The Impact of Factors of Market and Technology on the Decision about Technology Innovation Investment*

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Abstract: The appearance order of technologies with different quality and level is affected by market preference, the differences in product quality and develop cost between two technologies or products. With considering factors of market preference, the differences in product quality, and develop cost, this paper constructs real options model to analyzes the equilibrium of investment strategy of two competing firms who are asymmetric in product quality and technology level, and to investigate the investment interval between two firms. This study explains the phenomena of PAS （Personal Access Phone System） from point view of technology innovation.

Key words: Technology innovation; Real Options; Game equilibrium

0 Introduction

In December 1997, the Personal Access Phone System came from Japan was introduced in small and middle cities in China. The PAS business has been operating formally in China since Information Industry Ministry granted it (2000 #604) in 2000.

PAS is one kind of small area, low speed mobile and tetherless aceessing technology, and is the extension and complementarity of fixed telephone. From the point of view of pure technology, there exists much shortage in PAS. For example, the emission power of PAS is small, PAS needs many bases. As consumers are in high-speed vehicles, in closed rooms and in high building areas, they can't receive signal some times, and they have difficulties in sending information and exploring on web. Comparing with mobile telephone, PAS belongs to low-quality technology. However, the low-quality technology is adopted in Chengdu, Sian, Yuhang, Kunming, Sining, Beihai, Baoding, Taizhou et al. The number of consumers amount to eighty million in 2005†.

We survey the PAS phenomena from the point of view of technology innovation, the order of technology appearance in market is that low quality technology or product are first to come into existence, and the high appears subsequently. However, PAS seems to violate the general law. If we explain the phenomena from the view of adoption of technology, because the price of the current technology is high, the adopter would adopt the relative current laggard technology when new technology is presented and the price of the current laggard technology is depreciated. But, China Telecom who introduced and is operating PAS is not short of money, and the constructing of PAS is not saving money. Many people attributed the phenomena to the result of monopoly. However, the fact of fury competition between China Mobile and China Unicom is true.

Further, PAS is growing fairness in under developed area and middle and small city. However, it is expanding slowly in developed area such as Peking, Shanghai and Wuhan. Additionally, in the market of mobile of China, the difference of technology and service between China Mobile corporation and China Unicom corporation is small. Although the two corporations compete in price of all sorts, the price of calling phone is high at all times. The strategy of ' enjoy wireless, and pay in wire' made by PAS caters for the need of many consumers with low preference who stay in local area all times. So, the factors of preference of market, the quality difference, and the price difference induce the phenomena of PAS, at last, have impact on the order of appearance of technology.

Without considering the factor of government policy, from the point view of development of technology, the general questions induced by the phenomena of PAS are that: how do the market preference, the difference of quality and develop cost impact on the timing of technology development? How do these factors impact on the equilibrium strategies of two competing firms? Whereas, considering the factors of market preference, the quality (level) of product (technology), and develop cost, this paper constructs decision model based on real option approach to investigate the equilibrium strategies of firms, who are asymmetric in cost and product quality,

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and the interval between two firms' investments, and to reveal the inherent law of evolution of technology.

The results show that, fixed other parameters (cost and quality), in the sequential equilibrium and preemptive equilibrium dominated by high-quality product firm, the investment time that low-quality product firm lags behind high-quality product firm is reduced with the difference of develop cost increasing between them. However, in the sequential equilibrium and preemptive equilibrium dominated by low-quality firm, the results is inverse.

The results also show that, in complete monopoly market, with market preference increasing, the complete monopoly revenue of low-quality product and high-quality product firms would increase. In order to get some period of complete monopoly revenue in high market preference, the two firms all have strong incentive to preempt to invest. In other words, in complete monopoly market, high market preference is in favor of the two firms, they all like to invest in development of new product and occupy market early. On the other one hand, with market preference increasing, the duopoly revenue of high-quality product firm would increase, and the duopoly revenue of low-quality product firm would decrease. So, when the market preference is high, the high-quality product firm has much strong incentive to invest early, the low-quality product firm is inhibited to invest as a follower. The result is that investment interval is expanded between the investment timing of two firms. Vice versa, with the market preference decreasing, the low-quality product firm would like to invest early after his opponent invested. Some areas of China are low preference market, the result above appropriately explain the phenomena of PAS. But, when the low-quality product firm preempts to invest before the high-quality product firm in order to get some period of complete monopoly revenue, the investment interval between the two firms is expanded with the market preference increasing. This result violates our intuition. In fact, the increase of market preference enhances the duopoly revenue of high-quality product firm, and also enhances the complete monopoly revenue of low-quality product firm. But, with the market preference increasing, the relative growing speed of the duopoly revenue of high-quality product firm to that of the complete monopoly revenue of low-quality product firm decreases.

Further, in the sequential and preemptive equilibrium dominated by each firm, the investment interval between two firms is reduced with the difference of quality between them increasing. For the reason that, in the aspect of low-quality product firm, the increase of difference of product quality has nothing to do with its revenue of complete monopoly , but can increase its duopoly revenue. In the aspect of high-quality product firm, its complete monopoly and duopoly revenue can all increase with the difference of product quality between two firms increasing. So, in the sequential equilibrium dominated by low-quality product firm, the investment interval between two firms is reduced because of the increase of the duopoly revenue of the high-quality product firm. However, in the sequential equilibrium dominated by high-quality product firm, the investment interval is reduced because that the relative growing speed of the complete monopoly revenue of high-quality product firm to that of the duopoly revenue of low-quality product firm decreases.

The results of this paper tell us, in particular case, low-quality product could appear behind high-quality product. The lower market preference is, and the larger the difference of quality is, the shorter of the interval that low-quality product is lagged behind high-quality product is. The market preference and the difference of product quality give explain to the phenomena of PAS. Although develop cost is one important factor that impacts on the order of appearance of technology, it is not the crucial factor.

