Asymmetric Information and Optimal Debt Maturity

PRELIMINARY and INCOMPLETE

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Abstract

Why were financial institutions so reliant on short-term borrowing before the great recession, thereby exposing themselves to a significant amount of rollover risk? Why did the market of short-term lending collapse after the recession began? This paper provides a general framework to discuss the optimal debt maturity structure with information asymmetry. We show that good firms are willing to borrow in the short term because the extra information released to outside creditors in the rollover stage could help to distinguish them from the bad firms, and thus lower the cost of refinancing. We construct the unique pooling equilibrium with an optimal mix of short-term and long-term debt, in which the good firms maximize their profits and bad firms find it profitable to mimic the good firms. We argue that when the quality of firms' assets starts to deteriorate (the share of good firms is diminishing) and creditors become more prudential, firms will first incur more short-term debts in order to exploit the value of intermediate information. However, when the asset qualities deteriorate further and creditors become very pessimistic, borrowing short-term is too costly and the market of short-term borrowing freezes.

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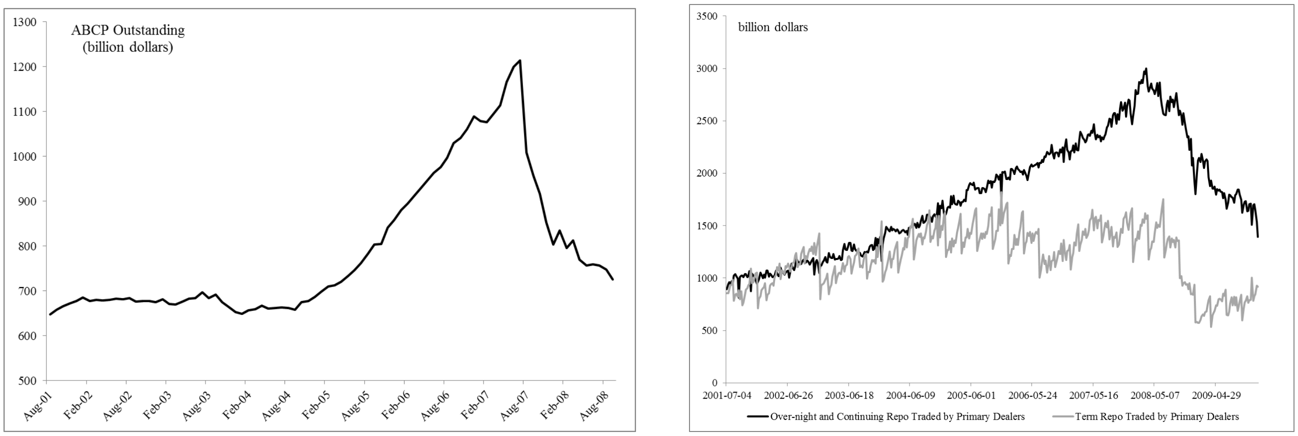


Figure 1: ABCP and Repo Markets (2001-2009)

1 Introduction

The maturity mismatch problem, or financing long-term investment projects by issuing short-term debt contracts, was at the nexus of the financial crisis from 2007 to 2010 (Brunnermeier (2009); Diamond and Rajan (2009); Hellwig (2009)). As Figure 1 shows, outstanding Asset Backed Commercial Papers (ABCP) and overnight Repo increased remarkably before the crisis started. In addition, there is data showing that the short-term borrowing and lending market collapsed shortly after the crisis began. Short-term borrowing exposes firms, especially financial firms, to a significant amount of rollover risk and thus is harmful to their financial stability. What are the benefits of incurring short-term debt? More importantly, why did firms incur more and more short-term debt before the financial crisis? Why did the short-term borrowing market collapse after the crisis began?

This paper aims to address all these questions under a framework of asymmetric information, following Flannery (1986) and Diamond (1991). If the creditors in the financial market do not have as accurate information about borrowers' financial strength as the borrowers themselves, short-term debt contracts, although costly because of the rollover risk, could be a signaling device allowing high type borrowers, e.g., borrowers with better assets, to distinguish themselves from the rest. However, the traditional signaling story cannot explain the trajectory of the debt structure before and after the recent financial crisis.

This paper establishes a unified framework incorporating an optimal pooling equilibrium with asymmetric information in order to understand the dynamics of debt maturity structures. In our framework, although creditors cannot distinguish the high type firms (firms with high quality assets) from the low type firms, there is some interim information that will partially reveal the type of firms when short-term debt is rolled over. This interim information could be revealed in firms' financial statements or other reports, and thus could be a piece of public information available to all market participants. Such interim information will help high type firms to (partially) differentiate themselves. The impact of interim information is referred as an "indirect revealing" mechanism because it does not depend on firm' behavior, as opposed to the mechanism of "direct revealing", i.e., the separating equilibrium where high-type and low-type firms choose different strategies. With this "indirect revealing" mechanism, incurring only short-term debt and fully relying on the "direct revealing" mechanism is costly for the high-type firms. For this reason, the separating equilibrium hardly exists.

We focus our attention on the optimal pooling equilibrium, where the high-type firms incur a mixture of short-term and long-term debt in order to maximize their expected profits and low-type firms find it optimal to mimic the high-type ones. In this equilibrium, the short-term debt exists in the optimal pooling equilibrium because the high-type firms could rely on the indirect revealing mechanism to lower their refinancing costs at the rollover stage. If high-type firms finance their investment opportunities purely with long-term debt, then the interim information becomes ineffective. Apart from the benefits, high-type firms face a clear trade-off when borrowing short-term because of the rollover risk. For this reason, the optimal pooling equilibrium always exists as an interior solution.

We find that the confidence of the creditors in the financial market is a key parameter that could well explain the dynamics of the debt maturity structure. The prior belief about asset market, or the confidence, of market participants is the proportion of high-type firms in the market.

Before the financial crisis began, financial market participants become more prudential after being over optimistic about the qualities of financial assets. There were asset pricing bubbles in the housing market, and the asset quality of sub-prime mortgages had decreased year by year since 2001. In particular, the fraction of low documentation loans increased (Demyanyk and Van Hemert (2011)). As the proportion of "bad" assets started to increase and financial market participants became more cautious about their investments, the benefits to the high-type firms of (partially) revealing their types increased. Thus, high-type firms relied more on the indirect revealing mechanism and incurred more short-term debts to make use of the interim information. This explains the increasing trend in short-term financing before the financial crisis.

However, when the asset quality decreased further and the market participants become more prudential, the increase in the rollover risk, or the difficulty of refinancing short-term debt, dominated the increase in the marginal benefit from the indirect revealing mechanism. Thus, the high-type firms took out less short-term debt in order to avoid the risk of failing to roll over debt. When the market became very pessimistic, short-term borrowing froze.

Short-term borrowing is an inefficient way to finance investment opportunities because of the rollover risk. Under our framework, this inefficiency arises from the indirect revealing mechanism and, ultimately, from the information friction in the financial market. Our analysis of government policies shows that restricting maturity choices or bailing-out all projects that are liquidated early could restore efficiency.

Related Literature

Short-term debt induces rollover risk by its maturity mismatch. Consequently, the debt maturity literature usually justifies the use of short-term debt by identifying its advantages over long-term debt. Most papers in this field follow the approach favored by the traditional capital structure literature, in the sense that they compare long-term and short-term debts with respect to the same issues that are relevant to a comparison of equity and debt.

For example, Brick and Ravid (1985) discuss the tax-shielding role of short-term and long-term debt, while Brick and Ravid (1991) extend the analysis to an environment with uncertain interest rates. Huberman and Repullo (2014) and Cheng and Milbradt (2012) focus on the role of short-term debt in alleviating the risk-shifting problem raised by Jensen and Meckling (1976). Diamond and He (2014) investigate how debt maturity affects the debt overhang problem first presented by Myers (1977). Finally, Leland and Toft (1996); Leland (1998) take both tax and the agency problem into consideration.

There is also banking literature that considers demand deposits to be the short-term debt issued by banks. Diamond and Dybvig (1983) argues that demand deposits are effective in satisfying consumers' liquidity needs, although they may induce bank runs. Calomiris and Kahn (1991) argue that demand deposits give depositors the right to withdraw their money if they detect misbehavior on the part of the bank, and thus can be seen as a disciplinary device.

By contrast to the literature emphasizing tax advantages and incentive problems, this paper analyzes the debt maturity problem from the perspective of asymmetric information and is closely related to the signaling hypothesis literature. Flannery (1986) pointed out that short-term debt provides a useful tool with which firms can reveal that they are of good type and thus lower the financial cost of future borrowing. So, if there is no cost incurred when issuing short-term debt, all firms will choose short-term debt in equilibrium. There is no room for discussing how the maturity structure changes in Flannerys model.

Diamond (1991) introduces the cost of short-term debt into Flannery's framework, namely, the liquidity risk induced by early liquidation. Thus, there is trade-off when using short-term debt in their model. They adopt a different definition of liquidity risk from the one used in this paper, namely that liquidity risk is the loss of control rights of the firm. This is independent of the firms credit rating (ex-ante proportion of good firms in our paper). Diamond predicts that firms with the best and worst credit ratings will utilize short-term debt, while firms with intermediate credit ratings will use long-term debt. This conclusion totally contradicts our results, and is not compatible with the facts.

In most theoretical papers, firms are only permitted to choose either long-term or short-term debt, and so the frameworks developed cannot be used to predict the effects of continuous changes in firms' debt maturity structures (e.g. Brick and Ravid (1985); Flannery (1986); Huberman and Repullo (2014)). To avoid this problem, we apply the framework of Brunnermeier and Oehmke (2013), which allows firms to use a mixture of short-term and long-term debt in any proportions. The key difference between their framework and ours is that they assume that the debt maturity structure is unobservable: the firm cannot commit to a maturity structure and thus there is a coordination problem among creditors. The negative externalities exerted by short-term debt holders on long-term debt holders will lead to a rat race of debt maturity. Their model predicts that whenever interim information is mostly about the probability of default, rather than recovery from default, there is a unique equilibrium in which all financing is short-term. This result provides a general explanation of the use of short-term debt, but provides no insight into the ways in which the maturity structure reacts to the environment.

In work that is similar to this paper, Eisenbach (2010) applies Brunnermeier and Oehmke's framework and also allows the maturity structure to be observable. He endogenizes the liquidation value so that it bears no relation to the value of any individual asset. Thus, in his model, the liquidation value can exceed the true asset value and liquidation need not be inefficient.

The rest of the paper is structured as follows. Section 2 presents the benchmark model without information asymmetry. Section 3 discusses the full model with asymmetric information. We characterize the conditions for both separating and pooling equilibria. In section 4, we solve the optimal equilibrium maturity structure for good firms, and discuss how this maturity structure changes with the ex-ante proportion of good firms. In section 5, some assumptions on our model are discussed, and we demonstrate the robustness of our results. Section 6 shows the policy implications of our model on government interventions. Section 7 concludes.

2 The Benchmark Model

2.1 Model setup

There are three dates $t = 0, 1, 2$. At date 0, a (financial) firm has access to a long-term asset which requires an initial cost normalized to 1. The asset pays no dividend, and it only yields payoff when it matures at date 2. The asset payoff is $\theta > 0$ with probability p $(0 < p < 1)$, and 0 with probability $1 - p$. The possible high outcome θ is unknown to the economy at date 0, and has ex ante distribution $\theta \in [0, \bar{\theta}] \sim F(\cdot)$ .We assume $F(\theta) = \theta/\bar{\theta}$ for the ease of calculation. And we require the asset has positive net present value, thus $p\mathbb{E}(\theta_H) > 1$ $(i.e., p\bar{\theta} > 2)$.

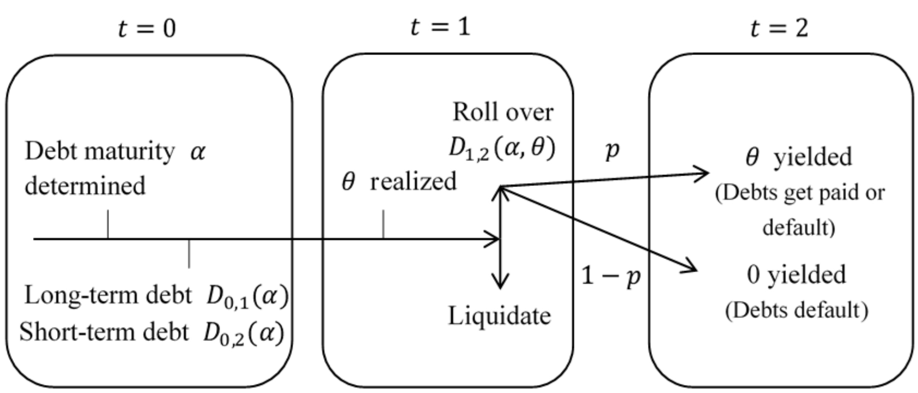


Figure 2: Time Line

We assume the firm has no initial capital, so the whole cost has to be financed. We allow the firm to finance the asset only in the form of debt, which means we don't consider the optimality problem of the debt or feasibility of other securities. There are a continuum of homogeneous creditors (with total measure 1). Each creditor is endowed with 1 unit of capital, so the firm has to borrow from all the creditors. At date 0, the firm can offer a short-term or long-term debt contract to borrow the the unit of capital from each creditor. Due to creditors' symmetry, all the short-term (and long-term) contracts are the same (i.e., have the same maturity and face value). Short-term debt matures at date 1 with face value $D_{0,1}$; long-term debt matures at date 2 with face value $D_{0,2}$. Denote the proportion of short-term creditors as α ($\alpha \in [0, 1]$), then we call this proportion the *debt maturity structure* of the firm. Although the firm offers debt contract to each creditor simultaneously, we allow the creditors to observe the whole maturity structure, which may be because the debt structure is also on the debt contract, or the firm promises informally to creditors about its own debt structure. This implicit assumption is also in Diamond (1991) and Eisenbach (2010), which allows us not to consider the coordination problem in Brunnermeier and Oehmke (2013). The main friction in our benchmark model is not the coordination problem among creditors, but the early liquidation risk of the firm.

