,t+1} \equiv \Pi(k_{i,t+1}, z_{i,t+1}) + (1-\delta_i)k_{i,t+1}$ at date $t+1$. In this two-period setup firms do not invest in period $t+1$. Period $t+1$'s cash flow is therefore the sum of operating revenues and of the liquidation value of capital, and the discount rate R_i is the gross expected equity return of firm i.[3]

Every firm in the economy faces an economic tradeoff between forgoing safe profits at time t in exchange of higher expected cash flows at time $t+1$. The first-order condition for firm i is

$$\frac{1}{R_i}\left(E[\Pi_k(k_{i,t+1}, z_{i,t+1})] + 1 - \delta_i\right) = 1 + \theta_i i_{i,t} \tag{1}$$

[2]Following several studies in the investment-based asset pricing literature, including Cochrane (1991), Berk, Green, and Naik (1999),Zhang (2005), Liu, Whited, and Zhang (2009), and Gomes and Schmid (2010), we adopt a partial equilibrium approach where firms take discount rates as given. This choice allows analytical tractability. Moreover, it is consistent with the goal of our work. Indeed, we let discount rates vary across firms, and we investigate how firms that behave optimally respond to differences in discount rates through their observed investment policy.

[3]Denote as $P_{i,t}$ and as $P_{i,t+1}$ the (ex-dividend) observed stock prices of firm i at time t and at time $t+1$ respectively, and as $R_{i,t+1}$ its realized gross stock return between dates t and $t+1$. By the definition of stock return:

$$R_{i,t+1} = \frac{P_{i,t+1} + d_{i,t+1}}{P_{i,t}} = \frac{V_{i,t+1}}{V_{i,t} - d_{i,t}}$$

where $V_{i,t+1} = \Pi(k_{i,t+1}, z_{i,t+1}) + (1-\delta_i)k_{i,t+1}$ is the cum-dividend observed market value of firm i at the beginning of period $t+1$. Taking expectations with respect to the sigma field at time t we obtain

$$E[R_{i,t+1}] \equiv \frac{E[V_{i,t+1}]}{V_{i,t} - d_{i,t}}$$

that, after firm i optimizes its investment $i_{i,t}$, yields

$$V_{i,t} \equiv d_{i,t} + \frac{1}{E[R_{i,t+1}]} E[V_{i,t+1}].$$

As a consequence, $R_i = E[R_{i,t+1}]$.

and has an intuitive interpretation. Firm i invests up to the point where the expected discounted marginal benefit of capital $k_{i,t+1}$ on the left-hand side equalizes its marginal cost on the right-hand side. The marginal benefit of investment is the sum of the expected marginal profit $E[\Pi_k(k_{i,t+1}, z_{i,t+1})]$ and of the liquidation value of one unit of capital $1-\delta_i$, while its marginal cost $1+\theta_i i_{i,t}$ is the expense to purchase one unit of capital net of adjustment costs.

The optimality condition (1) can be reformulated to unfold the relationship between inflexibility and expected equity returns and derive the empirical predictions that we test in Section 4. Isolating $k_{i,t+1}$ on the left-hand side, approximating the production function with a second-order Taylor expansion $f_i(k_{i,\tau}) \simeq f_{k,i} k_{i,\tau} - \frac{f_{kk,i}}{2} k_{i,\tau}^2$, with $f_{k,i} \geq 0$, $f_{kk,i} \leq 0$, $\tau \equiv \{t, t+1\}$, and subtracting $k_{i,t}$ from both sides yields

$$k_{i,t+1} - k_{i,t} \simeq \lambda_i (k_{i,t+1}^* - k_{i,t}) \tag{2}$$

where $\lambda_i \equiv \frac{\theta_i \delta_i R_i + f_{kk,i} E[z_{i,t+1}]}{\theta_i R_i + f_{kk,i} E[z_{i,t+1}]}$ and $k_{i,t+1}^* \equiv \frac{f_{k,i} E[z_{i,t+1}] + 1 - \delta_i - R_i}{\theta_i \delta_i R_i + f_{kk,i} E[z_{i,t+1}]}$. Equation (2) characterizes firm i's optimal investment policy as a partial adjustment model. In particular, as in the infinite-horizon settings of Sargent (1978), Caballero and Engel (1999), andKing and Thomas (2006), firms adjust their observed capital stock from $k_{i,t}$ to $k_{i,t+1}$ by closing a fraction $\lambda_i \in (0, 1]$ of the gap $k_{i,t+1}^* - k_{i,t}$ between the current capital stock $k_{i,t}$ and the target level of capital $k_{i,t+1}^*$. Notice that the target $k_{i,t+1}^*$ is dynamic in that it adjusts to changes in firm i's expected future productivity $E[z_{i,t+1}]$.[4]

The parameter λ_i is key to our analysis, as it measures the degree of operating flexibility of firm i. λ_i can be interpreted as the speed of adjustment towards the target $k_{i,t+1}^*$. Firms with high speed of adjustments are faster in filling the gap towards their dynamic targets. On the contrary, firms with low values of λ_i are sluggish in adjusting their investment policy to changes in their investment opportunities. The limiting case of $\lambda_i = 1$ corresponds to a perfectly flexible firm that faces no adjustment frictions ($\theta_i = 0$). In this scenario, equation (2) predicts that firm i is able to instantaneously fill the entire gap towards the target, such that the observed capital stock $k_{i,t+1}$ coincides with $k_{i,t+1}^*$.

[4] Technically, the dynamic target $k_{i,t+1}^*$ can be interpreted in an infinite-horizon model as the capital stock firm i would choose if adjustment costs were removed for one period. In the literature, this is also known as "frictionless" target. In contrast, a static target can be defined as the capital stock firm i would choose if adjustment costs were permanently removed. For a in-depth discussion, see Cooper and Willis (2004).

Based on the characterization of firms' optimal investment policies in equation (2), we derive three hypotheses that we test in Section 4. Our tests aim at establishing whether the inflexibility mechanism is a key driver of cross-sectional differences in expected equity returns consistent with the investment-based asset pricing paradigm of Zhang (2015) and with the q-theory model of Hou, Xue, and Zhang (2014).