Our study is related to the following literatures. Grenadier (1996) investigated the competition strategies and investment interval between two symmetric firms in real estate. Weeds (2002) studied the competition interaction behaviors of two symmetric firms under complete information and one stage R&D investment. Huisman et al (2001) studied that the technology adoption behaviors of two asymmetric firms in cost. Xiahui & Zengyong (2004) focused on the sequential, preemptive and simultaneous equilibrium, and the interval of technology adoption between two asymmetric firms in cost. Our study differs from them. We consider two firms which are asymmetry in cost and quality at the same time. Further, we pay more attention to the impact of the market structure, the market preference, quality and uncertainty from economic environment on the investment interval of two competition firms. Grenadier (1996), Huisman et al (2001), and Xiahui & Zengyong (2004) only made use of an abstract market demand function to depict the market structure, they could not accurately depict the impacts of market factors on the

firms' investment timing and behaviors. To explain the similar phenomena of PAS from the point of view of technology innovation, we must go deep into the market structure and the properties of innovation. Dasgupta & Stiglitz (1980) stressed that the properties of market structure and the activities of firms' innovation are endogenous, the innovation speed must trace back to the more basic factors, such as the demand environment, the research and development technology and the property of capital market.

This paper first adopts the model of market preference used by Shaked & Sutton (1982), Tirole (1988) and Rosenkranz (1995), and gets the revenue of complete monopoly and duopoly of two firms with different technology level or quality product. And then, this paper incorporates the market uncertainty into the model and analyzes the competition equilibrium strategies and interval of two firms' investment.

1 The assumption and model framework

We suppose a market in which there are two firms l and h, who choose to develop low and high quality product respectively according to the demand of market. For convenience, the set of firms, $i = \{ l, h \}$, in this paper, the subscripts of each variable denote different firm, i- denotes i's opponent. For example, $i = h$, thus, $i- = l$.

There is Bertrand competition between the two firms in market. According to the model of Shaked & Sutton (1982) simplified by Tirole (1989), and Rosenkranz's (1995) model, we suppose that the qualities of product two firms which choose is S and αS respectively. The difference of qualities between two products is ΔS, $\Delta S = (\alpha - 1)S$, $\alpha > 1$. For convenience, we denote α the difference of quality, which is the relative difference to low-quality product. The demand of product is also affected by the preference of market (consumer), so we suppose the utility of consumer is u.

$$u = \Phi q - P \tag{1}$$

Where, q is described as the quality of some product, $q = \{S, \alpha S\}$, P is the price of some product. The preference type is denoted by Φ, which can be regarded as the converting factor which can convert product quality to the form measured by money.

The consumers with different parameters Φ decide to purchase the product according to equation (1): they convert the product with quality q to the willingness to pay, Φq, less the product price set by firm, and get their utility which can be measured by money. For example, the telephone mobile in market is high-quality products, any consumers with different parameter Φ would like to own it. But, because of the impact of the structure of market and other factors, the higher the quality of technology is, the higher its price is, and the higher using cost. Although PAS is low-quality product, its operation cost is low. Thus, for the consumers with low preference, after synthetically comparing quality and use-cost, they would like to choose PAS,.

We suppose that there exist two products, high quality product and low quality product, which belong to the same category. In market, the preference parameter Φ is uniformly distributed across on the interval $[a, a + 1]$. Where a is the lowest preference type, $0 \le a \le 1$. In this paper, we let a be described directly as market preference because it determines the distributing range of consumer preference. According to the product prices set by two firms, there exists a consumer who is indifferent for the two products. He divides the market into two parts: low preference market and high preference market. According to the purpose of the paper, based on the assumption that there exists only one consumer in market, we slightly adjusts Rosenkranz (1995) model and get the prices and revenues of complete monopoly and duopoly of the two firms. The complete result is showed in lemma 1.

Lemma 1:

1) *The indifference consumer's preference parameter is*

$$\Phi^* = (P_h - P_l) / (\alpha - 1) S \tag{2}$$

Where, $P_l = (1-a)(\alpha-1)S/3$, $P_h = (a+2)(\alpha-1)S/3$.

2) *The demand curves faced by two firms are*

$$D_l(P_h, P_l) = \frac{P_h - P_l}{(\alpha-1)S} - a \tag{3}$$

$$D_h(P_h, P_l) = (a+1) - \frac{P_h - P_l}{(\alpha - 1)S} \tag{4}$$

Under duopoly condition, in order to ensure the consumer with lowest preference purchase the low-quality product, the difference of product quality would satisfy:

$$1 \le \alpha \le \left(\frac{3a}{1-a} + 1 \right) \tag{5}$$

3) *In duopoly market, the revenues of the two firms are:*

$$\pi_i^D = \begin{cases} \pi_l^D = \dfrac{(1-a)^2}{9}(\alpha - 1)S \\ \pi_h^D = \dfrac{(a+2)^2}{9}(\alpha - 1)S \end{cases} \tag{6}$$

4) *Under the situation of complete monopoly market, the complete monopoly prices and revenues for the two firms are*

$$P_i^M = \begin{cases} P_l^M = \dfrac{a+1}{2}S \\ P_h^M = \dfrac{a+1}{2}\alpha S \end{cases} \tag{7}$$

$$\pi_i^M = \begin{cases} \pi_l^M = \dfrac{(a+1)^2}{4}S \\ \pi_h^M = \dfrac{(a+1)^2}{4}\alpha S \end{cases} \tag{8}$$

According to lemma 1, we can get lemma 2:

Lemma 2: $\pi_h^M > \pi_h^D$, $\pi_l^M > \pi_l^D$.

According to lemma 1, the consumer's preference and the product quality of firms impact on each firm's price and demand. Firm *h* can require high price, but his market demand would be lowered. His product price and market demand could be increased with his product quality increasing. For firm *l*, the more higher the quality of his opponent's product is, the more different the two products means, and the more finely the market partition, and so, the more weakly the two firms compete. Moreover, the price of firm *l* would increase. However, the more high the consumers' preference is, the less the price of the low quality product and the market demand are. In the aspect of revenue of the two firms, with the market preference increasing, the complete monopoly revenues of the two firms would increase, and the duopoly revenue of firm *h* would also increase, however, the duopoly revenue of firm *l* would decrease. With the difference of quality between two firms' product enlarging, the revenues of complete monopoly and duopoly of firm *h* would also increase, and the monopoly revenue of firm *l* would not change but his duopoly revenue would increase.