At the beginning of date 1, the possible high payoff θ realizes. At this date, the expected payoff of the asset is $p\theta$, so the realization θ is actually a signal about the asset quality. The higher is θ, the higher asset payoff creditors perceive. After this realization, the short-term debt matures, and the firm has to find new short-term creditors (and we call them roll-over creditors for the rest of this paper) for financing. Based on this signal, roll-over debt creditors can decide whether or not to lend capital to the firm. If they decide to lend, the asset goes on and the firm signs a new contract from date 1 to date 2 at the end of date 1, with face value $D_{1,2}(\theta)$ with roll-over creditors; if they don't, the asset has to be early liquidated. We assume that the liquidation value is 0 for simplicity. In section 5.2, we will discuss the case that the liquidation value is a fraction of the asset expected value, and it will be found that our results will not be changed in that case.

At date 2, the asset pays off. And if 0 is yielded or the asset value θ can not cover all the debt face values, the firm goes default. As in Brunnermeier and Oehmke (2013), we assume that long-term creditors and short-term creditors have equal weights on the remaining value of the asset if it defaults, so they equally split the value of the firm by face values if the firm defaults at date 2. Actually, our results will not be changed if we assume that short-term creditors have seniority (Eisenbach (2010)).

2.2 The firm's problem

To specify the firm's problem, we have to determine face values of short-term and long-term debts given any certain debt maturity structure α . And by backward induction, we need to solve the face value of debt issued at date 1, i.e., $D_{1,2}(\theta)$, for the first. At date 1, a roll-over creditor is willing to lend her capital only if she can break even. Given the face values of the debts issued at time 0 ($D_{0,1}$ and $D_{0,2}$), the debt maturity structure α and the realization of high payoff θ, there could be two possible cases:

1. If $\alpha D_{1,2}(\theta) + (1-\alpha)D_{0,2} > \theta$, then at date 2 the firm is certain to default, and the roll-over creditors split the asset payoff with long-term creditors by face values, and thus the roll-over break even condition is:

$$D_{0,1} = \frac{D_{1,2}(\theta)}{\alpha D_{1,2}(\theta) + (1-\alpha)D_{0,2}} p\theta \tag{1}$$

The condition for there existing a solution to equation (1) is: $\frac{\alpha D_{0,1}}{p} = \tilde{\theta} < \theta < \hat{\theta} = \frac{\alpha D_{0,1}}{p} + (1-\alpha)D_{0,2}$

2. If $\alpha D_{1,2}(\theta) + (1-\alpha)D_{0,2} \leq \theta$, the all the creditors can get promised face values, and the roll-over break even condition for short-term creditors is:

$$D_{0,1} = p \cdot D_{1,2}(\theta) \tag{2}$$

The condition for there existing a solution to equation (2) is: $\hat{\theta} < \theta < \bar{\theta}$.

Basically, as long as equation (1) or (2) has a solution for $D_{1,2}(\theta)$, the roll-over creditors are willing to lend, so the roll-over condition is

$$\theta > \tilde{\theta} = \frac{\alpha D_{0,1}}{p} \tag{3}$$

The intuition for this condition is that, at date 1, the face value can designed to be arbitraily large so that roll-over creditors get all the asset value at date 2. Thus as long as the expected asset value at date 1 $p\theta$ is no less than the cost of roll-over creditors ($\alpha D_{0,1}$), they are willing to lend. Note that if $\alpha D_{0,1} \geqslant p\bar{\theta}$, the short-term debt will never be rolled over and $\tilde{\theta} = \bar{\theta}$, and if $\alpha D_{0,1} + (1-\alpha)D_{0,2} \geqslant \bar{\theta}$, the firm will certainly default at date 2 and $\hat{\theta} = \bar{\theta}$.

Now we turn to the borrowing conditions at date 0. For short-term creditors, there is only one kind of risk: if the realization θ at date 1 is too low ($\theta < \tilde{\theta}$), the debt will not be rolled over and the asset will be early liquidated and all creditors get nothing paid back. Only if the realization is high enough ($\theta > \tilde{\theta}$), the debt will be rolled over and the short-term debt can get the face value $D_{0,1}$ paid. The break-even condition for short-term debt is:

$$D_{0,1} \int_{\tilde{\theta}}^{\bar{\theta}} f(\theta) d\theta = 1 \tag{4}$$

For long-term creditors, they have to consider two kinds of risks: one is the early liquidation risk at date 1, which is the same with that of short-term creditors; and the other is default risk at date 2, if the realization is large enough for the debt to roll over, but not so large that all creditors get their face values paid at date 2 ($\tilde{\theta} < \theta < \hat{\theta}$), the firm will be certain to default, so long-term creditors have to split the default value with short-term creditors by face value. Only if the realization is high enough ($\theta > \hat{\theta}$), the long-term debt can get the face value $D_{0,2}$ paid. Thus the break-even condition for long-term creditors is:

$$\int_{\tilde{\theta}}^{\hat{\theta}} \frac{D_{0,2}}{\alpha D_{1,2}(\theta) + (1-\alpha) D_{0,2}} p\theta f(\theta) d\theta + D_{0,2} \int_{\hat{\theta}}^{\bar{\theta}} p f(\theta) d\theta = 1 \tag{5}$$

It is possible that for some debt maturity structure, the break-even conditions can not be all satisfied, so it is necessary to define feasible debt maturity structure.

Definition 1. *A debt maturity structure α is feasible, if there exists non-negative face values $D_{0,1}$, $D_{0,2}$ such that short-term and long-term creditors can both break even at date 0.*

Now we can discuss the profit of the firm. For a given feasible maturity structure α, and corresponding face values $\{D_{0,1}(\alpha), D_{0,2}(\alpha), D_{1,2}(\theta, \alpha)\}$, we can write down the expected net payoff for the firm at date 0:

$$\pi(\alpha) = \int_{\hat{\theta}(\alpha)}^{\bar{\theta}} p(\theta - \alpha D_{1,2}(\theta) - (1-\alpha) D_{0,2}) f(\theta) d\theta$$

This expression is quite intuitive: since all creditors break even, the profit of the firm can be seen as the social net benefit of the whole economy from the asset. The social cost is the starting cost of the asset, which is 1; the social benefit is the payoff of the asset, minus liquidation loss, which happens when $\theta < \tilde{\theta}$. By this expression, we can easily get $\frac{\partial \pi}{\partial \tilde{\theta}} < 0$: the more difficult for the firm to roll over, the less profit it earns. Now we know that to solve the optimization problem for the firm, we only have to calculate $\partial\tilde{\theta}/\partial\alpha$. Actually, we can solve equation (4) (see Appendix A) and get:

$$D_{0,1} = \frac{2}{1+\sqrt{1-4\alpha/p\bar{\theta}}}$$

Apparently $\frac{dD_{0,1}}{d\alpha} > 0$ because more short-term debt means higher roll-over risk and short-term creditors is willing to require a higher face value of debt. With this property, we can easily see that the optimal choice of the firm is to use no short-term debt. Before we get to this result, we need to ocnfirm the feasibility of the maturity structures.The following lemma shows the condition that all maturity structures are feasible.

Assumption 1. *$p\bar{\theta} \geqslant 4$.*

Lemma 1. *Under assumption A, at date 0, there exists positive face value $D_{0,1}$ and $D_{0,2}$ such that short-term and long-term creditors can both break even for all maturity structures $\alpha \in [0,1]$.*

Proof. See Appendix A 1 □

Lemma 1 indicates that as long as the project has a reasonable return in expectation, the project could get funding from creditors. Lemma 1 also confirms that the firm can always get non-zero profit no matter what maturity structure is chosen.

Proposition 1. *Without asymmetric information, if assumption A holds, then all the maturity structures $\alpha \in [0,1]$ are feasible, and $\alpha = 0$ is optimal for the firm among the feasible maturity structures.*

Proposition 1 implies that firms will only borrow long-term debt in the benchmark model. The reason is that short-term debt is associated with possible loss of early liquidation, while there is no rollover risk for the long-term debt. Without asymmetric information, there is no room for short-term borrowing in this model.

3 Maturity Structure Equilirium under Asymmetric Information

In this section, we introduce asymmetric information to the benchmark model. There are two types of firms (assets), denoted as Good firms (G) and Bad firms (B). The difference between the two types is the probability of high payoff: p_G for good firms and p_B for bad firm, $0 < p_B < p_G \leqslant 1$. The type is firms' private information and the prior belief of creditors about the two types are: $Prob(G) = \mu$, $Prob(B) = 1 - \mu$, where $0 < \mu < 1$. For the ease of discussing equlibrium maturity structure, we assume that both types of firm satisfy assumption A so that all the maturity structures are feasible. Although there is information asymmetry at date 0, we assume that there will be public information correlated with the firms' types disclosed at the interim date, which can alleviate the asymmetric information problem. There could be two states at date 1: good state ($s = g$) and bad state ($s = b$), and good firms are more possible to reach the good state, which also means conditional on good state, creditors will preceive a better quality of the firm. We assume that

$$P(s = g|G) = 1 \quad P(s = b|B) = q$$

where $q \in [0, 1]$ indicates the precision of the state: the larger is q, the more precise is the information conveyed by the state. For example, $q = 0$ means the interim information is totally useless and $q = 1$ means creditors can tell the types of firms by only observing the interim state. With this expression, we know that at date 1, the good state is reached in probability $P(g) = \mu + (1-\mu)(1-q)$ and bad state $P(b) = 1 - P(g)$. According to Bayesian rule, we can calculate the creditors' belief about the firm's type conditional on the state at date 1:

$$\mu(g) = P(G|s = g) = \frac{\mu}{\mu + (1-\mu)(1-q)}$$

$$\mu(b) = P(G|s = b) = 0$$

apparently $\mu(g) > \mu > \mu(b) = 0$, which means at good (bad) state, the creditors believe the firm is more likely to be good (bad).

At date 0, different types of firms can choose different maturity structures, which mights convey some information to creditors. Since this a dynamic game with incomplete information, we apply PBE as our equilibrium concept, and we only consider pure strategy equilibria of this game. Denote the maturity structures by good and bad firms as α_G and α_B. We could have two possible types of equilibrium: separating ($\alpha_G \neq \alpha_B$) and pooling ($\alpha_G = \alpha_B$). Since in separating equilibirum, the interim information is useless, which conflicts the reality and is not economically interesting, we will focus on pooling equilibrium and discuss equilibrium properties on the parameter area that separating equilirium can not be supported.

3.1 Separating equilibrium

In separating equilibrium, good firms and bad firms choose different maturities of debt$\alpha_G \neq \alpha_B$, so creditors can tell the two types of firms from their choices at date 0, and the belief at date 0 of creditors on equilibrium path is: $Prob(type = G \mid \alpha_G) = 1$, $Prob(type = G \mid \alpha = \alpha_B) = 0$. This belief will not change after the state at date1 is realized. Thus creditors act as if there is no asymmetric information, and the profit of the firm can be expressed in the form of benchmark model.

$$\pi_t(\alpha_{t'}) = p_t \int_{\hat{\theta}(\alpha_{t'})}^{\overline{\theta}} \left[\theta - \alpha_{t'} D_{1,2}(\alpha_{t'}, \theta) - (1 - \alpha_{t'}) D_{0,2}(\alpha_{t'})\right] f(\theta) d\theta$$

where t and t' indicate the firm's types $(t, t' = G, B)$ and $\pi_t(\alpha_{t'})$ denotes the profit of the firm with type t acting as a type t' firm. The face values$D_{0,1}(\alpha_{t'})$, $D_{1,2}(\alpha_{t'}, \theta)$ and $D_{0,2}(\alpha_{t'})$ can be calculated exactly as the benchmark model. Since the only difference between the two types is probability of high payoff, we can express the profit function in another way:

$$\pi_t(\alpha_{t'}) = p_t \left(\int_{\tilde{\theta}(\alpha_{t'})}^{\overline{\theta}} f(\theta) d\theta - \frac{1}{p_{t'}} \right)$$

In equilibrium, firms can not earn positive gains from deviating their equilibrium strategies, so we must have $\pi_G(\alpha_G) \geqslant \pi_G(\alpha_B)$ and $\pi_B(\alpha_B) \geqslant \pi_B(\alpha_G)$, these two inequalities imply that:

$$\int_{\tilde{\theta}(\alpha_G)}^{\overline{\theta}} f(\theta) d\theta - \frac{1}{p_G} = \int_{\tilde{\theta}(\alpha_B)}^{\overline{\theta}} f(\theta) d\theta - \frac{1}{p_B} \tag{6}$$

Thus there existing separating equilibrium is equal to there existing some maturity structures α_G and α_B that make the above equation holds. Note that $\frac{1}{p_G} < \frac{1}{p_B}$, so if the equation holds we must have $\tilde{\theta}(\alpha_G) > \tilde{\theta}(\alpha_B)$, which means $\alpha_B < \alpha_G$. In separating equilibrium, good firms have to take more risk to signal themselves out.

Lemma 2. *In any separating equilibrium, $\alpha_B < \alpha_G$.*

With this lemma, the left hand side of equation (6) can reach its minimum when $\alpha_G = 1$ and the right hand side of the equation can reach its maximum when $\alpha_B = 0$, thus to check the existence of separating equilibrium, we only have to compare these two extreme values. Note that in separating equilibrium, equation (6) holds means $\pi_G(\alpha_G) = \pi_G(\alpha_B)$ and $\pi_B(\alpha_B) = \pi_B(\alpha_G)$, i.e., both types of firms are indifferent whatever maturity structure they choose, this kind of equilibrium is somewhat trivial.

Assumption 2. *:* $\frac{1}{p_B} > \frac{1}{p_G} + \frac{1}{\bar{\theta} p_G^2} \frac{2}{\left(1+\sqrt{1-4/p_G\bar{\theta}}\right)^2}$.

Assumption 2 confirms acting as a good firm is best for both good firms and bad firms, so whatever maturity structure the good firms choose, to pick the same maturity structure with good firms is always the bad firms' best choice, i.e., there is no solution $0 \leqslant \alpha_B < \alpha_G \leqslant 1$ to equation (5).

