Are Bull and Bear Markets Economically Important?

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ABSTRACT

As fluctuations of the stock market have long been classified into bull and bear markets, investors encounter regime-switching uncertainty in investing. In this paper, we propose a novel way of incorporating regime-switching and model uncertainties into portfolio choice decisions. We find that risks and returns vary greatly across bull and bear regimes, and the certainty-equivalent losses associated with ignoring bull and bear markets are fairly large. Therefore, the economic value of incorporating regime-switching is substantial from an investment perspective.

THE STOCK MARKET GOES through periods of time when equity prices generally rise and other periods when they generally fall, and so investors have long recognized that the market's movements are more like bull and bear markets. Based on the belief that the stock price behavior in a bull market should be different from that in a bear market, Levy (1974) suggests calculating the beta coefficients for bull and bear markets separately. However, Fabozzi and Francis (1977) find that the betas of a market model are not significantly different across bull and bear markets. Based on a different approach, Kim and Zumwalt (1979) reach similar conclusions, although they do find that some securities respond differently in up- and down-markets. Because the bull/bear state is not observable, many of the existing studies use the realized returns to determine whether the market is in a bull or bear state at a given time. This ad hoc way of determining the market state is problematic. For instance, Elton (1999) finds that there are large differences in realized versus expected stock returns over specific periods. Therefore, the unknown bull/bear state and the associated expected returns should be modelled rather than being solely determined by the realized return at certain times.

Hamilton's (1989) path-breaking work provides a rigorous econometric model for analyzing bull and bear markets and the switching between the two regimes. Based on this framework, Schwert (1989) and Hamilton and Susmel (1994) study regime changes and market volatility, while Turner, Startz, and Nelson (1989) find that the S&P 500 index can have different mean and variance across bull and bear markets. Recently, Ang and Bekaert (2002) and Guidolin and Timmermann (2004) examine portfolio decisions when asset returns are subject to regime-switching that provide insights on investments across bull and bear markets. However, existing studies are limited to either the single market portfolio or the market portfolio plus one or two other assets.

In this article, we propose a Bayesian framework for analyzing the regime-switching model, and we study how regime-switching uncertainty affects investors' portfolio decisions.

Our approach distinguishes itself from existing ones in three major ways. First, our approach applies to a large number of assets. In particular, it is feasible to apply it to the Fama-French's (1993) three factors and to the 25 book-to-market and size portfolios, a total of 28 assets. In contrast, it would be very difficult if not possible to apply the existing approaches to such a high-dimensional regime-switching model because the likelihood function is too complex to be evaluated and optimized. In contrast, using a Bayesian approach, we can estimate the model with hundreds of parameters fairly easily, and our approach allows also a deeper understanding of the regime model by obtaining small sample distributions for various functions of interests, such as the probability of the market in a bull market and the standard errors of the associated bull risk-premium. The small sample inference is important because the asymptotic results of the classical approach are either unavailable or questionable in a regime-switching model.

Secondly, the approach in this article provides a way to incorporate regime-switching into portfolio decisions under asset pricing model uncertainty. In financial decision-making, asset pricing models often have useful assertions and suggestions, but they may not be 100% correct. For example, consider the portfolio choice problem when the investment universe consists of the Fama-French three factors and the 25 book-to-market and size portfolios. Under the Fama-French three-factor model, a mean-variance investor would only hold the three factor portfolios and none of the 25 assets because any mean-variance efficient portfolio is a combination of the three factors. The Fama-French three-factor model is, however, likely to be neither perfect nor useless. Therefore, investors encounter asset pricing model uncertainty when they are uncertain about the pricing ability of an asset pricing model. Pástor and Stambaugh (2000) and Avramov(2004), among others, investigate how mispricing uncertainty associated with an asset pricing model can fundamentally change the way we make portfolio decisions in the i.i.d. context. However, it is unknown how such mispricing uncertainty might change the asset allocation decisions when the data follow a more reason-able regime-switching process. The approach of this paper makes answers to this question possible.

Thirdly, our Bayesian framework fully recognizes and incorporates parameter uncertainty into the computation of the portfolio weights and the utility gain of investors. Barberis (2000), among others, shows that estimation errors can alter investors' optimal portfolios significantly. Therefore, besides taking into account the asset pricing model and the regime-switching uncertainties, our approach also incorporates parameter uncertainty into portfolio decisions. In contrast, the existing approaches are classical and ignore estimation errors.

We apply the proposed approach to study the portfolio choice problem of a risk-averse investor faced with a collection of benchmark and non-benchmark assets, and we examine the economic gains that accrue from incorporating regime-switching in light of an asset pricing model, regime-switching, and parameter uncertainties. Because the Fama-French three-factor model is now the standard benchmark model in the current literature, their three factors are chosen as the benchmark assets. The investment universe, in addition, consists of three alternative sets of non-benchmark assets, the Fama-French 25 assets sorted by book-to-market and size, the 10 standard size portfolios of the Center for Research in Security Prices (CRSP), and a set of Fama-French's 17 industry portfolios. In the spirit of Kandel and Stambaugh (1996) and Pástor and Stambaugh (2000), we question what utility gains an investor can achieve if he switches from a belief in no regime-switching return distributions to a belief in regime-switching ones. If the investor, does not believe in regime-switching returns, assuming the normal data-generating process is the true data-generating process (DGP), he would choose his optimal portfolio in the usual way, based on the Fama-French three-factor model. On the other hand, if he believes in regime-switching returns, he would choose his optimal portfolio based on the regime-switching model that incorporates the regime-switching characteristics into decision making. Thus, any utility gains beyond that of using the normal data-generating process may be interpreted as a measure of the economic gain of an investor who switches from a belief in no regime-switching returns to a belief in regime-switching ones.

Empirically, we find strong evidence of regime-switching between bull and bear markets. First, a bull regime has a market expected return of 9.0% per year, substantially different from the market expected return of -10.4% per year in a bear regime.[1] Second, not only does the market portfolio perform very differently across the two regimes, but the other two Fama-French factors, the size and value factors, do the same. The size premium is only -0.1% per year during the bear market, though it shows up significantly at 3.4% per year in the bull market. The value premium alters significantly across regimes, too. It shows up strongly in the downturn, at 13.8% per year, but only weakly in the upturn, at 2.9% per year. All the Fama-French 25 assets have significant positive returns in the bull market, but insignificant, small positive or even negative returns in the bear market. For instance, the first asset S1B1 has a negative return, -17.0% per year, in the bear market, but has a significantly positive return, 13.0 % per year, in the bull market. As for higher moments, the bear market generally has much higher volatilities than the bull market. For instance, the market volatility is 23.9% per year in the bear market, while it is only 12.3% per year in the bull market, an almost 50% drop. For many other assets, such as the size factor (SMB) and the value factor (HML), standard deviations in the bear market are more than twice as high as those in the bull markets.

Due to the statistically significant differences in asset returns across the bull and bear markets, we find that a power utility maximization investor can achieve over 2% certainty-equivalence gains per year in general when he switches from a belief in no regime-switching

[1] If investors believed that the return of the market index portfolio were characterized by the regime-switching model and if they were able to observe the current state, it would be hard to understand why they held the index portfolio in the bear market. Turner, Startz, and Nelson (1989) offer an explanation for this, based on the idea that investors are unable to observe the current state. In fact, the posterior standard deviation of the bear market premium is quite high and may be considered insignificant here. On the other hand, Pesaran and Potter (1993) analyze unobservable stochastic discount factors and economic structures that are compatible with predictions of negative market excess returns.

returns to a belief in regime-switching ones. Hence, not only are bull and bear regimes pronounced in the US stock market, but also there is substantial economic value to incorporate regime-switching into portfolio decisions. Moreover, the economic value depends heavily on the degree of the asset pricing model uncertainty. For example, when the non-benchmark assets are the Fama-French's 17 industry portfolios, the certainty-equivalence gain can be either 4.2% or 10.5% per year, depending on whether the investor dogmatically believes in the Fama-French three-factor model or thinks of it as entirely useless in the bull/bear market. Furthermore, the results are qualitatively similar when the transition probabilities are allowed to be time-varying with the changing conditions of the economy.

The remainder of the paper is organized as follows. Section 2 provides the model to capture regime-switching property in the data and then discusses the framework for making Bayesian portfolio choice decisions. Section 3 applies the proposed approach to the data and reports the empirical results. Section 4 extends the investigation to the case when the transition probabilities are time varying. Section 5 concludes.

I. The Regime-switching Model

To examine regime-switching in assets returns, we adopt a two-regime Markov regime-switching model that is parsimoniously specified, yet flexible enough to capture the idea of bull and bear markets.

A. Model Specification

Assume that the investment opportunity set consists of n risky assets and one riskless asset. In the case without regime-switching, the excess returns of the risky assets over the riskless asset are assumed to follow a multivariate normal distribution:

$$r_t \backsim MVN(E, V), \tag{1}$$

where MVN represents a multivariate normal distribution, E is an $n \times 1$ vector of means, and V is an $n \times n$ variance-covariance matrix.

When a regime-switching model is assumed to be the data-generating process, the data are drawn from two or more possible distributions (regimes). The transition from one regime to another is driven by the realization of a discrete variable (the regime), which follows a Markov chain process. Therefore, at each point of time, the process might stay in the same regime next period for a certain probability. Alternatively, it might switch to one of the other regimes in the next period. Now we decide how many regimes should be in our model. Generally speaking, the more states are allowed in the model, the less restriction is imposed, and the better the model fits the data. Nevertheless, like many other papers, such as Ang and Bekaert (2002), we assume in the rest of the paper, only two states because, with a limited data set, it is difficult to identify three or more states econometrically. In addition, the two-state assumption also parallels the dichotomy between bull and bear markets in investment practices.

Formally, in the case with regime-switching, the returns of the assets are assumed to follow a two-regime Markov regime-switching model with a multivariate normal distribution in each regime:

$$r_t \backsim MVN(E^{s_t}, V^{s_t}), \tag{2}$$

where MVN represents a multivariate normal distribution, E^{s_t} is an $n \times 1$ vector, V^{s_t} is an $n \times n$ matrix, and both are associated with the state at time t, $s_t \in S = (1, 2)$. The transition probabilities are determined by

$$\Pi = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}, \tag{3}$$

where $P = Pr(s_t = 1|s_{t-1} = 1)$ and $Q = Pr(s_t = 2|s_{t-1} = 2)$. Therefore, at each period of time, r_t follows a distribution associated with one of the states in S. Over time, the distribution of r_t switches from the distribution $MVN(E^{s_t}, V^{s_t})$, indicated by the current state s_t, to the distribution $MVN(E^{s_{t+1}}, V^{s_{t+1}})$, indicated by s_{t+1}, the state of the next period. Here, we assume that the transition probabilities are constant. Later, in Section 4, we will study the case when the transition probabilities are allowed to be time-varying.

As shown in the literature, mispricing uncertainty associated with an asset pricing model can fundamentally change the way investors make portfolio decisions. To incorporate asset pricing uncertainty, it is useful to cast the problem into a regression setting. Let $r_t = (y_t, x_t)$, where y_t contains the returns of m non-benchmark positions, and x_t contains the returns of k $(= n - m)$ benchmark positions. In our regime-switching framework, consider the familiar multivariate regression

$$y_t = \alpha^{s_t} + B^{s_t} x_t + u_t, \tag{4}$$

where $s_t \in S = (1, 2)$. Asset pricing models impose restrictions on α^{s_t}. For example, a factor-based pricing model, such as the Fama-French three-factor model, restricts α^{s_t} to be zero. To allow for mispricing uncertainty, following Pástor and Stambaugh (2000) and Pástor (2000), we specify, in a Bayesian framework, the prior distribution of α^{s_t} as a normal distribution conditional on Σ^{s_t},

$$\alpha^{s_t}|\Sigma^{s_t} \sim N\left(0, \sigma_\alpha^2 \left(\frac{1}{(s^{s_t})^2}\Sigma^{s_t}\right)\right), \tag{5}$$

where $(s^{s_t})^2$ is a suitable prior estimate for the average diagonal elements of Σ^{s_t}. The above alpha-Sigma link is also explored by MacKinlay and Pástor (2000) in the frequentist set-up. The numerical value of σ_α represents an investor's level of uncertainty about a given model's pricing ability. When $\sigma_\alpha = 0$, the investor believes dogmatically in the model, and there is no mispricing uncertainty. On the other hand, when $\sigma_\alpha = \infty$, the investor believes that the pricing model is entirely useless.[2]

To relate α^{s_t} and B^{s_t} to the earlier parameters E^{s_t} and V^{s_t}, consider the corresponding partition

$$E^{s_t} = \begin{pmatrix} E_1^{s_t} \\ E_2^{s_t} \end{pmatrix}, \; V^{s_t} = \begin{pmatrix} V_{11}^{s_t} & V_{12}^{s_t} \\ V_{21}^{s_t} & V_{22}^{s_t} \end{pmatrix}. \tag{6}$$

Under the usual multivariate normal distribution, it is clear that the distribution of y_t conditional on x_t and s_t is also normal, and the conditional mean is a linear function of x_t. Hence

$$E(y_t|x_t, s_t) = E_1^{s_t} + V_{12}^{s_t}(V_{22}^{s_t})^{-1}(x_t - E_2^{s_t}), \tag{7}$$

$$Var(y_t|x_t, s_t) = V_{11}^{s_t} - V_{12}^{s_t}(V_{22}^{s_t})^{-1}V_{21}^{s_t}. \tag{8}$$

Therefore, the parameters α^{s_t}, B^{s_t}, and the earlier parameters E^{s_t}, V^{s_t} obey the following relationship:

$$\alpha^{s_t} = E_1^{s_t} - B^{s_t}E_2^{s_t}, \qquad B^{s_t} = V_{12}^{s_t}(V_{22}^{s_t})^{-1}. \tag{9}$$

The covariance matrix of the regression residual at time t is,

$$\Sigma^{s_t} = V_{11}^{s_t} - B^{s_t}V_{22}^{s_t}(B^{s_t})'. \tag{10}$$

[2]We assume that investors have the same prior belief in the pricing ability of an asset pricing model across bull and bear markets.

And the returns of the benchmark positions follow a normal distribution,

$$x_t \backsim N(E_2^{s_t}, V_{22}^{s_t}).$$

The remaining priors are fairly standard and detailed in the appendix. Then, the complete prior on all the parameters can be written as

$$p_{0c}(\theta) = p_0(\alpha^1|\Sigma^1)p_0(\alpha^2|\Sigma^2)p_0(\Sigma^1)p_0(\Sigma^2)p_0(B^1)p_0(B^2)p_0(E_2^1)p_0(E_2^2)p_0(V_{22}^1)p_0(V_{22}^2)p_0(P, Q). \tag{11}$$

With the priors given above, the investor forms his posterior belief $p_c(\theta|R)$ in light of the data $\{R : r_t, t = 1, \ldots, T\}$,

$$p_c(\theta|R) \propto p_{0c}(\theta) \times p_c(R|\theta), \tag{12}$$

where $p_c(R|\theta)$ is the likelihood of the data.

When the investor solves his expected utility maximization problem, he integrates over the predictive distribution of the asset returns. Therefore, in a Bayesian decision context, as also demonstrated by Zellner and Chetty (1965), the predictive densities are of primary interest. In our case, the predictive distribution of $r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}$ can be computed as

$$p_c(r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}|R) = \int_\theta p_c(r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}|R, \theta)p_c(\theta|R)d\theta, \tag{13}$$

where $\hat{T}$ is the number of periods of the investor's investment horizon. For the one-regime model, we can obtain the predictive distribution by simplifying the above procedure through dropping the regime-switching part.

The two-regime model is quite parsimoniously specified. Nonetheless, in a high-dimensional setting, the model can contain hundreds of parameters. For instance, with the Fama-French 25 assets and the three factors, the total number of parameters in the model is 870. In addition, the state variable s_t is not observable. To deal with such complex high-dimensional models, we propose, for the first time, a Bayesian estimation procedure based on a Gibbs sampling procedure, a special case of the Markov chain Monte Carlo (MCMC) methods.[3] The basic idea of this approach is as follows. First, the model parameters are divided into a few blocks. Then, draw samples from the conditional densities of each block of the parameters conditioning on the values of other parameters. In particular, applying the data augmentation technique, the unobservable state is treated as an unknown parameter and is simulated conditioning on the other parameters of the model. The sampling procedure is repeated for a large number of times. After a transient or burn-in period, the Markov chain is assumed to have converged, and the sampled draws are collected as variates from the posterior distribution. The detail of our MCMC algorithm is in the appendix. Once we have the sampled draws from the posterior distribution, it is straightforward to estimate the parameters, such as the means and variances, and to produce the predictive distribution of the asset returns.

B. Investing under Regime-switching

Applying the proposed regime-switching model to the data, we find that regime-switching is statistically present in the US stock market (as shown later in Section 3). So, the question arises as to how important the regime-switching is from an investor's portfolio decision point of view. In this section, we analyze how to make portfolio decisions with the existence of regime-switching in asset returns.