H1. More inflexible firms earn higher average stock returns than more flexible firms, that is

$$\frac{\partial R_i}{\partial \lambda_i} < 0.$$

This hypothesis derives directly from the expression of λ_i into equation (2), and represents our main test of the inflexibility mechanism. The intuition behind it is capital budgeting. All else equal, firms with lower values of λ_i are less responsive in adjusting their capital stock to the target level $k^*_{i,t+1}$. Accordingly, they are willing to forgo (future and uncertain) expected profits to save (current and certain) adjustment expenses. High expected returns are required because, if the discount rates were not high enough to counteract the expected future profitability $E[z_{i,t+1}]$, firms would be better off closing a larger fraction of the gap to seize investment opportunities with a higher present value.

H2. Firms with larger gaps earn higher average stock return than firms with smaller gaps if firms are investing, that is

$$\frac{\partial R_i}{\partial (k^*_{i,t+1} - k_{i,t})} > 0, \qquad \text{if } k_{i,t+1} - k_{i,t} > 0.$$

H3. Firms with larger gaps earn lower average stock return than firms with smaller gaps if firms are disinvesting, that is

$$\frac{\partial R_i}{\partial (k^*_{i,t+1} - k_{i,t})} < 0, \qquad \text{if } k_{i,t+1} - k_{i,t} < 0.$$

The last two hypotheses derive from plugging $\lambda_i \equiv \frac{\theta_i \delta_i R_i + f_{kk,i} E[z_{i,t+1}]}{\theta_i R_i + f_{kk,i} E[z_{i,t+1}]}$ into (2), and taking the partial derivative with respect to the gap. Conditional on measuring the observed change in capital $k_{i,t+1} - k_{i,t}$ and the gap $k^*_{i,t+1} - k_{i,t}$, hypotheses H2 and H3 impose a stark restriction on the data by predicting the existence of a "kink". More precisely, the gap is supposed to affect expected equity returns of firms that are expanding ($k_{i,t+1} > k_{i,t}$) and shrinking ($k_{i,t} < k_{i,t+1}$) in opposite directions. Once again, capital budgeting provides an intuitive interpretation for H2 and H3. First, since $\lambda_i > 0$, equation (2) predicts that observed changes in firm's i capital stock are in a direction consistent with closing the gap. If firms are under-capitalized such that their target is above their current capital stock and $k^*_{i,t+1} - k_{i,t} > 0$, they are expected to invest such that $k_{i,t+1} - k_{i,t} > 0$ as well. Vice versa, over-capitalized firms are expected to disinvest. Thus, other conditions equal, given an observed change in capital $k_{i,t+1} - k_{i,t} > 0$, inflexible firms maintain larger gaps to bear lower adjustment costs. H2 predicts that discount rates must be therefore increasing with the gap, otherwise firms could earn larger net present values by getting closer to their target. Symmetrically, H3 predicts that firms that are downsizing ($k_{i,t+1} - k_{i,t} < 0$) must have a higher cost of capital if their gap $k^*_{i,t+1} - k_{i,t}$ is larger is absolute value, that is more negative.

Both the degree of inflexibility λ_i and the gap $k^*_{i,t+1} - k_{i,t}$ are inherently unobservable and must be estimated. This poses a challenge for testing the three aforementioned hypotheses empirically. In the next section we describe our estimation procedure.

3. Estimating Inflexibility

In this section we describe the estimation methodology for the key quantities of our analysis, namely the flexibility parameter λ_i and the target capital $k^*_{i,t+1}$. To keep notation parsimonious, we omit the firm's index i in this section. We provide a more detailed description of the algorithm in Appendix A.

3.1. State-Space Representation

We start from the state-space representation, which is based on the characterization of firms' investment policy of equation (2) in Section 2, that is

$$k_{t+1} - k_t = \lambda(k^*_{t+1} - k_t).$$

In this equation, the target capital $k^*_{t+1} \geq 0$ is not observable. Thus, it represents the latent variable of the model. In the following steps of the estimation, it is convenient not to restrict the latent variable to be positive. Therefore, we perform a change of latent variable and rewrite the previous equation as

$$k_{t+1} = \lambda k^*_t e^{x_{t+1}} + (1 - \lambda)k_t, \tag{3}$$

where $x_{t+1} = log i^*_{t+1}$ is the new latent variable, and $i^*_{t+1} = \frac{k^*_{t+1}}{k^*_t}$ is the gross growth rate of target capital. To proceed in the definition of the state space, we assume the following AR(1) dynamics for x_{t+1}:

$$x_{t+1} = \rho^* x_t + \eta_t. \tag{4}$$

The choice of the AR(1) dynamics, albeit discretionary, has the advantage of being parsimonious and tractable.

Finally, since the partial adjustment representation in (2) holds only approximately, we add an error term ε_t to it. Our state-space model is therefore

$$
\begin{aligned}
x_{t+1} &= \rho^* x_t + \eta_{t+1} && (5)\\
k_{t+1} &= \lambda k_t^* e^{x_{t+1}} + (1-\lambda) k_t + \varepsilon_{t+1}, && (6)
\end{aligned}
$$

where $\eta_t \sim N(0, \sigma_\eta)$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$, and η_t and ε_t are independent. As standard, we refer to the first equation as the state equation, and to the second equation as the observation equation.

3.2. Choice of the Estimation Technique

Given the state-space representation (5), the main challenges for the estimation are the non-linearity of the state space model and, most important, the need of estimates that are free of look ahead bias. We therefore adopt a sequential procedure that effectively deals with unknown parameters and a non-linear state space, namely the sequential Monte Carlo method in the version proposed by Liu and West (2001). This method is relatively simple to implement and it is computationally efficient. Specifically, it allows to update the posterior distribution of the unobservable state variables and of the unknown parameters in (5) through an efficient simulation-based on-line analysis. The estimation is performed sequentially, every time new observations become available.