Proposition 2. *1. When Assumption 2 holds, there is no separating equilibrium;*

2. When Assumption 2 is vialated, there is unique separating equilibrium in which $\alpha_G > 0$ satisfying $\pi_G(\alpha_G) = p_G(\frac{\bar{\theta}}{2} - \frac{1}{p_B})$ and $\alpha_B = 0$.

Proof. See Appendix B1. □

From this lemma, we know that when the difference of quality between the two types ($\frac{1}{p_B} > \frac{1}{p_G}$) is large enough, or the upper bound of potential high payoff ($\bar{\theta}$) is large enough, the bad firms will always mimic good firms, and thus there is no separating equilibrium. In other words, if the difference in expected returns is high between good firm and bad firm, bad firms are willing to take the risk of early liquidation to lower the cost of borrowing short-term debt by mimicking the good firm. But if condition B is not satisfied, mimicking is not profitable enough, so separating equilibrium is possible. In separating equilibrium bad firms will only borrow long-term debt and the good firms will have to take some rollover risk in the future in order to signal out itself at time 0. Figure 3 presents the conditions that make separating equilibrium feasible. In the figure, the northwest part is the area of non-separating equilibrium, and separating equilibrium is possible in the area between blue line and the 45 degree line. We also tested numerically for the possible area of $\bar{\theta}$, p_G to construct a separating equilibrium for given p_B. Separating equilibrium is possible only when p_G is very close to p_B and $\bar{\theta}$ is low. The area for generating a separating equilibrium in this model is rather slim.

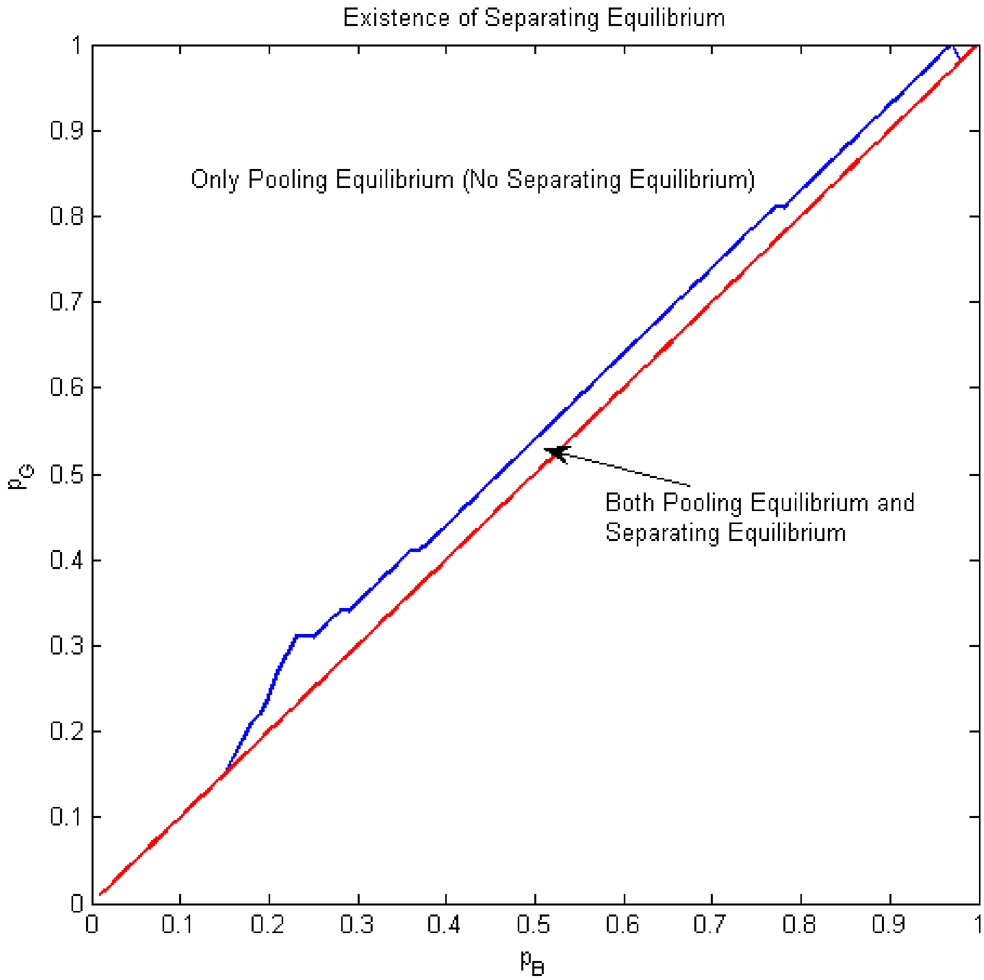


Figure 3: The Existence of Separating Equilibrium ($\lambda = 0.5$ $\bar{\theta} = 10$)

3.2 Pooling equilibrium

Pooling equilibrium is the focus of our for two reasons. First, there is always some ambiguity in the information available. Creditors cannot distinguish two types by observing the maturity structure at date 0 and they have to rely on the interim state of nature to know the quality of asset. And the problem of information only matters in pooling equilibrium. Second, according to the empirical research of Custódio et al. (2013), high quality firms do not experience a signicantly different evolution of debt maturity from low-quality firms, which means they are more likely to choose the same maturity. In the rest part of this paper, we only discuss the paramters that satisfy condition B.

In pooling equilibrium, both types of firms choose the same maturity structure $\alpha_G = \alpha_B = \alpha$, and the belief on the equilibrium path is equal to prior distribution at date 0 is:

$$Prob(type = G \mid \alpha' = \alpha) = \mu \tag{7}$$

At date 1, the belief of creditors will be updated with the realized state. Denote the probability of success in high payoff at state s as $p(s)$, we know from the belief that $p(s) = \mu(s)p_G + (1-\mu(s))p_B$ for $s = g, b$. With this expression, we can characterize the break-even conditions for creditors as in the benchmark model. At date 1, and given the realized state s, there could be two cases:

1. If $\alpha D^s_{1,2}(\theta) + (1-\alpha)D_{0,2} > \theta$, the roll-over break even condition for short-term creditors is:

$$D_{0,1} = \frac{D^s_{1,2}(\theta)}{\alpha D^s_{1,2}(\theta) + (1-\alpha)D_{0,2}} p(s)\theta \tag{8}$$

The condition for there existing a solution $D^s_{1,2}(\theta)$ to this equation is: $\frac{\alpha D_{0,1}}{p(s)} = \tilde{\theta}_s < \theta < \hat{\theta}_s = \frac{\alpha D_{0,1}}{p(s)} + (1-\alpha)D_{0,2}$.

2. If $\alpha D^s_{1,2}(\theta) + (1-\alpha)D^s_{0,2} \leq \theta$, the roll-over break even condition for short-term creditors is:

$$D_{0,1} = p(s) \cdot D^s_{1,2}(\theta) \tag{9}$$

The condition for there existing a solution $D^s_{1,2}(\theta)$ to this equation is : $\hat{\theta}_s < \theta < \bar{\theta}$.

The roll-over condition of short-term creditors at state s is that equation (9) or (10) has a positive solution: $\theta > \tilde{\theta}_s$.

At date 0, the break even condition for short-term creditors is:

$$\sum_{s=g,b} P(s) \left[\left(1 - F(\tilde{\theta}_s)\right) D_{0,1} \right] = 1 \tag{10}$$

At date 0, the break even condition for long-term creditors is:

$$\sum_{s=g,b} \left[P(s) \left(\int_{\tilde{\theta}_s}^{\hat{\theta}_s} \frac{D_{0,2}}{\alpha D^s_{1,2}(\theta) + (1-\alpha)D_{0,2}} p(s)\theta f(\theta) d\theta + D_{0,2} \int_{\hat{\theta}_s}^{\bar{\theta}} p(s) f(\theta) d\theta \right) \right] = 1 \tag{11}$$

In a pooling equilibrium with $\alpha_G = \alpha_B = \alpha$, the profit of good firms on equilibrium path is:

$$\pi_G(\alpha;\mu) = \int_{\hat{\theta}_g}^{\bar{\theta}} \left[\theta - \alpha D_{1,2}^g - (1-\alpha)D_{0,2}\right] p_G f(\theta) d\theta \tag{12}$$

Similarly, the profit of bad firms on equilibrium path is:

$$\pi_B(\alpha;\mu) = (1-q)\int_{\hat{\theta}_g}^{\bar{\theta}} \left[\theta - \alpha D_{1,2}^g - (1-\alpha)D_{0,2}\right] p_B f(\theta) d\theta + q\int_{\hat{\theta}_b}^{\bar{\theta}} \left[\theta - \alpha D_{1,2}^b - (1-\alpha)D_{0,2}\right] p_B f(\theta) d\theta \tag{13}$$

We introduce μ is the profit function to emphasize the importance of the prior asset quality.

For $\alpha_G = \alpha_B = \alpha > 0$ to be a pooling equilibrium, it requires both types of firms not to deviate from this choice. On off-equilibrium path, the belief that $Prob(type = G \mid \alpha' \neq \alpha) = 0$ can make the no-deviation condition most likely to hold, so we discuss the pooling equilibrium given this belief. Under this belief, the best choice of a firm if deviating from equilibrium strategy is choosing $\alpha = 0$, so the highest profit on off equilibrium path of a type t firm is:

$$p_t\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$$

It can be observed that if $\pi_B(\alpha) \geqslant p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$ holds, then $\pi_G(\alpha) \geq \frac{p_G}{p_B}\pi_B(\alpha) \geqslant p_G\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$ holds, so we only have to consider the no-deviation condition for the bad firms. It is easy to see that $\alpha_G = \alpha_B = 0$ is always a pooling equilibrium, because both types of firms get the highest profits given the belief of creditors. For $\alpha_G = \alpha_B = \alpha > 0$, we can show that the smaller α is, the more possible it can construct a pooling equilibrium.

Lemma 3. *Given creditors' belief on equilibrium path at date 0 described by equation (7), the profit of bad firms on equilibrium path strictly decreases with proportion of short-term debt and increaqses with the proportion of good firms:* $\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha} < 0$ *and* $\frac{\partial \pi_B(\alpha;\mu)}{\partial \mu} > 0$

Proof. See Appendix B2. □

Now we can find which maturity structures can construct a pooling equilibrium by only comparing $\pi_B(\alpha;\mu)$ and $p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$. According to lemma 4, when μ is very low, α also has to be very low to construct a pooling equilibrium. The following condition can confirm that $\alpha_B = \alpha_G = 1$ could always be possible to construct a pooling equilibrium when μ is large enough.

Condition C:

$$1+\sqrt{1-\frac{4}{\bar{\theta}}\frac{1}{p_G}} \geq \left(\frac{1-q}{p_G}+\frac{q}{p_B}\right)p_B+\sqrt{\left(\frac{1-q}{p_G}+\frac{q}{p_B}\right)^2 p_B^2-2\left(\frac{1-q}{p_G^2}+\frac{q}{p_B^2}\right)\frac{p_B}{\bar{\theta}}}.$$

If Condition C is violated, $\alpha_B = \alpha_G = 1$ can not construct a pooling equilibrium even μ approaches 1. With this condition, we can characterize the pooling equilibrium set.

Proposition 3 (**Pooling Equilibrium set**). *Denote $\alpha_e(\mu)$ as the upper bound of the maturity structure that can construct a pooling equilibrium.*

1. *If condition C is violated, then for any $0 < \mu < 1$, $\alpha_e(\mu) \in (0,1)$ satisfies $\pi_B(\alpha_e(\mu);\mu) = p_B\left(\frac{\bar{\theta}}{2}-\frac{1}{p_B}\right)$ and $\alpha_e'(\mu) > 0$;*
2. *If condition C is satisfied, then there exists $\mu_1 \in (0,1)$ and*
   (a) *when $0 < \mu < \mu_1$, $\alpha_e(\mu) \in (0,1)$ satisfies $\pi_B(\alpha_e(\mu);\mu) = p_B\left(\frac{\bar{\theta}}{2}-\frac{1}{p_B}\right)$ and $\alpha_e'(\mu) > 0$;*
   (b) *when $\mu_1 \leqslant \mu < 1$, $\alpha_e(\mu) = 1$.*

Proof. See Appendix B3. □

Proposition 3 gives an upbound $\alpha_e(\mu)$ for each μ to construct a pooling equilibrium and shows this upbound is (weakly) increasing in μ. It is because as the proportion of good firms increases, the average quality of firms' assets perceived by creditors also increases, creditors will require a lower face value to firms at date 0,the profit of bad firms will increase and their incentive to pool with good firms is higher. Figure 4 plots the upper bounds for different parameters which satisfy or violate condition C.