Following the framework of Pástor and Stambaugh (2000), we consider an investment universe that contains cash plus n spread positions. The Pástor and Stambaugh set-up

[3]The MCMC methods are simulation-based methods designed to sample densities that are otherwise intractable. The methods generate sample draws from the target distribution, the posterior distribution, by a recursive Monte Carlo sampling process where the transition kernel of this Markov process is constructed such that its limiting invariant distribution is the target distribution. In the article, we use a special case of the MCMC method, the Gibbers sampling procedure.

defines spread position i, constructed at the end of period $t-1$, as a purchase of one asset coupled with a short sale of an equal amount of another. The two assets are denoted as L_i and S_i, and their rates of return in period t are denoted as $R_{L_i,t}$ and $R_{S_i,t}$. Then a spread position of size X_i has a dollar payoff $X_i(R_{L_i,t} - R_{S_i,t})$. Since Regulation T requires the use of margins for risky investments, a constant $c > 0$ is used to characterize the degree of margin requirements. The spread position involves at least one risky asset which, without loss of generality, is designated as asset L_i. If the other asset of position i, S_i, of size X_i is risky as well, then $(2/c)|X_i|$ dollars of capital are required. Otherwise, $(1/c)|X_i|$ dollars of capital are required. For example, if $c = 5$, the set-up implies a 20% margin. We assume that there are I spreads that have risky assets on both sides and J spreads that have risky assets on the long side and the risk-free asset on the short side.

The total capital required to establish the spread positions must be less than or equal to the investor's initial wealth at time T, W_T. That is,

$$\sum_{i\in\Lambda}(2/c)|X_i| + \sum_{i\notin\Lambda}(1/c)|X_i| \leqslant W_T, \tag{14}$$

where Λ denotes the set of positions in which S_i is risky, or

$$\sum_{i\in\Lambda} 2|w_i| + \sum_{i\notin\Lambda} |w_i| \leqslant c, \tag{15}$$

where $w_i = X_i/W_T$. Let r_t denote an n-vector with i-th element $r_{i,t}$ $(= R_{L_i,t} - R_{S_i,t})$ representing the return of the i-th risky position at time t. At time T, a buy-and-hold power utility investor with $\hat{T}$ investment period solves

$$\max_w \int \frac{W_{T+\hat{T}}^{(1-\gamma)}}{1-\gamma} p_c(r_{T+\hat{T}}|R)dr_{T+\hat{T}}, \tag{16}$$

subject to the constraint in equation (15), where $p_c(r_{T+\hat{T}}|R)$ is the predictive density of the returns and $W_{T+\hat{T}}$ is the wealth at $T+\hat{T}$ when time T wealth W_T is assumed to be 1\$. More specifically,

$$W_{T+\hat{T}} = (1+R_f)^{\hat{T}} + \sum_{i=1}^{I} w_i r_{i_{T+\hat{T}}} + \sum_{j=1}^{J} w_j r_{j_{T+\hat{T}}}, \tag{17}$$

where

$$r_{i_{T+\hat{T}}} = (1+R_{iL_{T+1}})(1+R_{iL_{T+2}})\cdots(1+R_{iL_{T+\hat{T}}}) - (1+R_{iS_{T+1}})(1+R_{iS_{T+2}})\cdots(1+R_{iS_{T+\hat{T}}}), \tag{18}$$

$$r_{j_{T+\hat{T}}} = (1+R_{j_{T+1}})(1+R_{j_{T+2}})\cdots(1+R_{j_{T+\hat{T}}}) - (1+R_f)^{\hat{T}}, \tag{19}$$

and R_f denotes the rate of return of a riskless asset.

When there is no margin requirement, $c=\infty$, the first order condition is

$$\int [W_{T+\hat{T}}^{-\gamma} r_{T+\hat{T}}] p(r_{T+\hat{T}}|R) dr_{T+\hat{T}} = 0. \tag{20}$$

Analytical solution does not seem feasible. To solve (16) subject to (15) or to solve (20) numerically, we have to evaluate high dimensional integrals, and Monte Carlo simulation seems the only tractable approach. Hence, given returns from their predictive density $p(r_{T+\hat{T}}|R)$, (16) and (20) can be well approximated by

$$\frac{1}{M}\sum_{q=1}^{M}\left\{\frac{(W_{T+\hat{T}}^{q})^{1-\gamma}}{1-\gamma}\right\}, \tag{21}$$

and

$$\sum_{q=1}^{M}\left\{(W_{T+\hat{T}}^{q})^{-\gamma} r_{T+\hat{T}}^{q}\right\} = 0, \tag{22}$$

respectively, where $W_{T+\hat{T}}^{q} = (1+R_f)^{\hat{T}} + \sum_{i=1}^{I} w_i r_{i_{T+\hat{T}}}^{q} + \sum_{j=1}^{J} w_j r_{j_{T+\hat{T}}}^{q}$,

$$r_{i_{T+\hat{T}}}^{q} = (1+R_{iL_{T+1}}^{q})(1+R_{iL_{T+2}}^{q})\cdots(1+R_{iL_{T+\hat{T}}}^{q}) - (1+R_{iS_{T+1}}^{q})(1+R_{iS_{T+2}}^{q})\cdots(1+R_{iS_{T+\hat{T}}}^{q}), \tag{23}$$

$$r^q_{j_{T+\hat{T}}} = (1+R^q_{j_{T+1}})(1+R^q_{j_{T+2}})\cdots(1+R^q_{j_{T+\hat{T}}}) - (1+R_f)^{\hat{T}}, \tag{24}$$

and $R^q_{iL_{T+t}}, R^q_{iS_{T+t}}, R^q_{j_{T+t}}$, $t = 1, 2, \cdots, \hat{T}$, are the q-th draw and M is the total number of draws.

The key issue, then, lies in how to draw returns from their predictive density. To obtain predictive distribution of future returns $R_{iL_{T+t}}, R_{iS_{T+t}}, R_{j_{T+t}}$, we first obtain the excess return $R_{iL_{T+t}} - R_{iS_{T+t}}$ and $R_{j_{T+t}} - R_f$.[4] Then we obtain the return of the asset on the short side of the i-th spread, $R_{iS_{T+t}}$. Finally, we add the excess return $R_{iL_{T+t}} - R_{iS_{T+t}}$ on $R_{iS_{T+t}}$ and the excess return $R_{j_{T+t}} - R_f$ on R_f to obtain the raw return $R_{iL_{T+t}}$ and $R_{j_{T+t}}$. As for the two-regime model, to obtain the excess return $R_{iL_{T+t}} - R_{iS_{T+t}}$, $R_{j_{T+t}} - R_f$, and the raw return $R_{iS_{T+t}}$, we need to forecast the state in the future first. Now we show how to obtain the states in the future. For each one of the sample draws of the parameters $\hat{\theta}$ from their posterior $p_c(\theta|R)$ and the associated state at the time T, we draw a series of states for the time periods from $T+1$ to $T+\hat{T}$ according to the two-regime model. Once the state at a future time period $T+t$ is given, the excess returns $R_{iL_{T+t}} - R_{iS_{T+t}}$, $R_{j_{T+t}} - R_f$ at $T+t$ can be drawn from the conditional distributions, which are multivariate normal distributions with parameters $\hat{\theta}$. The raw return $R_{iS_{T+t}}$ can be drawn from its conditional distribution, a normal distribution with parameters drawn from the posterior $p(\theta_S|R_S)$, where $R_S = (R_{iS_t})$, $i = 1, 2, \cdots, I$, and $t = 1, 2, \cdots, T$.

C. Performance Measures

Clearly different predictive distributions may be derived from the one-regime model than from the two-regime model. Because the investment strategy optimal to one predictive distribution is not guaranteed to be optimal to the other predictive distribution, two different optimal investment strategies may be implied. One is optimal to the predictive distribution

[4] To simplify the notation, we drop the superscript q here.

derived from the one-regime model. The other is optimal to the predictive distribution derived from the two-regime model. Once these two different optimal portfolios are derived, we can evaluate the importance of incorporating regime-switching by gauging the differences between the two optimal portfolios. If the two optimal portfolios are very different, we can conclude that the regime-switching has a significant effect on portfolio decisions and that it is important to incorporate regime-switching. However, the differences cannot be simply evaluated based on the portfolio weights alone. Because of the correlations among the payoffs of risky positions, the performance of two portfolio choices can be similar even when the portfolios are quite different in position-by-position allocations. In this section, based on Pástor and Stambaugh (2000), we construct an economic measure to gauge the differences between two portfolios.

A portfolio allocation w_{2R}, which is optimal under the predictive distribution from the two-regime model (given a certain mispricing uncertainty), is easily computed and implies an expected utility of u_{2R}, computed by plugging w_{2R} into (21). Another allocation, w_{1R}, which is optimal under the predictive distribution from the one-regime model (with the same mispricing uncertainty), gives rise to an expected utility of u_{1R}, computed by plugging w_{1R} into (21).[5] Then the difference between the certainty-equivalent excess returns is

$$CE = [(1-\gamma)\hat{u}_{2R}]^{\frac{1}{1-\gamma}} - [(1-\gamma)\hat{u}_{1R}]^{\frac{1}{1-\gamma}} \,. \tag{25}$$

This is the 'perceived' certainty-equivalent gain to a power utility investor with a relative risk aversion coefficient equal to γ when he switches from a belief in the one-regime model to a belief in the two-regime model.[6] In other words, the certainty-equivalent gain is the amount

[5] Notice that both expected utilities are evaluated based on the same $r^q_{i_{T+\hat{T}}}$ and $r^q_{j_{T+\hat{T}}}$, drawn from the predictive distribution of the two-regime model, which is the assumed true data-generating process capturing the fluctuations of the stock market.

[6] In the same spirit, Fleming, Kirby and Ostdiek (2001) provide a similar but different measure in the classical framework.

of loss to the investor when he is forced to hold the portfolio optimal to the one-regime model yet not optimal to the regime-switching model.

Nevertheless, one potential problem is that the simple buy-and-hold strategy may not be able to fully capture the economic value of incorporating regime-switching into portfolio decisions. This is clearly a valid point since optimally exploiting regime-switching may require re-balancing the portfolio compositions if the original compositions are no longer optimal with respect to the new expectation of or belief in future regime-switching, based on newly arrived information.[7] But the goal here is to show that there is value in incorporating regime-switching. If we find that a simple buy-and-hold strategy can already yield substantial utility gain, that strategy is enough to achieve the goal since it provides a lower bound to the full value. Of course, if the simply strategy cannot yield a significant value, further studies on better and more complex strategies are necessary.

[7]Recent studies such as Aït-Sahalia and Brandt (2001) and Ang and Bekaert (2002) suggest that the inter-temporal hedging demands are typically very small components of the optimal portfolio allocations.

II. Empirical Results

In this section, we report the empirical results of applying the proposed methodology in Section 2 to the data. First, we examine whether there exist bull and bear regimes in the data. In particular, we provide estimates of the means, variances, covariances, correlations, betas and alphas for the bull market and the bear market, and show that many of them can be drastically different across regimes. Second, we illustrate how the bull market and the bear market switch from one to the other in the history. Finally, we investigate the economic importance of incorporating regime-switching into portfolio selections in terms of certainty-equivalent gains.

A. Regimes in Means and Variances

First, the data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios formed on size and book-to-market from July 1963 through December 2002, available from French's website.[8] To visually illustrate how different the means and variances can be across the bull market and the bear market, we plot the posterior distribution curves of the means and standard deviations of the Fama-French three factors in Figure 1. It is obvious that there exist two regimes with very different means and volatilities for the Fama-French three factors. In particular, as shown in graph (c) and (f), one regime has higher market mean return and lower market volatility than the other regime.

Insert Figure 1 about here.

From now on, we label the regime associated with the lower market return as the bear market, and the regime associated with the higher market return as the bull market. Table I reports the means of the Fama-French 25 portfolios and three factors in the bull market

[8] We are grateful to Ken French for making this data and many others used below available at his website: www.mba.tuck.dartmouth.edu/pages/faculty/ken.french.

and the bear market for a variety of mispricing errors $\sigma_\alpha = 0$, 3%, and ∞, respectively. Generally speaking, whatever the mispricing errors, all assets in the bear market have lower returns and even negative returns in some cases.[9] In particular, the market return is 9.0% per year in the bull market, but it is -10.4% per year in the bear market. The difference is -19.4%. The posterior standard deviation (pstd) is 9.9%. Therefore, the market index performs much better in the bull regime than in the bear regime. In addition, in the bear market, the size premium almost disappears, becoming a small negative number, -0.1% per year, but it is 3.4% per year in the bull market. The difference is -3.5% with a pstd of 7.8%. The difference seems not very significant statistically. The reason is that the pstd for the mean return in the bear market is very high, 7.3%, although the pstd is only 1.7% in the bull market. As for the value premium, it is 13.8% per year in the bear regime, but it is much smaller in the bull market, 2.9% per year. Again, the pstd for the mean return in the bear market is very high, 7.1%, compared to 1.6% in the bull market.

Insert Table I about here.

As for the Fama-French 25 non-factor assets, they generally have a significantly positive mean in the bull, with a low pstd, but a negative or a small positive mean in the bear, with a high pstd. For instance, under a dogmatic belief in the Fama-French three-factor model, the first asset S1B1 has a negative return of -17.0% per year with a high pstd of 17.2% per year in the bear, while a significantly positive return of 13.0% per year with a small pstd of 4.1% per year in the bull. As shown by Table I, the results are very similar across different beliefs in the Fama-French three-factor model.

As for the volatilities, Table II shows that the bear market generally has much higher volatilities than the bull market across different beliefs in the Fama-French 3-factor model. For instance, the market volatility is 23.9% per year in the bear while 12.3% per year in the

[9] The only exception is the value factor, HML, which has a higher return in the bear market.

bull, an almost 50% drop. For many other assets, such as the size factor, SMB, the value factor, HML, and the first Fama-French 25 assets, S1B1, the standard deviations in the bear market are even more than two times bigger than those in the bull markets. In addition, the pstd's of the volatilities (and the pstd's of the difference in volatilities) are much smaller than those of the mean returns reported in Table I. This is consistent with the common belief that variances (and differences in variances) are easier to estimate than means (and differences in means). As a result, the differences in volatilities are more significant than the differences in means across bull and bear markets.

Insert Table II about here.

B. Regimes in Covariances, Correlations, Betas and Alphas

Recently, there have been a number of studies on asymmetric co-movements between asset returns and market indices that suggest stocks move more often with the market when the market goes down than when it goes up. In this paper, we examine whether the co-movements of the returns of non-factor assets with the Fama-French three factors are different across bull and bear markets. We first consider the differences in covariances across the bull market and the bear market since covariances are usually direct inputs of parameters for optimal portfolio choice.[10] Then, we examine the differences in correlations across regimes because of the importance of correlations from the risk management perspective of hedging exposures. Finally, we analyze betas (and also alphas) since they are closely related to asset pricing theories.

First, we consider the differences in the covariances between the Fama-French 25 assets and the Fama-French three factors across the bull and bear markets. Table III reports the covariances with the Fama-French three factors under a diffuse prior belief in the Fama-

[10]Chamberlain (1983) and Owen (1983) investigate the distributions that imply mean-variance analysis.

French 3-factor model.[11] From the third and fourth columns, we can see that the covariances with the market portfolio are generally three to four times larger in the bear market than in the bull market. In addition, the differences in covariances across bull and bear markets are significant since the pstd's are relatively small compared to the mean differences. For instance, the covariance between the market index and S1B1, the first Fama-French 25 asset, is on average 56.1% higher in the bull market than in the bear market. This mean difference in covariance is almost four times the pstd of 15.6%, the biggest pstd of the differences in covariances across bull and bear markets. As for the covariances with the size factor SMB, as shown by the seventh and eighth columns, the covariances are in general three to five times larger in the bear market than in the bull market except for the last two assets, S5B4 and S5B5. In addition, the difference becomes less significant when the size increases. For instance, the difference is 5.7% while the pstd is 4.7% for S5B1.

Insert Table III about here.

The second and third columns from the right report the covariances with the value factor HML for the bear market and the bull market, respectively. In each of these two columns, from the top to the bottom, the covariances in general increase from a bigger negative number to a smaller negative number or a small positive number for each group of five assets within the same size category and with the book-to-market from low to high. However, the spread of the covariances in the bear market is much wider than that in the bull market for each of the five groups. For example, for the largest size group, the covariances increase from -20% to 7% in the bear market but only increase from -4.8% to 0.5% in the bull. In addition, the difference across regimes becomes less significant within the same size category and with the book-to-market from low to high.

[11] We omit the very similar results under informative priors on the Fama-French three-factor model, results that may be due to the fact that the priors on pricing ability are primarily about the expected returns and not much about the volatilities and covariances.

Secondly, Table IV and Table V report the correlations and betas with the three factors under a diffuse prior belief in the Fama-French three-factor model.[12] Unlike the large differences across regimes in the covariances, the correlations and the betas between the Fama-French 25 assets and the three factors are quite similar across the bull market and the bear market. For instance, the correlation between S1B1, the asset with the smallest size and book-to-market, and the market portfolio is 0.783 in the bull and 0.784 in the bear.