Other methods might be available to implement our estimation. We briefly discuss the rationale behind our choice hereafter. To do so, we first define the main objects of our analysis: the vector of latent variables $\theta_t = \{x_t\}$ and the vector of unknown parameters $\psi = \{\rho^*, \lambda, \sigma_\eta, \sigma_\varepsilon\}$. The ultimate goal of the estimation is to reconstruct all the unknown state-variable history $\theta_{0:t}$ from time 0 to time t. To make inference on $\theta_{0:t}$ and ψ given available data $y_{1:t} = \{k_{1:t}\}$ we need to compute their joint posterior density

$$\pi(\theta_{0:t}, \psi | y_{1:t}) = \pi(\theta_{0:t} | \psi, y_{1:t}) \pi(\psi | y_{1:t}). \tag{7}$$

In theory, the posterior density (7) can be computed using Bayes' rule. In simple models, $\pi(\theta_{0:t}, \psi | y_{1:t})$ can be computed in closed form. Usually, as in this case, computations are not analytically tractable. However, Bayesian inference provides computationally efficient tools for approximating the posterior distribution. For this reason, we opt for a Bayesian estimation technique. In this subset of methods, we favor a sequential algorithm over a standard Markov Chain Monte Carlo (MCMC) because we are not only filtering $\theta_{0:t}$, but also with the unknown parameters ψ. Thus, sequential metholds allow to exploit the samples generated from $\pi(\theta_{0:t}, \psi|y_{1:t})$ when simulating from the posterior in following period $\pi(\theta_{0:t+1}, \psi|y_{1:t+1})$. In other words, when a new observation becomes available, there is no need to re-run the entire estimation as in MCMC. This is the case due to the presence of unknown parameters ψ. In the case of known parameter, one could instead compute $\pi(\theta_{0:t+1}|y_{1:t+1})$ from $\pi(\theta_{0:t}|y_{1:t})$ by the estimation-error correction formulation provided by the Kalman filter. In addition, the sequential method we are using is suitable to deal with the non-linearity of our state space. For example, the standard Kalman filter would not be a viable choice.

3.3. The Algorithm

We now sketch our estimation algorithm. We provide additional details in Appendix A. The approach of Liu and West (2001) constructs an approximation of the true distribution of $\theta_{0:t}$ and ψ at time t that is continuous not only in θ_t, but also in ψ. In this way, importance sampling allows to draw values of ψ from a continuous importance density, without relying on the values of ψ used in the discrete approximation at time $t-1$. Consider the discrete approximation available at time $t-1$

$$\hat{\pi}_{t-1}(\theta_{0:t}, \psi) = \sum_{i=1}^{N} w_{t-1}^{(i)} \delta_{(\theta_{0:t-1}^{(i)}, \psi^{(i)})} \approx \pi(\theta_{0:t-1}, \psi|y_{1:t-1}), \tag{8}$$

where the index i refers to the particle i and N is the total number of particles, and $w_{t-1}^{(i)}$ is the weight associated to the $i-th$ particle. The marginal can then be computed as

$$\hat{\pi}_{t-1}(\psi) = \sum_{i=1}^{N} w_{t-1}^{(i)} \delta_{\psi^{(i)}} \approx \pi(\psi|y_{0:t-1}). \tag{9}$$

Liu and West (2001) suggest to replace each point mass $\delta_{\psi^{(i)}}$ with a Normal distribution, such that the resulting mixture becomes a continuous distribution. To preserve the mean and the variance of the vector ψ, the mixture is defined as follows:

$$\tilde{\pi}_{t-1}(\psi) = \sum_{i=1}^{N} w_{t-1}^{(i)} N(\psi; m^{(i)}, h^2 \Sigma), \tag{10}$$

with $m^{(i)} = a\psi^{(i)} + (1-a)\overline{\psi}$ and $a^2 + h^2 = 1$. Liu and West (2001) recommend to set $a = (3\delta - 1)/(2\delta)$ for a discount factor δ in $(0.95, 0.99)$, which corresponds to a in $(0.974, 0.995)$. The same criterion can be applied even in the presence of $\theta_{0:t-1}$ to the discrete distribution $\hat{\pi}_{t-1}(\theta_{0:t-1}, \psi)$, leading to the extension of $\tilde{\pi}_{t-1}$ to a joint distribution for $\theta_{0:t-1}$ and ψ, namely

$$\tilde{\pi}_{t-1}(\theta_{0:t-1}, \psi) = \sum_{i=1}^{N} w_{t-1}^{(i)} N(\psi; m^{(i)}, h^2 \Sigma) \delta_{\theta_{0:t-1}^{(i)}}. \tag{11}$$

This distribution is discrete in $\theta_{0:t-1}$, but continuous in ψ. From this point, the method parallels the development of a standard auxiliary particle filter. In particular, after the new data point y_t is observed, the distributions of interest become

$$\begin{aligned} \pi(\theta_{0:t}, \psi | y_{1:t}) &\propto \pi(\theta_{0:t}, \psi, y_t | y_{1:t-1}) && (12) \\ &\approx \pi(y_t | \theta_t, \psi) \cdot \pi(\theta_t | \theta_{t-1}, \psi) \cdot \tilde{\pi}_{t-1}(\theta_{0:t-1}, \psi) && (13) \\ &= \sum_{i=1}^{N} w_{t-1}^{(i)} \pi(y_t | \theta_t, \psi) \pi(\theta_t | \theta_{t-1}^{(i)}, \psi) N(\psi; m^{(i)}, h^2 \Sigma) \delta_{\theta_{0:t-1}^{(i)}}. && (14) \end{aligned}$$

Importantly, the approximation of the posterior distribution at time $t-1$ based on a mixture of Normals is well defined only if the vector ψ is expressed in a way that is consistent with such a distribution. A more efficient alternative is to use a mixture of non-normal distributions, appropriately selected so that their support is the same as that of the distribution of the parameter vector. In our case, we choose a Gamma distribution for the two variances σ_η and σ_ε, a Beta distribution for ρ^* and λ, and a Normal distribution for the latent variable x_t. The estimation procedure is applied to each firm individually.

3.4. Data and Priors

We use annual data from the Compustat/CRSP merged database. We consider the sample period 1980-2011. We exclude firms with SIC codes between 4900 and 4999, 6000 and 6999, and larger than 9000. We delete firm-year observations with missing data, and those for which total assets (item [at]), the capital stock (item [ppegt]), or sales (item [sale]) are either zero or negative. For us the variable k_t is represented by the capital stock net of depreciation (item [ppegt]). We exclude firms with less than five years of observations for estimation purposes.