4 The Optimal Maturity Structure in Pooling Equilibrium

An important issue is put up by proposition 3: the pooling equilibrium is not unique for any proportion of good firms $\mu \in (0,1)$, so we are involved in equilibrium selection problem. As in Flannery (1986) and Diamond (1991), we focus on the equilibrium the good firms like most among all the pooling equilibrium. More formally, as justified in Gorton and Metrick (2012), if the equilibrium

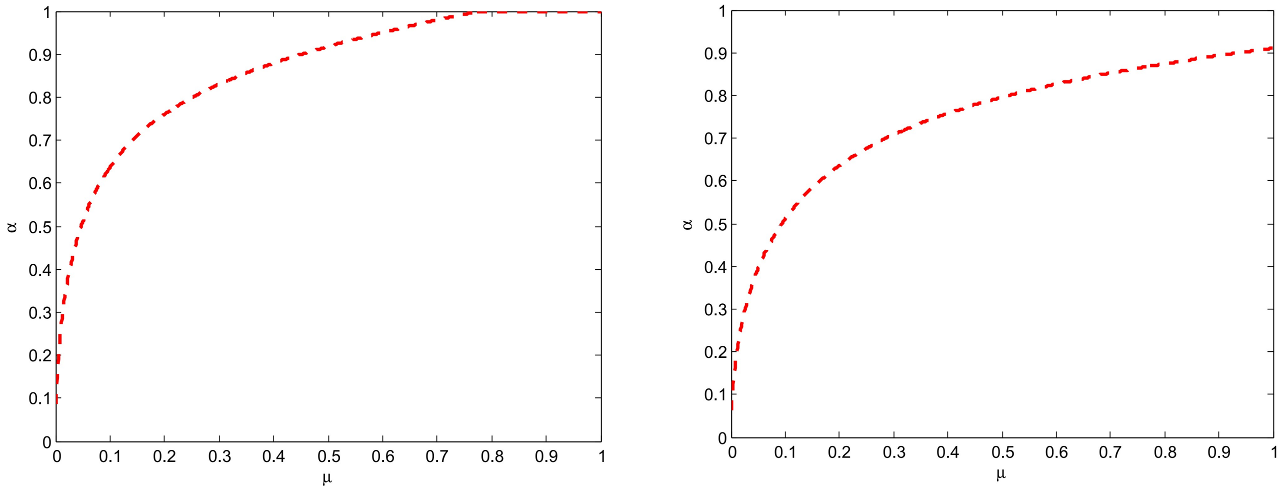


Figure 4: Pooling Equilibrium Set ($p_G = 0.9$, $p_B = 0.5; 0.7$, $\bar{\theta} = 10$, $q = 0.9$)

which is not optimal for good firms is realized, then deviating to the maturity structure optimal for good firms could (slightly) increase the belief that the firm is good. Considering this, both types of firms will have incentives to choose the good firms' favorite maturity structure. Applying this refinement, we will show that the equilibrium is unique. To make the discussion more clear, we don't directly derive the optimal equilibrium maturity structure among all pooling equilibria. Instead, we find the optimal choice for the good firm by ignoring the pooling equilibrium constraint first, and then combine the optimal choice with equilibrium set constraint.

Taking first derivative of the good firms' profit with respect to the proportion of short-term debt α, we can show (see Appendix B4) that $\frac{\partial \pi_G(\alpha;\mu)}{\partial \alpha}$ has the same sign with:

$$\Xi(\alpha;\mu) \triangleq (1-P(g))\, p_B \int_{\hat{\theta}_b}^{\overline{\theta}} f(\theta)d\theta \left(\frac{1}{p_B} - \frac{1}{p(g)}\right) \frac{\partial\left(\alpha D_{0,1}\right)}{\partial \alpha} - \left[P(g)p(g)\tilde{\theta}_g f(\tilde{\theta}_g)\frac{\partial \tilde{\theta}_g}{\partial \alpha} + (1-P(g))\tilde{\theta}_b f(\tilde{\theta}_b)\frac{\partial \tilde{\theta}_b}{\partial \alpha}\right]$$

This expression is quite intuitive: the first term represents the benefit of the short-term debt. At date 1, roll-over creditors perceive firms' quality as $p(g)$ in good state and p_B in bad state, and the face values of roll-over debt (when $\theta > \hat{\theta}_s$) are $\frac{\alpha D_{0,1}}{p(g)}$ and $\frac{\alpha D_{0,1}}{p_B}$. Their difference is the value that interim information brings good firms. Note that the interim information for good firms is always good, so the benefit comes from the cost of debt saved in bad state, which is expected to happen in probability $(1-P(g))\, p_B \int_{\hat{\theta}_b}^{\overline{\theta}} f(\theta)d\theta$ from date 0; the second term represents the cost of short-term debt, which is the expected liquidation loss perceived by investors at date 0. It is easy to see from this expression that $\frac{\partial \pi_G(0;\mu)}{\partial \alpha} > 0$, because when there is no early liquidation risk when there is no short-term debt issued by the firm. This result is shown in the following lemma.

Lemma 4. *The good firm will always use a positive fration of short-term debt:* $\frac{\partial \pi_G(0;\mu)}{\partial \alpha} > 0$.

Proof. See Appendix B4. □

Comparing this lemma with proposition 1, it is clear that asymmetric information is the key element leading to short-term debt use. Although creditors can not tell the types of firms at date 0 in pooling equilibria, they can indeed update their beliefs according to interim information contained in different states. Good firms always go to good state. They can be perceived as better at date 1 than date 0, which helps the them to reduce their cost of debt at date 1. Thus short-term debt plays an informational role for good firms at date 1. We call this effect as "indirect revealing mechnisam" (compared to the direct revealing mechnism at date 0 in separating equilibrium). The "indirect revealing mechnisam" and the roll-over risk of short-term debt form a trade-off faced by good firms. To further explore which the proportion of short-term debt will good firms choose under asymmetric information, we define the following function, which also has the same sign with $\frac{\partial \pi_G(\alpha;\mu)}{\partial \alpha}$.

$$\Lambda(\alpha;\mu) \triangleq \frac{p(g) - p_B}{p(g)} q(1-\mu) - \left[P(g)\tilde{\theta}_g f(\tilde{\theta}_g) + (1 - P(g))\tilde{\theta}_b f(\tilde{\theta}_b) \right] \frac{1}{\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta) d\theta}$$

and this function has the following properties.

Lemma 5. $\frac{\partial \Lambda(\alpha;\mu)}{\partial \alpha} < 0$ *and* $\frac{\partial^2 \Lambda(\alpha;\mu)}{\partial \mu^2} < 0$.

Proof. See Appendix B5. □

With this lemma, we know that if $\Lambda(\alpha;\mu) \geq 0$, good firms will choose only short-term debt for financing; and if $\Lambda(\alpha;\mu) < 0$, we get an interior solution. More importantly, this lemma shows how the prior proportion of good firms μ affects their debt maturity choice. It is intuitive that the cost of short-term debt is decreasing in μ because more good firms results in a higher average quality of firms' assets perceived by short-term creditors at date 0 and thus lower debt face value required by them, which means lower roll-over risk and expected liquidation loss. However, the relationship between benefit of short-term debt and μ is ambiguous: on one hand, $p(g)$ increases with μ and thus the cost of debt at good state is smaller, which enhances the "indirect revealing mechnisam"; on the other hand, as the proportion of μ increases, the average quality of firms' asset also increases, so the value that good firms being recogized as good at the interim date decreases, which weakens the "indirect revealing mechnisam". Particularly, as μ tends to 1, all firms are good, and "indirect revealing mechnisam" makes no sense in this case. Although which effect domintes is not for certain, lemma 6 shows that the overall effect tends to be more negative as μ increases. With this property, we can derive the optimal maturity structure and its monotonicity for good firms.

Proposition 4 (**Good firms' optimal choice**). *There are two possible cases of good firms' optimal debt maturity structure in pooling equilibrium:*

1. *If* $\underset{\mu}{Max}\Lambda(1;\mu) \geq 0$, *there exists* $0 < \mu_2 \leq \mu_3 < 1$, *the optimal maturity structure for good firms* $\alpha_o(\mu) = 1$ *when* $\mu \in [\mu_3, \mu_4]$ *and* $\alpha_o(\mu) \in (0,1)$ *satisfying* $\Lambda(\alpha_o(\mu);\mu) = 0$ *otherwise. Further,* $\alpha_o'(\mu) > 0$ *for* $\mu \in (0, \mu_2)$ *and* $\alpha_o'(\mu) < 0$ *for* $\mu \in (\mu_3, 1)$*;*
2. *If* $\underset{\mu}{Max}\Lambda(1;\mu) < 0$, *the optimal maturity structure for good firms* $\alpha_o(\mu) \in (0,1)$ *satisfies* $\Lambda(\alpha_o(\mu);\mu) = 0$ *for all* $\mu \in (0,1)$. *Further, there exists* $\mu_4 \in (0,1)$, $\alpha_o'(\mu) < 0$ *for* $\mu \in (0, \mu_4)$ *and* $\alpha_o'(\mu) > 0$ *for* $\mu \in (\mu_4, 1)$

Proof. See Appendix B6. □

The optimal maturity structure of good firms $\alpha_o(\mu)$ is hump-shaped in μ. The intuition is: if the proportion of good firms is too high, the quality difference between good assets and averaged assets perceived by investors is too small and good firms earn little even they are recogized as good, so the benefit from short-term debt is little; and if the proportion of good firms is too low, the difference between good state and bad state is too small and there is also little benefit for good firms to use short-term debt. Figure 5 shows this relationship between $\alpha_o(\mu)$ and μ.

So far, we have found all pooling equiliria and derived the optimal maturity structure for good firms without considering whether this optimal one can construct a equilibrium. Actually, we can easily combine these two results to find the good firms' optimal maturity structure among all structures that can construct a pooling equlibrium. . Denote the optimal (pooling) equilibrium debt maturity structure for the good firm as $\alpha^*(\mu)$. If $\alpha_o(\mu) < \alpha_e(\mu)$, then $\alpha_o(\mu)$ could construct a pooling equilibrium and thus feasible for good firms, so $\alpha^*(\mu) = \alpha_o(\mu)$; if $\alpha_o(\mu) > \alpha_e(\mu)$, then $\alpha_o(\mu)$ is no longer feasible. According to the proof proposition 4, we know that given any μ, $\pi_G'(\alpha) > 0$

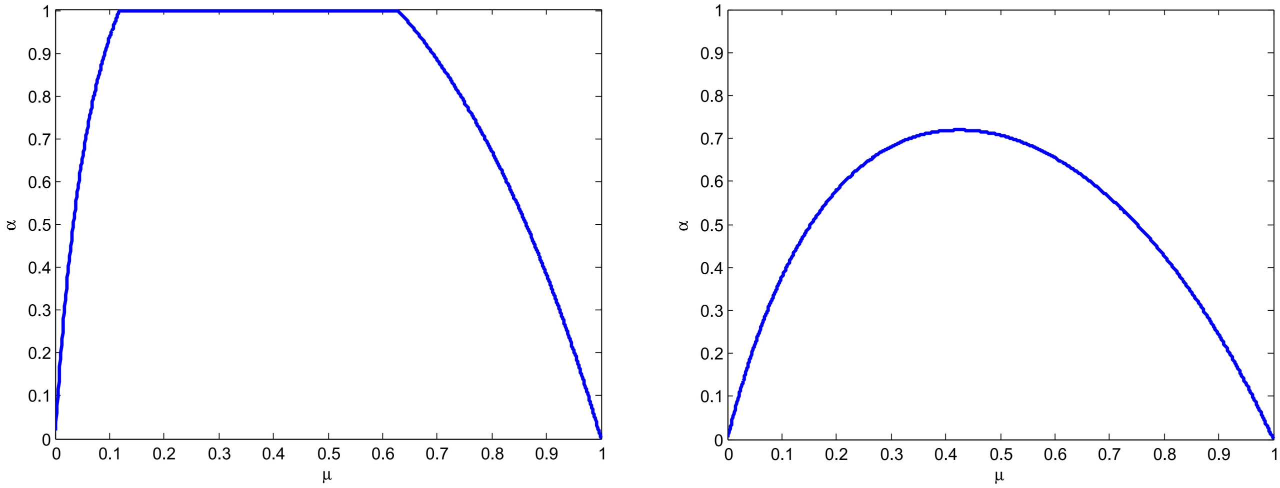


Figure 5: Good Firms' Optimal Choice ($p_G = 0.9$, $p_B = 0.5$, $\bar{\theta} = 15$, $q = 0.8; 0.5$)

for any $\alpha < \alpha_o(\mu)$, so good firms have to simply choose $\alpha_e(\mu)$ as optimum: $\alpha^*(\mu) = \alpha_e(\mu)$. In conclusion, we have the following proposition.

Proposition 5 (Optimal equilibrium maturity structure). *The optimal equilibrium structure for good firms is $\alpha^*(\mu) = min\{\alpha_o(\mu), \alpha_e(\mu)\}$. And there eixsts $\mu_5 \in (0, 1)$, such that $\alpha^*(\mu)$ increases with μ for $\mu \in (0, \mu_5)$ and $\alpha^*(\mu)$ decreases with μ for $\mu \in (\mu_5, 1)$.*

Proof. See Appendix B7. □

It is easy to see that $\alpha^*(\mu)$ also has the property that $\pi'_G(\alpha) > 0$ for any $\alpha < \alpha^*(\mu)$, which makes our way of equilibrium selection partly similar to the intuitive criterion initiated by Cho and Kreps (1987): if for some μ, $\alpha_o(\mu)$ is feasible but some maturity structure $\alpha < \alpha_o(\mu)$ is the realized in pooling equilibrium, then the belief $Prob(type = G \mid \alpha' = \alpha_o(\mu)) = 1$ must hold because only good firms have the incentive to increase short-term borrowing.

Figure 6 depicts the shape of $\alpha^*(\mu)$. Apparently, this optimal equilibrium maturity structure is also hump-shaped in the proportion of good firms μ. Thus the result of this model has potential to shed lights on the cause of increasing use of short-term debt before financial crisis and the market freeze after the financial crisis. Before the financial crisis starting from 2007, there are asset pricing bubbles in housing market, and the asset quality has decreased year by year since 2001 (Demyanyk and Van Hemert (2011)). Thus more and more short-term debts are issued. As the asset quality decreases further, market collapse and financial crisis happens, which is the turning point in the graph. After financial crisis happens, the asset quality perceived by creditors has deteriorated so

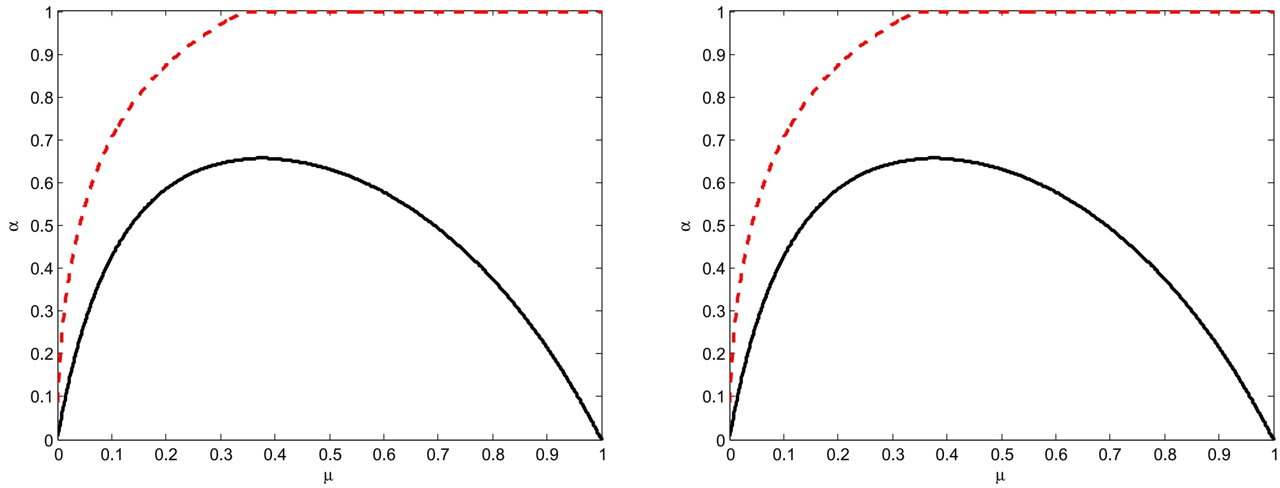


Figure 6: Optimal Equilibrium structure ($p_G = 0.9$, $p_B = 0.5$, $\bar{\theta} = 10$, $q = 0.7; 0.9$)

much that short-term debt use are decreased, because either good firms are unwilling to borrow short-term debt or it is strictly constrained by the pooling equilibrium condition, which is exactly the market freeze.