Insert Table IV about here.

Insert Table V about here.

Finally, we consider the mispricing α's. If the investor has a dogmatic belief in the Fama-French three-factor model in the bull/bear market, then the data will not be able to change his prior belief. Therefore we have zero posterior mispricing α's in both the bull market and the bear market. This is illustrated by the third and fourth columns in Table VI. However, when the investor has a undogmatic or even a diffuse prior on the pricing ability of the Fama-French three-factor model, then he will update his prior in light of the data to form his posterior distribution of the mispricing α's.

Insert Table VI about here.

In general, the mispricing α's are significant in many cases in the bull with low pstd's, while they are less significant in the bear because of high pstd's. In addition, the differences in the mispricing α's are not very significant because of high pstd's as well. Over all, the bull and bear markets are very different in variances and covariances, though less significantly different in means, correlations, betas and mispricing α's.

C. How do the Bull and Bear Markets Switch from One to the Other?

[12] Again, we omit the very similar results under informative priors on the Fama-French three-factor model.

Knowing the bull and bear markets are very different from each other in volatilities and means, we now turn our attention to examining how they switch from one to the other. Because the regime-switching is not observable, we assume that the investor has a diffuse prior on the probability of switching between the bull market and the bear market before looking at the data. Then he updates his prior based on the observed data to obtain the posterior using the Bayesian rule. Under a diffuse prior on the probability of switching between the two regimes, the probabilities of switching from the bull to the bear and from the bear to the bull are both 50%. Because there are only two states, the prior probabilities of staying in the bull and bear market are both 50% as well.[13] In light of the data, the Fama-French three factors from July 1963 through December 2002, the probability of staying in the bull increases from 50% to 92.6%, while the probability of staying in the bear increases from 50% to 74.4%, as shown in Panel A of Table VII. Correspondingly, the probability of switching from the bull to the bear drops from 50% to only 7.4%, while the probability of switching from the bear to the bull drops from 50% to 25.74 %. Therefore, the two regimes are persistent, in particular for the bull market.[14] In contrast, the iid one-regime model does not capture this serial dependence in the time series. As a result, besides the significant difference in variances, covariances, and mean returns across the states, this persistence of regimes may also add to the superiority of the regime-switching model to the one-regime model.

Insert Table VII about here.

[13]There are some related articles about regime changes. For example, Pesaran and Timmermann (1995), Pastor and Stambaugh (2001), and Bekaert, Harvey and Lumsdaine (2002), find that the asset return dynamics are unstable and subject to possible structural breaks. But their studies examine irreversible regime-switching or structural shifts where the data cannot switch back to a regime once they have left. In this article, the data can switch back to a regime, such as a bear market, after switching to another regime, such as a bull market, and staying in the bull market for certain periods.

[14]Das and Uppal (2001) model jumps in correlation using a continuous time jump model when international equity returns are affected by correlated jumps across countries. However, their approach with transitory jumps cannot capture the persistence of the bull and bear markets.

In Figure 2, we plot the empirical probability of being in the bear market from July 1963 through December 2002. We compute this probability by dividing the number of draws associated with the bear market by the number of total sample draws. This figure shows that almost all the recession periods, between a NBER peak and the following NBER trough, have high bear market probabilities. This coincidence of recessions and bear markets implies that we usually have a bear market during the bad times of the US economy.[15] However, there are quite a few short periods associated with high bear market probabilities but not classified as NBER recessions. One possible reason may be that stock markets also react to sectoral or shorter-lived contractions in the economy not designated recessions by the NBER dating, and so the frequency of stock market fluctuations between bull and bear markets can be higher than that of economy fluctuations between expansions and contractions. In addition, from Figure 2, it seems that bear markets are also likely to occur during some extreme events, such as the oil price shocks in the 1970s, the October 1987 stock market crash, and the 1997 Asian flu.[16]

Insert Figure 2 about here.

D. Performance Measures

In this section, we examine the economic costs of ignoring regime-switching in asset returns. We assume that the power utility maximizing investor has a relative risk aversion equal to 5. Table VIII reports the percentage points of $u_{2R} - u_{1R}$, referred to as certainty-equivalent gains in the table. The certainty-equivalent gain is the "perceived" gain of utilizing

[15] Hamilton and Lin (1996) find that economic recessions are the single largest factor driving the variances of stock returns. Bry and Boschan (1971) and the applications of their approach in King and Plosser (1994), Watson (1994), and Harding and Pagan (2002) detect turning points in business cycles.

[16] Santa-Clara and Valkanov (2003) find that the excess return in the stock market is higher under Democratic than Republican presidencies, and there is no difference in the riskiness of the stock market across presidencies that could justify a risk premium. Therefore, they conclude that the difference in returns through the political cycle is a puzzle.

regime-switching or the loss to the investor when he is forced to hold the portfolio optimal in the one-regime model that may be no longer optimal in the regime-switching model.

With varying degrees of pricing errors on the Fama-French three-factor model, columns 2, 3, and 4 of Panel A of Table VIII report the gains for a variety of investment horizons from one month to 12, 36, and 60 months, respectively, in the case of constant transition probabilities (C-RS). It is seen that, without margin requirements ($c = \infty$), the gains can be fairly large. With a 36-month horizon and a diffuse belief on the Fama-French three-factor model, the gain is 65.6%. When the investment horizon is shortened from 60 months to 12 months, the gain becomes 12.0%, smaller but still significant. With a dogmatic belief in the Fama-French three-factor model, the gain shrinks further to 1.4%. In additon, the gain is 18.9% with a modest 3% mispricing uncertainty on the Fama-French three-factor model when the Fama-French 3-factor model is not assumed either useless or 100% correct.

Insert Table VIII about here.

Now we consider the results when margin requirements are imposed. With a 20% margin requirement ($c = 5$), the gains become much smaller in general, as shown by the columns 2, 3 and 4 of Panel D of Table VIII. However, the gains are still significant in most cases. For instance, with a 12-month horizon and 3% mispricing uncertainty on the Fama-French three-factor model, the gain is 5.3%, certainly a significant number, though not so impressive as 18.9% in the case of no constraint. Overall, these results show that it is too costly to ignore the regime-switching in the Fama-French three factors when the investment universe consists of the Fama-French three factors and the Fama-French 25 assets.[17]

[17]A noteworthy fact is that the regimes in the non-factor Fama-French 25 portfolios are assumed to be the same as the regimes of the three factors. Although it is possible that the regimes of the Fama-French 25 portfolios are different from the regimes of the three factors, the difference between the regimes of the Fama-French 25 assets and those of the three factors may be primarily driven by idiosyncratic forces. In this article, we focus on the regime-switching in the Fama-French three factors, which can be consider as a proxy for the 'systematic' regime-switching in the US stock market.

Now we examine the sensitivity of the above results on the economic value of incorporating regime-switching to alternative investment universes. For this purpose, we apply the above analysis to two alternative investment universes. The first consists of the Fama-French three factors and the 10 Center for Research in Security Prices (CRSP) size portfolios. The second consists of the Fama-French three factors and the Fama-French's 17 industry portfolios. The columns 2, 3, and 4 of Panel B and C of Table VIII report the gains without margin requirements for the two alternative investment universes, respectively. In general, the gains for the two alternative investment universes are smaller than those for the Fama-French 25 assets. For instance, the largest gains are 17.7% and 68.5% for the 10 CRSP size portfolios and the Fama-French's 17 industry portfolios, respectively, while the largest gain is 307.5% for the Fama-French 25 assets. Nevertheless, the gains are in general still economically important. For instance, at a 12-month horizon and with a 3% mispricing error on the Fama-French 3-factor model, the gain for the 10 CRSP size portfolios is 3.7%, and the gain for the Fama-French's 17 industry portfolios is 12.2%. In addition, columns 2, 3 and 4 of Panel E and F of Table VIII report the gains with a 20% margin for the two alternative investment universes, respectively. It is easy to see that, with a 20% margin requirement, the gains are smaller but, in general, still significant. Hence, we can conclude that it is too costly to ignore the regime-switching in the Fama-French three factors when the investment universe consists of the Fama-French three factors and any of the three sets of assets, the Fama-French 25 assets, the 10 CRSP size portfolios, or the Fama-French's 17 industry portfolios.

III. Time-varying Transition Probabilities

The constant transition probabilities assumption in the above analysis can be relaxed to allow the transition probabilities to be time-varying to capture investors' information on state transition probabilities. For parsimony, following Perez-Quiros and Timmermann

(2000), we use a simple summary statistic to predict the future transition probabilities as follows:

$$\Pi = \begin{bmatrix} P_t & 1 - P_t \\ 1 - Q_t & Q_t \end{bmatrix}, \tag{26}$$

where $P_t = Pr(s_t = 1|s_{t-1} = 1) = \frac{1}{1+exp(a_1+b_1 LEI_{t-2})}$, $Q_t = Pr(s_t = 2|s_{t-1} = 2) = \frac{exp(a_2+b_2 LEI_{t-2})}{1+exp(a_2+b_2 LEI_{t-2})}$, and LEI_{t-2} is the two-month lagged value of the Leading Indicator reported by Conference Board.

We cast the problem into a regression setting and assume the priors on model parameters to be the same as their counterparts assumed in the case of constant transition probabilities. In addition, we assume a diffuse prior, $p_0(a_1, b_1, a_2, b_2)$, to be the prior on regime-switching transition probabilities parameters. Hence, the complete prior on all the parameters can be written as:

$$\begin{aligned} p_{0v}(\theta) &= p_0(\alpha^1|\Sigma^1)p_0(\alpha^2|\Sigma^2)p_0(\Sigma^1)p_0(\Sigma^2)p_0(B^1)p_0(B^2) && (27) \\ &\times p_0(E_2^1)p_0(E_2^2)p_0(V_{22}^1)p_0(V_{22}^2)p_0(a_1, b_1, a_2, b_2). && (28) \end{aligned}$$

With the priors given above, the investor forms his posterior belief $p_v(\theta|R)$ in light of the data $\{R : r_t, t = 1, \ldots, T\}$,

$$p_v(\theta|R) \propto p_{0v}(\theta) \times p_v(R|\theta), \tag{29}$$

where $p_v(R|\theta)$ is the likelihood of the data. Then, the predictive distribution of $r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}$ can be computed as

$$p_v(r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}|R) = \int_\theta p_v(r_{T+1}, r_{T+2}, \cdots, r_{T+\hat{T}}|R, \theta)p_v(\theta|R)d\theta. \tag{30}$$

As in the constant transition probabilities case, the key issue lies in how to draw returns from the predictive density when solve (17) subject to (15) or to solve (21) numerically in the portfolio optimizations. To obtain a predictive distribution of future returns, $R_{iL_{T+t}}, R_{iS_{T+t}}, R_{j_{T+t}}$, again, we first need to obtain the excess returns, $R_{iL_{T+t}} - R_{iS_{T+t}}$ and $R_{j_{T+t}} - R_f$. Then we obtain the return of the asset on the short side of the i-th spread, $R_{iS_{T+t}}$. Finally, we add the excess return $R_{iL_{T+t}} - R_{iS_{T+t}}$ to $R_{iS_{T+t}}$ and the excess return $R_{j_{T+t}} - R_f$ to R_f to obtain the raw returns $R_{iL_{T+t}}$ and $R_{j_{T+t}}$. To obtain the excess return $R_{iL_{T+t}} - R_{iS_{T+t}}$, $R_{j_{T+t}} - R_f$ and the raw return $R_{iS_{T+t}}$, we need to forecast the state in the future first. Once the state at a future time period $T + t$ is given, we can draw the excess returns $R_{iL_{T+t}} - R_{iS_{T+t}}$, $R_{j_{T+t}} - R_f$ at $T + t$ from their conditional distributions, which are multivariate normal distributions with parameters drawn from the posterior $p_v(\theta|R)$. Then we can draw the raw return $R_{iS_{T+t}}$ from its conditional distribution, a normal distribution with parameters drawn from the posterior $p(\theta_S|R_S)$, where $R_S = (R_{iS_t}), i = 1, 2, \cdots, I$, and $t = 1, 2, \cdots, T$.

Now we show how to obtain the states in the future in the case of time-varying transition probabilities. We must first forecast the values of the Leading Indicator in the future to determine the transition probabilities and the states in the future. We assume that the Leading Indicator follows an autocorrelation process:

$$L_t = a + bL_{t-1} + \epsilon_t, \tag{31}$$

where $\epsilon_t \sim N(0, \sigma^2)$. Then, we can simulate M sample draws from the posterior distribution of the parameters (a, b, σ^2). In addition, we have,

$$
\begin{aligned}
L_{T+1} &= a + bL_T + \epsilon_{T+1}, \\
L_{T+2} &= a + ba + b^2 L_T + \epsilon_{T+2} + b\epsilon_{T+1}, \\
&\vdots \\
L_{T+\hat{T}} &= (1 + b + b^2 + \cdots + b^{\hat{T}-1})a + b^{\hat{T}} L_T + \epsilon_{T+\hat{T}} + b\epsilon_{T+\hat{T}-1} + \cdots + b^{\hat{T}-2}\epsilon_{T+2} + b^{\hat{T}-1}\epsilon_{T+1}.
\end{aligned} \tag{32}
$$

Therefore, L_{T+t} is normally distributed with the mean and variance given by

$$\mu_{L_{T+t}} = (1 + b + b^2 + \cdots + b^{t-1})a + b^t L_T, \tag{33}$$

$$\Sigma_{L_{T+t}} = (1 + b^2 + b^4 + \cdots + b^{2(t-1)})\sigma^2, \tag{34}$$

where $t = 1, 2, \cdots, \hat{T}$. Given each set of values of the M draws of the parameters (a, b, σ^2), we can simulate a series of $L_{T+1}, L_{T+2}, \cdots, L_{T+\hat{T}-2}$ for the fixed historical value of L_T. Then, we can obtain the transition probabilities at time period $t = T+1, T+2, \cdots, T+\hat{T}$ from the formulas $P_t = Pr(s_t = 1|s_{t-1} = 1) = \frac{1}{1+exp(a_1+b_1 LEI_{t-2})}$ and $Q_t = Pr(s_t = 2|s_{t-1} = 2) = \frac{exp(a_2+b_2 LEI_{t-2})}{1+exp(a_2+b_2 LEI_{t-2})}$. Then, based on the state at time T and the series of time-varying transition probabilities obtained as above, we draw a series of states for the time periods from $T+1$ to $T+\hat{T}$.

In Panel B of Table VII, we report the estimates to the parameters in (26). Both b_1 and b_2 are positive. This finding implies that a higher level of LEI indicates a higher probability to stay in the bull market (State 2) and a lower probability to stay in the bear market (State 1). Because there are only two regimes, it also implies that a higher level of LEI indicates a lower probability of switching from the bull market to the bear market and a higher probability of switching from the bear market to the bull market. In addition, in

Panel C of Table VII, we report the estimates to the parameters in (31), the autocorrelation process of the Leading Indicator. The positive value of b indicates that the Leading Indicator is positively correlated with its one period lag.

Recall that in the case of constant transition probabilities, the two regimes are persistent. In particular, the bull market is highly persistent, since the probability of staying in the bull is 92.6%, as shown in Panel A of Table VII. When the transition probabilities are allowed to be time-varying and determined by LEI through (26), at each time period t, we obtain one sample for the probability of staying either in the bear market or in the bull market by plugging a draw from the posterior of a_1, b_1, a_2, b_2 and the historical data LEI_{t-2} into (26). Then, we obtain the posterior mean of the probability of staying in either the bear market or in the bull market at the time period t by taking the average of the samples. The two graphs in Figure 3 present the time series of the posterior means of the probabilities of staying in the bear market (Graph (a)) and the bull market (Graph (b)) from July 1963 through December 2002. As shown by Graph (b) of Figure 3, the probability of staying in the bull market is quite stable. It slightly fluctuates between the lower 90% and the middle 90%. The only huge drop is around 1980, probably due to the recession in the early 1980s. In addition, as shown by Graph (a) of Figure 3, while the probability of staying in the bear market is less stable and fluctuates over a wider range between the vicinities of 60% and 75%, it is always above 50%. Therefore, the two regimes are still persistent when transition probabilities are not assumed constant but allowed to be time-varying, in particular for the bull market.

Insert Figure 3 about here.

Now, we examine the economic costs of ignoring regime-switching in asset returns in the case of time varying transition probabilities. We assume, as before, that the power utility maximizing investor has a relative risk aversion equal to 5. Columns 5, 6, and 7 of Table

VIII report the percentage points of $u_{2R} - u_{1R}$, referred to as certainty-equivalent gains in the table, in the case of time-varying transition probabilities (T-RS). Again, the certainty-equivalent gain is the "perceived" gain of utilizing the regime-switching, or the amount of loss to the investor when he is forced to hold the portfolio that is optimal in the one-regime model.