The revenues of the two firms would change with such uncertainty factor, Y_t, as macro economy, industry policy, or the amount of consumers et al fluctuating. We adopt the similar method of Grenadier (1996) and Huisman et al (2001), the duopoly revenues of the two firms under market uncertainty are:

$$\widetilde{\pi}_i^D = \pi_i^D Y_t \tag{9}$$

The complete monopoly revenues of the two firms are:

$$\widetilde{\pi}_i^M = \pi_i^M Y_t \tag{10}$$

Where, Y_t follows Geometrical Brown Motion:

$$dY_t = \mu Y_t dt + \sigma Y_t dz \tag{11}$$

Where, μ is drift, σ is deviation, μ and σ are constant, dz is standard Winner process.

The relationship of develop costs between the two firms satisfies:

$$K_h = cK_l \tag{12}$$

Where, c is described as the relative difference of develop cost between the two firms, $c > 0$.

Because there exists competition between the two firms, there may be monopolizer, leader who first invests, follower who invests after leader investing in the market, or the two firms invest simultaneously. So, the superscripts, such as *M*, *L*, *F* and *s*, of all variables respectively denote monopolizer, leader, follower, or simultaneous investor.

2 The analysis of technology development investment

The development of technology is one kind of innovation process that is based on the current patent and aims at the demand of market. Meanwhile, the technology uncertainty may be backseat or inessential. For example, in Schwartz's model (2004) of research and development model of patent, in the post-patent stage, there exists no uncertainty of success, and there is only one problem that when the development of technology would be finished. Without considering whether to be successful, we analyze the investment behaviors of competition firms in this section.

2.1 The decision of competing firms

For the purpose of discussion later, we first get the investment threshold of firm *i* and his value in complete monopoly market. According to the method of Dixit & Pindyck (1994), if firm invests at time *t*, firm *i*'s value is:

$$V_i^M = \int_t^\infty e^{-r(\tau-t)}\pi_i^M Y_\tau d\tau - K = \frac{\pi_i^M Y_t}{r-\mu} - K_i \tag{13}$$

Where, *r* is discount rate.

In order to maximize firm *i*'s value, he would choose the appropriate time, *T*, to develop new product. The Bellman decision equation is followed in complete monopoly market:

$$V_i^M = \max_T \left\{ \frac{\pi_i^M Y_T}{r-\mu} - K_i, e^{-rdt} E\left[V_i^M\left(Y_t + dY_t\right)\right] \right\}$$

The equation above is actually optimal stopping problem, the first term of the right hand side of equation is described as the value of firm *i* if he invests immediately. The second term denoted the value of firm *i* if he keeps on waiting for investing. According to the equation above, we can get the value of firm in two states:

$$V_i^M(Y) = \begin{cases} A_0 Y^{\beta_0} & Y < Y_i^M \\ \dfrac{\pi_i^M Y}{r-\mu} - K_i & Y \geq Y_i^M \end{cases} \tag{14}$$

Where, $A_0 = \dfrac{\pi_i^M}{\beta_0(r-\mu)} {Y_i^M}^{1-\beta_0}$, $\beta_0 = 0.5 - \dfrac{\mu}{\sigma^2} + \sqrt{\left(\dfrac{\mu}{\sigma^2} - 0.5\right)^2 + \dfrac{2r}{\sigma^2}}$, the investment critical value is:

$$Y_i^M = \frac{\beta_0}{\beta_0 - 1} \frac{(r-\mu)K_i}{\pi_i^M} \tag{15}$$

When the leader has invested, the follower only gets duopoly revenue if he invests. According to corresponding Bellman equation, we get the follower's value:

$$V_i^F(Y) = \begin{cases} A_i^F Y^{\beta_0} & Y < Y_i^F \\ \dfrac{\pi_i^D Y}{r-\mu} - K_i & Y \geq Y_i^F \end{cases} \tag{16}$$

where, $A_i^F = \dfrac{\pi_i^D}{\beta_0(r-\mu)} {Y_i^F}^{1-\beta_0}$, the follower's investment threshold is:

$$Y_i^F = \frac{\beta_0}{\beta_0 - 1} \frac{(r-\mu)K_i}{\pi_i^D} \tag{17}$$

The motivation of firm's preempting to invest is that he can get a period of complete monopoly revenue before the follower invests. This motivation is one most important factor,

which motivates firms to innovate. When the leader decides to invest, on the one hand, he must pursue to complete monopoly revenue as possible, on the other one hand, he must also consider the effect on his value caused by the investment of the follower.

The value of leader is:

$$V_i^L(Y)=\begin{cases}\dfrac{\pi_i^M Y}{r-\mu}-\dfrac{\left(\pi_i^M-\pi_i^D\right)}{r-\mu}Y_{i-}^F\left(\dfrac{Y}{Y_{i-}^F}\right)^{\beta_0}-K_i & Y<Y_{i-}^F\\ \dfrac{\pi_i^D Y}{r-\mu}-K_i & Y\geq Y_{i-}^F\end{cases} \tag{18}$$

Note, Y_{i-}^F is the follower investment threshold.

The first term of the first line of equation (18) is the complete monopoly revenue if the leader invests, the second term is the reduced revenue caused by the investment of the follower. $\left(Y/Y_{i-}^F\right)^{\beta_0}$ is a random discount rate. The second line of equation (18) tells us that the follower would also invest once $Y\geq Y_{i-}^F$, so, the leader would also get duopoly revenue.

If the two firms invest simultaneously, the value of them is:

$$V_i^s(Y)=\begin{cases}A_i^s Y^{\beta_0} & Y<Y_i^s\\ \dfrac{\pi_i^D Y}{r-\mu}-K_i & Y\geq Y_i^s\end{cases} \tag{19}$$

Where, $A_i^F=A_i^s$. According to equation (19), we can see that, $Y_i^F=Y_i^s$, and $V_i^s(Y)=V_i^F(Y)$

2.2 Equilibrium analysis

If firm wants to be leader, he must satisfy two condition: on the one hand, he would like to be leader, that is, the value of firm when he first invests excesses the value when he secondly invests. There exists one critical value Y_i^{P1}, which satisfies that the value of leader equals to the that of follower. On the other one hand, he would be able to be leader, that is, he can invest before his opponent invests.