Finally we use the entire period 1961-1979 to initialize our priors. We need to define four different priors. The two standard deviations σ_η and σ_ε should be non-negative. Thus, we choose a uniform distribution with a zero lower bound. The upper bound of σ_η is set equal to ten, which guarantees a non-restrictive range since standard deviations are expressed as percentage rates. The upper bound is the same for every firm. The upper bound of σ_ε is set to one-hundred times the initial value of the capital stock. Thus, it is firm specific, and it accomodates possibly huge variations in the observation equation. We then consider the auto-regressive coefficient of the state equation ρ^*. Its prior is defined by estimating, for each firm, the auto-correlation of the gross growth rate of its capital stock, and then averaging out the resulting values. We then use the standard deviation of this point estimate to set the upper and lower bound of the uniform distribution. We then have a uniform distribution in the interval $[0.6, 0.8]$. The underlying assumption is that the auto-correlation of the observable capital stock is comparable in size to the auto-correlation of the target capital. Finally, we need to define the prior for the flexibility parameter λ. To this purpose we regress the capital stock on the lagged capital stock and a set of standard accounting variables in a panel setting. We then use the complement to one of the coefficient on the lagged capital stock as the midpoint of the uniform distribution for the prior. Finally, the standard deviation of the point estimates is used to define the lower and the upper bound of the distribution.

3.5. Descriptive Statistics

In this sub-section we report descriptive statistics of the estimates of our procedure. Table 1 shows that firms with a capital stock below their target level of capital adjust the capital stock to fill the gap. In particular, the growth rate of capital in the following period is positive. This is apparent for the first two groups of firms, sorted according to their gap, and it is consistent with the characterization in equation 2. Symmetrically, firms that are above their target level of capital tend to downsize and converge toward their targets.

[Insert Table 1 Here]

4. Empirical Results

Section 4.1 describes the data and the sample, and Sections 4.2 and 4.3 test the inflexibility hypotheses we developed in Section 2.

4.1. Data and Sample

In our asset pricing tests we use monthly stock returns for firms on NYSE, AMEX, Nasdaq covered by the Center of Research in Security Prices (CRSP) from 1980 to 2012. Since the estimation procedure for inflexibility and the gap described in Section 3 is initialized with full-sample information from 1961 to 1979, 1980 is chosen as the starting year to prevent look-ahead biases from contaminating our results. Delisting returns are included in monthly returns. We match monthly data on returns to the annual estimates of inflexibility and of the gap, and to accounting data from the CRSP/COMPUSTAT merged database. The gap and capital growth are scaled by the one-year lagged stock of capital. We follow the matching procedure of Fama and French (1992), which guarantees a gap of at least six months between fiscal year-ends and returns. Accordingly, we match monthly stock returns from July of calendar year t to June of calendar year $t+1$ with data from each company's latest fiscal year ending in calendar year $t-1$.

We consider (log) market capitalization and (log) book-to-market equity as control variables in some of our analyses. We compute the latter two variables using standard procedures. Market capitalization is measured at June of calendar year t for the returns between July of calendar year t and June of calendar year $t+1$. Market capitalization is defined as the product of a company's outstanding shares and the company's stock price. Book-to-market equity is the ratio between a firm's book value equity and the firm's market capitalization at the end of December of calendar year $t-1$. We compute book equity as the sum of shareholders' equity, balance sheet deferred taxes and investments, and tax credits if available, minus the book value of preferred stocks. Depending on data availability, we estimate the book value of preferred stocks using, in this order, their redemption, liquidation or par value. To compute excess returns we use the one-month Treasury Bill rate from Kenneth French's website.

4.2. Inflexible Returns

Table 2 considers univariate portfolio sorts involving our estimates of flexibility and of the gap, and the observed capital growth. The table reports average annualized excess returns and Sharpe ratios of 10 equally weighted and 10 value-weighted portfolios sorted on each of the three variables, and the average return spread between the lowest and the highest decile portfolios (L-H). Consistent with the inflexibility hypothesis H1, the top row shows that average returns decline almost monotonically from low-flexibility to high-flexibility portfolios, with an average spread of 4.88% for the equally-weighted portfolios and of 3.76% for the value-weighted portfolios. In this univariate analysis both the gap and capital growth display similar decreasing average returns across their respective decile portfolios, to the extent that they are positively correlated. Specifically, in the equally-weighted sorts the returns spreads for the two variables are respectively 9.25% (Sharpe ratio = 1.05) and 9.87% (Sharpe ratio = 1.06), while in the corresponding value-weighted sorts the spreads are 4.43% (Sharpe ratio = 0.38) and 4.87% (Sharpe ratio = 0.40). The decreasing return associated with capital growth is akin to the well-known asset growth anomaly documented by Cooper, Goolen, and Schill (2008).

[Insert Table 2 Here]

Table 3 reports estimates from Fama and MacBeth (1973) cross-sectional regressions of monthly excess returns on estimated flexibility, gap, and observed capital growth. The rightmost column of Table 3 strongly supports the inflexibility hypothesis and confirms the sizeable spread in average returns in Table 2. The slope for estimated flexibility is -0.0097, more than 10 standard errors from zero, after controlling for size and book-to-market equity. The premium for inflexibility is economically sizeable. A ten-percent decrease in the speed of adjustment towards the target leads to a premium of approximately 10 basis points per month over the riskfree rate after "zeroing out" the effects of size and book-to-market equity, with an implied annualized Sharpe ratio of 1.25.[5]

[5]In Fama and MacBeth (1973) regressions, the estimated loading on a given regressor can be interpreted as the average monthly return of a self-financing portfolio with a unit value for the regressor, and that hedges the effects of control variables. The corresponding standard error is the standard deviation of monthly returns of such a portfolio, divided by the square root of the number of months considered (389 in this

[Insert Table 3 Here]

Table 4 and Table 5 consider the interaction of inflexibility and capital growth, and document that the return spread associated with inflexibility is stable across groups of capital growth. Table 4 examines sequential portfolio sorts that allow variation in inflexibility which is unrelated to capital growth. Similarly to Fama and French (1992), we first subdivide data in quintiles based on capital growth and, within each quintile, we sort observations in five groups of flexibility. As in Table 2, we report average annualized excess returns and Sharpe ratios for value-weighted portfolios, and the return spread (L-H).[6] In all quintiles of capital growth there is evidence of a sizeable spread between the low-flexibility and the high-flexibility portfolios, ranging from 2.11% for the firms with low capital growth to 8.77% for the firms with high capital growth.