With varying degrees of pricing errors on the Fama-French three-factor model, the first panel reports the gains for different investment horizons in the case of T-RS, respectively. Similar to the case of C-RS, the gains in the case of T-RS, without margin requirements ($c = \infty$), can be fairly large. With a 60-month horizon and a diffuse belief on the Fama-French three-factor model, the gain can be as high as 75.1% in the case of T-RS. When the investment horizon is shortened from 60 months to 12 months, the gain becomes 9.4%, smaller but still significant. With a dogmatic belief in the Fama-French three-factor model, the gains shrink further, to 0.9%. In addition, when there is a modest 3% pricing error on the Fama-French three-factor model, the gain is 6.9%. With a 20% margin requirement ($c = 5$), the gains become much smaller in general, as shown by columns 5, 6 and 7 of Panel D of Table VIII. However, the gains are still significant in many cases, in particular when the investment horizon is 36 or 60 months. For instance, with a 60-month horizon and 3% mispricing uncertainty on the Fama-French 3-factor model, the gain can be as high as 26.5%. In addition, we examine the sensitivity of the above results on the economic value of incorporating regime-switching to the use of alternative investment universes in the case of T-RS. For this purpose, again, we apply the above analysis to two alternative investment universes. The first consists of the Fama-French three factors and the 10 CRSP size portfolios. The second consists of the Fama-French three factors and the Fama-French's 17 industry portfolios. Columns 5, 6 and 7 of Panel B and C of Table VIII report the gains without margin requirements for these two alternative sets of assets, respectively, in T-RS. In general, similar to C-RS, the gains for 10 CRSP size portfolios and Fama-French's 17 industry portfolios are smaller than those for the Fama-French 25 assets. However, the gains, such as 10.7% and 43.4%, are still economically important in general. For instance, the largest gains are 10.7% and 43.4% for the 10 CRSP size portfolios and the Fama-French's 17 industry portfolios, respectively, while the largest gain is 134.4% for the Fama-French 25 assets. In addition, columns 5, 6 and 7 of the Panel E and F report gains with a 20% margin for the two investment universes, respectively, in T-RS. With a 20% margin requirement, the gains are still significant in T-RS in many cases, in particular for the 36-month and the 60-month investment horizons. For instance, at the 60-month horizon and with a 3% mispricing error on the Fama-French three-factor model, the gain for Fama-French's 17 industry portfolios is 19.2% in T-RS.

Therefore, we can conclude that, in general, it is too costly to ignore regime-switching in the Fama-French three factors when the investment universe consists of the Fama-French three factors and any one of the three sets of assets, the Fama-French 25 assets, the 10 CRSP size portfolios or Fama-French's 17 industry portfolios, whether we assume constant transition probabilities or time varying transition probabilities.[18]

IV. Conclusions

Many important empirical questions in finance are studied under a one-regime assumption for the underlying data generating process, though investors have long recognized that the market's movements are more like bull and bear markets. The question is how investors can incorporate the shifts in returns into their portfolio decision making. We propose a Bayesian framework for the decision problem under a regime-switching model. Our framework captures not only the regime-switching feature of the data but also the asset pricing model

[18]Overall, we have found that the other results of time-varying transition probabilities compared to those of constant transition probabilities are in general very similar. For instance, the bull and bear markets are very different in variances and covariances in the case of time varying transition probabilities. To save the space, however, we do not report those similar results, which are available from the authors upon request.

uncertainty and the parameter estimation uncertainty associated with investors' portfolio decisions. In addition, our method provides an exact inference on parameter and on estimating functions of interest that might otherwise be impossible in the classical framework. Therefore, it applies to a large number of assets that better represent invertors' investment opportunities.

Applying the proposed approach to the data, we find that risks and returns vary greatly across bull and bear markets. For instance, the bull regime has a much higher market return than the bear regime with an average annual difference of 19.4%! Besides this strong evidence of the existence of regimes, we also find that, in terms of the certainty-equivalent (CE) measure, the CE loss to an investor who is forced to hold the portfolio that is optimal under a one-regime model is substantial, suggesting that the incorporation of regime-switching is very important from an investment perspective.

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Appendix A: Predictive Distribution

We first show the procedure to simulate samples from the predictive distribution in the two-regime model with constant transition probabilities in Appendix A. In Appendix B, we show the necessary adjustments for simulating samples from the predictive distribution when transition probabilities are time-varying. Because the historical states are not observable, we first show how to simulate these states based on our proposed model. Based on Chib (1996), we simulate states from the joint posterior conditional density of states conditioning on $\theta = [(\alpha^i,\ B^i, \Sigma^i,\ E_2^i, V_{22}^i, i = 1, 2),\ P, Q]$:

$$\begin{aligned} p(S_T|X_T, \theta) = p(s_T|X_T, \theta) \times \cdots \times p(s_t|X_T, S^{t+1}, \theta) \\ \times \cdots \times p(s_1|X_T, S^2, \theta), \end{aligned} \tag{A1}$$

where $S^{t+1} = (s_{t+1}, \ldots, s_T)$, and $X_T = (X_1, X_2, \cdots, X_T)$, the historical returns of the Fama and French's (1993) three factors. By Bayes theorem,

$$\begin{aligned} p(s_t|X_T, S^{t+1}) &\propto p(s_t|X_t, \theta) \times f(X^{t+1}, S^{t+1}|X_t, s_t, \theta) \\ &\propto p(s_t|X_t, \theta) \times p(s_{t+1}|s_t, \theta) \times f(X^{t+1}, S^{t+2}|X_t, s_t, s_{t+1}, \theta) \\ &\propto p(s_t|X_t, \theta) \times p(s_{t+1}|s_t, \theta), \end{aligned} \tag{A2}$$

since $f(X^{t+1}, S^{t+2}|X_t, s_t, s_{t+1}, \theta)$ is independent of s_t, where $X^{t+1} = (x_{t+1}, \ldots, x_T)$ and $X_t = (x_1, \ldots, x_t)$. To obtain $p(s_t|X_t, \theta)$, we first have a prediction step. By the law of total probability,

$$p(s_t|X_{t-1}, \theta) = \sum_{k=1}^{2} p(s_t|s_{t-1} = k, \theta) \times p(s_{t-1} = 1|X_{t-1}, \theta), \tag{A3}$$

since $p(s_t|X_{t-1}, s_{t-1}, \theta) = p(s_t|s_{t-1}, \theta)$. Then, we have a update step. By Bayes theorem,

$$p(s_t|X_t, \theta) \propto p(s_t|X_{t-1}, \theta) \times f(y_t|X_{t-1}, \theta_{s_t}). \tag{A4}$$

Initialize the *prediction* and *update* steps by setting $p(s_1|X_0, \theta)$ to be the stationary distribution of the chain and then recursively compute the mass function $p(s_t|X_t, \theta), t = 1, 2, \ldots, T$. Now we can simulate the last state s_T from the mass function $p(s_T|X_T, \theta)$. Then simulate

$s_{T-1}, s_{T-2}, \ldots, s_1$ backward from

$$p(s_t|X_T, S^{t+1}) \propto p(s_t|X_t,\theta) \times p(s_{t+1}|s_t,\theta), \quad t = T-1, T-2, \ \ldots, 1. \tag{A5}$$

Assume that we have Q different draws of states $S^q = (s_1^q, s_2^q, \cdots, s_T^q)$, where $q = 1, 2, \cdots, Q$. For each draw of states, we group the data into two sets, R^1 and R^2, according to the states associated, where $R^i = \{Y^i, X^i\}$, $i = 1, 2$, $Y^i = \{y_t|s_t^q = i\}'$, a $T^i \times m$ matrix, T^i is the number of observations in data set R^i, $X^i = \{x_t|s_t^q = i\}'$, a $T^i \times k$ matrix. In addition, define $Z^i = (\iota_T^i \ X^i)$, a $T^i \times (k+1)$ matrix, where ι_T^i denotes a T^i-vector of ones. Also define $A^i = (\alpha^i, B^i)'$, a $(k+1)\times m$ matrix and $a^i = vec(A^i)$. Then the regression model (4) can be written as

$$Y^i = Z^i A^i + U^i, \tag{A6}$$

where $U^i = \{u_t|s_t^q = i\}'$, a $T^i \times m$ matrix. Conditioning on each draw of states, the likelihood function of R^i can be factored as

$$p(Y^i, \ X^i \mid E^i, \ V^i) = p(Y^i \mid A^i, \ \Sigma^i, \ X^i)p(X^i \mid E_2^i, \ V_{22}^i), \tag{A7}$$

where

$$\begin{aligned}
p(Y^i \mid A^i, \ \Sigma^i, \ X^i) &\propto |\Sigma^i|^{-\frac{T}{2}} exp\left\{-\frac{1}{2}tr\left[(Y^i - Z^iA^i)'(Y^i - Z^iA^i)(\Sigma^i)^{-1}\right]\right\} \\
&\propto |\Sigma^i|^{-\frac{T}{2}} exp\left\{-\frac{T}{2}tr\widehat{\Sigma}^i(\Sigma^i)^{-1} - \frac{1}{2}tr\left(\left(A^i - \widehat{A}^i\right)'(Z^i)'Z^i\left(A^i - \widehat{A}^i\right)(\Sigma^i)^{-1}\right)\right\} \\
&\propto |\Sigma^i|^{-\frac{T}{2}} exp\left\{-\frac{T}{2}tr\widehat{\Sigma}^i(\Sigma^i)^{-1} - \frac{1}{2}tr\left[\left(a^i - \hat{a}^i\right)'\left((\Sigma^i)^{-1} \otimes (Z^i)'Z^i\right)\left(a^i - \hat{a}^i\right)\right]\right\},
\end{aligned} \tag{A8}$$

and

$$\begin{aligned}
p(X^i \mid E_2^i, \ V_{22}^i) &\propto |V_{22}^i|^{-\frac{T}{2}} exp\left\{-\frac{1}{2}tr\left((X^i - \iota_{T^i}(E_2^i)')'(X^i - \iota_{T^i}(E_2^i)')\right)(V_{22}^i)^{-1}\right\} \\
&\propto \ |V_{22}^i|^{-\frac{T^i}{2}} exp\left\{-\frac{T}{2}tr\widehat{V_{22}}^i V_{22}^{-1} - \frac{T^i}{2}tr\left(\left(E_2^i - \widehat{E_2}^i\right)\left(E_2^i - \widehat{E_2}^i\right)'(V_{22}^i)^{-1}\right)\right\},
\end{aligned} \tag{A9}$$

The joint prior distribution of all parameters is

$$p_0(\theta) = p_0(\alpha^1|\Sigma^1)p_0(\alpha^2|\Sigma^2)p_0(\Sigma^1)p_0(\Sigma^2)p_0(B^1)p_0(B^2)p_0(E_2^1)p_0(E_2^2)p_0(V_{22}^1)p_0(V_{22}^2)p_0(P,Q), \tag{A10}$$

where

$$p_0(\alpha^i|\Sigma^i) \propto |\Sigma^i|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}(\alpha^i)'\left(\frac{\sigma_\alpha^2}{(s^i)^2}\Sigma^i\right)^{-1}(\alpha^i)\right\}, \tag{A11}$$

$$p_0(\Sigma^i) \propto |\Sigma^i|^{-\frac{\nu_\Sigma+m+1}{2}} exp\left\{-\frac{1}{2}trH^i(\Sigma^i)^{-1}\right\}, \tag{A12}$$

$$p_0(B^i) \propto \ 1, \tag{A13}$$

$$p_0(E_2^i) \propto \ 1, \tag{A14}$$

$$p_0(V_{22}^i) \propto \ |V_{22}^i|^{-\frac{k+1}{2}}, \tag{A15}$$

$H^i = (s^i)^2(\nu_\Sigma - m - 1)I_m, \quad \nu_\Sigma = 30, \ (s^i)^2 = tr((Y^i - Z^i\widehat{A}^i)'(Y^i - Z^i\widehat{A}^i)/T^i)/m, \ \widehat{A}^i = ((Z^i)'Z^i)(Z^i)'Y^i$, and we assume that the prior distribution of $(P, 1-P)$, independent of $(1-Q, Q)$, is a Dirichlet on the two-dimensional simplex, i.e.,

$$(P, 1-P) \backsim D(1,1), \tag{A16}$$

and

$$(1-Q, Q) \backsim D(1,1).$$

In addition, consider the transformation

$$(\alpha^i)'\left(\frac{\sigma_\alpha^2}{(s^i)^2}\Sigma^i\right)^{-1}\alpha^i = (a^i)'\left((\Sigma^i)^{-1} \otimes D^i\right)a^i, \tag{A17}$$

where $a^i = vec(A^i)$ and D^i is a $(k+1)\times(k+1)$ matrix whose $(1,1)$ element is $(s^i)^2/\sigma_\alpha^2$ and whose other elements are all zero. Then, it follows that the likelihood in (A7) – (A9) can be combined with the prior in (A11) – (A16) to obtain the posterior distribution

$$p(E, V, P, Q|R) \propto p(R|E, V, P, Q)p_0(E, V, P, Q).$$

Because both the likelihood function conditioning on the states and the prior can be factored into two independent parts on (a^i, Σ^i) and (E_2^i, V_{22}^i), respectively. Therefore, the posteriors on (a^i, Σ^i) and (E_2^i, V_{22}^i) are independent as well. Hence, the joint posterior of the regression parameters is

$$p(a^i, \Sigma^i \,|R^i) \propto |\Sigma^i|^{-\frac{k+1}{2}} exp\left\{-\frac{1}{2}(a^i)' \left((\Sigma^i)^{-1} \otimes D^i\right) a^i - \frac{1}{2} tr\left(\left(a^i - \hat{a}^i\right)' \left((\Sigma^i)^{-1} \otimes (Z^i)'Z^i\right) \left(a^i - \hat{a}^i\right)\right)\right\}$$
$$\times |\Sigma^i|^{-\frac{T^i+\nu_\Sigma+m-k+1}{2}} exp\left\{-\frac{1}{2} tr\left(H^i + T^i \widehat{\Sigma}^i\right) (\Sigma^i)^{-1}\right\}. \tag{A18}$$

Let $F^i = D^i + (Z^i)'Z^i$, and $Q^i = (Z^i)'Z^i - (Z^i)'Z^i(F^i)^{-1}(Z^i)'Z^i$. By completing the square on a^i, we have

$$\begin{aligned} p(a^i, \Sigma^i \,|R^i) \propto & |\Sigma^i|^{-\frac{k+1}{2}} exp\left\{-\frac{1}{2}\left[\left(a^i - \tilde{a}^i\right)' \left((\Sigma^i)^{-1} \otimes F^i\right) \left(a^i - \tilde{a}^i\right)\right]\right\} \\ & \times |\Sigma^i|^{-\frac{T^i+\nu_\Sigma+m-k+1}{2}} exp\left\{-\frac{1}{2} tr\left(H^i + T^i \widehat{\Sigma}^i + (\widehat{A}^i)'Q^i \widehat{A}^i\right) (\Sigma^i)^{-1}\right\}, \end{aligned} \tag{A19}$$

where $\tilde{a}^i = (I_m \otimes (F^i)^{-1}(Z^i)'Z^i)\,\hat{a}^i$. Hence,

$$(\Sigma^i)^{-1} \mid R \sim\ W\left(T^i + \nu_\Sigma - k,\ \left(H^i + T^i \widehat{\Sigma}^i + (\widehat{A}^i)'Q^i \widehat{A}^i\right)^{-1}\right), \tag{A20}$$

and

$$a^i \mid (\Sigma^i)^{-1},\ R^i \sim\ N\left(\tilde{a}^i,\ \Sigma^i \otimes (F^i)^{-1}\right). \tag{A21}$$

In addition, the joint posterior distribution of E_2^i and V_{22}^i is

$$p\left(E_2^i,\ V_{22}^i | R^i\right)\ \propto\ |V_{22}^i|^{-\frac{T^i+k+1}{2}} exp\left\{-\frac{T^i}{2} tr \widehat{V_{22}}^i (V_{22}^i)^{-1} - \frac{T^i}{2} tr\left(\left(E_2^i - \widehat{E_2}^i\right)\left(E_2^i - \widehat{E_2}^i\right)' (V_{22}^i)^{-1}\right)\right\}. \tag{A22}$$

As a result, we have

$$(V_{22}^i)^{-1} \mid R^i\ \sim\ W\left(T^i - 1,\ \left(T^i \widehat{V_{22}}^i\right)^{-1}\right) \tag{A23}$$

and

$$E_2^i \mid V_{22}^i,\ R^i,\ \sim N\left(\widehat{E_2}^i,\ \frac{1}{T^i} V_{22}^i\right). \tag{A24}$$

Finally, the posterior distributions of P, Q are,

$$(P, 1-P) \backsim D(S_{11}+1, S_{12}+1), \tag{A25}$$

and

$$(1-Q, Q) \backsim D(S_{21}+1, S_{22}+1),$$

where S_{ij}, $i, j = 1, 2$, is the total number of one-step transitions from state i to state j in each draw of states. To carry out the posterior evaluation, we draw samples from the joint posterior distribution as follows:

1) draw a series of states S according to (A1) to (A5),

2) $(\Sigma^i)^{-1} \mid R \sim\ W\left(T^i + \nu_\Sigma - k,\ \left(H^i + T^i\widehat{\Sigma}^i + (\widehat{A}^i)'Q^i\widehat{A}^i\right)^{-1}\right),$

3) $a^i \mid (\Sigma^i)^{-1},\ R^i \sim\ N\left(\tilde{a}^i,\ \Sigma^i \otimes (F^i)^{-1}\right),$

4) $(V_{22}^i)^{-1} \mid R^i\ \sim\ W\left(T^i - 1,\ \left(T^i\widehat{V_{22}}^i\right)^{-1}\right),$

5) $(V_{22}^i)^{-1} \mid R\ \propto\ W\left(T - 1,\ \left(T\widehat{V_{22}}^i\right)^{-1}\right),$

6) $E_2^i \mid V_{22}^i,\ R^i,\ \sim N\left(\widehat{E_2}^i,\ \frac{1}{T^i}V_{22}^i\right),$

7) $(P, 1-P) \backsim D(S_{11}+1, S_{12}+1)$, and, $(1-Q, Q) \backsim D(S_{21}+1, S_{22}+1)$,

8) Repeat steps $1) - 7)$.