5 Robustnesses and Discussions

5.1 Information Accuracy

In our model, the key element is the informational role of short-term debt, which roots in the accuracy of the interim information. As long as $q > 0$, good firms are more likely to reach the good state and different states are informative. Intuitively, the more accurate the interim information is, the more good firms benefit from short-term debt, and thus they are more willing to shorten their debt maturity. However, roll-over risk is also increasing in q because more accurate interim information means bad firms are less likely to get the good state. Creditors expect that good state is less likely to be reached at date 1, so they will raise the face value of short-term debt, and thus the cost of using short-term debt. The two driving forces lead to opposite effects on optimal equilibrium maturity structures. Figure 7 plots how α^* is affected by the information accuracy q. It can be observed that in most cases, the benefit increase effect dominates and short-term debt use is increasing in interim information accuarcy.

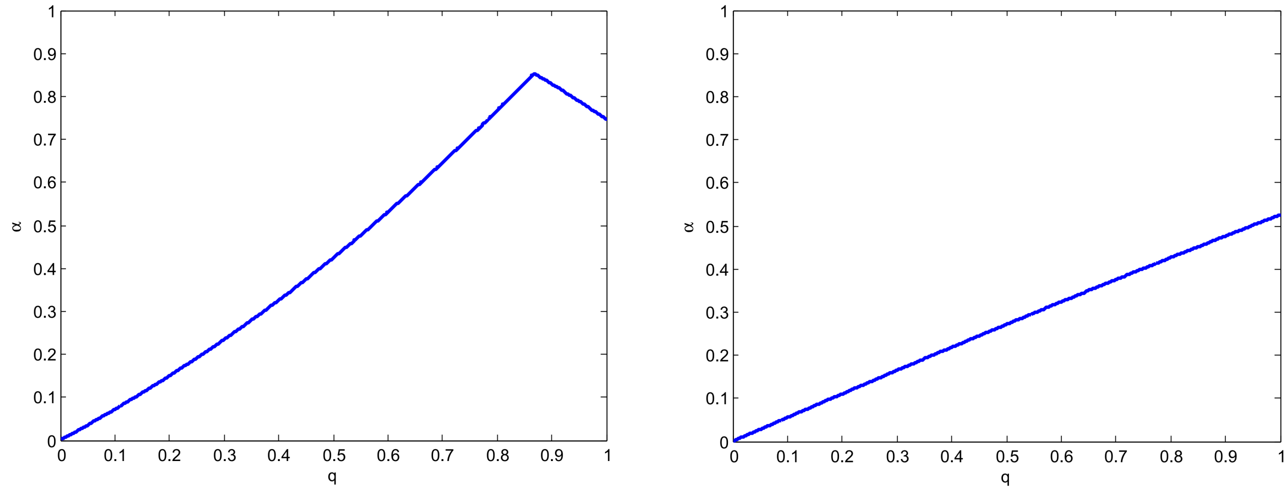


Figure 7: Information Accuracy and Maturity Structure ($p_G = 0.9$, $p_B = 0.5$, $\bar{\theta} = 10$, $\mu = 0.3; 0.8$)

5.2 Liquidation cost

We assume in our model that the liquidation value at date 1 is 0. Actually, our result is not affected by this assumption. As in Brunnermeier and Oehmke (2013) and Huberman and Repullo (2014), the liquidation value can be a fraction of the conditional expected payoff of the asset $\lambda \mathbb{E}(\theta|S_1)$, where $\lambda \in [0, 1)$ indicates the recovery rate: the lower is λ, the larger is the liquidation cost. With this expression, the break even conditions at date 0 for short-term and long-term creditors are changed.:

$$\sum_{s=g,b} P(s) \left[\lambda \int_0^{\tilde{\theta}_s} p(s)\theta f(\theta) d\theta + \left(1 - F(\tilde{\theta}_s)\right) D_{0,1} \right] = 1$$

$$\sum_{s=g,b} \left[P(s) \left(\lambda \int_0^{\tilde{\theta}_s} p(s)\theta f(\theta) d\theta + \int_{\tilde{\theta}_s}^{\hat{\theta}_s} \frac{D_{0,2}}{\alpha D_{1,2}^s(\theta) + (1-\alpha) D_{0,2}} p(s)\theta f(\theta) d\theta + D_{0,2} \int_{\hat{\theta}_s}^{\bar{\theta}} p(s) f(\theta) d\theta \right) \right] = 1$$

All other conditions and objective functions of firms remain the same. By the same process in section 4, we can derive the optimal equilibrium maturity structure chosen by good firms. Figure 8 plots the relationship between this maturity structure and the proportion of good firms when liquidation value is 30% of expected asset value, which is similar with Figure 6. We can also see how the maturity structure changes with recovery rate. It is quite intuitive that the higher is the liquidation value, the lower is the cost of short-term debt, and thus the more short-term debt is used. This result is shown in Figure 9.

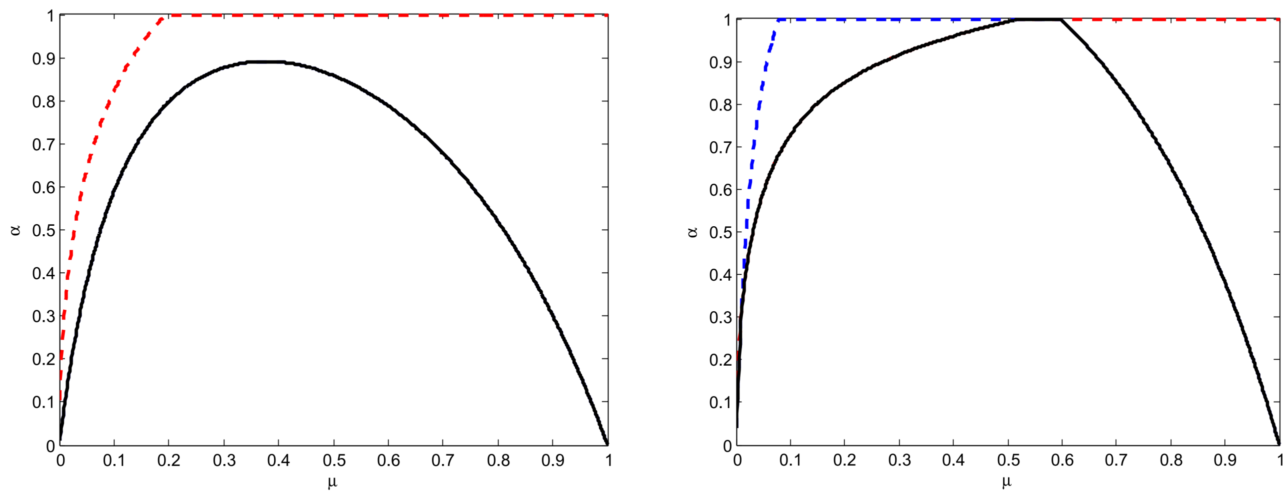


Figure 8: Optimal Equilibrium structure ($p_G = 0.9$, $p_B = 0.5$, $\lambda = 0.3$, $\bar{\theta} = 10$, $q = 0.7; 0.9$)

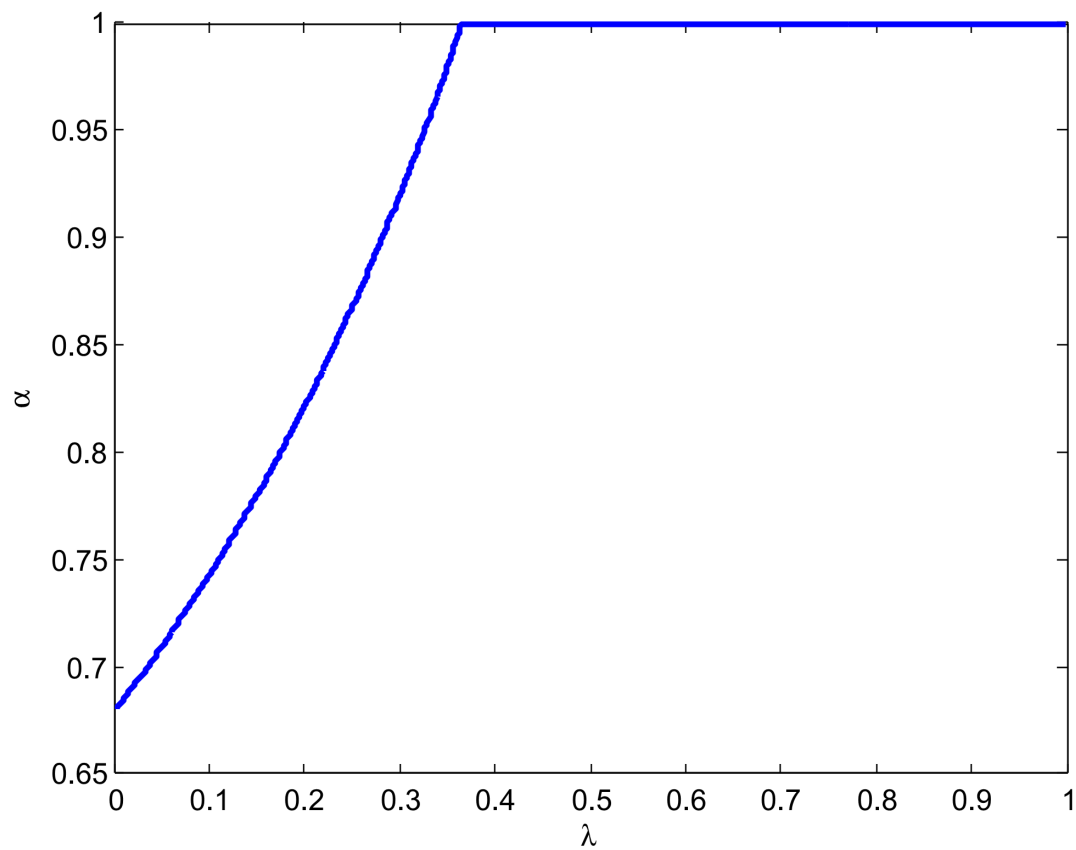
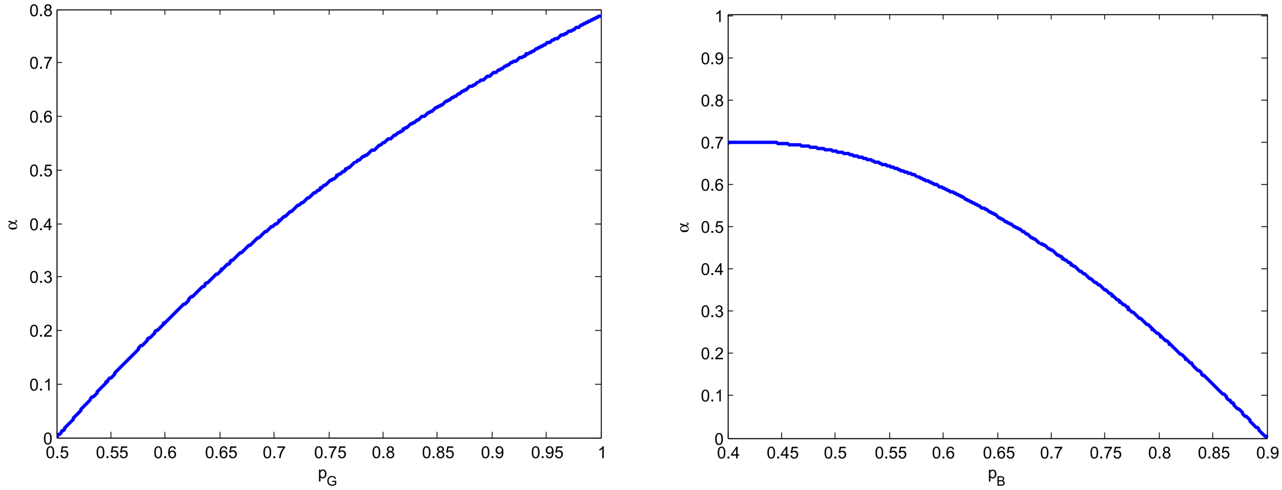


Figure 9: Recovery Rate and Maturity Structure ($p_G = 0.9$, $p_B = 0.5$, $\bar{\theta} = 10$, $q = 0.75$, $\mu = 0.5$)



(a) a

(b) b

Figure 10: Asset Quality and Maturity structure ($\bar{\theta} = 10$, $q = 0.75$, $\mu = 0.5$)

5.3 Asset Quality

Drop in the quality of asset could be a potential reason for financial crisis. Could it be a factor that causes dramatic increase of short-term? Our model is able to reject this hypothesis. Figure 10 shows that, if the good asset has lower probability to get a high return (p_G decreases from 1 to 0.5), the proportion of short-term debt will decline as well. The reason lies in the fact that, if the quality of good project is getting worse, short-term borrowing is more costly as higher rollover risk. More than that, short-term debt is not as attractive as before because, even if the good firm can partially reveal its type by using short-term debt, the expected marginal decrease in cost of borrowing is lower. So, the incentive to borrow short term is weaker in this case. We also show that as the bad asset becomes better (p_B increases from 0.4 to $p_G = 0.9$), the proportion of short-term debt decreases. The reason is similar: that the difference between good firms and bad firms declines makes the "indirect revealing meachnism" less profitable and thus good firms are less willing to use short-term debt.