According to the first condition, we can get Y_i^{P1} from equation (20):

$$R_i(Y_i^{P1})=V_i^L(Y_i^{P1})-V_i^F(Y_i^{P1})=0 \tag{20}$$

If the two firms' value of preempting to invest excess their following value, there must exist Y_i^{P1} respectively for them. When $Y\geq Y_i^{P1}$, $V_i^L-V_i^F>0$, the firm *i* would like to be leader. When firm *i* invests at Y_i^{P1}, he can get temporary complete monopoly revenue, which can compensate his cost of preempting to invest.

According to the second condition, if $Y_i^{P1}<Y_{i-}^{P1}$, the firm *i* needs not invest at Y_i^{P1}, he can be leader when he invests before Y_t reaches the investment critical value, Y_{i-}^{P1}, of his opponent, the firm *i*- only gets the following value. Thus, the firm *i* has chance to invest at leisure at Y_i^L, which is the investment critical value that firm *i* maximizes his value. Because that firm *i* preempts to invest and firm *i*- is not able to be leader, *i*-'s optimal strategy is to invest at Y_{i-}^F and to get the following value. If firm *i*- preempts to invest before his opponent *i* invests in order to prevent *i* from getting complete monopoly revenue, firm *i*- would endure huge loss. It is not rational for him.

Additionally, if *i*-'s value of being leader always less than his value of being follower, he would not like to be leader and would be most willing to be follower. Thus, firm *i*- invests at Y_{i-}^F. Here, if firm *i* satisfies the first condition, he doesn't worry his opponent invest before his investing, and could invest at the optimal investment threshold, Y_i^L.

Based on the analysis above, if firm *i* can at leisure invest at Y_i^L, the competition equilibrium between the two firms is called as sequential equilibrium. If only if the firm preempts his opponent at Y_i^{P1}, the competition equilibrium is called as preemptive equilibrium.

Because the two firms are asymmetric in product quality and develop cost, firm *l* can take advantage of his low develop cost and preempts to invest, firm *h* can take advantage of his high quality and preempt to invest. Thus, firm *h* or *l* all has chance to invest before their opponents do. According to condition of sequential equilibrium, we can get proposition 1:

Proposition 1: *Given a and α,*

(1) For firm h, there exists only one c_h *:*

$$c_h = \frac{1}{\pi_l^D}\left[\frac{{\pi_h^M}^{\beta_0} - {\pi_h^D}^{\beta_0}}{\beta_0\left(\pi_h^M - \pi_h^D\right)}\right]^{\frac{1}{\beta_0 - 1}} \tag{21}$$

1) *When* $c > c_h$, $V_h^L(Y) < V_h^F(Y)$.

2) *When* $c = c_h$, *there exists only one* Y_h^P, *so* $V_h^L(Y_h^P) = V_h^F(Y_h^P)$.

3) *When* $c < c_h$, *there exist* Y_h^{P1} *and* Y_h^{P2}. *If* $Y_h^{P1} < Y < Y_h^{P2}$, $V_h^L(Y) > V_h^F(Y)$. *If* $Y < Y_h^{P1} \cup Y > Y_h^{P2}$, $V_h^L(Y) < V_h^F(Y)$.

(2) *For firm l, there exists only one* c_l *:*

$$c_l = \frac{K_h}{K_l} = \pi_h^D\left[\frac{{\pi_l^M}^{\beta_0} - {\pi_l^D}^{\beta_0}}{\beta_0\left(\pi_l^M - \pi_l^D\right)}\right]^{\frac{1}{1-\beta_0}} \tag{22}$$

1) *When* $c < c_l$, $V_l^L(Y) < V_l^F(Y)$.

2) *When* $c = c_l$, *there exists only one* Y_l^P, *so* $V_l^L(Y_l^P) = V_l^F(Y_l^P)$.

3) *When* $c > c_l$, *there exist* Y_l^{P1} *and* Y_l^{P2}, *if* $Y_l^{P1} < Y < Y_l^{P2}$, $V_l^L(Y) > V_l^F(Y)$. *If* $Y < Y_l^{P1} \cup Y > Y_l^{P2}$, $V_l^L(Y) < V_l^F(Y)$.

Given market preference, *a*, and difference of products quality, *α*, for firm *h* and *l*, Case 1) and 2) suggest that, when firm *i*'s value of being leader is less than his value of being follower, he would only invest at Y_i^F, his opponent could be leader and at leisure invests at optimal time. Thus, c_h and c_l mark clearly two firms' sequential equilibrium regions. Case 3) suggest that, when $c \in (c_l, c_h)$, the two firms' values of being leader are larger than their values of being follower, that is, $V_i^L(Y) > V_i^F(Y)$, the two firms all have incentive to preempt to invest. In the case of preempting to invest, if $Y_i^{P1} < Y_{i-}^{P1}$, firm *i* needn't invest at Y_i^{P1}, he has chance to invest at leisure at Y_i^L. Thus, in the preemptive equilibrium, firm *i* can invest at threshold $\min\{Y_{i-}^{P1}, Y_i^L\}$ before firm *i-* invests. Firm *i-* can but invest at Y_{i-}^F. For $c \in (c_l, c_h)$, there could possible exists optimal investment threshold, $Y_i^L = Y_i^M$, for the two firms.

The following proposition 2 and 3 give the conditions of marking the sequential and preemptive regions through Y_i^L.

Proposition 2: *Given a, when α < c, there exists only one* $c_m \in (0, c_h)$, *for firm l:*

(1) $c \in (0, c_m)$ 时, $Y_l^L > Y_h^{P1}$; *When* $c \in (0, c_m)$, $Y_l^L > Y_h^{P1}$.

(2) $c \in [c_m, c_h)$ 时, $Y_l^L \le Y_h^{P1}$。 *When* $c \in [c_m, c_h)$, $Y_l^L \le Y_h^{P1}$.

Where $c_m = 1/z^*$, $z^* \in (0, \bar{z})$, z^* *is determined by equation* (23).

$$z^*\alpha\beta_0{\pi_l^M}^{\beta_0} - z^*\beta_0\left(\pi_h^M - \pi_h^D\right){\pi_l^D}^{\beta_0-1} - \left(\beta_0 - 1\right){\pi_l^M}^{\beta_0} - {z^*}^{\beta_0}\,{\pi_h^D}^{\beta_0} = 0 \qquad (23)$$

where, $\bar{z} = \left[\dfrac{\alpha{\pi_l^M}^{\beta_0} - \left(\pi_h^M - \pi_h^D\right){\pi_l^D}^{\beta_0-1}}{{\pi_h^D}^{\beta_0}}\right]^{\frac{1}{\beta_0-1}}$。

Proposition 3: *Given market preference a, when c < α, there exists only one* $c_n \in (c_l, \infty)$, *for firm h:*

(1) *when* $c \in (0, c_n]$, $Y_h^L \le Y_l^{P1}$.