We can, following Geweke and Zhou (1996), start the above Gibbs sampling procedure from any arbitrary initial value in the support of the posterior density. Let $g = M + Q$ denote the total number of iterations of the above loop. To eliminate the impact of the initial value, we disregard the first M draws of the burning period, and use the other Q draws as the draws from the predictive distribution.

Appendix B: Predictive Distribution with Time-varying Transition Probabilities

The likelihood function and the prior on $(\alpha^i, B^i, \Sigma^i, E_2^i, V_{22}^i)_{i=1}^2$ are the same as those in the above section for the two-regime model with constant transition probabilities. The only difference here is that we allow the transition probabilities to be time-varying and determined by the LEI through $P_t = Pr(s_t = 1|s_{t-1} = 1) = \frac{1}{1+exp(a_1+b_1 LEI_{t-2})}$ and $Q_t = Pr(s_t = 2|s_{t-1} = 2) = \frac{exp(a_2+b_2 LEI_{t-2})}{1+exp(a_2+b_2 LEI_{t-2})}$. We assume a multivariate normal distribution $N(0, 50I)$ to be the prior on a_1, a_2, b_1, b_2, where I is a 4×4 identity matrix and 0 is a 4×1 zero vector. The number 50 is big enough to make the prior on the location of a_1, a_2, b_1, b_2 to be not restrictive. Then, an important sampling method is used to obtain the draws of a_1, a_2, b_1, b_2 as follows. First, obtain the maximum likelihood estimate of a_1, a_2, b_1, b_2 by maximizing the likelihood function, $L = \sum_{t=1}^T p_t$, where $p_t = Pr(s_t = S_t^q | s_{t-1} = S_{t-1}^q)$. Then, we draw a sample from a multivariate normal distribution $N(\hat{b}, \hat{V_b})$, where $\hat{b} = argmaxL$ and $\hat{V_b}$ is the information matrix of L. Compare this draw with $\hat{b}$, and keep the one which has the higher value when plugged into the posterior distribution of a_1, a_2, b_1, b_2 which is obtained by combining the likelihood function, L, and the prior on a_1, a_2, b_1, b_2. The rest of the parameters are drawn through the corresponding procedures listed in the above section.

Table I: Mean Estimations

This table presents the annualized posterior means and posterior standard variations (in percentage points) of the mean returns of the Fama-French 25 book-to-market and size portfolios and the Fama-French three factors, the market index (MKT), the size factor (SMB), and the value factor (HML). The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing errors imposed on the Fama-French 3-factor model are $\sigma_\alpha = 0$, 3% or ∞, respectively.

Portfolio	$\sigma_\alpha = 0$				$\sigma_\alpha = 3\%$				$\sigma_\alpha = \infty$			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	6.7 (4.4)	13.0 (4.1)	-17.0 (17.2)	-30.0 (18.3)	2.0 (4.6)	7.5 (4.3)	-17.7 (17.1)	-25.2 (18.2)	1.6 (4.5)	7.0 (4.3)	-18.0 (17.2)	-25.0 (18.4)
S1B2	8.4 (3.8)	13.3 (3.6)	-8.8 (14.7)	-22.1 (15.7)	8.4 (3.9)	12.6 (3.7)	-7.5 (14.7)	-20.1 (15.7)	8.4 (3.9)	12.5 (3.7)	-6.2 (14.7)	-18.8 (15.8)
S1B3	8.9 (3.3)	12.9 (3.2)	-4.9 (12.2)	-17.8 (13.2)	9.3 (3.4)	12.4 (3.3)	-3.0 (12.3)	-15.4 (13.3)	9.4 (3.3)	12.4 (3.3)	-1.3 (12.4)	-13.7 (13.5)
S1B4	9.5 (3.1)	12.8 (3.0)	-2.2 (11.4)	-15.0 (12.4)	11.7 (3.2)	14.4 (3.1)	0.3 (11.5)	-14.1 (12.6)	11.9 (3.1)	14.6 (3.1)	2.4 (11.6)	-12.1 (12.6)
S1B5	11.1 (3.2)	14.1 (3.2)	-0.4 (12.1)	-14.6 (13.1)	12.6 (3.3)	15.8 (3.2)	0.8 (12.2)	-15.0 (13.2)	12.8 (3.3)	15.9 (3.2)	1.8 (12.2)	-14.1 (13.3)
S2B1	5.8 (4.1)	12.0 (4.0)	-16.2 (14.9)	-28.3 (16.0)	3.8 (4.2)	9.7 (4.1)	-17.0 (15.0)	-26.6 (16.2)	3.7 (4.2)	9.4 (4.1)	-17.5 (15.2)	-26.9 (16.4)
S2B2	8.1 (3.3)	12.3 (3.2)	-6.9 (11.7)	-19.3 (12.7)	6.9 (3.4)	11.1 (3.3)	-7.8 (11.9)	-18.9 (13.0)	6.8 (3.3)	11.0 (3.3)	-8.5 (12.2)	-19.5 (13.2)
S2B3	9.0 (2.9)	12.4 (2.9)	-3.1 (10.4)	-15.5 (11.3)	9.8 (3.0)	13.9 (3.0)	-4.1 (10.6)	-18.0 (11.5)	9.9 (3.0)	14.1 (3.0)	-4.9 (10.8)	-18.9 (11.7)
S2B4	9.7 (2.8)	12.6 (2.8)	-1.0 (10.1)	-13.6 (11.0)	10.6 (2.9)	13.8 (2.8)	-1.0 (10.2)	-14.8 (11.2)	10.7 (2.8)	13.9 (2.8)	-1.1 (10.4)	-15.0 (11.3)
S2B5	11.4 (3.1)	14.5 (3.1)	0.2 (11.4)	-14.4 (12.4)	11.1 (3.2)	14.3 (3.1)	-0.1 (11.5)	-14.4 (12.5)	11.1 (3.2)	14.3 (3.1)	-0.5 (11.6)	-14.7 (12.6)
S3B1	4.8 (3.7)	10.9 (3.6)	-17.0 (13.9)	-27.9 (14.9)	4.0 (3.9)	9.6 (3.7)	-16.6 (13.9)	-26.2 (14.9)	4.0 (3.8)	9.5 (3.7)	-16.2 (14.1)	-25.7 (15.1)
S3B2	7.7 (2.9)	11.4 (2.9)	-6.6 (10.2)	-18.1 (11.1)	7.7 (3.1)	11.9 (3.0)	-7.2 (10.6)	-19.1 (11.6)	7.7 (3.0)	11.9 (3.0)	-7.7 (11.0)	-19.7 (12.0)
S3B3	8.7 (2.6)	11.6 (2.6)	-2.2 (9.3)	-13.9 (10.1)	7.9 (2.8)	11.7 (2.7)	-4.3 (9.5)	-16.0 (10.4)	7.9 (2.7)	11.7 (2.7)	-6.1 (9.9)	-17.8 (10.8)
S3B4	9.4 (2.5)	12.0 (2.5)	0.3 (8.8)	-11.8 (9.6)	9.4 (2.6)	12.5 (2.6)	-0.7 (9.0)	-13.2 (9.9)	9.5 (2.6)	12.5 (2.6)	-1.5 (9.3)	-14.0 (10.1)
S3B5	11.1 (2.8)	13.7 (2.8)	0.6 (10.4)	-13.0 (11.3)	11.2 (3.0)	14.0 (2.9)	0.9 (10.5)	-13.1 (11.4)	11.3 (3.0)	14.0 (2.9)	1.2 (10.5)	-12.8 (11.4)
S4B1	3.9 (3.3)	9.6 (3.1)	-17.0 (12.6)	-26.6 (13.4)	5.3 (3.5)	9.9 (3.2)	-13.7 (12.7)	-23.6 (13.5)	5.5 (3.4)	9.9 (3.2)	-10.9 (12.5)	-20.7 (13.4)
S4B2	7.2 (2.7)	10.6 (2.6)	-6.5 (9.6)	-17.1 (10.5)	5.4 (2.9)	8.6 (2.8)	-6.7 (9.9)	-15.4 (10.8)	5.2 (2.9)	8.4 (2.9)	-6.9 (10.2)	-15.4 (11.2)
S4B3	8.4 (2.5)	11.3 (2.5)	-2.9 (9.1)	-14.2 (9.8)	7.6 (2.7)	10.8 (2.7)	-3.5 (9.3)	-14.3 (10.2)	7.6 (2.7)	10.7 (2.7)	-4.0 (9.5)	-14.8 (10.3)
S4B4	8.8 (2.5)	11.8 (2.5)	-1.3 (8.5)	-13.1 (9.4)	9.0 (2.6)	11.3 (2.6)	-0.1 (8.7)	-11.4 (9.6)	9.1 (2.6)	11.3 (2.6)	1.0 (8.9)	-10.3 (9.7)
S4B5	10.8 (2.8)	13.7 (2.8)	-0.0 (10.1)	-13.7 (11.0)	10.2 (3.0)	13.1 (2.9)	-0.1 (10.2)	-13.2 (11.2)	10.2 (3.0)	13.0 (3.0)	-0.1 (10.5)	-13.1 (11.4)
S5B1	2.3 (2.6)	6.5 (2.6)	-14.1 (9.7)	-20.6 (10.4)	4.4 (2.7)	8.9 (2.6)	-13.1 (9.9)	-22.1 (10.6)	4.6 (2.7)	9.1 (2.6)	-12.4 (10.0)	-21.5 (10.7)
S5B2	5.3 (2.4)	8.7 (2.4)	-7.5 (8.6)	-16.2 (9.3)	5.0 (2.6)	8.7 (2.5)	-8.3 (8.9)	-17.0 (9.6)	5.0 (2.5)	8.7 (2.5)	-9.1 (9.2)	-17.8 (9.9)
S5B3	5.8 (2.2)	8.6 (2.1)	-6.0 (8.4)	-14.6 (9.0)	5.5 (2.4)	8.2 (2.3)	-5.5 (8.6)	-13.7 (9.3)	5.5 (2.4)	8.2 (2.3)	-5.1 (8.8)	-13.3 (9.5)
S5B4	7.7 (2.2)	10.1 (2.2)	-0.3 (8.0)	-10.4 (8.7)	6.8 (2.4)	9.2 (2.3)	-1.4 (8.2)	-10.5 (8.9)	6.7 (2.4)	9.1 (2.4)	-2.1 (8.4)	-11.2 (9.1)
S5B5	9.0 (2.3)	11.4 (2.3)	1.1 (8.4)	-10.3 (9.2)	6.5 (2.7)	8.9 (2.7)	-1.1 (8.9)	-9.9 (9.8)	6.3 (2.7)	8.7 (2.7)	-2.7 (9.5)	-11.4 (10.4)
MKT	4.9 (2.5)	9.0 (2.4)	-10.4 (9.2)	-19.4 (9.9)	4.9 (2.5)	9.0 (2.4)	-10.4 (9.2)	-19.4 (9.9)	4.9 (2.5)	9.0 (2.4)	-10.4 (9.2)	-19.4 (9.9)
SMB	2.5 (1.8)	3.4 (1.7)	-0.1 (7.3)	-3.4 (7.8)	2.5 (1.8)	3.4 (1.7)	-0.1 (7.3)	-3.4 (7.8)	2.5 (1.8)	3.4 (1.7)	-0.1 (7.3)	-3.4 (7.8)
HML	5.2 (1.7)	2.9 (1.6)	13.8 (7.1)	10.9 (7.6)	5.2 (1.7)	2.9 (1.6)	13.8 (7.1)	10.9 (7.6)	5.2 (1.6)	2.9 (1.6)	13.8 (7.1)	10.9 (7.6)

Table II: Standard Deviation Estimations

This table presents the annualized posterior means and posterior standard variations (in percentage points) of the standard deviations of the Fama-French 25 book-to-market and size portfolios and the Fama-French three factors, the market index (MKT), the size factor (SMB), and the value factor (HML). The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing errors imposed on the Fama-French 3-factor model are $\sigma_\alpha = 0$, 3% or ∞, respectively.