[Insert Table 4 Here]

Table 5 shows estimates from Fama-MacBeth regressions for each quintile of capital growth. The slope for our estimate of flexibility is negative in all quintiles, with values of -0.0170 (t-statistic = -8.68), -0.0097 (t-statistic = -5.90), -0.0065 (t-statistic = -5.93), -0.0080 (t-statistic = -5.86), and -0.0170 (t-statistic = -8.46) moving from the lowest to the highest quintile of capital growth.

[Insert Table 5 Here]

The results in Tables 4 and 5 show that the premium for inflexibility does not appear to subsumed by the observable changes in a firm's capital stock. Thus, as equation (2) suggests, while firm's investment and inflexibility are deeply related, our estimates for inflexibility provide additional price-relevant information besides that reflected in observed corporate investment.

case). Assuming an average monthly risk-free rate of 30 basis points, a t-statistic of 10.31 approximately corresponds to an annualized Sharpe ratio of 1.25.

[6] Results for equally-weighted portfolios are qualitatively similar.

Overall, the results in this section provide strong support to the inflexibility hypothesis H1 we formulate in Section 2.

4.3. Investment, Gap, and Returns

As discussed in Section 4.2, the gap and capital growth are positively correlated and exhibit similar patterns in the sorts of Table 2. This is consistent with the dynamics described by equation (2), that prescribes that firms with a positive gap ($k^*_{i,t+1} > k_{i,t}$) invest to increase their capital stock. However, hypotheses H2 and H3 suggest nontrivial empirical predictions when the effects of the gap and capital growth on expected returns are considered jointly. The first four columns of Table 3 report estimates of the slopes of the gap and capital growth when they are simultaneously included in Fama-Macbeth regressions, also controlling for size (column 2), book-to-market equity (column 3), and both of them (column 4). The estimated slopes for capital growth and the gap are fairly stable across the four specifications, with point estimates between 0.00021 (t-statistic = 2.60) and 0.00027 (t-statistic = 3.03) for the gap, and between -0.00077 (t-statistic = -6.16) and -0.00110 (t-statistic = -7.26) for capital growth. Hence, despite their positive correlations, none of the two variables subsumes the effect of the other on average returns.

Table 6 and Table 7 report evidence from sequential portfolio sorts and Fama-MacBeth regressions to test hypotheses H2 and H3 empirically. In particular, Table 6 considers variation in the gap which is unrelated to capital growth by sorting observations in quintiles of gap within each quintile of capital growth. Similarly to Fama and French (1992), we first subdivide data in quintiles based on capital growth and, within each quintile, we sort observations in five groups based on the gap. Table 6 reports average annualized excess returns and Sharpe ratios for value-weighted portfolios, and the return spread (H-L) between firms the high and the low gap portfolios. The results support H2 and H3 in that the gap is negatively related to average returns in groups of firms with negative capital growth, and positively related to returns in groups of firms with positive capital growth. In particular, the first two rows of Table 6 refer to negative capital growth firms, with average values of -0.25 for group 1 and -0.06 for group 2. Firms with low gaps outperform those with high gaps within these clusters, with sizeable spreads of 6.86% in group 1 (Sharpe ratio = 0.43) and

9.14% in group 2 (Sharpe ratio = 0.69). Vice versa, the last two rows refer to positive capital growth firms, with average values on 0.14 in group 4 and 0.41 in group 5. Remarkably, in the latter two groups firms with high gaps outperform firms with low gaps, with spreads of 9.96% in group 4 (Sharpe ratio = 0.61) and 6.55% in group 5 (Sharpe ratio = 0.44). Group 3 contains firms with approximately zero capital growth and is associated with no significant gap spreads. Thus, a "kink" in correspondence of zero capital growth seems to emerge in the data, consistent with our hypotheses.

[Insert Table 6 Here]

Table 7 confirms the patterns we document in Table 6 by means of Fama-MacBeth regressions, after controlling for size and book-to-market equity. As in Table 5, regressions are estimated for each quintile of capital growth, defined as in the sorts of Table 6. The estimated slopes for the gap are positive and statistically significant for the fourth and the fifth quintile of capital growth (0.0015, t-statistic = 2.58 for quintile 4; 0.0010, t-statistic = 2.96 for quintile 5), negative and statistically significant for the first and the second quintile (-0.0031, t-statistic = -2.75 for quintile 1; -0.0034, t-statistic = -1.95 for quintile 2), and indistinguishable from zero for the third quintile (0.0010, t-statistic = 1.02).

[Insert Table 7 Here]

In sum, the evidence in Table 6 and Table 7 appears to be strongly in favor of the economic mechanism captured by equation (2), and is consistent with hypotheses H2 and H3.

5. Conclusions

Investment-based asset pricing models get traction from a common tenet, namely firms' limited flexibility in adjusting their physical capital. Is the inflexibility mechanism a first-order driver of expected equity returns? In this work, we provide an empirical foundation for it and, ultimately, for the investment-based asset pricing paradigm. We propose a standard modeling framework where firms optimally invest by closing a fraction of the gap between their existing capital stock and a target level of capital. Inflexible firms are sluggish in filling the gap, are more risky, and earn higher expected returns. We use sequential Monte Carlo techniques to estimate inflexibility and the gap for a large sample of US listed firms, and we test how they affect expected equity returns. Our evidence strongly corroborates the inflexibility mechanism as a first-order driver of cross-sectional differences in returns. Ultimately, our results support the investment-based paradigm as a mainstream avenue to turn anomalies into empirical regularities.