(2) *when* $c \in (c_n, c_h)$, $Y_h^L > Y_l^{P1}$.

Where $c_n = z^*$, $z^* \in (0, \bar{z})$, z^* *is determined by equation* (24).

$$z^*\left(1/\alpha\right)\beta_0{\pi_h^M}^{\beta_0} - z^*\beta_0\left(\pi_l^M - \pi_l^D\right){\pi_h^D}^{\beta_0-1} - \left(\beta_0 - 1\right){\pi_h^M}^{\beta_0} - {z^*}^{\beta_0}\,{\pi_l^D}^{\beta_0} = 0 \qquad (24)$$

Where, $\bar{z} = \left[\dfrac{\left(1/\alpha\right){\pi_h^M}^{\beta_0} - \left(\pi_l^M - \pi_l^D\right){\pi_h^D}^{\beta_0-1}}{{\pi_l^D}^{\beta_0}}\right]^{\frac{1}{\beta_0-1}}$.

According to proposition 1, 2 and 3, figure 1 depicts one of equilibria.

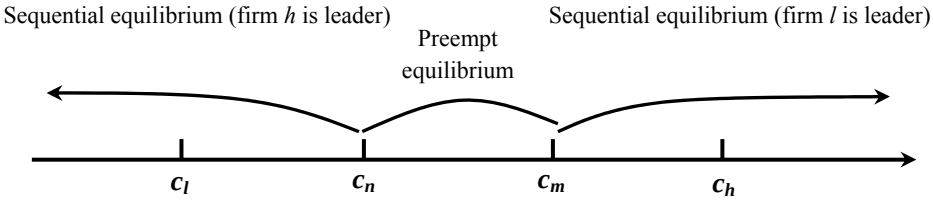

Figure 1 Possible equilibrium

According to figure 1, given market preference, *a*, and product difference, *α*, the change of cost difference, *c*, would cause the two firms' roles change in competition. Because *c* is determined by *a* and *α* in competition, the investment thresholds would change correspondingly. According to figure 1, there are several possible cases:

Case 1: $c_l = c_m$, that is, when $c > c_l$, the equilibrium would change from the sequential region dominated by frim *h* into the sequential region dominated by firm *l*.

Case 2: that is, when $c > c_l$, the two firms would go into preemptive region.

Case 3: $c_n = c_m$, that is, there exists not preemptive region, the equilibrium changes directly from one sequential equilibrium to the other.

Case 4: $c_n = c_h$, that is, when $c < c_h$, firm *h* become leader immediately.

According to case 1, we can get corollary 1.

Corollary 1: *There exists a difference of quality* α^*, *when* $\alpha = \alpha^*$, $c_l = c_m$. α^* *is determined by equation* (22) *and* (23).

Because the quality difference, *α*, is restricted by equation (5) and there exists one of maximal value, $\dfrac{3a}{1-a} + 1$, so we can deduce that there exists a^* and $\alpha^* = \dfrac{3a^*}{1-a^*} + 1$ is hold. Thus, we can get corollary 2:

Corollary 2: *There exists market preference,* $a^* = 0.391$, *when* $a = a^*$, $\alpha^* = \dfrac{3a^*}{1-a^*} + 1$. a^* *is*

determined by $\alpha^* = \frac{3a^*}{1-a^*} + 1$, *equation* (22) *and* (23).

From corollary 2, we know, when $a > a^*$, there exist two cases for a: $\alpha > \alpha^*$ and $\alpha < \alpha^*$. however, when $a < a^*$, there exists only one case, $\alpha < \alpha^*$. These different cases could lead the positions of c_l , c_n, c_m and c_h to change in figure 1, and lead to competition equilibria change.

According to case 2, we can get corollary 3:

Corollary 3: *When* $c < \alpha$, *there exists quality difference*, α^{**}. *When* $\alpha = \alpha^{**}$, $c_l = c_n$. α^{**} *is determined by equation* (22) *and* (24).

According to proposition 2 and 3, if firm i invests at Y_i^L , he must satisfy $Y_i^L \le Y_{i-}^L$. Futher, according to equation (15), firm h must satisfy $c < \alpha$, and firm l must satisfy $c \ge \alpha$. if $c_n \ge c_m$, firm h doesn't satisfy $c < \alpha$, whereas, firm l satisfies $c \ge \alpha$. Thus, when case 3 and 4 exist, the two firms is in the preemptive equilibrium if $c_l < c < c_m$. firm l is in sequential equilibrium if $c > c_m$.

According to numerical solution, we get figure 2, which depicts the changes of c_l , c_n and c_m with change of α, when $a = a^*$ and $a > a^*$.

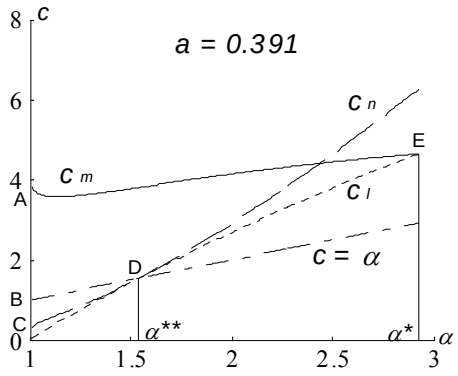

Figure 2-a

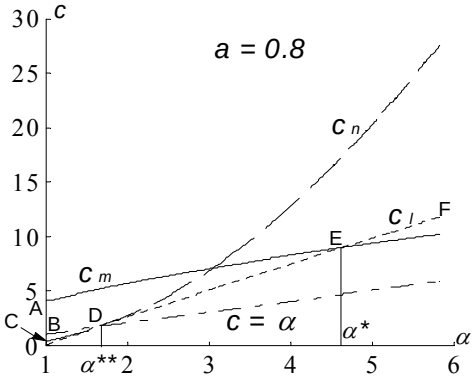

Figure 2-b

In figure 2-a, the region of under CDE is sequential equilibrium dominated by firm h. The region of ACDE is preemptive equilibrium region. The region of above AE is sequential equilibrium dominated by firm l. In figure 2-b, the region of under CDEF is sequential equilibrium dominated by firm h. The region of ACDE is preemptive equilibrium region. The region of above AEF is sequential equilibrium dominated by firm l.