Portfolio	$\sigma_\alpha = 0$				$\sigma_\alpha = 3\%$				$\sigma_\alpha = \infty$			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	28.8 (0.9)	21.1 (1.2)	46.5 (4.9)	25.3 (4.5)	28.8 (0.9)	21.4 (1.2)	46.4 (4.9)	25.1 (4.5)	28.8 (0.9)	21.4 (1.2)	46.5 (4.9)	25.1 (4.5)
S1B2	24.7 (0.8)	18.3 (1.0)	39.8 (4.3)	21.5 (4.0)	24.7 (0.8)	18.4 (1.0)	39.8 (4.2)	21.5 (4.0)	24.7 (0.8)	18.4 (1.0)	39.9 (4.3)	21.5 (4.0)
S1B3	21.3 (0.7)	16.4 (1.0)	33.3 (3.4)	16.9 (3.3)	21.3 (0.7)	16.5 (0.9)	33.3 (3.4)	16.9 (3.3)	21.3 (0.7)	16.5 (0.9)	33.3 (3.4)	16.9 (3.2)
S1B4	19.9 (0.7)	15.4 (0.9)	31.1 (3.2)	15.7 (3.1)	19.8 (0.7)	15.3 (0.9)	31.1 (3.2)	15.8 (3.1)	19.8 (0.7)	15.3 (0.9)	31.1 (3.2)	15.8 (3.1)
S1B5	20.8 (0.7)	16.0 (0.9)	32.8 (3.5)	16.8 (3.4)	20.8 (0.7)	15.9 (0.9)	32.8 (3.5)	16.9 (3.4)	20.8 (0.7)	15.9 (0.9)	32.8 (3.5)	16.9 (3.4)
S2B1	26.3 (0.9)	20.5 (1.2)	40.3 (3.8)	19.8 (3.6)	26.3 (0.9)	20.6 (1.2)	40.3 (3.8)	19.6 (3.6)	26.3 (0.9)	20.6 (1.2)	40.2 (3.8)	19.6 (3.6)
S2B2	21.3 (0.7)	16.7 (0.9)	32.5 (3.1)	15.7 (3.0)	21.3 (0.7)	16.8 (0.9)	32.5 (3.1)	15.7 (3.0)	21.3 (0.7)	16.8 (0.9)	32.4 (3.1)	15.6 (3.0)
S2B3	18.9 (0.6)	15.1 (0.8)	28.6 (2.8)	13.6 (2.8)	18.9 (0.6)	14.9 (0.9)	28.6 (2.8)	13.7 (2.7)	18.9 (0.6)	14.9 (0.9)	28.6 (2.8)	13.6 (2.7)
S2B4	18.1 (0.6)	14.3 (0.7)	27.6 (2.8)	13.2 (2.7)	18.0 (0.6)	14.3 (0.8)	27.6 (2.8)	13.3 (2.7)	18.0 (0.6)	14.3 (0.8)	27.6 (2.8)	13.3 (2.7)
S2B5	20.1 (0.6)	15.7 (0.8)	31.0 (3.3)	15.3 (3.2)	20.1 (0.6)	15.8 (0.8)	31.0 (3.3)	15.3 (3.2)	20.1 (0.7)	15.8 (0.8)	31.0 (3.3)	15.3 (3.2)
S3B1	24.0 (0.8)	18.6 (1.1)	37.1 (3.6)	18.6 (3.3)	24.1 (0.8)	18.6 (1.1)	37.1 (3.6)	18.5 (3.3)	24.0 (0.8)	18.6 (1.1)	37.2 (3.6)	18.5 (3.3)
S3B2	19.2 (0.6)	15.4 (0.8)	28.8 (2.7)	13.4 (2.6)	19.2 (0.6)	15.4 (0.8)	28.8 (2.7)	13.4 (2.6)	19.2 (0.6)	15.4 (0.8)	28.8 (2.7)	13.4 (2.6)
S3B3	17.3 (0.6)	14.0 (0.8)	26.0 (2.5)	12.0 (2.4)	17.4 (0.6)	14.0 (0.8)	26.0 (2.5)	12.0 (2.4)	17.3 (0.6)	14.0 (0.8)	25.9 (2.5)	12.0 (2.4)
S3B4	16.5 (0.5)	13.4 (0.7)	24.5 (2.5)	11.1 (2.4)	16.5 (0.5)	13.4 (0.7)	24.6 (2.5)	11.2 (2.4)	16.5 (0.5)	13.4 (0.7)	24.5 (2.5)	11.1 (2.4)
S3B5	18.7 (0.6)	14.8 (0.8)	28.8 (3.1)	14.0 (3.0)	18.7 (0.6)	14.8 (0.8)	28.8 (3.1)	14.0 (3.0)	18.7 (0.6)	14.8 (0.8)	28.8 (3.1)	14.0 (3.0)
S4B1	21.5 (0.7)	16.5 (0.9)	33.4 (3.3)	16.9 (3.0)	21.5 (0.7)	16.5 (0.9)	33.5 (3.3)	17.0 (3.1)	21.5 (0.7)	16.5 (0.9)	33.6 (3.3)	17.1 (3.1)
S4B2	18.1 (0.6)	14.5 (0.8)	27.1 (2.5)	12.6 (2.4)	18.1 (0.6)	14.6 (0.8)	27.1 (2.5)	12.5 (2.4)	18.1 (0.6)	14.6 (0.8)	27.1 (2.5)	12.5 (2.4)
S4B3	17.0 (0.6)	13.8 (0.7)	25.5 (2.5)	11.7 (2.4)	17.1 (0.6)	13.8 (0.7)	25.5 (2.5)	11.7 (2.4)	17.1 (0.6)	13.8 (0.7)	25.5 (2.5)	11.6 (2.4)
S4B4	16.3 (0.5)	13.5 (0.7)	24.0 (2.3)	10.5 (2.3)	16.3 (0.5)	13.5 (0.7)	24.0 (2.3)	10.5 (2.3)	16.3 (0.5)	13.5 (0.7)	24.0 (2.3)	10.5 (2.3)
S4B5	18.8 (0.6)	15.3 (0.7)	28.1 (2.9)	12.7 (2.8)	18.8 (0.6)	15.4 (0.7)	28.1 (2.9)	12.7 (2.8)	18.9 (0.6)	15.4 (0.7)	28.1 (2.9)	12.7 (2.8)
S5B1	17.0 (0.6)	13.8 (0.8)	25.2 (2.3)	11.4 (2.2)	17.0 (0.6)	13.7 (0.8)	25.2 (2.3)	11.6 (2.2)	17.0 (0.6)	13.6 (0.8)	25.3 (2.3)	11.6 (2.2)
S5B2	16.0 (0.5)	13.1 (0.7)	23.5 (2.2)	10.3 (2.1)	16.0 (0.5)	13.1 (0.7)	23.4 (2.1)	10.3 (2.1)	16.0 (0.5)	13.1 (0.7)	23.4 (2.1)	10.3 (2.1)
S5B3	15.2 (0.5)	12.0 (0.6)	23.3 (2.3)	11.3 (2.1)	15.2 (0.5)	12.0 (0.7)	23.3 (2.3)	11.3 (2.2)	15.2 (0.5)	12.0 (0.6)	23.4 (2.3)	11.3 (2.2)
S5B4	15.0 (0.5)	12.3 (0.6)	22.1 (2.1)	9.9 (2.1)	15.0 (0.5)	12.3 (0.6)	22.1 (2.1)	9.8 (2.0)	15.0 (0.5)	12.3 (0.6)	22.1 (2.1)	9.8 (2.1)
S5B5	16.6 (0.5)	13.7 (0.7)	24.4 (2.3)	10.7 (2.3)	16.7 (0.5)	13.8 (0.7)	24.4 (2.3)	10.5 (2.3)	16.7 (0.5)	13.9 (0.7)	24.3 (2.3)	10.5 (2.3)
MKT	15.7 (0.5)	12.3 (0.7)	23.9 (2.3)	11.6 (2.1)	15.7 (0.5)	12.3 (0.7)	23.9 (2.3)	11.6 (2.1)	15.7 (0.5)	12.3 (0.7)	23.9 (2.3)	11.6 (2.1)
SMB	11.4 (0.4)	8.3 (0.5)	18.8 (2.0)	10.5 (1.9)	11.4 (0.4)	8.3 (0.5)	18.8 (2.0)	10.5 (1.9)	11.4 (0.4)	8.3 (0.5)	18.8 (2.0)	10.5 (1.9)
HML	10.5 (0.3)	7.5 (0.5)	17.1 (1.8)	9.6 (1.6)	10.4 (0.3)	7.5 (0.5)	17.1 (1.8)	9.6 (1.6)	10.5 (0.3)	7.5 (0.5)	17.1 (1.8)	9.6 (1.6)

Table III: Covariance Estimations

This table presents the posterior means and the posterior standard variations (in percentage points) of the covariances between the Fama-French 25 book-to-market and size portfolios and the Fama-French three factors, the market index (COV_{MKT}), the size factor (COV_{SMB}), and the value factor (COV_{HML}). The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing error imposed on the Fama-French 3-factor model is $\sigma_\alpha = \infty$.

Portfolio	COV_{MKT}				COV_{SMB}				COV_{HML}			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	29.6 (2.2)	17.2 (2.1)	73.3 (16.2)	56.1 (15.6)	20.5 (1.6)	10.6 (1.5)	56.6 (13.8)	46.0 (13.4)	-13.0 (1.3)	-5.9 (1.2)	-37.7 (10.2)	-31.8 (9.9)
S1B2	25.1 (1.9)	15.1 (1.8)	60.5 (13.5)	45.4 (13.1)	18.3 (1.4)	9.2 (1.3)	52.0 (12.7)	42.9 (12.3)	-8.6 (1.1)	-4.0 (0.9)	-24.4 (7.7)	-20.4 (7.6)
S1B3	22.1 (1.7)	13.7 (1.6)	51.9 (11.3)	38.2 (11.0)	15.3 (1.2)	8.1 (1.1)	42.0 (10.1)	33.9 (9.8)	-5.8 (0.9)	-2.9 (0.8)	-15.6 (5.8)	-12.7 (5.8)
S1B4	20.3 (1.5)	12.4 (1.5)	48.2 (10.7)	35.8 (10.5)	13.9 (1.1)	7.5 (1.0)	37.3 (9.0)	29.8 (8.8)	-3.9 (0.8)	-2.1 (0.7)	-9.9 (5.1)	-7.9 (5.2)
S1B5	20.7 (1.6)	12.5 (1.5)	50.0 (11.4)	37.4 (11.2)	14.2 (1.1)	7.7 (1.1)	37.7 (9.3)	30.0 (9.0)	-2.5 (0.8)	-1.2 (0.7)	-6.4 (5.2)	-5.2 (5.3)
S2B1	29.5 (2.1)	18.1 (2.1)	69.1 (14.1)	51.0 (13.5)	16.8 (1.4)	9.0 (1.3)	45.0 (11.2)	35.9 (10.9)	-12.9 (1.2)	-7.0 (1.2)	-33.2 (8.3)	-26.2 (8.1)
S2B2	23.9 (1.7)	15.0 (1.7)	55.2 (11.3)	40.2 (11.0)	13.6 (1.1)	7.5 (1.1)	35.8 (8.8)	28.3 (8.6)	-6.9 (0.9)	-4.0 (0.9)	-15.9 (5.5)	-11.9 (5.6)
S2B3	21.1 (1.5)	13.3 (1.5)	48.1 (10.0)	34.8 (9.8)	11.4 (1.0)	6.4 (1.0)	29.3 (7.4)	23.0 (7.3)	-4.0 (0.8)	-2.5 (0.7)	-8.1 (4.6)	-5.6 (4.7)
S2B4	19.7 (1.4)	12.4 (1.4)	45.5 (9.7)	33.1 (9.4)	10.4 (0.9)	5.8 (0.9)	26.8 (7.0)	21.0 (6.9)	-2.3 (0.7)	-1.5 (0.6)	-4.1 (4.3)	-2.6 (4.4)
S2B5	21.4 (1.6)	13.5 (1.5)	49.5 (10.9)	36.0 (10.7)	11.7 (1.0)	6.3 (1.0)	31.2 (8.1)	24.9 (7.9)	-1.6 (0.8)	-0.9 (0.6)	-3.0 (4.9)	-2.1 (5.0)
S3B1	27.9 (1.9)	17.0 (1.9)	66.0 (13.4)	49.1 (12.9)	13.8 (1.3)	7.1 (1.1)	37.9 (9.9)	30.7 (9.7)	-12.4 (1.1)	-6.4 (1.1)	-32.8 (8.0)	-26.5 (7.8)
S3B2	22.7 (1.6)	14.1 (1.6)	52.3 (10.4)	38.2 (10.1)	9.6 (1.0)	5.8 (0.9)	23.1 (6.5)	17.3 (6.5)	-5.6 (0.8)	-3.6 (0.8)	-11.5 (4.7)	-7.9 (4.8)
S3B3	19.9 (1.4)	12.8 (1.4)	44.6 (8.9)	31.8 (8.7)	7.8 (0.8)	4.9 (0.8)	18.1 (5.5)	13.3 (5.5)	-2.5 (0.7)	-2.0 (0.6)	-3.0 (4.2)	-0.9 (4.3)
S3B4	18.3 (1.3)	12.1 (1.3)	40.0 (8.4)	27.9 (8.2)	6.7 (0.8)	4.3 (0.7)	15.0 (5.1)	10.7 (5.1)	-0.8 (0.7)	-1.1 (0.5)	1.5 (4.1)	2.6 (4.2)
S3B5	20.2 (1.5)	12.7 (1.4)	46.9 (10.2)	34.2 (10.0)	8.2 (0.9)	5.0 (0.8)	20.0 (6.3)	15.1 (6.3)	-0.4 (0.8)	-0.5 (0.6)	1.2 (4.7)	1.7 (4.8)
S4B1	25.8 (1.8)	15.6 (1.7)	61.8 (12.5)	46.2 (12.1)	9.9 (1.1)	4.9 (0.9)	28.2 (8.3)	23.3 (8.2)	-11.3 (1.0)	-5.6 (1.0)	-30.8 (7.5)	-25.1 (7.3)
S4B2	21.9 (1.5)	13.9 (1.6)	50.0 (9.8)	36.1 (9.4)	6.3 (0.8)	3.9 (0.8)	14.4 (5.4)	10.5 (5.4)	-4.6 (0.8)	-3.1 (0.7)	-8.7 (4.4)	-5.6 (4.6)
S4B3	19.9 (1.4)	13.0 (1.4)	44.4 (9.0)	31.4 (8.7)	5.0 (0.8)	3.5 (0.7)	10.4 (4.8)	6.9 (4.9)	-2.1 (0.7)	-1.8 (0.6)	-1.8 (4.2)	0.0 (4.3)
S4B4	18.6 (1.3)	12.3 (1.3)	41.1 (8.4)	28.8 (8.2)	4.9 (0.8)	3.0 (0.7)	12.0 (4.8)	9.0 (4.9)	-1.0 (0.7)	-0.9 (0.5)	-0.6 (3.8)	0.3 (4.0)
S4B5	20.4 (1.5)	13.2 (1.4)	45.8 (9.7)	32.6 (9.5)	5.6 (0.9)	3.5 (0.8)	12.7 (5.4)	9.2 (5.5)	0.1 (0.8)	-0.3 (0.6)	2.4 (4.6)	2.7 (4.8)
S5B1	20.8 (1.4)	12.9 (1.5)	48.2 (9.4)	35.4 (9.0)	2.4 (0.8)	1.1 (0.6)	6.8 (4.6)	5.7 (4.7)	-8.4 (0.8)	-4.8 (0.8)	-20.0 (5.0)	-15.2 (4.9)
S5B2	19.4 (1.3)	12.6 (1.4)	43.0 (8.3)	30.4 (8.0)	1.5 (0.7)	1.0 (0.6)	3.0 (4.2)	1.9 (4.3)	-4.1 (0.7)	-2.8 (0.7)	-7.7 (3.9)	-4.9 (4.0)
S5B3	17.6 (1.2)	11.0 (1.3)	40.9 (8.3)	29.9 (8.1)	0.9 (0.7)	0.6 (0.6)	1.9 (4.1)	1.2 (4.3)	-2.5 (0.6)	-1.5 (0.5)	-5.3 (3.8)	-3.8 (3.9)
S5B4	16.0 (1.2)	11.0 (1.2)	33.6 (6.9)	22.6 (6.9)	0.3 (0.7)	0.7 (0.6)	-1.3 (3.9)	-2.0 (4.1)	0.6 (0.6)	-0.4 (0.5)	4.9 (4.0)	5.2 (4.1)
S5B5	16.3 (1.3)	11.2 (1.3)	33.8 (7.2)	22.5 (7.2)	1.0 (0.7)	1.1 (0.7)	0.6 (4.3)	-0.5 (4.5)	1.6 (0.7)	0.5 (0.5)	7.0 (4.5)	6.5 (4.7)

Table IV: Correlation Estimations

This table presents the posterior means and the posterior standard variations (in percentage points) of the correlations between the Fama-French 25 book-to-market and size portfolios and the Fama-French three factors, the market index ($CORR_{MKT}$), the size factor ($CORR_{SMB}$), and the value factor ($CORR_{HML}$). The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing error imposed on the Fama-French 3-factor model is $\sigma_\alpha = \infty$.