Table 1. Descriptive Statistics

Table 1 reports descriptive statistics for stocks sorted in quintiles with respect to the gaps we estimate using the procedure of Section 3. We report the average of the following variables: gap, capital growth, cash holdings (computed as the ratio of cash and cash equivalents and total assets), return on asset (computed as the ratio of the operating income to total assets), the book-to-market ratio and the real value of sales, computed using the 1960 deflator. All variables are defined in Appendix B.

Gap Quintile	Gap	Capital Growth	Cash	ROA	BM	Size (real)
1	-1.011	-0.457	0.160	-0.121	0.109	9,915
2	-0.338	-0.157	0.133	-0.006	0.111	8,455
3	0.042	0.003	0.120	0.031	0.096	18,791
4	0.621	0.241	0.143	0.040	0.072	15,875
5	2.156	0.676	0.186	0.028	0.067	11,584

Table 2. Inflexible Returns: Univariate Sorts

Table 2 stocks are sorted independently every June in deciles based on their estimated values of flexibility, gap and capital growth. We report excess returns and Sharpe ratios for the bottom decile (L), the top decile (H) and for the third, fifth and seventh decile. We also report the difference for the excess returns and the Sharpe ratio between the top decile and the bottom decile (H-L). The left panel reports equally-weighted returns, while the right panel reports value-weighted returns. Data are from January 1980 to December 2011. All variables are defined in Appendix B.

Sorting Variable		Equally-Weighted						Value-Weighted					
		H-L	L	3	5	7	H	H-L	L	3	5	7	H
Flexibility	R^e	-4.88	14.29	14.85	11.88	10.99	9.41	-3.76	8.43	10.01	5.28	8.86	4.66
	SR	-0.53	0.72	0.68	0.52	0.48	0.53	-0.38	0.50	0.45	0.29	0.45	0.29
Gap	R^e	-9.25	16.86	14.20	11.93	11.64	7.61	-4.43	9.96	8.63	7.63	7.64	5.53
	SR	-1.05	0.66	0.72	0.68	0.59	0.31	-0.38	0.51	0.53	0.50	0.43	0.24
Capital Growth	R^e	-9.87	16.68	14.43	12.93	11.62	6.81	-4.87	9.58	8.74	7.82	7.80	4.72
	SR	-1.06	0.66	0.75	0.70	0.60	0.28	-0.40	0.48	0.51	0.52	0.45	0.20

Table 3. Inflexible Returns: Fama-MacBeth Regressions

Table 3 reports coefficient estimates from Fama-MacBeth regressions. The dependent variable is the monthly return. Column (1) includes gap and capital growth. Column (2) includes gap, capital growth and the logarithm of size as defined in Fama and French (1992). Column (3) includes gap, capital growth and the logarithm of book-to-market as defined in Fama and French (1992). Column (4) includes gap, capital growth, the logarithm of size and the logarithm of book-to-market as defined in Fama and French (1992). Column (5) includes flexibility, the logarithm of size, and the logarithm of book-to-market as defined in Fama and French (1992). T-statistics are reported in parentheses. The symbols ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. Data are from January 1980 to December 2011. All variables are defined in Appendix B.

	(1)	(2)	(3)	(4)	(5)
Gap	0.00027***	0.00022***	0.00023***	0.00021***	
	(3.03)	(2.68)	(2.86)	(2.60)	
Capital Growth	-0.11***	-0.0093***	-0.0080***	-0.0077***	
	(-7.26)	(-6.61)	(-6.22)	(-6.16)	
Flexibility					-0.0097***
					(-10.31)
Log(size) - in June - as in F&F 1992		-0.0015***		-0.0011***	-0.0019***
		(-3.86)		(-2.74)	(-4.34)
Log(book-to-market) - in June - as in F&F 1992			0.0032***	0.0017*	0.0023**
			(4.82)	(1.91)	(2.57)
Constant	0.014***	0.032***	0.038***	0.042***	0.043***
	(5.31)	(4.82)	(6.39)	(5.81)	(6.04)
Observations	1,153,324	1,153,324	1,153,324	1,153,324	1,153,324
R^2	0.004	0.019	0.011	0.029	0.029

Table 4. Inflexible Returns: Bivariate Sorts

In Table 4 stocks are sorted independently every June in quintiles based on their value of capital growth. Then, stocks in each quintile are further sorted independently every June in quintiles based on their value of flexibility. We report the excess value-weighted return and the Sharpe ratio for the bottom quintile (L), the top quintile (H) and for the second, third and fourth quintile. We also report the difference for the excess return and the Sharpe ratio between the top quintile and the bottom quintile (H-L). Data are from January 1980 to December 2011. All variables are defined in Appendix B.

Group of Capital Growth		H-L	L	2	3	4	H
Group 1	R^e	-2.11	16.47	16.30	16.84	16.11	14.37
	SR	-0.18	0.71	0.60	0.62	0.67	0.72
Group 2	R^e	-4.43	15.62	15.04	13.56	12.96	11.19
	SR	-0.41	0.79	0.72	0.65	0.65	0.63
Group 3	R^e	-4.94	14.39	14.65	11.01	12.39	9.45
	SR	-0.49	0.75	0.70	0.54	0.61	0.53
Group 4	R^e	-3.99	14.81	13.21	10.59	12.39	10.82
	SR	-0.40	0.76	0.63	0.50	0.61	0.58
Group 5	R^e	-8.77	14.70	9.74	7.79	5.63	5.93
	SR	-0.69	-0.65	0.41	0.32	0.24	0.29

Table 5. Inflexible Returns: Fama-MacBeth Regressions for Quintiles of Capital Growth

Table 5 reports coefficient estimates from Fama-MacBeth regressions for each quintile of capital growth. The dependent variable is the monthly return. Using data from January 1980 to December 2011, stocks are sorted independently every June in quintiles based on their value of capital growth. Then, for each quintile, we run a Fama-MacBeth regression of the monthly returns on flexibility, the logarithm of size and the logarithm of book-to-market as defined in Fama and French (1992). Column (i) refers to the i-th quintile of capital growth. T-statistics are reported in parentheses. The symbols ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. All variables are defined in Appendix B.