Suppose firm i has synthesis advantage, the existence of the simultaneous equilibrium dominated by firm i must satisfy two conditions: 1) firm i would not like to be leader, that is, the value of simultaneously investing is not less than the value of being leader. 2) His opponent i- would not like to be leader, that is, the i-'s value of being leader is less than the value of being follower, or the value of being leader is less than his value of simultaneously investing. Thus, when initial value of Y_t is very small, given the market preference, a, and the quality difference, α, to satisfy the condition of simultaneous equilibrium, according to 1), when $Y_i^{P1} < Y < Y_{i-}^F < Y_i^s$, the condition $V_i^s(Y) > V_i^L(Y) > V_i^F(Y)$ should be hold. But, according to equation (19), $V_i^s(Y) = V_i^F(Y)$, and $Y_l^s = Y_l^F$, $Y_h^s = Y_h^F$. Thus, there exists contradiction. Based on the analysis, we get proposition 4.

Proposition 4: *When the initial value of* Y_t *is samall, there exists not simultaneous equilibrium between two firms.*

So, if the initial value of Y_t satisfies $Y_t > Y^F = \max\left(Y_l^F, Y_h^F\right)$, there exists simultaneous equilibrium between two firms.

Based on proposition 1, 2, 3, 4, corollary 1, 2, 3, and above analysis, we get corollary 4:

Corollary 4: *In imperfect competition, the equilibrium strategies of two firms differ from product quality is determined by market preference*, *a*, *the difference of product quality,* α, *and develop cost*, *c*. *The relationships of equilibrium strategies and the variables are stated in table* 1.

Table 1 *The relationships of equilibrium strategies and the differences of product quality and develop cost*

<table>
<tr><td rowspan="4">$a < a^*$
the equilibrium is the same as the case of $\alpha < \alpha^*$ when $a > a^*$.</td><td colspan="5">$a > a^*$</td></tr>
<tr><td colspan="3">$\alpha < \alpha^*$</td><td>$\alpha = \alpha^*$</td><td>$\alpha > \alpha^*$</td></tr>
<tr><td>$\alpha < \alpha^{**}$</td><td>$\alpha = \alpha^{**}$</td><td>$\alpha > \alpha^{**}$</td><td rowspan="2">$c \geq c_l = c_m$
Sequential equilibrium dominated by firm with low quality product</td><td rowspan="2">$c > c_l$
Sequential equilibrium dominated by firm with low quality product</td></tr>
<tr><td>$c > c_m$
Sequential equilibrium dominated by firm with low quality product</td><td>$c > c_m$
Sequential equilibrium dominated by firm with low quality product</td><td>$c > c_m$
Sequential equilibrium dominated by firm with low quality product</td></tr>
<tr><td>$c_m > c > c_n$
Preemptive equilibrium</td><td>$c_m > c > c_l = c_n$
Preemptive equilibrium</td><td>$c_m > c > c_n$
Preemptive equilibrium</td><td rowspan="2">$c < c_l = c_m$
Sequential equilibrium dominated by firm with high quality product</td><td rowspan="2">$c < c_l$
Sequential equilibrium dominated by firm with high quality product</td></tr>
<tr><td>$c \leq c_n$:
1) $c < \alpha$
Sequential equilibrium dominated by firm with high quality product
2) $c > \alpha$
Preemptive equilibrium</td><td>$c \leq c_l = c_n$
Sequential equilibrium dominated by firm with high quality product</td><td>$c < c_n$
Sequential equilibrium dominated by firm with high quality product</td></tr>
</table>

2.3 The investment interval between asymmetric firms

We assume that the market initial demand is very low, and there are not new products in market. The two firms observe the change of Y_t, and choose the optimal investment time according to his own and opponent situation. Thus, we can investigate the impacts of market preference, the differences of product quality and develop cost between two firms on the interval between their investments. If firm *i* first invests, firm *i*- can but invest at time T_{i-}^F, when Y_t reach Y_{i-}^F. The expectation interval is described as E (T).

Using equation 1.11 in Harrison (1985), and the application of a simple change in variables, the cumulative distribution function of expectation arrival time, *T*, which Y_0 reach Y^* can be written as

$$\begin{aligned}\mathrm{Prob}[T \leq t] = \mathrm{N}\left[\frac{-\ln\left(Y^*/Y_0\right)+(\mu-0.5\sigma^2)t}{\sigma\sqrt{t}}\right] \\ +\left(\frac{Y^*}{Y_0}\right)^{(2/\sigma^2)(\mu-0.5\sigma^2)} \mathrm{N}\left[\frac{-\ln\left(Y^*/Y_0\right)-(\mu-0.5\sigma^2)t}{\sigma\sqrt{t}}\right]\end{aligned} \tag{25}$$

Where N (Z) denotes the cumulative standard normal distribution function. For $\mu - 0.5\sigma^2 > 0$, E (T) exists and is equal to

$$\mathrm{E}(T) = \frac{\ln\left(Y_{i-}^F/Y_i^P\right)}{\mu-0.5\sigma^2} \tag{26}$$

where $Y_i^P = \left\{Y_i^{P1}, Y_i^L\right\}$.

The investment interval of sequence equilibrium is obtained by proposition 5.

Proposition 5:

1) *If market preference and the product quality difference is fixed, in the region of sequential equilibrium dominated by firm h, the investment interval between two firms is shorten with the develop cost difference increasing. However, in the region of sequential equilibrium dominated by firm l, the result is inverse*.
2) *If develop cost difference is fixed, in the regions of each of firms' sequential equilibrium, the investment interval between two firms is reduced with the increasing of product quality differenc, and is expanded with the increasing of market preference*.