5.4 Noisy interim information

So far we have assumed that good firms always go to good state at date 1, so the interim information is partially revealing because only bad firms will go to the bad state. In fact, our result can be easily extended to the case that the interim information is noisy at both states. Similar with Huberman and Repullo (2014), we can assume that $P(s=g|G)=P(s=b|B)=m$, where $m\in[\frac{1}{2},1]$ indicates the precision of the state: the larger is m, the more precise is the information conveyed by the state. Here we require $m\geqslant\frac{1}{2}$ because we need to maintain the property that good firms get to the good state with a higher probability. With this expression, we know that at date 1, the good state is reached in probability $P(g)=\mu m+(1-\mu)(1-m)$ and bad state $P(b)=1-P(g)$. According to Bayesian rule, we can calculate the creditors' belief about the firm's type conditional on the state at date 1:

$$\mu(g)=P(G|s=g)=\frac{\mu m}{\mu m+(1-\mu)(1-m)}\quad \mu(b)=P(G|s=b)=\frac{\mu(1-m)}{\mu(1-m)+(1-\mu)m}$$

apparently $\mu(g)>\mu>\mu(b)$, which means at good (bad) state, the creditors believe the firm is more likely to be good (bad). Break-even conditions are also changed in accordance with these beliefs. And the profits of firms are :

$$\pi_G(\alpha)=m\int_{\hat{\theta}_g}^{\overline{\theta}}\left[\theta-\alpha D_{1,2}^g-(1-\alpha)D_{0,2}\right]p_G f(\theta)d\theta+(1-m)\int_{\hat{\theta}_b}^{\overline{\theta}}\left[\theta-\alpha D_{1,2}^b-(1-\alpha)D_{0,2}\right]p_G f(\theta)d\theta$$

$$\pi_B(\alpha)=(1-m)\int_{\hat{\theta}_g}^{\overline{\theta}}\left[\theta-\alpha D_{1,2}^g-(1-\alpha)D_{0,2}\right]p_B f(\theta)d\theta+m\int_{\hat{\theta}_b}^{\overline{\theta}}\left[\theta-\alpha D_{1,2}^b-(1-\alpha)D_{0,2}\right]p_B f(\theta)d\theta$$

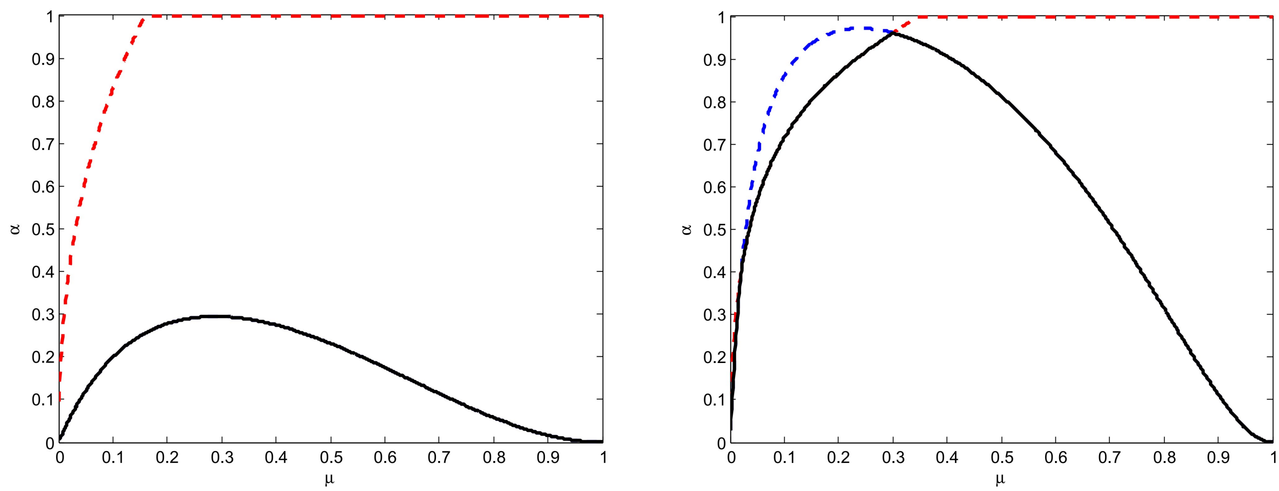
The bad firms' profit function is similar with expression (13) while the good firms' profit function is different from that in (12) because good firms are possible to reach the bad state in this case.

Although interim information is noisy, all intuitions remain in this case. We plot the relationship between optimal equilibrium maturity structure and the prior proportion of good firms in Figure 9, which is still hump-shaped.

6 Efficiency and Government policy

6.1 Efficiency

As in the benchmark model, the only source of inefficiency in the model is coming from the early liquidation. Early liquidation will force the on-going positive NPV project to be stopped. If there is short-term borrowing, the possibility of early liquidation is positive at $t=0$. Thus, the social planner's optimal solution, no matter with or without information asymmtry, is $\alpha=0$. In other



(a) a (b) b

Figure 11: Optimal Equilibrium Maturity Structure with Noisy Interim Information ($p_G = 0.9$, $p_B = 0.5$, $\bar{\theta} = 15$, $m = 0.7; 0.9$)

words, borrowing long-term debt is the social optimal, which is different from the decentralized optimal pooling equilibrum we constructed. The next section will discuss some potential government policy to improve efficiency.

6.2 Government policy

6.2.1 Debt maturity restriction

If government could restrict the borrowing of short-term debt, within this framework, any restriction $\alpha_R < \alpha^\star$ lowered the share of short-term borrowing will restore efficiency. Less short-term borrowing means the less likely the early liquidation would take place. Compared to the optimal pooling equilibrium, the restricted pooling equilibrium α_R will benefit the bad firms and a net transfer from the good firms.

6.2.2 Bailout

Early liquidation is the only source of inefficiency in the model. Government could bailout the otherwise early liquidated projects by helping the borrowers to rollover their short-term borrowing. We assume that government will have priority in the project return once they lend money to roll over short-term borrowings. This policy will restore efficiency because there is no early liquidation

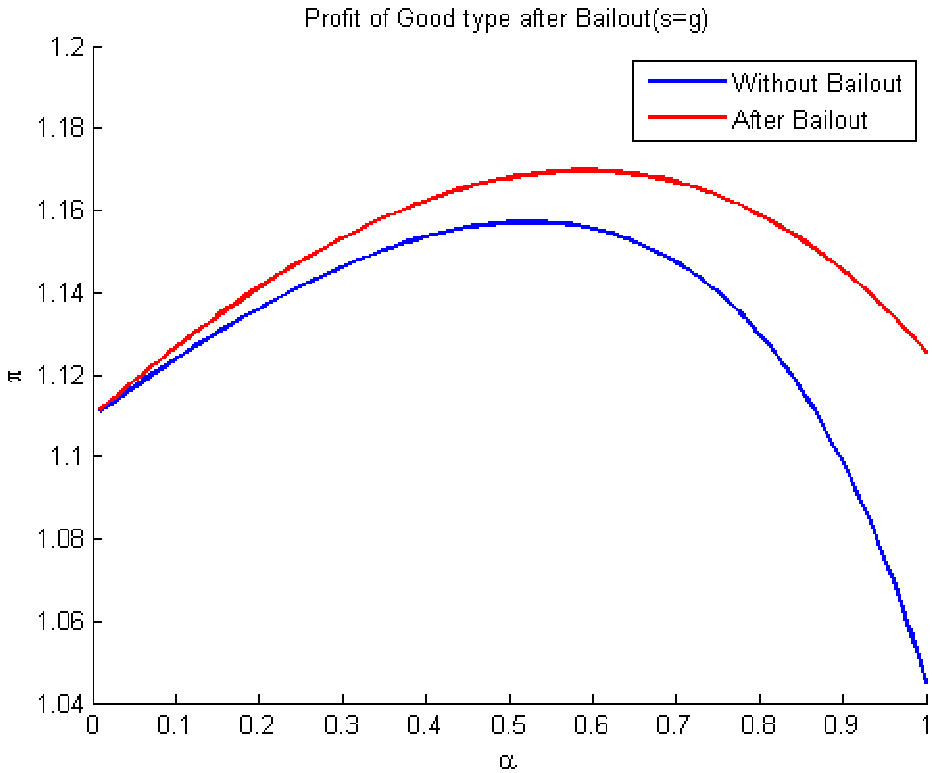


Figure 12: Profit of Good type with (without) bailout in pooling equilibrium

any more. However, the complete bailout policy will provide incentive to good firms to borrow more short-term debt, as good firms could enjoy the benefits from revealing their type without taking rollover risk. The optimal pooling equilibrium in this case is $\alpha^{\star} = 1$. Both good firms and bad firms are better off under this policy but there is a net transfer from government.

Assume bailout is costly for government, another reasonable policy to restore efficiency is to bailout only if the extra information in the interim stage is good, i.e. $s = g$. If $s = g$, it is more likely that this project is a good project with higher NPV. Goverment may just want to save better projects in expectation as implementing this bailout is costly. Under this policy, good firms will borrow more short-term debt and earn higher profit, which can be seen in Figure 7. But, as in the complete bailout case, there is a net loss for government. To summarize, given the pooling equilibrium we constructed, the bailout policy will lower the cost of short-term borrowing and thus increase the share of it. It could restore efficiency but there is a net transfer from government to borrowers.

7 Conclusion

Our paper provides a framework in which we can discuss how debt maturity structure changes as the economic environment. Different from most previous literatures which compare short-term debt and long-term debt directly (Flannery (1986); Huberman and Repullo (2014); Diamond and He (2014)), we allow the financial institution to use a combination of different debts.

Under this framework, we show that as the market confidence, or the prior belief about the share of good firms starts to decrease, the optimal equilibrium maturity structure shortens and get longer as this proportion further decreases. This provides a potential explanation for the increasing short-term debt before financial crisis and market freeze in short-term financing market after the crisis happens.

Appendix

Appendix A: Solutions and Properties of Face Values and Cut-offs

A1. Solutions and properties of face values without asymmetric information .

First we slove $D_{0,1}$. From equation (4) we have:

$$\frac{\alpha}{p\bar{\theta}}D_{0,1}^2 - D_{0,1} + 1 = 0$$

The condition for there existing positive solution $D_{0,1}$ to this equation is $1 \geqslant \frac{4\alpha}{p\bar{\theta}}$. So under assumption $p\bar{\theta} \geqslant 4$ holds, there is positive solution for $D_{0,1}$ for any $\alpha \in [0, 1]$.

We know that there are two solutions for each $\alpha \in [0, 1)$. Since $\frac{\partial \pi}{\partial \tilde{\theta}} < 0$ and $\tilde{\theta} = \frac{\alpha D_{0,1}}{p}$, the firm will certainly choose the smaller face value to make creditors break even, and thus we pick the smaller one as the solution of $D_{0,1}$, so

$$D_{0,1} = \frac{2}{1 + \sqrt{1 - 4\alpha/p\bar{\theta}}}$$

Apparently we have $\frac{dD_{0,1}}{d\alpha} > 0$.

Then we prove that $D_{0,2}$ exists as long $D_{0,1}$ exists. Eliminating $D_{0,1}(\theta)$ by inserting equation (1) and (4) into equation (5) and we get:

$$\int_{\tilde{\theta}}^{\hat{\theta}} p\theta f(\theta)d\theta + \alpha D_{0,1}\int_{\hat{\theta}}^{\overline{\theta}} f(\theta)d\theta + (1-\alpha)D_{0,2}\int_{\hat{\theta}}^{\overline{\theta}} pf(\theta)d\theta - 1 = 0$$

thus solving equation (5) is equal to solving the above equation. Define the left hand side of this equation as $L(D_{0,2}|D_{0,1}, \alpha)$, then $L(0|D_{0,1}, \alpha) = (\alpha - 1)D_{0,1}\int_{\hat{\theta}}^{\overline{\theta}} f(\theta)d\theta \leqslant 0$ and $\frac{\partial L}{\partial D_{0,2}} = (1-\alpha)\int_{\hat{\theta}}^{\overline{\theta}} pf(\theta)d\theta \geqslant 0$, so we only need to confirm that $L(D_{0,2}|D_{0,1}, \alpha)$ is non-negative when $D_{0,2} \to +\infty$. Note that $\hat{\theta} = \overline{\theta}$ for $D_{0,2} \geqslant (\bar{\theta} - \alpha D_{0,1})/(1-\alpha)$, so we have

$$L(D_{0,2} = +\infty|D_{0,1}) = \int_{\tilde{\theta}}^{\overline{\theta}} p\theta f(\theta)d\theta - 1 = \pi(\alpha) = \frac{p\bar{\theta}}{2} - 1 - \frac{\alpha^2 D_{0,1}^2}{2p\bar{\theta}}$$

where $\pi(\alpha) \geqslant 0$ is equal to $\sqrt{2p\bar{\theta}(\frac{p\bar{\theta}}{2} - 1)} \geqslant \alpha D_{0,1}$, which holds with the assumption $p\bar{\theta} \geqslant 4$.

A2. Solutions and Properties of face values $D_{0,1}$, $D_{0,2}$ with asymmetric information .

1. The existence of face values. From equation (10) we can solve $D_{0,1}$:

$$D_{0,1} = \frac{2}{1 + \sqrt{1 - \frac{4\alpha}{\bar{\theta}} \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)}} \tag{14}$$

Since p_G and p_B both satisfy assumption $p\bar{\theta} \geqslant 4$, $D_{0,1}$ and thus $D_{0,2}$ always exist.