	(1)	(2)	(3)	(4)	(5)
Flexibility	-0.017***	-0.0097***	-0.0065***	-0.0080***	-0.017***
	(-8.68)	(-5.90)	(-5.93)	(-5.86)	(-8.46)
Log(size) - in June - as in F&F 1992	-0.0041***	-0.0024***	-0.0015***	-0.0015***	-0.0021***
	(-6.01)	(-4.36)	(-3.67)	(-3.39)	(-3.83)
Log(book-to-market) - in June - as in F&F 1992	0.0020**	0.0021**	0.0016**	0.0016*	0.0028***
	(2.21)	(2.23)	(1.97)	(1.95)	(3.00)
Constant	0.055***	0.046***	0.035***	0.033***	0.036***
	(6.54)	(5.65)	(4.57)	(4.48)	(4.28)
Observations	228,196	234,754	238,150	254,229	169,350
R^2	0.029	0.039	0.035	0.034	0.037

Table 6. Mind the Gap: Bivariate Sorts on Capital Growth and Gap

In Table 6 stocks are sorted independently every June in quintiles based on their value of capital growth. Then, stocks in each quintile are further sorted independently every June in quintiles based on their value of the gap. We report the excess value-weighted return and the Sharpe ratio for the bottom quintile (L), the top quintile (H) and for the second, third and fourth quintile. We also report the difference for the excess return and the Sharpe ratio between the top quintile and the bottom quintile (H-L). Data are from January 1980 to December 2011. All variables are defined in Appendix B.

Group of Capital Growth		H-L	L	2	3	4	H
Group 1 -0.25	R^e	-6.86	11.93	10.71	11.26	7.41	5.07
	SR	-0.43	0.53	0.53	0.54	0.33	0.27
Group 2 -0.06	R^e	-9.14	13.57	8.54	8.58	5.84	4.43
	SR	-0.69	0.72	0.47	0.50	0.30	0.27
Group 3 0.01	R^e	-0.69	9.99	8.17	8.78	10.23	9.30
	SR	-0.05	0.61	0.49	0.51	0.58	0.49
Group 4 0.14	R^e	9.96	4.59	8.12	6.16	7.15	14.55
	SR	0.61	0.24	0.45	0.31	0.36	0.76
Group 5 0.41	R^e	6.55	3.34	6.85	6.26	10.49	9.90
	SR	0.44	0.15	0.32	0.29	0.44	0.42

Table 7. Mind the Gap: Fama-MacBeth Regressions for Quintiles of Capital Growth

Table 7 reports coefficient estimates from Fama-MacBeth regressions for each quintile of capital growth. Using data from January 1980 to December 2011, stocks are sorted independently every June in quintiles based on their value of capital growth. Then, for each quintile, we run a Fama-MacBeth regression of the monthly returns on gap, the logarithm of size and the logarithm of book-to-market as defined in Fama and French (1992). Column (i) refers to the i-th quintile of capital growth. T-statistics are reported in parentheses. The symbols ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. All variables are defined in Appendix B.

	(1)	(2)	(3)	(4)	(5)
Gap	-0.0031***	-0.0034*	0.0010	0.0015***	0.0010***
	(-2.75)	(-1.95)	(1.02)	(2.58)	(2.96)
Log(size) - in June - as in F&F 1992	-0.0025***	-0.0015***	-0.0011***	-0.00096**	-0.00070
	(-4.30)	(-3.33)	(-2.80)	(-2.28)	(-1.37)
Log(book-to-market) - in June - as in F&F 1992	0.0021**	0.0021**	0.0013	0.0015*	0.0026***
	(2.34)	(2.20)	(1.57)	(1.74)	(2.72)
Constant	0.058***	0.046***	0.035***	0.034***	0.037***
	(6.83)	(5.69)	(4.55)	(4.62)	(4.31)
Observations	228,196	234,754	238,150	254,229	169,350
R^2	0.030	0.041	0.034	0.033	0.035

Appendix A. Estimation Procedure

In this section we provide a detailed description of the estimation algorithm. We start by defining the objects involved in the estimation procedure. θ_t is the latent variable, in our case x_t, the logarithm of gross growth rate of target capital i_t^*.

The algorithm boils down to the following iterative procedure

- Initialize
  Draw $(\theta_0^{(1)}, \psi^{(1)}), ..., (\theta_0^{(N)}, \psi^{(N)})$ independently from $\pi(\theta_0)\pi(\psi)$. Set $w_0^{(i)} = N^{-1}$ for $i = 1, ..., N$, and

$$\hat{\pi}_0 = \sum_{i=1}^{N} w_0^{(i)} \delta_{(\theta_0^{(i)}, \psi^{(i)})}$$

- For $t = 1, ..., T$

  - Compute $\overline{\psi} = E_{\hat{\pi}_{t-1}}(\psi)$ and $\Sigma = Var_{\hat{\pi}_{t-1}}(\psi)$. For $i = 1, ..., N$, set

$$\begin{aligned} m^{(i)} &= a\psi^{(i)} + (1-a)\overline{\psi} \\ \hat{\theta}_t^{(i)} &= E(\theta_t | \theta_{t-1} = \theta_{t-1}^{(i)}, \psi = m^{(i)}) \end{aligned}$$