The proposition 5 shows that, in the sequent equilibrium and preemptive equilibrium dominated by firm *h*, fixed other parameters (cost and quality), the investment timing of firm *l* lags behind that of firm *h* is reduced with the difference of develop cost increasing. However, in the sequential equilibrium and preemptive equilibrium dominated by firm *l*, the results is inversed.

Our general intuition is that the high quality and market preference would enhance the advantage of firm *h* and weaken the advantage of firm *l*. That is, if firm *h* invests first, the more high his product quality is, the larger the interval between the two firms investment timing is. If firm *l* invests first, the higher market preference is, the shorter the investment interval between two firms is. Proposition 5 tell us the fact, which the impacts of the difference of product quality and market preference on investment interval between two firms are same. In fact, the purpose of firm's preempting invest is to get a period of complete monopoly return, and the purpose of follower is to get duopoly return. But, the impacts of the difference of product quality and market preference on the complete monopoly return and duopoly return are different for firms with different quality product. We must analyze concretely.

In the aspect of the difference of product quality, in the Bertrand type price competition, if two firms are the same in every respect, the results of competition between two firms are that each firm gets zero return. Thus, to avoid the internecine case, firm *h* should do his best to differentia product quality. According to equation (6), (8), (15) and (17), for firm *l*, the enlarge of difference of quality can't change his complete monopoly return, but can enhance his duopoly return. However, for firm *h*, the enlarge of difference of quality can enhance not only his complete monopoly return but also his duopoly return. Thus, in the region of sequential equilibrium dominated by firm *l*, the reduction of investment interval between two firms is due to the increase of duopoly return of firm *h*. However, in the region of sequential equilibrium dominated by firm *h*, the reduction of investment interval is due to relative speed, the increase of complete monopoly return of firm *h* to the increase of duopoly return of firm *l*, weaken gradually.

In the aspect of market preference, given develop cost and product quality fixed, in the region of sequential equilibrium dominated by each firm, according to equation (6), (8), (15) and (17), in complete monopoly market, the increase of market preference would enhance complete monopoly return of two different firms. That is, based on the trade off return and cost, the increase of market preference would reduce two firms' investment thresholds. However, in duopoly market, the increase of market preference would reduce duopoly return of firm *l* and increase duopoly return of firm *h*, moreover, would raise investment threshold of firm *l* and decrease that of firm *h*. In the region of sequential equilibrium dominated by firm *h*, the increase of market preference makes firm *h* has strong incentive to invest early, at the same time, restrains firm *l*'s incentive to invest as follower. As a result, in the region of sequential equilibrium dominated by firm *h*, with the increase of market preference, the investment interval would enlarge. But, with the market preference increase, firm *l* also has strong incentive to preempt to invest, his preemptive purpose is to get a period of complete monopoly return. In the region of sequential equilibrium dominated by firm *l*, although the increase of market preference can improve the duopoly return of firm *h*, the relative speed of his improvement of duopoly return to firm *l*'s improvement of complete monopoly return enlarge gradually. Thus, the increase of market preference would enlarge the investment interval between two firms.

In a word, the impact of the market preference on the sequential equilibrium dominate by each firm is due to the purpose of them to get a period of monopoly return, but, the market preference and quality difference impact on follower in different way.

Because the impacts of the relationships between every factors on Y_i^{P1} are very complicated in preemptive equilibrium, we will analyze them in the next subsection by numerical analysis.

2.4 Numerical analysis

In this subsection, by numerical analysis, we will investigate characteristic of impacts of the differences of develop cost and product quality, and of market preference on investment interval between firm *h* and *l* in preemptive region.

In figure 3, 4, and 5, we set $\sigma = 0.2$, $\mu = 0.04$, $r = 0.06$. Note, E $(T) > 0$ means that firm *l* invests first, E$(T) < 0$ means that firm *h* invests first.

We first analyze figure 3. Figure 3 pictures the change of E(T) with the difference of product quality changing in the case of $c = 4.5$, $a = 0.5$ (solid line) and $a = 0.7$ (dashed line). Taking example for $a = 0.7$, when $\alpha = 1$, E(T) is positive, according to equation (26). Initially, firm *l* invests first, so the equilibrium is sequential equilibrium dominated by him. With the difference of product quality raising from value one, E(T) is reduced gradually. The results of Proposition 5, which the increase of difference of product quality would decrease the investment interval between two firms in the region of sequential equilibrium, is turned out. While $\alpha = 1.44$, E(T) jumps into his preemptive equilibrium region. With the increase of α, the investment interval, E(T), in preemptive equilibrium region decreases gradually. When $\alpha = 2.63$, firm *h* comes to being leader because of his high quality, here, E(T) drops into the preemptive equilibrium region of him. The advantage of high quality product of firm *h* enhances obviously with the product quality further increasing. When $\alpha = 2.77$, E(T) jumps into the sequential equilibrium region dominated by firm *h*. We can see, in the sequential equilibrium and preemptive equilibrium regions dominated by firm *h*, the absolute value of E(T) decreases gradually with α increasing. The conclusions on quality difference and investment interval in proposition 5 are further turned out. The change trends of E(T) when $a = 0.5$ are similar to the change trends when $a = 0.7$. The different is that investment interval of the former is smaller than that of the latter.