2. The partial derivative properties of face values with respect to α. Apparently, $\frac{\partial D_{0,1}}{\partial \alpha} > 0$ and $\frac{\partial(\alpha D_{0,1})}{\partial \alpha} > 0$. Eliminate $D_{1,2}^{s}(\theta)$ in equation (11) with equation (8) and (10) and we get

$$\sum_{s=g,b} \left[P(s) \left(\int_{\tilde{\theta}_s}^{\hat{\theta}_s} p(s)\theta f(\theta) d\theta + \alpha D_{0,1} \int_{\hat{\theta}_s}^{\bar{\theta}} f(\theta) d\theta + (1-\alpha) D_{0,2} \int_{\hat{\theta}_s}^{\bar{\theta}} p(s) f(\theta) d\theta \right) \right] = 1$$

Take first derivative with respect to α:

$$\frac{\partial \left[(1-\alpha) D_{0,2} \right]}{\partial \alpha} = - \frac{\left[P(g) \left(-\tilde{\theta}_g f(\tilde{\theta}_g) + \int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta) d\theta \right) + (1 - P(g)) \left(-\tilde{\theta}_b f(\tilde{\theta}_b) + \int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta) d\theta \right) \right]}{P(g) p(g) \int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta) d\theta + (1 - P(g)) p(b) \int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta) d\theta} \frac{\partial (\alpha D_{0,1})}{\partial \alpha} \tag{15}$$

3. The partial derivative properties of face values with respect to μ. From expression (14), we can calculate:

$$\frac{\partial (\alpha D_{0,1})}{\partial \mu} = - \frac{ - \frac{\partial \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)}{\partial \mu} \frac{4\alpha^2}{\bar{\theta}} }{ \sqrt{1 - \frac{4\alpha}{\bar{\theta}} \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)} \left(1 + \sqrt{1 - \frac{4\alpha}{\bar{\theta}} \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)} \right)^2 }$$

where

$$\frac{\partial \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)}{\partial \mu} = -q \left(\frac{1}{p(b)} - \frac{1}{p(g)} \right) - P(g) \frac{(1-q)(p_G - p_B)}{\left[\mu p_G + (1-\mu)(1-q) p_B \right]^2}$$

+so $\frac{\partial(\alpha D_{0,1})}{\partial \mu} < 0$. Further, we can calculate:

$$\frac{\partial^2 \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right)}{\partial \mu^2} = 2q \frac{\partial \left(\frac{1}{p(g)} \right)}{\partial \mu} + P(g) \frac{\partial^2 \left(\frac{1}{p(g)} \right)}{\partial \mu^2} = \frac{2(1-q)^2 (p_G - p_B)^2}{\left[\mu p_G + (1-\mu)(1-q) p_B \right]^2} > 0$$

so $\frac{\partial^2 \left[- \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)} \right) \right]}{\partial \mu^2} < 0$ and we can easily see that $\frac{\partial^2 (\alpha D_{0,1})}{\partial \mu^2} > 0$.

To get the properties of $(1-\alpha)\,D_{0,2}$, eliminate $D^s_{1,2}(\theta)$ in equation (11) with equation (8) and we get

$$(\mu p_G + (1-\mu)p_B)\left(\overline{\theta}\,[(1-\alpha)D_{0,2}] - \frac{[(1-\alpha)D_{0,2}]^2}{2}\right) - (\alpha D_{0,1})\,[(1-\alpha)D_{0,2}] = (1-\alpha)\bar{\theta}$$

Take first and second derivative with respect to μ:

$$\frac{\partial\,[(1-\alpha)D_{0,2}]}{\partial\mu} = \frac{-(p_G-p_B)\left[\overline{\theta}(1-\alpha)D_{0,2} - \frac{(1-\alpha)^2D_{0,2}^2}{2}\right] + [(1-\alpha)D_{0,2}]\,\frac{\partial(\alpha D_{0,1})}{\partial\mu}}{(\mu p_G + (1-\mu)p_B)\left(\overline{\theta} - [(1-\alpha)D_{0,2}]\right) - (\alpha D_{0,1})} < 0$$

$$\begin{aligned}\frac{\partial^2[(1-\alpha)D_{0,2}]}{\partial\mu} \quad = \quad & \frac{-\left((p_G-p_B)\left(\overline{\theta}-[(1-\alpha)D_{0,2}]\right)-(\mu p_G+(1-\mu)p_B)\frac{\partial\left[(1-\alpha)D_{0,2}\right]}{\partial\mu}-\frac{\partial\left(\alpha D_{0,1}\right)}{\partial\mu}\right)\frac{\partial\left[(1-\alpha)D_{0,2}\right]}{\partial\mu}}{(\mu p_G+(1-\mu)p_B)\left(\overline{\theta}-[(1-\alpha)D_{0,2}]\right)-(\alpha D_{0,1})} \\ & + \frac{[(1-\alpha)D_{0,2}]\frac{\partial^2\left(\alpha D_{0,1}\right)}{\partial\mu}+\frac{\partial\left[(1-\alpha)D_{0,2}\right]}{\partial\mu}\frac{\partial\left(\alpha D_{0,1}\right)}{\partial\mu}-(p_G-p_B)\left(\overline{\theta}-(1-\alpha)D_{0,2}\right)\frac{\partial\left[(1-\alpha)D_{0,2}\right]}{\partial\mu}}{(\mu p_G+(1-\mu)p_B)\left(\overline{\theta}-[(1-\alpha)D_{0,2}]\right)-(\alpha D_{0,1})} > 0\end{aligned}$$

A3. Properties of cut-off values $\tilde{\theta}_s$, $\hat{\theta}_s$ with asymmetric information .

1. The partial derivative properties of $\tilde{\theta}_s = \frac{\alpha D_{0,1}}{p(s)}$ $(s = g, b)$ and $\hat{\theta}_b = \frac{\alpha D_{0,1}}{p_B} + (1-\alpha)D_{0,2}$ with respect to α. Apparently, $\frac{\partial\hat{\theta}_s}{\partial\alpha} > 0$ $(s = g, b)$ since $\frac{\partial(\alpha D_{0,1})}{\partial\alpha} > 0$. Using expression (15), we can calculate:

$$\frac{\partial\hat{\theta}_b}{\partial\alpha} = \frac{P(g)\left(\frac{p(g)}{p_B}-1\right)\int_{\hat{\theta}_g}^{\overline{\theta}} f(\theta)d\theta + \left[P(g)\left(\tilde{\theta}_g f(\tilde{\theta}_g)\right) + (1-P(g))\left(\tilde{\theta}_b f(\tilde{\theta}_b)\right)\right]}{P(g)p(g)\int_{\hat{\theta}_g}^{\overline{\theta}} f(\theta)d\theta + (1-P(g))p(b)\int_{\hat{\theta}_b}^{\overline{\theta}} f(\theta)d\theta}\frac{d\,(\alpha D_{0,1})}{d\alpha} > 0 \quad (16)$$

2. The partial derivative properties of $\tilde{\theta}_s = \frac{\alpha D_{0,1}}{p(s)}$ $(s = g, b)$ with respect to μ. It is easy to calculate that $\frac{\partial p(g)}{\partial\mu} = \frac{(1-q)(p_G-p_B)}{[\mu+(1-\mu)(1-q)]^2} > 0$, $\frac{\partial p(b)}{\partial\mu} = 0$. We have also shown that $\frac{\partial(\alpha D_{0,1})}{\partial\mu} < 0$, so $\frac{\partial\tilde{\theta}_s}{\partial\mu} < 0$ $(s = g, b)$. To get the second derivative, we calculate that:

$$\frac{\partial^2 p(g)}{\partial^2\mu} = -\frac{2q(1-q)(p_G-p_B)}{[\mu q+(1-\mu)(1-q)]^3} < 0 \quad \frac{\partial p^2(b)}{\partial^2\mu} = 0$$

and with $\frac{\partial^2(\alpha D_{0,1})}{\partial\mu^2} > 0$, we have

$$\frac{\partial^2\tilde{\theta}_s}{\partial\mu^2} = \frac{p(s)\left[\frac{\partial^2(\alpha D_{0,1})}{\partial\mu^2}p(s) - (\alpha D_{0,1})\,\frac{\partial^2 p(s)}{\partial\mu^2}\right] - 2\frac{\partial p(s)}{\partial\mu}\left[\frac{\partial(\alpha D_{0,1})}{\partial\mu}p(s) - (\alpha D_{0,1})\,\frac{\partial p(s)}{\partial\mu}\right]}{p^3(s)} > 0$$

3. The partial derivative properties of $\hat{\theta}_s = \tilde{\theta}_s + (1-\alpha)D_{0,2}$ $(s = g, b)$ with respect to μ. Combining the properties of $\tilde{\theta}_s$ and $(1-\alpha)D_{0,2}$, we have $\frac{\partial \hat{\theta}_s}{\partial \mu} < 0$ and $\frac{\partial^2 \hat{\theta}_s}{\partial \mu^2} > 0$ $(s = g, b)$.

Appendix B: Proofs of Propositions and Lemmas

B1. Proof of Proposition 2 . Define $\pi_t(\alpha_{t'}) = p_t A(\alpha_{t'})$, then

$$A(\alpha_B = 0) = \int_{\tilde{\theta}(0)}^{\bar{\theta}} f(\theta)d\theta - \frac{1}{p_B} = \frac{\bar{\theta}}{2} - \frac{1}{p_B}$$

$$A(\alpha_G = 1) = \int_{\tilde{\theta}(1)}^{\bar{\theta}} f(\theta)d\theta - \frac{1}{p_G} = \frac{\bar{\theta}}{2} - \frac{1}{p_G} - \frac{1}{\bar{\theta}p_G^2}\frac{2}{\left(1+\sqrt{1-4/p_G\bar{\theta}}\right)^2}$$

When assumption B holds $\frac{1}{p_B} > \frac{1}{p_G} + \frac{1}{\bar{\theta}p_G^2}\frac{2}{\left(1+\sqrt{1-4/p_G\bar{\theta}}\right)^2}$, $A(\alpha_G = 1) > A(\alpha_B = 0)$, then there is no separating equilibrium.□

B2. Proof of Lemma 4 . First we prove $\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha} < 0$.

$$\begin{aligned}
\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha} &= -\frac{\partial(\alpha D_{0,1})}{\partial \alpha}\left[\frac{(1-q)p_B}{p(g)}\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + \frac{qp_B}{p(b)}\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right] \\
&\quad -\frac{\partial[(1-\alpha)D_{0,2}]}{\partial \alpha}\left[(1-q)p_B\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + qp_B\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right] \\
&= -\frac{\partial(\alpha D_{0,1})}{\partial \alpha}\frac{p_B\left(\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right)\left(\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right)}{P(g)p(g)\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + (1-P(g))p(b)\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta} \times \\
&\quad \left\{\frac{p(g)-p(b)}{p(g)p(b)}\mu(2q-1)p_G + \left[P(g)\tilde{\theta}_g f(\tilde{\theta}_g) + (1-P(g))\tilde{\theta}_b f(\tilde{\theta}_b)\right]\left[\frac{1-q}{\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta} + \frac{q}{\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta}\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha} &= -\frac{\partial(\alpha D_{0,1})}{\partial \alpha}\left[\frac{(1-q)p_B}{p(g)}\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + \frac{qp_B}{p(b)}\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right] \\
&\quad +\frac{\left[P(g)\left(-\tilde{\theta}_g f(\tilde{\theta}_g) + \int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right) + (1-P(g))\left(-\tilde{\theta}_b f(\tilde{\theta}_b) + \int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right)\right]}{P(g)p(g)\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + (1-P(g))p(b)\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta}\left[(1-q)p_B\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + qp_B\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right]\frac{d(\alpha D_{0,}}{d\alpha} \\
&= -\frac{\partial(\alpha D_{0,1})}{\partial \alpha}\frac{p_B\left(\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right)\left(\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right)}{P(g)p(g)\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + (1-P(g))p(b)\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta} \times \\
&\quad \left\{\frac{p(g)-p_B}{p(g)p_B}\mu q p_G + \left[P(g)\tilde{\theta}_g f(\tilde{\theta}_g) + (1-P(g))\tilde{\theta}_b f(\tilde{\theta}_b)\right]\left[\frac{1-q}{\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta} + \frac{q}{\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta}\right]\right\}
\end{aligned}$$

where the second equal sign comes from replacing $\frac{\partial[(1-\alpha)D_{0,2}]}{\partial \alpha}$ with expression (16). Since $\frac{\partial(\alpha D_{0,1})}{\partial \alpha} > 0$ and $q \geq 0$, we have $\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha} < 0$.