  - For $k = 1, ..., N$

    * Draw I_k with $P(I_k = i) \propto w_{t-1}^{(i)} E(y_t | \theta_{t-1} = \hat{\theta}_{t-1}^{(i)}, \psi = m^{(i)})$
    * Draw $\rho^{*(k)}$ from $Beta(\mu_{\rho^*}^{(k)}, \sigma_{\rho^*}^{2(k)})$ where $\mu_{\rho^*}^{(k)} = \mu(\alpha_{\rho^*}^{(k)}, \beta_{\rho^*}^{(k)}) = a\rho^{*(k)} + (1 - a)\bar{\rho}^{*(k)}$ and $\sigma_{\rho^*}^{2(k)} = \sigma^2(\alpha_{\rho^*}^{(k)}, \beta_{\rho^*}^{(k)}) = h^2\Sigma$ with $\alpha_{\rho^*}^{(k)} = \frac{(\mu_{\rho^*}^{(k)})^2(1-\mu_{\rho^*}^{(k)})}{\sigma_{\rho^*}^{2(k)}} - \mu_{\rho^*}^{(k)}$ and $\beta_{\rho^*}^{(k)} = \frac{\mu_{\rho^*}^{(k)}(1-\mu_{\rho^*}^{(k)})^2}{\sigma_{\rho^*}^{2(k)}} - (1 - \mu_{\rho^*}^{(k)})$
    * Draw $\lambda^{(k)}$ from $Beta(\mu_\lambda^{(k)}, \sigma_\lambda^{2(k)})$ where $\mu_\lambda^{(k)} = \mu(\alpha_\lambda^{(k)}, \beta_\lambda^{(k)}) = a\lambda^{(k)} + (1-a)\bar{\lambda}^{(k)}$ and $\sigma_\lambda^{2(k)} = \sigma^2(\alpha_\lambda^{(k)}, \beta_\lambda^{(k)}) = h^2\Sigma$ with $\alpha_\lambda^{(k)} = \frac{(\mu_\lambda^{(k)})^2(1-\mu_\lambda^{(k)})}{\sigma_\lambda^{2(k)}} - \mu_{\rho^*}^{(k)}$ and $\beta_\lambda^{(k)} = \frac{\mu_\lambda^{(k)}(1-\mu_\lambda^{(k)})^2}{\sigma_\lambda^{2(k)}} - (1 - \mu_\lambda^{(k)})$

* Draw $\sigma_\eta^{(k)}$ from $Gamma(\mu_{\sigma_\eta}^{(k)}, \sigma_{\sigma_\eta}^{2(k)})$ where $\mu_{\sigma_\eta}^{(k)} = \mu(\alpha_{\sigma_\eta}^{(k)}, \beta_{\sigma_\eta}^{(k)}) = a\sigma_\eta^{(k)} + (1 - a)\bar{\sigma}_\eta^{(k)}$ and $\sigma_{\sigma_\eta}^{2(k)} = \sigma^2(\alpha_{\sigma_\eta}^{(k)}, \beta_{\sigma_\eta}^{(k)}) = h^2 \Sigma$ with $\alpha_{\sigma_\eta}^{(k)} = \frac{(\mu_{\sigma_\eta}^{(k)})^2}{\sigma_{\sigma_\eta}^{2(k)}}$ and $\beta_{\sigma_\eta}^{(k)} = \frac{\mu_{\sigma_\eta}^{(k)}}{\sigma_{\sigma_\eta}^{2(k)}}$
* Draw $\sigma_\varepsilon^{(k)}$ from $Gamma(\mu_{\sigma_\varepsilon}^{(k)}, \sigma_{\sigma_\varepsilon}^{2(k)})$ where $\mu_{\sigma_\varepsilon}^{(k)} = \mu(\alpha_{\sigma_\varepsilon}^{(k)}, \beta_{\sigma_\varepsilon}^{(k)}) = a\sigma_\varepsilon^{(k)} + (1 - a)\bar{\sigma}_\varepsilon^{(k)}$ and $\sigma_{\sigma_\varepsilon}^{2(k)} = \sigma^2(\alpha_{\sigma_\varepsilon}^{(k)}, \beta_{\sigma_\varepsilon}^{(k)}) = h^2 \Sigma$ with $\alpha_{\sigma_\varepsilon}^{(k)} = \frac{(\mu_{\sigma_\varepsilon}^{(k)})^2}{\sigma_{\sigma_\varepsilon}^{2(k)}}$ and $\beta_{\sigma_\eta}^{(k)} = \frac{\mu_{\sigma_\varepsilon}^{(k)}}{\sigma_{\sigma_\varepsilon}^{2(k)}}$
* Draw $\theta_t^{(k)}$ from $\pi(\theta_t|\theta_{t-1} = \theta_{t-1}^{(I_k)}, \psi = \psi^{(k)})$ and set $\theta_{0:t}^{(k)} = (\theta_{0:t-1}^{(I_k)}, \theta_t^{(k)})$
* Set

$$\tilde{w}_t^{(k)} = \frac{\pi(y_t|\theta_{t-1} = \theta_t^{(k)}, \psi = \psi^{(k)})}{\pi(y_t|\theta_{t-1} = \hat{\theta}_t^{(I_k)}, \psi = m^{(I_k)})}$$

– Normalize the weights

$$w_t^{(i)} = \frac{\tilde{w}_t^{(i)}}{\sum_{j=1}^N \tilde{w}_t^{(j)}}$$

– Compute the effective sample size

$$N_{eff} = \left(\sum_{i=1}^N (w_t^{(i)})^2 \right)^{-1}$$

– If $N_{eff} < N_0$, re-sample.

* Draw a sample of size N from the discrete distribution

$$P((\theta_{0:t}, \psi) = (\theta_{0:t-1}^{(i)}, \psi^{(i)})) = w_t^{(i)}, \qquad i = 1, ..., N$$

and re-label this sample

$$(\theta_{0:t}^{(1)}, \psi^{(1)}), ..., (\theta_{0:t}^{(N)}, \psi^{(N)})$$

* Reset the weights: $w_t^{(i)} = N^{-1}, \qquad i = 1, ..., N$

– Set $\hat{\pi}_t = \sum_{i=1}^N w_t^{(i)} \delta_{(\theta_{0:t}^{(i)}, \psi^{(i)})}$

We set a equal to $(0.95)^2$, N equal to $5,000$ and N_{eff} equal to 100. The resampling procedure is needed to deal with the degeneracy phenomenon, where after a few iterations, all but one particle will have a negligible weight. Doucet (1998) shows that the variance of the weights can only increase over time. As a consequence it is impossible to avoid such a phenomenon. We allow for resampling even though in our case the phenomenon is not severe to the extent that we deal with relatively short time series.

Appendix B. Variable Definition

The following table reports the definition of the variables used in the analyses. All variables are defined with reference to Compustat items.

Variable	Variable Definition
Capital Stock	Property, plant and equipment net of depreciation (PPENT)
Size	log(Sales net (SALE))
Market-to-Book	(Market value (CSHO * PRCC) + Book debt)/Assets - Total (AT)
Market Cap	Market value (CSHO * PRCC) (in 1990 dollars)
Cash	Cash and Short-Term Investments (CHE)/ Assets - Total (AT)

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