Portfolio	$CORR_{MKT}$				$CORR_{SMB}$				$CORR_{HML}$			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	78.5 (1.8)	78.3 (2.5)	78.4 (4.0)	0.1 (5.0)	74.7 (2.1)	71.7 (3.4)	76.7 (4.5)	5.0 (6.2)	-51.5 (3.4)	-43.3 (5.6)	-56.1 (7.2)	-12.8 (9.6)
S1B2	77.7 (1.8)	80.1 (2.3)	75.4 (4.5)	-4.7 (5.2)	77.9 (1.8)	72.1 (3.3)	82.0 (3.7)	9.9 (5.4)	-40.0 (3.9)	-34.6 (5.8)	-42.4 (8.5)	-7.8 (10.8)
S1B3	79.2 (1.7)	81.0 (2.2)	77.5 (4.2)	-3.6 (4.9)	75.5 (2.0)	70.7 (3.5)	79.2 (4.1)	8.4 (5.9)	-31.3 (4.2)	-27.8 (6.0)	-32.5 (9.2)	-4.7 (11.6)
S1B4	78.2 (1.8)	79.3 (2.4)	77.1 (4.3)	-2.1 (5.2)	73.6 (2.1)	70.8 (3.4)	75.5 (4.6)	4.7 (6.2)	-22.7 (4.4)	-21.2 (6.0)	-22.2 (9.8)	-1.0 (12.1)
S1B5	76.3 (2.0)	76.8 (2.7)	75.7 (4.5)	-1.1 (5.6)	71.6 (2.2)	70.2 (3.4)	72.4 (5.0)	2.2 (6.5)	-14.0 (4.5)	-11.8 (6.3)	-13.6 (10.1)	-1.8 (12.5)
S2B1	85.7 (1.2)	85.4 (1.7)	85.5 (2.8)	0.1 (3.5)	67.0 (2.6)	63.3 (4.2)	70.2 (5.6)	6.9 (7.8)	-56.3 (3.1)	-53.1 (5.0)	-57.2 (7.1)	-4.0 (9.4)
S2B2	85.9 (1.2)	86.8 (1.6)	84.7 (3.0)	-2.2 (3.6)	67.1 (2.5)	64.4 (4.1)	69.4 (5.6)	5.0 (7.6)	-36.8 (4.0)	-37.5 (5.7)	-34.2 (9.4)	3.3 (11.8)
S2B3	85.4 (1.3)	86.7 (1.6)	83.6 (3.1)	-3.0 (3.7)	63.2 (2.8)	61.5 (4.3)	64.5 (6.2)	3.1 (8.2)	-24.4 (4.3)	-26.7 (5.9)	-19.9 (10.1)	6.8 (12.4)
S2B4	83.7 (1.4)	85.0 (1.8)	82.1 (3.4)	-2.9 (4.0)	60.3 (2.9)	59.1 (4.3)	61.1 (6.6)	2.0 (8.5)	-14.7 (4.5)	-16.7 (6.0)	-10.6 (10.3)	6.2 (12.6)
S2B5	81.4 (1.6)	83.4 (2.1)	79.3 (3.9)	-4.1 (4.6)	61.0 (2.9)	57.8 (4.6)	63.3 (6.3)	5.5 (8.4)	-9.1 (4.6)	-9.0 (6.2)	-6.8 (10.3)	2.2 (12.7)
S3B1	88.8 (1.0)	88.7 (1.3)	88.5 (2.3)	-0.3 (2.8)	60.1 (2.9)	55.3 (4.8)	63.9 (6.4)	8.7 (8.8)	-59.0 (3.0)	-54.1 (4.9)	-61.3 (6.6)	-7.2 (8.8)
S3B2	90.3 (0.8)	89.7 (1.2)	90.6 (1.9)	0.9 (2.3)	52.4 (3.3)	54.4 (4.9)	50.5 (7.9)	-3.9 (10.1)	-33.4 (4.1)	-36.6 (5.8)	-28.0 (10.0)	8.6 (12.5)
S3B3	87.7 (1.1)	89.0 (1.3)	85.7 (2.8)	-3.3 (3.2)	47.1 (3.6)	50.5 (5.1)	44.1 (8.5)	-6.4 (10.8)	-16.8 (4.5)	-23.1 (6.1)	-8.4 (10.8)	14.7 (13.3)
S3B4	84.8 (1.3)	87.9 (1.5)	81.2 (3.7)	-6.7 (4.1)	42.5 (3.7)	46.8 (5.3)	38.5 (8.9)	-8.3 (11.2)	-5.6 (4.6)	-13.0 (6.0)	3.7 (10.7)	16.7 (13.0)
S3B5	82.4 (1.5)	83.6 (1.9)	80.9 (3.6)	-2.7 (4.3)	46.1 (3.6)	48.4 (5.3)	43.7 (8.5)	-4.7 (10.8)	-2.4 (4.6)	-5.5 (6.1)	2.6 (10.5)	8.1 (12.8)
S4B1	92.0 (0.7)	92.2 (0.9)	91.6 (1.7)	-0.6 (2.0)	48.3 (3.5)	42.6 (5.7)	52.6 (7.7)	10.0 (10.5)	-60.2 (2.9)	-54.0 (4.9)	-63.4 (6.3)	-9.4 (8.5)
S4B2	92.4 (0.7)	92.7 (0.9)	92.0 (1.6)	-0.7 (2.0)	36.2 (4.0)	38.9 (5.9)	33.5 (9.4)	-5.4 (12.1)	-28.9 (4.2)	-33.7 (5.8)	-22.7 (10.3)	11.0 (12.8)
S4B3	89.2 (0.9)	91.3 (1.0)	86.8 (2.7)	-4.5 (3.0)	30.9 (4.1)	36.2 (5.9)	25.7 (9.8)	-10.5 (12.3)	-14.0 (4.5)	-20.8 (6.1)	-5.3 (10.9)	15.5 (13.5)
S4B4	87.0 (1.1)	88.7 (1.3)	85.3 (3.0)	-3.4 (3.4)	31.6 (4.1)	32.1 (6.1)	31.2 (9.4)	-0.9 (12.3)	-7.0 (4.6)	-10.1 (6.1)	-2.1 (10.6)	8.0 (12.9)
S4B5	82.6 (1.5)	83.7 (1.9)	81.3 (3.6)	-2.4 (4.2)	31.0 (4.1)	33.1 (6.1)	28.6 (9.7)	-4.4 (12.5)	0.4 (4.6)	-2.6 (6.1)	5.5 (10.6)	8.1 (13.0)
S5B1	93.6 (0.6)	91.7 (1.0)	95.1 (1.0)	3.5 (1.5)	14.8 (4.5)	11.7 (6.7)	16.8 (10.2)	5.1 (13.2)	-56.4 (3.2)	-55.7 (4.8)	-55.0 (7.6)	0.7 (9.8)
S5B2	92.8 (0.6)	93.4 (0.8)	91.7 (1.8)	-1.7 (2.1)	9.8 (4.5)	11.3 (6.8)	8.0 (10.4)	-3.3 (13.6)	-29.2 (4.2)	-33.0 (5.9)	-23.0 (10.6)	10.0 (13.2)
S5B3	88.1 (1.0)	88.7 (1.3)	87.2 (2.6)	-1.5 (3.0)	6.4 (4.5)	7.8 (6.9)	4.9 (10.4)	-2.8 (13.5)	-18.7 (4.4)	-19.3 (6.3)	-16.0 (10.5)	3.3 (13.3)
S5B4	81.6 (1.5)	87.0 (1.5)	75.6 (4.9)	-11.4 (5.2)	2.4 (4.6)	8.6 (6.9)	-3.5 (10.5)	-12.1 (13.7)	4.2 (4.6)	-4.7 (6.1)	14.7 (10.9)	19.4 (13.4)
S5B5	74.5 (2.0)	79.0 (2.4)	69.1 (5.9)	-9.9 (6.7)	6.6 (4.5)	11.6 (6.9)	1.6 (10.6)	-10.0 (13.8)	11.3 (4.5)	5.4 (6.3)	19.3 (10.7)	14.0 (13.5)

Table V: β Estimations

This table presents the posterior means and the posterior standard variations (in percentage points) of the β's between the Fama-French 25 book-to-market and size portfolios and the Fama-French three factors, the market index (β_{MKT}), the size factor (β_{SMB}), and the value factor (β_{HML}). The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing error imposed on the Fama-French 3-factor model is $\sigma_\alpha = \infty$.

Portfolio	β_{MKT}				β_{SMB}				β_{HML}			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	1.44 (0.05)	1.36 (0.07)	1.53 (0.13)	0.16 (0.16)	1.89 (0.08)	1.86 (0.12)	1.90 (0.17)	0.04 (0.22)	-1.42 (0.11)	-1.23 (0.16)	-1.53 (0.23)	-0.30 (0.29)
S1B2	1.22 (0.05)	1.20 (0.06)	1.26 (0.12)	0.06 (0.15)	1.69 (0.06)	1.60 (0.11)	1.74 (0.13)	0.14 (0.18)	-0.95 (0.10)	-0.84 (0.14)	-0.99 (0.22)	-0.15 (0.27)
S1B3	1.08 (0.04)	1.09 (0.06)	1.08 (0.10)	-0.00 (0.12)	1.41 (0.06)	1.41 (0.10)	1.41 (0.11)	-0.00 (0.17)	-0.64 (0.09)	-0.61 (0.13)	-0.63 (0.19)	-0.03 (0.24)
S1B4	0.99 (0.04)	0.99 (0.06)	1.01 (0.09)	0.02 (0.12)	1.28 (0.05)	1.31 (0.09)	1.25 (0.11)	-0.06 (0.16)	-0.43 (0.09)	-0.43 (0.12)	-0.40 (0.18)	0.02 (0.23)
S1B5	1.01 (0.04)	0.99 (0.06)	1.04 (0.10)	0.05 (0.13)	1.30 (0.06)	1.35 (0.10)	1.26 (0.13)	-0.09 (0.17)	-0.28 (0.09)	-0.25 (0.13)	-0.26 (0.20)	-0.01 (0.25)
S2B1	1.44 (0.04)	1.44 (0.06)	1.44 (0.09)	0.01 (0.12)	1.55 (0.08)	1.58 (0.13)	1.51 (0.16)	-0.07 (0.22)	-1.42 (0.10)	-1.46 (0.14)	-1.35 (0.20)	0.11 (0.26)
S2B2	1.17 (0.03)	1.19 (0.05)	1.15 (0.08)	-0.04 (0.10)	1.25 (0.06)	1.31 (0.11)	1.20 (0.13)	-0.11 (0.18)	-0.75 (0.09)	-0.84 (0.13)	-0.65 (0.19)	0.19 (0.24)
S2B3	1.03 (0.03)	1.05 (0.05)	1.00 (0.07)	-0.05 (0.10)	1.04 (0.06)	1.11 (0.10)	0.99 (0.12)	-0.13 (0.17)	-0.44 (0.08)	-0.53 (0.12)	-0.33 (0.17)	0.20 (0.22)
S2B4	0.96 (0.03)	0.99 (0.05)	0.95 (0.07)	-0.04 (0.10)	0.95 (0.06)	1.02 (0.10)	0.90 (0.12)	-0.12 (0.17)	-0.25 (0.08)	-0.32 (0.11)	-0.17 (0.17)	0.14 (0.21)
S2B5	1.04 (0.03)	1.07 (0.05)	1.03 (0.09)	-0.04 (0.12)	1.07 (0.06)	1.10 (0.11)	1.05 (0.13)	-0.06 (0.19)	-0.17 (0.09)	-0.19 (0.13)	-0.12 (0.19)	0.06 (0.24)
S3B1	1.36 (0.03)	1.35 (0.04)	1.38 (0.08)	0.03 (0.09)	1.27 (0.08)	1.25 (0.13)	1.27 (0.16)	0.02 (0.22)	-1.36 (0.09)	-1.34 (0.13)	-1.34 (0.18)	0.00 (0.23)
S3B2	1.11 (0.02)	1.12 (0.04)	1.09 (0.05)	-0.03 (0.07)	0.88 (0.07)	1.01 (0.11)	0.78 (0.14)	-0.24 (0.19)	-0.61 (0.08)	-0.75 (0.12)	-0.47 (0.18)	0.27 (0.23)
S3B3	0.97 (0.02)	1.01 (0.04)	0.93 (0.06)	-0.08 (0.08)	0.72 (0.06)	0.85 (0.10)	0.61 (0.13)	-0.24 (0.18)	-0.28 (0.08)	-0.43 (0.11)	-0.13 (0.17)	0.30 (0.21)
S3B4	0.89 (0.03)	0.96 (0.04)	0.84 (0.06)	-0.12 (0.08)	0.61 (0.06)	0.76 (0.10)	0.50 (0.13)	-0.26 (0.18)	-0.09 (0.07)	-0.23 (0.11)	0.05 (0.16)	0.28 (0.20)
S3B5	0.98 (0.03)	1.01 (0.05)	0.98 (0.08)	-0.03 (0.10)	0.76 (0.07)	0.87 (0.11)	0.67 (0.15)	-0.20 (0.20)	-0.04 (0.08)	-0.11 (0.12)	0.04 (0.18)	0.15 (0.23)
S4B1	1.26 (0.02)	1.24 (0.03)	1.29 (0.06)	0.05 (0.07)	0.91 (0.08)	0.85 (0.12)	0.94 (0.16)	0.09 (0.21)	-1.24 (0.08)	-1.18 (0.11)	-1.25 (0.16)	-0.07 (0.20)
S4B2	1.07 (0.02)	1.10 (0.03)	1.04 (0.05)	-0.06 (0.06)	0.57 (0.07)	0.69 (0.11)	0.49 (0.14)	-0.20 (0.20)	-0.50 (0.08)	-0.65 (0.11)	-0.36 (0.17)	0.29 (0.22)
S4B3	0.97 (0.02)	1.03 (0.03)	0.93 (0.06)	-0.10 (0.07)	0.46 (0.06)	0.61 (0.11)	0.35 (0.14)	-0.26 (0.19)	-0.23 (0.07)	-0.38 (0.11)	-0.08 (0.16)	0.30 (0.21)
S4B4	0.91 (0.02)	0.97 (0.03)	0.86 (0.06)	-0.12 (0.07)	0.45 (0.06)	0.53 (0.11)	0.40 (0.13)	-0.13 (0.18)	-0.11 (0.07)	-0.18 (0.11)	-0.03 (0.15)	0.15 (0.20)
S4B5	0.99 (0.03)	1.05 (0.05)	0.96 (0.07)	-0.09 (0.10)	0.51 (0.07)	0.62 (0.12)	0.43 (0.15)	-0.19 (0.21)	0.01 (0.08)	-0.05 (0.13)	0.09 (0.18)	0.14 (0.23)
S5B1	1.02 (0.02)	1.02 (0.03)	1.01 (0.04)	-0.01 (0.05)	0.22 (0.07)	0.19 (0.11)	0.23 (0.14)	0.03 (0.19)	-0.92 (0.06)	-1.01 (0.09)	-0.82 (0.14)	0.19 (0.18)
S5B2	0.95 (0.02)	1.00 (0.02)	0.90 (0.04)	-0.10 (0.05)	0.14 (0.06)	0.18 (0.11)	0.10 (0.13)	-0.08 (0.19)	-0.45 (0.07)	-0.58 (0.11)	-0.32 (0.15)	0.26 (0.20)
S5B3	0.86 (0.02)	0.87 (0.03)	0.85 (0.05)	-0.02 (0.06)	0.08 (0.06)	0.11 (0.10)	0.06 (0.13)	-0.05 (0.18)	-0.27 (0.07)	-0.31 (0.10)	-0.22 (0.15)	0.09 (0.19)
S5B4	0.78 (0.03)	0.87 (0.03)	0.70 (0.06)	-0.17 (0.07)	0.03 (0.06)	0.13 (0.10)	-0.04 (0.12)	-0.17 (0.18)	0.06 (0.07)	-0.08 (0.10)	0.19 (0.14)	0.27 (0.19)
S5B5	0.79 (0.03)	0.89 (0.04)	0.71 (0.08)	-0.18 (0.09)	0.10 (0.07)	0.19 (0.12)	0.02 (0.14)	-0.17 (0.20)	0.18 (0.07)	0.10 (0.12)	0.28 (0.16)	0.18 (0.21)

Table VI: α Estimations

This table presents the posterior means and the posterior standard variations (in percentage points) of the α's of the Fama-French 25 book-to-market and size portfolios against the Fama-French three factor model. The posterior standard variations are reported in parentheses. The data are the monthly returns of the Fama-French three factors and the Fama-French 25 portfolios from July 1963 through December 2002. The columns under NRS reports the results for the one-regime model. The columns under RS reports the results for the two regimes, Bull and Bear, and the difference between the two regimes, Bear-Bull, for the two-regime model. The mispricing errors imposed on the Fama-French 3-factor model are $\sigma_\alpha = 0$, 3% or ∞, respectively.