Then we prove $\frac{\partial \pi_B(\alpha;\mu)}{\partial \mu} > 0$. We can calculate that $\frac{\partial \pi_B(\alpha;\mu)}{\partial \mu} = -\frac{(1-q)p_B}{\bar{\theta}}\frac{\partial \hat{\theta}_g}{\partial \mu} + \frac{qp_B}{\bar{\theta}}\frac{\partial \hat{\theta}_b}{\partial \mu}$. Since we have shown in Appendix A3 that $\frac{\partial \hat{\theta}_s}{\partial \mu} < 0$ $(s = g, b)$, clearly we have $\frac{\partial \pi_B(\alpha;\mu)}{\partial \mu} > 0$.□

B.3 Proof of Proposition 3. We can calculate:

$$\pi_B(1;\mu) \geqslant p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right) \Leftrightarrow \left(\frac{1-q}{p^2(g)} + \frac{q}{p^2(b)}\right)\frac{D_{0,1}^2(1;\mu)}{2\bar{\theta}} - \left(\frac{1-q}{p(g)} + \frac{q}{p(b)}\right)D_{0,1}(1;\mu) + \frac{1}{p_B} \geq 0$$

Define $M(x;\mu) = \left(\frac{1-q}{p^2(g)} + \frac{q}{p^2(b)}\right)\frac{x^2}{2\bar{\theta}} - \left(\frac{1-q}{p(g)} + \frac{q}{p(b)}\right)x + \frac{1}{p_B}$, then

$$\frac{\partial M(x;\mu)}{\partial \mu} = x\left[\left(1 - \frac{1}{p(g)}\frac{x}{\bar{\theta}}\right)\frac{1-q}{p^2(g)}\frac{\partial p(g)}{\partial \mu} + \left(1 - \frac{q}{p(b)}\frac{x}{\bar{\theta}}\right)\frac{q}{p^2(b)}\frac{\partial p(b)}{\partial \mu}\right] > 0$$

and $\frac{\partial M(x;\mu)}{\partial x} < 0$ at the smaller solution x_1 to $M(x) = 0$, where

$$x_1 = \frac{2}{\left(\frac{1-q}{p(g)} + \frac{q}{p(b)}\right)p_B + \sqrt{\left(\frac{1-q}{p(g)} + \frac{q}{p(b)}\right)^2 p_B^2 - 2\left(\frac{1-q}{p^2(g)} + \frac{q}{p^2(b)}\right)\frac{p_B}{\bar{\theta}}}}$$

so $\frac{\partial x_1}{\partial \mu} > 0$ and $\pi_B(1;\mu) \geqslant p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right) \Leftrightarrow D_{0,1}(1;\mu) \leqslant x_1(\mu)$. We have shown that $\frac{\partial D_{0,1}(1;\mu)}{\partial \mu} < 0$, so if $D_{0,1}(1;1) < x_1(1)$ holds, there is the unique solution μ_1 to $D_{0,1}(1;\mu) = x_1(\mu)$ and $D_{0,1}(1;\mu) \leq x_1(\mu)$ for $\mu \in [\mu_1, 1)$,$D_{0,1}(1;\mu) > x_1(\mu)$ for $\mu \in (0, \mu_1)$; and if $D_{0,1}(1;1) > x_1(1)$, $D_{0,1}(1;\mu) > x_1(\mu)$ for $\mu \in (0, 1)$. It can be calculated that $D_{0,1}(1;1) \leq x_1(1)$ is equal to condition C.

If condition C holds, for $\mu \in [\mu_1, 1)$, $\pi_B(1;\mu) \geqslant p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$ for any $\alpha \in [0, 1]$; for $\mu \in (0, \mu_1)$, the maximum α (denoted as $\alpha_e(\mu)$) that makes the $\pi_B(1;\mu) \geqslant p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$ hold satisfies $\pi_B(\alpha;\mu) = p_B\left(\frac{\bar{\theta}}{2} - \frac{1}{p_B}\right)$. We know that $\frac{\partial \pi_B(\alpha;\mu)}{\partial \alpha}\alpha'(\mu) + \frac{\partial \pi_B(\alpha;\mu)}{\partial \mu} = 0$ and we have $\alpha_e'(\mu) > 0$ in this case. If condition C is violated, it is the same with the latter case ($\mu \in (0, \mu_1)$) when condition C holds.□

B.4 Proof of Lemma 5.

$$\begin{aligned}
\frac{\partial \pi_G(\alpha;\mu)}{\partial \alpha} &= -\frac{\partial(\alpha D_{0,1})}{\partial \alpha}\left(\frac{p_G}{p(g)}\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right) - \frac{\partial[(1-\alpha)D_{0,2}]}{\partial \alpha}\left(p_G\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right) \\
&= \frac{\partial(\alpha D_{0,1})}{\partial \alpha}\frac{p_G\left(\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta\right)\left(\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta\right)}{P(g)p(g)\int_{\hat{\theta}_g}^{\bar{\theta}} f(\theta)d\theta + (1-P(g))p(b)\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta}\times \\
&\left\{\frac{p(g)-p_B}{p(g)}q(1-\mu) - \left[P(g)\tilde{\theta}_g f(\tilde{\theta}_g) + (1-P(g))\tilde{\theta}_b f(\tilde{\theta}_b)\right]\frac{1}{\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta}\right\}
\end{aligned}$$

Thus $\frac{\partial \pi_G(\alpha;\mu)}{\partial \alpha}$ has the same sign with $\Lambda(\alpha;\mu)$. Apparently $\Lambda(0;\mu) = \frac{p(g)-p_B}{p(g)} q(1-\mu) > 0$.$\square$

B.5 Proof of Lemma 6. We have shown in Appendix A that $\frac{\partial \tilde{\theta}_s}{\partial \alpha} > 0$ $(s = g, b)$ and $\frac{\partial \hat{\theta}_b}{\partial \alpha} > 0$, so $\frac{\partial \Lambda(\alpha;\mu)}{\partial \alpha} < 0$.

To prove $\frac{\partial^2 \Lambda(\alpha;\mu)}{\partial \mu^2}$, re-write $\Lambda(\alpha;\mu) = \Gamma(\mu)(1-\mu) - \Phi(\alpha;\mu)\Psi(\alpha;\mu)$, $\Lambda(\alpha;\mu) = \Gamma(\mu)(1-\mu) - \Omega(\alpha;\mu)$where $\Gamma(\mu) = \frac{p(g)-p_B}{p(g)} q(1-\mu)$, $\Omega(\alpha;\mu) = \Phi(\alpha;\mu)\Psi(\alpha;\mu)$ and $\Phi(\alpha;\mu) = P(g)\tilde{\theta}_g f(\tilde{\theta}_g) + (1-P(g))\tilde{\theta}_b f(\tilde{\theta}_b)$, $\Psi(\alpha;\mu) = \frac{1}{\int_{\hat{\theta}_b}^{\bar{\theta}} f(\theta)d\theta}$. Apparently $\frac{d\Gamma(\mu)}{d\mu} > 0$ because $p(g)$ is increasing in μ, and we can calculate that$\frac{\partial^2 \left(\frac{1}{p(g)}\right)}{\partial \mu^2} = \frac{2(1-q)(p_G-p_B)(p_G-(1-q)p_B)}{[\mu p_G + (1-\mu)(1-q)p_B]^3} > 0$, so $\frac{d^2\Gamma(\mu)}{d\mu^2} < 0$. Then observe that $\Phi(\alpha;\mu) = \left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)}\right) \frac{\alpha D_{0,1}}{\bar{\theta}}$. Since we have shown in Appendix A2 that $\frac{\partial\left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)}\right)}{\partial \mu} < 0$, $\frac{\partial^2\left(\frac{P(g)}{p(g)} + \frac{1-P(g)}{p(b)}\right)}{\partial \mu^2} > 0$, $\frac{\partial(\alpha D_{0,1})}{\partial \mu} < 0$ and $\frac{\partial^2(\alpha D_{0,1})}{\partial \mu^2} > 0$, we can get $\frac{\partial \Phi(\alpha;\mu))}{\partial \mu} < 0$ and $\frac{\partial^2 \Phi(\alpha;\mu)}{\partial \mu^2} > 0$. Further, we have shown in Appendix A3 that $\frac{\partial \hat{\theta}_s}{\partial \mu} < 0$ and $\frac{\partial^2 \hat{\theta}_s}{\partial \mu^2} > 0$ $(s = g, b)$, so $\frac{\partial \Psi(\alpha;\mu)}{\partial \mu} < 0$ and $\frac{\partial^2 \Psi(\alpha;\mu)}{\partial \mu^2} > 0$. Combining the analysis of $\Phi(\alpha;\mu)$ and $\Psi(\alpha;\mu)$, we have $\frac{\partial \Omega(\alpha;\mu)}{\partial \mu} < 0$ and $\frac{\partial^2 \Omega(\alpha;\mu)}{\partial \mu^2} > 0$. With these properties, it is easy to see that: $\frac{\partial \Lambda(\alpha;\mu)}{\partial \mu} = \frac{d\Gamma(\mu)}{d\mu}(1-\mu) - \Gamma(\mu) - \frac{\partial \Omega(\alpha;\mu)}{\partial \mu}$, $\frac{\partial^2 \Lambda(\alpha;\mu)}{\partial \mu^2} = \frac{d^2\Gamma(\mu)}{d\mu^2}(1-\mu) - \frac{d\Gamma(\mu)}{d\mu} - \frac{\partial^2 \Omega(\alpha;\mu)}{\partial \mu^2} < 0$.$\square$

B.6 Proof of Proposition 4. It can be observed that $\Lambda(0;0) = \Lambda(1;0) = 0$, and $\Lambda(0;\mu) > 0$ for any $\mu \in (0,1)$. By concavity of $\Lambda(\alpha;\mu)$ in μ, we know that $\underset{\mu}{Max}\Lambda(0;\mu) > 0$. And $\underset{\mu}{Max}\Lambda(\alpha;\mu)$ is decreasing in α since $\frac{\partial \Lambda(\alpha;\mu)}{\partial \alpha} < 0$. We can calculate that $\Lambda(\alpha;0) < 0$, $\Lambda(\alpha;1) < 0$ for any $\alpha \in (0,1]$. So there may two cases:

(1) If $\underset{\mu}{Max}\Lambda(1;\mu) \geq 0$, then $\underset{\mu}{Max}\Lambda(\alpha;\mu) > 0$ for any $\alpha \in [0,1]$. And by concavity of $\Lambda(\alpha;\mu)$ in μ, for each $\alpha \in [0,1]$, there exists two values of $\mu \in (0,1)$ such that $\Lambda(\alpha;\mu) = 0$ (there is only one solution for $\alpha = 1$ if $\underset{\mu}{Max}\Lambda(1;\mu) = 0$). Denote $\mu_2, \mu_3 \in (0,1)$ $(\mu_2 < \mu_3)$ as the two solutions to $\Lambda(1;\mu) = 0$ ($\mu_2 = \mu_3$ if $\underset{\mu}{Max}\Lambda(1;\mu) = 0$), and we have $\Lambda(\alpha;\mu) \geq 0$ for any $\mu \in [\mu_2, \mu_3]$ and $\alpha \in [0,1]$. Thus $\alpha_o(\mu) = 1$ for $\mu \in [\mu_2, \mu_3]$ and $\alpha_o(\mu) < 1$ satisfying $\Lambda(\alpha_o(\mu);\mu) = 0$ for $\mu \in (0,\mu_2) \cup (\mu_3, 1)$. Further, for any $\mu' \in (0,\mu_2)$, there exists with $\mu'' \in (\mu_3, 1)$ satisfying $\alpha_o(\mu') = \alpha_o(\mu'')$, which means $\frac{\partial \Lambda(\alpha_o(\mu');\mu')}{\partial \mu} > 0$ and $\frac{\partial \Lambda(\alpha_o(\mu'');\mu'')}{\partial \mu} < 0$ by concavity of $\Lambda(\alpha;\mu)$. Thus we have $\frac{\partial \Lambda(\alpha_o(\mu;\mu)}{\partial \mu} > 0$ for $\mu \in (0,\mu_2)$ and $\frac{\partial \Lambda(\alpha_o(\mu;\mu)}{\partial \mu} < 0$ for $\mu \in (\mu_3, 1)$. Take derivative of μ in equation $\Lambda(\alpha_o(\mu);\mu) = 0$, we have:

$$\frac{\partial \Lambda(\alpha_o(\mu);\mu)}{\partial \alpha}\alpha_o'(\mu) + \frac{\partial \Lambda(\alpha_o(\mu);\mu)}{\partial \mu} = 0$$

and it is easy to see that $\alpha_o'(\mu) > 0$ *for* $\mu \in (0, \mu_2)$ *and* $\alpha_o'(\mu) < 0$ *for* $\mu \in (\mu_3, 1)$.

(2) If $\underset{\mu}{Max}\Lambda(1;\mu) < 0$, then there exists $\alpha_1 \in (0,1)$ such that $\underset{\mu}{Max}\Lambda(\alpha_1;\mu) = 0$. For each $\alpha \in [0, \alpha_1)$, there exists two values of $\mu \in (0,1)$ such that $\Lambda(\alpha;\mu) = 0$, by concavity of $\Lambda(\alpha;\mu)$ in μ. Denote $\underset{\mu}{argmax}\Lambda(\alpha_1;\mu)$ as $\mu_4 \in (0,1)$, then $\Lambda(\alpha_1;\mu_4) = 0$, and we have $\alpha_o(\mu) \leq \alpha_1$ for any $\mu \in (0,1)$. Similar with the former case, $\frac{\partial\Lambda(\alpha_o(\mu;\mu)}{\partial\mu} > 0$ for $\mu \in (0, \mu_4)$ and $\frac{\partial\Lambda(\alpha_o(\mu;\mu)}{\partial\mu} < 0$ for $\mu \in (\mu_4, 1)$. Thus $\alpha_o'(\mu) < 0$ *for* $\mu \in (0, \mu_4)$ *and* $\alpha_o'(\mu) > 0$ *for* $\mu \in (\mu_4, 1)$.$\square$

B.7 Proof of Proposition 5. Denote μ_d as the largest μ satisfying $\alpha_o'(\mu) = 0$, then $\alpha_o'(\mu) \geq 0$ for any $\mu \in (0, \mu_d)$ and $\alpha_o'(\mu) < 0$ for any $\mu \in (\mu_d, 1)$. Thus there is at most one intersection between $\alpha_e(\mu)$ and $\alpha_o(\mu)$ in the interval $(\mu_d, 1)$. If there isn't, $\alpha^*(\mu) = min\{\alpha_o(\mu), \alpha_e(\mu)\}$ (weakly) increases with μ for $\mu \in (0, \mu_d)$ because $\alpha_o(\mu), \alpha_e(\mu)$ are both increasing. Besides, $\alpha^*(\mu) = \alpha_o(\mu)$ for $\mu \in (\mu_d, 1)$ (otherwise there will be intersection between $\alpha_o(\mu)$ and $\alpha_e(\mu)$) and thus it strictly decreases with μ in this interval. If there is intersection, denote this one as $\mu_s > \mu_d$. Apparently, $\alpha^*(\mu)$ still (weakly) increases with μ for $\mu \in (0, \mu_d)$. For $\mu \in (\mu_d, \mu_s)$, it must be $\alpha^*(\mu) = \alpha_e(\mu)$ and it also (weakly) increases with μ. For $\mu \in (\mu_s, 1)$, $\alpha^*(\mu) = \alpha_o(\mu)$ and thus it strictly decreases with μ.$\square$

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