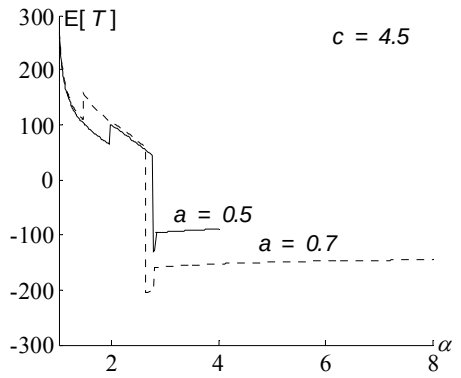

Figure 3

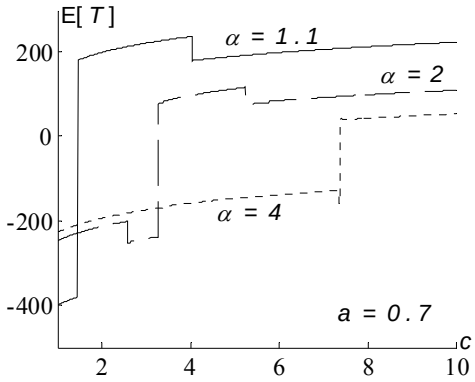

Figure 4

Given market preference fixed, figure 4 investigates the change trends of E(T) when develop cost difference changes when $\alpha = 1.1$ (solid line), $\alpha = 2$ (dashed line) and $\alpha = 4$ (dot line). Taking example for $\alpha = 2$, when $c = 1$, the equilibrium is initially the sequential equilibrium dominated by firm *h*. With *c* increasing, the absolute value of E(T) decreases gradually, when $c = 2.59$, E(T) drops from sequential equilibrium into preemptive equilibrium of firm *h*, in the preemptive equilibrium region, the increase of *c* reduces E(T). When $c = 3.26$, the two firms' advantages are inversed. Equilibrium is changed from preemptive equilibrium dominated by firm *h* into preemptive equilibrium dominated by firm *l*. In this preemptive equilibrium region, E(T) becomes larger gradually. When $c = 5.26$, equilibrium goes from firm *l*'s preemptive equilibrium into his sequential equilibrium, in this sequential equilibrium, E(T) continuously enlarge with the increase of *c*. The change trends of E(T) when $\alpha = 1.1$and $\alpha = 4$ are similar to the change trends when $\alpha = 2$.

According to equation (5), because the market preference restrains the maximal value of difference of product quality, convenient for analyzing, in figure 5, we let the initial value of market preference, $a = 0.2$, the difference of product quality, $\alpha = 1.5$. While analyzing the change trends of E(T) in equilibrium dominated by each firm, we set $c = 4$ for firm *l*, and set $c = 1.4$ for firm *h*. First, for firm *l*, when $a = 0.2$, equilibrium is sequential equilibrium of firm *l*, in this equilibrium, E(T) increases gradually with *a* increasing. When $a = 0.47$, firm *l* jumps form his sequential equilibrium into his preemptive equilibrium. In this equilibrium, E(T) also increases

gradually with *a* increasing. For firm *h*, his equilibrium drops from sequential equilibrium into his preemptive equilibrium with *a* increasing. In his two equilibrium regions, the absolute value of E(*T*) increases with *a* increasing.

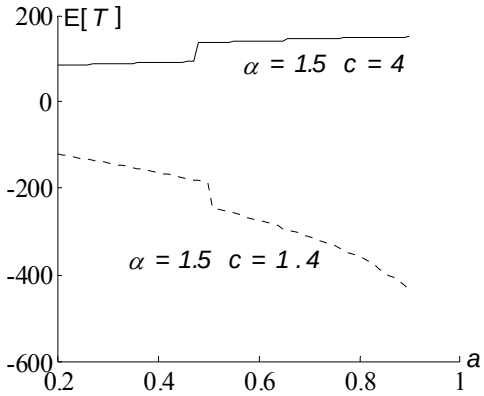

Figure 5

Based on the synthetical analysis of figure 3, 4 and 5, we can get the conclusion 1:

Conclusion 1:

1) *Given market preference and the product quality difference fixed, in the region of preemptive equilibrium dominated by firm h, the investment interval between two firms is shorten with develop cost difference increasing. However, in the region of preemptive equilibrium dominated by firm l, the result is inverse*.
2) *Given develop cost difference fixed, in the regions of each of firms' preemptive equilibrium, the investment interval of two firms is reduced with the increasing of product quality difference between two firms, and is expanded with the increasing of market preference*.

In conclusion, the impacts of the differences of product quality and develop cost, and market preference on the investment interval in the preemptive equilibrium are similar to cases of these factors on that in the sequential equilibrium.

We can see from figure 3, 4 and 5, in particular case, low quality product could appear behind high quality product. The lower market preference is, the larger the difference of quality is, the shorter of the interval that low quality product is lagged behind high quality product. The market preference and the difference of quality give explain to the phenomena of PAS. Although cost is one important factor that impacts on the order of appearance of technology, it is not the crucial factor.

3 Conclusion

Considering the factors of market preference, the differences of quality (level) of product (technology) and develop cost, based on real option approach, this paper constructs decision model to investigate the equilibrium strategy of firms which are asymmetric in cost and quality of product, and time interval, and to reveal the inherent law of evolution of technology.

The results show that, fixed other parameters (cost and quality), in the sequential equilibrium and preemptive equilibrium dominated by firm with high-quality product, the invest timing of firm with low-quality product lags behind that of firm with high-quality product is reduced with the difference of develop cost between them increasing. However, in the sequential equilibrium and preemptive equilibrium dominated by firm with low-quality product, the results is inverse.

This paper shows further that, given develop cost difference fixed, in the regions of each of firms' preemptive and sequential equilibrium, the investment interval between two firms is reduced with the increasing of product quality difference, and is expanded with the increasing of market preference.

References

[1] Grenadier S R. (1996) Strategic exercise of options development cascades and overbuilding in real estate markets[J]. Journal of Finance, 51(5): 1653-1679

[2] Weeds H. (2002) Strategic delay in real options model of R&D competition. Review of Economic Studies, 69(3): 729-747

[3] Huisman K J M. (2001) Technology investment: A game theoretic real options approach. Boston: Kluwer Academic Publishers.

[4] Xiahui, Zengyong. (2005) The strategy of technology innovation of asymmetric firms in imperfect competition. Management Science Journal, 8(1): 30-41. In Chinese.

[5] Dasgupta P, Stiglitz J. Uncertainty, industrial structure, and the speed of R&D. Bell Journal of Economics, 1980, 11:1-28

[6] Shaked A, Sutton J. (1982) Relaxing price competition through production differentiation. Review of Economic Studies, 49: 3-13.

[7] Tirole J. (1988) The theory of industrial organization. Massachusettts Institute of Technology.

[8] Rosenkranz S. (1995) Innovation and cooperation under vertical production differentiation. International Journal of Industrial Organization, (13): 1-22.

[9] Schwartz E S. (2004) Patents and R&D as Real Options Economic Notes by Banca Monte dei Paschi di Siena SpA, 33(1): 23−54

[10] Dixit A K, Pindyck R S. (1994) Investment under Uncertainty. Princeton, NJ: Princeton University Press.

[11] Harrison, J M. (1985) Brownian motion and stochastic flow systems. New York: John Wiley & Sons, Inc.

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