Portfolio	$\sigma_\alpha = 0$				$\sigma_\alpha = 3\%$				$\sigma_\alpha = \infty$			
	NRS	RS			NRS	RS			NRS	RS		
		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull		Bull	Bear	Bear-Bull
S1B1	0 (0)	0 (0)	0 (0)	0 (0)	-4.9 (1.2)	-6.0 (1.3)	-0.7 (3.3)	-5.3 (3.7)	-5.4 (1.3)	-6.6 (1.4)	-1.1 (4.9)	-5.5 (5.3)
S1B2	0 (0)	0 (0)	0 (0)	0 (0)	-0.0 (0.9)	-0.8 (0.9)	1.5 (2.4)	-2.2 (2.6)	-0.0 (1.0)	-0.8 (1.0)	2.8 (3.5)	-3.6 (3.7)
S1B3	0 (0)	0 (0)	0 (0)	0 (0)	0.5 (0.8)	-0.5 (0.8)	2.1 (1.8)	-2.7 (2.1)	0.5 (0.8)	-0.6 (0.9)	3.9 (2.7)	-4.4 (2.9)
S1B4	0 (0)	0 (0)	0 (0)	0 (0)	2.3 (0.7)	1.8 (0.8)	2.7 (1.8)	-0.9 (2.1)	2.5 (0.8)	2.0 (0.8)	5.0 (2.7)	-3.0 (2.9)
S1B5	0 (0)	0 (0)	0 (0)	0 (0)	1.6 (0.8)	1.8 (0.8)	1.3 (1.9)	0.5 (2.2)	1.8 (0.8)	1.9 (0.8)	2.4 (2.8)	-0.4 (3.0)
S2B1	0 (0)	0 (0)	0 (0)	0 (0)	-2.1 (0.9)	-2.6 (1.0)	-0.8 (2.2)	-1.8 (2.5)	-2.3 (1.0)	-2.9 (1.0)	-1.4 (3.1)	-1.5 (3.4)
S2B2	0 (0)	0 (0)	0 (0)	0 (0)	-1.3 (0.8)	-1.4 (0.8)	-0.9 (2.1)	-0.5 (2.3)	-1.4 (0.8)	-1.5 (0.8)	-1.7 (3.1)	0.2 (3.3)
S2B3	0 (0)	0 (0)	0 (0)	0 (0)	0.9 (0.7)	1.7 (0.8)	-1.0 (1.8)	2.7 (2.0)	1.0 (0.8)	1.8 (0.8)	-1.9 (2.7)	3.7 (2.9)
S2B4	0 (0)	0 (0)	0 (0)	0 (0)	0.9 (0.7)	1.3 (0.8)	0.0 (1.6)	1.3 (1.9)	1.0 (0.7)	1.4 (0.8)	-0.1 (2.3)	1.5 (2.6)
S2B5	0 (0)	0 (0)	0 (0)	0 (0)	-0.3 (0.7)	-0.2 (0.8)	-0.3 (1.7)	0.1 (1.9)	-0.4 (0.8)	-0.3 (0.9)	-0.7 (2.4)	0.4 (2.7)
S3B1	0 (0)	0 (0)	0 (0)	0 (0)	-0.8 (0.8)	-1.4 (0.9)	0.4 (2.0)	-1.8 (2.3)	-0.8 (0.9)	-1.5 (1.0)	0.9 (2.8)	-2.4 (3.1)
S3B2	0 (0)	0 (0)	0 (0)	0 (0)	0.1 (0.9)	0.5 (0.9)	-0.7 (2.4)	1.2 (2.6)	0.1 (1.0)	0.5 (0.9)	-1.2 (3.5)	1.8 (3.7)
S3B3	0 (0)	0 (0)	0 (0)	0 (0)	-0.8 (0.9)	0.1 (0.9)	-2.3 (2.2)	2.3 (2.4)	-0.9 (0.9)	0.1 (0.9)	-4.2 (3.3)	4.3 (3.4)
S3B4	0 (0)	0 (0)	0 (0)	0 (0)	0.1 (0.8)	0.5 (0.8)	-1.1 (2.0)	1.6 (2.3)	0.1 (0.9)	0.5 (0.9)	-1.9 (3.0)	2.5 (3.2)
S3B5	0 (0)	0 (0)	0 (0)	0 (0)	0.2 (0.9)	0.3 (1.0)	0.3 (2.2)	0.0 (2.5)	0.2 (1.0)	0.4 (1.0)	0.6 (3.2)	-0.3 (3.5)
S4B1	0 (0)	0 (0)	0 (0)	0 (0)	1.5 (0.9)	0.3 (0.9)	3.5 (2.1)	-3.2 (2.4)	1.7 (0.9)	0.3 (1.0)	6.6 (3.3)	-6.3 (3.5)
S4B2	0 (0)	0 (0)	0 (0)	0 (0)	-1.9 (1.0)	-2.2 (1.0)	-0.3 (2.4)	-1.9 (2.7)	-2.1 (1.0)	-2.4 (1.0)	-0.5 (3.6)	-1.9 (3.9)
S4B3	0 (0)	0 (0)	0 (0)	0 (0)	-0.7 (0.9)	-0.6 (0.9)	-0.7 (2.3)	0.1 (2.6)	-0.8 (1.0)	-0.7 (1.0)	-1.3 (3.4)	0.6 (3.6)
S4B4	0 (0)	0 (0)	0 (0)	0 (0)	0.3 (0.9)	-0.5 (0.9)	1.3 (2.0)	-1.8 (2.3)	0.3 (0.9)	-0.6 (1.0)	2.4 (3.0)	-3.0 (3.2)
S4B5	0 (0)	0 (0)	0 (0)	0 (0)	-0.6 (1.1)	-0.7 (1.2)	-0.1 (2.5)	-0.5 (2.9)	-0.6 (1.1)	-0.8 (1.3)	-0.1 (3.6)	-0.6 (4.0)
S5B1	0 (0)	0 (0)	0 (0)	0 (0)	2.3 (0.7)	2.7 (0.8)	1.0 (1.5)	1.6 (1.8)	2.5 (0.7)	2.9 (0.8)	1.8 (2.2)	1.0 (2.5)
S5B2	0 (0)	0 (0)	0 (0)	0 (0)	-0.3 (0.8)	0.0 (0.8)	-0.9 (2.0)	0.9 (2.2)	-0.3 (0.8)	0.0 (0.9)	-1.7 (2.9)	1.8 (3.1)
S5B3	0 (0)	0 (0)	0 (0)	0 (0)	-0.3 (0.9)	-0.4 (1.0)	0.6 (2.4)	-1.0 (2.7)	-0.3 (1.0)	-0.4 (1.0)	1.0 (3.5)	-1.4 (3.7)
S5B4	0 (0)	0 (0)	0 (0)	0 (0)	-1.0 (0.8)	-1.0 (0.9)	-1.1 (2.0)	0.1 (2.2)	-1.1 (0.9)	-1.1 (0.9)	-1.9 (2.8)	0.8 (3.0)
S5B5	0 (0)	0 (0)	0 (0)	0 (0)	-2.6 (1.2)	-2.7 (1.3)	-2.3 (3.0)	-0.5 (3.3)	-2.8 (1.2)	-3.0 (1.3)	-4.1 (4.3)	1.1 (4.7)

Table VII: Estimates of Some Parameters

Panel A of this table presents the posterior means and the posterior standard deviations (in the parentheses) of the constant transition probabilities, P and Q. In the two-regime model with constant transition probabilities (C-RS), at any given time t, the underlying state s_t can be either 1 or 2. The transition probabilities are determined as follows:

$$\Pi = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix},$$

where $P = Pr(s_t = 1|s_{t-1} = 1)$ and $Q = Pr(s_t = 2|s_{t-1} = 2)$. Panel B presents the posterior means and the posterior standard deviations (in the parentheses) of the parameters, a_1, b_1, a_2, and b_2, in the two-regime model with time-varying transition probabilities (T-RS). When the transition probabilities are time-varying, they are defined as follows:

$$\Pi = \begin{bmatrix} P_t & 1-P_t \\ 1-Q_t & Q_t \end{bmatrix},$$

where $P_t = Pr(s_t = 1|s_{t-1} = 1) = \frac{1}{1+exp(a_1+b_1 LEI_{t-2})}$, $Q_t = Pr(s_t = 2|s_{t-1} = 2) = \frac{exp(a_2+b_2 LEI_{t-2})}{1+exp(a_2+b_2 LEI_{t-2})}$, and LEI_{t-2} is the two-month lagged value of the Leading Economic Indicator (LEI) reported by Conference Board. Panel C presents the posterior means and the posterior standard deviations (in the parentheses) of the parameters, a, b, and σ^2, in the process of LEI. We assume that the process of LEI follows an autocorrelation process:

$$L_t = a + bL_{t-1} + \epsilon_t,$$

where $\epsilon_t \sim N(0, \sigma^2)$.

Panel A: Transition Probabilities in C-RS

P	(std)	Q	(std)
0.926	(0.028)	0.744	(0.075)

Panel B: Parameters of the Transition Probabilities in T-RS

a_1	(std)	b_1	(std)	a_2	(std)	b_2	(std)
-0.899	(0.476)	0.265	(0.740)	2.963	(0.397)	0.379	(0.569)

Panel C: Parameters in the LEI Process

a	(std)	b	(std)	σ^2	(std)
0.068	(0.022)	0.313	(0.044)	0.222	(0.015)

Table VIII: Utility Gains of Incorporating Regime-switching

This table presents the utility gains (in percentage points) of incorporating regime-switching when the transition probabilities are constant (C-RS) or time-varying (T-RS), with mispricing uncertainty $\sigma_\alpha = 0$, 3%, or ∞, on the Fama-French 3-factor model. The investment horizons are 1-month, 12-month, 36-month, and 60-month, respectively. The are three alternative investment opportunity sets. The first investment opportunity set consists of the Fama-French three factors and the Fama-French 25 book-to-market and size portfolios. The second one consists of the Fama-French three factors and the 10 CRSP size portfolios. The third one consists of the Fama-French three factors and and the Fama-French's 17 industry portfolios. Panel A, B, and C presents the utility gains for the three alternative investment opportunity sets, respectively, in the case without constraint while Panel D, E, and F presents the utility gains for the three alternative investment opportunity sets, respectively, in the case with constraint, a 20% margin requirement (c=5). The investor has a power utility function with a risk averse coefficient equal to 5.

Month		C-RS			T-RS	
	σ_α	0	3%	∞	0	3%	∞
				Without Constraint			
				Panel A: Fama-French 25 book-to-market and size portfolios			
1		0.8	2.6	3.3	0.1	0.2	0.1
12		1.4	18.9	12.0	0.9	6.9	9.4
36		1.3	59.5	65.6	2.8	84.8	95.8
60		2.9	118.6	307.5	16.2	134.4	75.1
				Panel B: 10 CRSP size portfolios			
1		1.2	1.2	1.7	0.1	0.1	0.1
12		2.3	3.7	3.3	0.6	1.2	1.5
36		5.7	8.8	8.3	3.0	3.9	2.9
60		10.0	11.8	17.7	8.6	10.7	7.6
				Panel C: Fama-French's 17 industry portfolios			
1		0.9	1.8	2.7	0.1	0.1	0.1
12		4.2	12.2	10.5	0.7	1.7	8.1
36		7.4	26.1	34.3	5.8	7.2	43.4
60		7.0	26.4	68.5	18.3	43.0	42.4
				With Constraint ($c = 5$)			
				Panel D: Fama-French 25 book-to-market and size portfolios			
1		0.8	1.3	1.2	0.0	0.3	0.1
12		1.4	5.3	4.7	0.7	0.3	3.6
36		1.2	6.4	14.4	2.5	7.9	12.7
60		2.9	28.5	16.9	10.7	26.5	8.0
				Panel E: 10 CRSP size portfolios			
1		1.1	0.8	1.1	0.0	0.0	0.0
12		2.3	1.5	1.5	0.6	0.2	0.2
36		3.2	4.6	2.7	2.8	3.6	0.8
60		9.4	5.0	7.7	8.3	6.4	3.2
				Panel F: Fama-French's 17 industry portfolios			
1		0.8	0.8	1.2	0.1	0.0	0.1
12		3.3	1.8	2.9	0.8	0.1	0.3
36		4.8	2.7	6.3	6.2	6.5	1.5
60		7.1	14.1	20.9	15.2	19.2	25.6

Figure 1: Posterior Distribution of Means and Standard Deviations

Based on the the monthly returns of the Fama-French three factors from July 1963 through December 2002, we obtain the posterior distributions of mean returns and standard deviations of the Fama-French three factors, the size factor (SMB), the value factor (HML) and the market index (MKT). The solid curves are posterior distributions for the bull market, while the dashed curves are posterior distributions for the bear market. The transition probabilities in the two-regime model are assumed constant. We show the posterior distributions of the mean returns for (a) the size factor, (b) the value factor, and (c) the market index, and the posterior distributions of the standard deviations for (d) the size factor, (e) the value factor, and (f) the market index.

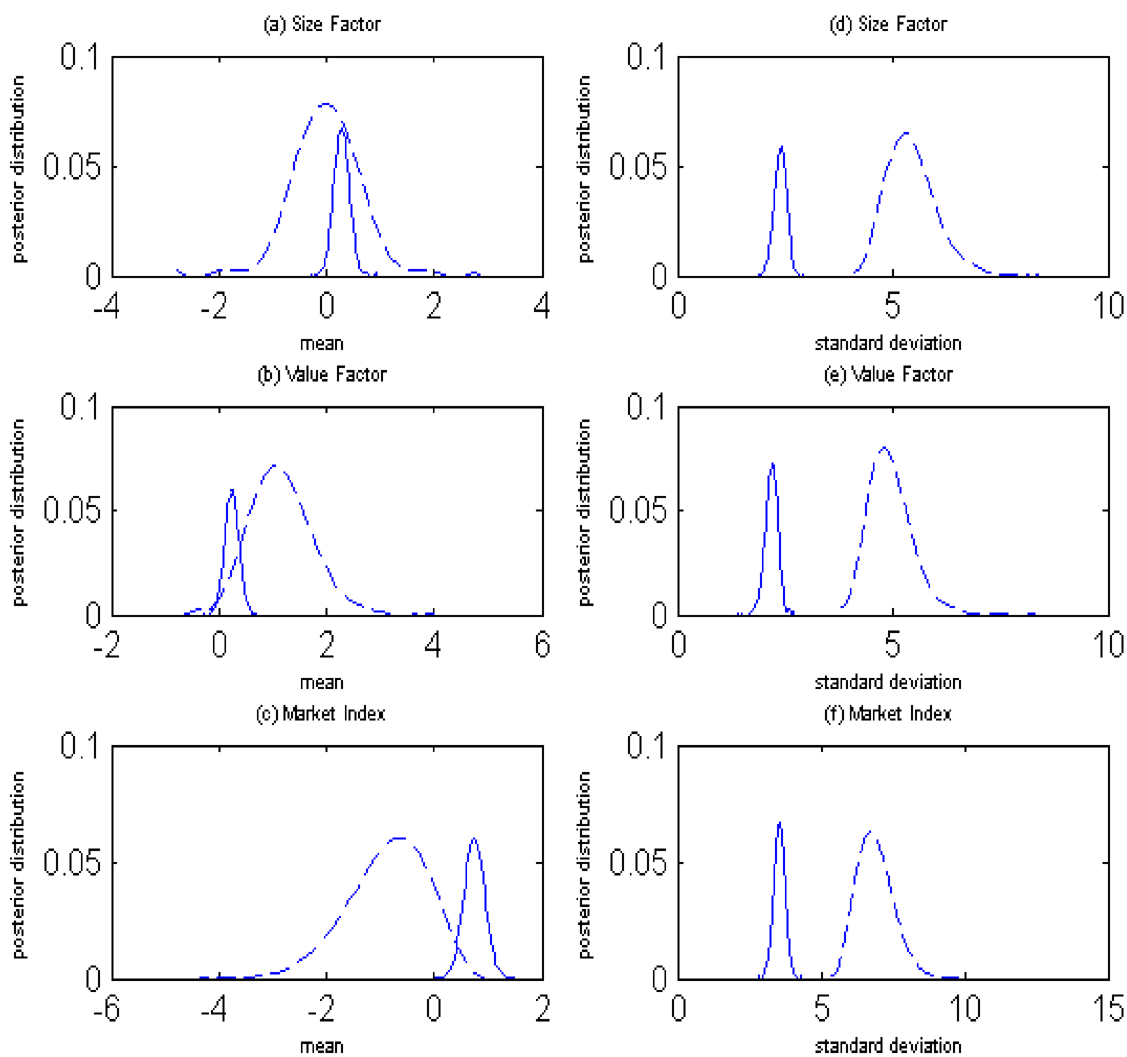

Figure 2: Probability in the Bear Market

This figure plots the time series of the posterior mean of the probability of being in the bear regime from July 1963 through December 2002. The vertical dashed and solid lines indicate the National Bureau of Economic Research (NBER) peaks and troughs, respectively. The transition probabilities in the two-regime model are assumed constant.

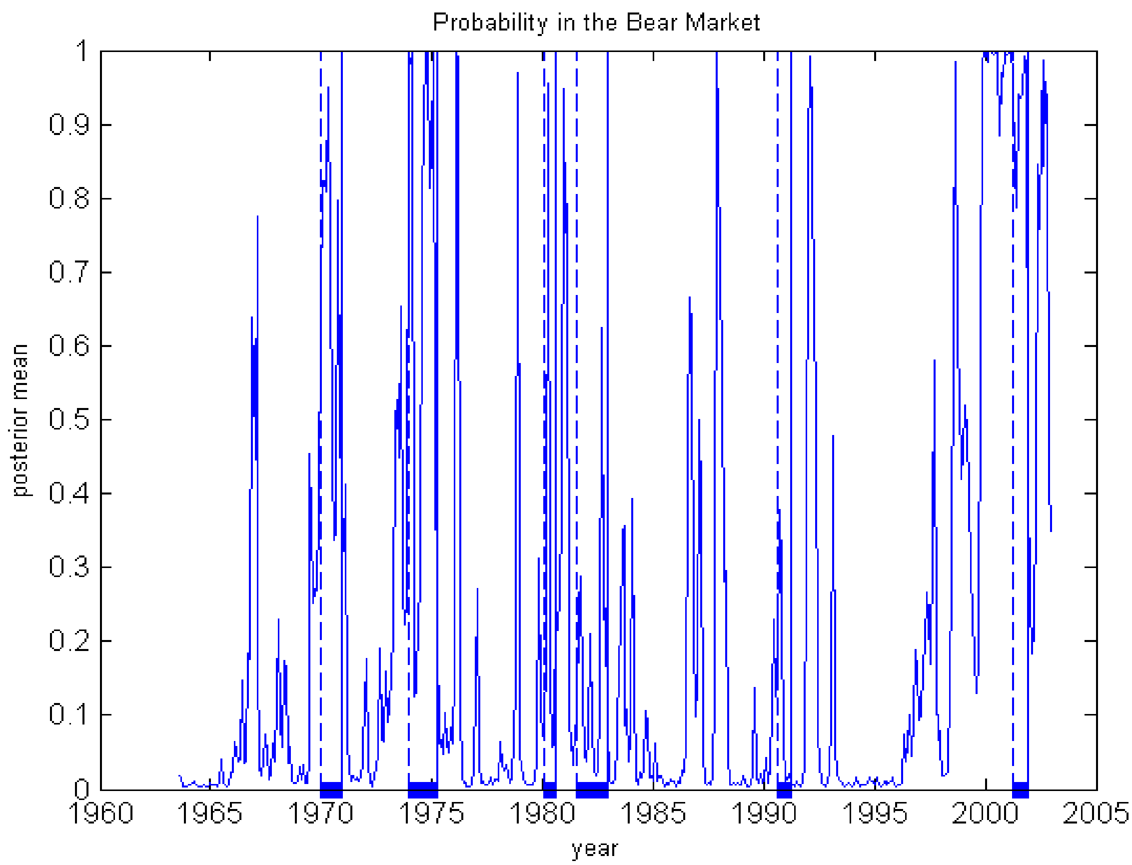

Figure 3: Probabilities of Staying in Bear and Bull markets

This figure plots the time series of the posterior mean of the probability of staying in: (a) the bear market, and (b) the bull market, from July 1963 through December 2002. The transition probabilities in the two-regime model are assumed time-varying.